Predatory Trading and Credit Freeze

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Abstract

This paper studies how predatory trading affects the ability of banks and large trading institutions to raise capital in times of temporary financial distress in an environment in which traders are asymmetrically informed about each others' balance sheets. Predatory trading is a strategy in which a trader can profit by trading against another trader's position, driving an otherwise solvent but distressed trader into insolvency. The predator, however, must be sufficiently informed of the distressed trader's balance sheet in order to exploit this position. I find that when a distressed trader is more informed than other traders about his own balances, searching for extra capital from lenders can become a signal of financial need, thereby opening the door for predatory trading and possible insolvency. Thus, a trader who would otherwise seek to recapitalize is reluctant to search for extra capital in the presence of potential predators. Predatory trading may therefore make it exceedingly difficult for banks and financial institutions to raise credit in times of temporary financial distress.

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1 Introduction

The inter-bank lending market and the discount window of the Fed are two facilities which allow banks to borrow short-term in order to meet temporary liquidity needs. However, these opportunities are not always availed by traders. Financial institutions often appear reluctant to borrow, even at times when liquidity is most needed. In this paper I study how strategic interactions among banks may deter financial institutions from raising money in times of temporary financial distress.

Financial markets are often modeled as interactions between small traders in perfectly competitive markets taking prices as given. However, in reality these markets are not devoid of large players with market impact. For this reason, a recent literature has begun to emphasize strategic behavior among large financial institutions. "Predatory trading" is one such strategic interaction. Brunnermeier and Pedersen (2005) define predatory trading as "trading that induces and/or exploits the need of other investors to reduce their positions." That is, predatory trading is a strategy in which a trader can profit by trading against another trader's position, driving an otherwise solvent but distressed trader into insolvency. The forced liquidation of the distressed trader leads to price swings from which the predator can then profit. Brunnermeier and Pedersen (2005) provide a framework to study this type of interaction, and show how predatory trading can in fact induce a distressed trader's need to liquidate.

In this paper I explore how predatory trading may affect the incentives of banks to seek loans in times of financial distress. In general, a distressed bank or trader may wish to raise money in order to temporarily bridge financial short-falls. However, in an environment in which banks have private information about their own finances, searching for extra capital from outside lenders may become a signal of financial weakness. Traders can then exploit this information, and predatorily trade against funds that they infer to be sufficiently week. Therefore, the mere act of searching for loans may expose a distressed firm to predatory trading and possible insolvency.

The key assumption behind this result is that there exists asymmetric information among traders–that is, ex ante, traders have private information about their own balance sheet that is not available to other traders. Within an asymmetric information environment, actions undertaken by banks to relieve financial distress may convey information about its underlying financial state. Hence, in deciding whether or not to search for a loan, a distressed bank faces a trade-off between the financial cushion provided by a loan and the information this act reveals. In equilibrium, I find that some distressed funds who would otherwise seek to recapitalize may be reluctant to search for extra capital in the presence of potential predators.

Predatory trading may therefore deter banks and financial institutions from raising funds in times when they need it the most.

Finally, I examine policy implications. In particular, I look at the policy implications of the Term Auction Facility (TAF). The Term Auction Facility is a facility which auctioned off loans to financial institutions. The collateral for these loans were the same as for those at the discount window, hence they should be no different. I show that even with the same types of informational assumptions, TAF can work very differently from the discount window, simply because it is an auction-format. If there is a strong stigma effect, the TAF can help alleviate this stigma effect and get funds to banks that need it. However, I also show that when the stigma effect is not strong, the TAF may in fact be problematic.

Anecdotal and (some) Empirical Evidence. The findings of this paper support certain historical and anecdotal evidence about strategic trading and the reluctance of financial institutions to find loans in times of distress. One of the most often-cited examples of predatory trading is the alleged front-running against the infamous hedge fund Long-Term Capital Management (LTCM) in the fall of 1998. After realizing losses in a number of markets, it is reported that LTCM began searching for capital from a number of Wall Street banks, most notably Goldman Sachs & Co. LTCM alleges that with this information Goldman then traded heavily against LTCM's positions in credit-default swaps, front-running LTCM's eventual unwinding. *Business Week* writes,[1]

> ..if lenders know that a hedge fund needs to sell something quickly, they will sell the same asset–driving the price down even faster. Goldman Sachs & co. and other counterparties to LTCM did exactly that in 1998. Goldman admits it was a seller but says it acted honorably and had no confidential information.

Similarly, in *When Genius Failed: The rise and fall of Long-Term Capital Management*, Lowenstein writes,[2]

> As it scavenged for capital, Long-Term had been forced to reveal bits and pieces and even the general outline of its portfolio... Meriwether bitterly complained to the Fed's Peter Fisher that Goldman, among others, was "front-running", meaning trading against it on the basis of inside knowledge. Goldman, indeed, was an extremely active trader in mid-September, and rumors that

[1] "The Wrong Way to Regulate Hedge Funds," *Business Week*, February 26, 2001, p. 90.

[2] According to the author, *When Genius Failed: The Rise and Fall of Long-Term Capital Management* was based on interviews with former employees and partners of the firm, as well as interviews conducted at the major Wall street invesment banks.

> Goldman was selling Long-Term's positions in swaps and junk bonds were all over Wall Street.

Furthermore, an interesting study by Cai (2007), uses a unique data set of audit transactions to examine the trading behavior of market makers in the Treasury bond futures market during LTCM's collapse. Cai finds strong evidence of predatory front running behavior by market makers, based on their informational advantages.[3]

The findings of this paper also provide a possible explanation as to why financial firms may not obtain loans in times of financial shortfall. During the 2008-2011 financial crisis, sources report that Lehman Brothers was reluctant to publicly raise liquidity, a month or so before its collapse. *The Wall Street Journal* writes,[4]

> As the credit crunch deepened, the Fed had set up a new lending facility for investment banks. Although the central bank doesn't reveal who borrows from it, the market generally figures it out, and there's a stigma associated with it. Lehman didn't do so over the summer, because it didn't want to be seen as needing Fed money, says one person familiar with the matter.

The *WSJ* further reports that Lehman eventually tried to secretly raise funds from the European Central Bank:

> In the weeks before it collapsed, Lehman Brothers Holdings Inc. went to great lengths to conceal how fast it was careening toward the financial precipice. The ailing securities firm quietly tapped the European Central Bank as a financial lifeline.

Eventually, any funds Lehman could acquire were apparently not enough, and the investment bank declared bankruptcy on September 15, 2008. The *Associated Press* writes,[5]

> If the mortgage meltdown is like a financial hurricane, then think of Lehman Brothers as a casualty that waited too long to cry for help.

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[3] Although identities are concealed in the transactions dataset, Cai finds one large clearing firm (coded "PI7") with large customer orders during the crisis period which closely match various features of LTCM's trades executed through Bear Stearns, including trade size, pattern and timing. More importantly, Cai finds that market makers traded on their own accounts in the same direction just one or two minutes before before PI7 customer orders were executed.

[4] "The Two Faces of Lehman's Fall." *The Wall Street Journal,* October 6, 2008.

[5] "Financial hurricane victim Lehman waited too long." *The Associated Press*, September 14, 2008.

Finally, there is some quantitative evidence that a stigma effect exists when firms borrow money from the Fed. This evidence is given by the premium firms paid for term loan auctions.

At the beginning of the financial crisis, the Fed encouraged banks to request loans at the discount window if they needed cash. It reduced the discount rate by half a percentage point, but very few banks borrowed discount loans, due to fear that this action would signal weakness (citation?) In response to the low level of discount window lending, the Fed made credit available through special lending facilities. In December 2007, the Fed created the Term Auction Facility (TAF). Under this program, the Fed lent to banks through auctions every two weeks it provided a predetermined level of loans ($25-75 billion) to banks that submitted the highest interest rate bids. Banks were more eager to bid in these auctions than to take out traditional discount loans. Participation was not publicized as widely, so these loans had less of a stigma effect.

> In August 2007, banks were reluctant to rely on discount window credit to address their funding needs. The banks' concern was that their recourse to the discount window, if it became known, might lead market participants to infer weakness—the so-called stigma problem. Ben Bernanke (2009)

Armentier et al (2011) find that at the height of the financial crisis, banks were willing to pay an average premium of at least 37 basis points (and 150 after Lehman's bankruptcy) to borrow from the Term Auction Facility rather than from the Fed discount window. Cassola, Hortacsu, and Kastl (2013) similarly find that in Europe, financial firms were willing to borrow at an interest rate premium exceeding a 30 basis points premium over EONIA by the end of 2007. They contend that this evidence is suggestive of a stigma effect of borrowing at the discount window.

This paper. In this paper, I build a simple model with two traders: a distressed trader and a (potential) predator. The distressed trader owns an illiquid asset which yields positive returns at a later date, but must liquidate this asset prematurely if its liquid (cash) holdings hit some lower bound. Cas holdings are subject to exogenous shocks which may put this trader in danger of the lower bound. Thus, in order to prevent premature liquidation, banks with low cash holdings have to option to search for loans and thereby increase their liquid holdings.

At the same time, the potential predator closely watches the actions of the distressed trader. The predator cannot observe the cash holdings of the distressed trader. However, it may observe whether or not the distressed trader searches for a loan on the interbank market. Observation of this action thereby reveals in equilibrium some information about the cash holdings of the distressed trader. After observing whether the distressed trader searches or does not search for a loan, the potential predator decides whether or not to predatorily trade. If it decides to predatorily trade, the two firms enter into a predatory trading war, which the predator wins if and only if its holdings are greater than that of the distressed. The losing trader must then prematurely liquidate their illiquid asset.

I make two important assumptions. First, distressed traders with cash holdings low enough without a loan always fail (due to the income shock), so that these traders always search for a loan. Similarly, distressed traders with large cash holdings are sufficiently far from the lower bound such that they never fail, even if they are predatorily traded against. In this case, these high-liquidity traders never search for a loan. Thus, it is the behavior of firms with intermediate levels of cash-holdings–it is their behavior which is the object of interest. Second, without obtaining a loan, these intermediate level firms may have difficulty surviving a predatory attack. Thus, if the predator happens to predatorily trade against an intermediate-level trader, the former will win and the latter will end up liquidating their investment.

I analyze the perfect-Bayesian equilibrium of this game. I show that the equilibrium (or equilbria) of this game depends on the parameters. First, if the exogenous probability that the distressed trader fails due to the exogenous liquidity shock is sufficiently small, then there exists an equilibrium in which distressed traders with intermediate values of wealth refrain from searching, and the predator only predatorily trades against low-wealth type traders who decide to search. This in contrast to the behavior of the distressed trader if there were no predator, in which case the distressed trader would search for a loan. Thus, searching for a loan provides a negative signal about the trader's viability, which the predator exploits. In order to guard against this, the trader voluntarily refrains from searching. Yet this implies the firm may instead fail from exogenous income shocks. Finally, I consider how some simple policies may affect the equilibria of this game.

Related Literature. This paper is related to two particular literatures–models of predatory trading, and analysis of the stigma effect of borrowing.

There are only a few papers on predatory trading. Brunnermeier and Pedersen (2005) provides the basic framework for predatory trading used in this paper. Brunnermeier and Pedersen (2005) show that if a distressed firm is forced to liquidate a large position, other traders have the incentive to trade in the same direction, in order to profit from large price swings. Furthermore, they show that predatory trading can even induce the distressed trader's need to liquidate. In their analysis, the predator is perfectly informed of the distressed trader's balance sheet, whereas in this paper I relax this assumption and allow traders to have private information about their own finances. This is motivated by the observation that banks often know more about their own balance sheet (and portfolio) than other institutions. Finally, Carlin, Lobo, and Viswanathan (2007) offer a complementary theory of predatory trading: they show how predation is a manifestation of a breakdown in cooperation between market participants. Although I find this theory also compelling, this paper adheres to the Brunnermeier and Pedersen version of predatory trading.

Second, there are also only a few recent papers that address the stigma effect of borrowing. Ennis and Weinberg (2009) study a model in which a bank may be sending a negative signal about its financial health to financial market participants when it accesses the Fed discount window. Kondo and Papanikolaou (2012) is a related paper which is concerned with the limits of arbitrage. In their model, financial firms cannot borrow from each other because of the worry that they might be front-run on their own assets by their creditor. Thus

Finally, this paper more generally emphasizes the importance of considering non-competitive markets in which large strategic traders do not take prices as given. Strategic trading based on private information about security fundamentals is studied by Glosten and Milgrom (1985) and Kyle (1985), while speculative trading by investors with no knowledge of fundamental values, but who do possess superior knowledge of the trading environment is studied by Madrigal (1996) and Vayanos (2001). Allen and Gale (1992), on the other hand, study stock price manipulation in which an investor buys and sells shares, incurring profits by convincing others that he is informed. While this is only a small sample of papers, there are many more papers considered with strategic trading based on the information about the trading environment.

This paper furthermore examines how lending problems may arise from the strategic interactions among banks. In this way, this paper is related to Acharya, Gromb, and Yorulmazer (2009), who study market power in the interbank lending market. They show that during crises episodes, the profits a surplus bank may gain from buying fire-sale assets and increasing market share may lead to a lower willingness to supply interbank loans. Similarly, this paper is related to the literature on the role of the central bank during episodes of aggregate liquidity shortages or interbank-lending market breakdown, see for example Allen and Gale (1998), Holmstrom and Tirole (1998), Diamond and Rajan (2005), and Gorton and Huang (2006). Finally, Bolton and Scharfstein (1990) show that an optimal lending contract may leave a firm unable to fully counter predation risk. They consider product market predation, not financial market predation. Finally, while all of these papers emphasize the provision of liquidity by banks and central banks, i.e. the suppliers of funds, they do not consider the signal value of searching for liquidity by distressed financial firms and how that endogenously affects the demand for funds.

Layout. This paper is organized as follows. Section 2 describes the model. In Section 3, I define the equilibrium of the economy and analyze the optimal decision for each type of trader. Section **??** presents the benchmark case in which there is no predator. Section **??** characterizes the equilibria in the full model with predatory trading and compares this to the benchmark case with no predatory trading. Section 6 concludes.

2 The Model

There are 3 periods: $t \in \{1, 2, 3\}$ and two liquid assets: a riskless bond and a risky asset. The risk-free rate is normalized to 0. The risky asset has an aggregate supply Q and a final payoff z at time $t = 3$, where z is a random variable with an expected value of $\mathbb{E}z = \bar{z} > 0$. The price of the risky asset at any time t is denoted s_t.

There are two strategic traders, the distressed trader and the (potential) predator, which are denoted by $i \in \{d, p\}$. Both traders are risk neutral and seek to maximize their expected wealth at time $t = 3$, which I denote as w_i. Each strategic trader is large, and hence, his trading impacts the equilibrium price. Traders can be thought of as hedge funds or the proprietary trading desks of large investment banks. Let x_{it} denote trader i's holding of the stock at time t. Each strategic trader has a given initial endowment, x_{i1}, of the risky asset and is restricted to hold $x_{it} \in [-\bar{x}, \bar{x}]$.[6] For simplicity I assume that each trader's initial endowment is equal to its maximum long position, that is, $x_{i1} = \bar{x}$, for all $i \in \{d, p\}$.

In addition to the two large strategic traders, the market is populated by long-term investors. The long-term traders are price-takers and have at each point in time an aggregate demand curve given by

$$Y(s_t) = \frac{1}{\lambda}(\bar{z} - s_t). \tag{1}$$

This demand schedule has two important attributes. First, it is downward sloping: in order for long-term traders to hold more of the risky asset, they must be compensated in terms of lower prices.[7] Second, the long-term traders' demand depends only on the current price s_t, that is, they do not attempt to profit from future price swings.[8]

The market clearing price solves $Q = Y(s_t) + x_{pt} + x_{dt}$. Market clearing implies that the equilibrium stock price is given by $s_t = \bar{z} - \lambda[Q - (x_{pt} + x_{dt})]$. Due to the constraint

[6] This position limit can be interpreted more broadly as a risk limit or a capital constraint.

[7] This could be due to risk aversion or due to institutional frictions that make the risky asset less attractive for long-term traders. For instance, long-term traders may be reluctant to buy complicated derivatives such as asset-backed securities.

[8] Long-term investors may be interpreted as pension funds and individual investors. Under this interpretation, long-term investors may not have sufficient information, skills, or time to predict future price changes.

on asset holdings, strategic traders cannot take unlimited positions. Assuming the case of limited capital, i.e. $2\bar{x} < Q$, the equilibrium price is always lower than the fundamental value: $s_t < \bar{z}$, $\forall t$. Therefore, strategic traders can expect positive profits from holding the asset until time $t = 3$.

In addition to the risky asset, each strategic trader is endowed with an illiquid investment. At time $t = 3$, this investment, if not liquidated, yields a payoff of u, where u is a random variable with an expected value of $\mathbb{E}u = \bar{u} > 0$. This investment is non-tradeable in the following sense: it cannot be sold by the trader at any point in time before the investment has materialized in the last stage. I let v_{it} represent the paper value at time t of this investment. For example, if the trader is an investment bank, v_{it} may be thought of as the value of investments made by the lending side of the bank which, perhaps due to agency reasons, cannot be securitized.

The paper value of the distressed's investment is subject to liquidity shocks, such that v_t is not necessarily equal to $\bar{u}$ at every point in time. In particular, v_{dt} at any point in time takes one of three values: $v_{dt} \in V_d \equiv \{v_l, v_m, v_h\}$, where without loss of generality $v_l < v_m < v_h$. The realizations of v_t are however independent of u, so that the trader's expected final payoff from his non-tradeable investment is always given by $\bar{u}$. On the other hand, the predator's valuation of non-tradeable assets is constant over time, and equal to its expected payoff: $v_{pt} = \bar{u}$, $\forall t$.

At any time t, a trader's "mark-to-market" wealth is given by $w_{it} = x_{it}s_t + v_{it}$. If the trader survives to period 2, its expected payoff from holding its portfolio is $\mathbb{E}[w_i] = \mathbb{E}[\omega_{i,3}] = x_{i,3}\bar{z} + \bar{u}$. Let $\bar{w}$ denote the maximum expected wealth of a trader's portfolio, that is, $\bar{w} \equiv \bar{x}\bar{z} + \bar{u}$. However, if at any time before the last period a trader's wealth drops below some threshold level $\underline{w}$, then the trader must liquidate all assets at fire sale prices. This assumption of forced liquidation could be due to margin constraints, risk management, or other considerations in connection with low wealth. Let $L < \bar{w}$ be the fire-sale value of the entire portfolio if the trader is forced to liquidate before the last stage, and let $\Delta \equiv \bar{w} - L$ denote the difference between the expected payoff from the portfolio and its fire sale value. One may think of Δ as the penalty the trader incurs for liquidating prematurely.

Timing and Information. There are three stages. Before the first stage, Nature draws a type for the distressed trader. In stage 1, the d. These events are summarized in Figure 1.

Before the stage 1, Nature draws an initial value $v_{d1} \in V_d = \{v_l, v_m, v_h\}$ of the distressed's

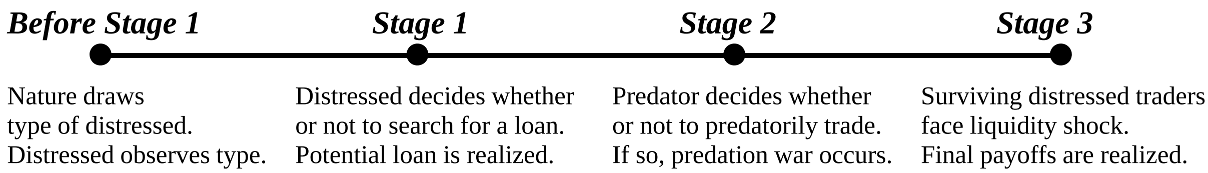

Figure 1: Timeline

non-tradeable holdings according to the following distribution

$$v_{d1} = \begin{cases} v_l & \text{with probability} \quad q_l \\ v_m & \text{with probability} \quad q_m \\ v_h & \text{with probability} \quad q_h \end{cases}$$

where $q_l + q_m + q_h = 1$. For simplicity, I set these probabilities equal, so that $q_l = q_m = q_h = 1/3$. One may think of this as the initial "type" of the distressed trader. That is, the distressed is initially a low type if $v_{d1} = v_l$, a medium type if $v_{d1} = v_m$, and a high type if $v_{d1} = v_h$.

Stage 1. In stage 1, the distressed trader learns his initial type v_1 (or valuation of his non-tradeable investment), but the distressed's type is not observed by the predator.[9] This can be interpreted as investors conducting a valuation of the financial firm, but this value is not released publicly. Once it observes v_{d1}, the distressed trader then has the option to search for additional resources from an outside lender. I let $a_d \in A_d \equiv \{S, NS\}$ denote the action taken by the distressed, where S denotes the decision to "search" for a loan, and NS denotes the decision to "not search". For this reason, I refer to stage 1 as the "loan-seeking stage".

If the distressed decides to search, he receives a loan which increases his liquid wealth to $\hat{v}$, where $v_\ell < v_m < \hat{v} < v_h$. Thus, it is beneficial for all types to obtain a loan except the high type. Before deciding to search for a loan, however, the distressed does not know the precise value of this loan. In particular, I assume that the loan is stochastic and with some probability $\hat{v} < v_p$, otherwise $\hat{v} > v_p$. In particular, I will assume $\hat{v}$ takes two values: $\hat{v} \in \{v_w, v_s\}$ where $v_w < v_p < v_s$; here v_w can be thought of as the weak type, and v_s as the strong type. Thus, the loan gives the bank higher liquidity, but it may not always be enough to escape the predator.

The probability of being able to beat the predator will be increasing in the trader's initial

[9] The predator's type v_p is constant and common knowledge throught out the game.

type. For example, if the distressed initially is a medium type v_m and decides to search for a loan, with probability π_m he receives a loan which makes him the strong type v_s, and with probability $1-\pi_m$ he is the weak type v_w.

$$\hat{v} = \begin{cases} v_w & \text{with probability} \quad 1-\pi_m \\ v_s & \text{with probability} \quad \pi_m \end{cases}$$

Similarly, if the distressed initially is a low type v_ℓ and decides to search for a loan, with probability π_ℓ he receives a loan which makes him the strong type v_s, and with probability $1-\pi_\ell$ he receives a loan which makes him the weak type v_w.

$$\hat{v} = \begin{cases} v_w & \text{with probability} \quad 1-\pi_\ell \\ v_s & \text{with probability} \quad \pi_\ell \end{cases}$$

Like the bond, the loan has zero interest. Finally, if the distressed decides to not search for a loan, his type remains constant; that is, $v_i' = v_i$.

After the distressed decides whether or not to search for a loan, the value v_{d2} is realized. This value is again observed by the distressed but not by the predator.

Stage 2. Although the predator does not observe the distressed's type v_{d2} directly, the predator does however observe whether the distressed decided to search or not. Specifically, the predator observes a_d. After observing a_d, the predator then decides whether or not to predatorily trade against the distressed. I let $a_p \in A_p \equiv \{P, NP\}$ denote the action taken by the potential predator, where P denotes the decision to "predatorily trade", and NP denotes the action to "not predatorily trade". For this reason, I refer to this stage as the "predatory phase".

If the predator decides to predatorily trade, then the strategic traders engage in a "predation war". The results of this predation war are derived from Brunnermeier and Pedersen (2005). The mechanics of this predation war are not the main focus of this paper. For this reason, in this section I only present the important (reduced-form) results that are pertinent to understanding the model. A more detail description of the predation war is given in the Appendix.

If the predator decides to predatorily trade, then the strategic traders engage in a "predation war" in which both traders sell the risky asset as fast as possible. This predation war continues until one of the traders is forced to leave the market. The trader who is forced to leave the market is the trader whose wealth falls below the minimum wealth threshold $\underline{w}$ first–that is, the trader with the lower amount of wealth will be forced to leave the market. The predator therefore wins the predation war and the distressed loses if and only if $v_p > v_{d2}$.

In this case, the predator buys back up to its optimal position $\bar{x}$, receives strictly positive profits $m > 0$ and moves on to stage 3, while the distressed trader is forced into liquidation and receives final payoff $w_d = L$. On the other hand, if the predator loses and the distressed wins the predation war, the predator must liquidate its assets at fire sale prices and receives final payoff $w_p = L$, while the distressed buys back up to its optimal position $\bar{x}$ and continues on to stage 3.[10]

If the predator decides not to predatorily trade, then there is no predation war. Both traders move on to the next period with no change to their current or expected wealth.

Stage 3. In stage 3, conditional on making it to this stage (either not engaging or winning the predation war in stage 2), the predator receives the final realized wealth from his portfolio, $w_p = x_{p,3}z + u$.

In addition, the distressed trader, conditional on making to this stage (either not engaging or winning the predation war in stage 2), is subject to an exogenous income shock. This income shock has two outcomes, either it results in stage 3 wealth below the threshold $\underline{w}$, forcing the distressed trader to liquidate, or it results in stage 3 wealth above the threshold.

The distressed in period 3, has valuation $v_{d,3}$ equal to its valuation in the previous period, $v_{d,2}$. The probability of hitting the lower bound on wealth after the income shock depends on the distressed's current type. In particular, the probability that the trader's wealth after the income shock is above the threshold is increasing in $v_{d,3}$. If the distressed trader is low type, then his wealth after the income shock is above the threshold $\bar{w}$ with probability p_l. If the trader is medium type, then his wealth after the income shock is above the threshold with probability p_m. If the trader is either v_w or v_s, then his wealth after the income shock is above the threshold with probability $\hat{p}$. Finally, if the trader is high type, then his wealth after the income shock is above the threshold with probability p_h. I assume $p_l < p_m < \hat{p} < p_h$, so that the high type has the lowest probability of hitting the lower bound on wealth, and the low type has the highest probability of hitting the lower bound.

If the distressed hits the lower bound on wealth after the income shock, he is forced to liquidate all assets at fire-sale prices and receives final payoff $w_d = L$. If instead the distressed's wealth is above the threshold after the income shock, then he receives the final payoffs from holding the portfolio, $w_d = x_{d3}z + u$.

The key decision for the distressed trader occurs in stage 1, the loan seeking stage. In this stage, after observing his initial value (or type), the distressed trader decides whether

[10] Note that the distressed does not make profits from winning the pedation war. This may be interpreted as the distressed isn't trying, or does not have the skills, to profit from the exit of the predator. Thus, in the event that the distressed wins a predation war, it receives zero gains: $m_d = 0$.

or not to search for a loan. In making this decision, there are two future risks that the distressed trader faces: predatory trading risk in stage 2 and exogenous income risk in stage 3. Searching for a loan is the distressed trader's only way of potentially protecting itself against these risks. In stage 3, the lower the trader's valuation, $v_{d,3}$, the greater the probability of hitting the lower bound on wealth. For this reason a loan would be desirable. However, the main caveat of searching for a loan is its possible signal value–that is, the potential predator sees whether the distressed searched for a loan, and hence infers some information from this action. Therefore, in deciding whether to search for a loan, a distressed bank faces a trade-off between the financial cushion provided by a loan and the information it conveys.

The key decision for the predator occurs in stage 2. While the predator does not observe the distressed's type, he does observe whether or not the distressed searched for a loan. This is motivated by the following. Financial firms must contact outside lenders, counterparties, or central banks when seeking loans. Although any loan amount received may not be observed by the market, the act of seeking liquidity is likely to become public. Thus, potential predators may be able to infer information from this action. In the next section, I show how the predator forms beliefs about the distressed's type optimally via Bayes rule.

Note that is optimal for each trader to always hold $\bar{x}$ of the risky asset, unless engaged in a predation war. This corner solution is due to the long-term investor's demand curve and to the fact that traders have limited capital, so that the equilibrium price is always lower than the fundamental value.

Finally, I make the following assumptions on parameter values.

Assumption. $v_m < v_p < v_h$

That is, if the predator predatorily trades in stage 2, he succeeds if $v_{d2} = v_l$ or v_m, but fails if $v_{d2} = v_h$. Note that even after receiving a loan, the distressed firm may not be a high type. Thus, this assumption implies that a loan will not always bring the firm into the range where it is not subject to predation risk. Bolton and Scharfstein (1990) show that an optimal financial contract may leave an agent cash constrained even if the agent is subject to predation risk.[11]

Assumption. $p_h = 1$ *and* $p_l = 0$

This simply states that in stage 3, the high type never hits the lower bound on wealth while the low type always hits the lower bound on wealth. This will imply that the distressed's type space has dominance regions. That is, for the two extreme types–the low

[11] They consider product market predation, not financial market predation. Furthermore, they do not consider the signal value of searching for liquidity when information is asymmetric.

type and the high type–searching and not searching, respectively, are strictly dominant. The dominant strategies will be proven and shown in the following section.

Assumption. $0 < \pi_\ell < \pi_m$

This assumption states that, conditional on searching for a loan, the probability of the medium type becoming a high type is strictly greater than the probability of the low type becoming a high type. Furthermore, the assumption that π_ℓ is strictly greater than zero means that the low type has some chance of beating the predator if he decides to search for a loan.

3 Equilibrium Definition

Both strategic traders are risk-neutral and expected payoff maximizers. There are two stages in this game in which the traders make choices, and the choices each trader makes may affect their final payoff w_i. In this section I define the equilibrium in this game and characterize the decision rules for each agent.

The type space of the distressed in period 1 is given by $V_d = (v_l, v_m, v_h)$, the action space of the distressed in stage 1 is given by $A_d = (S, NS)$, and the action space of the predator in stage 2 is given by $A_p = (P, NP)$. With this in mind, we define the strategies of the traders as follows.

Definition 1. *A strategy f of the distressed trader is a mapping from the distressed's type space to an action, that is $f : V_d \to A_d$. A strategy g of the predator is a mapping from its information set to an action, that is, $g : A_d \to A_p$.*

Note that in this game, each agent–the distressed and the predator–may face the risk of liquidating prematurely and receiving final payoff $w_i = L$. For the predator, this could be the outcome of the predation war. For the distressed, this could either be the outcome of the predation war, or the outcome of the exogenous income shock in stage 3. In terms of final outcomes of the game, I say that a particular trader "survives" if he is never forced to liquidate. That is, "survival" refers to the event that the trader makes it through the entire game without liquidating and receives the final value of holding its portfolio, $w_i = \omega_{i3} = x_{i3}z + u$.

In stage 0, the distressed trader, after observing its initial type, v_{d1}, decides whether or not to search for a loan. To make this decision the distressed trader forms beliefs about his survival probability that depend not only on his chosen action and its initial type, but also on the strategy of the predator. Given an initial type $v_{d,1}$, let $\alpha\left(a_d|v_{d1}, g\right)$ denote the distressed's belief it survives if it chooses action a_d, conditional on having type v_{d1} and that the predator following strategy g. Thus, given initial type v_{d1}, the distressed's expected payoff conditional on not searching is given by

$$\mathbb{E}_{d1}\left[w_d|NS, v_{d1}, g\right] = \bar{w}\alpha\left(NS|v_{d1}, g\right) + \left(\bar{w} - \Delta\right)\left(1 - \alpha\left(NS|v_{d1}, g\right)\right), \tag{2}$$

since he gets expected payoff $\bar{w}$ if he survives, and liquidation value $L = \bar{w} - \Delta$ otherwise. On the other hand, given initial type v_d, the distressed's expected payoff conditional on searching is given by

$$\mathbb{E}_{d1}\left[w_d|S, v_{d1}, g\right] = \bar{w}\alpha\left(S|v_{d1}, g\right) + \left(\bar{w} - \Delta\right)\left(1 - \alpha\left(S|v_{d1}, g\right)\right) \tag{3}$$

since he gets expected payoff $\bar{w}$ if he survives, minus the fixed cost of the loan, and liquidation value $L = \bar{w} - \Delta$ otherwise.

Likewise, in stage 1, the predator, after observing the action of the distressed, a_d, decides whether or not to predatorily trade. To make this decision the predator forms beliefs about his survival probability that depend not only on his chosen action, but also on the observed action of the distressed and the distressed's strategy. Given an observed action a_d, let $\beta\left(a_p|a_d, f\right)$ denote the predator's belief he survives if he chooses action a_p, conditional on the distressed following strategy f. If the distressed chooses to predatorily trade, then the survival probability is merely the posterior probability that $v_{d2} < v_p$, i.e. $\beta\left(P|a_d, f\right) = \Pr\left[v_{d2} < v_p|a_d, f\right]$. On the other hand, if the predator does not predatorily trade, then $\beta\left(NP|a_d f\right) = 1$, for any a_d, f. Therefore, given the observed action a_d, the predator's expected payoff conditional on predatorily trading is given by

$$\mathbb{E}_{p2}\left[w_p|P, a_d, f\right] = \left(\bar{w} + m\right)\beta\left(P|a_d, f\right) + \left(\bar{w} - \Delta\right)\left(1 - \beta\left(P|a_d, f\right)\right) \tag{4}$$

since he gets expected payoff $\bar{w}$ if it survives, plus profits m, and liquidation value $L = \bar{w} - \Delta$ otherwise. On the other hand, given observed action a_d, the predator's expected payoff conditional on not predatorily trading is given by

$$\mathbb{E}_{p2}\left[w_p|NP, a_d, f\right] = \bar{w} \tag{5}$$

The equilibrium of this game is then defined as follows.

Definition 2. *An equilibrium is a strategy for the distressed* $f : V_d \rightarrow A_d$, *a strategy for* *the predator $g : A_d \to A_p$, and a survival belief $\alpha : A_d \times V_d \to [0,1]$ of the distressed and a survival belief $\beta : A_p \times A_d \to [0,1]$ of the predator, such that*

(i) For each $v_{d1} \in V_d$, the distressed of initial type v_d searches for a loan if and only if his expected payoff from doing so is greater than his expected payoff from not searching

$$f\left(v_{d1}\right) = S \quad \textit{if and only if} \;\; \mathbb{E}_{d1}\left[w_d | S, v_{d1}, g\right] > \mathbb{E}_{d1}\left[w_d | NS, v_{d1}, g\right], \tag{6}$$

conditional on the predator following strategy g.

(ii) For each $a_d \in A_d$, the predator who observes a_d predatorily trades if and only if his expected payoff from doing so is greater than his expected payoff from not predating

$$g\left(a_d\right) = P \quad \textit{if and only if} \;\; \mathbb{E}_{p2}\left[w_p | P, a_d, f\right] > \mathbb{E}_{p2}\left[w_p | NP, a_d, f\right] \tag{7}$$

conditional on the distressed following strategy f.

(iii) The survival belief of the distressed, α, is based on the predator following strategy g.

(iv) The survival belief of the predator β is formed using Bayes rule and based on the distressed following strategy f.

Conditions (i) and (ii) of Definition 2 require that the strategies of the distressed and the predator are sequentially rational given their beliefs. Condition (iii) states that the belief system must be consistent given the strategy profile of the players. Thus, the equilibrium definition is that of a standard perfect-Bayesian equilibrium, in which the distressed is the sender and the predator is the receiver. Finally, I prove shortly that in this game there are no out-of-equilibrium beliefs.

Decision rule for the distressed trader. I first consider the decision for the distressed trader in stage 1. The expected payoffs for the distressed from searching and from not searching are given in (3) and (2), respectively. Combining these with the distressed's decision rule stated in (6), it follows that optimal action for the distressed trader may be expressed as follows.

Lemma 1. *Given initial valuation $v_{d,1}$ and conditional on the predator following strategy g, the distressed trader searches for a loan if and only if*

$$\alpha\left(NS | v_{d1}, g\right) < \alpha\left(S | v_{d1}, g\right) \tag{8}$$

The above Lemma gives a simple rule, in terms of the distressed's beliefs, for when it is optimal for the distressed to search for a loan. Lemma 1 states that the distressed trader will search if and only if the probability he faces of survival from not searching is less than the probability of survival from searching. This seems fairly intuitive–searching is optimal if it increases the distressed's perceived chances of survival.

Lemma 1 gives a simple decision rule for the distressed trader, given his initial type. Using this decision rule, it is now clear that for the two extreme types–the low type v_d and the high type v_h–find searching and not searching, respectively, strictly dominant. This is stated in the following lemma.

Lemma 2. *For any strategy of the predator, the low type always finds it optimal to search. Likewise, for any strategy of the predator, the high type always finds it optimal to not search.*

The proof of Lemma 2 is very simple. Consider first the low-type's decision. For any strategy of the predator, if the low type decides not to search, his probability of survival is zero, since no matter what happens in stage 2, the exogenous income shock in stage 3 will force the firm to liquidate ($p_l = 0$). On the other hand, for any strategy of the predator if the low type decides to search for a loan, his probability of survival is strictly positive. Condition (8) is hence satisfied for all g. Therefore, no matter the strategy of the predator, the low type always finds it optimal to search. This is due in particular to Assumption 2, i.e. that

Similarly, consider the decision of the high type in stage 1. For any strategy of the predator, the high type's probability of survival, whether it searches or not, is always equal to 1. This is due to the fact that neither the predator in stage 2 nor the income shock in stage 3 can force the high type to liquidate. Therefore, according to condition (8), the high type always finds it optimal to not search.

Lemma 2 clarifies the type of equilibria that may exist in this game. The property that the low type always searches and the high type never searches, i.e. that there are dominance regions in the type space, implies that any possible equilibrium in this game *must* be a separating (or semi-separating) equilibrium. Any action observed by the predator is consistent with an equilibrium path, and hence no off-the-equilibrium beliefs need be specified.

Finally, note that Lemma 2 also contributes to understanding the signalling nature in this game. Because the predator does not observe the type of the distressed, it can only infer information from the distressed's action. From Lemma 2, we see that regardless of the predator's strategy, it is strictly dominant for the low type to search, and it is strictly dominant for the high type to not search. Therefore, when the medium type makes its decision whether or not to search, part of its trade-off is whether to be pooled with the low types or to be pooled with the high types, and in this way convey information to the predator.

The medium type is thus the "marginal trader".

Decision rule of the predator. I now consider the decision for the predator in stage 2. The expected payoffs for the predator from trading and from not trading are given in (4) and (5), respectively. Combining these with the predator's decision rule stated in (7), it follows that optimal action for the predator may be expressed as follows.

Lemma 3. *Conditional on observing action a_d and on the distressed following strategy f, the predator predatorily trades if and only if*

$$\frac{1-\beta\left(P|a_d,f\right)}{\beta\left(P|a_d,f\right)}<\frac{m}{\Delta} \tag{9}$$

Much like Lemma 1, Lemma 3 gives a simple cut-off rule, in terms of the predator's beliefs, for when it is optimal for the predator to predatorily trade. To see this, note that the left-hand side of equation (8) is merely the ratio of the probability of failing to the probability of succeeding, if the trader decides to predatorily trade. Lemma 3 states that the predator will predatorily trade if and only if this ratio is sufficiently low. The cut-off for this ratio, i.e. the right-hand side of equation (8), is a constant which is increasing in the gain from winning a predation war, m, but decreasing in the liquidation penalty Δ. Therefore, the predator finds it optimal to predatorily trade if the probability of surviving conditional on predatorily trading is high enough. However, a lower gain from predatorily trading or a higher liquidation penalty makes it less likely that the predator will find it optimal to predatorily trade.

The decision rules stated in Lemmas 1 and 3 greatly simplify the equilibrium analysis, which I move to next.

4 Equilibrium

In this section I now characterize the equilibrium or equilibria of this game. I first start with the benchmark case in which there is no predator. Afterwards, I turn to the equilibrium of the full game with predation. At the end of this section I discuss some policy recommendations.

4.1 Equilibrium without Predator

I first analyze the equilibrium in a benchmark case in which there is no predator. That is, I consider a setting identical to that described in Section 2, but without stage 2, the predatory stage. Within this predator-less setting, I need only to consider the optimal strategy of the distressed.

In this environment, Lemma 2 continues to hold; that is, it is optimal for the low types to search and for the high types to not search. Thus, in terms of the distressed's strategy, one needs only to find what is optimal for the medium type. Although there is no predation risk, the medium type still faces income shock risk. Hence if the medium type decides not to search, his probability of survival is given by $\alpha(NS|v_m) = p_m$. On the other hand, if the medium decides to search for a loan, he gets a new probability of surviving $\alpha(S|v_m) = \hat{p}$. Combining these probabilities with the decision rule in (8), I find that the medium-type searches if and only if

$$p_m < \hat{p} \tag{10}$$

This is always true, as the loan helps the trader have a more liquid position. Therefore, the medium type always searches for a loan in the absence of predators.

The focus of this paper is to study the effect of predatory trading on the incentives of financial firms to seek liquidity in times of distress. In this environment, there is a clear incentive for a bank to seek out a loan in the absence of predators. In the absence of predation risk, searching for a loan increases the distressed's chances of avoiding the lower wealth bound, making it preferable for him to search. In any case there is a clear incentive for the medium type to seek out a loan when there are no predators.[12]

Thus, the following proposition characterizes the optimal strategy for the distressed in the benchmark with no predatory trading.

Proposition 1. *When there are no predators, the distressed follows a strategy in which the low and medium types search for a loan and the high type does not search.*

4.2 Equilibrium with Predator

I now study the equilibrium (or equilibria) of the full game with predatory trading, as laid out in Section 2. As this is a signalling game, there can in principle be multiple equilibria. In order to characterize the set of *all possible equilibria*, I consider the entire set of possible strategies of one of the traders. Here I choose to focus on the set of strategies of the predator.

[12] Another way to justify this condition is to imagine there were a continuum of types. Then there would exist a type, strictly greater than the low type that would search.

For each of the predator's strategies, I determine under what conditions the strategy may be compatible with an equilibrium. By systematically considering each of the predator's strategies, this procedure allows me to characterize the set of all possible equilibria.

A strategy of the predator is merely a mapping from its information set to an action, $g : A_d \rightarrow A_p$. Since A_d and A_p each contain only two elements, there are only four possible strategies for the predator. Let G be the space of all of these strategies. I label these strategies as $\{g_1, g_2, g_3, g_4\}$ and describe each as follows:

(i) Let g_1 be strategy in which the predator never predatorily trades

$$g_1(a_d) \equiv \ NP \quad \text{for all} \quad a_d \in A_d$$

(ii) Let g_2 be strategy in which the predator always predatorily trades

$$g_2(a_d) \equiv \ P \quad \text{for all} \quad a_d \in A_d$$

(iii) Let g_3 be strategy in which the predator predatorily trades if and only if he observes that the distressed did not search.

$$g_3(a_d) \equiv \begin{cases} NP & \text{if} \quad a_d = S \\ P & \text{if} \quad a_d = NS \end{cases}$$

(iv) Finally, let g_4 be strategy in which the predator predatorily trades if and only if he observes that the distressed did search.

$$g_4(a_d) \equiv \begin{cases} P & \text{if} \quad a_d = S \\ NP & \text{if} \quad a_d = NS \end{cases}$$

In this section, I consider each strategy $g \in G$ separately. For a given proposed strategy g of the predator, I find the best response of the distressed trader. That is, I find the survival probabilities of the distressed based on the belief that the predator is following strategy g, and given these survival beliefs I find the optimal strategy of the distressed. I denote this strategy $f' = BR_d(g)$, where BR_d signifies that is the best response of the distressed. Next, given the distressed's best response strategy f', I then find the best response of the predator. That is, I find the survival probabilities of the predator based on the belief that the distressed is following strategy f', and then find the optimal strategy of the predator based on these survival beliefs. I then denote this strategy as $g' = BR_p(f')$, where BR_p signifies that is is the best response of the predator.

I then characterize under what conditions the predator's best response strategy coincides with the proposed strategy. If $g' = g$, then there exists a fixed point in the traders' best responses: $f' = BR_d(g')$ and $g' = BR_p(f')$. In this case, the strategy profile $\{f', g'\}$ and corresponding survival beliefs therefore constitute an equilibrium. On the other hand, if under no conditions the fixed point exists, I then conclude that an equilibrium in which the predator follows the proposed strategy does not exist.

I now consider each strategy.

Suppose the predator follows strategy g_1. That is, the predator follows a strategy in which he never predatorily trades.

Distressed trader's best response. If the predator follows a strategy in which he never predatorily trades, in terms of the distressed trader's decision making process it is as if the predator did not exist. In other words, the distressed never faces any predation risk. The distressed trader will thus follow the same strategy outline above in Section **??** for the benchmark case in which there is no predator. Proposition 1 implies that the distressed's best response to g_1 is a strategy in which the low and medium types search, while the high type does not search.

Predatory trader's best response. The predator forms its survival beliefs based on the presumption that the distressed is following a strategy in which the low and medium types search, while the high type does not search. If the predator observes that the distressed did not search, the predator infers that he must be facing a high type. Hence, the predator knows that if he predatorily trades, his probability of surviving is zero. On the other hand, if he does not predatorily trade he receives a final payoff of $\bar{w}$. Therefore, if the predator observes that the distressed did not search, it is optimal for the predator to not predatorily trade.

On the other hand, if the predator observes that the distressed searched for a loan, then by Bayes rule the probability of the predator surviving a predation war is given by

$$\beta(P, S|f') = \frac{(1 - \pi_l)\, q_l + (1 - \pi_m)\, q_m}{q_l + q_m}$$

This is simply the probability that the distressed's wealth after seeking a loan is less than the wealth of the predator. Combining this with the predator's decision rule in (9), the predator finds it optimal to predatorily trade if and only if

$$\kappa_{\ell m} < m/\Delta$$

where

$$\kappa_{\ell m} \equiv \frac{\pi_l + \pi_m}{(1-\pi_l) + (1-\pi_m)} \tag{11}$$

where I have used the fact that $q_l = q_m$. Therefore the proposed equilibrium in which the predator never predatorily trades exists if and only if the above condition is not satisfied, that is, when $m/\Delta < \kappa_{\ell m}$. I label this as $\kappa_{\ell m}$ since it is ratio of strong types to weak types in a pool of both orginally low and medium types.

Suppose the predator follows strategy g_2. That is, the predator follows a strategy in which he always predatorily trades.

Distressed trader's best response. From Lemma 2, we know that the low type chooses to search and the high type chooses to not search. Now consider the optimal choice of the medium type. Given that the predator is following a strategy in which he always predatorily trades, if the medium type chooses to not search, then he will be engaged in a predation war which he will surely lose, since $v_m < v_p$. Hence, the medium type's probability of survival from not searching, $\alpha(NS|v_m, g_2)$, is equal to zero. On the other hand, if the medium type chooses to search, then he still faces a predation war. In this case, however, there is some positive probability that the distressed receives a loan that allows him to win the predation war. Therefore, the medium type's probability of survival from searching for a loan is given by $\alpha(S|v_m, g_2) = \pi_m \hat{p}$, which is strictly greater than zero. According to the distressed's decision rule (8), it is optimal for the medium type to search. The distressed's best response to g_2 is therefore a strategy in which the low and medium types search, while the high type does not search.

Predatory trader's best response. The predator forms its survival beliefs based on the presumption that the distressed is following a strategy in which the low and medium types search, while the high type does not search. If the predator observes that the distressed did not search, the predator infers that he must be facing a high type. Hence, the predator knows that if he predatorily trades, his probability of surviving is zero. On the other hand, if he does not predatorily trade he receives a final payoff of $\bar{w}$. Therefore, if the predator observes that the distressed did not search, it is optimal for the predator to not predatorily trade. Thus, given the strategy of the distressed, under no conditions does the best response of the predator coincide with g_2. Therefore, no equilibrium exists in which the predator follows a strategy in which he always predatorily trades.

Suppose the predator follows strategy g_3. Suppose the predator follows a strategy in which he predatorily trades if and only if the distressed does not search.

Distressed trader's best response. The low type searches and the high type does not. The medium type forms his beliefs based on the strategy of the predator. Given that the predator is following a strategy in which it predatorily trades if and only if the distressed does not search, if the medium type chooses to not search, then he will be engaged in a predation war which he will surely lose, since $v_m < v_p$. Hence, the medium type's probability of survival from not searching, $\alpha(NS, v_m|g_3)$, is equal to zero. On the other hand, given the predator's strategy, if the medium type chooses to search, then he will not face any predation risk and the only risk he faces is the exogenous income shock in stage 3. In this case, his probability of survival is given by $\alpha(S, v_m|g_3) = \hat{p}$ which is strictly greater than zero. According to the distressed's decision rule (8), it is optimal for the medium type to search. The distressed's best response to g_3 is therefore a strategy in which the low and medium types search, while the high type does not search.

Predatory trader's best response. The predator forms its survival beliefs based on the presumption that the distressed is following a strategy in which the low and medium types search, while the high type does not search. If the predator observes that the distressed did not search, the predator infers that he must be facing a high type. Hence, the predator knows that if he predatorily trades, his probability of surviving is zero. On the other hand, if he does not predatorily trade he receives a final payoff of $\bar{w}$. Therefore, if the predator observes that the distressed did not search, it is optimal for the predator to not predatorily trade. Thus, given the strategy of the distressed, under no conditions does the best response of the predator coincide with g_3. Therefore, no equilibrium exists in which the predator follows a strategy in which he predatorily trades if and only if the distressed does not search.

The predator follows strategy g_4. Finally suppose the predator follows a strategy in which he predatorily trades if and only if the distressed searches.

Distressed trader's best response. The low type searches and the high type does not. The medium type forms his beliefs based on the strategy of the predator. Given that the predator is following a strategy in which he predatorily trades if and only if the distressed searches, if the medium type chooses to not search, then he will not face any predation risk and the only risk he faces is the exogenous income shock in stage 3. Hence, the medium type's probability of survival from not searching is given by $\alpha(NS, v_m|g_4) = p_m$. On the other hand, given the predator's strategy, if the medium type chooses to search, then he will be engaged in a predation war with the predator in stage 2, which the distressed will lose if he is still a medium type, but will win if he receives a loan that makes him a high type. Therefore, the medium type's probability of survival from searching for a loan is given by $\alpha(S, v_m|g_4) = \pi_m\hat{p}$. According to the distressed's decision rule (8), the medium type searches if and only if

$$p_m < \pi_m \hat{p}$$

Therefore, depending on parameter values, the medium type could find either choice optimal. If $p_m < \pi_m \hat{p}$, then conditional on the predator's strategy it is optimal for the medium type to search. In this case, the distressed's best response to g_4 is a strategy in which the low and medium types search, while the high type does not search. On the other hand, if $p_m > \pi_m \hat{p}$, then conditional on the predator's strategy, it is optimal for the medium type to not search. In this case, the distressed's best response to g_4 is a strategy in which the low type searches for a loan, and the medium and high types do not.

Predatory trader's best response. To characterize the best response of the predator, I consider separately two cases: first, the case in which $p_m < \pi_m \hat{p}$ and second the case in which $p_m > \pi_m \hat{p}$.

First, suppose that $p_m < \pi_m \hat{p}$. In this case, the predator forms his survival beliefs based on the presumption that the distressed is following a strategy in which the low and medium types search, while the high type does not search. Note that this strategy of the distressed is identical to the distressed's best response to g_1. Using the findings of that discussion, one may infer that if the predator observes that the distressed does not search, it is optimal for the predator to not predatorily trade. On the other hand, if the predator observes that the distressed does search for a loan, then the predator finds it optimal to predatorily trade if and only if $\kappa_{\ell m} < m/\Delta$, where $\kappa_{\ell m}$ is given in (11). In this case, the predator's best response coincides with g_4. Therefore the proposed equilibrium in which the predator predatorily trades if and only if the distressed searches exists whenever $\kappa_{\ell m} < m/\Delta$ and $p_m < \pi_m \hat{p}$.

Second, suppose that $p_m > \pi_m \hat{p}$. In this case, the predator forms his survival beliefs based on the presumption that the distressed is following a strategy in which the low type searches for a loan, but the medium and high types do not. If the predator observes that the distressed does not search, he infers that it must be facing either a medium or high type. In this case, the probability of the predator surviving a predation war is given by $\beta\left(P, NS|f'\right) = \frac{q_m}{q_m+q_h}$. Combining this with the predator's decision rule in (9), I find that it is optimal for the predator to not predatorily trade. On the other hand, if the predator observes that the distressed does search for a loan, then the probability of the predator surviving a predation war is given by $\beta\left(P, S|f'\right) = 1 - \pi_\ell$. Combining this with the predator's decision rule in (9), the predator finds it optimal to predatorily trade if and only if

$$\kappa_\ell < m/\Delta$$

where

$$\kappa_\ell \equiv \frac{\pi_\ell}{1-\pi_\ell} \tag{12}$$

This is the ratio of strong types to weak types in a pool of originally all low types. Note that, given the assumptions on parameter values π_ℓ and π_m,

$$\kappa_\ell < \kappa_{\ell m} \text{ as long as } \pi_\ell < \pi_m \tag{13}$$

There is a short proof of this in the Appendix. Therefore the proposed equilibrium in which the predator predatorily trades if and only if the distressed searches exists whenever $\kappa_\ell < m/\Delta$ and $p_m > \pi_m \hat{p}$.

Results. Given the above analysis, I first state the following non-existence result.

Lemma 4. *(i) No equilibrium exists in which the predator follows a strategy in which he always predatorily trades.*

(ii) No equilibrium exists in which the predator follows a strategy in which he predatorily trades if and only if he observes that the distressed did not search.

This lemma is useful in that it implies that in any equilibrium, the predator is either playing a strategy in which he never predatorily trades, or one in which he predatorily trades if and only if he observes that the distressed searched for a loan. In the former case, the equilibrium of the game will be similar to that in the benchmark with no predator. In the latter case, the fact that the predator predatorily trades if and only if he observes that the distressed searched for a loan, implies that the presence of predators creates an incentive for the distressed to refrain from searching.

I now characterize the set of all possible equilibria in this game in the following proposition.

Proposition 2. *In this game there are three types of equilibria, each of which exist in particular regions of the parameter space*

Type I. Consider an equilibrium in which the low-type distressed trader searches for a loan, the medium and high-type distressed trader do not search, and the predator predatorily trades if and only if the distressed searches. An equilibrium of this type exists whenever $m/\Delta > \kappa_\ell$ *and* $p_m > \pi_m \hat{p}$.

Type II. Consider an equilibrium in which the low and medium-type distressed trader searches for a loan, the high-type distressed trader does not search, and the predator predato-

rily trades if and only if the distressed searches. An equilibrium of this type exists whenever $m/\Delta > \kappa_{\ell m}$ and $p_m < \pi_m \hat{p}$.

Type III. Consider an equilibrium in which the low and medium-type distressed trader searches for a loan, the high-type distressed trader does not search, and the predator never predatorily trades. An equilibrium of this type exists whenever $m/\Delta < \kappa_{\ell m}$.

These regions of equilibria are presented in a diagram in Figure 2.

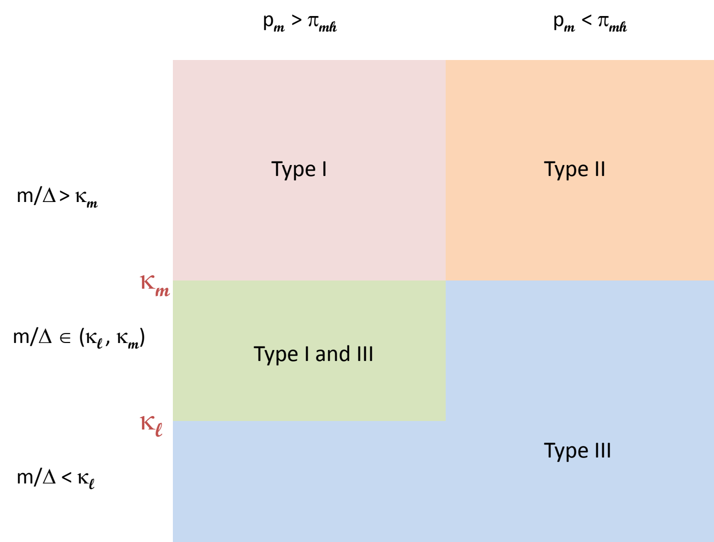

Figure 2: Equilibria Regions

As one can see from the figure, in most regions of the parameter space the equilibrium is unique. The only non-generic region in which there is not a unique equilibrium is when $p_m > \pi_m \hat{p}$, and $\kappa_\ell < m/\Delta < \kappa_{\ell m}$. This is possible because $\kappa_{\ell m}$ is strictly greater than κ_ℓ. Here, two types of pure-strategy equilibria exist: type I and type III.[13]

Note that the existence of this region of multiple equilibria depends on $\kappa_{\ell m}$ being strictly greater than κ_ℓ, which occurs when π_m is strictly greater than π_l. If instead $\pi_m = \pi_l$, then $\kappa_{\ell m} = \kappa_\ell$, and in this case, there would be no region of multiplicity. In all parts of the parameter space, the equilibrium would be generically unique.

Let us consider each type of equilibrium.

Type I. Under some parameter values, there exists an equilibrium in which only the low type distressed trader searches for a loan, the medium and high types do not search, and the

[13] Of course, there also exists a third equilibria in mixed strategies. In this equilibrium, the distressed strategy will be one in which the low type searches, the high type does not search, and the medium type randomizes between searching and not searching, while the predator's strategy is one in which it randomizes between predatorily trading and not predatorily trading.

predator predatorily trades if and only if he observes that the distressed trader searches for a loan.

Consider the predator's decision. When the predator observes that the distressed trader searches for a loan, he knows that this trader must be of low type. Using Bayes rule, the predator forms posterior beliefs over the probability of winning in a predation war against this trader. Given that the predator knows that only the low types search, the predator knows it has a high chance of winning in a predation war. Thus, as long as the expected return to predatorily trading is high relative to the expected loss, the predator finds it optimal to predatorily trade when the distressed trader searches. This condition is met when $m/\Delta > \kappa_\ell$. The left-hand side is the ratio of the gain from winning to the loss from losing, while the right-hand side is the ratio of the probability of losing to the probability of winning the predation war. If only low types search, κ_ℓ is rather low, and hence this condition may be easily satisfied.

Thus, even though the predator cannot directly observe types, searching for a loan is a strong signal that the distressed has a very weak financial status. For this reason the predator finds it optimal to predatorily trade when it observes the distressed trader searching for a loan.

The predator's equilibrium strategy provides a strong incentive for the distressed medium-type trader to refrain from searching. Consider the decision of the medium type distressed trader. If the medium type chooses to not search, then he will not face any predation risk; the only risk he faces is in the exogenous income shock in stage 3. On the other hand, if the medium type chooses to search for a loan, he then engages in a predation war in stage 2, which he can win only if he receives a large enough loan to become a high type. Therefore, when the probability of surviving the income shock as a medium type is high relative to the transition probability of becoming a high type after searching for a loan, that is, when the ratio p_m/π_{mh} is sufficiently high, then the medium type finds it optimal to not search. In other words, the medium type prefers to pool himself with high types by not searching and consequently facing greater income risk, over pooling himself with low types but consequently facing predation risk. One can think of this as a financially weak firm that tries to ride out a temporary financial shortfall on its own, without signalling any weakness to predators by seeking outside liquidity.

Type II. Starting from the type I equilibrium, suppose instead that $p_m < \pi_m \hat{p}$. Now, when the ratio $p_m/\pi_m \hat{p}$ is sufficiently low, that is, when the medium type's probability of surviving the exogenous income shock is low relative to the transition probability of becoming a high type after searching for a loan, then the medium-type distressed trader finds it optimal to search for a loan. In this case, the medium type knows it's unlikely to survive the income shock without getting a loan, and thus must search for a loan despite the fact that this means it will be subject to a predation war. That is, when the ratio $p_m/\pi_m\hat{p}$ is sufficiently low, the increase in survival probability the medium type gains from searching is high enough to compensate for the increased predation risk. Therefore, despite the danger of predatory trading, the medium finds it optimal to pool with the low-types and search for a loan.

One must also consider the decision of the predator. Now, when the predator observes that the distressed trader searches for a loan, he knows that this trader must be either a low *or* a medium type. Using Bayes rule, the predator forms posterior beliefs over the probability of winning in a predation war against this trader. Again, as long as the expected return to predatorily trading is higher than the expected loss, the predator finds it optimal to predatorily trade when the distressed trader searches. This condition is now met when $m/\Delta > \kappa_{\ell m}$. The left-hand side is still the ratio of the gain from winning to the loss from losing, while the right-hand side is the ratio of the probability of losing to the probability of winning the predation war, conditional on both low and medium types searching. Note that this condition is stricter than in the type I equilibria, because $\kappa_{\ell m} > \kappa_{\ell}$. This is due to the fact that in this equilibrium, *both* medium and low types search, whereas above, only low types search. Thus, the probability of losing the predation war is higher when medium types also search, and hence the ratio of gains to losses, m/Δ, must be even higher in order for the predator to find it optimal to predatorily trade.

In summary, when $p_m < \pi_m\hat{p}$ and $m/\Delta > \kappa_{\ell m}$ there exists an equilibrium in which both the low and medium type distressed traders search for a loan, the high type does not search, and the predator predatorily trades if and only if he observes that the distressed trader searches for a loan. In fact, in this region, this equilibrium is unique.

Type III. Finally, starting from the type II equilibrium, suppose instead that $m/\Delta < \kappa_{\ell m}$. In this case, the predator finds it optimal to never predatorily trade–as long as both the low and medium types search, the ratio of probabilities of losing the predation war to winning the predation war is too high relative to the relative gain from winning. As for the distressed trader then, it is as if the predator did not exist. Hence, the distressed trader behaves in the same way as it does in the benchmark case when there is no predator–that is, the low and medium types search for a loan while the high type does not. This is true for all values of $p_m/\pi_m\hat{p}$.

Thus, when $m/\Delta < \kappa_{\ell m}$, an equilibrium exists in which the low and medium types search, the high type does not search, and the predator never predatorily trades.

These equilibria illustrate how predatory trading may affect the incentives of banks to seek loans in times of financial distress. In the benchmark without predation, the medium-type distressed trader searches for a loan in order to protect itself against exogenous income risk. However, when there are predators who cannot directly observe the distress of traders, actions undertaken by these traders to relieve financial distress often convey information about their underlying financial state. For this reason, predators have the incentive to predatorily trade when they see a distressed trader searching for a loan.

The potential of predatory trading clearly affects the incentives of distressed banks to search for loans. In deciding whether or not to search for a loan, a distressed financial firm now faces a trade-off between the financial cushion provided by a loan and the information that searching for loans reveals. In particular, in the Type I equilibrium, the medium-type distressed traders who would otherwise seek to recapitalize are reluctant to search for loans in the presence of predators. Instead, in order to not signal any weakness, they decide to pool themselves with the high-type banks and refrain from seeking outside liquidity. In this sense, these banks try to ride out a temporary financial shortfall on their own, but without any loan, they are subject to greater income risk.

5 Policy Implications of the Term Auction Facility

In December of 2007, in response to the low use of the discount window lending, the Federal Reserve introduced a policy called the Term Auction Facility. The Term Auction Facility, or TAF, was a facility in which the Fed auctioned off loans to banks. This was seen as an attempt to limit the stigma associated with accessing central bank liquidity. In this section, I ask whether in my model a policy such as TAF may help to alleviate the stigma problem.

In order to incorporate a TAF-like facility, I augment the model in the following way. First, I say there is a unit mass of distressed traders. Nature chooses each distressed trader's types as before, but 1/3 are high type, 1/3 are medium type, and 1/3 are low type.

Next, I assume that after Nature chooses types, but before the loan searching stage, there is another stage–this stage is the TAF stage. After learning their types the distressed traders may now participate in TAF, in which the Fed auctions loans with a total loan supply of S. Each trader that wants to participate submits a bid specifying the rate and the quantity at which it is willing to transact. Each bank is allowed only one rate-quantity offer.[14] That is, banks submit a step-like demand function as follows.

$$d_i(r) = \begin{cases} 0 & \text{if} \quad r > r_i \\ \hat{v} - v_i & \text{if} \quad r \leq r_i \end{cases}$$

[14] The actual TAF allowe each bidder to make two rate-quantity offers.

where r is the interest paid on the loan of size $\hat{v} - v_i$. Aggregate demand for TAF loans is then given by the sum of all bids.

$$D(r) = \int_I d_i(c)\, di$$

where I is the set of all bidders.

Once bids are submitted, the Fed then collects the bids and determines the maximum rate at which demand weakly exceeds supply. Specifically, market clearing occurs at the maximum r^* where $S \leq D(r^*)$. The market clearing rate r^* is called the "stop-out" rate.

As with the true TAF, here I assume the auction is a uniform-price auction. This means that all bidders pay the stop-out price for all units they request at prices exceeding the stop-out-price.[15] For simplicity, I assume no rationing of leftover funds. Finally, the bank does not have to pay the interest on the loan until the very end of period 3, that is, until after the income shock is realized.

I assume that bids in the auction are not observed by outsiders. However, the identity of the winners of the loan are observed. Thus, the predator by observing the winners of the TAF, may be able to information about the winners' types. After the auction, the distressed traders who did not win may still search for loans in the non-TAF market and the rest of the game is the same as before.

Note that the loan rules are the same as if the bank had borrowed at the discount window. The only difference between searching for a loan in the non-TAF market is that in TAF the interest rate is set competitively through the auction format. According to Armantier, Krieger, McAndrews (2008), "borrowing is fully collateralized; assets used as collateral are those eligible to be pledged at the discount window."

Equilibrium Analysis. First, note that the high type never uses the TAF since it is in no danger of failing. This implies that we need only look at the medium and low type's decisions.

I restrict analysis to the region where $p_m > \pi_m \hat{p}$, so that a stigma effect could be possible. Here there could either be type I or type III equilibria without TAF. Suppose first that we are originally in a type I equilibrium where the medium type does not search.

Proposition 3. *Suppose $p_m > \pi_m \hat{p}$ and $m/\Delta > \kappa_\ell$ so that we are in a type I equilibrium without TAF.*

[15] "The Federal Reserve also chose a uniform-price (or single-price) auction rather than a discriminatory (pay-your-bid) auction in part to spur participation further. By using the uniform-price structure common in Treasury auctions, the Fed reasoned that banks would be more comfortable with bidding." Armantier, Krieger, McAndrews (2008)

If $m/\Delta < \kappa_m$ and v_m is sufficiently greater than v_ℓ, then there exists $\underline{S}, \bar{S}$ such that for all $S \in \left(\underline{S}, \bar{S}\right)$ there exists an equilibrium that looks as follows. The predator does not predatorily trade against traders that do not search, winners of loans in the TAF market, but it does predatorily trade against agents who search for loans in the outside market. High types do not search for a loan nor participate in the TAF market. Medium and low types submit bids to TAF, the medium types win loans in the TAF market, and low types search for loans in the non-TAF market.

The proof is in the Appendix.

Proposition 3 shows that TAF can be helpful in reducing the stigma effect of obtaining loans, so that funds are funneled to medium type firms. Why is this? Medium type needs less loan quantity, and hence are willing to pay a higher interest rate for their loans than the low type. Thus, the medium type submits higher bids than low type. As long as the loan supply isn't too large, the medium type wins the TAF loans.

Thus, the signal from winning the TAF loan is a positive signal. The winners of TAF are more likely to be stronger than those that borrow at the discount window. The predator then decides not to predatorily trade against the banks who win the TAF loan, because he knows that these banks are in general strong. Thus, the medium types are now able to get extra cash by separating themselves from the loan type. The conditions that $\kappa_\ell < m/\Delta < \kappa_m$ simply means that the predator is not willing to trade against traders he knows are medium types who win TAF, but instead wants to trade against the low types who received loans.

Before, without TAF, the medium type chose to pool themselves with high types. Here, the medium type separates both from the low types as well as the high types. In this case, they signal that they are better than the low types–they can afford to pay a higher price for the loan. This is similar the original signalling intuition in Spence ().

Thus TAF can help alleviate the stigma problem when it exists. However, next I show that a lending facility such as TAF can also be problematic in some circumstances.

Proposition 4. *Suppose $m/\Delta < \kappa_{\ell m}$ so that we are in a type III equilibrium without TAF: the low and medium type both search, and the predator never predatorily trades.*

If $m/\Delta > \kappa_\ell$ and v_m is sufficiently greater than v_ℓ, then there exists $\underline{S}, \bar{S}$ such that for all $S \in \left(\underline{S}, \bar{S}\right)$ there exists an equilibrium that looks as follows. The predator does not predatorily trade against traders that do not search, winners of loans in the TAF market, but do predatorily trade against agents who search for loans in the outside market. High types do not search for a loan nor participate in the TAF market. Medium and low types submit bids to TAF, the medium types win loans in the TAF market, and low types search for loans in the non-TAF market.

For intermediate values of S, there can now exisst an equilibrium in which the predator predatorily trades against those who search in the outside market, but not those in TAF. The firms which win the TAF auction are the medium type, while those who obtain a loan in the non TAF market are low type.

Thus, if the stigma effect is strong, as it is in the type I equilibrium without TAF, proposition 3 shows that TAF can be helpful in getting loans to medium types who would otherwise not receive funding. On the other hand, if the stigma effect is not very strong it can be problematic. Here, in the type III equilibrium without TAF, the low types benefit from pooling with medium types. Both search for loans and obtain funding. The predator doesn't predatorily trade against these traders because of the presense of the stronger medium types.

In this type of regime, TAF can potentially be harmful. Proposition 4, shows that TAF may cream skim the medium types, leaving the low types to face the predator. The low types are worse off because now they must fight a predation war.

This section thus show how it may be possible that a policy such as TAF improves on the equilibrium outcome and alleviates the stigma problem. However, under certain circumstances

Here, even if it is observed whether the distressed trader uses TAF, there is still a stigma effect.

Of course, it would be easier to obtain this solution if it were unobservable that firms win TAF. I show that even with the same types of informational assumptions, TAF can work very differently from the discount window, simply because it is an auction-format.

Finally, this paper abstracts from insolvency. This can be another problem with lending cash to all firms. Here, firms are merely liquidity constrained, rather than insolvent. They can all pay the interest rate even if they fail. Thus, these results show that even without insolvent firms, TAF can be problematic for some types of equilibria.

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Perhaps would like to add: Finally–one would thnk that increasing S is good. Suppose now that S is sufficiently high so that all bidders in the auction will win. Suppose the predator now predatorily trades against the TAF winners. This could then potentially add stigma also to the TAF market. Need to add this.

6 Conclusion

This paper analyzes how predatory trading may affect the incentives of banks to seek loans in times of financial distress. I find that when a distressed trader is more informed than other traders about its own balances, searching for extra capital from lenders can become a signal of financial need, thereby opening the door for predatory trading and possible insolvency. I find equilibria in which some distressed traders who would like borrow short-term in order to meet temporary liquidity needs, may be reluctant to do so in the presence of potential predators. Predatory trading may therefore deter banks and financial institutions from raising funds in times when they need it the most.

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Appendix

In this appendix I provide a more detailed analysis of the predation war discussed in Section 2. In stage 2, if the predator decides to predatorily trade, then the strategic traders engage in a "predation war". The results of this predation war are derived directly from Brunnermeier and Pedersen (2005).

Suppose that time is continuous within this stage and denoted by $\tau \in [0, \bar{\tau}]$. That is, traders may now trade continuously in the asset. Let $x_i(\tau)$ denote the position of trader i in the asset at time τ, and let $s(\tau)$ denote the price of that asset. At the beginning of stage 2, each trader has an initial position, $x_i(0) = \bar{x}$, of the risky asset. Within stage 2 the trader can now continuously trade the asset by choosing his trading intensity, $a_i(\tau)$. Hence, at time τ the trader's position in the risky asset is given by

$$x_i(\tau) = x_i(0) + \int_0^\tau a_i(u)du$$

As mentioned previously, each strategic trader is restricted to hold $x_i(\tau) \in [-\bar{x}, \bar{x}]$. Finally, I consider the case of limited capital, such that $2\bar{x} < Q$. In addition to the two large strategic traders, the market is populated by long-term investors, whose aggregate demand curve is given in (1).

Furthermore, it is assumed that traders cannot sell infinitely fast. Strategic traders can as a whole can trade at most $A \in R$ shares per time unit at the current price. That is, at any moment τ, the aggregate amount of trading must satisfy $a_d(\tau) + a_p(\tau) \leq A$. This implies that if both strategic traders are trading, the greatest intensity at which each trader may trade is $A/2$.[16]

Trader i's within-stage objective is to maximize his expected wealth at the end of the stage. His earnings from investing in the risky asset is given by the final value of his stock holdings, $x_i(\bar{\tau})z$, minus the cost of buying shares. That is, a strategic trader's objective is to choose a trading process so as to maximize

$$\max E\left[x_i(\bar{\tau})z + \int_0^{\bar{\tau}} a_i(\tau)s(\tau)d\tau\right]$$

Due to limited capital of the strategic traders, $s(\tau) < \bar{z}$ at any time, and hence, any optimal trading strategy satisfies $x_i(\bar{\tau}) = \bar{x}$ for the surviving trader. That is, any surviving

[16] Brunnermeier and Pedersen (2005) assume that strategic traders can as a whole trade at most $A \in R$ shares per time unit at the current price. Rather than simply assuming that orders beyond A cannot be executed, they assume that traders suffer temporary impact costs if orders exceed this bound. They then show that it is optimal for traders to trade as fast as possible without incurring this cost.

trader ends up with the maximum capital in the arbitrage position. The qualitative results presented in this appendix will then depend on the following: (i) strategic traders have limited capital, that is $2\bar{x} < Q$, otherwise $s(\tau) = \bar{z}$ and (ii) markets are illiquid in the sense that large trades move prices ($\lambda > 0$) and traders cannot trade arbitrarily fast ($A < \infty$).

Let τ_d and τ_p denote the amount of time it takes for the distressed trader and the predator to hit their lower bounds on wealth, respectively, if both were trading simultaneously at their highest intensity. That is,

$$\tau_d \equiv \frac{w_{d,2}(0) - \underline{w}}{A/2} \qquad \text{and} \qquad \tau_p \equiv \frac{w_{p,2}(0) - \underline{w}}{A/2}$$

where $w_{d,2} = \bar{x}s(0) + v_{d,2}$ and $w_{p,2} = \bar{x}s(0) + v_p$. In equilibrium, both traders sell as fast as possible until one of the traders is forced to leave the market. Specifically, both traders trade at constant speed $-A/2$ from from time 0 to time τ^*, where $\tau^* \equiv \min\{\tau_d, \tau_p\}$. Therefore, the pivotal time τ^* is determined by the wealth of the trader who is closest to the threshold; in other words, the trader who begins the period with lower wealth (i.e. the lower v) is the trader who is forced to leave the market. I assume that $\underline{w}$ is high enough such that at least one trader hits the lower bound.

More precisely, the trader which is forced to leave the market trades according to the following process

$$a_i(\tau) = \begin{cases} -A/2 & \text{for} \quad \tau \in [0, \tau^*] \\ 0 & \text{for} \quad \tau \geq \tau^* \end{cases}$$

While the surviving trader trades according to the following process

$$a_i(\tau) = \begin{cases} -A/2 & \text{for} \quad \tau \in [0, \tau^*] \\ A & \text{for} \quad \tau \in \left[\tau^*, \tau^* + \frac{\bar{x} - x(\tau^*)}{A}\right] \\ 0 & \text{for} \quad \tau \geq \tau^* + \frac{\bar{x} - x(\tau^*)}{A} \end{cases}$$

Thus, both traders trade as fast as they can at constant speed $-A/2$ for τ^* periods, at which point one trader is forced to leave the market. This liquidation strategy is known by both strategic traders. At time τ^*, the surviving trader then buys at a constant rate back up to the original arbitrage position $\bar{x}$; this takes $\frac{\bar{x} - x(\tau^*)}{A}$ periods. From then on, the surviving trader remains in this position.

Finally, the equilibrium price follows the following trajectory

$$s(\tau) = \begin{cases} s(0) - \lambda A \tau & \text{for} \quad \tau \in [0, \tau^*] \\ s(0) - \lambda 2\bar{x} + \lambda A (\tau - \tau^*) & \text{for} \quad \tau \in \left[\tau^*, \tau^* + \frac{\bar{x} - x(\tau^*)}{A}\right] \\ \bar{z} - \lambda(\bar{x} - Q) & \text{for} \quad \tau \geq \tau^* + \frac{\bar{x} - x(\tau^*)}{A} \end{cases}$$

The simultaneous selling by both strategic traders leads to price "overshooting." This implies that the surviving trader may yield a gain from winning the predation war. This gain is given by

$$m = \int_0^{\bar{\tau}} a_i(\tau) s(\tau)\, d\tau.$$

This is because the surviving trader sells his assets for an average price that is higher than the price at which he buys them back after the other trader has left the market. Therefore, the predator has an incentive to predatorily trade in order to profit from the price swings that occur in the wake of the liquidation. Furthermore, the overshooting price due to simultaneous selling makes liquidation excessively costly for the trader who is ultimately forced to leave the market.

Proof of Condition 13 One may quickly prove this via contradiction. Suppose instead that $\kappa_\ell > \kappa_m$

$$\frac{\pi_{lh}}{\pi_{ll} + \pi_{lm}} > \frac{\pi_{lh} + \pi_{mh}}{\pi_{ll} + \pi_{lm} + \pi_{mm}}$$

Multiplying both sides by the demoninators we obtain

$$\pi_{lh}(\pi_{ll} + \pi_{lm} + \pi_{mm}) > (\pi_{lh} + \pi_{mh})(\pi_{ll} + \pi_{lm})$$

Cancelling terms on both sides yields $\pi_{lh}\pi_{mm} > \pi_{mh}(\pi_{ll} + \pi_{lm})$. Next, substituting in for $\pi_{mh} = 1 - \pi_{mm}$ and $\pi_{ll} + \pi_{lm} = 1 - \pi_{lh}$, we get

$$\pi_{lh}\pi_{mm} > 1 - \pi_{mm} - \pi_{lh} + \pi_{mm}\pi_{lh}$$

which implies that $0 > 1 - \pi_{mm} - \pi_{lh}$. Finally, again using the fact that $\pi_{mh} = 1 - \pi_{mm}$, the above condition may be re-written as $\pi_{lh} > \pi_{mh}$. But this is contradicted by our assumption that $\pi_{lh} \leq \pi_{mh}$. Therefore, $\kappa_\ell \leq \kappa_m$.

Proof of Proposition 3 If the medium type doesn't win TAF, it finds it beneficial to not search. Decides between (not searching) and using TAF. The medium type bids in TAF if and only if

$$\bar{w}\alpha\left(NS|v_m,g\right)+\left(\bar{w}-\Delta\right)\left(1-\alpha\left(NS|v_m,g\right)\right)\leq\left(\bar{w}-\left(1+r_m\right)\left(\hat{v}-v_m\right)\right)\alpha\left(TAF|v_m,g\right)+\left(\bar{w}-\Delta-\left(1+r_m\right)\right.$$

$$\begin{aligned}
-\Delta\left(1-\alpha\left(NS|v_m,g\right)\right) &= -\left(1+r_m\right)\left(\hat{v}-v_m\right)-\Delta\left(1-\alpha\left(TAF|v_m,g\right)\right) \\
\Delta\left(1-\alpha\left(NS|v_m,g\right)\right) &= \left(1+r_m\right)\left(\hat{v}-v_m\right)+\Delta\left(1-\alpha\left(TAF|v_m,g\right)\right) \\
\Delta\left(\alpha\left(TAF|v_m,g\right)-\alpha\left(NS|v_m,g\right)\right) &= \left(1+r_m\right)\left(\hat{v}-v_m\right)
\end{aligned}$$

$$\begin{aligned}
1+r_m &= \frac{\Delta\left(\alpha\left(TAF|v_m,g\right)-\alpha\left(NS|v_m,g\right)\right)}{\hat{v}-v_m} \\
1+r_m &= \frac{\Delta\left(\hat{p}-p_m\right)}{\hat{v}-v_m}
\end{aligned}$$

If the medium type searches for a loan, but doesn't get it, it just does what it did before–it doesn't search, and gets its payoff from not searching.

The indifference conditions of the low type. In the original equilibrium, the low type finds it beneficial to search for a loan. He decides between searching and obtaining a loan w some probability and using TAF. If the low type doesn't win TAF, it searches for a loan in the non-TAF market, gets predatorily traded against. survivies with prob $\pi_\ell\hat{p}$

$$\bar{w}\alpha\left(S|v_\ell,g\right)+\left(\bar{w}-\Delta\right)\left(1-\alpha\left(S|v_\ell,g\right)\right)\leq\left(\bar{w}-\left(1+r_\ell\right)\left(\hat{v}-v_\ell\right)\right)\alpha\left(TAF|v_m,g\right)+\left(\bar{w}-\Delta-\left(1+r_\ell\right)\left(\hat{v}-v_\ell\right)\right.$$

$$\begin{aligned}
1+r_l &= \frac{\Delta\left(\alpha\left(TAF|v_\ell,g\right)-\alpha\left(S|v_\ell,g\right)\right)}{\hat{v}-v_\ell} \\
&= \frac{\Delta\left(\hat{p}-\pi_\ell\hat{p}\right)}{\hat{v}-v_\ell}
\end{aligned}$$

thus

$$\begin{aligned}
\frac{\Delta\left(\hat{p}-p_m\right)}{\hat{v}-v_m} &> \frac{\Delta\left(\hat{p}-\pi_\ell\hat{p}\right)}{\hat{v}-v_\ell} \\
\frac{\hat{p}-p_m}{\hat{p}-\pi_\ell\hat{p}}\left(\hat{v}-v_\ell\right) &> \hat{v}-v_m
\end{aligned}$$

Proposition 5. *If $p_m > \pi_m \hat{p} > \pi_\ell \hat{p}$, then for all values of v_m and v_ℓ that satisfy*

$$\hat{v} - v_m < \chi (v_h - v_\ell)$$

for some constant $\chi < 1$, $r_m > r_\ell$

That is, $v_h - v_m$ must be sufficiently small relative to $v_h - v_\ell$. Otherwise, it may be the case that $r_m < r_\ell$. For now, suppose it is the case that $r_m > r_\ell$. Thus, the medium types submit higher bids than the low types. If S is sufficiently low, then only the medium types win the TAF auction.

Thus, in terms of outcomes for the distressed trader, the medium types win the TAF auction, and the low types have no choice but to search in the non-TAF market.

Finally, what remains to be shown is that the predator finds it optimal to predatorily trade against those in the non-TAF market, but not against those in the TAF market. The predator chooses to trade against those in the non-TAF market as long as

$$m/\Delta > \kappa_\ell$$

On the other hand, if the predator observes that the distressed won the TAF auction, then the predator knows that this is a medium type. Thus, the probability of the predator surviving a predation war is given by $\beta (P, S|f') = 1 - \pi_m$. Combining this with the predator's decision rule in (9), the predator finds it optimal to predatorily trade if and only if

$$\kappa_m < m/\Delta$$

where

$$\kappa_m \equiv \frac{\pi_m}{1 - \pi_m} \tag{14}$$

Note that, given the assumptions on parameter values π_ℓ and π_m, $\kappa_\ell < \kappa_m$ as long as $\pi_\ell < \pi_m$.

Proof of Proposition 4 Well, first the medium type has the same problem as above, as does the low type. If the medium type doesn't win TAF, it doesn't search. If the low type doesn't win TAF, it searches. Also, the predator has the same problem. Thus, the parameter restrictions are the same.