Algorithmic and High Frequency Trading in Dynamic Limit Order Markets

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Abstract

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JEL classification: C63, C73, D47, D53, D83, G11, G12, G14.
Keywords: High frequency trading, algorithmic trading, limit order market, low-latency trading, dynamic equilibrium model, asynchronous endogenous decisions.

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1 Introduction

Financial markets have undergone a major technological transformation during the past decade: from human-led transactions to algorithmic trading (henceforth, AT), in which sophisticated computers quickly process information, and algorithms automatically submit and modify orders utilizing superfast connections to the exchanges. However, this financial innovation has generated a relatively favourable position for investors with algorithmic trading features, over the rest of the market participants. On the one hand, algorithmic traders have an 'informational advantage' since AT technology allows fast access and quick analysis of market information. On the other hand, algorithmic traders have a 'trading speed advantage' since they have a low-latency transmission of orders.[1] Currently, there is a growing theoretical literature on understanding the impact of AT on market quality and stability as well as possible damage to 'traditional' investors. These studies have independently characterized AT through either the informational advantage or the trading speed advantage.[2] The goal of our study is to fill this gap by presenting a dynamic equilibrium model with algorithmic trading in a limit order market, where algorithmic traders have effectively both trading advantages. Our objective is to understand which of these AT trading advantages induces more damage to 'traditional' slow traders, to observe potential synergies of the AT trading features, to study possible dangers and benefits of this technology for market quality and to analyse potential regulations.

Currently, the exchanges in which we can find AT are fully, or at least partially, organized as limit order markets (e.g., BATS U.S. stock exchange, NYSE, NASDAQ, London Stock Exchange,

[1] The trading speed advantage is associated with a sub-group of the algorithmic traders, who are known as high frequency traders.

[2] For studies in which algorithmic traders are characterized using the informational advantage, see among others Martinez and Roşu (2011), Biais *et al*. (2012a), and Foucault *et al*. (2012); while for studies in which AT technology is modelled using the trading speed advantage, see Hoffmann (2013).

NYSE Euronext and BATS Chi-X Europe).[3] Consequently, the microstructure characteristics and particularities of these types of trading venues should be considered when evaluating the effects of AT on market quality.[4] Therefore, we consider a limit order market in our dynamic equilibrium model. Traders can submit market orders or limit orders.[5] As in a real limit order market, the limit order book is characterized by a set of discrete prices, and respects the time and price priorities for the execution of limit orders. In addition, traders can cancel unexecuted limit orders depending of potential changes in market conditions. Thus, given the dynamic features of our equilibrium model, we can generate transactions and the evolution of the order book, which represents an additional contribution by our paper. In fact, a recent study of AT technology sponsored by the British government states that: "*simulation tools and techniques could enable central regulatory authorities to judge the stability of particular financial markets, given knowledge of the structure of those markets*".[6] In our study, we reproduce the behaviour of a complete limit order book as in reality; hence we are able to simultaneously evaluate the impact of AT from multiple edges and scenarios, analyse the dynamic interactions between different types of traders, examine several market quality measures, and evaluate potential policy instruments in a controlled environment. We would like to answer the following questions: is always AT good (or bad) for market performance and the profits of 'traditional' investors? In particular, when do AT agents induce benefits (and when do not) for market quality? Furthermore, how can we regulate this technology?

[3] In fact, 85% of the leading stock exchanges around the world are now entirely electronic limit order markets with no floor-trading (Jain, 2005).

[4] AT traders have to take into account the microstructure characteristics of markets when they design their investment strategies, which also makes features of limit order markets relevant to evaluate the impact of AT on market quality and stability.

[5] A limit order is a commitment made by a trader at time t to trade the asset in the future at a pre-specified price p; while a market order is a request to trade immediately at the best price available (i.e., at the bid or ask prices depending the direction of the order). In addition, a limit order from a given trader is always executed through the submission of a market order.

[6] This study involved 150 leading experts from more than 20 countries. The name of the study is "Foresight: The Future of Computer Trading in Financial Markets (2012) Final Project Report".

We present a dynamic equilibrium model in continuous-time with a single asset. The model is a stochastic asynchronous game with endogenous trading decisions. The common value of the asset, v_t, follows a random walk and reflects its fundamental valuation.[7] There are two types of risk-neutral agents: fast traders and slow traders (also called algorithmic traders and less-skilled 'traditional' traders, respectively). Agents arrive at the market following a Poisson process at rate λ, where a proportion β^{FT} of the agents are fast traders while the rest of the market participants are slow investors. Fast traders have an informational advantage and a low-latency transmission of orders in relation to slow traders. First, fast traders can contemporaneously observe v_t, while slow traders observe the fundamental value of the asset with a time lag (i.e., at any instant t slow traders only know $v_{t-\Delta_t}$).[8] Second, traders can re-enter the market multiple times to revise and to modify previous trading strategies. However, agents cannot instantaneously modify trading decisions due to the fact that cognition limits prevent them from continuously monitoring the market; thus trading plans are 'sticky' (see, e.g., Biais *et al*., 2012b). Nevertheless, algorithmic traders have the possibility of evaluating market changes and modifying previous trading strategies much faster than slow traders. Thus, fast traders and slow traders re-enter the market according to two Poisson processes at rate λ_r^{FT} and λ_r^{ST}, respectively, where $\lambda_r^{FT} > \lambda_r^{ST}$.[9] As a first step, we analyse the effect of AT on market quality when fast traders only have either the informational advantage or the trading speed advantage; afterwards we analyse the case when AT agents have both advantages together.

[7] The fundamental value of the asset can be thought of as the discounted value of expected future dividends.

[8] This assumption is supported by previous empirical studies on AT, which show that fast traders are better informed than other market participants (see, e.g., Hendershott and Riordan, 2010; Brogaard, 2010; Kirilenko *et al*., 2011; and Brogaard *et al*., 2012). In addition, similar assumptions have already been used in AT theoretical models by Biais *et al*. (2012a), Foucault *et al*. (2012), and Martinez and Roşu (2011).

[9] The expected time between re-entries for algorithmic traders is lower than for slow traders, since the expected value of an exponentially distributed variable x, $E(x)$, with parameter λ is $E(x) = 1/\lambda$.

We find that AT induces changes in the trading behaviour of 'traditional' investors, which depends of the trading preferences of fast traders. Fast traders have two main options to make profits. On the one side, AT traders can make profits through the liquidity provision which is reflected in the difference between the bid and ask prices. On the other side, fast traders can profit by picking-off limit orders, when the fundamental value unexpectedly moves against the limit orders submitted by other agents. In the case where the market participation of ‘less-skilled’ slow investors is small, slow traders prefer to execute more market orders (they have a tendency to be liquidity demanders), while fast traders execute more limit orders (fast traders prefer to behave more as liquidity suppliers). Slow traders prefer to execute more market orders because limit orders have the risk of being ‘picked-off’ when market conditions change unfavourably against them and when many fast traders are present; while AT agents use informational and trading speed advantages to provide liquidity which induces a reduction on the bid-ask spread. Moreover, the reduction in the bid-ask spread generate additional incentives to slow traders for the submission of market orders since they are less costly. However, in the case where the market participation of ‘less-skilled’ investors is high, we report evidence that AT traders may induce more damages than benefits to the market. In this scenario, instead of using their advantages for liquidity provision, fast traders exhibit a ‘predatory’ behaviour through market orders by ‘picking-off’ limit orders coming from the big crowd of slow agents in the market.

We find that AT technology reduces the waiting costs for slow traders, but fast traders require a payment for this service that is larger than the reduction in the waiting cost for traditional investors. Thus, fast traders induce economic damage to slow trader’s profits. In relation to the welfare of the system, fast traders with only an *informational advantage* increase the global welfare. Conversely, fast traders with only a *trading speed advantage* induce welfare reductions. Nevertheless, there is a positive synergy between the informational and trading speed advantages of fast traders; when these are combined, the welfare of the system increases even more than when fast traders have only an informational superiority. Additionally, the market participation of ‘less-skilled’ agents and fast traders in the market has a non-linear effect on market welfare, due to the different trading behaviour of traders when the proportion of ‘less-skilled’ investors changes in the market. The maximum system welfare is obtained when fast traders constitute around 70% of the market participation, which in fact is congruent with the current U.S. stock trading volume reported in the empirical literature. For instance, the SEC in 2009 and Brogaard (2010) report that 73% and 77% of the trading volume on the U.S. stock market can be attributed to AT technology, respectively.[10]

We show that AT reduces microstructure noise, especially when fast traders have informational advantages, since it mitigates the cognitive limits of human beings. In addition, despite the fact that slow traders can observe the fundamental value of the asset with a time lag Δ_t, in the model we allow them to capture and to learn the information revealed in the market activity by fast traders. The learning process followed by slow traders helps them to make more precise estimations about the contemporaneous fundamental value of the asset, and thus to make better trading decisions. Consequently, the cognitive capacities of slow traders combined with the presence of fast traders in the market (who submit informative and competitive orders) induce a reduction in the slow trader’s errors in beliefs in relation to v_t. Our findings are consistent with the empirical evidence reported by Hendershott and Riordan (2010), Brogaard (2010), and Brogaard *et al*. (2012) regarding improvements in informational efficiency generated by AT technology.[11]

We report that AT improves liquidity when the market participation of less-skilled 'traditional' traders is smaller than participation of fast traders in the market. The increase in market liquidity is reflected in reductions in the quoted and effective spreads and in a

[10] See “SEC runs eye over high-speed trading,” *Financial Times*, July 29, 2009. Similar results have been obtained in empirical studies for foreign exchange markets (see, e.g., Chaboud *et al*., 2011).

[11] Our findings are also related to Kirilenko *et al*. (2011), who provide empirical evidence that algorithmic traders may have informational advantages, since they can make orders in the right direction in relation to price changes.

reduction in the time between the instant in which a trader arrives and her first order submission. Our results are also congruent with the results of empirical studies which show that there is a positive relationship between AT technology and market liquidity (see, e.g., Hendershott *et al*., 2012; Hasbrouck and Saar, 2012; and Riordan and Storkenmaier, 2012). Nevertheless, in the case in which the market participation of less-skilled 'traditional' traders is larger than the participation of fast traders, AT produces liquidity damages. In this environment, as we explained previously, fast traders prefer to execute market orders following a 'predatory' behaviour to 'pick-off' the limit orders of slow traders. This reduces the potential liquidity provision generated by AT technology.

Agents maximize their utility for the dynamic decision problem, which allows us to calculate the welfare gains for different traders' decisions. We obtain the equilibrium numerically, as the model is analytically intractable. Given the asynchronous nature of the game, we solve the equilibrium using the algorithm introduced by Pakes and McGuire (2001), which was originally proposed for industrial organization problems with sequential decisions. This algorithm provides a Markov-perfect equilibrium which has been successfully implanted into dynamic models for limit order markets by Goettler *et al*. (2005, 2009), although without exploring the effects of AT on market quality and stability as in our study.

We also perform two policy exercises through our model. Firstly, we analyse the impact on market quality of potential regulations to control AT market activity, such as a latency restriction and a cancellation fee applied to fast traders; and secondly, we examine the potential effect of an increase in market volatility on trading behaviour and market performance. A latency restriction and a cancellation fee for fast traders have harmful impacts on market quality, since both regulations represent additional frictions that affect negatively the market functioning. Nevertheless, a cancellation fee for fast traders generates a more direct effect on reductions in the adverse selection faced by slow traders than the latency restriction regulation. In addition, a cancellation fee produces a positive change in the behaviour of fast traders; this policy instrument induces fast traders to behave more as liquidity suppliers than when the regulation is not applied. This is in line with the empirical evidence reported recently by Malinova *et al*. (2013), where AT agents trade more limit orders than the rest of the agents after the implementation of a cancelation fee in the Toronto Stock Exchange. However, the decision of the 'right' level for the cancellation fee is crucial. A small cancellation fee may have no impact on the liquidity supply from fast traders; while a high cancellation fee could induce some traders not to acquire the AT technology since there is a large implicit participation cost. This is important in terms of market regulations, since a high cancellation fee could induce a reduction in the number of fast traders. A reduction in the number of AT traders may damage market quality especially when 'less-skilled' agents are predominant in the market, as explained previously.

In relation to the effect of an increase in market volatility, we find that fast traders may have incentives to trade in assets that are more volatile or during a period of high economic volatility, because AT agents can make larger profits in these market conditions. This finding is also consistent with Kirilenko *et al*.'s (2011) study in relation to the 'flash crash' where a high intraday volatility was observed on May 6th, 2010. Kirilenko *et al*. (2011) find that the 'flash crash' was due to a wrongly executed selling plan by a large fundamental trader; nevertheless they also observed abnormal trading behaviours by AT traders.

The study of the effects of algorithmic trading on traders' behaviours and market quality is extremely relevant given the large proportion of trading activity that is generated by traders with AT technology. The paper is organised as follows. Section 2 presents a literature review. Section 3 introduces the model and describes the algorithm used to solve the asynchronous trading game. Section 4 shows the effects of AT on the trading behaviour of market participants. Section 5 presents the impact of AT on the payoffs and gains from trade. Section 6 analyses the microstructure noise and errors in slow traders' beliefs when there are AT traders in the market. Section 7 examines the relationship between AT and market liquidity.

Section 8 reports two policy analyses (the effect of potential regulations to control AT activity and the impact of an increase in market volatility on market quality). Finally, Section 9 concludes.

2 Literature review

Our work is connected to the growing theoretical literature on AT. However, previous studies do not fully include the relationship between AT and the multiple microstructure features of dynamic limit order markets, as our study does. In addition, there have been a limited number of efforts at developing a dynamic model to explore the impact on, and potential synergies in, market performance of informational advantages and the effective low-latency transmission of orders that AT technology provides to some traders. Biais *et al*. (2012a) present a 3-period model of AT, in which fast traders know the fundamental value before slow traders in a similar way to our approach. Biais *et al*. (2012a) find that fast traders can generate adverse selection costs for slow traders; and thus AT may induce negative externalities. They argue that adverse selection appears due to the superior information of fast traders, given that algorithmic traders can process public information faster than slow agents. Foucault *et al*. (2012) present two dynamic models with a market maker and an informed trader who can only submit market orders. In the first model, the market maker and the trader receive information at the same time (although with different precision levels); while in the second model the informed trader receives information a moment before the market maker. Foucault *et al*. (2012) find that the advantage in information increases trading volume, decreases liquidity, induces price changes that are more correlated with fundamental value movements, and reduces informed order flow autocorrelations.

Martinez and Roşu (2011) introduce a model with a dealer and informed fast traders. Fast traders only submit market orders and have an informational advantage, but they are also uncertainty averse regarding the level of the asset value. Martinez and Roşu (2011) find that AT generates most of the volatility and trading volume in the market, and present evidence that AT makes the markets more efficient as fast traders incorporate their information advantages in transaction prices.[12] Hoffmann (2013) presents a dynamic model, in which AT traders have a 'pure' trading speed advantage. In his model, traders can submit market or limit orders, while only fast traders can modify limit orders after the arrival of new information. Hoffmann (2013) shows that slow traders strategically submit limit orders with a lower execution probability which in equilibrium is always welfare-reducing, given their loss of bargaining power. Jovanovic and Menkveld (2012) present a model of AT liquidity suppliers with access to public information. Jovanovic and Menkveld (2012) show that fast liquidity suppliers may reduce informational friction due to the superior speed in information analysis and execution of AT technology, but that AT can also reduce welfare because of the adverse selection that slow traders face.

The paper most closely related to ours is the study developed by Bongaerts and Van Achter (2013). They introduce a dynamic model with slow and fast traders where fast traders have informational and trading speed advantages, although the trading speed advantage is modelled differently to our modelling setup. Bongaerts and Van Achter (2013) present a restricted limit order book, in which slow and fast traders can only submit limit orders, and where unexecuted limit orders cannot be modified or cancelled. They model the trading speed advantage by allowing fast traders to arrive at the market at a more intense rate of arrival than slow traders. Bongaerts and Van Achter's (2013) model is complementary to ours, since they make the choice of being fast traders an endogenous one, while it is an exogenous decision in our model (though we analyse the endogenous acquisition equilibrium using the differences in payoffs for fast traders and slow traders in Section 5). Bongaerts and Van

[12] Additionally, Pagnotta and Philippon (2012) study exchanges' incentives to invest in faster platforms. They show that exchange competition in speed reduces prices further, leads to more fragmentation, improves investor participation and increases the trading volume.

Achter (2013) find that when traders have decided to adopt AT technology with both informational and trading speed advantage, if the trading speed advantage is efficient enough, the adoption rate can be large and liquidity may evaporate when it is more needed, which may induce market freezes.

Our study is also methodologically associated with the state-of-the-art microstructure models for limit order markets developed by Goettler *et al*. (2005, 2009). Goettler *et al*. (2005, 2009) introduce dynamic models in which investors have to make asynchronous trading decisions, depending on their information set and the market structure, in which the equilibrium is obtained numerically, as in our study. Their model represents a step forward in terms of realism in relation to previous multi-period models of limit order markets.[13] Even though there is a methodological connection between our paper and the microstructure study conducted by Goettler *et al*. (2005, 2009), our research focus differs in exploring the impacts of AT in relation to market quality and integrity; and thus our objective is to answer a different set of questions. Furthermore, and differently to Goettler *et al*. (2005, 2009), we consider a more developed model for AT that includes traders with different speeds in relation to the low-latency transmission of orders. Additionally, we perform policy analyses by including a cancellation fee in the model to avoid anticompetitive tactics by algorithmic traders (which has already been implemented by some exchanges), and we evaluate the effects of different volatility levels on market quality and stability when there are investors with AT technology.[14]

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[13] Early work on multi-period equilibrium models for limit order markets imposed some restrictive assumptions to make the models analytically tractable (see, e.g., Parlour, 1998; Foucault, 1999; Foucault *et al*., 2005; and Roşu, 2009).

[14] Our paper is also closely related to Biais *et al*. (2012b), who present a model in which investors have sticky plans due to limited cognition. Although Biais *et al*. (2012b) do not specifically study the interaction between slow and fast traders and the possible informational advantages of AT technology, they analyse the effects of sticky trading decisions in a limit order market. They show that sticky trading plans lengthen market price recovery and induce round trip trades which increase volume. See Lynch (1996), Reis (2006a,b), Mankiw and Reis (2002), Alvarez *et al*. (2011), and Alvarez *et al*. (2012) for additional studies regarding the economic impact of infrequent updating on investment decisions.

3 The model

3.1 *The market characteristics*

We consider a dynamic continuous-time model of algorithmic trading in a limit order market with a single financial asset. The fundamental value of the asset, v_t, follows a random walk with drift zero and volatility σ. The model is an asynchronous dynamic trading game in which there are two types of risk-neutral traders: fast traders and slow traders. Agents (fast traders and slow traders) arrive at the market following a Poisson process at rate λ, where a proportion β^{FT} of the agents are fast traders while the rest of the market participants are slow investors. Similar to Biais *et al*. (2012a), Foucault *et al*. (2012) and Martinez and Roşu (2011), we assume that fast traders can process new information faster than slow traders. Thus, we assume that at time t fast traders know the current fundamental value of the asset v_t; while slow traders only know the fundamental value with a lag Δ_t (i.e., fast traders have an informational advantage).

Traders can submit limit orders and market orders. Traders can also revise and modify their unexecuted limit orders multiple times. However, due to cognition limits, agents cannot immediately modify their previous limit orders after a change in market conditions; thus trading decisions are 'sticky'. Nevertheless, fast traders have more tools and resources to evaluate possible cancellations and they can make modifications faster than slow agents (i.e., AT traders have a trading speed advantage). Therefore, fast traders re-enter the market following a Poisson process at rate λ_r^{FT} to revise unexecuted limit orders, while slow traders also re-enter according to a Poisson process at rate λ_r^{ST}, where $\lambda_r^{FT} > \lambda_r^{ST}$.[15]

All traders observe the evolution of the order book until time t, which generates two informational effects. On one hand, slow traders can use the historical trading activity to update their expectations of the fundamental value of the asset, and hence to make a better

[15] We use a similar notation to Goettler *et al*. (2005 and 2009) regarding the microstructure features of the model for the dynamic limit order book market.

prediction of the current value of v_t. On the other hand, algorithmic traders also observe the trading history of the market and can also estimate the expected value of slow traders regarding v_t; and thus fast agents can predict the trading strategies of slow investors, which enables them to further increase their payoffs related to AT technology.

Each trader has an intrinsic private value to trade the asset, α, which is drawn from a discrete distribution F_α and is known before making any trading decision. The private value is idiosyncratic and constant to each agent. The private value arises from differences in terms of intrinsic benefits from trading such as tax exposures, wealth shocks, hedging needs, or differences in investment horizons, amongst others. This private value gives additional heterogeneity to the different agents in the dynamic trading game. For instance, traders with higher absolute values in their intrinsic benefits to trade are likely to be liquidity demanders (they would like to trade immediately using market orders) since the main benefits are coming from their private values rather than the trading activity by itself. However, they have to pay an 'immediacy' cost for market orders implicit in the bid-ask spread. Conversely, traders with α equal to zero (and hence with no intrinsic benefits to trade) are indifferent in taking either side of the market and hence maximize their benefits depending on the available trading possibilities; consequently they are likely liquidity suppliers since they prefer to submit limit orders to capture the 'immediacy' cost paid by agents with market orders.[16]

As in real limit order markets, the limit order book L_t is described by a discrete set of prices, denoted as $\{p^i\}_{i=-\infty}^{\infty}$, where $p^i < p^{i+1}$ and the tick size, d, is the distance between any two consecutive prices. There is a backlog of outstanding orders to buy or to sell, l_t^i, which are associated with each price p^i. A positive (negative) number in l_t^i denotes buy (sell) orders in the book, and it represents the depth at price p^i. Therefore, the bid price is $B(L_t) =$

[16] Fast traders with zero private value are equivalent to the AT liquidity suppliers in Jovanovic and Menkveld (2012).

$\max\{p^i|l_t^i > 0\}$ while the ask price is $A(L_t) = \min\{p^i|l_t^i < 0\}$, and if the order book is empty on the bid side or on the ask side $B(L_t) = -\infty$ or $A(L_t) = \infty$, respectively.

The limit order book respects the time and price priorities for the execution of limit orders. Buy (sell) orders at higher (lower) prices are executed first, and limit orders submitted earlier have priority in the queue when they have the same price. In addition, when a trader submits an order, the order price identifies whether the order is a market order or a limit order. This means that an order to buy (sell) at a price above (below) the ask (bid) price is executed immediately at the ask (bid) price; and thus this order is a market order.

Each agent can trade one share and has to make three main trading decisions after arriving in the market: i) to submit an order or to wait until the market conditions change; ii) to buy or to sell the asset; and iii) to choose the price at which she will submit the order, which implies the decision to submit a market order or a limit order, depending on whether the price is inside or outside the quotes.[17] Therefore, despite the fact that traders arrive following the Poisson process with parameters λ, the submission rate is different as agents can decide to submit or to wait in the market, which depends endogenously on the market conditions given that trading decisions are state-dependent.

As we mentioned previously, traders can re-enter the market and modify their previous unexecuted limit orders. Therefore, traders have to make additional trading decisions after re-entering: i) whether to cancel an unexecuted limit order or retain the order without changes; ii) if she decides to cancel the order, whether to submit immediately a new order (a limit or a market order) or to wait for different market conditions in the future to submit her new order; and iii) if she decides to submit a new order after a cancellation, she has to choose the type of order and its price. Therefore, agents have to take the possibility of re-entry into account in the utility maximization problem.

[17] We can include additional shares per agent in the trading decision. However, similarly to Goettler *et al*. (2009), we assume one share per trader to make the model computationally tractable.

Once a trader submits a limit order, she remains part of the trading game by revising her order until it is executed; however the trader exits the market permanently after execution of her order. Consequently, there is a random number of active market participants at each instant, who are monitoring their unexecuted limit orders in the book.

Traders have to pay a cancellation fee c_{canc} when they cancel an unexecuted limit order. [18] In the case of a re-entry, a trader can leave the order without changes, which has the benefit of keeping her priority time in the queue.[19] The negative side of leaving an order in the book is that the asset value could move in directions that affect future payoffs. For instance, in the scenario of a growth in the asset value, some limit sell orders could be priced too low, and a quick trader could make profits from the difference. This possibility represents an implicit transaction cost of being 'picked-off' when prices change unexpectedly after limit orders have been submitted. Conversely, when the asset value decreases, a sell limit order has the risk of not resulting in a trade. To take into account the risk that a limit order may not result in a trade, we include a cost of 'delaying' by a discount rate ρ_d, which reflects the cost of not executing immediately. This 'delaying' cost does not represent the time value of money; instead ρ_d reflects opportunity costs and the cost of monitoring the market until a limit order is executed. Thus, the payoffs of order executions are discounted back to the trader's arrival time at rate ρ_d where $0 < \rho_d < 1$.

3.2 The trader's dynamic maximization problem

Let $\Upsilon \in \{0,1\}$ be a trader indicator, where $\Upsilon = 0$ if an agent is a fast trader and $\Upsilon = 1$ if the agent is a slow trader. Suppose that a trader arrives at the trading game and observes state s

[18] We include a cancellation cost in the model with the objective of evaluating recent regulations on some exchanges (e.g. NYSE Euronext), where there is a fee for cancellations with the objective of controlling AT activity (see Section 8).

[19] It is important to point out that the order priority could have changed, depending on the shape of the book, which should be taken into account in the decision to cancel and re-submit.

of the market. For convenience, let the entry time be equal to zero. The state that a given trader observes includes: i) her private value α; ii) her trading speed features Υ; iii) the contemporaneous limit order book L_t that is a consequence from the previous trading activity; iv) her current beliefs concerning the fundamental value of the asset, v_0^{Υ}, that depends on Υ; and v) the status of her previous action in the case that the trader has already previously submitted a limit order to the market, which includes the original submission price, the priority in the book given that price, and the type of order (a sell order or a buy order). Recall that in the case of a fast trader, v_0^{Υ} is equal to the contemporaneous fundamental value of the asset (i.e., $v_0^0 = v_0$). In the case of a slow trader, she can only know the fundamental value of the asset with a lag Δ_t, but she can also observe the trading activity up to the present, which allows her to capture and to learn from the information disclosed by fast traders and thus to improve her accuracy concerning v_0^1 in relation to v_0.

Let $\Gamma(s)$ be the possible set of actions that a trader can take given the state s (e.g., to wait until market conditions change, to buy or to sell the asset, or the submission prices, amongst others). Let $\eta(h|\tilde{a}, s)$ be the probability that an order is executed at time h given that the trader takes the action $\tilde{a} \in \Gamma(s)$ when she faces the state s. It is important to notice that $\eta(\cdot)$ incorporates all possible future states and strategic actions adopted by other traders until h. If the decision $\tilde{a}$ is the submission of a market order $\eta(0|\tilde{a}, s) = 1$, while $\eta(h|\tilde{a}, s)$ converges asymptotically to zero when the trader decides to submit a limit order with a price far away from the fundamental value. In addition, let $\gamma(v|\sigma, s)$ be the density function of v at time h that depends on the volatility σ of the fundamental value of the asset and the state s. The density function $\gamma(\cdot)$ depends on s because it takes into account the trader's current belief regarding the fundamental value of the asset, which is particularly important in the case of slow traders. Therefore, the expected value of an order that is executed prior to a re-entry at time h_r is:

$$\pi(h_r, \tilde{\alpha}, s) = \int_0^{h_r} \int_{-\infty}^{\infty} e^{-\rho_d h_r} \big((\alpha + v - \tilde{p})\tilde{x}\big)\, \eta(h_r|\tilde{\alpha}, s)\gamma(v|\sigma, s) dv dh_r. \tag{1}$$

Here, $(\alpha + v - \tilde{p})\tilde{x}$ is the instantaneous payoff of the order where $\tilde{p}$ is the submission price which is part of the decision $\tilde{a}$; while $\tilde{x}$ is also a component of the decision $\tilde{a}$ and reflects whether the trader decides to submit a buy order ($\tilde{x} = 1$), to submit a sell order ($\tilde{x} = -1$) or to submit nothing ($\tilde{x} = 0$). This payoff is transformed to a present value at the rate ρ_d which is the cost of 'delaying' previously defined in this section.

Let $R(\cdot)$ be the probability distribution of the re-entry time which is exogenous and follows an exponential distribution at rates λ_r^{FT} or λ_r^{ST} if the agent is a fast trader or a slow trader, respectively. In addition, let $\psi(s_{h_r}|\tilde{a}, s, h_r)$ be the probability that the state s_{h_r} takes place at time h_r given the previous state s and the action $\tilde{a}$, which also includes all potential states and strategic decisions followed by other traders until h_r. Therefore, the value to an agent of arriving at the state s, $V(s)$, is given by the Bellman equation of the trader's optimization problem:

$$\begin{aligned} V(s) = \max_{\tilde{a} \in \Gamma(s)} \int_0^{\infty} \Bigg[\pi(h_r, \tilde{a}, s) + e^{-\rho_d h_r} \int_{s_{h_r} \in S} \Big(V\big(s_{h_r}\big) - \tilde{z}_{s_{h_r}} c_{canc} \Big) \\ \cdot \psi\big(s_{h_r} \big| \tilde{a}, s, h_r\big) ds_{h_r} \Bigg] dR(h_r|s) \end{aligned} \tag{2}$$

where S is the set of possible states; the first term is defined in equation (1), while the second term reflects the subsequent payoff in the case of re-entries. In the case of re-entry, fast traders pay a cancellation fee c_{canc} when they cancel an unexecuted limit order; thus $\tilde{z}_{s_{h_r}} = 1$ if the optimal decision of an AT agent in the state s_{h_r} is a cancellation and $\tilde{z}_{s_{h_r}} = 0$ in any other case.

3.3 *Solving the asynchronous trading game*

We solve the model using a numerical method due to the analytical intractability of the trading game. Nevertheless, a solution using traditional numerical approaches is also difficult to obtain given the large dimension of the state space of the model. For that reason, following Goettler *et al*. (2005, 2009), we obtain a stationary Markov-perfect equilibrium using the algorithm introduced by Pakes and McGuire (2001), which resolves a large state space size problem by reaching the equilibrium only on the recurring states class. In this subsection, we will explain this algorithm which is used to reach the equilibrium of our model.

The model reflects a dynamic trading game in which traders asynchronously arrive and select optimal actions (i.e., trading decisions) that maximize their expected utility given the observed state. Therefore, optimal trading decisions are state-dependent. Moreover, trading decisions are Markovian, since the market condition reflected in the observed state is a consequence of the history of events and previous states that define the game.

The intuition behind Pakes and McGuire's (2001) algorithm is that we can initially see the trading game as a Bayesian learning process in which traders learn how to behave in each state. Thus, traders follow a learning-by-doing mechanism by playing in the game until we reach the equilibrium. In this learning-by-doing process, the trading game starts with each type of trader having initial beliefs about the expected payoffs of different actions and states. Afterwards, traders update their beliefs dynamically by playing in the game when they observe their realized payoffs from their actions. The equilibrium is reached when the expected payoff and the optimal trading decision of each trader type in a given state, s^*, are exactly the same expected payoff and decision if a similar trader observes s^* in the future (i.e., there is nothing to learn anymore). Therefore, we obtain a Markov-perfect Bayesian equilibrium which is also a symmetric equilibrium since it is time independent, because optimal trading decisions from each type of trader are the same when they face the same state in the present or in the future.[20]

Once we obtain the equilibrium after making traders play in the game for a couple of billion trading events, we fix the traders' beliefs and simulate a further 300 million events. These last 300 million simulated events allow us to evaluate the behaviour of the different agents without the effects of the learning process described in the previous paragraph. Our objective in this paper is to evaluate the effects of AT technology on market quality and integrity without capturing additional noise due to the algorithm updating process. Consequently, all the results and analysis presented in this paper are obtained from the last 300 million simulated events.

We also fix the speed condition for each type of trader to solve the equilibrium of the model. Therefore, the cost of being 'fast' (i.e., the cost of having AT technology) is given by the differences in payoffs for fast and slow traders. In the appendix, we explain in detail the algorithm, the convergence criteria used to obtain the model equilibrium, and the *learning capacity* of slow traders.

Simplifying the state space: In principle, the dimensionality of the state space is unbounded given that the fundamental value of the asset: i) is a continuous variable, and ii) may evolve to diverse levels. The unbounded limits of the fundamental value of the asset also affect the structure of the limit order book (i.e., we could have an infinite grid of prices). Nevertheless, following Goettler *et al*. (2009), we use some model properties to make the problem computationally tractable by simplifying the state space in the trading game. Firstly, although the fundamental value of the asset follows a random walk, agents do not take into account all

[20] We induce trembles in the traders' decisions to ensure that the updating process considers all possible actions in each state when we run the trading game to solve the equilibrium. Specifically, we disturb the traders' decisions with a small probability ξ to select actions that are suboptimal while the algorithm converges. We set ξ equal to 0.50%. In the case of a tremble, the trader selects among all suboptimal actions with equal probability. Once we reach the equilibrium of the model, we make ξ equal to zero to generate further 300 million events, which are the events that are reflected in the results of the different analyses in our study.

the numbers to the right of the decimal separator of v (i.e., agents observe 95.01 instead of 95.0100236). This discretization is in fact what limit order markets do in reality, since the limit order book is a discrete grid of prices with increments of size of one tick. For instance, the tick size of a stock in NYSE Euronext is equal to €0.01 when its price is between €50 and €99.99. Thus, we create a discrete vector of prices $\{\hat{v}^j\}_{j=0}^{\infty}$ which reflects the levels of the fundamental value that a trader observes, with $\hat{v}^0 = 0$ and $\hat{v}^{j+1} - \hat{v}^j = d$ where d is equal to the tick size of the limit order book in our model. Therefore, if the fundamental value of the asset at a given time is v, then the level that a trader observes is $\hat{v}^* \in \{\hat{v}^j\}_{j=0}^{\infty}$ given by $\hat{v}^* - d/2 \leq v < \hat{v}^* + d/2$.

Nevertheless, the vector $\{\hat{v}^j\}_{j=0}^{\infty}$ still has an infinite set of prices, which means that the limit order book's grid of prices has to be unbounded. For that reason, and secondly, we always put the centre of the book at the discrete level of the fundamental value of the asset that a trader observes. Consequently, given that the limit order book is also a discrete set of prices $\{p^i\}_{i=-\infty}^{\infty}$ with a tick size d, we put the centre of the book p^0 equal to $\hat{v}^*$ which was defined in the previous paragraph. For instance, imagine the scenario in which the discrete level of the fundamental value today is $\hat{v}^*$ but after a while v changes enough to be closer to $\hat{v}^{**}$ (i.e., $\hat{v}^{**} - d/2 \leq v < \hat{v}^{**} + d/2$). Suppose that $\hat{v}^{**} - \hat{v}^* = md$ where m is an integer that can be positive or negative. In this scenario, we move the book in m ticks to centre the book again at the discrete level of the current fundamental value $\hat{v}^{**}$. Hence, since we move the centre of book, we modify the existing limit orders in the book to take into account the new relative difference with respect to $\hat{v}^{**}$.[21] Thus, the prices of all orders are always relative to the current discrete level of the fundamental value of the asset, which makes traders think about 'relative' prices in relation to the new $\hat{v}^{**}$ rather than its effective 'level' value. Moreover, previous

[21] It is important to note that slow traders and fast traders observe the book differently given their informational differences. Fast traders observe the book centred on the discrete value of v_t. However, slow traders know $v_{t-\Delta t}$ but they can also observe the trading activity until t to improve the accuracy of their expectations regarding v_t. Thefreore, slow traders observe the book centred on the discrete level of their expected value of v_t which also corresponds to one of the prices of $\{\hat{v}^j\}_{j=0}^{\infty}$.

transaction prices are also expressed relative to the new value of $\hat{v}^{**}$. This allows us to enormously reduce the dimensionality of the state space.[22]

3.4 *Existence*

A state s is defined by the four-tuple: (α, Υ, $L_t(\hat{v}^*)$, *status of previous action*), where the agent's intrinsic private value to trade the asset, α, is drawn from a discrete distribution F_α; Υ is a trader indicator ($\Upsilon = 0$ or $\Upsilon = 1$ if there is a fast trader or a slow trader, respectively); the limit order book L_t is described by a discrete price-set which is centred on the discrete level of the fundamental value of the asset $\hat{v}^*$; and the *status of previous action* comes from a discrete set of possible actions given the market conditions.[23] Furthermore, each new action $\tilde{a}$ is taken from $\Gamma(s)$ which is the set of potential actions given the state s, where $\Gamma(s)$ is a discrete and finite set of decisions. Therefore, the action-state space is countable and finite, thus the model game has a Markov perfect equilibrium (see Reeder, 1979).[24] Additionally, although we do not prove uniqueness, we check whether the equilibrium is computationally unique. For each parameter setup, we use different initial values at the beginning of the algorithm with the objective of being sure that we obtain the same computational equilibrium.

3.5 *The model parameterization*

We assume the following plausible parameter values to be used in the model setup of our base case. In the random walk process of the fundamental value of the asset we use a volatility, σ,

[22] We impose an additional constraint on the limit order book. Given that the limit order book is centred on v, the number of ticks in the book around v is set high enough so that even very 'unaggressive' strategies can never go outside the grid of prices. We also use additional features of the model and we impose some specific restrictions, explained in detail in the appendix, with the objective of making the problem computationally tractable.

[23] The discrete level of the fundamental value of the asset $\hat{v}^*$ is not directly part of the variables that identify a state given that the limit order book L_t is centred on $\hat{v}^*$. Hence, all prices are expressed in relative terms with respect to $\hat{v}^*$.

[24] Similar arguments are used in Goettler *et al*. (2005) to show the existence of a similar dynamic model for limit order markets although without traders with trading speed advantages.

equal to 0.50 on an annual basis. The value of the volatility is based on the analysis of Zhang (2010), who presents a daily volatility for U.S. stock returns of 0.033, which is equivalent to an annual volatility of 0.524. In Section 8, we modify the value of σ to evaluate the impact of a high market volatility on the agents' behaviour and market quality measures; we observe that the results with a different volatility level are robust and comparable.

Similar to Goettler *et al*. (2009), we assume that the distribution of the private value is discrete with support $\{-8, -4, 0, 4, 8\}$ measured in ticks and with a cumulative distribution function $\{0.15, 0.35, 0.65, 0.85, 1.00\}$, which are the same for AT traders and slow traders. The distribution values are based on the findings of Hollifield *et al*. (2006) regarding the private values of stocks on the Vancouver Stock Exchange. In addition, given that the subgroup of algorithmic traders called 'pure' high frequency traders may not have private valuations (since they often carry small inventories), in a supplementary appendix we report the case where fast traders have only a private value equal to zero. The results presented in the supplementary appendix are also consistent with the findings reported in the body of our study.

We assume that traders arrive on average every 40 milliseconds and randomly 70% of them are fast traders. Although the current trading speed in some markets is at the level of microseconds rather than milliseconds; we select this arrival rate for computational tractability. This rate of arrivals for fast traders is also consistent with the timescale of some of the first papers regarding AT technology. For instance, Cont (2011) works at a millisecond scale to analyse the impact of fast traders on market quality. Moreover, we assume that 70% of the agents are fast traders, which is consistent with some empirical studies. For example, Brogaard (2010) presents evidence that 77% of the stock trading volume in the U.S. market can be associated with AT strategies, and the SEC reports that 73% of the stock trading volume in the U.S. stock comes from this technology.[25] Nevertheless, in the following sections, we modify the proportion of fast traders to analyse the impact of changes in this parameter on trading behaviour and market quality.

Limited cognition, due to the fact that traders are engaged in other tasks or because there is a noisy environment, may affect the modification speed of previous trading decisions. For instance, Trimmel and Poelzl (2006) show that in human beings, background noise lengthened reaction time by inhibiting parts of the cerebral cortex, which may increase cognitive limits when multiple stimuli are received. In the model we assume that slow traders can re-enter the market to modify limit orders on average every 600 milliseconds ($\lambda_r^{ST} = 1/0.60$). This is consistent with the literature on human behaviour in relation to reaction times. Reaction times of human beings are in the order of 200 milliseconds for a single stimulus to 700 milliseconds for six stimuli (see Kosinski, 2012). Fast traders have more tools and resources for evaluating and monitoring their orders in the book than slow traders. Thus, we assume that AT traders re-enter the market five times faster than slow traders (i.e., $\lambda_r^{ST} = 1/0.12$). This reflects the fact that even fast traders cannot monitor the market continuously, because there are processing times even with this technology, there is noise in the signals received, and it is costly. The assumption that fast traders are only five times faster than slow traders may be unrealistic given the current trading speed in markets. However, our 'theoretical' model aims to mainly reflect the 'relative' trading speed difference, and its potential impact on market quality and stability, rather than to reproduce the 'exactl' current speed difference that we can observe in the market, which is computationally intractable. Nevertheless, we change the 'relative' speed between slow and fast traders in the supplementary appendix; the results reflect the robustness of the model in relation to changes in the trading speed advantage of AT traders.

[25] See footnote 10.

Fast traders also have informational advantages since they can observe the contemporaneous level of the fundamental value of the asset, while slow traders can observe it with a lag. We assume that the time lag in which slow traders observe the fundamental value of the asset, Δ_t, is equal to 800 milliseconds. Similar to the other parameters, we examine the robustness of the model with a different value of Δ_t; these results are also presented in the supplementary appendix.

We assume that the 'delaying' cost, ρ_d, is reflected in a continuous discount rate equal to 0.03 for all agents.[26] Similar to Goettler *et al*. (2009), we experimented with different values for ρ_d (see the supplementary appendix) and obtained results qualitatively equivalent to the ones presented in the following sections.

We obtain the model equilibrium and reproduce a historical limit order book with a cancellation fee for fast traders equal to zero; thus the results between Section 4 and Section 7 are obtained assuming $c_{canc} = 0$. However, in Section 8 we include a cancellation cost in the model with the objective of evaluating recent regulations on some exchanges, in which a fee is imposed for cancellations to try control the trading activity of agents with AT technology. In Section 8, we set the cancellation cost for fast traders, c_{canc}, equal to 0.1 in ticks to observe the impact of such a measure on the market and the behaviour of the different agents.[27]

[26] Foucault *et al*. (2005) also use a similar 'delaying' cost which is called an 'impatience' rate in their study.

[27] The value assumed for the cancellation cost is consistent with the value imposed in NYSE Euronext, in which above an order-trade ratio of 100:1 a charge of €0.10 fee is applied to cancellations. Suppose that a trader in NYSE Euronext submits and cancels 100 consecutive orders and, immediately after that, she submits and cancels another one. The cancellation fee for the 101st cancellation can be shared with the previous uncharged cancellations to 'distribute' the cost, which makes a cancellation cost per order of €0.001 (i.e., €0.10/101= €0.001). The cancellation cost per order of €0.001 represents a lower bound since after the 101st cancellation; additional cancellation costs (from unexecuted limit orders) will be divided by the first 100 uncharged cancellations plus the new charged cancellations. Therefore, if we assume that the tick size is €0.01 our cancellation fee of 0.1 ticks (i.e., €0.001) is similar to what we can observe currently in the market.

4 The trading behaviour of market participants

A systematic investigation into the consequences of algorithmic trading for market functioning through a dynamic equilibrium model, which takes into account the most important features of this new trading technology, is highly relevant, not only from an academic point of view, but also from a policy perspective. For instance, in 2010, the SEC stated in relation to high frequency traders (a subset of AT traders) that "*By any measure, high frequency trading is a dominant component of the current market structure and is likely to affect nearly all aspects of its performance*". In this section, we start the analysis with the effects of AT on the trading behaviour of market participants.

How do slow traders and fast traders optimally act? Do fast traders prefer to provide or to consume liquidity? Do slow and fast traders submit more aggressive or more cautious limit orders? How many unexecuted limit orders are cancelled? The optimal trading behaviour is very complex in a limit order market due to the particular characteristics of each type of order. On the one hand, market orders do not have associated any 'waiting' cost because they are executed immediately. However, traders have to pay a cost for the 'immediacy' that a market order provides (e.g., a trade may wish to buy a share 'immediately' for hedging her portfolio, which represent an exogenous reason to trade). In the case that a trader submits a buy market order, she has to buy on the sell side of the book; and thus the trader has to pay (as 'immediacy' cost) the difference between the ask price and the fundamental value of the asset: – $(\tilde{p}^{ask} - v)$. Conversely, if a trader submits a sell market order, she has to sell on the buy side of the book and to pay the difference between the fundamental value of the asset and the bid price: $-(v - \tilde{p}^{bid})$.

On the other hand, limit orders have a 'waiting' cost since they are not immediately executed. This cost is characterized in our model in the 'delaying' cost; payoffs from trades are discounted back to the trader arrival time at rate ρ_d. Limit orders are the ones that are reflected (and are queuing) on the sell and buy side of the book; thus limit orders are 'waiting' to be executed when another trader submits a market order. The most competitive limit orders in the buy and sell sides give the bid and ask prices, respectively. Thus, trades always involve a market order and a limit order (at the bid or ask prices depending of the side of the book). Traders who submit a limit order provide liquidity to the market; and thus they receive the 'immediacy' cost paid by other traders with market orders. Consequently, liquidity providers post limit orders which will be executed later by liquidity demanders through their market orders.

Limit orders also have an associated risk of being 'picked-off' when the fundamental value of the asset moves against them. For example, when the fundamental value of the asset decreases, some limit orders on the buy side could be priced too high and a fast trader (or a trader with more knowledge of the fundamental value of the asset) could make profits from the difference. Thus, to avoid the 'picking-off risk', some investors may prefer to submit market orders (which have an 'immediacy' cost), or to submit less aggressive orders to get protection behind other orders that are queuing in the book (but paying larger 'waiting' costs).

> **Observation 1.** *Algorithmic trading induces changes in the trading behaviour of slow traders since the 'picking-off risk' of limit orders increases when there are fast traders in the market.*

We reach this first observation through the results presented in Table 1. Table 1 reports trading behaviour statistics per trader type. These trading behaviour statistics are the percentage of limit orders *executed*, the probabilities of being 'picked-off', the number of limit orders *submitted*, the number of limit order cancellations, the time between the instant in which a trader arrives and the execution of her limit order, and the probability of submitting a limit sell order at the ask price which is an aggressive limit order (we only report in Table 1, the probability of submitting a limit sell order at the ask price since the model is symmetric on both sides of the book). The probability of being 'picked-off' is calculated with executed limit orders: we take the number of limit sell (buy) orders that are executed when their execution price is below (above) the fundamental value of the asset, which is divided by all the limit orders executed in the market. Results in Table 1 are reported for five scenarios. The first scenario is when slow traders (STs) and fast traders (FTs) have exactly the same characteristics; both do not observe contemporaneously the fundamental value of the asset and both are 'slow' in re-entering and modifying unexecuted limit orders. The first scenario is equivalent to saying that there are *only* slow traders in the market. The second scenario envisages that FTs have only an informational advantage; they observe the contemporaneous level of fundamental value of the asset. The third scenario is when FTs have only a trading speed advantage; they can modify unexecuted limit orders quicker than STs. In the fourth scenario, FTs have both the informational and the trading speed advantages. The fifth scenario reflects the case where that STs and FTs have the informational and the trading advantages; thus the fifth scenario is equal to saying that there are *only* fast traders in the market. In Table 1, we assume that the market participation of AT traders is 70% when slow and fast traders coexist.

[Insert Table 1 here]

Table 1 shows that slow traders change their behaviour when there are fast traders in the market. STs execute more market orders when FTs have either an informational advantage or a trading speed advantage (see first panel in Table 1). The effect is stronger when FTs have both advantages, where the percentage of executed market orders by STs increases to 53.509% from 50.000% in the case where there are only slow traders in the market. It is important to note that standard errors for all market quality measures are sufficiently small since we use a large number of simulated events. The intuition behind this result is related to an increase in the ‘picking-off risk’ that slow traders face when they send limit orders in an environment with fast traders. Fast traders have an advantage in analysing information (they have more information about the level of the fundamental value which is essential to detect limit orders to be ‘picked-off’), and fast traders have an advantage in quickly modifying previous trading decisions (they can modify unexecuted limit orders to market orders with the objective of ‘picking-off’ limit orders from other traders). Hence, the ‘picking-off risk’ of limit orders is particularly high for 'traditional' slow agents when there are traders with AT technology in the market (see second panel in Table 1). For instance, the 'picking-off risk' for slow traders increases from 21.580% in the scenario when there are only slow traders in the market to 42.406% when there are fast traders with informational and trading speed advantages. Thus, slow traders prefer to trade more through market orders which do not have any 'picking-off risk' since they are executed immediately.

Fast traders *execute* more limit orders than market orders (see first panel in Table 1), and thus they prefer to be liquidity suppliers which induce a reduction in the bid-ask spread thanks to the market liquidity improvements (in Section 7 Table 8 we will present results regarding to improvements in market quality in some scenarios when there are AT traders in the market). Thus, and in relation to the market order preferences of slow traders explained in the previous paragraph, the reductions in the bid-ask spread generate additional incentives to slow traders for the submission of market orders since they pay on average lower ‘immediacy’ costs.

Slow traders *submit* more limit orders, but also cancel more, when there are fast traders in the market (second scenario to fourth scenario) than when slow traders are alone (first scenario). For example, Table 1 third and fourth panels show that the number of limit order submissions and cancellations by slow traders increases from 1.054 and 0.554 per trader in the first scenario to 1.379 and 0.914 per trader in the fourth scenario, respectively. This is explained because STs have the 'fear' of being 'picked-off'; therefore as soon as they have a small signal that the fundamental value changes unfavourably against them, they cancel immediately their unexecuted limit orders.

Fast traders also *submit* more limit orders and also have more cancellations than slow traders (see third and fourth panel in Table 1). For instance, in Table 1 the number of limit orders submitted and cancellations per fast trader are 1.911 and 1.396, while slow traders have as values 1.379 and 0.914, respectively, when fast traders have both the informational and trading advantage (fourth scenario). Nevertheless, the reason for the large number of limit order executions, submissions and cancellations is different for fast traders than the explanation for slow traders' preferences in relation to market orders. Since fast traders have informational advantages, they will know whether the fundamental value will move against them. In addition, they can react quickly to changes in v_t because they have a trading speed advantage to modify unexecuted limit orders, which reduces their probabilities of being 'picked-off' in relation to slow traders. Thus, limit orders are highly attractive to traders with AT technology. Our results are consistent with the findings presented in empirical studies, which show evidence that AT traders supply liquidity to the market (see, e.g., Hendershott *et al*., 2012; Hasbrouck and Saar, 2012; Riordan and Storkenmaier, 2012; and Malinova *et al*., 2013).

The slow traders' 'fear' of being 'picked-off' is also reflected in the increase in their time to execute limit orders when there are fast traders in the market. The time between the instant in which a trader arrives and the execution of her limit order for a slow trader increases from 0.705 sec. when there are only slow traders in the market to 1.059 sec. when there are fast traders with informational and trading speed advantages. The increase in the time to execution for slow traders is explained by two reasons. First, they cancel their unexecuted limit orders when they observe any potential signal that the fundamental value moves negatively against them; and second, slow traders submit less aggressive orders to also reduce the probability of being 'picked-off' by getting protection behind other orders in the book. For example, the probability of submitting a limit sell order at the ask price (see sixth panel in Table 1) for slow traders is reduced from 33.868% in the first scenario when there are only slow traders in the market to 14.846% in the fourth scenario when AT traders have informational and trading speed advantages.[28]

Table 2 confirms the evidence provided in Table 1 regarding the changes in the agents' trading behaviour. Table 2 reports the percentage of limit orders executed, probabilities of being 'picked-off'' and the time between the instant in which a trader arrives and the execution of her limit order. Table 2 show the results differentiated per trader and private values, in three scenarios: i) when slow and fast traders coexist in the market; ii) when there are only slow traders in the market; and iii) when there are only fast traders. In Table 2, fast traders have both informational and trading speed advantages, and the market participation of AT traders is 70% when slow and fast traders are present. We combine the results for positive and negative private values given that the model is symmetric on both sides of the book.

[Insert Table 2 here]

Table 2 shows that traders (STs and FTs) with a high absolute intrinsic value to trade (i.e., $|\alpha| = 8$) are willing to execute more market orders than limit orders. They prefer to quickly execute market orders since the main part of their payoffs is coming from exogenous reasons reflected in their high private value (i.e., α is an important part of the instantaneous payoff in equation (1)). Therefore, traders with a high absolute intrinsic value to trade are in general liquidity demanders. They do not want to pay a 'waiting' cost with limit orders by discounting their intrinsic α values at rate ρ_d, instead they are willing to pay the 'immediacy' cost implicit in market orders to capture their private values as soon as possible. For instance, on average

[28] Furthermore, in the supplementary appendix, we modify different model parameters to evaluate the impact of these changes on the agents' behaviour and market quality measures. The results presented in the supplementary appendix are robust and comparable to the findings presented in this section.

just 25.176%, 30.727% and 22.209% of all traders with $|\alpha| = 8$ execute limit orders when there are STs and FTs in the market, only STs in the market and only FTs in the market, respectively. Conversely, traders with no private values (i.e., $\alpha = 0$) are eager to trade through more limit orders than market orders. Traders with $\alpha = 0$ do not have an exogenous reason to trade immediately; they can wait to obtain benefits from the liquidity provision. Hence, traders with $\alpha = 0$ prefer to be liquidity suppliers, and by doing so, they can capture the 'immediacy' cost paid by traders with market orders. For example, 73.977%, 71.256% and 73.234% of the traders with $\alpha = 0$ execute limit orders when there are slow and fast traders in the market, only slow traders and only fast traders, respectively.

Table 2 shows that slow traders with a non-zero intrinsic value to trade (slow traders with $|\alpha| = 4$ and $|\alpha| = 8$) reduce the execution of limit orders when there are fast traders (42.636% and 14.054%, respectively) in relation to the scenario when there are only slow traders in the market (48.513% and 30.727%, respectively). Slow traders with a non-zero intrinsic value to trade know that their main profits are coming from their private values; thus they prefer to avoid the risk of being 'picked-off' coming from limit orders when there are AT agents. Table 2 also reports that slow traders with $\alpha = 0$ execute more limit orders (84.069%) than fast traders (69.647%) when there are slow and fast traders in the game. On the one side, fast traders with $\alpha = 0$ do not only obtain profits by providing liquidity through limit orders; they also submit market orders to 'pick-off' limit orders coming from slow traders (which reduces the execution of limit orders by fast traders). On the other side, slow traders with a zero private value can also make profits through market orders by 'picking-off' other traders; however, the probability is low when there are traders with informational and trading speed advantages. Instead, slow traders with $\alpha = 0$ prefer to execute more limit orders, but they submit unaggressive orders to get protection against being 'picked-off'; which can be observed in the rise in the time to execute their limit orders. For instance, the time between the instant in which a trader arrives and the execution of her limit order for a slow trader with $\alpha = 0$ is 1.842 sec. while for fast traders it is 1.084 sec. when there are both STs and FTs in the market.[29,30]

The changes in the behaviour of agents are not only affected by the informational and trading speed advantages of AT traders, but also by the proportion of market participation of 'less-skilled' agents and fast investors. Table 3 presents the same trading behaviour statistics as in Table 1, but here the percentage of market participation of fast traders in the market are 20%, 40%, 60% and 80% (in which fast traders have both informational and trading speed advantages).[31]

[Insert Table 3 here]

> **Observation 2.** (i) *AT traders prefer to act as liquidity suppliers when they represent the majority of the market participation.*
>
> (ii) *In the case that the market participation of 'less-skilled' investors is larger than the participation of fast traders, fast traders may induce more damage than benefits to the market. In this scenario, instead of using their advantages to provide liquidity, fast traders exhibit 'predatory' behaviour through market orders by 'picking-off' limit orders coming from the big crowd of slow traders.*

Table 3 presents evidence to confirm Observation 2. Table 3 first panel shows that fast traders execute more market orders while slow traders execute more limit orders when the market participation of 'less-skilled' investors is predominant (i.e., when the percentage of market

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29 In addition, in unreported results, we find that the probability of submitting a limit sell order at the ask price for fast traders with $\alpha = 0$ is 31.071% while that for slow traders with $\alpha = 0$ is 9.626% when fast traders have both trading advantages.

30 Fast traders with no intrinsic value to trade can be classified in a sub-group of algorithmic traders: 'pure' high frequency traders, since they do not have an exogenous reason to trade (they carry small inventories). Therefore, as a robustness check, in Table A1 in the supplementary appendix we get the equilibrium with a model in which all fast traders have $\alpha = 0$. We observe that the results reported in the supplementary appendix are robust and qualitatively similar to the results presented in this section.

31 It is important to notice that in our model AT traders compete with each other; however the equilibrium may be different if there is only one monopolist fast trader who submits several orders to the market.

participation of fast traders is lower than 50%), which is opposite to the observations reported in Table 1 where the market participation of fast traders is 70%. For instance, slow traders (fast traders) execute 50.652% (47.468%) of limit orders when fast traders constitute 20% of the market. The explanation for these results is straight-forward. Fast traders can make profits using either market orders to 'pick-off' limit orders mainly from less-skilled' traders, or using limit orders to capture the 'immediacy' cost paid by traders with market orders. When the percentage of market participation of fast traders is lower than 50%, there is a big mass of slow traders in the market who are exposed to the risk of being 'picked-off'. Therefore, fast traders prefer to be 'predators' by executing more market orders to 'pick-off' limit orders from slow investors; and thus fast agents reduce their preferences of supplying liquidity. Slow traders know that there a large number of 'other' slow traders with whom they can trade, which reduces their probability of being 'picked-off'. Therefore, when the percentage of market participation of 'less-skilled' investors is predominant, slow traders prefer to execute more limit orders to capture the 'immediacy' cost paid by other agents, although through unaggressive limit orders. Thus, this predatory behaviour of AT traders, in the case of a big mass of slow traders, may damages the market liquidity (as we will show in the following sections).

5 The effects of AT on the payoffs of traders and gains from trading

What are the economic benefits of fast traders? Do fast traders generate economic damage for traditional market participants? What are the gains from trading when some investors have AT technology? Are there economic benefits for the financial system coming from fast traders? On the one side, a significant amount of money has to be paid to acquire the AT technology because it requires the purchase of computational equipment, subscriptions to real time data providers, highly trained professionals to design AT algorithms, IT teams to develop connections and platforms, and co-location services to rent a space in the exchange for computers to submit faster orders than competitors.[32] Therefore, investment in this technology should be justified by an increase in the trading profits of AT traders.[33]

On the other side, there is some consensus among academics and practitioners regarding the potential economic detriment to slow traders due to the presence of AT traders. For instance, Biais *et al*. (2012a) show evidence that when some traders become fast, there is an increase of adverse selection costs for other agents, hence AT may generate negative externalities. In addition, in a debate carried out by *The Economist* on March 7th, 2012 Seth Merrin (founder and CEO, Liquidnet) stated in relation to algorithmic and high frequency trading that: "*High-frequency traders are, by design, trading ahead of market orders, to the detriment of long-term investors. HFT benefits very few, at the expense of very many, which defies the purpose of why a market exists and, as a result, has lessened the overall quality of the markets*".

In order to examine the motivation of some agents to invest in AT technology, and to analyse the effects of fast traders on the profits of 'traditional' slow traders, we calculate the average payoff per trade (i.e., the surplus or utility) for the different agents. Additionally, with the objective of understanding the different elements of the trading profits, we decomposed the payoffs of investors in order to analyse the gains and losses from the transaction process.

Suppose a trader with the private value α_{buy} arrives to the market, and waits a time t_{buy} until to submit a market buy order at price $\tilde{p}$. The realized payoff for this trader is given by $(\alpha_{buy} + v - \tilde{p})e^{-\rho_d t_{buy}}$ where v is the fundamental value of the asset and ρ_d is the 'delaying' discount rate described in Section 3. Since this trader submits a market order, this order

[32] The purpose of co-location services provided by exchanges is to decrease the time between order submission by AT computers and order reception in the exchange servers; and hence to further reduce trading latency.

[33] On July 4, 2007, an article in the International Herald Tribune, entitled "Citigroup to expand electronic trading capabilities by buying Automated Trading Desk" wrote: "*Goldman spends tens of millions of dollars on this stuff. They have more people working in their technology area than people on the trading desk…The nature of the markets has changed dramatically*".

represents a transaction. Therefore, this market order has to be executed with a limit sell order in the book. Suppose that the trader who submitted this limit sell order has a private value α_{sell} and was waiting a time t_{sell} until her limit orders is executed. The realized payoff for the second trader, who submitted the limit order which is executed with the market order from the first trader, is given by $(-\alpha_{sell} - v + \tilde{p})e^{-\rho_d t_{sell}}$. Thus, the total gains from trade (GFT) can be written as:

$$GFT = \left(\alpha_{buy} + v - \tilde{p}\right)e^{-\rho_d t_{buy}} + (-\alpha_{sell} - v + \tilde{p})e^{-\rho_d t_{sell}} \tag{3}$$

The first and second components on the right side of equation (3) reflect the buyer's gains and the seller's gains from trade, respectively. Similar examples can be described for a market sell order executed with a limit buy order. Hence, we can rewrite equation (3) as:

$$\begin{aligned} GFT &= \alpha_{buy} - \alpha_{sell} + \alpha_{buy}(e^{-\rho_d t_{buy}} - 1) - \alpha_{sell}(e^{-\rho_d t_{sell}} - 1) \\ &+ (v - \tilde{p})\, e^{-\rho_d t_{buy}} - (v - \tilde{p})e^{-\rho_d t_{sell}}, \end{aligned} \tag{4}$$

or

$$\begin{aligned} GFT &= Private\ value\ buyer + Private\ value\ seller \\ &+ Waiting\ cost\ buyer + Waiting\ cost\ seller \\ &+ Money\ Transfer\ buyer + Money\ transfer\ seller, \end{aligned} \tag{5}$$

where

$$Private\ value\ buyer = \alpha_{buy}$$

$$Private\ value\ seller = -\alpha_{sell}$$

$$Waiting\ cost\ buyer = \alpha_{buy}(e^{-\rho_d t_{buy}} - 1)$$

$$Waiting\ cost\ seller = -\alpha_{sell}(e^{-\rho_d t_{sell}} - 1)$$

$$Money\ Transfer\ buyer = (v - \tilde{p})\, e^{-\rho_d t_{buy}}$$

$$Money\ transfer\ seller = -(v - \tilde{p})e^{-\rho_d t_{sell}}$$

The first and second elements in equation (5) are the private value of the buyer and the seller in the transaction.[34] In general, traders cannot execute orders immediately in the market to obtain their intrinsic private values to trade (i.e., α_{buy} or $-\alpha_{sell}$). Instead, traders have to 'wait', which is costly given that there is a discount rate ρ_d. Traders have to wait either because there is a lack of liquidity, the market conditions are no ideal or due to their own decision when a trader submits a limit order.[35] Therefore, the second and third elements reflect the 'waiting' cost for the buyer and the seller: $\alpha_{buy}(e^{-\rho_d t_{buy}} - 1)$ and $-\alpha_{sell}(e^{-\rho_d t_{sell}} - 1)$, respectively. Additionally, when there is a trade, one trader gains some money and her counterparty losses the same amount, which is simply a 'money transfer'. However, the money gained and lost is discounted back differently depending on when each trader arrived at the market. Consequently, the fifth and sixth components in equation (5) represent the money transfer of the buyer, $(v - \tilde{p})\, e^{-\rho_d t_{buy}}$, and the money transfer of the seller, $-(v - \tilde{p})e^{-\rho_d t_{sell}}$, respectively.

Observation 3. (i) *Fast traders induce economic damage for slow traders.*

(ii) *AT reduces the waiting costs for slow traders, but fast traders require a payment for this service which is reflected in a more negative money transfer from traditional agents.*

Table 4 presents the average payoffs, waiting costs and money transfer per trade for each agent and in the same five scenarios as in Table 1. The market participation of AT traders in Table 4 is assumed to be 70%. Table 4 first panel shows that fast traders have higher payoffs than slow traders either when AT agents have only an informational advantage (second scenario), when fast traders have only a trading speed advantage (third scenario), or when fast traders have both informational and trading speed advantages (fourth scenario). For

[34] The expression presented in equation (5) is inspired by the analysis of gains from trade in a limit order market presented in Hollifield *et al*. (2006).

[35] In our model, traders can wait before submitting an order when they arrive for the first time at the game, and traders can cancel unexecuted limit orders. Therefore, the waiting costs also include the 'no submission' cost and the 'no execution' cost, as in Hollifield *et al*. (2006). The 'no submission' cost and the 'no execution' cost are reflected in additional waiting time (i.e., t_{buy} or t_{sell}) for traders to execute an order.

instance, fast traders make 3.826 ticks while slow traders make 3.662 ticks, in the case where fast traders have both informational and trading speed superiority. Moreover, Table 4 reports that there is a reduction in the payoffs for slow traders when there are fast traders (second scenario to the fourth scenario) in relation to the case in which there are only slow traders in the market (first scenario). For example, the average payoff for slow traders decreases from 3.764 ticks when slow traders are alone to 3.662 ticks when there are fast traders who have information and trading speed advantages in the market. Thus, we show evidence that fast traders generate adverse selection costs for other market participants.[36]

[Insert Table 4 here]

Nevertheless, it is also important to answer the following question: Is there any economic gain that fast traders may 'indirectly' generate with regard to the profits of slow traders? Table 4 second panel shows that fast traders induce some benefits to 'traditional' slow investors by reducing their waiting costs. For example, the waiting cost for slow traders is reduced in absolute value from -0.164 ticks when there are no fast traders in the market (first scenario) to -0.104 ticks when fast traders have informational and trading speed advantages (fourth scenario). In addition, in all cases in which slow and fast traders coexist (second scenario to fourth scenario) the waiting cost for fast traders is higher than the waiting cost for slow traders. The high waiting cost for fast traders also supports the findings presented in Table 1 and Table 2, in which we show that fast traders have an important role in liquidity provision. Fast traders are eager to submit and execute more limit orders than slow traders (when the market participation of fast traders is predominant), which have to wait in the book to be executed. Furthermore, it is important to note that there is a reduction in the absolute value of

[36] Despite the profits generated by fast traders who have access to resources to invest in AT technology, other agents may not be able to afford such a large initial investment (e.g. small investors or long-term traders who are not trading constantly and thus cannot justify such investment). However, agents who decide to be 'slow' can anticipate that fast traders may be present in the market, and hence they can trade strategically to take this into account as we reported in the previous section. This is consistent with the findings of Biais *et al*. (2012a), who show that in some scenarios slow and fast traders may co-exist.

waiting costs for the complete system when fast traders have informational and trading speed advantages (-0.158 ticks) in relation to the case when there are only slow traders in the market (-0.164 ticks). The reduction in the absolute value of the waiting costs of the game supports the liquidity improvements that can be observed in certain scenarios thanks to AT, which will be reported in the following sections.

Although the money gained by a given trader in a transaction is equal to the money lost by her counterparty, and hence the model is a zero-sum game, it is important to note that the total money transfer is not equal to zero in all scenarios in Table 4. This is explained by the optimal strategic behaviour of the different agents, and the discount rate ρ_d, which reduces the money transfer depending on the arrival time of each trader. A trade is always performed through the execution of a market order with a limit order in the book. Suppose that a trade consists of a limit buy order and a market sell order. In this case we can write the total money transfer of this trade as (see equation (5)): $(v - \tilde{p})\ e^{-\rho_d t_{buy,limit}} - (v - \tilde{p})e^{-\rho_d t_{sell,market}}$, where $t_{buy,limit}$ and $t_{sell,market}$ are the time periods since the buyer and the seller arrived at the market, respectively. However, the time to execute a market order is typically shorter than the time to execute limit orders given that limit orders have to be submitted before execution; hence $t_{buy,limit} > t_{sell,market}$.[37] Therefore, the money transfer paid by market orders for a quick execution (i.e., with an 'immediacy' cost which is a negative value) is higher in absolute terms than the money received by limit orders for providing liquidity (which is a positive value): $|(v - \tilde{p})\ e^{-\rho_d t_{buy,limit}}| < |(v - \tilde{p})e^{-\rho_d t_{sell,market}}|$. Consequently, given that all transactions in the game consist of exactly 50% of market orders and 50% of limit orders, and the discounted

[37] The time to execute market orders is usually lower than the time to execute limit orders; however we could have some cases in the model where $t_{buy,limit} < t_{sell,market}$. For instance, when a trader arrives at the market in our model, she can wait before submitting an order. Therefore, if a seller (with a market order) arrived a long time ago (waiting before submitting her market sell order), it may be the case that the buyer with the limit buy order arrived after the seller and thus: $t_{buy,limit} < t_{sell,market}$. In general this case is not common since traders who submit market order are the ones with high absolute private values to trade, as shown in Table 2 (i.e., with high α absolute values); these traders prefer to execute market orders quickly, since the main part of their payoffs comes from exogenous trading reasons.

value gained through limit orders is on average less than the money paid through market orders; then the 'total money transfer' is on average less than zero.

Table 4 shows that although slow traders experience a reduction in the absolute value of the waiting costs when there are fast traders in the market, in relation to the scenario when there are no fast traders present, slow traders also experience a more negative value in the money transfer (see third panel in Table 4). For instance, the money transfer for slow traders is −0.234 ticks (−0.072 ticks) in the fourth scenario when there are fast traders with both informational and trading speed advantages (in the first scenario when there are only slow traders in the market). Moreover, fast traders have a positive value in the money transfer, 0.007 ticks, in the case when they have more information and are quicker in modifying unexecuted limit orders. The differences in the money transfer between slow and fast traders provides direct evidence of the adverse selection that slow traders suffer when there are fast traders with informational advantages (0.189 ticks), when fast traders have trading speed advantages (0.117 ticks) and when fast traders have both advantages (0.241 ticks).

> **Observation 4.** (i) *Fast traders with informational advantages improve the global welfare of the system when they represent the majority of the market participation.*
> (ii) *Fast traders with only a trading speed advantage induce a welfare reduction because in this scenario fast traders have high waiting costs which are not compensated with a sufficient increase in their money transfer.*
>
> (ii) *Nevertheless, there is a positive synergy between the informational and trading speed advantages of fast traders when these are combined, system welfare increases even more than when fast traders have only informational superiority. This is explained by a more efficient use of the information which is incorporated into orders by fast traders when they have higher trading speed.*

Table 4 first panel reports that the existence of fast traders with informational advantages improves the total average agent's payoff per trade, which is evidence of an improvement in the global welfare of the system (we use as measure of welfare the 'total' average payoff per trade.). The second scenario in Table 4 shows that when fast traders have only an informational advantage, there is an increase in the total average payoff per trade (3.771 ticks) in relation to the first scenario, in which there are only slow traders in the market (3.764 ticks). However, the third scenario in Table 4 reports that fast traders with 'pure' trading speed advantage reduce the total average payoff (3.741 ticks). In fact, we will show in the following sections that when fast traders have only a trading speed advantage, they also induce a decline in some market quality measures in relation to informational efficiency and liquidity.[38] The reduction of the total average payoff in the third scenario in Table 4 is due to the fact that 'pure' speed, when there are no full information about the fundamental value of the asset (there is a lag Δ_t to observe v for all traders), induce a suboptimal equilibrium in terms of the total average payoff per trade, since traders cancel and resubmit more than they should.[39] Thus, fast traders have high waiting costs (-0.195 ticks) which are not compensated with an adequate increase in their money transfer (-0.052 ticks). For instance, fast traders have a total payoff (3.753 ticks) that is even lower than the level when there are only slow traders in the market (3.764 ticks).

Nevertheless, there is a positive synergy between the informational and trading speed advantage of fast traders. Table 4 first panel shows that when fast traders have both advantages, the total average payoff per trade increases even more (3.777 ticks) than when fast traders have only an informational superiority (3.771 ticks), which is explained by a more

[38] This is consistent with Hoffmann (2013), who shows that in a model in which there are fast traders with only trading speed advantages, the welfare is reduced.

[39] In our model, traders individually maximize their expected payoffs which is in fact what happens in the reality; however the equilibrium may be different if there is a social planer who try to maximize the best result for the complete system.

efficient use of the information incorporated in orders by fast traders when they are quicker in reacting and modifying their unexecuted limit orders.[40]

Table 5 corroborates the results presented in Table 4 in relation to the changes in the average payoffs for the different traders. Table 5 presents the average payoff, waiting costs and money transfer for diverse types of traders differentiated by private values, for three scenarios: i) when slow and fast traders coexist in the market; ii) when there are only slow traders in the market; and iii) when there are only fast traders. In this table, fast traders have both informational and trading speed advantages, and the market participation of AT traders is 70% when slow and fast traders are present. Table 5 shows that all slow traders have a lower average payoff when there are fast traders in the market (0.159 ticks, 3.520 ticks and 7.354 ticks for $\alpha = 0$, $|\alpha| = 4$ and $|\alpha| = 8$, respectively) than in the case when slow traders are alone (0.367 ticks, 3.570 ticks and 7.420 ticks for $\alpha = 0$, $|\alpha| = 4$ and $\alpha| = 8$, respectively). This is consistent with Table 4, where we present evidence that fast traders induce adverse selection costs for traditional investors.

[Insert Table 5 here]

For all private values in Table 5, fast traders always have higher payoffs than slow traders in the scenario where there are slow and fast traders in the market. However the difference in average payoff between fast traders and slow traders is largest for agents with $\alpha = 0$ (0.302 ticks). This big difference in the average payoff between slow and fast traders when $\alpha = 0$ shows that this type of trader is the most willing to pay for AT technology. Agents with $\alpha = 0$ do not have other exogenous private reasons to trade; they make profits only by the trading activity by itself. Therefore, traders with $\alpha = 0$ are eager to pay more for having additional trading advantages in relation to competitors; and thus to increase the chances to obtain

[40] Furthermore, in the supplementary appendix, we modify different model parameters to evaluate the impact of other model setups on average payoffs, waiting cost and money transfer. We can see that the outcomes of the analyses presented in the supplementary appendix are robust and congruent with the findings presented here.

positive profits in their transactions. In contrast, the difference in average payoff between slow and fast traders is lowest (0.097 ticks) for agents with a high private value (i.e., $|\alpha| = 8$). Traders with $|\alpha| = 8$ have exogenous reasons to trade in the market; in general they execute their orders through market orders (see Table 2) to capture their intrinsic private values as soon as possible. Consequently, traders with high private values are not interested in making profits from the trading activity *per se*, hence AT technology is less valuable for them.

Observation 5. *Agents without an intrinsic value to trade are the most willing to pay for AT technology, since they make profits only by the trading activity per se, and hence they are eager to pay more for having some trading advantages in relation to the competition.*

Endogenous acquisition equilibria can be observed from Table 5. For example, in the scenario when there are slow and fast traders in the market, if the cost per trade for acquiring AT technology is between zero and 0.097 ticks, there is an equilibrium in which all agents pay for this technology. Similarly, if the acquisition cost per trade for being fast is between 0.112 ticks and 0.302 ticks, in equilibrium only agents with $\alpha = 0$ would acquire the AT machinery.

The results presented in Table 5 are consistent with the trading behaviour of the different agents reported in Table 2. On the one hand, traders with high intrinsic values to trade (i.e., $|\alpha| = 8$) generally execute market orders to capture their private values quickly in the payoffs (i.e., their $|\alpha|$ values are the most important part in their profits). However, traders with $|\alpha| = 8$ pay the 'immediacy' cost of market orders which is reflected in a more negative money transfer. For instance, Table 5 first panel shows that agents with $|\alpha| = 8$ have a value of -0.429 ticks as a total average for the money transfer. On the other hand, traders with a zero private value, $\alpha = 0$, do not have exogenous reasons to trade; thus they can only make profits through the trading activity. Moreover, agents with $\alpha = 0$ have zero waiting costs (see equation (5) and Table 5); hence they are willing to submit limit orders and thus to receive the 'immediacy' cost paid by agents with market orders through a 'money transfer'. For example, Table 5 first panel reports that traders with $\alpha = 0$ have a total average money transfer of 0.371 ticks.[41]

Observation 6. *The market participation of 'less-skilled' agents and AT traders in the market has a non-linear effect on market welfare. The maximum system welfare is obtained when the percentage of fast trader participation is higher than 50% but lower than 100%. This is due to the changes in the trading behaviour of traders when the proportion of market participation of 'less-skilled' agents and fast traders changes in the game.*

Another key element in the increase in market welfare is the market participation of 'less-skilled' investors and AT traders, which affects the strategic trading behaviour of agents in different scenarios; this can be observed in Figure 1. We report in Figure 1 the average payoffs, waiting costs and money transfer per trade when the percentage of fast traders is 0%, 10%, 20% and so on until 100%, at intervals of 10%. The upper panel of Figure 1 shows that the total average payoff per trade reaches the maximum value at around 70% of market participation of fast traders, which is in fact consistent to the current U.S. stock AT trading volume reported in the empirical literature. Brogaard (2010) shows evidence that 77% of the trading volume on the U.S. stock market can be attributed to AT activity while the SEC reports a 73% of the trading volume on the U.S. stock market is coming from this technology.

[Insert Figure 1 here]

[41] As a robustness check, and similarly to the previous section, we get a model equilibrium in which all fast traders have $\alpha = 0$ (see Table A2 in the supplementary appendix). Table A2 shows that despite waiting cost for fast traders are zero since $\alpha = 0$ (see equation (5)), fast traders have a positive value in the money transfer. However, the money transfer of fast traders small due to the long time that they have to wait to execute limit orders (See Table A1). In addition, slow traders will have a worse average payoff than in the scenario without fast traders. Fast traders need to make profits with the trading activity since they have $\alpha = 0$. Therefore, fast traders exhibit an important predatory behaviour by 'picking-off' limit orders from slow traders, which induces a more negative value in the money transfer from traditional investors. In addition, slow traders will increase the execution time for their limit orders because they submit less aggressive orders; thus the absolute value of the waiting cost for slow traders also increases.

On the left side of Figure 1, when there is a big market participation of 'less-skilled' agents, fast agents prefer to make profits through market orders (which was reported in Section 4 Table 3). In this case, fast traders prefer to execute market orders since there is a big mass of slow traders with potential limit orders that can be 'picked-off'. This induces more negative waiting costs for the system, because fast traders wait longer before executing to find a mispriced limit order. For instance, the time between the instant in which an AT trader arrives and her first order submission is 0.734 sec. (0.285 sec.) when fast traders represent 20% (80%) of the market participation. In fact, when fast investors represent 20% of the agents, the time waiting before submitting a market order for AT traders is higher (0.813 sec.) than their time to execute a limit order (0.402 sec.). Thus, the main part of the waiting cost of fast traders is coming from the waiting process before submitting market orders. Moreover, there are additional liquidity damages since liquidity provision is mainly supplied by slow traders with unaggressive limit orders, which also generates negative waiting costs coming from the 'traditional' investors.

However, as the market participation of AT agents increases, limit orders start to be more attractive for fast traders (see also Section 4 Table 3) since there are not enough slow traders to 'pick-off' and there are more 'fast' competitors in the game. When the market participation of fast traders is higher than 50%, fast traders prefer to use their informational and trading speed advantages with limit orders, and hence to receive the 'immediacy' cost paid by other traders with market orders. This induces a liquidity improvement and a decrease in the total waiting cost, since fast traders wait less before submitting orders (their main business is not to 'detect' slow traders to 'pick-off' their orders). This waiting cost reduction when there are more fast traders is also helped by the fact that slow traders, who has less tools to provide competitive limit order, reduce their liquidity provision.

On the right side of Figure 1, where there is a high market participation of fast traders, the probability of being 'picked-off' is enormous, even for investors with AT technology, since many agents are informed and have trading speed advantages. Thus, fast traders submit several limit orders which are cancelled rapidly when they receive even a small signal that they can be 'picked-off' by other quick agents.[42] This reduces the potential positive benefits that liquidity providers can obtain since they have to stay in the market for longer to obtain some profits (they have to discount the potential positive money transfer to the rate ρ_d for a longer period), which increases the negativity of the total money transfer. Therefore, when there are many fast traders, the game reach a suboptimal equilibrium in terms of the total average payoff which is reduced.[43]

It is important to note that reductions in the total average payoff when there are only fast traders, which are due to the change in the trading behaviour of AT agents, do not imply a decline in market quality. In fact, we will show in the following sections that market microstructure noise is reduced and some marker liquidity measures improve in the scenario when AT traders are alone in the market.

6 Microstructure noise and errors in slow traders' beliefs when there are fast traders in the market

The microstructure features of financial markets may induce some friction that makes the transaction price, p_t, depart from the fundamental value of the asset, v_t. Thus, the transaction price can be decomposed into two components: the fundamental value of the asset v_t plus microstructure noise ξ_t, hence $p_t = v_t + \xi_t$.[44] In a frictionless world, microstructure noise should be zero; however ξ_t can be an important component of prices in real markets.

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[42] We can see in the last column of Table 3 (Section 4) that when fast traders constitute 80% of the market, there is a large number of cancellations by fast traders and they take longer to execute orders.

[43] In addition, fast traders with a high private value ($|\alpha| = 8$) submit fewer limit orders, and if they submit limit orders they reduce their execution time (see Panel 3 in Table 2 from Section 4). This induces a smaller waiting cost, although the reduction in the absolute level of the waiting cost does not compensate for the more negative value in the money transfer when there are many fast traders.

[44] See Hasbrouck (2002) for a discussion of microstructure noise.

One important point of friction in financial markets is the limited cognition of market participants. Currently, a large amount of information has to be analysed by agents in order to make optimal trading decisions. However, the analysis of all this information is not perfect due to cognition limits because, for instance, investors can be busy completing other tasks.

The existence of traders with AT technology may mitigate the cognitive limits of human beings, as computers can rapidly analyse and process a large amount of information. Fast traders have superior speed in processing news and signals, which can be quickly used in trading strategies by the low-latency transmission of orders. Thus, prices should rapidly reflect the improvements in the informational analysis. Consequently, fast traders may improve informational efficiency by reducing the difference between the fundamental value and the transaction price, and hence decreasing microstructure noise.

Observation 7. *Microstructure noise is reduced by the presence of participants with AT technology .*

In the first and second panels of Table 6, we present the levels of microstructure noise for the same five scenarios as in Table 1, in which the market participation of AT traders is 70% when slow and fast traders coexist. The first and second panels of Table 6 report the mean of the absolute value of the difference between v_t and p_t and its standard deviation, respectively. The market is observed every 10 minutes to obtain the values in this table. In Table 6, we can observe reductions in microstructure noise when there are fast traders in the market. For instance, the absolute value of the microstructure noise is reduced to 0.503 ticks when fast traders have informational and trading speed advantages (fourth scenario) from 1.328 ticks when there are only slow traders in the market (first scenario). Moreover, microstructure noise is lowest when there are only fast traders in the market. However, reductions in microstructure noise are not very important when fast traders have only a trading speed advantage (third scenario), where the absolute value of the microstructure noise is only reduced to 1.315 ticks.

[Insert Table 6 here]

Our results are supported by previous empirical studies, which present evidence that algorithmic traders can submit orders in the same direction as price movements due to the advantages that they have in analysing information (e.g., Kirilenko *et al*., 2011). Hendershott and Riordan (2010) and Brogaard *et al*. (2012) also show that fast traders play a beneficial role in price discovery, as fast investors trade in the direction of permanent price movements, and in the reverse direction of transitory pricing errors. In addition, Brogaard (2010) finds that fast traders add more to price discovery than traditional market participants.[45]

The reduction in microstructure noise due to the trading activity of fast traders also has an informational impact on slow traders' beliefs. Slow traders are eager to capture and to learn from any information disclosed in trades, with the objective of improving the accuracy of their estimations about the fundamental value. They do so to make better trading decisions and to decrease the adverse selection that they face when there are fast traders in the market.

> **Observation 8.** (i) *The learning process followed by slow traders reduces their belief errors regarding v_t in the presence of AT participants.*
>
> (ii) *AT technologies improve market informational efficiency when fast traders have advantages in analysing and processing information.*

The third and fourth panels in Table 6 report the errors in slow traders' beliefs regarding the current fundamental value of the asset. The errors in slow traders' beliefs are defined as $\mathrm{E}(v_t) - v_t$, where $\mathrm{E}(v_t)$ reflects their estimations of v_t. The third and fourth panels report the mean of the absolute value and the standard deviation of $\mathrm{E}(v_t) - v_t$, respectively. Recall that

[45] In addition, reductions in microstructure noise can be related to reductions in transaction price volatility, as reported in the empirical analysis of Hasbrouck and Saar (2012), because $p_t = v_t + \xi_t$ and hence microstructure noise is an important component of the realized volatility obtained from market traded prices.

in the scenario with only slow traders there is 'nothing to learn', since there are no agents with informational advantages. Table 6 shows that errors in beliefs decrease when there are fast traders with informational advantages in the market. Table 6 third panel reports that the absolute value of the errors in slow traders' beliefs is reduced from the case with only slow traders (1.189 ticks) to the case when fast investors have informational and trading speed advantages (0.398 ticks). An accurate estimation of the fundamental value is crucial for slow traders to determinate the expected payoff for potential trading strategies in the presence of fast traders. In this context, it is also important to note in Table 6 column 3 that when there are fast traders in the market with only a trading speed advantage, the belief errors of slow traders increase marginally. Therefore, Table 6 column 3 shows that when there is 'nothing to learn', a pure trading speed advantage for fast traders may negatively affect the perception of slow traders regarding the real level of the fundamental value of the asset.

In Table 7 we examine the impact of changes in the market participation of 'less-skilled' agents and AT traders on microstructure noise and the belief errors of slow traders. Table 7 shows that when there are more market participation of fast traders with informational and trading speed advantages in the market, both microstructure noise and the belief errors of slow traders decrease. The results presented in Table 7 confirm our findings that AT technology improves market informational efficiency. Microstructure noise is reduced by two effects: first, when there are more fast agents who trade at higher speed there is more 'information' regarding the fundamental value of the asset in transactions and quotes, which diminishes microstructure noise; and second, since there is more 'information' on market activity, slow traders can use that information to improve the accuracy of their estimations about the contemporaneous value of v_t, thus their trading decisions are based on more precise information, which also reduces microstructure noise. [46]

[46] As a robustness check, we present the same measure for microstructure noise and for the belief errors of slow traders in a model in which all fast traders have α = 0 (see Table A3 in the

[Insert Table 7 here]

7 The impact of fast traders on market liquidity

What is the impact of AT on market liquidity? In Section 4, we observed in Table 1 that fast traders are likely to execute and submit limit orders (when they represent the majority of the market participation) since AT traders use informational and trading speed advantages to provide liquidity. Therefore, the liquidity supply is directly affected by the trading activity of AT players.

Table 8 examines the effect of AT on the bid-ask spread, the effective spread, the number of limit orders at the ask price (total and effectively traded), the number of limit orders on the sell side of the book (total and effectively traded), and the time between the instant in which a trader arrives and her first order submission (for slow traders and fast traders). Table 8 presents liquidity measures for the same five scenarios as in Table 1, where the market participation of AT traders is 70% when slow and fast traders coexist. The effective spread is calculated as: $\tilde{y}(\tilde{p} - m)$, where $\tilde{p}$ is the transaction price, m is the midpoint between the bid and ask quotes, and $\tilde{y}$ is an indicator variable in which $\tilde{y} = 1$ or $\tilde{y} = -1$ if the transaction involves a market buy order or a market sell order, respectively. Differently to the bid-ask spread, in which the value reflects posted positions, the effective spread reflects the conditions of 'effective' transactions.

[Insert Table 8 here]

Observation 9. *Fast traders improve market liquidity when the market participation of fast traders is predominant. The liquidity improvement is reflected in reductions in the*

supplementary appendix). Moreover, in the supplementary appendix, we change different model parameters to evaluate the impact of these modifications on microstructure noise and the belief errors of slow traders. The results presented in all the robustness checks are consistent and comparable to the results presented in Table 6 and Table 7.

quoted and effective spread, an increase in market depth, and in a decline in the time between the instant in which a trader arrives and her first order submission.

Table 8 first and second panels show that the bid-ask spread and the effective spread are reduced to 1.453 ticks and 0.816 ticks when only fast traders have both informational and trading speed advantages (fourth scenario), in relation to the 1.615 ticks and 1.095 ticks when there are only slow traders in the market (first scenario), respectively. The reductions in the bid-ask spread and effective spread, reported in the fourth scenario in Table 8, are due to three main reasons. First, there are more chances to find a counterpart for all investors when there are fast traders in the market. Second, quotes are 'more informative' and also more competitive since fast traders prefer to submit more limit orders than slow traders. Third, the quotes that come from the limit order submissions of slow traders are also 'more informative' when there are fast traders in the market, because slow traders can capture and learn from the information revealed by the fast traders' activity.

This means that there is improvement in the process of 'matching' buyers and sellers. This 'matching' process improvement is also reflected in Table 8 (see the last two panels) in the reduction of time between the instant in which a trader arrives and her first order submission. For instance, the time waited before submitting the first order is 0.336 sec. for slow traders and 0.293 sec. for fast traders when fast traders have informational and trading speed advantages, while the time before submitting the first order is 0.668 for all traders in the scenario where there are only slow traders in the game. Our results are consistent with the findings presented in empirical studies, in which AT technologies are associated with improvements in market liquidity. For instance, Hendershott *et al*. (2012), Hasbrouck and Saar (2012) and Riordan and Storkenmaier (2012) report evidence that fast traders may reduce market spreads.

Nevertheless, fast traders with a 'pure' trading speed advantage enlarge quoted and effective spreads. Table 8 column 3 (see first and second panels) shows that the bid-ask spread and the effective spread increase to 1.834 ticks and 1.153 ticks when fast traders have only a trading speed advantage, from the scenario when there are only slow traders with 1.614 ticks and 1.095 ticks, respectively. The third scenario in Table 8 provides evidence that the potential reduction observable in the in bid-ask spread and effective spread are mainly driven by an improvement in informational efficiency (see, e.g., Copeland and Galai, 1983; Glosten and Milgrom, 1985; and Kyle, 1985).[47]

The fourth scenario in Table 8 (third panel) also shows that there is an increase in the number of limit orders at the ask price when there are fast traders with informational and trading speed advantages (2.163), in relation to the first scenario where there are only slow traders in the market (1.967). However, the number of limit orders at the ask price 'effectively traded' (see fourth panel in Table 8) decreases to 0.617 in the fourth scenario, from 0.934 in the first scenario. Similar pattern is observed for the 'effectively traded' and the total number of limit orders on the sell side of the book (see Table 8 fifth and sixth panels). Fast traders are willing to exploit their informational and trading speed advantages through limit orders (they prefer to be liquidity suppliers); they submit several limit although the level of cancellations is also high. Given their advantages over slow traders, fast traders quickly submit limit orders which increase the number of limit orders in the book. Nevertheless, these limit orders are rapidly modified depending on the evolution of market conditions; thus the number of orders effectively traded is smaller.

We also analyse the effects on different liquidity measures when we modify the market participation of 'less-skilled' agents and fast traders, which is presented in Table 9. Interestingly, the bid-ask spread is larger when the market participation of 'less-skilled'

[47] Furthermore, Table 8 also shows that quoted spread and effective spread are reduced when there are only fast traders in the market (fifth column).

agents is important (see Table 9 column 1 and column 2) than when slow traders are alone in the market (see Table 8 column 1). For instance, the bid-ask spread in Table 9 column 1 is 2.070 ticks while the bid-ask spread in Table 8 column 1 is 1.614 ticks. The results presented in Table 9 column 1 and column 2 are consistent with the changes in market participants' trading behaviour reported in Section 4 Table 3, when the percentage of market participation of fast traders is less than 50%. Table 3 showed that fast traders execute more market orders while slow traders execute more limit orders when there is a prevalent participation of 'less-skilled' investors (see Table 3 column 1 and column 2). This is explained by the fact that when the percentage of market participation of fast traders is lower than 50%, there is a large number of slow traders who can be potentially 'picked-off' by fast traders. In this scenario, instead of providing liquidity with the execution of limit orders, fast traders prefer to submit market orders with the objective of 'picking-off' the limit orders submitted by 'traditional' agents. Nevertheless, slow traders (who are the main liquidity suppliers in this case) submit unaggressive orders to reduce their 'picking-off risk', which represents a damage to market liquidity and is reflected in an increase in the bid-ask spread.

> **Observation 10.** *In the case in which the market participation of 'less-skilled' agents is larger than the participation of fast traders, AT damages market liquidity. In this environment, fast traders prefer to execute market orders following predominantly 'predatory' behaviour to 'pick-off' the limit orders of slow traders. This reduces the potential liquidity provision generated by AT.*

[Insert Table 9 here]

The change in the trading behaviour of different agents is also reflected in the depth of the book, when the market participation of 'less-skilled' traders in the market is large. The number of limit orders at the ask price and the number of limit orders on the sell side of the book are 1.491 and 4.769 (1.967 and 6.139) in Table 9 column 1 when there fast traders constitute 20% of the market participants (in Table 8 column 1 when there are only slow traders in the market). This reduction in the depth of the book is because, as we explained in the previous paragraph, several fast traders only prefer to wait instead of providing liquidity to the market; and thus to execute market orders with the objective of 'picking-off' the limit orders of 'less-skilled' agents.

8 Policy analysis

An important characteristic of our study is that we can evaluate the impact of possible market regulations and potential changes on market conditions, as the model is able to reproduce the evolution of a true order market book in a controlled environment. In this section, we consider two policy exercises. On the one side, we examine the impact on market performance of a latency restriction and a cancellation fee applied to fast traders. On the other side, we analyse market quality measures, trading behaviours and gains from trade for market participants in a scenario with an increase in market volatility, which may be the case of some particular economic periods or for some securities in the market.

8.1 The effect of a latency restriction and cancellation fee applied to fast traders

Policy makers have been analyzing the potential implementation of regulations to control the market activity of AT traders. The main objective of such regulations is to reduce the damage for traditional slow traders and to avoid unethical trading practices coming from fast traders such as 'quote stuffing'.[48] For instance, on September 22, 2010, the SEC Chairman, Mary

[48] For example, fast traders may submit a large number of limit orders that are cancelled in a very short period, which is called 'quote stuffing' (see, e.g., Egginton *et al.*, 2012). The main objective of 'quote stuffing' is to flood the book with a high number of fictitious and uninformative orders, and thus to obscure current market conditions for competitors. This affects slow traders in particular, since they

Schapiro, said that "...*high frequency trading firms have a tremendous capacity to affect the stability and integrity of the equity markets. Currently, however, high frequency trading firms are subject to very little in the way of obligations, either to protect that stability by promoting reasonable price continuity in tough times, or to refrain from exacerbating price volatility*". Two important regulations have been mooted: a latency restriction and a cancellation fee for fast traders. In fact, some markets have already implemented some of these regulations, especially in relation to cancellation fees for AT agents (e.g., NYSE Euronext and the Toronto Stock Exchange). However, some regulators and exchanges have argued that potential measures to control AT activity may induce more disadvantages than advantages. For example, in 2011 the London Stock Exchange stated, referring to cancellation fees, that: "*Any measure that increases transaction costs will reduce liquidity in markets, increasing the cost of capital for companies, and may also exacerbate volatility*".

Observation 11. (i) *A latency restriction and a cancellation fee for fast traders have negative impacts on market quality since both regulations represent additional market friction that affects market performance unfavourably.*

(ii) *A cancellation fee for fast traders reduces adverse selection by slow traders more directly than the latency restriction regulation. In addition, a cancellation fee generates a positive change in the behaviour of fast traders; this regulation may induce a preference for fast traders to behave as liquidity suppliers.*

Table 10 shows several market quality measures (used previously for Table 1 to Table 9 to evaluate the impact of AT technology on market performance) when a latency restriction (second column) and a cancellation fee (third column) are applied to fast traders. The latency restriction is modelled by changing the average re-entry speed for fast traders. In Table 10, the average re-entry time for fast traders increases from 120 milliseconds (first column) to

are not able to process a large amount of information quickly. In addition, 'quote stuffing' also induces congestion in the IT networks of exchanges, which may cause the systems to collapse.

300 milliseconds (second column). In relation to the cancellation fee regulation, we assumed a cancellation fee equal to zero for Table 1 to Table 9; however in Table 10 column 3 we impose a cancellation fee for fast traders equal to 0.1 ticks (i.e., $c_{canc} = 0.1$ in equation (2)).

[Insert Table 10 here]

Table 10 column 2 and column 3 show that a latency restriction and a cancellation fee have the same negative effect in a first group of market quality measures, the same positive impact in a second group of values for market performance, and divergent outcomes in a third group of market indicators. In the first group of market measures in Table 10, in which the latency restriction and a cancellation fee for fast traders have a similar negative effect, both regulations induce a reduction in global market welfare, increase the microstructure noise, and augment the quoted and effective spreads. For instance, the total average payoff is reduced from 3.777 ticks (first column) when there are no regulations, to 3.774 ticks (second column) and 3.701 ticks (third column) when the latency restriction and the cancellation fee are applied to fast traders, respectively. This negative impact on market quality is expected, since both policy instruments represent additional market friction that negatively affects market performance.

One of the objectives of a latency restriction and a cancellation fee for fast traders is to reduce the flood of uninformative limit orders that are cancelled in a short period (i.e., 'quote stuffing'), which obscures market conditions for the rest of the market participants. In fact, both regulations induce a reduction in the number of cancellations coming from fast traders. Nevertheless, Table 10 shows that when a latency restriction or a cancellation fee are applied, both regulations can also induce informational damage for traditional slow traders (see slow trader’s errors in beliefs: $\mathrm{E}(v_t) - v_t$) which should be taken into account for the design of these types of market policies.

The application of a latency restriction and a cancellation fee to AT agents also has a positive impact on some market measures. Both policy instruments diminish the adverse selection faced by slow traders: they increase the average payoff for 'traditional' agents, because these regulations induce a reduction in the probability of being 'picked-off' for limit orders coming from slow traders. For example, the average payoff for slow traders in Table 10 increases from 3.662 ticks (first column) when there are no regulations to 3.665 ticks (second column) and 3.667 ticks (third column) when fast traders are subject to the latency restriction and the cancellation fee, respectively.

Furthermore, a latency restriction and a cancellation fee for AT traders have different outcomes in a third group of market quality indicators. In relation to a latency restriction for agents with AT technology, when this regulation is applied, some of the positive effects of fast traders could disappear. For example, the liquidity provision behaviour of fast traders is affected negatively when the latency restriction is imposed. Table 10 reports that fast traders prefer to execute fewer limit orders when there is a latency restriction: 51.090% (51.505%) of fast traders execute limit orders when there is a (there is no) latency restriction. This reduction in liquidity provision by investors with AT generates a global system slowdown. Thus, a latency restriction induces an increase in the time to execution for limit orders coming from slow and fast traders, an augment in the waiting cost for slow traders, and it reduces market liquidity, which is reflected for example in a rise in the time between a trader's arrival and her first order submission.

A cancellation fee for fast traders affects the reduction in adverse selection by slow traders more directly than the latency restriction regulation. Moreover, a cancellation fee modifies the behaviour of fast traders in a way that is much healthier for the system. On the one hand, in the case that a cancellation fee is imposed on fast traders and when market conditions change against a limit order coming from a 'traditional' investor, a fast trader with an unexecuted limit order in the book has to decide between two options. The first option is to cancel her unexecuted limit order and pay a cancellation fee, and then to submit a new market order and thus to ‘pick-off’ the slow trader. The second option is to keep her original limit order, thus avoiding the cancellation fee, and to wait for the profits coming from her liquidity provision. Therefore, a cancellation fee for fast traders provides a 'threshold', at which a cancellation is only undertaken if the additional expected benefit of this action is higher than the cancellation fee level.

On the other hand, when a latency restriction regulation is in place, fast traders with limit orders are 'only' slower than before to cancel limit orders, and hence to re-submit market orders to 'pick-off' limit orders from slow traders. In the case of a latency restriction for fast traders, they will still 'pick-off' slow traders if they can, but more slowly, which is not the case for a cancellation fee. This effect is observed in Table 10 where fast traders execute more limit orders when they face a cancellation fee; 51.861% (51.505%) of fast traders execute limit orders when they have (do not have) to pay a cancellation fee. Consequently, a cancellation fee induces on fast traders to behave as liquidity suppliers, which improves some market liquidity measures. For example, the time between the instant in which a trader arrives and the execution of her limit order for a slow trader is reduced, the absolute value for the waiting cost for slow traders decreases, and the number of limit orders (and the number of limit orders effectively traded) at the ask prices also increases. This is consistent with the empirical evidence reported recently by Malinova *et al*. (2013), where AT agents trade more limit orders than the rest of the agents after the implementation of a cancelation fee in the Toronto Stock Exchange.

Nevertheless, deciding on the ‘right’ level of the cancellation fee for fast traders is crucial. A small cancellation fee may have no impact on the liquidity supply of fast traders. Conversely, a high cancellation fee could induce some traders not to purchase the AT technology, since there is a large implicit participation cost. Therefore, a high cancellation fee may cause a reduction in the number of fast traders, which could generate more damage than benefits to market quality (especially when the market participation of 'less-skilled' traders is predominant). The choice of the level of the cancellation fee for fast traders is also related to a third potential regulation that has not been analysed in this study: a transaction tax for agents with AT (which has already been applied in some countries such as France). When the level of the transaction tax is high enough, it is also equivalent to have a large participation cost for fast traders; which could induce a reduction in the number of AT participants that may be unfavourable for market performance.

9.2 The effect of an increase in market volatility

There has been intense debate whether AT can affect market stability given the large trading volume generated by fast traders. This debate was particularly fervent after a brief period of extreme intraday volatility in the U.S. financial market, on May 6th, 2010, which is commonly called the 'flash crash'. Initially, this phenomenon was attributed to a downward spiral of orders from quick algorithmic traders.[49] However, Kirilenko *et al*. (2011) provide evidence that a wrongly executed selling plan from a large fundamental trader triggered this event. Nevertheless, Kirilenko *et al*. (2011) state that "*During the Flash Crash, the trading behaviour of HFTs appears to have exacerbated the downward move in prices*".

> **Observation 12.** *Fast traders may have incentives to trade in assets that are more volatile or during economic period in which there is high volatility, since they can make larger profits when market volatility is high and they have informational and trading speed advantages.*

Table 11 reports the effect of an increase in market volatility on traders' behaviour and market quality. In the second column in Table 11, there is an increase in the volatility of the

[49] For instance, a perception survey conducted by Market Strategies International, between June 23, 2010 and June 29, 2010, shows that 46% of 380 retail advisors believed that AT was the biggest contributor to the 'flash crash', while 83% of the sample believed that AT was an important contributor.

fundamental value of the asset in which $\sigma = 0.60$, in relation to our base case reflected in the first column where $\sigma = 0.50$.

[Insert Table 11 here]

Table 11 reports that the average payoff for fast traders increases from 3.826 ticks when $\sigma = 0.50$ to 3.834 ticks when $\sigma = 0.60$. The increase in the average payoff for fast traders is due to large changes in the fundamental value of the asset when market volatility increases, which is valuable if fast traders have informational and trading speed advantages. In the case of high volatility, fast traders can make more profit by supplying liquidity given the non-linear features of limit orders. Limit orders have a similar structure to option contracts since both are ex-ante commitments to trade in the future at a specified price (see Copeland and Galai, 1983). As with option contracts, the average payoff of a limit order increases as the volatility grows if you can react quickly to market changes, which is the case for fast traders. In addition, a high volatility increases the price movements and the chances to 'pick-off' mispriced limit orders from 'less-skilled' agents. This phenomenon has an important implication, since fast traders may have incentives to trade in assets which are more volatile, or when there is an elevated volatility level in some particular economic periods. This finding is also consistent with Kirilenko *et al.*'s (2011) study in relation to the 'flash crash', in which they report abnormal trading behaviour by fast traders when this event happened.

Finally, in relation to other market quality measures, greater market volatility causes an increase in the probability of being 'picked-off' for all traders, an augment in the number of limit orders submitted and cancelled for both types of trader, a reduction in the time to execution of limit orders, a rise in microstructure noise, an increase in slow trader's errors in beliefs, a decrease in the probability of submitting competitive orders for either slow and fast traders, a rise in the quoted and effective spreads, and a decrease in the depth of the book.

9 Conclusions

Algorithmic trading represents one of the most important market technological transformations since NASDAQ was the world's first electronic stock market in 1971. Therefore, a systematic study of the effects of AT on market quality and stability is highly relevant, given that the potential benefits and dangers of this new technology have not yet been fully understood. In this study, we introduce a dynamic equilibrium model with AT which describes a stochastic sequential game in a limit order market. AT traders have an informational advantage and an effective superiority in terms of a low-latency transmission of orders. The dynamic model includes the main features of a real limit order market. Thus, we can generate the evolution of a complete limit order book which represents an additional contribution by our study. Consequently, we can analyse the effect of AT in multiple scenarios and evaluate regulation in a controlled environment.

We find that AT induces changes in the trading behaviour of 'traditional' traders. AT traders prefer to act as liquidity suppliers when they represent the majority of the market participation; thus AT improves liquidity. In the case where the market participation of 'less-skilled' traders is predominant, fast traders behave as liquidity demanders; instead of using their advantages for liquidity provision, fast traders exhibit 'predatory' behaviour through market orders by 'picking-off' limit orders coming from the big crowd of slow traders. This 'predatory' behaviour by fast traders, when there is a big mass of 'less-skilled' invesors, damages market liquidity.

We show that AT reduces waiting costs but finally damages slow traders' profits. Fast traders with *only* informational (trading speed) advantages, increase (reduce) global welfare. Nevertheless, there is a positive synergy between the informational and trading speed advantages of fast traders; when they have both advantages, market welfare increases even more than when fast traders have only informational superiority. The maximum system welfare is obtained when the market participation of fast traders constitute around 70% of all market agents, which is congruent with the current U.S. stock trading volume reported in the empirical literature. Moreover, we show that in general, AT reduces microstructure noise since it mitigates the cognitive limits of human beings.

We also perform some policy exercises using the dynamic features of our model. We show that a latency restriction and a cancellation fee for fast traders have harmful impacts on market quality, given that both regulations represent additional market friction that negatively affects market performance. However, a cancellation fee may be a better policy instrument to control AT activity than a latency restriction, since a cancellation fee may induce a preference for fast traders to behave as liquidity suppliers. Moreover, we find that fast traders may have incentives to trade in assets that are more volatile or during an economic period in which there is a high volatility. This is due to the fact that AT traders can make larger profits when market volatility is high, which explains the abnormal trading behaviour of agents with AT technology observed in the 'flash crash'.

Finally, the model presented in our paper is intuitive since we can reproduce over time the behaviour of a complete limit order market as in reality. However other interesting issues remain to be addressed. For instance, the study of the effect of AT on the correlation of different assets, the study of the impact of fast traders on multiple markets, and the analysis when dark pools are present have been left for future research.

Appendix

The updating process to reach the equilibrium: For any state s of the economy, there is a set of possible actions, $\Gamma(s)$, that a trader can follow. Suppose that a given trader arrives for the first time or re-enters the market at time t and observes the state s. In our model setup, the trader has beliefs about the expected payoff of each possible action that could be taken given the observed state s. Suppose that $U_t(\tilde{a}|s)$ is the expected payoff at time t that is associated with the action $\tilde{a} \in \Gamma(s)$. Suppose that the trader decides at time t to take the optimal action $\tilde{a}^*$ that provides the maximum expected payoff out of all possible actions. As a first case, suppose that the optimal action $\tilde{a}^*$ is not a market order (e.g., a limit order, or a cancellation and resubmission). Later on at time t_r the same trader re-enters the market, but the market conditions have changed. The trader observes a new state s_{t_r} in which she follows the optimal strategy $\tilde{a}^{**}$ that also gives a maximum payoff given the new market conditions. Consequently, the original decision $\tilde{a}^*$ induces a realized continuation of optimal actions and expected payoffs; and thus the updating process of beliefs can be written as:

$$U_{t_r}(\tilde{a}^*|s) = \frac{n_{\tilde{a}^*,s}}{n_{\tilde{a}^*,s}+1} U_t(\tilde{a}^*|s) + \frac{1}{n_{\tilde{a}^*,s}+1} e^{-\rho_d(t_r-t)} \left(U_{t_r}\left(\tilde{a}^{**}|s_{t_r}\right) - \tilde{z}_{\tilde{a}^{**}} c_{canc} \right), \qquad \text{(A1)}$$

where $n_{\tilde{a}^*,s}$ is a counter that increases by one when the action $\tilde{a}^*$ is taken in the state s.[50]

Alternatively, as a second case, suppose that the optimal decision $\tilde{a}^*$ is a market order (i.e., there is no future time t_r as in the previous case). Then, the updating process of the expected payoff of the optimal action $\tilde{a}^*$ in this scenario can be expressed as:

$$U_t(\tilde{a}^*|s) = \frac{n_{\tilde{a}^*,s}}{n_{\tilde{a}^*,s}+1} U_t(\tilde{a}^*|s) + \frac{1}{n_{\tilde{a}^*,s}+1} (\alpha + v_t - \tilde{p})\tilde{x} \qquad \text{(A2)}$$

Here $\tilde{p}$ is the submission price, α is the private value of the trader, v_t is the fundamental value of the asset, and $\tilde{x}$ is equal to one (minus one) when the trader submits a buy (sell) order.[51]

As a third case, suppose that the optimal decision $\tilde{a}^*$ is a limit order; however, later on at time t_r this limit order is executed due to the fact that another trader submits a market order. The

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[50] The value of $n_{\tilde{a}^*,s}$ affects how quickly we reach the model equilibrium (a large value in $n_{\tilde{a}^*,s}$ is associated with a slow convergence). Therefore, we reset $n_{\tilde{a}^*,s}$ after a while to improve the convergence speed.

[51] Equation (A2) is also valid for slow traders who do not know the contemporaneous fundamental value of the asset, since they will eventually know it after a lag Δ_t and thus they can also update their beliefs regarding the expected payoffs of their actions.

updating process for the first trader with the optimal action $\tilde{a}^*$ can be reflected in the following equation:

$$U_{t_r}(\tilde{a}^*|s) = \frac{n_{\tilde{a}^*,s}}{n_{\tilde{a}^*,s}+1} U_t(\tilde{a}^*|s) + \frac{1}{n_{\tilde{a}^*,s}+1} e^{-\rho_d(t_r-t)}\left(\alpha + v_{t_r} - \tilde{p}\right)\tilde{x}, \tag{A3}$$

where α is the private value for the first trader. Similarly, for the second trader who submits the market order that executes the limit order of the first trader, the updating process can be expressed as:

$$U_{t_r}\left(\tilde{a}\middle|s_{t_r}\right) = \frac{n_{\tilde{a},s_{t_r}}}{n_{\tilde{a},s_{t_r}}+1} U_{t_r}\left(\tilde{a}\middle|s_{t_r}\right) + \frac{1}{n_{\tilde{a},s_{t_r}}+1}(\alpha' + v_t - \tilde{p})\tilde{x}, \tag{A4}$$

in which α' and $\tilde{a}$ are the private value and the optimal decision of the second trader, respectively. In this case, $\tilde{a}$ is a market order which is chosen at time t_r by the second trader when the state s_{t_r} is found. It is important to observe in this last case that any market order implies the execution of a previously submitted limit order. Thus, in the presence of market orders the updating process in beliefs always involves two traders: the trader who submits the market order, and the trader who submitted the limit order which is executed by the market order (see equation (A3) and equation (A4)).[52]

The learning capacity of slow traders: As mentioned previously, even though slow traders can observe the fundamental value of the asset with a time lag Δ_t, the model takes into account that slow traders can improve the accuracy of their beliefs regarding v_t by observing the market trading activity. Suppose that a slow trader arrives at the market at time t. The slow trader observes the fundamental value $v^*_{t-\Delta t}$ (which is the discretized level of $v_{t-\Delta t}$, as

[52] The initial beliefs about the expected payoffs $U_0(\tilde{a}|s)$ of the possible actions $\tilde{a} \in \Gamma(s)$ that a trader can take given that she faces state s are set as follows. Suppose one of the possible actions for a trader with private value α in the state s is to submit a limit sell order at price p when the fundamental value is v. We set the initial expected payoff of this action as $p - v - \alpha$ discounted by ρ_d until the expected time that a new fast trader arrives at the market. This value is only a first approximation since we assume that v is constant, which is not true in the model, and there is a chance that the next trader may submit another limit order instead of a market order that executes the limit order of the previous trader. In the case of a market sell order the expected payoff is simply $p - v - \alpha$ without any discount. Similar values are obtained for buy orders.

explained in the Subsection 3.3), the current limit order book L_t that is the result of the previous trading activity, the price of the most recent transaction $\hat{p}_t$, and the direction of that transaction $\hat{b}_t$ which is one (minus one) if the last transaction was the product of a buy (sell) order.[53] Let $\mathrm{E}\left[v_t \middle| v^*_{t-\Delta t}, L_t, \hat{p}_t, \hat{b}_t\right] = v_{t-\Delta t} + \phi(v^*_{t-\Delta t}, L_t, \hat{p}_t, \hat{b}_t)$ be the expected value of the fundamental value of the asset given $v^*_{t-\Delta t}$, L_t, $\hat{p}_t$, and $\hat{b}_t$, in which $\phi(\cdot)$ is the adjustments that a slow trader has to apply to $v_{t-\Delta t}$ in order to improve the accuracy of her beliefs about v_t. Let $j(v^*_{t-\Delta t}, L_t, \hat{p}_t, \hat{b}_t)$ be the number of times in which the combination of values for $v^*_{t-\Delta t}$, L_t, $\hat{p}_t$, and $\hat{b}_t$ are observed in the trading game. We suppress the dependent variables in j for notational convenience. Therefore, we can update $\phi(\cdot)$ as follows:

$$\phi_{j+1}\left(v^*_{t-\Delta t}, L_t, \hat{p}_t, \hat{b}_t\right) = \frac{j}{j+1}\phi_j\left(v^*_{t-\Delta t}, L_t, \hat{p}_t, \hat{b}_t\right) + \frac{1}{j+1}(v_t - v_{t-\Delta t}), \tag{A5}$$

where $\phi_0(\cdot) = 0$. Thus, for a slow trader the expected fundamental value is $\mathrm{E}[v_t | \cdot\,] = v_{t-\Delta t} + \phi(\cdot)$, while for a fast trader $\mathrm{E}[v_t | \cdot\,] = v_t$.

Convergence criteria: We check for convergence after running the trading game for a couple of billion trading events. Afterwards, we check for convergence the evolution of agents' beliefs every 300 million simulations. Suppose that the first group of 300 million simulations after we start checking for convergence finishes at time t_1. Suppose that the subsequent second group of 300 million simulations finishes at time t_2. Let $U_{t_1}(\tilde{a}|s)$ and $U_{t_2}(\tilde{a}|s)$ be the expected payoffs that are associated with the action $\tilde{a}$ when the state s is present at time t_1 and t_2, respectively. In addition, suppose that $k^{t_1,t_1}_{\tilde{a},s}$ is the number of times that the action $\tilde{a}$ was taken between t_1 and t_2 when traders face s. Similar to Goettler *et al.* (2009), we evaluate the change in the expected value $\left|U_{t_2}(\tilde{a}|s) - U_{t_1}(\tilde{a}|s)\right|$ of all pairs $(\tilde{a}, s)$ weighted by $k^{t_1,t_1}_{\tilde{a},s}$ every 300 million simulations. Once this weighted absolute difference is smaller than 0.01 (which

[53] Previous transaction prices are implicit in the information of the current limit order book L_t which is the outcome of previous trades.

suggests that the model has converged), we apply other two convergence criteria in line with Pakes and McGuire (2001).

After reaching a small weighted absolute difference in the change in the expected values as described in the previous paragraph, we fix the agents' beliefs concerning the expected payoffs, $U^{*}(\cdot)$, and simulate the trading game for another 300 million events. Then, we calculate the realized payoffs of all order submissions after they have been executed. Let $\tilde{J}(\cdot)$ be the realized payoffs. It is important to observe that $\tilde{J}(\cdot)$ is a direct measure of the benefits to trade which is not "averaged" as in equation (A1) to equation (A4). First, we require that the correlation between $U^{*}(\cdot)$ and $\tilde{J}(\cdot)$ is higher than 0.99. Second, we calculate the mean absolute difference between $U^{*}(\cdot)$ and $\tilde{J}(\cdot)$ weighted by the number of times that a specific action is selected in a given state in the last 300 million simulated events, which also has to be below 0.01 (i.e., in a similar way to the previous paragraph when we evaluate the change in the expected value between $U_{t_2}(\tilde{a}|s)$ and $U_{t_2}(\tilde{a}|s)$ weighted by $k_{\tilde{a},s}^{t_1,t_1}$). If any convergence criterion is not reached we continue simulating the trading game and updating the beliefs through equation (A1) to equation (A4) until all convergence criteria are satisfied.

Simplifications for computational tractability. In spite of the reductions in the dimensionality of the state space of the model achieved by the preceding simplifications, the state space is still very large and computationally intractable. This is due to the many possible shapes and features that the order book can present (which, if they are combined with the different types of traders, private values, and beliefs about the fundamental value of the asset, makes the dimensionality of the state space enormous). Ideally, we would like to include all the futures of the market book but this would make the problem unmanageable computationally. Consequently, similar to Goettler *et al*. (2009), we characterize the book with the following combination of variables: i) the current bid and ask prices; ii) the depth at the bid price and the depth at the ask price; iii) the total depth (including all the orders) on the buy side and the total depth on the sell side; and iv) the features of the previous transaction (price and whether it was due to a sell or a buy market order).

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Table 1
Trading behaviour of market participants

The table contains trading behaviour statistics for slow traders (STs) and fast traders (FTs). The table reports the percentage of limit orders *executed* per trader, the probability of being 'picked-off', the number of limit orders *submitted*, the number of limit order cancellations, the time between the instant in which a trader arrives and the execution of her limit order and the probability of submitting a limit sell order at the ask price (i.e., an aggressive limit sell order). The probability of being 'picked-off' is calculated with executed limit orders: we take the number of limit sell (buy) orders that are executed when their execution price is below (above) the fundamental value of the asset, which is divided by all the limit orders executed in the market. The time between the instant in which a trader arrives and the execution of her limit order is expressed in seconds. The model is symmetric on both sides of the book therefore it is not necessary to also report the probability of submitting a limit buy order at the bid price. Results are reported for five scenarios. In the first scenario STs and FTs have exactly the same characteristics; both do not observe contemporaneously the fundamental value of the asset and both are 'slow' in re-entering and modifying unexecuted limit orders (which is equivalent to saying that there are only slow traders in the market). The second scenario envisages that FTs have only an informational advantage (they observe the contemporaneous level of the fundamental value of the asset). The third scenario envisages that FTs have only a trading speed advantage (they can modify unexecuted limit orders quicker than STs). In the fourth scenario, FTs have both advantages (FTs have the informational as well as the trading speed advantages). The fifth scenario envisages that both STs and FTs have the informational and the trading advantage (the fifth scenario is equivalent to saying that there are only fast traders in the market). In this table, the market participation of AT traders is 70% when slow and fast traders coexist. Standard errors for all market quality measures are sufficiently small since we use a large number of simulated events. The Markov equilibrium is obtained independently for each case.

	FT: No advantages	FT: Only inform. advan.	FT: Only trad. speed.	FT: Both advantages	FT and ST: Both advan.
	ST: $\Delta_t = 0.8$; $\lambda_r = 1/0.6$ FT: $\Delta_t = 0.8$; $\lambda_r = 1/0.6$ (Scenario: Only ST in the market)	ST: $\Delta_t = 0.8$; $\lambda_r = 1/0.6$ FT: $\mathbf{\Delta_t = 0.0}$; $\lambda_r = 1/0.6$ (Scenario: ST and FT in the market)	ST: $\Delta_t = 0.8$; $\lambda_r = 1/0.6$ FT: $\Delta_t = 0.8$; $\mathbf{\lambda_r = 1/0.12}$ (Scenario: ST and FT in the market)	ST: $\Delta_t = 0.8$; $\lambda_r = 1/0.6$ FT: $\mathbf{\Delta_t = 0.0}$; $\mathbf{\lambda_r = 1/0.12}$ (Scenario: ST and FT in the market)	ST: $\mathbf{\Delta_t = 0.0}$; $\mathbf{\lambda_r = 1/0.12}$ FT: $\mathbf{\Delta_t = 0.0}$; $\mathbf{\lambda_r = 1/0.12}$ (Scenario: Only FT in the market)
			Percentage of limit orders 'executed' per trader		
ST	50.000%	48.336%	49.299%	46.491%	50.000%
FT	50.000%	50.714%	50.301%	51.505%	50.000%
Total	50.000%	50.000%	50.000%	50.000%	50.000%
			Prob. of being picked-off after submitting a limit order		
ST	21.580%	37.934%	23.035%	42.406%	36.097%
FT	21.580%	26.856%	18.403%	27.159%	36.097%
Total	21.580%	30.071%	19.773%	31.416%	36.097%
			Number of limit orders 'submitted' per trader		
ST	1.054	1.309	1.425	1.379	4.819
FT	1.054	1.041	2.270	1.911	4.819
Total	1.054	1.122	2.017	1.752	4.819
			Number of limit order cancellations per trader		
ST	0.554	0.825	0.932	0.914	4.319
FT	0.554	0.534	1.767	1.396	4.319
Total	0.554	0.622	1.517	1.252	4.319
			Time between the instant in which a trader arrives and the execution of her limit order		
ST	0.705	1.165	0.937	1.059	1.623
FT	0.705	0.635	0.603	0.539	1.623
Total	0.705	0.789	0.702	0.684	1.623
			Prob. of submitting a limit sell order at the ask price (an aggressive order)		
ST	33.868%	13.306%	28.878%	14.846%	21.159%
FT	33.868%	32.076%	33.371%	31.910%	21.159%
Total	33.868%	25.506%	32.421%	27.893%	21.159%

Table 2
Trading behaviour of traders differentiated by private values

The table reports trading behaviour statistics of investors differentiated by private values. The table presents results for slow traders (STs) and fast traders (FTs). The table reports the percentage of limit orders executed, the probability of being 'picked-off'' and the time between the instant in which a trader arrives and the execution of her limit order per trader. The results reflect three scenarios: i) when slow and fast traders coexist in the market; ii) when there are only slow traders in the market; and iii) when there are only fast traders. The probability of being 'picked-off' is calculated with executed limit orders: we take the number of limit sell (buy) orders that are executed when their execution price is below (above) the fundamental value of the asset, which is divided by all the limit orders executed in the market. The time between the instant in which a trader arrives and the execution of her limit order is expressed in seconds. In this table, fast traders have both informational and trading speed advantages, and the market participation of AT traders is 70% when slow and fast traders are present. We combine the results for positive and negative private values given that the model is symmetric on both sides of the book. Standard errors for all market quality measures are sufficiently small since we use a large number of simulated events. The Markov equilibrium is obtained independently for each case.

	% Limit orders 'executed' per trader type				Prob. of being picked-off after submitting a limit order				Time between the instant in which a trader arrives and the execution of her limit order			
	Private value |α|				Private value |α|				Private value |α|			
	0	4	8	Total	0	4	8	Total	0	4	8	Total
	Slow traders and fast traders in the market											
ST	84.069%	42.636%	14.054%	46.491%	64.054%	18.132%	11.098%	42.406%	1.842	0.143	0.077	1.059
FT	69.647%	54.068%	29.948%	51.505%	38.561%	22.342%	12.276%	27.159%	1.084	0.198	0.091	0.539
Total	73.977%	50.636%	25.176%	50.000%	47.262%	21.278%	12.078%	31.416%	1.343	0.184	0.088	0.684
	Only slow traders in the market											
ST	71.256%	48.513%	30.727%	50.000%	32.829%	14.834%	9.762%	21.580%	1.434	0.190	0.101	0.705
	Only fast traders in the market											
FT	73.234%	53.419%	22.209%	50.000%	54.530%	24.467%	12.486%	36.097%	3.502	0.169	0.068	1.623

Table 3
Agents' trading behaviour for different levels of fast trader market participation
The table presents the same trading behaviour statistics reported in Table 1 when the percentage of fast traders in the market are 20%, 40%, 60% and 80%. The table shows results for slow traders (STs) and fast traders (FTs). In this table, fast traders have both informational and trading speed advantages. The Markov equilibrium is obtained independently for each case.

	% of Traders in the ST: 80% and FT: 20%	% of Traders in the ST: 60% and FT: 40%	% of Traders in the ST: 40% and FT: 60%	% of Traders in the ST: 20% and FT: 80%
	Percentage of limit orders 'executed' per trader			
ST	50.652%	50.403%	47.720%	44.036%
FT	47.468%	48.955%	51.522%	51.492%
Total	50.000%	50.000%	50.000%	50.000%
	Prob. of being picked-off after submitting a limit order			
ST	22.797%	31.037%	38.324%	45.580%
FT	19.516%	21.520%	24.872%	29.465%
Total	22.176%	27.348%	30.012%	32.304%
	Number of limit orders 'submitted' per trader			
ST	1.082	1.119	1.245	1.630
FT	1.274	1.326	1.640	2.645
Total	1.120	1.158	1.482	2.442
	Number of limit order cancellations per trader			
ST	0.574	0.615	0.768	1.189
FT	0.799	0.836	1.124	2.130
Total	0.620	0.658	0.982	1.942
	Time between the instant in which a trader arrives and the execution of her limit order			
ST	0.598	0.688	0.878	1.350
FT	0.402	0.379	0.452	0.702
Total	0.561	0.568	0.615	0.816
	Prob. of submitting a limit sell order at the ask price (an aggressive order)			
ST	21.029%	22.754%	18.793%	13.319%
FT	23.006%	27.646%	31.246%	29.204%
Total	21.478%	24.814%	27.077%	27.089%

Table 4
Average payoffs, waiting costs and money transfer

The table reports the average payoffs, waiting costs and money transfer per trade for slow traders (STs) and fast traders (FTs). The results in this table are presented using the same five scenarios as in Table 1. Waiting costs and money transfer are described in equation (5). In relation to payoffs, suppose a trader with private value α submits an order (a limit or a market order) to buy at price $\tilde{p}$, which is not modified until the execution at time t. The realized payoff for this trader is given by $e^{-\rho_d t}(\alpha + v - \tilde{p})$. In the case that the trader submits a limit buy order and after a while re-enters and cancels the order, then the payoff is given by the same logic as described in equation (1). Similar examples can be described for the realized payoff of sell orders, but in an opposite direction. In this table, the market participation of AT traders is 70% when slow and fast traders coexist. The Markov equilibrium is obtained independently for each case. Standard errors for payoffs are less than 0.0003 for fast traders while standard errors for slow traders are less than 0.009 for all scenarios. All measures are expressed in ticks.

	FT: No advantages	FT: Only inform. advan.	FT: Only trad. speed.	FT: Both advantages	FT and ST: Both advan.
	ST: Δ_t = 0.8; λ_r = 1/0.6 FT: Δ_t = 0.8; λ_r = 1/0.6 (Scenario: Only ST in the market)	ST: Δ_t = 0.8; λ_r = 1/0.6 FT: $\boldsymbol{\Delta_t}$ **= 0.0**; λ_r = 1/0.6 (Scenario: ST and FT in the market)	ST: Δ_t = 0.8; λ_r = 1/0.6 FT: Δ_t = 0.8; $\boldsymbol{\lambda_r}$ **= 1/0.12** (Scenario: ST and FT in the market)	ST: Δ_t = 0.8; λ_r = 1/0.6 FT: $\boldsymbol{\Delta_t}$ **= 0.0**; $\boldsymbol{\lambda_r}$ **= 1/0.12** (Scenario: ST and FT in the market)	ST: $\boldsymbol{\Delta_t}$ **= 0.0;** $\boldsymbol{\lambda_r}$ **= 1/0.12** FT: $\boldsymbol{\Delta_t}$ **= 0.0;** $\boldsymbol{\lambda_r}$ **= 1/0.12** (Scenario: Only FT in the market)
			Average payoff per trader		
ST	3.764	3.668	3.711	3.662	3.738
FT	3.764	3.815	3.753	3.826	3.738
Total	3.764	3.771	3.741	3.777	3.738
FT - ST	0.000	0.146	0.042	0.164	0.000
			Waiting cost per trader		
ST	-0.164	-0.135	-0.120	-0.104	-0.153
FT	-0.164	-0.177	-0.195	-0.181	-0.153
Total	-0.164	-0.164	-0.172	-0.158	-0.153
FT - ST	0.000	-0.043	-0.075	-0.076	0.000
			Money transfer per trader		
ST	-0.072	-0.197	-0.169	-0.234	-0.109
FT	-0.072	-0.008	-0.052	0.007	-0.109
Total	-0.072	-0.065	-0.087	-0.065	-0.109
FT - ST	0.000	0.189	0.117	0.241	0.000

Table 5
Market participant gains from trading differentiated by private values

The table contains the average payoffs, waiting costs and money transfer per trade for slow traders (STs) and fast traders (FTs) differentiated by private values. Payoffs, waiting costs and money transfer were defined in Table 4. The results reflect three scenarios: i) when slow and fast traders coexist in the market; ii) when there are only slow traders in the market; and iii) when there are only fast traders. In this table, fast traders have both informational and trading speed advantages, and the market participation of AT traders is 70% when slow and fast traders are present. We combine the results for positive and negative private values given that the model is symmetric on both sides of the book. The Markov equilibrium is obtained independently for each case. Standard errors for payoffs are less than 0.0003 for fast traders while standard errors for slow traders are less than 0.0009 for all scenarios. All measures are expressed in ticks.

	Average payoff per trader				Waiting cost per trader				Money transfer per trader			
	Private value |α|				Private value |α|				Private value |α|			
	0	4	8	Total	0	4	8	Total	0	4	8	Total
					Slow traders and fast traders in the market							
ST	0.159	3.520	7.354	3.662	0.000	-0.199	-0.082	-0.104	0.159	-0.280	-0.564	-0.234
FT	0.462	3.632	7.450	3.826	0.000	-0.318	-0.178	-0.181	0.462	-0.050	-0.371	0.007
Total	0.371	3.598	7.421	3.777	0.000	-0.282	-0.150	-0.158	0.371	-0.119	-0.429	-0.065
FT - ST	0.302	0.112	0.097	0.164	0.000	-0.119	-0.096	-0.076	0.302	0.231	0.193	0.241
					Only slow traders in the market							
ST	0.367	3.570	7.420	3.764	0.000	-0.267	-0.191	-0.164	0.367	-0.163	-0.389	-0.072
					Only fast traders in the market							
FT	0.180	3.628	7.442	3.738	0.000	-0.295	-0.119	-0.153	0.180	-0.077	-0.439	-0.109

Table 6

Microstructure noise and the slow trader's errors in beliefs

The table reports statistics on microstructure noise and the slow trader's errors in beliefs. The results in this table are presented using the same five scenarios as in Table 1. Microstructure noise is $p_t - v_t$, where p_t and v_t are the transaction price and the fundamental value of the asset, respectively. The errors in beliefs of slow traders, $\mathrm{E}(v_t) - v_t$, concern the fundamental value of the asset which is observed by slow traders with a lag Δ_t. Nevertheless, slow traders can learn from the information revealed in trading activity by fast traders, and hence can improve the accuracy of their beliefs $\mathrm{E}(v_t)$. In this table, the market participation of AT traders is 70% when slow and fast traders coexist. The market is observed every 10 minutes. Standard errors for all market quality measures are sufficiently small since we use a large number of simulated events. The Markov equilibrium is obtained independently for each case. All measures are expressed in ticks.

FT: No advantages	FT: Only inform. advan.	FT: Only trad. speed.	FT: Both advantages	FT and ST: Both advan.
ST: Δ_t = 0.8; λ_r = 1/0.6 FT: Δ_t = 0.8; λ_r = 1/0.6 (Scenario: Only ST in the market)	ST: Δ_t = 0.8; λ_r = 1/0.6 FT: **Δ_t = 0.0**; λ_r = 1/0.6 (Scenario: ST and FT in the market)	ST: Δ_t = 0.8; λ_r = 1/0.6 FT: Δ_t = 0.8; **λ_r = 1/0.12** (Scenario: ST and FT in the market)	ST: Δ_t = 0.8; λ_r = 1/0.6 FT: **Δ_t = 0.0; λ_r = 1/0.12** (Scenario: ST and FT in the market)	ST: **Δ_t = 0.0; λ_r = 1/0.12** FT: **Δ_t = 0.0; λ_r = 1/0.12** (Scenario: Only FT in the market)
Microstructure noise: Mean $\lvert v_t - p_t \rvert$				
1.328	0.571	1.315	0.503	0.409
Microstructure noise: Std. Dev. $(v_t - p_t)$				
1.736	0.825	1.720	0.754	0.643
Belief errors of slow traders regarding the fundamental value: Mean $\lvert \mathrm{E}(v_t) - v_t \rvert$				
1.189	0.484	1.191	0.398	0.000
Belief errors of slow traders regarding the fundamental value: Std. Dev. $(\mathrm{E}(v_t) - v_t)$				
1.578	0.678	1.582	0.588	0.000

Table 7

Microstructure noise and the slow trader's errors in beliefs for different levels of fast trader market participation

The table presents statistics on microstructure noise $(p_t - v_t)$ and the slow trader's errors in beliefs $(\mathrm{E}(v_t) - v_t)$. The table reports scenarios when the percentage of fast traders in the market are 20%, 40%, 60% and 80%. In this table, fast traders have both informational and trading speed advantages. The market is observed every 10 minutes. Standard errors for all market quality measures are sufficiently small since we use a large number of simulated events. The Markov equilibrium is obtained independently for each case. All measures are expressed in ticks.

% of Traders in the Market ST: 80% and FT: 20%	% of Traders in the Market ST: 60% and FT: 40%	% of Traders in the Market ST: 40% and FT: 60%	% of Traders in the Market ST: 20% and FT: 80%
Microstructure noise: Mean $\lvert v_t - p_t \rvert$			
1.028	0.768	0.573	0.465
Microstructure noise: Std. Dev. $(v_t - p_t)$			
1.398	1.091	0.841	0.705
Belief errors of slow traders regarding the fundamental value: Mean $\lvert \mathrm{E}(v_t) - v_t \rvert$			
1.010	0.743	0.514	0.326
Belief errors of slow traders regarding the fundamental value: Std. Dev. $(\mathrm{E}(v_t) - v_t)$			
1.316	1.006	0.720	0.497

Table 8
Market liquidity
The table contains statistics of diverse market liquidity measures. The results in this table are presented using the same five scenarios as in Table 1. The table reports the bid-ask spread, the effective spread, the number of limit orders at the ask price (total and effectively traded), the number of limit orders on the sell side of the book (total and effectively traded), and the time between the instant in which a trader arrives and her first order submission (for slow traders and fast traders). The bid-ask spread and the effective spread are expressed in ticks. The time between the instant in which a trader arrives and her first order submission is expressed in seconds. The effective spread is calculated as: $\tilde{y}(\tilde{p} - m)$, where $\tilde{p}$ is the transaction price, m is the midpoint between the bid and ask quotes, and $\tilde{y}$ is an indicator variable in which $\tilde{y} = 1$ or $\tilde{y} = -1$ if the transaction involves a market buy order or a market sell order, respectively. Differently to the bid-ask spread, in which the value reflects posted positions, the effective spread reflects the conditions of 'effective' transactions. In addition, the 'effectively traded' limit orders in the order book are the limit orders submitted and traded without any modification. In this table, the market participation of AT traders is 70% when slow and fast traders coexist. The bid-ask spread, the number of limit orders at the ask price (total and effectively traded), and the number of limit orders on the sell side of the book (total and effectively traded) are obtained by observing the market every 10 minutes. Standard errors for all market quality measures are sufficiently small since we use a large number of simulated events. The Markov equilibrium is obtained independently for each case.

FT: No advantages	FT: Only inform. advan.	FT: Only trad. speed.	FT: Both advantages	FT and ST: Both advan.
ST: Δ_t = 0.8; λ_r = 1/0.6 FT: Δ_t = 0.8; λ_r = 1/0.6 (Scenario: Only ST in the market)	ST: Δ_t = 0.8; λ_r = 1/0.6 FT: **Δ_t = 0.0**; λ_r = 1/0.6 (Scenario: ST and FT in the market)	ST: Δ_t = 0.8; λ_r = 1/0.6 FT: Δ_t = 0.8; **λ_r = 1/0.12** (Scenario: ST and FT in the market)	ST: Δ_t = 0.8; λ_r = 1/0.6 FT: **Δ_t = 0.0**; **λ_r = 1/0.12** (Scenario: ST and FT in the market)	ST: **Δ_t = 0.0; λ_r = 1/0.12** FT: **Δ_t = 0.0; λ_r = 1/0.12** (Scenario: Only FT in the market)
		Bid-ask spread		
1.614	1.501	1.834	1.453	1.402
		Effective spread		
1.095	0.850	1.153	0.816	0.630
		N. of limit orders at the ask		
1.967	1.666	2.417	2.163	2.775
		N. of limit orders at the ask (effectively traded)		
0.934	0.743	0.599	0.617	0.288
		N. of limit orders on the sell side of the book		
6.139	6.532	6.284	6.335	10.297
		N. of limit orders on the sell side of the book (effectively traded)		
2.914	2.912	1.558	1.728	1.068
		Time between the instant in which a ST arrives and her first order submission		
0.668	0.660	0.328	0.336	0.335
		Time between the instant in which a FT arrives and her first order submission		
0.668	0.681	0.307	0.293	0.335

Table 9
Market liquidity for different levels of fast trader market participation

The table presents the same statistics for the market liquidity measures reported in Table 8 when there are different levels of fast trader market participation . The table reports scenarios when the percentage of fast traders in the market is 20%, 40%, 60% and 80%. In this table, fast traders have both informational and trading speed advantages. Standard errors for all market quality measures are sufficiently small since we use a large number of simulated events. The Markov equilibrium is obtained independently for each case.

% of Traders in the Market ST: 80% and FT: 20%	% of Traders in the Market ST: 60% and FT: 40%	% of Traders in the Market ST: 40% and FT: 60%	% of Traders in the Market ST: 20% and FT: 80%
Bid-ask spread			
2.070	1.657	1.533	1.417
Effective Spread			
1.054	0.918	0.870	0.788
N. of limit orders at the ask			
1.491	1.645	1.889	2.348
N. of limit orders at the ask (effectively traded)			
1.054	0.918	0.870	0.588
N. of limit orders on the sell side of the book			
4.769	4.584	5.259	6.832
N. of limit orders on the sell side of the book (effectively traded)			
2.129	1.980	1.775	1.399
Time between the instant in which a ST arrives and her first order submission			
0.734	0.510	0.403	0.285
Time between the instant in which a FT arrives and her first order submission			
0.525	0.426	0.363	0.278

Table 10

The effects on market performance of a latency restriction and a cancellation fee

The table reports several market quality measures (used previously for Table 1 and Table 9) to evaluate the impact of a latency restriction (second columns) and a cancellation fee (third column), both applied to fast traders, on market performance. The latency restriction is modelled by changing the average re-entry speed for fast traders. In this table, the average re-entry time for fast traders increases from 120 milliseconds (first column) to 300 milliseconds (second column). In relation to the cancellation fee regulation, we previously assumed a cancellation fee equal to zero for Table 1 to Table 9; however in this table we impose a cancellation fee on fast traders equal to 0.1 ticks (third column). In this table, fast traders have both informational and trading speed advantages, and the market participation of AT traders is 70%. Standard errors for all market quality measures are sufficiently small since we use a large number of simulated events. The Markov equilibrium is obtained independently for each case.

	The effects of a latency restriction and a cancellation fee		
	FT: No regulations	FT: latency restriction	FT: cancellation fee
	ST: Canc. fee = 0 ; λ_r = 1/0.6 FT: Canc. fee = 0 ; λr = 1/0.12 (Scenario: ST and FT in the market)	ST: Canc. fee = 0 ; λ_r = 1/0.6 FT: Canc. fee = 0 ; **λ_r = 1/0.3** (Scenario: ST and FT in the market)	ST: Canc. fee = 0 ; λ_r = 1/0.6 FT: **Canc. fee = 0.1** ; λ_r = 1/0.12 (Scenario: ST and FT in the market)
% of limit orders executed per ST	46.491%	47.456%	45.661%
% of limit orders executed per FT	51.505%	51.090%	51.861%
Prob. of being picked-off (limit orders) ST	42.406%	40.147%	39.638%
Prob. of being picked-off (limit orders) FT	27.159%	27.040%	27.061%
# limit orders submitted per ST	1.379	1.345	0.944
# limit orders submitted per FT	1.911	1.474	1.701
# limit order cancellations per ST	0.914	0.870	0.488
# limit order cancellations per FT	1.396	0.963	1.183
Time until to execute a limit order per ST	1.059	1.111	0.690
Time until to execute a limit order per FT	0.539	0.587	0.513
Prob. of subm. a limit sell order at the ask per ST	14.846%	15.076%	29.792%
Prob. of subm. a limit sell order at the ask per FT	31.910%	32.034%	31.806%
Average payoff per ST	3.662	3.665	3.667
Average payoff per FT	3.826	3.820	3.715
Average payoff per total	3.777	3.774	3.701
Waiting cost per ST	-0.104	-0.119	-0.096
Waiting cost per FT	-0.181	-0.179	-0.184
Waiting cost per total	-0.158	-0.161	-0.158
Money transfer per ST	-0.234	-0.215	-0.237
Money transfer per FT	0.007	-0.001	0.017
Money transfer per total	-0.065	-0.065	-0.059
Microstructure noise: Mean $\lvert v_t - p_t \rvert$	0.503	0.538	0.507
Belief errors of ST: Mean $\lvert E(v_t) - v_t \rvert$	0.398	0.440	0.407
Bid-ask spread	1.453	1.478	1.464
Effective spread	0.816	0.883	0.981
N. of limit orders at the ask	2.163	1.915	2.167
N. of limit orders at the ask (effectively traded)	0.617	0.680	0.680
N. of limit orders on the sell side	6.335	6.309	4.869
N. of limit orders on the sell side (effectively traded	1.728	2.323	1.528
Time until the first order submission per ST	0.336	0.509	0.653
Time until the first order submission per FT	0.293	0.476	0.289

Table 11

The impact of an increase in the volatility of the fundamental value of the asset on market quality

The table contains several market quality measures (used previously for Table 1 to Table 9) to evaluate the effect of an increase in the volatility of the fundamental value of the asset on market quality. In the second column there is an increase in the volatility of the fundamental value of the asset ($\sigma = 0.60$) in relation to our base case reflected in the first column ($\sigma = 0.50$). In this table, fast traders have both informational and trading speed advantages, and the market participation of AT traders is 70%. Standard errors for all market quality measures are sufficiently small since we use a large number of simulated events. The Markov equilibrium is obtained independently for each case.

	Different volatility levels of the fundamental value (σ)	
	σ = 0.50 (Case: ST and FT in the market)	**σ = 0.60** (Case: ST and FT in the market)
% of limit orders executed per ST	46.491%	45.330%
% of limit orders executed per FT	51.505%	52.001%
Prob. of being picked-off (limit orders) ST	42.406%	43.441%
Prob. of being picked-off (limit orders) FT	27.159%	30.070%
# limit orders submitted per ST	1.379	1.406
# limit orders submitted per FT	1.911	2.133
# limit order cancellations per ST	0.914	0.953
# limit order cancellations per FT	1.396	1.613
Time until to execute a limit order per ST	1.059	0.986
Time until to execute a limit order per FT	0.539	0.470
Prob. of subm. a limit sell order at the ask per ST	14.846%	11.757%
Prob. of subm. a limit sell order at the ask per FT	31.910%	19.085%
Average payoff per ST	3.662	3.606
Average payoff per FT	3.826	3.834
Average payoff per total	3.777	3.766
Waiting cost per ST	-0.104	-0.101
Waiting cost per FT	-0.181	-0.197
Waiting cost per total	-0.158	-0.168
Money transfer per ST	-0.234	-0.294
Money transfer per FT	0.007	0.031
Money transfer per total	-0.065	-0.066
Microstructure noise: Mean $\lvert v_t - p_t \rvert$	0.503	0.546
Belief errors of ST: Mean $\lvert E(v_t) - v_t \rvert$	0.398	0.485
Bid-ask spread	1.453	1.725
Effective spread	0.816	0.885
N. of limit orders at the ask	2.163	1.539
N. of limit orders at the ask (effectively traded)	0.617	0.402
N. of limit orders on the sell side	6.335	5.397
N. of limit orders on the sell side (effectively traded)	1.728	1.409
Time until the first order submission per ST	0.336	0.322
Time until the first order submission per FT	0.293	0.296

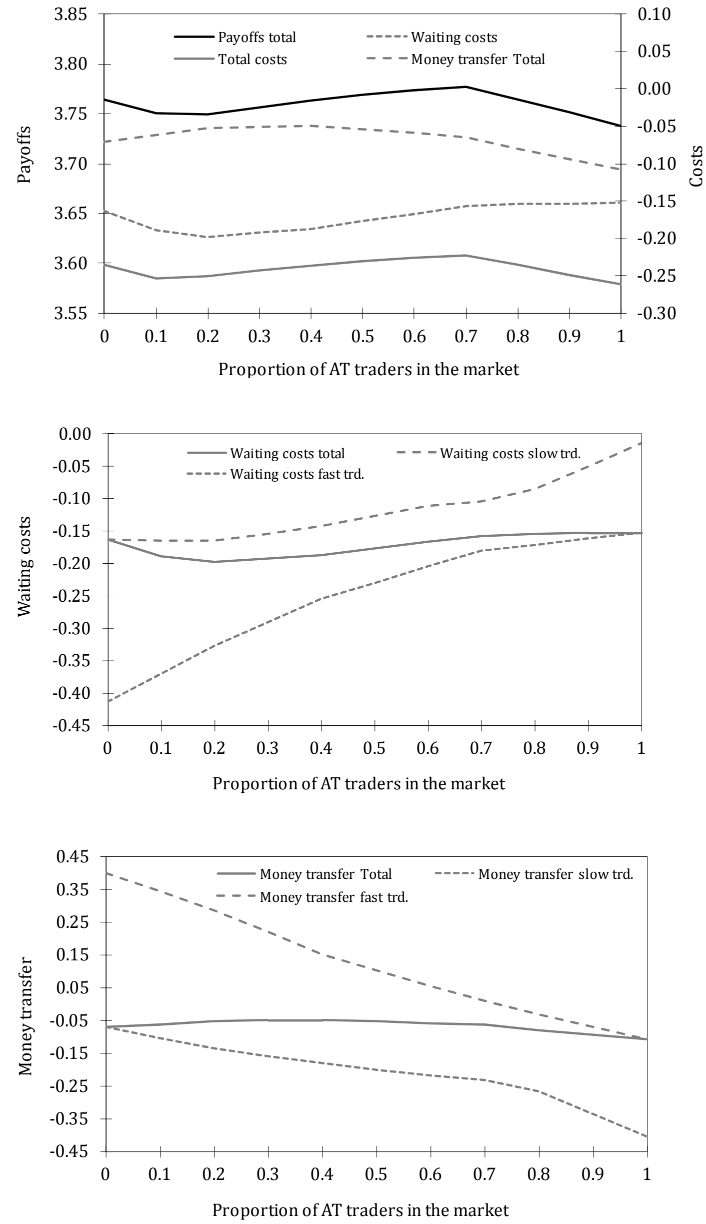

Figure 1. Gains from trading for different levels of fast trader market participation . This figure reports the average payoffs, waiting costs and money transfer per trade when the percentage of fast traders is 0%, 10%, 20% and so on until 100% with intervals of 10%. Payoffs, waiting costs and money transfer were defined in Table 4. In this table, fast traders have both informational and trading speed advantages. In the extreme case in which there are 0% fast traders and 100% fast traders, we include a small proportion of fast traders (0.001% of fast traders) and slow traders (0.001% of slow traders), respectively, with the objective of observing the marginal payoffs of a small group of fast traders and slow traders in each case (we repeat the same strategy for the waiting costs and the money transfer). Standard errors for all market quality measures are sufficiently small since we use a large number of simulated events. The Markov equilibrium is obtained independently for each case. All measures are expressed in ticks.