Globalization, Trade Imbalances and Inequality

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Abstract

We investigate the role of trade imbalances for the distributional consequences of globalization. We do so through the lens of a quantitative, general equilibrium, multi-country, multi-sector model of trade with four key ingredients: (a) workers with different levels of skills are organized into separate representative households; (b) endogenous trade imbalances arise from households' consumption and saving decisions; (c) production exhibits capital-skill complementarity; (d) labor market frictions across sectors and non-employment. We conduct a series of counterfactual experiments that illustrate the quantitative importance of both trade imbalances and capital-skill complementarity for the evolution of the skill premium.

1 Introduction

Is a trade deficit evidence of some workers losing for the benefit of others? Does it reflect opportunities shifting away from manufacturing workers and towards professionals? Policy makers routinely voice such concerns. Curiously, economists have spent little time on these questions—assuming away trade imbalances in models of trade and inequality. We work towards remedying this situation by offering a framework to analyze the distributional consequences of trade when imbalances—trade over time—are subject to the same shocks that determine trade patterns—trade over space.

To accomplish this task we build a general equilibrium, multi-country, multi-sector model of trade with four crucial ingredients. First, workers have different education levels—allowing us to discuss inequality in earnings and non-employment between these groups. Second, workers of each education level are organized into separate representative families within each country. Each representative family makes collective consumption and savings decisions, allowing us to parsimoniously and tractably discuss trade imbalances and consumption differences across—but not within—groups. Third, production features capital-skill complementarity—an empirically relevant

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amplification mechanism for inequality (Krusell et al., 2000; Parro, 2013; Burstein et al., 2013). Fourth and finally, workers face mobility frictions à la Artuç et al. (2010) across sectors, and the possibility of entering non-employment.

Our modeling strategy is not without its tradeoffs, but has several appealing properties for calibration and simulation of counterfactuals. First, despite featuring many countries, many sectors, and a rich input-output structure, calibration can be performed country-by-country, facilitating estimation of parameters that are not easily observed in the data, nor estimated elsewhere. Second, despite modeling consumption dynamics as an endogenous choice, our model is amenable to the "hat algebra" of Dekle et al. (2007). This makes solving counterfactual equilibria straightforward, and allows us to evade estimating initial trade costs and productivities. The costs of our approach are twofold. First, we do not allow for within-group inequality, and so can only discuss the skill premium and differences in non-employment and consumption across workers with different levels of education. Second, we have to assume perfect foresight over all aggregate variables in any counterfactual, foreclosing on possibly important questions related to uncertainty and imperfect information. Nevertheless, we believe that our model provides an important starting point to address the questions posed at the paper's onset.

After calibrating the model, we explore the quantitative importance of allowing for trade imbalances and capital-skill complementarity for the skill premium, non-employment by group, and consumption by group. To do so, we simulate a steady, 15 year rise in productivity in China in all sectors. China, finding itself growing, borrows in the short run against future income. The interaction between trade costs and the desire to shift consumption over time generates novel dynamics in our model. Specifically, because manufacturing and agricultural goods are easier to trade across borders compared to services, these are the goods which the US and other countries supply to China when it borrows, while China diverts resources towards services. Thus, in the short run, manufacturing and agricultural sectors tend to expand outside of China, and this pattern reverses in the long run.

The dynamic patterns in production induced by consumption smoothing lead to dynamics in the skill premium through two channels. There is a *labor demand* channel—as low skilled workers tend to be intensive in agriculture and manufacturing, but high skilled workers tend to be intensive in services, the initial shift towards tradable goods lowers the skill premium outside of China, and then raises it in the long run. By way of contrast, in a world of balanced trade that features only the labor demand channel, we find a mostly negligible impact on the skill premium outside of China at any horizon, as shifting production across tradable industries does not matter as much. There is also *capital-skill complementarity*, which amplifies the demand for skilled relative to unskilled labor if capital prices fall. This channel tends to push up the skill premium everywhere. Interestingly, there is a non-trivial interaction between these two forces. Capital-skill complementarity, by shifting the long-run distribution of income, also changes motives for shifting resources across time, feeding back into the labor demand channel. We find that in our model with both imbalances and capital-skill complementarity, the long-run skill premium rises in all countries, and the short-run skill premium declines *more* than in a model with only the labor demand channel.

In addition to wage inequality, we investigate the role of endogenous imbalances in shaping non-employment and consumption. Non-employment for all workers responds more strongly to shocks than in a balanced trade world—but in most countries it is especially responsive for unskilled workers. As we describe in the main text, this is a consequence of trade imbalances allowing consumption to adjust more quickly than wages do. Growth in China generates higher long-run consumption for all parties. Unlike the case of the skill premium, consumption inequality grows much less (but still increases) after the shock compared to the case without imbalances. In our simulation, capital-skill complementarity ultimately ensures that skilled workers benefit relative to unskilled workers everywhere. However, the ability to smooth consumption allows skilled and unskilled workers to insure each other over the transition path—compressing consumption differences.

The remainder of this paper fleshes out the discussion schematized above. In section 2 we lay out our model—an extension of Dix-Carneiro et al. (2021) with the additional ingredients that let us speak to inequality. We then briefly outline how to bring our model to data in section 3. In section 4 we look into detail at the shock described above; in ongoing work, we are using our model to explore the effects of "the China shock" on American workers from 2000 to 2014. Finally, in section 6 we reflect on our results and their interpretation, also offering our thoughts on directions for future research.

2 Model

Our model builds on existing workhorse models of globalization, trade imbalances, capital-skill complementarity and labor market adjustment. Trade imbalances are modeled according to the inter-temporal approach of Obstfeld and Rogoff (1995), and the trade block is based on Caliendo and Parro (2015). The production structure with capital-skill complementarity is based on Krusell et al. (2000) and Parro (2013), and we adopt the framework in Artuç et al. (2010) to model labor mobility frictions across sectors. We put these frameworks together similarly to Dix-Carneiro et al. (2021).

2.1 Environment

We consider a multi-sector, multi-country world unfolding in discrete time, indexed by t, with no aggregate uncertainty and perfect foresight of aggregate variables. The world is comprised of

$i = 1, ..., N$ countries. Each country i has a constant supply of skilled (S) workers, $\bar{L}_i^S$, and of unskilled (U) workers, $\bar{L}_i^U$. Workers within skill groups are perfect substitutes in production, but there is imperfect substitutability across skill groups. There are $k = 1, ..., K$ sectors. Each sector k is characterized by a unit continuum of varieties $j \in [0, 1]$ that can be traded across countries. Varieties are aggregated by perfectly competitive domestic firms into non-tradable sector-specific composite goods according to:

$$Q_{k,i,t}^I = \left(\int_0^1 Q_{k,i,t}(j)^\epsilon dj \right)^{\frac{1}{\epsilon}}, \tag{1}$$

where $Q_{k,i,t}(j)$ is the quantity employed of variety j of sector k in country i at time t, and $\frac{1}{1-\epsilon}$ is the elasticity of substitution across varieties within a sector. Composite goods can either be used in production of the final consumption good, or as inputs into the production of individual varieties. The price of one unit of sector k's composite good is given by the price index associated to (1), and is denoted by $P_{k,i,t}^I$.

A non-tradable final good is produced by perfectly competitive domestic firms that aggregate the sector-specific composite goods according to:

$$Q_{i,t}^F = \prod_{k=1}^K \left(Q_{k,i,t}^I\right)^{\mu_{k,i}}, \tag{2}$$

where $\mu_{k,i}$ are expenditure shares, and sum to 1. The price of one unit of the final good is given by the price index associated with (2), and is denoted by $P_{i,t}^F$.

Workers earn wages on competitive spot markets for labor. Denote the wage paid to skill type $s \in \{S, U\}$ in sector k of country i at time t by $w_{k,i,t}^s$. At the *end* of each period, t, workers in sector k may choose to switch to work in some other sector k' by incurring a mobility cost we describe below. Workers may also enter a non-employment state, indexed by $k = 0$.[1] We will also set $w_{0,i,t}^s = 0$ for $s \in \{S, U\}$.

2.2 Households and Labor Supply

Within each country, workers of skill type $s = U, S$ are aggregated into a representative family. The household head of skill group s, taking prices and wages as given, determines consumption, savings, and labor supply decisions for each member of the household, maximizing aggregate utility. We first describe the utility of individual workers, then we will show how the household planner aggregates individual member utilities. Finally, we explain how the household's problem can be decentralized to the worker level and written recursively. For ease of notation, we temporarily omit

[1] As will become clear, this non-employment state encompasses a wide range of reasons why workers may not be employed, including non-employment or exit from the labor force. In section *Data Section* we describe exactly what is and is not included in our measure of employment and non-employment.

country and skill subscripts, i and s, and index individuals by ℓ.

At the end of each period t, workers are allocated to a sector k_{t+1} (including possibly non-employment, $k_{t+1}=0$) for the next period. In order to move from sector k to k', the worker incurs a common cost of mobility, $C_{kk'}$ (with $C_{kk}=0$), and an additive stochastic component, $\omega_{k',\ell,t}$. The $\omega_{k,\ell,t}$ shocks are assumed to be *iid* across individuals and time, and are distributed according to a Gumbel distribution with parameters $(-\nu^{EM}\zeta,\zeta)$, where ν^{EM} is the Euler-Mascheroni constant and ζ is a shape parameter. This structure closely follows Artuç et al. (2010). Given this structure, the *flow* utility for worker ℓ at time t, $\mathcal{U}_{\ell,t}$, is given by:

$$\mathcal{U}_{\ell,t} \equiv \mathcal{U}(c_{\ell,t}, k_t, \widetilde{\mathbf{d}}_{\ell,t}, \boldsymbol{\omega}_{\ell,t}) = \log(c_{\ell,t}) + \eta_{k_t} + \sum_{k'=0}^{K} \widetilde{d}_{k',\ell,t}\left[-C_{k_t,k'} + \omega_{k',\ell,t}\right], \tag{3}$$

where $c_{\ell,t}$ is worker ℓ's consumption at time t, η_{k_t} is a sector-specific utility term, and $\widetilde{\mathbf{d}}_{\ell,t} = \left(\widetilde{d}_{1,\ell,t},...,\widetilde{d}_{K,\ell,t}\right)$ is a vector with time $t+1$ sectoral choice indicators. That is, $\widetilde{d}_{k,\ell,t}=1$ if worker ℓ chooses to work in sector k at $t+1$ and $\widetilde{d}_{k,\ell,t}=0$ if the worker chooses to work in a sector different from k. Note that $\widetilde{\mathbf{d}}_{\ell,t}$ is indexed by t to highlight that this decision is made at time t. We impose that workers can only work in one sector per period, so that $\sum_{k'=0}^{K}\widetilde{d}_{k',\ell,t}=1$. Thus, the total supply of workers in the family to industry k at time $t+1$ is given by:

$$L_{k,t+1} = \int_0^{\bar{L}} \widetilde{d}_{k,\ell,t}\, d\ell. \tag{4}$$

The household head's objective is to maximize the net present value of (3) integrated across family members subject to her budget constraint. In addition consumption and employment decisions, the planner has access to financial markets by means of buying and selling one-period, riskless bonds. Bonds can be traded by families within the same country (across skill groups) as well as internationally, and are availably in zero net supply globally. International bond markets are frictionless, with a nominal return that is equalized across space and equal to R_t.

The household head of a given skill group chooses the path of consumption $c_{\ell,t}$, labor supply $\widetilde{d}_{k,\ell,t}$, and bonds, B_t, to solve:

$$\max_{\{c_{\ell,t}\},\{\widetilde{d}_{k,\ell,t}\},\{B_t\}} \quad E_0\left\{\delta^t \phi_t \int_0^{\bar{L}} \mathcal{U}_{\ell,t}\, d\ell\right\} \tag{5}$$

$$\text{subj. to} \quad P_t^F \int_0^{\bar{L}} c_{\ell,t}\, d\ell + B_{t+1} = \sum_{k=1}^{K} w_{k,t} \int_0^{\bar{L}} \widetilde{d}_{k,\ell,t-1}\, d\ell + R_t B_t, \tag{6}$$

where E_0 refers to the expectation over future idiosyncratic shocks $\{\boldsymbol{\omega}_{\ell,t}\}$. There is no aggregate uncertainty, and households have perfect foresight over the evolution of aggregate variables. The discount factor, δ, is common across all workers, but is subject to family-level shifters, denoted by ϕ_t. These shifters can differ across countries, skill levels, and over time.[2] The right hand side of the budget constraint (6) reflects total income available to the household at time t. The first term is the wage income aggregated across all individuals, and the second term is the revenue accruing from interest payments on bonds purchased in the previous year. The first term in the left hand side of the budget constraint is total expenditure of the household on final goods. The gap between total expenditures and income must be equal to total bonds purchases.

Let $\widetilde{\lambda}_t$ be the Lagrange multiplier on the family's budget constraint. The first order condition on consumption implies that $c_{\ell,t}^{-1} = P_{F,t}\widetilde{\lambda}_t$ for all ℓ.[3] That is, consumption is equalized across a skill group's family members within a period. We denote this *per capita* consumption by c_t.

Before discussing savings and the worker's individual labor supply decision, we will bring back country and skill subscripts, i and s, but drop individual subscripts, ℓ. Turning to the savings behavior of the household, the first order condition on bonds implies the following Euler Equation:

$$\frac{P_{i,t+1}^{F} c_{i,t+1}^{s}}{P_{i,t}^{F} c_{i,t}^{s}} = \delta R_{t+1} \widehat{\phi}_{i,t+1}^{s}, \tag{7}$$

where $\widehat{\phi}_{i,t+1}^{s} \equiv \frac{\phi_{i,t+1}^{s}}{\phi_{i,t}^{s}}$ and will be referred to as (skill group s specific) inter-temporal shocks. It is now apparent that these inter-temporal shocks are wedges to the Euler equation, giving flexibility for our model to match the path of expenditures in the data. Given a path of income and initial conditions on bold holdings $\left\{B_{i,0}^{s}\right\}$, equations (6) and (7) determine the path of bonds $\left\{B_{i,t}^{s}\right\}$ for each type of household $s = U, S$.

Similarly to Dix-Carneiro et al. (2021), the labor supply decision of workers can be decentralized and written recursively from the perspective of an individual worker. In the remainder of this subsection, we outline this recursive problem and the implied transition dynamics for labor supply.

The optimal labor supply decision, $\widetilde{d}_{k,i,\ell,t}^{s}$, of worker ℓ of skill level s in sector k in country i at time t facing idiosyncratic shocks $\boldsymbol{\omega}_t$ solves:

$$\widetilde{V}_{k,i,t}^{s}(\boldsymbol{\omega}_t) = \widetilde{\lambda}_{i,t}^{s} w_{k,i,t}^{s} + \eta_{k,i}^{s} + \max_{k'} \left\{ -C_{kk',i}^{s} + \omega_{k',i,t}^{s} + \delta \widehat{\phi}_{i,t+1}^{s} E_{\boldsymbol{\omega}} \left[\widetilde{V}_{k',i,t}^{s}(\boldsymbol{\omega}_t) \right] \right\}, \tag{8}$$

where $\widetilde{V}_{k,i,t}^{s}(\boldsymbol{\omega}_t)$ is the value function of a worker of skill level s in sector k in country i at t

[2]The use of these shifters is common in the international macroeconomics literature (Stockman and Tesar, 1995; Bai and Ríos-Rull, 2015). As we illustrate in equation (7), these sifters lead to wedges in the Euler equation commanding how households trade off current with future consumption. The fact that these shifters lead to wedges in Euler equations implies that they can also be viewed as generated by asset markets frictions. Allowing for these wedges is important for the model to match the dynamics of aggregate expenditures with final goods. In any case, these parameters are not allowed to respond to shocks in the global economy.

[3]In an abuse of terminology we will continue to refer to $\widetilde{\lambda}_t$ as the Lagrange multiplier. However, the correct shadow price associated with period's t budget constraint is given by $\delta^t \phi_t \widetilde{\lambda}_t$.

facing shocks $\boldsymbol{\omega}_t$. Observe that in equation (8), wages are multiplied by the household head's Lagrange multiplier on the budget constraint $\widetilde{\lambda}^s_{i,t}$. To understand the role of the Lagrange multiplier, note that $\widetilde{\lambda}^s_{i,t} w^s_{k,i,t}$ is the marginal utility accrued to the whole household from the additional consumption brought in by a worker employed in sector k. Therefore, for the household problem to be decentralized, individual workers must internalize the effect of their labor supply decisions on the whole family's utility. This is a key difference between our setup and the hand-to-mouth setup in Artuç et al. (2010), and similar papers.

It is convenient to work directly with $E_{\boldsymbol{\omega}}\left[\widetilde{V}^s_{k,i,t}(\boldsymbol{\omega}_t)\right]$, and we denote this integrated value function by $V^s_{k,i,t}$. The Gumbel structure on shocks implies that there is a simple recursive formula for $V^s_{k,i,t}$ given by:

$$V^s_{k,i,t} = \widetilde{\lambda}^s_{i,t} w^s_{k,i,t} + \eta^s_{k,i} + \zeta_i \log\left(\sum_{k'=0}^{K} \exp\left(\frac{-C^s_{kk',i} + \delta\widehat{\phi}^s_{i,t+1} V^s_{k',i,t+1}}{\zeta_i}\right)\right). \tag{9}$$

If we aggregate individual policy rules solving (8) across the distribution of idiosyncratic shocks $\boldsymbol{\omega}$, we obtain inter-sectoral transition rates for skill group s following the familiar multinomial logit form:

$$s^s_{kk',i,t,t+1} = \frac{\exp\left(\frac{-C^s_{kk',i} + \delta\widehat{\phi}^s_{i,t+1} V^s_{k',i,t+1}}{\zeta_i}\right)}{\sum_{k''=0}^{K} \exp\left(\frac{-C^s_{kk'',i} + \delta\widehat{\phi}^s_{i,t+1} V^s_{k'',i,t+1}}{\zeta_i}\right)}. \tag{10}$$

Armed with inter-sectoral transition rates, labor allocations across sectors is governed by:

$$L^s_{k,i,t+1} = \sum_{k'=0}^{K} L^s_{k',i,t} s^s_{k'k,i,t,t+1} \tag{11}$$

2.3 Production

2.3.1 Capital Goods

Capital goods are produced by perfectly competitive domestic firms in a similar way to final consumption goods. They are non-tradable and composed of sector-specific composite goods according to:

$$Q^K_{i,t} = \prod_{k=1}^{K} (Q^I_{k,i,t})^{\alpha_{k,i}}, \tag{12}$$

where $\alpha_{k,i}$ are expenditure weights that sum to one. This is a flexible way of modeling capital that is common in the literature. For example, by letting $\alpha_{k,i} = 1$ and $\mu_{k,i} = 0$ for some specific-sector k, we capture the idea of a capital sector as in Parro (2013). Alternatively, by letting $\alpha_{k,i}$ vary across sectors we can capture the idea that investment goods and consumption goods have different compositions by aggregating similar goods, as in Eaton et al. (2016). We denote the price index associated with (12) by $P^K_{i,t}$.

2.3.2 Intermediate Goods

In each sector, there is a continuum of tradable varieties indexed by $j \in [0, 1]$. Perfectly competitive firms in country i at time t can produce variety j according to a nested CES production function as in Parro (2013). The lowest tier nest aggregates capital and skilled labor into a composite, $h_{k,i,t}(j)$, according to:

$$h_{k,i,t}(j) = \left[\chi_{k,i}^{\frac{1}{\rho}} \left[x^K_{k,i,t}(j)\right]^{\frac{\rho-1}{\rho}} + (1-\chi_{k,i})^{\frac{1}{\rho}} \left[S_{k,i,t}(j)\right]^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}}, \tag{13}$$

where $x^K_{k,i,t}(j)$ is the quantity of capital used in production, $S_{k,i,t}(j)$ is the quantity of skilled labor, $\chi_{k,i}$ is a time and variety invariant share parameter, and ρ is the elasticity of substitutions between capital and skilled labor. This first composite good is then combined with unskilled labor in the second tier according to:

$$\upsilon_{k,i,t}(j) = \left[\xi_{k,i}^{\frac{1}{\sigma}} \left[h_{k,i,t}(j)\right]^{\frac{\sigma-1}{\sigma}} + (1-\xi_{k,i})^{\frac{1}{\sigma}} \left[U_{k,i,t}(j)\right]^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \tag{14}$$

where $U_{k,i,t}(j)$ is the quantity of unskilled labor used in production, $\xi_{k,i}$ is a time and variety invariant share parameter, and σ is the elasticity of substitution between the capital-skill composite and unskilled labor. Finally, this composite is combined with inputs in a Cobb-Douglas fashion to make the final output according to:

$$Y_{k,i,t}(j) = z_{k,i,t}(j)\upsilon_{k,i,t}(j)^{\gamma_{k,i}} \prod_{l=1}^{K} M_{l,i,t}(j)^{(1-\gamma_{k,i})\nu_{kl,i}}, \tag{15}$$

where $z_{k,i,t}(j)$ is a common productivity across all producers of variety j of sector k in country i at time t, $M_{l,i,t}(j)$ is the quantity of employed inputs from sector l (given by equation (1)), $\gamma_{k,i}$ is the value-added share in production in sector k and country i, and the ν's summarize the input-output structure of the economy in country i.

The above production structure makes clear the role of capital in our setup. Despite the fact that capital fully depreciates in our model, it is different from standard intermediate inputs as it differentially substitutes for skilled and unskilled labor. In particular, so long as $\sigma > \rho$, the production function exhibits capital-skill complementarity: a reduction in the price of capital raises demand for skilled labor relative to unskilled labor.[4]

[4]One can derive this property following Shepard's Lemma. In particular, and temporarily omitting time, country, and sector subscripts, if P^K is the price of capital, and p^h is the price of the capital-skill composite, then the demand

2.4 Prices and International Trade

Only intermediate varieties j can be traded across countries, but they are subject to trade costs.[5] Trade costs are sector, but not variety, specific. We denote the cost of shipping *from* origin o to destination i in sector k at time t by $d_{k,oi,t}$. In contrast to Parro (2013), we abstract from tariff revenues, but will allow for trade imbalances to adjust endogenously to changes in fundamentals. However, we plan to add tariffs to our model in the next version of this paper, allowing us to study impacts of trade wars.

Given wages, $w^s_{k,i,t}$, and prices of intermediates and capital, $P^I_{k,i,t}$ and $P^K_{i,t}$ respectively, we can write the unit cost of an input bundle in country i in sector k at t as:

$$c_{k,i,t} = \left(p^{\upsilon}_{k,i,t}/\gamma_{k,i}\right)^{\gamma_{k,i}} \times \left(\prod_{l=1}^{K} \left(P^I_{l,i,t}/\nu_{kl,i}\right)^{\nu_{kl,i}}\right)^{1-\gamma_{k,i}}, \tag{16}$$

where

$$p^{\upsilon}_{k,i,t} = \left(\xi_{k,i}[p^h_{k,i,t}]^{1-\sigma} + (1-\xi_{k,i})[w^U_{k,i,t}]^{1-\sigma}\right)^{\frac{1}{1-\sigma}}, \tag{17}$$

and

$$p^h_{k,i,t} = \left(\chi_{k,i}[P^K_{i,t}]^{1-\rho} + (1-\chi_{k,i})[w^S_{k,i,t}]^{1-\rho}\right)^{\frac{1}{1-\rho}}. \tag{18}$$

Since there is perfect competition, the price of variety j in sector k at time t going from o to i will be given by the marginal cost:

$$p_{k,oi,t}(j) = \frac{c_{k,o,t}}{z_{k,o,t}(j)} d_{k,oi,t}.$$

Consumers and firms in country i buy from the lowest cost producer. Hence, the price of variety j in sector k in country i at time t is given by:

$$p_{k,i,t}(j) = \min_{o} \left\{p_{k,oi,t}(j)\right\}.$$

Following Eaton and Kortum (2002), we assume that $z_{k,i,t}(j) \sim Frechet(A_{k,i,t}, \lambda)$, where $A_{k,i,t}$ is a scale parameter akin to sector-specific total factor productivity (TFP) and λ is the shape parameter that determines comparative advantage within sectors. The scale parameters are allowed to be country, sector, and time specific. However, the scale parameters are common across countries and

of skilled relative to unskilled labor as a function of capital prices is given by,

$$\frac{d\log(S/U)}{d\log P^K} = (\rho - \sigma) \times \left(\frac{P^K}{p^h}\right)^{1-\rho}.$$

This is a decreasing function iff $\rho < \sigma$.

[5] This is without loss of generality, since one could include non-tradables by setting trade costs to infinity, and could include freely traded goods by setting (proportional) trade costs to 1.

assumed to be constant over time. Given this assumption, the price of sector k goods in country i at time t is given by:

$$P^I_{k,i,t} = B_{k,i} \times \left(\sum_{o=1}^{N} A_{k,o,t} [c_{k,o,t} d_{k,oi,t}]^{-\lambda} \right)^{-1/\lambda}, \tag{19}$$

where $B_{k,i}$ is a constant. Given wages, equations (16) to (19), together with the definition of $P^K_{i,t} = \prod_{l=1}^{K} \left(P^I_{l,i,t} / \alpha_{l,i} \right)^{\alpha_{l,i}}$, define a system of equations that can be used to solve for all goods prices in the economy. Moreover, it can be shown that under the Frechet assumption, the expenditure share of total sectoral output, k, in o by i at t, takes on the familiar gravity form:

$$\pi_{k,oi,t} = \frac{A_{k,o,t} [c_{k,o,t} d_{k,oi,t}]^{-\lambda}}{\sum_{o'=1}^{N} A_{k,o',t} [c_{k,o',t} d_{k,o'i,t}]^{-\lambda}}. \tag{20}$$

2.5 Market Clearing

Given an allocation of labor across sectors, $L^s_{k,i,t}$, labor market clearing requires:

$$w^s_{k,i,t} L^s_{k,i,t} = e^s_{k,i,t} \gamma_{k,i} Y_{k,i,t}, \tag{21}$$

where $Y_{k,i,t}$ is gross output in sector k in i at t, and $e^s_{k,i,t}$ is the share of value-added paid to labor of type s. Because we do not assume that $\sigma = \rho = 1$, the expenditure share is endogenous to prices. Goods market clearing requires that gross output in sector k in each country and period is equal to the sum of demand across trade partners. Denote total expenditure on sector k by country i by $X_{k,i,t}$. From equation (20), goods market clearing can be written as:

$$Y_{k,o,t} = \sum_{i=1}^{N} \pi_{k,oi,t} X_{k,i,t}. \tag{22}$$

Expenditure $X_{k,i,t}$ is the sum of final consumption by households, capital expenditure, and expenditure on inputs. To obtain the equation determining $X_{k,i,t}$, first define $NX_{i,t}$ as the value of total net exports in country i at time t, $I_{i,t}$ to be total disposable income across all households in country i at time t, and $X^K_{i,t}$ to be total expenditure on capital goods in i at t. With this notation in hand, $X_{k,i,t}$ can be written as:

$$X_{k,i,t} = \mu_{k,i} [I_{i,t} - NX_{i,t}] + \alpha_{k,i} X^K_{i,t} + \sum_{l=1}^{K} (1 - \gamma_{l,i}) \nu_{lk,i} Y_{l,i,t}, \tag{23}$$

where

$$X^K_{i,t} = \sum_{l=1}^{K} e^K_{l,i,t} \gamma_{l,i} Y_{l,i,t}, \tag{24}$$

and $e^K_{l,i,t}$ is the endogenous share of payments to capital in sector l.

Aggregate disposable income $I_{i,t}$ is given by the sum of disposable income across skill groups $I^s_{i,t}$, which is given by:

$$I^s_{i,t} = w^s_{k,i,t} L^s_{k,i,t} = e^s_{l,i,t} \gamma_{l,i} Y_{l,i,t}. \tag{25}$$

Finally, bonds market clearing requires that net aggregate exports are equal to changes in bonds net of interest,

$$NX_{i,t} = I^S_{i,t} + I^U_{i,t} - \left(E^{C,S}_{i,t} + E^{C,U}_{i,t}\right) = B^S_{i,t+1} + B^U_{i,t+1} - R_t(B^S_{i,t} + B^U_{i,t}), \tag{26}$$

and that bonds are in global zero net supply,

$$\sum_{i=1}^{N} \sum_{s \in \{U,S\}} B^s_{i,t} = 0. \tag{27}$$

In equation (26), $E^{C,s}_{i,t} \equiv \overline{L}^s_i P^F_t c^s_t$ is the total expenditure of skill group s in final goods.

2.6 Equilibrium

An equilibrium in this model is a set of initial steady-state allocations $\{L^s_{k,i,0}, B^s_{i,0}, \}$, a set of final steady-state allocations $\{L^s_{k,i,\infty}, B^s_{i,\infty}, \}$ and sequences of policy functions for workers/firms $\{s^s_{kk',i,t,t+1}\}$, value functions for workers $\{V^s_{k,i,t}\}$, bond decisions by the households $\left\{B^s_{i,t}\right\}$, bond returns $\left\{R^t\right\}$, allocations $\{L^s_{k,i,t}\}$, household consumption per capita $\{c^s_{i,t}\}$, trade shares $\{\pi_{k,io,t}\}$, and price indices $\left\{P^{I,t}_{k,i}, P^{K,t}_{k,i}, P^{F,t}_{k,i}\right\}$ such that: (a) Workers' value functions solve (9); (b) Consumption and bonds decisions solve (5) subject to (6), given an initial distribution of bonds $\left\{B^s_{i,0}\right\}$; (c) Allocations evolve according to (11); (d) Trade shares are given by (20); (e) Prices are set competitively and goods markets clear: (22) to (25); (f) Labor markets clear: $\sum_{k=1}^K L^s_{k,i,t} = \overline{L}^s_i$ for $s = U, S$; (g) Bonds market clears: $\sum_{i=1}^N \sum_{s=U,S} B^s_{i,t} = 0$. Appendix D displays the algorithms we designed to compute the steady-state equilibrium and to compute transitional dynamics.

2.7 Discussion

To understand how trade imbalances arise in our model, assume that there are no inter-temporal preference shocks, and so $\widehat{\phi}^s_{i,t} = 1$ for all i, s and t. In this case, equation (7) implies that $E^{C,s}_{i,t+1} = \delta R^{t+1} E^{C,s}_{i,t}$ for all i and s over the transition path. Normalizing $\sum_{i=1}^N \sum_{s=U,S} E^{C,s}_{i,t} = 1$—so that all nominal variables are expressed as a fraction of world expenditure on final goods—we obtain that $R_t = 1/\delta$ for all t. In turn, this implies that individual countries' expenditures on final goods are constant as a share of world expenditure following a shock. Therefore, for any path of shocks, countries immediately smooth final expenditures as a share of global expenditures. To fix ideas, suppose that China realizes that it will gradually become more productive and richer. In this case, our model predicts that China will consume above production in the short run and then below in the long run, leading to short-run trade deficits and long-run trade surpluses. Nonetheless, in the data, we rarely observe this stark version of expenditure smoothing we have just discussed. As we previously discussed, the inter-temporal preference shocks $\widehat{\phi}^{s}_{i,t} = 1$ are wedges that reconcile our model with the observed data.

3 Calibration and Data

3.1 Preliminaries

We calibrate our model to a global economy with six sectors and six countries. We consider a world comprised of the United States, China, and four country aggregates: Europe, Asia/Oceania, the Americas, and the Rest of the World. Each country's economic activity consists of six sectors: Agriculture; Low-, Mid- and High-Tech Manufacturing; Low- and High-Tech Services. Tables A.1 and A.2 in Appendix A detail these divisions.

Table 1 summarizes the parameters we need to numerically solve the model. We split them into three categories: (i) parameters that are fixed at values previously reported in the literature, as they are difficult to identify given available data (Panel A); (ii) parameters that can be determined without having to solve the model (Panel B); and (iii) parameters that are estimated by the method of simulated moments. We calibrate our model using a variety of datasets for year 2000 or closest year available.

We start by discussing parameters fixed according to values reported in the literature, which are listed in Panel A. First, we calibrate the model at the annual frequency. In this case, annual steady-state international bonds' returns are given by $1/\delta$, so we set $\delta = 0.95$ implying annual returns of 5%.[6] The estimation of the dispersion of $\boldsymbol{\omega}$ shocks typically requires panel data and instrumental variable strategies. As a result, we impose this parameter to be common across countries and set $\zeta_i = 1.61$ based on the estimate Artuç and McLaren (2015) obtained using US data. The Frechet scale parameter $\lambda = 4$ comes from Simonovska and Waugh (2014). We follow Parro (2013) and set the elasticity of substitution between skilled labor and capital goods $\rho = 0.67$, and the elasticity of substitution between unskilled labor and capital goods $\sigma = 1.67$. These elasticities have been originally estimated by Krusell et al. (2000).

[6]This choice is based on the fact that both the Federal Funds and T-Bill rates in 1999-2000 were between 5% and 6%: https://fred.stlouisfed.org/series/FEDFUNDS and https://fred.stlouisfed.org/series/DTB1YR.

Table 1: Summary of Parameters

Panel A. Fixed According to the Literature			
Parameter	Value	Description	Source
δ	0.95	Discount factor	5% annual interest rate
ζ_i	1.61	Dispersion of $\boldsymbol{\omega}$ shocks	Artuç and McLaren (2015)
λ	4.00	Frechet Scale Parameter	Simonovska and Waugh (2014)
ρ	0.67	Capital-skilled labor EoS	Krusell et al. (2000)
σ	1.67	Composite-unskilled labor EoS	Krusell et al. (2000)

Panel B. Calibrated Outside of the Model		
Parameter	Description	Source
$\mu_{k,i}$	Final Expenditure Shares	WIOD
$\alpha_{k,i}$	Capital Expenditure Shares	WIOD
$\gamma_{k,i}$	Value-Added Expenditure Shares	WIOD
$\nu_{k\ell,i}$	Input-Output Matrix	WIOD

Panel C. Calibrated Using the Model Structure	
Parameter	Description
$C^s_{kk'}, i$	Mobility Costs
$\eta^s_{k,i}$	Sector-Specific Utility

Notes:

Turning to Panel B, we can directly calibrate final expenditure shares $\mu_{k,i}$, capital expenditure shares $\alpha_{k,i}$, labor expenditure shares $\gamma_{k,i}$, and input-output shares $\nu_{k\ell,i}$, without having to solve the model. To that aim, we employ the World Input Output Database (WIOD), which compiles data from national accounts combined with bilateral international trade data for a large collection of countries. These data cover 56 sectors and 44 countries, including a Rest of the World aggregate, between 2000 and 2014. In the next section, we describe how we obtain the labor supply parameters (mobility costs and sector-specific utilities).

3.2 Labor Supply Parameters

The calibration procedure for mobility costs $C^s_{kk',i}$ and sector-specific utilities $\eta^s_{k,i}$ proceeds in two steps. First, we show that given values for ζ and δ, and data on skill-group-specific wages and transition rates across sectors we can invert the model to obtain these parameters. This is the procedure we follow for US-specific parameters. For the remaining countries, we do not have data on transition rates by skill groups, and, sometimes, these data are only available at a more aggregate level than what we have in Table (A.2)—see Dix-Carneiro et al. (2021) for details. Therefore, we impose $C^s_{kk',i} = \psi_i \times C^s_{kk',US}$ for the remaining countries, where ψ_i is calibrated to target the average sectoral persistence rate of workers in country i.

3.2.1 Inverting the Model for Parameters in the US

To calibrate the labor supply parameters for the US, we assume that the economy is in steady state in the year 2000. Given that we treat ζ and δ as known, we show that mobility costs, $C^s_{kk',US}$, and sector-specific utilities, $\eta^s_{k,US}$ can be obtained as a function of observed inter-sectoral transition rates and wages. In the remainder of this subsection, we omit the country and time subscripts, since we focus on the US in steady state. We also omit skill superscripts for easy of exposition, but note that we calibrate different parameters for each skill group.

Inverting the model proceeds similarly to the estimation strategy of Artuç et al. (2010). In particular, manipulating a steady state version of (10) leads to the following equation:

$$\log\left(\frac{s_{kk'}}{s_{kk}}\right) - \delta \log\left(\frac{s_{kk'}}{s_{k'k'}}\right) = -\frac{(1-\delta)C_{kk'}}{\zeta} + \frac{\delta}{\zeta}\widetilde{\lambda}\left(w_{k'} - w_k\right) + \frac{\delta}{\zeta}\left(\eta_{k'} - \eta_k\right), \tag{28}$$

with the convention that wages in the non-employment sector are zero, and with the normalization that $C_{kk} = 0$. Both transitions and wages differentials are observable for the US, and so this equation can be used to recover the mobility costs and sector-specific utilities conditional on a choice of discount factor, δ, and switching elasticity, ζ.[7] Solving for η and C in (28) requires several normalizations. This is especially clear from the term $\eta_{k'} - \eta_k$, since shifting the vector of η's by a constant would leave this difference unchanged. In addition to setting $C_{kk} = 0$, we also set the utility of non-employment $\eta_0 = 0$. With these normalizations in hand, (28) describes a system of linear equations that can be solved for both $C_{kk'}$ and η_k. Central to this procedure is access to full data on transitions and wages. This is the case for the United States, but data from other countries is often more limited. In the next subsection, we discuss how we estimate the labor supply parameters for other countries.

3.2.2 Calibration Routine for Non-US Countries

For countries besides the US, we also observe labor allocations and wages in each sector from the WIOD. However, we currently only have inter-sectoral transition matrices that are common across skill levels. In addition, for some countries (for example, China), we only have data on transitions

[7] We set $\widetilde{\lambda}^s_i = \frac{\overline{L}^s_i}{E^{C,s}_i}$, where $E^{C,s}_i$ is computed in three steps. First, we compute steady-state output $Y_{k,i}$ across countries using equations (22) to (25) conditional on $\{\pi^{Data}_{k,oi}\}$, $\left\{e^{U,Data}_{l,i}\right\}$, $\left\{e^{S,Data}_{l,i}\right\}$, and $\left\{e^{K,Data}_{l,i}\right\}$. See Appendix D.2 for details. Second, we compute total expenditures as $E^C_i = \sum_k (1 - e^{K,Data}_{k,i})\gamma_{k,i} * Y_{k,i} - NX^{Data}_i$. Third, we impose that $E^{C,s}_i = \frac{I^s_{i,t}}{I_{i,t}}$.

between more aggregated sectors, which don't include Agriculture. Therefore, we proceed by jointly calibrating a scaling factor ψ_i (common across skill levels) for mobility costs, so that $C^s_{kk',i} = \psi_i \times C^s_{kk',US}$, and a vector of sector-specific utilities $\boldsymbol{\eta}^s_i$ for each country.

To recover these parameters we similarly assume that the global economy is initially in steady state and employ a nested procedure. First, we compute output across sectors and countries, $\{Y_{k,i}\}$—see Appendix D.2 for details. Next, we compute wages implied by $\{Y_{k,i}\}$, expenditure shares on skilled and unskilled workers $\left\{e^{s,Data}_{k,i}\right\}$, the observed labor allocations in the data $\left\{L^{s,Data}_{k,i}\right\}$: $w^{s,0}_{k,i} = \frac{\gamma_{k,i} e^{s,Data}_{k,i} Y_{k,i}}{L^{s,Data}_{k,i}}$. In the inner nest, given a value of ψ_i, we recover the $\boldsymbol{\eta}^s_i$ vector that exactly replicates $w^{s,0}_{k,i}$. Appendix D.1 details an iterative procedure solving for this problem. Here, we outline the basic idea. Given a current guess of ψ_i and $w^{s,0}_{k,i}$, we solve the Bellman equation in (9) and obtain transition rates (10). These steady-state transition rates imply labor allocations $L^{s,Model}_{k,i}$. We then compute a model-implied wage that rationalizes these allocations: $w^{s,Model}_{k,i} = \frac{\gamma_{k,i} e^{s,Data}_{k,i} Y_{k,i}}{L^{s,Model}_{k,i}}$. At this point, we update $\boldsymbol{\eta}^s_i$ based on the deviations between the model-implied wage and $w^{s,0}_{k,i}$. Intuitively, if the guessed value for $\eta^s_{k,i}$ is too low, then labor supply, evaluated at $w^{s,0}_{k,i}$, will be too low in that sector—and the iterative algorithm will raise $\eta^s_{k,i}$.

The outer loop of our calibration routine finds a value of ψ_i that minimizes the distance between some function of the observed subset of transitions, and their data counterpart. In principle, one could use any function of the observed subset of transitions, and it may depend on the data one has at hand. In practice, we match the average persistence of workers in a sector, $\sum_k \sum_s s^{s,Data}_{kk,i} \frac{L^s_{k,i}}{\overline{L}^s_i}$.[8] We use golden section search to minimize the squared distance between model and data persistence.

3.3 Data

We use data from the Current Population Survey (CPS) in the United States to obtain employment allocations, including non-employment. For the remaining countries, we obtain unemployment rates from ILOSTAT and employment allocations from WIOD. Average wages across countries and sectors are similarly drawn from the WIOD.[9]

To be able to identify mobility costs, we make use of micro-data from several countries. Except for the US and China, all the remaining countries are country aggregates. In these cases, we select one country or set of countries as "representative" for which we measure yearly worker transition rates across sectors. Table 3 lists the representative countries and the datasets we have used to

[8] In some cases, we only observe a subset of sectors or aggregated sectors. For example, in China we only observe total manufacturing. In these cases, we replicate the observed transition matrix in the model, and apply the function to these replicated transition matrix. The details of what we observe for each country can be found in the Data Appendix.

[9] Given that workers are homogeneous in our model, we adjust the wage data from WIOD to control for differences in skill composition across sectors. We also adjust wages for differences in industrial composition across countries in each of our four country aggregates. Our Data Appendix provides the details behind this procedure.

Table 2: Mobility Costs in the US – $C^s_{US}/(\widetilde{\lambda}^s_{US} \times \overline{w}^s_{US} \times \zeta)$ for $s \in \{Non-College, College\}$

From ↓ To →	Agr.	LT Manuf.	MT Manuf.	HT Manuf.	LT Serv.	HT Serv.
Panel A: Non-College						
Agriculture	0	6.05	5.91	6.20	3.54	5.19
LT Manufacturing	4.91	0	3.86	3.70	2.98	4.19
MT Manufacturing	5.91	4.12	0	4.15	2.83	4.42
HT Manufacturing	4.91	3.88	4.01	0	2.84	3.80
LT Services	5.60	5.66	5.36	5.38	0	3.96
HT Services	5.32	5.34	5.41	4.80	2.39	0
Non-Employment	4.70	5.90	5.45	5.72	2.96	3.97
Panel B: College						
Agriculture	0	5.18	6.42	4.42	4.46	3.47
LT Manufacturing	7.56	0	5.64	4.42	3.65	4.20
MT Manufacturing	6.36	4.02	0	5.15	3.73	4.38
HT Manufacturing	6.86	4.75	4.49	0	3.14	2.88
LT Services	6.81	5.27	5.73	5.54	0	3.31
HT Services	6.67	5.75	5.42	5.27	3.53	0
Non-Employment	6.19	7.56	6.76	6.64	3.99	3.51

Notes: Estimates of mobility costs in the literature, such as Artuç et al. (2010) and Artuç and McLaren (2015) (a) normalize the average wage in the US $\overline{w}_{US} = 1$; and (b) have $\widetilde{\lambda}_{US} = 1$. To be able to compare our estimates to those, we express C^s_{US} as a fraction of $\widetilde{\lambda}^s_{US} \times \overline{w}^s_{US} \times \zeta$ for each skill group.

obtain transition rates.[10] As previously anticipated, we only use skill-specific transition rates for the US. Transition rates for the remaining countries aggregate across skill groups.

3.4 Calibration Results

We map college workers to the skilled labor group and non-college workers to the unskilled labor group. Tables 2 to 5 contain estimates of the labor supply parameters, while Tables C.1 to C.4 in Appendix C contain estimates of the various preference and production function parameters. We begin with Table 2, which contains estimated mobility costs in the US. We report the values of mobility costs as a fraction of ζ, the dispersion of idiosyncratic preference shocks for sectors. In addition, to make our estimates more directly comparable to those in Artuç et al. (2010) and Artuç and McLaren (2015), we express C^s_{US}/ζ relative to $\widetilde{\lambda}^s_{US} \times \overline{w}^s_{US}$, where $\overline{w}^s_{US}$ is the average wage of skill group s in the US.[11] From now on, we refer to the numbers in Table 2 as normalized mobility costs. Our estimates of normalized mobility costs are remarkably similar in magnitude

[10] The Brazilian *Relação Anual de Informações Sociais* and the Turkish *Entrepreneur Information System* (EIS) are administrative datasets. See Dix-Carneiro (2014) and Demir et al. (2021) for a description of these data. We are extremely grateful to Wei Huang and Banu Demir for their very generous help with China's Urban Household Survey and with Turkey's EIS data, respectively.

[11] Artuç et al. (2010) and Artuç and McLaren (2015) normalize the average wage in the US, $\overline{w}_{US} = 1$, and have $\widetilde{\lambda}_{US} = 1$.

Table 3: Micro Data Used to Compute Inter-Sectoral Transition Rates

Country Aggregate (Representative Country)	Source	Year
United States	Current Population Survey (CPS)	1999-2000
China	Urban Household Survey	2004
Europe (United Kingdom)	Labour Force Survey	1999-2001
Asia/Oceania (Korea, Australia)	Korean Labor and Income Panel Study	1999-2000
	Household, Income and Labour Dynamics in Australia	2001-2002
Americas (Brazil)	Relação Anual de Informações Sociais	1999-2000
Rest of World (Turkey)	Entrepreneur Information Survey	2014

Notes: For Asia/Oceania, we target the population-weighted average of transition rates and coefficient of variation of wages for South Korea and Australia. We were not able to gather information for the year 2000 for all the datasets we employ. In these cases, we selected the closest possible year for which the relevant data are available.

across skill groups. If anything, they are slightly larger for college educated workers. There are potentially many reasons for this finding, ranging from incorrect estimates of ζ to the possibility of un-modeled specific human capital. Nevertheless, the numbers are similar in magnitude to other estimates (Artuç et al., 2010; Artuç and McLaren, 2015).

While the magnitudes are similar across skill groups, there are some notable differences in the patterns of mobility costs. For example, mobility costs for college educated workers to enter Agriculture are much higher than almost any other cost, with only costs of exiting non-employment at the same level. This is not the case for Non-College workers. These steep costs are needed to rationalize the low rates at which College educated workers switch into the Agriculture sector. While the average off-diagonal element of the transition matrix is on the order of 2%, the average transition rate into Agriculture is less than 1%. These moving costs can matter for the transition dynamics of the skill premium, as differences in the willingness of workers to reallocate across certain sectors can shape the dynamics of wages adjustments.

Table 4 compares mobility costs around the world, appropriately normalized, $C_i/(\widetilde{\lambda}_i \overline{w}_i)$, to those in the US. The appropriately normalized values of ψ_i are relatively close to 1, with the Rest of the World (RoW) being a notable exception. Chinese mobility costs are estimated to be smaller than those in the US, but the implied magnitudes are all still within 2 to 4 times marginal-utility-adjusted average wages. However, RoW's costs are much lower.

Sector-specific utilities are shown in Table 5. Given that workers of skill group s choose sectors based on wages scaled by the Lagrange multiplier $\widetilde{\lambda}_i^s$ (see equation (8)), we compare our estimates of $\eta_{k,i}^s$ to the model-implied values of $\widetilde{\lambda}_i^s \times \overline{w}_i^s$ across countries. Generally the appropriately normalized value of η is positive—suggesting that employment tends to be more attractive than non-employment, above and beyond wages. There are some exceptions to this conclusion—mostly in the Americas, where η's are often negative. This result likely reflects very high persistence in

Table 4: Mobility Costs Around the World Relative to the US's $\frac{C_i/(\widetilde{\lambda}_i\overline{w}_i)}{C_{US}/(\widetilde{\lambda}_{US}\overline{w}_{US})} = \psi_i \times \frac{\widetilde{\lambda}_{US}\overline{w}_{US}}{\widetilde{\lambda}_i\overline{w}_i}$

	Country					
	US	China	Europe	Asia/Oc.	Americas	RoW
$\psi_i \times \frac{\widetilde{\lambda}_{US}\overline{w}_{US}}{\widetilde{\lambda}_i\overline{w}_i}$	1	0.71	1.05	0.91	1.18	0.54

Notes: C_i stands for the skill-group size weighted mean of mobility costs in country i, $\widetilde{\lambda}_i\overline{w}_i$ stands for the skill-group size weighted mean of marginal utility adjusted wages in country i. This table reports $\frac{C_i/(\widetilde{\lambda}_i\overline{w}_i)}{C_{US}/(\widetilde{\lambda}_{US}\overline{w}_{US})} = \psi_i \times \frac{\widetilde{\lambda}_{US}\overline{w}_{US}}{\widetilde{\lambda}_i\overline{w}_i}$ so that we are better able to compare estimated mobility costs relative to the US.

Table 5: Sector-Specific Utilities $\eta^s_{k,i}/(\widetilde{\lambda}^s_i \times \overline{w}^s_i)$ for $s \in \{Non-College, College\}$

	Country					
Sector	US	China	Europe	Asia/Oc.	Americas	RoW
Panel A: No College						
Agriculture	1.85	1.30	0.45	1.80	0.70	2.12
LT Manufacturing	1.14	-0.36	0.03	0.95	-0.32	1.11
MT Manufacturing	1.67	0.40	0.24	1.20	0.29	1.55
HT Manufacturing	0.94	-1.14	-0.54	0.70	-2.71	-1.32
LT Services	1.63	0.09	-0.21	0.98	-0.43	-0.02
HT Services	1.72	0.74	0.20	1.07	-0.03	0.97
Non-Employment	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00
Panel B: College						
Agriculture	3.01	2.01	0.67	1.78	0.51	0.82
LT Manufacturing	1.72	1.03	0.24	0.75	-0.06	0.37
MT Manufacturing	2.09	1.35	0.51	0.87	0.30	0.12
HT Manufacturing	1.30	0.09	-0.38	0.43	-1.99	-2.27
LT Services	2.56	1.41	0.09	0.77	-0.22	-1.06
HT Services	2.78	2.07	0.57	0.90	0.30	0.32
Non-Employment	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00

Notes: Workers decide in what sector to search partly based on wages scaled by $\widetilde{\lambda}^s_i$. To aid the interpretation of the magnitude of the estimates of $\eta^s_{k,i}$, we express them as a fraction of $\widetilde{\lambda}^s_i \times \overline{w}^s_i$, where $\overline{w}^s_i$ is the average wage of skill group s in country i.

non-employment in Brazil (our representative country for the Americas), which is around 97%. This region also has high non-employment rates relative to other countries. For example, non-employment among Non-College Workers is 9%, second only to Europe. These terms can pick up additional, un-modeled labor market frictions, non-employment benefits, or non-compensating differentials. We are agnostic on what exactly is absorbed into η, but we do assume that these are held constant across different counterfactuals.

Another pattern that stands out is that η's in Agriculture are generally high—especially in the US, China, and RoW. This is how the model can rationalize the size of this sector in these countries despite their relatively low wages. For example, in the US, the wage in Agriculture for College Workers is nearly half of the average wage in the services sectors, and less than one third of the average wage in the manufacturing sectors. Finally, looking across skill groups, the sector-specific utilities are larger for college-educated workers in the US, China, Europe and the Americas, and lower in Asia/Oceania and the Rest of the World. This suggests that differences in non-employment between groups are unlikely to be explained by wage differences alone. With these parameters in hand, we are in a position to perform counterfactual experiments. In the next section we turn to the model's response to productivity shocks in order to illuminate and quantity the model's key mechanisms, then in section 5 we turn to our analysis of the rise of China.

4 Mechanisms

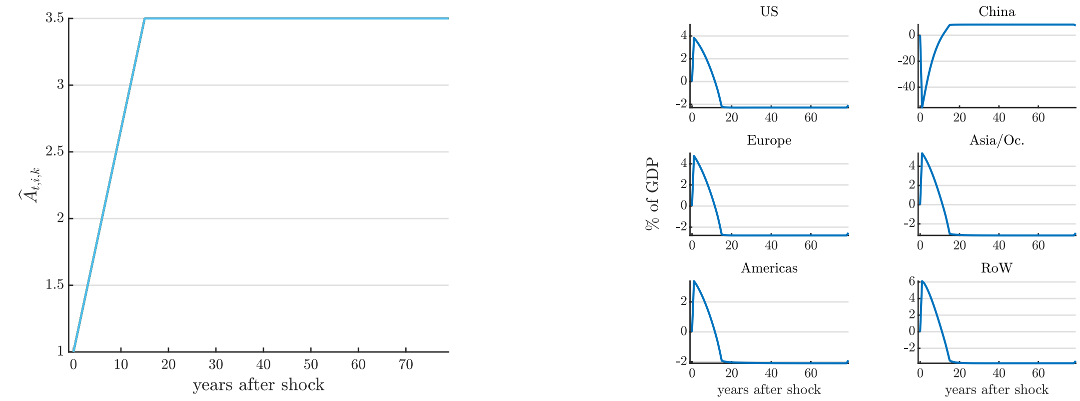
In this section, we simulate a slow a slow linear increase in Chinese productivity to shed light on key mechanisms in our model. We focus attention on three outcomes: (1) the skill premium; (2) non-employment; (3) consumption inequality. The skill premium is measured as the average wage of College workers relative to Non-College workers, and consumption inequality is measured as the ratio between aggregate consumption of College workers and Non-College workers. We first outline how slow productivity growth in China impacts the global economy over time: we study its impact on trade imbalances, output, and labor allocations across sectors. Next, we turn our main the outcomes of interest. Throughout, we benchmark the quantitative implications of our model contrasting it to a model where countries balance trade in each period, and with a model featuring Cobb-Douglas production function in capital, college educated workers, and non-college workers.[12] These two alternative models serve as a vantage point for understanding the importance of trade imbalances alone, capital-skill complementarity alone, and their interaction. That said, we proceed with the caveat that as our model is non-linear, this procedure does not provide an exact decomposition.

Figure 1a plots the changes in Chinese productivity we feed into the model. There is a linear

[12] More precisely, we set $\sigma = .99$ and $\rho = 1.01$.

Figure 1: Slow Productivity Growth in China and Resulting Trade Imbalances

(a) Productivity Growth in China $\widehat{A}_{k,i,t}$ (b) Net Exports - Full Model



rise in $\widehat{A}_{k,China,t}$ that is common across all sectors and plateaus at 3.5 after 15 years—roughly in line with the productivity changes China actually experienced between 2000 and 2014, which we extract in the next section.[13] Figure 1b plots the resulting change in net exports across countries. China, finding itself richer in the future, borrows in the short run—running a trade deficit. In the long run, this deficit is paid down by running a small surplus in perpetuity. The behavior of trade imbalances in the remaining countries is symmetric: they run an initial trade surplus, followed by a perpetual deficit. This evolution of trade imbalances is consistent with what we would expect from a consumption-smoothing point of view. However, as we now describe, this simple shift in consumption over time can have rich dynamic impacts on workers of different skills.

To understand how allocations respond to the shock, Figures 2a to 2d plot the evolution of gross output shares after the shock. We begin with the version of our model with balanced trade in each period, and Cobb-Douglas production functions, shown in Figure 2a. As China is steadily growing richer, countries increase their exports to China, global production patterns tilt towards Chinese demand, and China demands more of the capital good.[14] This leads to a mostly monotonic convergence towards a new steady state: the United States expands output in Agriculture, in which it has a comparative advantage and Chinese demand is high; consequently, China balances trade by expanding its relatively tradable manufacturing sectors, in which it enjoys a comparative advantage.[15]

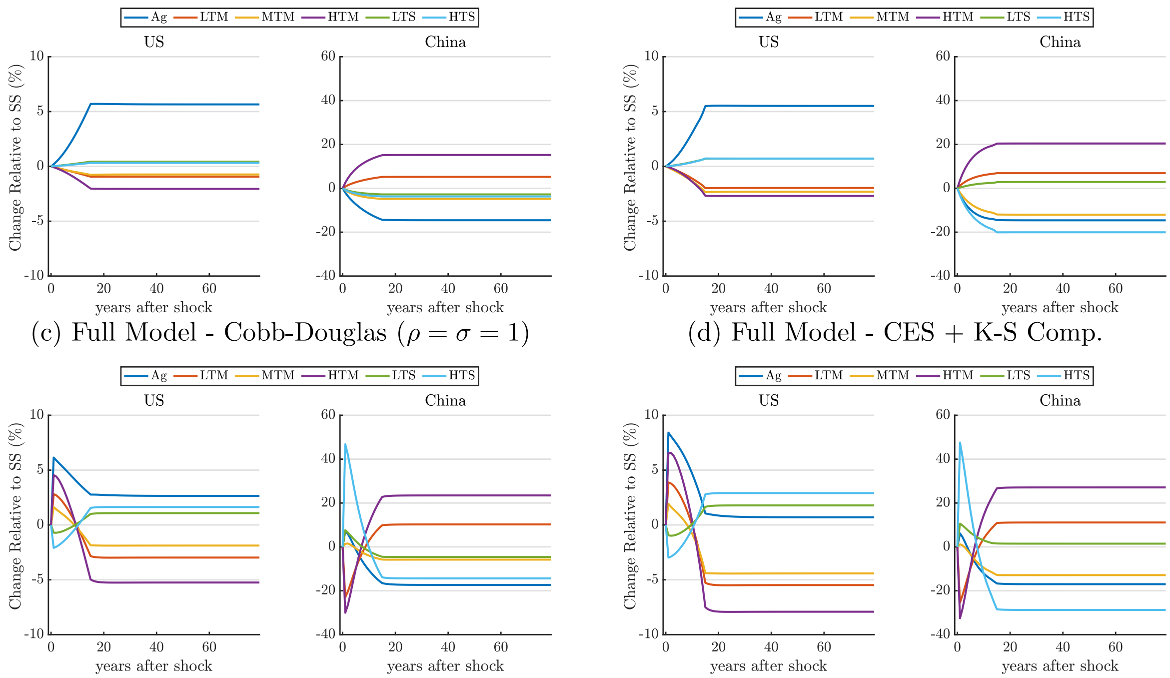
With Cobb-Douglas technology, capital is essentially another input—there are no differences

[13] Recall that $A_{k,i,t}$ is the location parameter of the Frechet productivity distribution. Total Factor Productivity growth in sector k will be approximately $\widehat{A}^{1/\lambda}_{k,China,t}$, where λ is the dispersion term in the distribution.

[14] With Cobb-Douglas production, this is equivalent to augmenting the input-output structure—the coefficient on inputs from sector l into sector k production would become, $(1-\gamma_{k,i})\nu_{kl,i} + \gamma_{k,i} e^{K}_{k,i,2000}\alpha_{l,i}$.

[15] Figure B.1 plots the $\alpha_{k,i}$ and $\mu_{k,i}$ parameters across countries and sectors. One can see the stark difference in China's expenditure on Agriculture relative to other countries, as well as the importance of Low-Tech Services and High-Tech Manufacturing for Chinese capital goods.

Figure 2: Evolution of Output Shares in Response to Slow Chinese Productivity Growth



(a) Balanced Trade - Cobb-Douglas ($\rho = \sigma = 1$)

(b) Balanced Trade - CES + K-S Comp.

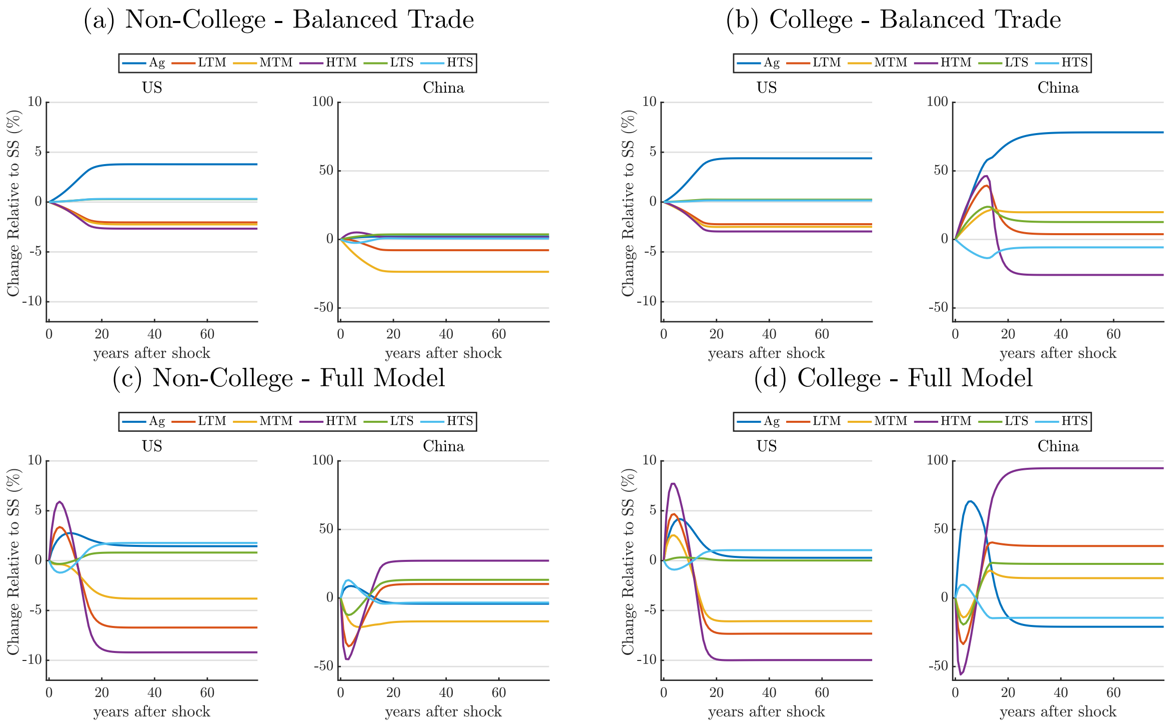
(c) Full Model - Cobb-Douglas ($\rho = \sigma = 1$)

(d) Full Model - CES + K-S Comp.

in substitutability across skills. Figure 2b relaxes this assumption and plots the evolution of output shares when production features capital-skill complementarity. While adjustment patterns remain monotonic, several long-run differences emerge. For example, in China, the output share of Low-Tech Services—which contains the construction sector, a crucial input into capital—increases, whereas it declines as a share of total output in the Cobb-Douglas case. In the US, the presence of capital-skill complementarity mildly amplifies initial patterns of reallocation—services grow by more in shares, and manufacturing shrinks.

Turning to the effects of trade imbalances, Figures 2c and 2d plot output shares for the full model, with Cobb-Douglas and capital-skill complementarity respectively. Long-run allocations across the two technologies are similar to the balanced-trade models. For example, in China, the output share of Low-Tech Services expands in the full model with the technology with capital-skill complementarity, but declines slightly with the Cobb-Douglas technology. More starkly, in both cases the evolution of output is not monotonic. As highlighted in Dix-Carneiro et al. (2021), production patterns undergo reversals. This is on account of the interplay between trade costs and trade imbalances. In the short run, when the US runs a trade deficit, they strongly expand their exports to China. However, as services are relatively non-tradable, they find it most efficient to export goods—agriculture and manufacturing. China, on the other hand, expands Agriculture and High-Tech Services in the short run. In the long run this is reversed—the US expands its services sectors, while China switches into manufacturing.

Figure 3: Labor Reallocation in Response to Slow Chinese Productivity Growth Across Skill Groups – Production with Capital-Skill Complementarity



The above discussion contrasts sharp differences in the short- and long- run predictions for the economy's evolution under the model we developed in section 2 (Figure 2d), and those more standard in the international trade literature (Figure 2a). We now ask what effect, if any, these differences have on worker outcomes. To this end, Figure 3 plots changes in labor allocations for each skill group for the balanced-trade and the full model with imbalances. In both cases, we focus on the CES technology with capital-skill complementarity.

Patterns of labor reallocation generally follow the patterns of output. However, there are noticeable exceptions, especially in China. General equilibrium forces can interact in complicated ways when there are multiple factors of production (Jones and Scheinkman, 1977), which has motivated much of the "quantitative" approach in trade (Costinot and Rodriguez-Clare, 2014). Rather than attempting to comprehensively explain reallocation patterns in response to the shock, we highlight three patterns that are important to understand wages, non-employment and consumption. First, regardless of assumptions on trade imbalances, reallocation patterns are similar across skill groups—especially in the US. Second, in the full model, labor reallocation follows the same non-monotonic patterns as output. Finally, there is more reallocation in the full model than in the model with trade balance. This can be seen through a simple comparison of the scale of the y-axis across the subplots in Figure 3. The amplified reallocation is a direct consequence of moving consumption over time. For example, when China promises the US more consumption in the future, the US becomes willing to dramatically shift production patterns away from the initial steady state, and towards the tradable components of the Chinese consumption basket.

We are now in a position to study the behavior of the skill premium—both in the long run, and along the transition to a new steady state. In our setting, as in Parro (2013), there are two channels through which changes in primitives can affect the skill premium. The first is the demand for skills. Since Stolper and Samuelson (1941) economists have understood that international trade favors abundant factors in each country. In the United States, this would be college educated workers—the ratio of college to non-college workers is 0.39, compared to 0.13 globally. In China, on the other hand, this would be non-college workers—the ratio of college to non-college workers is only 0.05. Trade costs and imbalances complicate the standard textbook story, and generate new dynamics in labor demand relative to a world of balanced trade.

In addition to the labor demand channel, capital-skill complementarity can also play an important role in shaping the skill premium. If global shocks lower the price of capital goods, demand for college workers increases *in all sectors*, pushing up the skill premium regardless of production patterns. Capital goods tend to be intensive in services sectors (for example, construction is part of Low-Tech Services), nevertheless there is a healthy tradable component. Hence, productivity shocks in China can have global repercussions for the price of capital goods. In order to separately gauge these two channels, we also study the evolution of the skill premium in the model with Cobb-Douglas technology—effectively shutting down capital-skill complementarity.

Table 6 contains changes in the skill premium in the short run (measured 4 periods after the shock is unveiled), and in the long run (at the final steady state). The first two columns contain results for the full model, with capital-skill complementarity (K-S) and with Cobb-Douglas (CD) technology respectively; the third and fourth column contain results from the model imposing balanced trade. In the full model, the skill premium initially declines in most countries, but rises dramatically in China. Both trade imbalances and capital-skill complementarity play a role in this response. To see this note that in the fourth column—the model with neither imbalances nor capital-skill complementarity—there is no appreciable change in the skill premium. This echoes findings by Parro (2013), and is in line with the broadly similar reallocation patterns across skill groups in Figure 3. Tellingly, in China there is a sizable impact on the skill premium, and this is where reallocation patterns tended to diverge. Adding capital-skill complementarity, but maintaining balanced trade, leads to a small increase in the skill premium in most places, and a large increase in the skill premium in China. This is a direct result of the lower cost of capital goods.

Adding back endogenous trade imbalances but maintaining Cobb-Douglas technology generates more sizable changes in the skill premium. This is because with imbalances, initial patterns of reallocation are amplified. This amplification makes the labor demand channel more important. In the US, there is a burst of movement into Agriculture and manufacturing. These industries are

Table 6: Changes (in %) in the Skill Premium in Response to Slow Chinese Productivity Growth

	Full Model		Balanced Trade	
	K-S	CD	K-S	CD
Panel A: Short Run ($t = 4$)				
US	-0.73	-0.37	0.07	-0.00
China	168.70	8.50	108.79	-0.63
Europe	-0.75	-0.45	0.10	-0.00
Asia/Oc.	-1.38	-1.01	0.15	-0.01
Americas	-0.25	-0.29	0.22	0.05
RoW	-0.25	-0.29	0.22	0.05
Panel B: Long Run				
US	1.10	0.22	0.36	-0.05
China	419.75	-4.43	421.35	-1.15
Europe	1.43	0.28	0.63	-0.06
Asia/Oc.	2.22	0.62	0.74	-0.06
Americas	2.68	0.51	1.67	0.13
RoW	2.68	0.51	1.67	0.13

Notes: K-S stands for model with capital-skill complementarity. CD stands for model with Cobb-Douglas production. The skill premium is measured as the wage of College workers relative to the wage of Non-College workers.

less skill intensive than the services sectors, and thus this movement favors Non-College workers.[16]

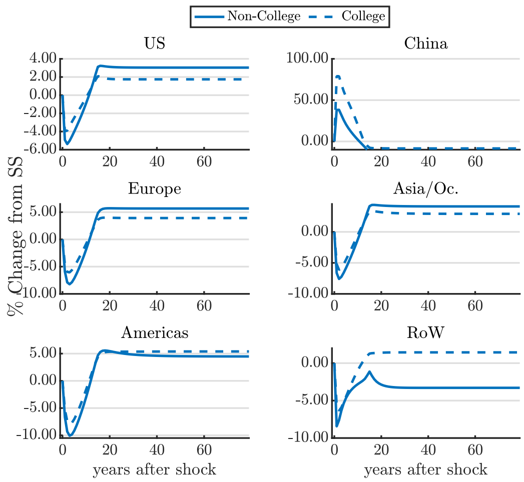
In the full model where we add back capital-skill complementarity, the skill premium initially declines by even more outside of China. This is puzzling since real capital expenditure rises on impact. However, with capital-skill complementarity, imbalances are magnified: the initial surpluses outside China are larger, and the labor demand channel *dominates* the effect of changes in capital prices. That is to say, there is even *more* reallocation towards industries that are intensive in Non-College workers. This is an important point of contrast with the findings of Parro (2013), who found a very limited role for industry composition to affect the skill premium. In China, in contrast with its partners, the skill premium is driven up even further because the declining cost of capital and the reallocation channel move in the same direction.

The long- and short- run impacts on the skill premium diverge in our full model, but not in a model ignoring trade imbalances. To the latter point, the third and fourth columns of Panel B of Table 6 show that under balanced trade, initial changes in the skill premium are magnified in the long-run, regardless of technological assumptions. However, in our full model, the changing composition of production reverses the short-run changes in the skill premium. For example, the US experiences a long-run rise in the skill premium, which is amplified by a factor of 5 in a model with

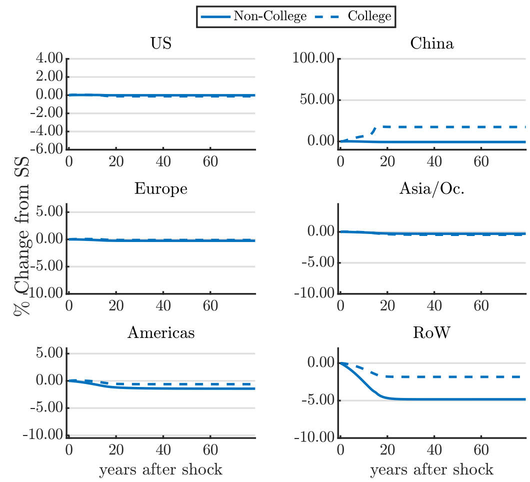
[16] For example, in the US, the initial ratio of expenditure on College to Non-College workers is 1.14 in High Tech Services, but only 0.25 in Agriculture.

Figure 4: Change in the Non-Employment Rate w.r.t the Initial Steady State (Proportional)

(a) Full Model



(b) Balanced Trade



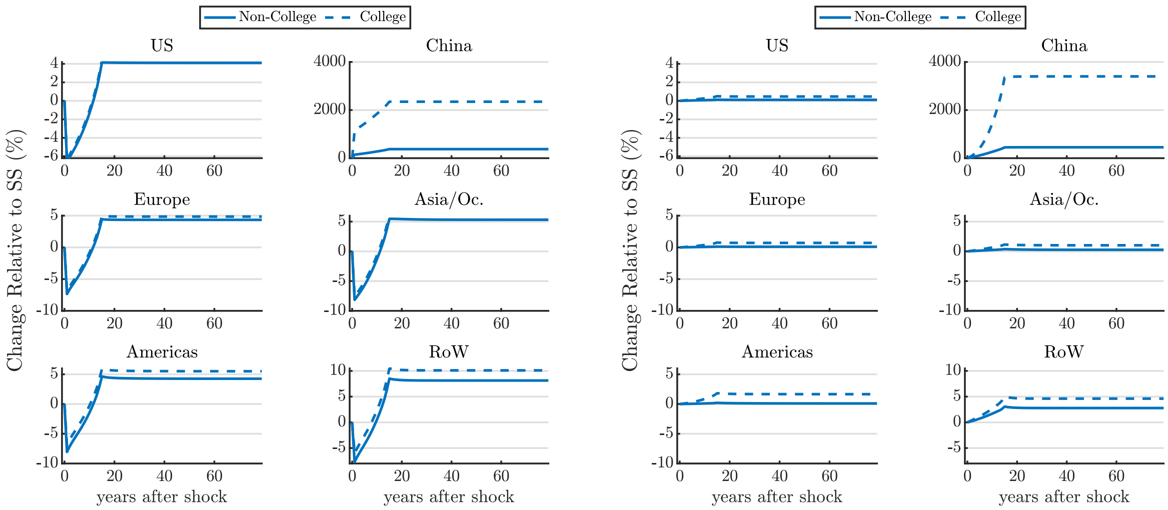
versus without capital skill complementarity. In China, the initial rise in the skill premium is fully reversed in a world without capital-skill complementarity. With this channel turned on, the skill premium rises even further in the long run—but less so than in the model without trade imbalances, highlighting that the capital-skill-complementarity channel and the labor-demand channel oppose each other. In short, trade imbalances and capital-skill complementarity together lead to dynamics for the skill premium after a trade shock that are distinct from allowing for either ingredient separately.

Welfare depends on more than just income, so we now describe the implications of the shock for non-employment and consumption across skill groups. Focusing first on non-employment, Figures B.3a and B.3b plot the evolution of non-employment in the full model and in the model with balanced trade, respectively. We highlight two patterns: (1) non-employment responses are much larger in a world with trade imbalances—for example, in the US, peak growth in non-employment is 4% in our Full Model, while it is negligibly different from zero with balanced trade; (2) Non-College workers tend to respond more than College workers—notable exceptions are the Rest of the World aggregate and China in the case of trade balance.

To understand these patterns note that the non-employment response in our model is governed by two forces. One is inter-sectoral reallocation, which shifts workers' expected mobility costs and sector-specific utilities in a way that can make non-employment more or less attractive. For example, shifting demand to high-η sectors will tend to lower non-employment rates. The second force is changes in marginal-utility-adjusted real wages, $\widetilde{\lambda}\times w$—which we shorten to real wages in this discussion. Changes in real wages can explain why the full model generates a larger non-employment response relative to the case with trade balance: consumption expenditure adjusts immediately,

Figure 5: Consumption Evolution After Productivity Shock in China

(a) Full Model (b) Balanced Trade



while wages adjust gradually.[17] Noting that $\widetilde{\lambda}^s_{i,t} = \frac{\overline{L}^s_i}{E^{C,s}_{i,t}}$, where $E^{C,s}_{i,t}$ is total expenditure of skill group s on final goods consumption, this generates dynamics in real wages that are very different with and without trade imbalances.[18]

The outsized response of Non-College to College workers is curious in light of the fact that the parameter governing mobility across sectors (including non-employment), ζ, is common across groups. Two mechanisms in the model explain this pattern. The first, as discussed above, is that wages, and hence real wages, move differently for the two groups. The second is that for most countries (the Rest of the World being an important exception), non-employment rates in steady state are higher for Non-College workers. Hence, there are more marginal workers that can be drawn in and out of employment.[19]

Finally, we discuss consumption inequality across skill groups. While our model features an assortment of shocks and non-pecuniary utility terms (the mobility costs, C, sector-specific utilities, η, and the idiosyncratic preference shocks, $\boldsymbol{\omega}$), we focus on real consumption in lieu of total welfare. This is because the other terms reflect an array of un-modeled frictions and benefits, which are

[17] To see that expenditures adjust immediately, note that equation (7) with no inter-temporal shocks implies that $\frac{E^{C,s}_{i,t+1}}{E^{C,s}_{i,t}} = \delta R_{t+1}$, where $E^{C,s}_{i,t}$ is the total expenditure of skill group s on final goods. Normalizing $\sum_i \sum_s E^{C,s}_{i,t} = 1$, so that all aggregate variables are expressed in terms of world final good expenditures, we obtain $E^{C,s}_{i,t+1} = E^{C,s}_{i,t}$, so that final good expenditures within each skill group, measured as a fraction of world expenditures, immediately adjust to any revealed path of shocks.

[18] Figure B.4 in Appendix B plots the change in $\widetilde{\lambda} \times w$ over time. Besides China there is a clear symmetry in the response of non-employment and the marginal-utility-adjust wage. In the case of China, the reallocation effects are very large and dominate the real wage channel.

[19] More formally, given our GEV preferences, the change in flows into and out of non-employment after a shock will be proportional to the initial flows multiplied by the changes in the value of each sector. For Non-College workers in most countries, the initial flows into (out of) non-employment are larger (smaller) on average.

hard to interpret. Moreover, consumption is the typical object of study in the International Trade literature. Figures 5a and 5a plot real consumption after the shock. Chinese consumption grows dramatically, as they are the main recipients of the productivity boost.[20] Comparing across models makes clear the trade-off facing China with access to bonds: the short-run rise in consumption is sudden and dramatic when China can borrow, whereas it is slow and protracted when they cannot; in the long-run, consumption tapers off at a lower level as China must repay their debt. Consumption in other countries on the other hand declines as they lend to China, and then rises in the long run. Notice that the long run change in consumption is much larger for lender countries in the full model versus the model with balanced trade—in the US, consumption growth is a full order of magnitude higher.

The presence of bond markets has strong implications for welfare inequality, as workers of different skill levels can insure themselves within countries. Thus, in the full model, the shock increases consumption inequality everywhere, but this jump is small and immediate. This is because, with perfect foresight, all agents immediately smooth to a new level of consumption expenditure, EC^s_{it}, but since local price indices, $P^F_{i,t}$, do not vary across skill groups within a country, the gap in expenditure between skill groups is the same as the gap in real consumption. In a world of balanced trade, the dynamics of consumption inequality are very different: there is an ever-widening gap between groups, and this gap is larger than when workers can insure each other temporally.

In this section, we have shown how a simple shock—a secular increase in productivity in one country—can lead to rich dynamics in allocations, wage inequality, non-employment, and consumption inequality. More importantly, despite these complex dynamics, much can be understood as a straight-forward consequence of shifting consumption over time and space. In the next section, we consider a more complicated shock, the so-called "China Shock" and one less amenable to the above analysis. Nevertheless, we hope this section has made a convincing case that imbalances and capital-skill complementarity will play a crucial, quantitative role in shaping workers' experiences after a trade shock.

5 The China Shock

In this section, coming soon, we extract shocks in productivities, trade costs, and inter-temporal preferences the economy experienced between 2000 and 2014. To do so, we use the same methods as Dix-Carneiro et al. (2021). Next, we study the implications of the China shock for the US trade

[20]The numbers for Chinese College workers almost strain credulity, but are a consequence of capital-skill complementarity and growth in China. Total consumption grows by a factor of ~ 650, and by less in our Cobb-Douglas model. Given the size of the shock, this is in line with simulations from Dix-Carneiro et al. (2021). The reason that skilled workers enjoy such high growth in China is because they are a very small share of the workforce, so the capital-skill complementarity is particularly valuable to them. In ongoing work we are pursuing robustness with respect to σ and ρ, to understand how cross-country heterogeneity in these parameters may affect our results.

deficit, labor allocations across sectors, the skill premium and consumption inequality.

6 Concluding Remarks

Coming Soon.

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Appendix

A Additional Tables

Table A.1: Country Definitions

1	USA
2	China
3	Europe
4	Asia/Oceania
5	Americas
6	Rest of the World (ROW)

Notes: Asia/Oceania = {Australia, Japan, South Korea, Taiwan}, Americas = {Brazil, Canada, Mexico}, Rest of the World ={Indonesia, India, Russia, Turkey, Rest of the World}. This partition of the world was dictated by data availability from the World Input Output Database.

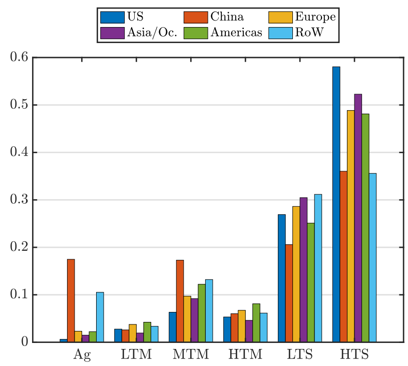
Table A.2: Sector Definitions

1	Agriculture/Mining	Agriculture, Forestry and Fishing; Mining and quarrying
2	Low-Tech Manufacturing	Wood products; Paper, printing and publishing; Coke and refined petroleum; Basic and fabricated metals; Other manufacturing
3	Mid-Tech Manufacturing	Food, beverage and tobacco; Textiles; Leather and footwear; Rubber and plastics; Non-metallic mineral products
4	High-Tech Manufacturing	Chemical products; Machinery; Electrical and optical equipment; Transport equipment
5	Low Tech Services	Utilities; Construction; Wholesale and retail trade; Transportation; Accommodation and food service activities; Activities of households as employers
6	Hi Tech Services	Publishing; Media; Telecommunications; Financial, real estate and business services; Government, education, health

B Additional Figures

Figure B.1: Aggregation Parameters in 2000

(a) Consumption: $\mu_{k,i}$



(b) Capital: $\alpha_{k,i}$

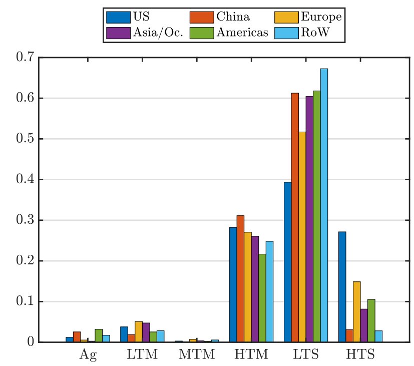
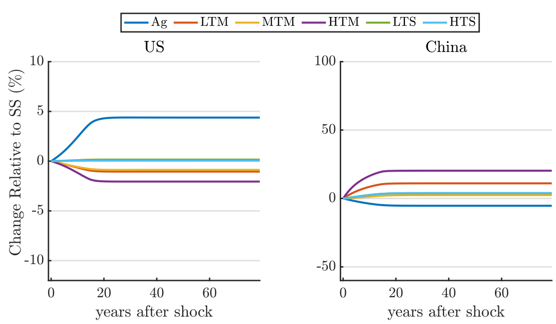
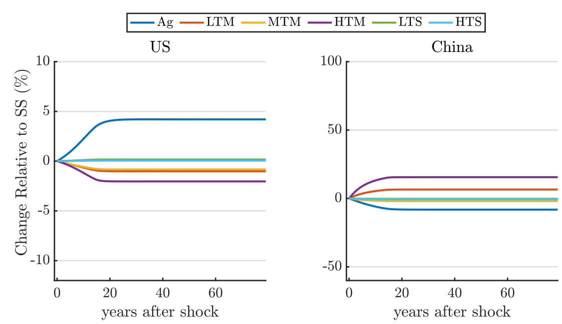


Figure B.2: Labor Reallocation in Response to Slow Chinese Productivity Growth Across Skill Groups – Cobb-Douglas Production

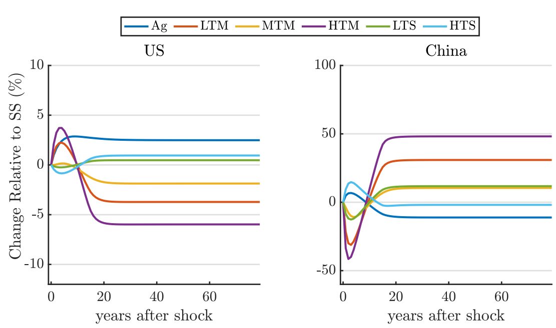
(a) Non-College - Balanced Trade



(b) College - Balanced Trade



(c) Non-College - Full Model



(d) College - Full Model

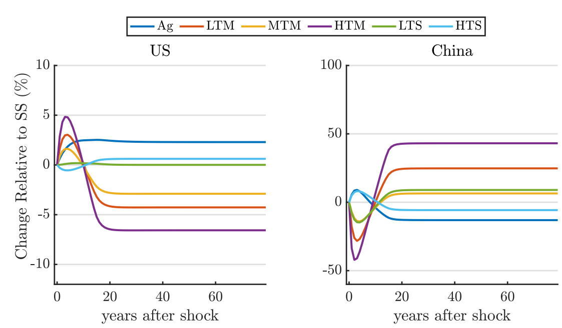


Figure B.3: Change in the Non-Employment Rate w.r.t the Initial Steady State (Proportional)

(a) Full Model (b) Balanced Trade

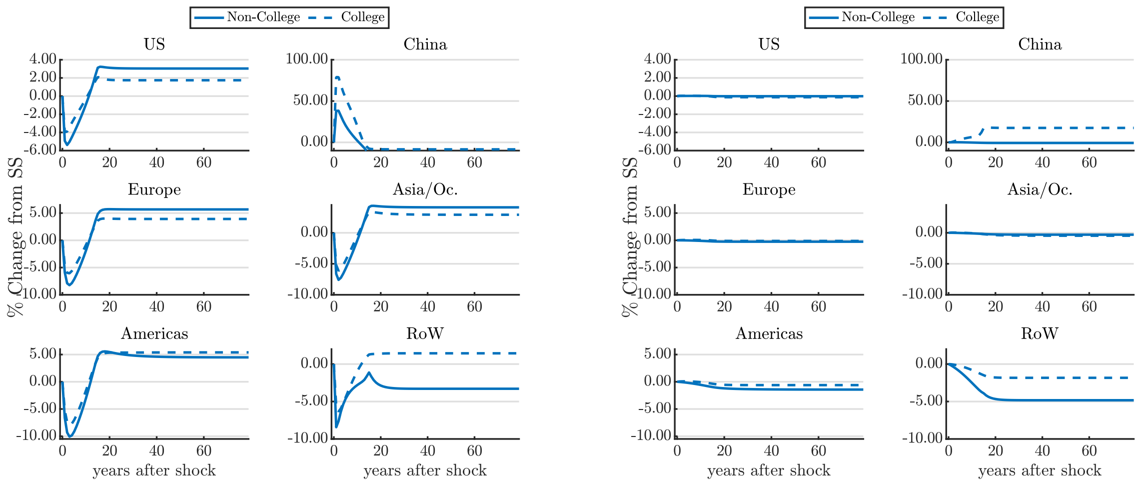
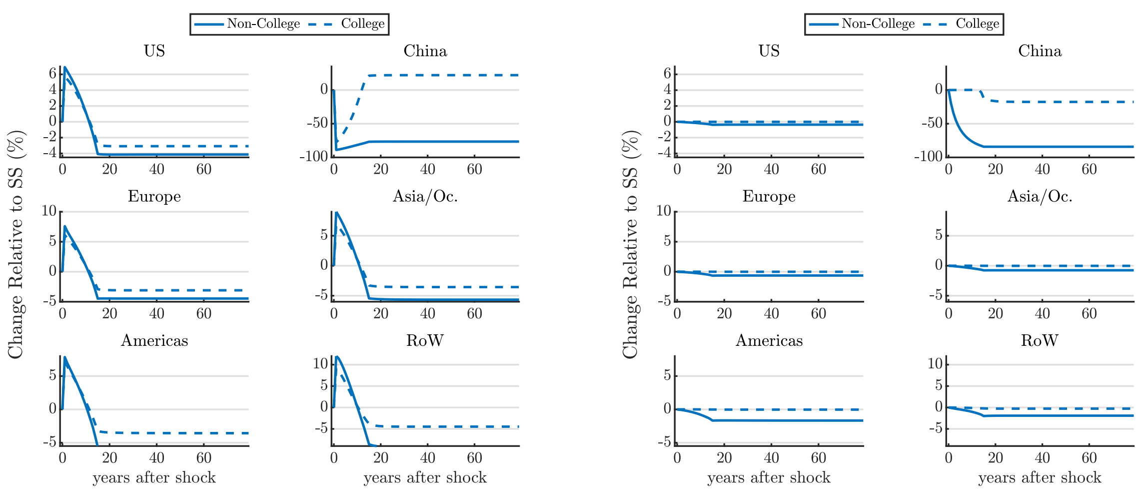


Figure B.4: Change in Marginal Utility Adjusted Wages

(a) Full Model - CES (b) Balanced Trade - CES



C Additional Parameter Estimates

In this section, we display the parameters displayed in Panel B of Table 1.

Table C.1: Final Expenditure Shares $\mu_{k,i}$

			Country			
Sector	US	China	Europe	Asia/Oc.	Americas	RoW
Agr.	0.01	0.17	0.02	0.01	0.02	0.11
LT Manuf.	0.03	0.03	0.04	0.02	0.04	0.03
MT Manuf.	0.06	0.17	0.10	0.09	0.12	0.13
HT Manuf.	0.05	0.06	0.07	0.05	0.08	0.06
LT Serv.	0.27	0.21	0.29	0.30	0.25	0.31
HT Serv.	0.58	0.36	0.49	0.52	0.48	0.36

Table C.2: Capital Good Shares $\alpha_{k,i}$

			Country			
Sector	US	China	Europe	Asia/Oc.	Americas	RoW
Agr.	0.01	0.03	0.01	0.00	0.03	0.02
LT Manuf.	0.04	0.02	0.05	0.05	0.03	0.03
MT Manuf.	0.00	0.00	0.01	0.00	0.00	0.01
HT Manuf.	0.28	0.31	0.27	0.26	0.22	0.25
LT Serv.	0.39	0.61	0.52	0.60	0.62	0.67
HT Serv.	0.27	0.03	0.15	0.08	0.11	0.03

Table C.3: Labor Shares in Production $\gamma_{k,i}$

	Country					
Sector	US	China	Europe	Asia/Oc.	Americas	RoW
Agr.	0.45	0.58	0.56	0.54	0.62	0.67
LT Manuf.	0.37	0.25	0.32	0.35	0.28	0.27
MT Manuf.	0.33	0.28	0.31	0.37	0.32	0.28
HT Manuf.	0.39	0.24	0.33	0.32	0.31	0.25
LT Serv.	0.61	0.37	0.49	0.54	0.56	0.48
HT Serv.	0.62	0.55	0.63	0.67	0.67	0.68

Table C.4: Input-Output Table – Averages Across Countries $\frac{1}{N}\sum_{i=1}^{N}\nu_{k\ell,i}$, Standard Dev. across Countries in Parentheses.

User ↓ Supplier →	Agr.	LT Manuf.	MT Manuf.	HT Manuf.	LT Serv.	HT Serv.
Agr.	0.27	0.08	0.12	0.14	0.26	0.14
	(0.05)	(0.02)	(0.03)	(0.03)	(0.05)	(0.06)
LT Manuf.	0.19	0.38	0.04	0.08	0.22	0.08
	(0.04)	(0.06)	(0.01)	(0.01)	(0.04)	(0.04)
MT Manuf.	0.22	0.07	0.29	0.11	0.22	0.09
	(0.03)	(0.02)	(0.04)	(0.02)	(0.05)	(0.04)
HT Manuf.	0.02	0.16	0.07	0.46	0.18	0.11
	(0.02)	(0.02)	(0.01)	(0.05)	(0.04)	(0.05)
LT Serv.	0.06	0.14	0.10	0.10	0.34	0.26
	(0.04)	(0.03)	(0.02)	(0.03)	(0.07)	(0.10)
HT Serv.	0.01	0.08	0.03	0.11	0.27	0.51
	(0.00)	(0.02)	(0.02)	(0.06)	(0.06)	(0.16)

D Solution Methods

D.1 Inner Loop on η for Calibration

Note that this can be performed separately by country and skill group.

Step 1. Given data on labor supply and gross output, parameters estimated outside the procedure, a current guess of ψ_i, and a current guess of $\eta_i^{(g)}$, construct the mobility costs:

$$C_i^s = C_{US}^s \times \psi.$$

Step 2. Solve the Bellman equation across sectors using wages computed from data on worker expenditure shares, labor allocations, and the model-implied allocation of gross output, $w_{k,i}^{s,0} = \frac{\gamma_{k,i} e_{k,i}^{s,Data} Y_{k,i}}{L_{k,i}^{s,Data}}$:

$$V_{k,i}^s = \widetilde{\lambda}_i^s w_{k,i}^{s,0} + \eta_{k,i}^{s,(g)} + \zeta_i \log\left(\sum_{k''=0}^{K} \exp\left(\frac{-C_{kk'',i}^s + \delta V_{k'',i}^s}{\zeta_i}\right)\right)$$

Step 3. Solve for the transition rule for workers given:

$$s_{kk',i}^s = \frac{\exp\left(\frac{-C_{kk',i}^s + \delta V_{k',i}^s}{\zeta_i}\right)}{\sum_{k''=0}^{K} \exp\left(\frac{-C_{kk'',i}^s + \delta V_{k'',i}^s}{\zeta_i}\right)}.$$

Step 4. Solve for the implied supply of labor, $\tilde{L}_{k,i}^s$, by solving for the unit eigenvector of s and scaling appropriately. I.e., solve

$$[s_{kk',i}^s]' \tilde{L}^s = \tilde{L}^s$$

subject to $\langle \mathbf{1}_{K+1}, \tilde{L}^s \rangle = \bar{L}^s$, where $\tilde{L}^s$ is a $K+1$-vector, $\mathbf{1}_{K+1}$ is a $K+1$-vector of ones, and $\langle , \rangle$ is the dot product.

Step 5. Solve for the *model* implied wage:

$$w_{k,i}^{s,MODEL} = \frac{\gamma_{k,i} e_{k,i}^s Y_{k,i}}{L_{k,i}^s}$$

Step 6. Update η according to the gap in wages:

$$\eta^{(g+1)} = \eta^{(g)} + \mathcal{D} \times (\log(w^{MODEL} - w^{0x}),$$

where $\mathcal{D}$ is a tuning parameter that prevents overly large jumps in η.

Step 7. If $||\eta^{(g+1)} - \eta^{(g)}|| < tol$, terminate. Else return to step 2.

D.2 Solving for the Initial Steady State

In order to solve for the initial steady state equilibrium we first need data on the following:

Symbol	Description
μ	Consumption Expenditure Shares
α	Capital Expenditure Shares
γ	Value-Added Shares
ν	Input-Output Matrix
e^K	Capital Expenditure in VA
e^S	Skilled Labor Expenditure in VA
e^U	Unskilled Labor Expenditure in VA
ϱ	Share of Skilled Consumption in National Consumption
π	Trade Matrices
NX	Trade Imbalances

From here we can solve for the initial steady state *country-by-country*. We do so according to the following algorithm:

Step 1. Given initial data solve for gross output according to

$$\begin{aligned} Y_{k,o} = & \sum_{i=1}^{N} \sum_{l=1}^{K} \pi_{k,oi}^{Data} \times \left[\mu_{k,i}\gamma_{l,i}(1-e_{l,i}^K) + \alpha_{k,i}\gamma_{l,i}e_{l,i}^K + (1-\gamma_{li})\nu_{lk,i}\right] Y_{l,i} \\ & - \sum_{i=1}^{N} \pi_{k,oi}^{Data}\mu_{k,i}NX_i \end{aligned}$$

and the additional restriction,

$$\sum_{k,o} Y_{k,o} = 1.$$

Step 2. Solve for consumption expenditure in each country by skill group. To do so, first calculate total consumption expenditure in each country:

$$E_i^C = \sum_{k=1}^{K} \gamma_{k,i}(1-e_{k,i}^K)Y_{k,i} - NX_i.$$

Then allocate this to each skill group according to,

$$\begin{aligned} E_i^{C,S} =& \varrho_i EC_i \\ EC_i^U =& (1-\varrho_i) EC_i. \end{aligned}$$

1. Calculate the Lagrange multiplier of each household:

$$\widetilde{\lambda}_i^s = L_i^s / E_i^{C,s}$$

   Steps 1 - 3 do not change within the algorithm to compute steady state. The loop below can be done country-by-country.

2. Guess $\{L_{k,i}^s\}$.

3. Given initial data and gross output solve for wages:

$$w_{k,i}^s = \frac{\gamma_{k,i} e_{k,i}^s Y_{k,i}}{L_{k,i}^s}$$

4. Solve the Bellman equation for each worker's labor supply decision:

$$V_{k,i}^s = \widetilde{\lambda}_i^s w_{k,i}^s + \eta_{k,i}^s + \zeta_i \log\left(\sum_{k''=0}^{K} \exp\left(\frac{-C_{kk'',i}^s + \delta V_{k'',i}^s}{\zeta_i}\right)\right)$$

5. Solve for the transition rule for workers given:

$$s_{kk',i}^s = \frac{\exp\left(\frac{-C_{kk',i}^s + \delta V_{k',i}^s}{\zeta_i}\right)}{\sum_{k''=0}^{K} \exp\left(\frac{-C_{kk'',i}^s + \delta V_{k'',i}^s}{\zeta_i}\right)}.$$

6. Solve for the implied supply of labor, $\tilde{L}_{k,i}^s$, by solving for the unit eigenvector of s and scaling appropriately. I.e., solve

$$[s_{kk',i}^s]' \tilde{L}^s = \tilde{L}^s$$

   subject to $\langle \mathbf{1}_{K+1}, \tilde{L}^s \rangle = \bar{L}^s$, where $\tilde{L}^s$ is a $K+1$-vector, $\mathbf{1}_{K+1}$ is a $K+1$-vector of ones, and $\langle , \rangle$ is the dot product.

7. Update the guess of labor using a penalty:

$$L_{k,i}^{s,New} = pL_{k,i}^s + (1-p)\tilde{L}_{k,i}^s$$

8. Go back to Step 4 with the new guess of $L^s_{k,i}$ until $||\tilde{L}^s_{k,i} - L^s_{k,i}|| < tol$, for some tolerance.

D.3 Algorithm for New Steady State

We will refer to $t = 0$ as the initial steady state and $t = 1$ as the final steady state.

This algorithm normalizes world output to 1: $\sum_{k=1}^{K}\sum_{i=1}^{N} Y_{k,i,1} = 1$.

Step 0: We start with an initial steady state characterized by $\{\pi_{k,oi,0}\}$, $\left\{e^U_{k,i,0}\right\}$, $\left\{e^S_{k,i,0}\right\}$, $\left\{e^K_{k,i,0}\right\}$, $\left\{NX^U_{i,0}\right\}$, $\left\{NX^S_{i,0}\right\}$. Consider a change in (long-run) productivity $\widehat{A}_{k,i} \equiv \frac{A_{k,i,1}}{A_{k,i,0}}$ and trade costs $\widehat{d}_{k,oi} \equiv \frac{d_{k,oi,1}}{d_{k,oi,0}}$.

Step 1: Guess new steady-state allocations $\left\{L^U_{k,i,1}\right\}$ and $\left\{L^S_{k,i,1}\right\}$.

Step 2: Guess new steady-state wages $\left\{w^U_{k,i,1}\right\}$ and $\left\{w^S_{k,i,1}\right\}$.

Step 3: Compute wage changes $\widehat{w}^U_{k,i} = \frac{w^U_{k,i,1}}{w^U_{k,i,0}}$, $\widehat{w}^S_{k,i} = \frac{w^S_{k,i,1}}{w^S_{k,i,0}}$.

Step 4: Obtain trade shares $\{\pi_{k,oi,1}\}$.
First, iteratively solve the system below, conditional on $\widehat{w}^U_{k,i}$ and $\widehat{w}^S_{k,i}$:

$$\widehat{P}^K_i = \prod_{k=1}^{K} \left(\widehat{P}^I_{k,i}\right)^{\alpha_{k,i}},$$

$$\widehat{p}^h_{k,i} = \left[\frac{e^K_{k,i,0}}{1 - e^U_{k,i,0}} \left(\widehat{P}^K_i\right)^{1-\rho} + \frac{e^S_{k,i,0}}{1 - e^U_{k,i,0}} \left(\widehat{w}^S_{k,i}\right)^{1-\rho}\right]^{\frac{1}{1-\rho}},$$

$$\widehat{p}^v_{k,i} = \left[e^U_{k,i,0} \left(\widehat{w}^U_{k,i}\right)^{1-\sigma} + \left(1 - e^U_{k,i,0}\right) \left(\widehat{p}^h_{k,i}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}},$$

$$\widehat{c}_{k,i} = \left(\widehat{p}^v_{k,i}\right)^{\gamma_{k,i}} \prod_{l=1}^{K} \left(\widehat{P}^I_{l,i}\right)^{(1-\gamma_{k,i})\nu_{kl,i}},$$

$$\widehat{P}^I_{k,i} = \left[\sum_{o=1}^{N} \pi_{k,oi,0} \widehat{A}_{k,o} \left[\widehat{c}_{k,o} \widehat{d}_{k,oi}\right]^{-\lambda}\right]^{-1/\lambda}.$$

Compute $\widehat{\pi}_{k,oi} = \widehat{A}_{k,o} \left(\frac{\widehat{c}_{k,o}\widehat{d}_{k,oi}}{\widehat{P}^I_{k,i}}\right)^{-\lambda}$.

And finally, $\pi_{k,oi,1} = \pi_{k,oi,0} \times \widehat{\pi}_{k,oi}$.

Step 5: Compute $\left\{e^K_{k,i,1}\right\}$, $\left\{e^U_{k,i,1}\right\}$, $\left\{e^S_{k,i,1}\right\}$.

First, obtain changes:

$$\widehat{e}^{K}_{k,i} = \frac{\left[\widehat{p}^{h}_{k,i}\right]^{\rho-\sigma}\left[\widehat{P}^{K}_{i}\right]^{1-\rho}}{\left[\widehat{p}^{v}_{k,i}\right]^{1-\sigma}},$$

$$\widehat{e}^{S}_{k,i} = \frac{\left[\widehat{p}^{h}_{k,i}\right]^{\rho-\sigma}\left[\widehat{w}^{S}_{k,i}\right]^{1-\rho}}{\left[\widehat{p}^{v}_{k,i}\right]^{1-\sigma}},$$

$$\widehat{e}^{U}_{k,i} = \frac{\left[\widehat{w}^{U}_{k,i}\right]^{1-\sigma}}{\left[\widehat{p}^{v}_{k,i}\right]^{1-\sigma}}.$$

Then obtain final levels.

$$e^{K}_{k,i,1} = e^{K}_{k,i,0} \times \widehat{e}^{K}_{k,i},$$

$$e^{S}_{k,i,1} = e^{S}_{k,i,0} \times \widehat{e}^{S}_{k,i},$$

$$e^{U}_{k,i,1} = e^{U}_{k,i,0} \times \widehat{e}^{U}_{k,i}.$$

Step 6: Solve for $\{Y_{k,i,1}\}$.

$$\begin{aligned} Y_{k,o,1} = &\sum_{i=1}^{N}\sum_{l=1}^{K} \pi_{k,oi,1}\left(\mu_{k,i}\gamma_{l,i}\left(1-e^{K}_{l,i,1}\right) + \alpha_{k,i}\gamma_{l,i}e^{K}_{l,i,1} + \left(1-\gamma_{l,i}\right)\nu_{lk,i}\right)Y_{l,i,1} \\ &- \sum_{i=1}^{N}\pi_{k,oi,1}\mu_{k,i}NX_{i,1}, \end{aligned}$$

$$\sum_{k=1}^{K}\sum_{i=1}^{N}Y_{k,i,1} = 1.$$

Step 7: Update wages:

$$\left(w^{U}_{k,i,1}\right)^{new} = \frac{e^{U}_{k,i,1}\gamma_{k,i}Y_{k,i,1}}{L^{U}_{k,i,1}},$$

$$\left(w^{S}_{k,i,1}\right)^{new} = \frac{e^{S}_{k,i,1}\gamma_{k,i}Y_{k,i,1}}{L^{S}_{k,i,1}}.$$

Go back to Step 3 until convergence of wages.

Step 8: Compute aggregate expendtures.

$$E^{C,U}_{i,1} = \sum_{k=1}^{K} w^{U}_{k,i,1}L^{U}_{k,i,1} - NX^{U}_{i,1},$$

$$E_{i,1}^{C,S} = \sum_{k=1}^{K} w_{k,i,1}^{S} L_{k,i,1}^{S} - NX_{i,1}^{S}.$$

Step 9: Compute Lagrange multipliers $\widetilde{\lambda}_{i,1}^{U} = \frac{\overline{L}_i^U}{E_{i,1}^{C,U}}$, $\widetilde{\lambda}_{i,1}^{S} = \frac{\overline{L}_i^S}{E_{i,1}^{C,S}}$.

Step 10: Solve the Bellman Equations $\left\{V_{k,i,1}^{s}\right\}$:

$$\left(V_{k,i,1}^{s}\right)^{g+1} = \widetilde{\lambda}_{i,t}^{s} w_{k,i,T_{SS}}^{s} + \eta_{k,i}^{s} + \zeta_i \log\left(\sum_{l=0}^{K} \exp\left(\frac{-C_{kl,i}^{s} + \delta\left(V_{l,i,1}^{s}\right)^{g} - \delta\left(V_{k,i,1}^{s}\right)^{g}}{\zeta_i}\right)\right) + \delta\left(V_{k,i,1}^{s}\right)^{g}.$$

Step 11: Update allocations $L_{k,i,1}$.

Compute transition rates for $s = U, S$:

$$s_{kl,i,1}^{s} = \frac{\exp\left(\frac{-C_{kl,i}^{s} + \delta V_{l,i,1}^{s}}{\zeta_i}\right)}{\sum_{k'=0}^{K} \exp\left(\frac{-C_{kk',i}^{s} + \delta V_{k',i,1}^{s}}{\zeta_i}\right)}.$$

Obtain their implied steady-state allocations by computing $\left(s_{kl,i,1}^{s}\right)^{\infty}$. Multiply these shares by $\overline{L}_i^s$ to obtain $\left(L_{k,i,1}^{s}\right)'$, and update:

$$\left(L_{k,i,1}^{s}\right)^{new} = \left(1 - \lambda_L\right) L_{k,i,1}^{s} + \lambda_L \left(L_{k,i,1}^{s}\right)'.$$

Step 12: Armed with $\left(L_{k,i,1}^{s}\right)^{new}$ go to Step 2 until $dist\left(L_{k,i,1}^{s}, \left(L_{k,i,1}^{s}\right)'\right)$ is very small.

D.4 Algorithm for Out of Steady State Transitional Dynamics

Outer Loop: Interation on $\left\{NX_{i,t}^{U}\right\}$ and $\left\{NX_{i,t}^{S}\right\}$

Step 0: Consider a given path of shocks $\left\{\widehat{A}_{k,i,t}\right\}_{t=0}^{T_{SS}}$, $\left\{\widehat{d}_{k,oi,t}\right\}_{t=0}^{T_{SS}}$ where $\widehat{A}_{k,i,t} \equiv \frac{A_{k,i,t}}{A_{k,i,0}}$ and $\widehat{d}_{k,oi,t} \equiv \frac{d_{k,oi,t}}{d_{k,oi,0}}$. World expenditure on final goods is normalized to 1 in every period: $\sum_{i=1}^{N}\sum_{s=U,S} E_{i,t}^{C,s} = 1$ for all t.

Step 1: Start with the estimated equilibrium at $t=0$. Change the normalization from $\sum_i \sum_k Y_{k,i} = 1$ to $\sum_{i=1}^{N}\sum_{s=U,S} E_{i}^{C,s} = 1$ for all nominal variables. Variables to be re-normalized: $\left\{E_{i,0}^{C,U}\right\}$, $\left\{E_{i,0}^{C,S}\right\}$, $\{Y_{k,i,0}\}$, $\left\{w_{k,i,0}^{U}\right\}$, $\left\{w_{k,i,0}^{S}\right\}$, $\left\{NX_{i,0}^{U}\right\}$, $\left\{NX_{i,0}^{S}\right\}$.

Step 2: Obtain $\left\{B_{i,0}^{U}\right\}$ and $\left\{B_{i,0}^{S}\right\}$ under normalization $\sum_{i=1}^{N}\sum_{s=U,S} E_{i,t}^{C,s} = 1$.

$$B_{i,0}^{s} = \frac{NX_{i,0}^{s}}{\left(1-\frac{1}{\delta}\right)}.$$

Step 3: Make initial guesses for $\left\{NX_{i,T_{SS}}^{U}\right\}_{i=1}^{N}$, $\left\{NX_{i,T_{SS}}^{S}\right\}_{i=1}^{N}$ (under normalization $\sum_{i=1}^{N}\sum_{s=U,S} E_{i}^{C,s} = 1$)

Step 4: Solve for the new steady state at T_{SS} conditional on $\left\{NX_{i,T_{SS}}^{U}\right\}_{i=1}^{N}$, $\left\{NX_{i,T_{SS}}^{S}\right\}_{i=1}^{N}$ and the change in parameter values $\left\{\widehat{A}_{k,i,T_{SS}}\right\}$, $\left\{\widehat{d}_{k,oi,T_{SS}}\right\}$.

Notice that the steady-state algorithm uses the normalization $\sum_i \sum_k Y_{k,i} = 1$. Within this algorithm, re-normalize $\left\{NX_{i,T_{SS}}^{U}\right\}_{i=1}^{N}$, $\left\{NX_{i,T_{SS}}^{S}\right\}_{i=1}^{N}$ with respect to this normalization. To perform such normalization, use revenues $\{Y_{k,i,0}\}$ in the initial steady state if this is the first outer loop iteration. Otherwise, use revenue $\{Y_{k,i,T_{SS}}\}$ obtained in Step 6 below.

After computing the final steady state, change the normalization from $\sum_i \sum_k Y_{k,i} = 1$ back to $\sum_{i=1}^{N}\sum_{s=U,S} E_{i}^{C,s} = 1$. Variables to be re-normalized: $\left\{E_{i,T_{SS}}^{C,U}\right\}$, $\left\{E_{i,T_{SS}}^{C,S}\right\}$, $\{Y_{k,i,T_{SS}}\}$, $\left\{w_{k,i,T_{SS}}^{U}\right\}$, $\left\{w_{k,i,T_{SS}}^{S}\right\}$, $\left\{NX_{i,T_{SS}}^{U}\right\}$, $\left\{NX_{i,T_{SS}}^{S}\right\}$.

Step 5: Normalize $\sum_s \sum_i E_{i,t}^{C,s} = 1$ for all t. Start at $t = T_{SS}-1$ and go backward until $t=1$ and sequentially compute:

$$R_{t+1} = \frac{\sum_{s=U,S}\sum_{i=1}^{N} \frac{E_{i,t+1}^{C,s}}{\widehat{\phi}_{i,t+1}}}{\delta \sum_{s=U,S}\sum_i E_{i,t}^{C,s}} = \frac{1}{\delta}\sum_{s=U,S}\sum_{i=1}^{N}\frac{E_{i,t+1}^{C,s}}{\widehat{\phi}_{i,t+1}},$$

$$E_{i,t}^{C,s} = \frac{E_{i,t+1}^{C,s}}{\delta \widehat{\phi}_{i,t+1} R_{t+1}} \text{ for } s = U, S,$$

to obtain paths for $\left\{ E_{i,t}^{C,U} \right\}$ and $\left\{ E_{i,t}^{C,S} \right\}$. Note that because $B_{i,1}^s$ are decided before the shock, $R^1 = R^0 = \frac{1}{\delta}$.

Step 6: Solve for out-of-steady-state dynamics conditional on paths for $\left\{ E_{i,t}^{C,U} \right\}$ obtained in Step 5 and $\left\{ E_{i,t}^{C,S} \right\}$. See Inner Loop algorithm below.

Step 7: Using the path for disposable income $I_{i,t}^s$ obtained in Step 6 compute:

$$\left(NX_{i,t}^s \right)' = I_{i,t}^s - E_{i,t}^{C,s} \text{ for } t = 0, ..., T_{SS} - 1,$$

$$\left(NX_{i,T_{SS}}^s \right)' = -\frac{1-\delta}{\delta} \frac{1}{\left(\prod_{\tau=1}^{T_{SS}-1} (R_\tau)^{-1} \right)} \left(B_{i,0}^s + \sum_{t=1}^{T_{SS}-1} \left(\prod_{\tau=1}^{t} (R_\tau)^{-1} \right) \left(NX_{i,t}^s \right)' \right).$$

Step 8: Compute $dist \left(\left\{ NX_{i,t}^s \right\}, \left\{ \left(NX_{i,t}^s \right)' \right\} \right)$.

Step 9: Update $\left(NX_{i,T_{SS}}^s \right)^{new} = (1 - \lambda_o) NX_{i,T_{SS}}^s + \lambda_o \left(NX_{i,T_{SS}}^s \right)'$. Go back to Step 4 until convergence of $\left\{ NX_{i,t}^s \right\}$ for both skill groups.

Inner Loop Algorithm: Conditional on paths $\left\{E^{C,U}_{i,t}\right\}$ and $\left\{E^{C,S}_{i,t}\right\}$ obtained in Step 5 of Outer Loop Algorithm.

Define $\widehat{x}_t \equiv \frac{x_t}{x_0}$.

Step 0: Compute multipliers $\widetilde{\lambda}^s_{i,t} = \frac{\overline{L}^s}{E^{C,s}_{i,t}}$ for $s = U, S$, $t = 1, ..., T_{SS}$.

Step 1: Guess wage paths $\left\{w^s_{k,i,t}\right\}_{t=1}^{T_{SS}}$ for $s = U, S$.

Step 2: Obtain series $\{\pi_{k,oi,t}\}_{t=1}^{T_{SS}}$.

For each $t = 1, ..., T_{SS}$, compute $\widehat{w}^U_{k,i,t} = \frac{w^U_{k,i,t}}{w^U_{k,i,0}}$, $\widehat{w}^S_{k,i,t} = \frac{w^S_{k,i,t}}{w^S_{k,i,0}}$ and iteratively solve the system:

$$\widehat{P}^K_{i,t} = \prod_{k=1}^{K} \left(\widehat{P}^I_{k,i,t}\right)^{\alpha_{k,i}},$$

$$\widehat{p}^h_{k,i,t} = \left[\frac{e^K_{k,i,0}}{1 - e^U_{k,i,0}} \left(\widehat{P}^K_{i,t}\right)^{1-\rho} + \frac{e^S_{k,i,0}}{1 - e^U_{k,i,0}} \left(\widehat{w}^S_{k,i,t}\right)^{1-\rho}\right]^{\frac{1}{1-\rho}},$$

$$\widehat{p}^v_{k,i,t} = \left[e^U_{k,i,0} \left(\widehat{w}^U_{k,i,t}\right)^{1-\sigma} + \left(1 - e^U_{k,i,0}\right) \left(\widehat{p}^h_{k,i,t}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}},$$

$$\widehat{c}_{k,i,t} = \left(\widehat{p}^v_{k,i,t}\right)^{\gamma_{k,i}} \prod_{l=1}^{K} \left(\widehat{P}^I_{l,i,t}\right)^{(1-\gamma_{k,i})\nu_{kl,i}},$$

$$\widehat{P}^I_{k,i,t} = \left[\sum_{o=1}^{N} \pi_{k,oi,0} \widehat{A}_{k,o,t} \left[\widehat{c}_{k,o,t} \widehat{d}_{k,oi,t}\right]^{-\lambda}\right]^{-1/\lambda}.$$

Compute $\widehat{\pi}_{k,oi,t} = \widehat{A}_{k,o,t} \left(\frac{\widehat{c}_{k,o,t}\widehat{d}_{k,oi,t}}{\widehat{P}^I_{k,i,t}}\right)^{-\lambda}$.

And finally $\pi_{k,oi,t} = \pi_{k,oi,0} \times \widehat{\pi}_{k,oi,t}$.

Step 3: Compute $\left\{e^K_{k,i,t}\right\}_{t=1}^{T_{SS}}$, $\left\{e^U_{k,i,t}\right\}_{t=1}^{T_{SS}}$, $\left\{e^S_{k,i,t}\right\}_{t=1}^{T_{SS}}$. For each $t = 1, ..., T_{SS} - 1$ compute:

$$\widehat{e}^K_{k,i,t} = \frac{\left[\widehat{p}^h_{k,i,t}\right]^{\rho-\sigma} \left[\widehat{P}^K_{i,t}\right]^{1-\rho}}{\left[\widehat{p}^v_{k,i,t}\right]^{1-\sigma}},$$

$$\widehat{e}^S_{k,i,t} = \frac{\left[\widehat{p}^h_{k,i,t}\right]^{\rho-\sigma} \left[\widehat{w}^S_{k,i,t}\right]^{1-\rho}}{\left[\widehat{p}^v_{k,i,t}\right]^{1-\sigma}},$$

$$\widehat{e}^U_{k,i,t} = \frac{\left[\widehat{w}^U_{k,i,t}\right]^{1-\sigma}}{\left[\widehat{p}^v_{k,i,t}\right]^{1-\sigma}},$$

$$e^K_{k,i,t} = e^K_{k,i,0} \times \widehat{e}^K_{k,i,t},$$

$$e^S_{k,i,t} = e^S_{k,i,0} \times \widehat{e}^S_{k,i,t},$$

$$e^U_{k,i,t} = e^U_{k,i,0} \times \widehat{e}^U_{k,i,t}.$$

Step 4: Solve for $\left\{V^s_{k,i,t}\right\}_{t=1}^{T_{SS}}$ for $s = U, S$.

Step 4a: Solve for Steady-State Bellman for $s = U, S$:

$$\left(V^s_{k,i,T_{SS}}\right)^{g+1} = \widetilde{\lambda}^s_{i,t} w^s_{k,i,T_{SS}} + \eta^s_{k,i} + \zeta_i \log\left(\sum_{l=0}^{K} \exp\left(\frac{-C^s_{kl,i} + \delta\left(V^s_{l,i,T_{SS}}\right)^g - \delta\left(V^s_{k,i,T_{SS}}\right)^g}{\zeta_i}\right)\right) + \delta\left(V^s_{k,i,T_{SS}}\right)^g$$

Step 4b: Solve Bellman Equations backwards for $t = T_{SS} - 1, ..., 1 \Rightarrow \left\{V^s_{k,i,t}\right\}_{t=1}^{T_{SS}-1}$:

$$V^s_{k,i,t} = \widetilde{\lambda}^s_{i,t} w^s_{k,i,t} + \eta^s_{k,i} + \zeta_i \log\left(\sum_{l=0}^{K} \exp\left(\frac{-C^s_{kl,i} + \delta\widehat{\phi}^s_{i,t+1} V^s_{l,i,t+1}}{\zeta_i}\right)\right)$$

Step 4c: Obtain transition rates for $t = 1, ..., T_{SS} - 1$:

$$s^s_{kl,i,t,t+1} = \frac{\exp\left(\frac{-C^s_{kl,i} + \delta\widehat{\phi}^s_{i,t+1} V^s_{l,i,t+1}}{\zeta_i}\right)}{\sum_{k'=0}^{K} \exp\left(\frac{-C^s_{kk',i} + \delta\widehat{\phi}^s_{i,t+1} V^s_{k',i,t+1}}{\zeta_i}\right)} \text{ for } s = U, S.$$

Step 5: Compute allocation paths going forward ($t = 0$ to $t = T_{SS} - 1$):

$$L^s_{k,i,t+1} = \sum_{l=1}^{K} L^s_{l,i,t} s^s_{lk,i,t,t+1}.$$

Step 6: Solve for $\{Y_{k,i,t}\}_{t=1}^{T_{SS}}$:

$$
\begin{aligned}
Y_{k,o,t} = &\sum_{i=1}^{N} \pi_{k,oi,t} \mu_{k,i} \left(\sum_{s=U,S} E_{i,t}^{C,s} \right) \\
&+ \sum_{i=1}^{N} \sum_{l=1}^{K} \pi_{k,oi,t} \left(\alpha_{k,i} \gamma_{l,i} e_{l,i,t}^{K} + \left(1 - \gamma_{l,i}\right) \nu_{lk,i} \right) Y_{l,i,t}.
\end{aligned}
$$

Step 7: Compute $\left(w_{k,i,t+1}^{s}\right)' = \frac{e_{k,i,t+1}^{s} \gamma_{k,i} Y_{k,i,t+1}}{L_{k,i,t+1}^{s}}$.

Step 8: Compute $dist\left(\left\{w_{k,i,t+1}^{s}\right\}, \left\{\left(w_{k,i,t+1}^{s}\right)'\right\}\right)$ and update $w_{k,i,t+1}^{s} = \left(1 - \lambda_w\right) w_{k,i,t+1}^{s} + \lambda_w \left(w_{k,i,t+1}^{s}\right)'$ until convergence.

Step 9: Compute disposable income (to be used in Step 7 of Outer Loop):

$$
I_{i,t}^{s} = \sum_{k=1}^{K} e_{k,i,t}^{s} \gamma_{k,i} Y_{k,i,t} \text{ for } s = U, S.
$$

D.5 Algorithm for Shocks Extraction

Important: Let T_{Data} denote the last period for which we have data. Let $\widetilde{T} > T_{Data}$ be the period after which there are no more shocks and $E_{i,t}^{C,s}$ is assumed to be constant across countries (according to the $\sum_i \sum_s E_{i,t}^{C,s} = 1$ normalization). We will denote $E_{i,t}^{C} \equiv \sum_s E_{i,t}^{C,s}$, aggreate expenditure across skill groups. The algorithm conditions on $\left\{E_{i,t}^{C}\right\}_{t=0}^{T_{Data}}$, $\{\widehat{\pi}_{k,oi,t}\}_{t=0}^{T_{Data}}$ and $\left\{\widehat{P}_{k,i,t}^{I}\right\}_{t=0}^{T_{Data}}$.

Outer Loop: iteration on $\left\{NX_{i,t}^{s}\right\}$

Step 0: Compute changes in trade costs $\left\{\widehat{d}_{k,oi,t}\right\}_{t=0}^{T_{Data}}$:

$$\widehat{d}_{k,oi,t} = \left(\frac{\widehat{\pi}_{k,oo,t}}{\widehat{\pi}_{k,oi,t}^{t}}\right)^{1/\lambda} \frac{\widehat{P}_{k,i,t}^{I}}{\widehat{P}_{k,o,t}^{I,t}}.$$

Set $\widehat{d}_{k,oi,t} = \widehat{d}_{k,oi,T_{Data}}$ for $t > T_{Data}$.

Step 1: Start with the estimated state equilibrium at $t = 0$. Remember that we used the normalization $\sum_{i=1}^{I}\sum_{k=1}^{K} Y_{k,i} = 1$ during the estimation procedure. **Change the normalization** from $\sum_{i=1}^{I}\sum_{k=1}^{K} Y_{k,i} = 1$ to $\sum_{i=1}^{I} E_{i}^{C} = 1$. Nominal variables to be renormalized: $\{Y_{k,i,0}\}$, $\left\{w_{k,i,0}^{U}\right\}$, $\left\{w_{k,i,0}^{S}\right\}$, $\left\{E_{i,0}^{C,U}\right\}$, $\left\{E_{i,0}^{C,S}\right\}$, $\left\{NX_{i,0}^{U}\right\}$, $\left\{NX_{i,0}^{S}\right\}$.

Step 2: Compute $E_{i,t}^{C,s} = E_{i,0}^{C,s} \frac{(E_{i,t}^{C})_{Data}}{(E_{i,0}^{C})_{Data}}$ for $t = 1, ..., T_{Data}$ and $s = U, S$, and where $E_{i,0}^{C}$ is aggregate consumption expenditure in the estimated steady state, and $\left(E_{i,t}^{C,s}\right)_{Data}$ comes directly from the data. Normalize $E_{i,t}^{C}$ to ensure that $\sum_{i=1}^{N} E_{i,t}^{C} = 1$ in every period.

Step 3: Normalize $\widehat{\phi}_{US,U,t} = 1$ for all $t = 1, ..., T_{SS}$. This yields:

$$R_{t+1} = \frac{E_{US,U,t+1}^{C}}{\delta E_{US,U,t}^{C}} \text{ for } t = 1, ..., T_{Data} - 1.$$

Obtain remaining shocks using:

$$\widehat{\phi}_{US,S,t+1} = \frac{E_{i,S,t+1}^{C}}{\delta E_{i,S,t}^{C} R_{t+1}} \text{ for } t = 1, ..., T_{Data} - 1$$
$$\widehat{\phi}_{i,s,t+1} = \frac{E_{i,s,t+1}^{C}}{\delta E_{i,s,t}^{C} R_{t+1}} \text{ for } t = 1, ..., T_{Data} - 1 \text{ and } s = U, S.$$

Step 4: Obtain $B^s_{i,0}$ with respect to the normalization $\sum_{i=1}^{I} E^C_i = 1$:

$$B^s_{i,0} = \frac{NX^s_{i,0}}{\left(1 - \frac{1}{\delta}\right)} \text{ for } s = U, S.$$

Step 5: Make initial guess for $NX^s_{i,T_{SS}}$ for $s = U,\ S$ (with respect to the normalization $\sum_{i=1}^{I} E^C_i = 1$).

Step 6: Compute steady state equilibrium at T_{SS}, conditional on $NX^s_{i,T_{SS}}$, $\widehat{A}_{k,i,T_{SS}}$ and $\widehat{d}_{k,oi,T_{SS}}$. If this is the first iteration of the outer loop, impose $\widehat{A}_{k,i,T_{SS}} = 1$. Otherwise, feed $\widehat{A}_{k,i,T_{SS}}$ resulting from Step 9 below.

- Step 6a: Notice that the steady-state algorithm uses the normalization $\sum_{i=1}^{I}\sum_{k=1}^{K} Y_{k,i} = 1$. Normalize $NX_{i,T_{SS}}$ with respect to normalization $\sum_{i=1}^{I}\sum_{k=1}^{K} Y_{k,i} = 1$. To perform such normalization, use revenue $\{Y_{k,i,T_{SS}}\}$ obtained in the initial steady state if this is the first outer loop iteration, otherwise use revenue $\{Y_{k,i,T_{SS}}\}$ obtained in Step 9 below.

- Step 6b: After computing the final steady state, **change the normalization** from $\sum_{i=1}^{I}\sum_{k=1}^{K} Y_{k,i} = 1$ to $\sum_{i=1}^{I} E^C_i = 1$ using $\left\{E^C_i\right\}$ obtained in Step 3a. Nominal variables to be renormalized: $\{Y_{k,i,T_{SS}}\}$, $\left\{w^U_{k,i,T_{SS}}\right\}$, $\left\{w^S_{k,i,T_{SS}}\right\}$, $\left\{E^{C,U}_{i,T_{SS}}\right\}$, $\left\{E^{C,S}_{i,T_{SS}}\right\}$, $\left\{NX^U_{i,T_{SS}}\right\}$, $\left\{NX^S_{i,T_{SS}}\right\}$.

Step 7: Impose $E^{C,s}_{i,t} = \begin{cases} E^{C,s}_{i,T_{Data}} + \frac{E^{C,s}_{i,T_{SS}} - E^{C,s}_{i,T_{Data}}}{\widetilde{T} - T_{Data}} \left(t - T_{Data}\right) \text{ for } t = T_{Data}+1, ..., \widetilde{T} \\ E^{C,s}_{i,T_{SS}} \text{ for } t > \widetilde{T} \end{cases}$.
That is, $E^{C,s}_{i,t}$ evolves linearly between T_{Data} and $\widetilde{T}$ when it reaches its steady state value determined in Step 6.

Step 8: Compute

$$R_{t+1} = \frac{E^C_{US,t+1}}{\delta E^C_{US,t}} \text{ for } t \geq T_{Data}.$$

And obtain remaining shocks $\left\{\widehat{\phi}_{i,t}\right\}_{t=T_{Data}+1}^{T_{SS}}$ using

$$\widehat{\phi}_{i,t+1} = \frac{E^C_{i,t+1}}{\delta E^C_{i,t} R_{t+1}} \text{ for } t \geq T_{Data}.$$

Step 9: Solve for the out-of-steady-state dynamics conditional on aggregate expenditures $\left\{E_{i,t}^{C,U}\right\}_{t=0}^{T_{SS}}$, $\left\{E_{i,t}^{C,S}\right\}_{t=0}^{T_{SS}}$, on preference shifters $\left\{\widehat{\phi}_{i,t}\right\}_{t=2}^{T_{SS}}$ and trade cost shocks $\left\{\widehat{d}_{k,oi,t}\right\}_{t=1}^{T_{SS}}$.

Step 10: Using the path for disposable income $\left\{I_{i,t}^{U}\right\}_{t=1}^{T_{SS}}$, $\left\{I_{i,t}^{S}\right\}_{t=1}^{T_{SS}}$ obtained in Step 9 compute:

$$\left(NX_{i,t}^{s}\right)' = I_{i,t}^{s} - E_{i,t}^{C,s} \text{ for } t = 0, ..., T_{SS} - 1,$$

$$\left(NX_{i,T_{SS}}^{s}\right)' = -\frac{1-\delta}{\delta}\frac{1}{\left(\prod_{\tau=1}^{T_{SS}-1}\left(R_{\tau}\right)^{-1}\right)}\left(B_{i,0}^{s} + \sum_{t=1}^{T_{SS}-1}\left(\prod_{\tau=1}^{t}\left(R_{\tau}\right)^{-1}\right)\left(NX_{i,t}^{s}\right)'\right).$$

Step 11: Compute $dist\left(\left\{NX_{i,t}^{s}\right\}, \left\{\left(NX_{i,t}^{s}\right)'\right\}\right)$.

Step 12: Update $\left(NX_{i,T_{SS}}^{s}\right)^{new} = \left(1-\lambda_{o}\right)NX_{i,T_{SS}}^{s} + \lambda_{o}\left(NX_{i,T_{SS}}^{s}\right)'$. Go back to Step 6 until convergence of $\left\{NX_{i,t}^{s}\right\}$ for both skill groups.

Inner Loop Algorithm: Conditional on paths $\left\{E_{i,t}^{C,U}\right\}_{t=0}^{T_{SS}}$ and $\left\{E_{i,t}^{C,S}\right\}_{t=0}^{T_{SS}}$, net exports $\left\{NX_{i,t}^{C,U}\right\}_{t=0}^{T_{SS}}$, $\left\{NX_{i,t}^{C,S}\right\}_{t=0}^{T_{SS}}$ and shocks $\left\{\widehat{\phi}_{i,t}\right\}_{t=0}^{T_{SS}}$ and $\left\{\widehat{d}_{k,oi,t}\right\}_{t=0}^{T_{SS}}$ obtained in the Outer Loop.

As before, we denote changes relative to $t=0$ by $\widehat{x}_t \equiv \frac{x_t}{x_0}$. This loop conditions on data $\{\widehat{\pi}_{k,oi,t}\}_{t=0}^{T_{Data}}$ and $\left\{\widehat{P}_{k,i,t}^{I}\right\}_{t=0}^{T_{Data}}$. Define $\widehat{d}_{k,oi} = \frac{d_{k,oi,T_{SS}}}{d_{k,oi,0}}$ and $\widehat{A}_{k,oi} = \frac{A_{k,i,T_{SS}}}{A_{k,i,0}}$

Step 1: Given paths $\left\{E_{i,t}^{C,U}\right\}_{t=0}^{T_{SS}}$ and $\left\{E_{i,t}^{C,S}\right\}_{t=0}^{T_{SS}}$, compute multipliers $\widetilde{\lambda}_{i,t}^{s} = \frac{\overline{L}^{s}}{E_{i,t}^{C,s}}$ for $s=U,S$, $t=0,...,T_{SS}$.

Step 2: Guess wage paths $\left\{w_{k,i,t}^{s}\right\}_{t=0}^{T_{SS}}$ for $s=U,S$.

Step 3: For each $t=1,...,T_{Data}$ compute $\widehat{w}_{k,i,t}^{U} = \frac{w_{k,i,t}^{U}}{w_{k,i,0}^{U}}$, $\widehat{w}_{k,i,t}^{S} = \frac{w_{k,i,t}^{S}}{w_{k,i,0}^{S}}$ and compute:

$$\widehat{P}_{i,t}^{K} = \prod_{k=1}^{K}\left(\widehat{P}_{k,i,t}^{I}\right)^{\alpha_{k,i,t}},$$

$$\widehat{p}_{k,i,t}^{h} = \left[\frac{e_{k,i,0}^{K}}{1-e_{k,i,0}^{U}}\left(\widehat{P}_{i,t}^{K}\right)^{1-\rho} + \frac{e_{k,i,0}^{S}}{1-e_{k,i,0}^{U}}\left(\widehat{w}_{k,i,t}^{S}\right)^{1-\rho}\right]^{\frac{1}{1-\rho}},$$

$$\widehat{p}_{k,i,t}^{v} = \left[e_{k,i,0}^{U}\left(\widehat{w}_{k,i,t}^{U}\right)^{1-\sigma} + \left(1-e_{k,i,0}^{U}\right)\left(\widehat{p}_{k,i,t}^{h}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}},$$

$$\widehat{c}_{k,i,t} = \left(\widehat{p}_{k,i,t}^{v}\right)^{\gamma_{k,i,t}}\prod_{l=1}^{K}\left(\widehat{P}_{l,i,t}^{I}\right)^{(1-\gamma_{k,i,t})\nu_{kl,i,t}}.$$

Compute $\widehat{A}_{k,i,t} = \frac{\widehat{\pi}_{k,ii,t}}{\left(\widehat{P}_{k,i,t}^{I}\right)^{\lambda}}\left(\widehat{c}_{k,i,t}\right)^{\lambda}$.
For $t \geq T_{Data}+1$ impose:

$$\widehat{A}_{k,i,t} = \widehat{A}_{k,i,T_{Data}}.$$

Step 4: Obtain $\{\pi_{k,oi,t}\}_{t=T_{Data}+1}^{T_{SS}}$. For each $t=T_{Data}+1,...,T_{SS}$, compute $\widehat{w}_{k,i,t}^{U} = \frac{w_{k,i,t}^{U}}{w_{k,i,0}^{U}}$, $\widehat{w}_{k,i,t}^{S} = \frac{w_{k,i,t}^{S}}{w_{k,i,0}^{S}}$ and iteratively solve the system:

$$\widehat{P}_{i,t}^{K} = \prod_{k=1}^{K}\left(\widehat{P}_{k,i,t}^{I}\right)^{\alpha_{k,i,t}},$$

$$\widehat{p}_{k,i,t}^{h} = \left[\frac{e_{k,i,0}^{K}}{1-e_{k,i,0}^{U}}\left(\widehat{P}_{i,t}^{K}\right)^{1-\rho} + \frac{e_{k,i,0}^{S}}{1-e_{k,i,0}^{U}}\left(\widehat{w}_{k,i,t}^{S}\right)^{1-\rho}\right]^{\frac{1}{1-\rho}},$$

$$\widehat{p}^{v}_{k,i,t} = \left[e^{U}_{k,i,0}\left(\widehat{w}^{U}_{k,i,t}\right)^{1-\sigma} + \left(1-e^{U}_{k,i,0}\right)\left(\widehat{p}^{h}_{k,i,t}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}},$$

$$\widehat{c}_{k,i,t} = \left(\widehat{p}^{v}_{k,i,t}\right)^{\gamma_{k,i,t}} \prod_{l=1}^{K}\left(\widehat{P}^{I}_{l,i,t}\right)^{(1-\gamma_{k,i,t})\nu_{kl,i,t}},$$

$$\widehat{P}^{I}_{k,i,t} = \left[\sum_{o=1}^{N}\pi_{k,oi,0}\widehat{A}_{k,o,t}\left[\widehat{c}_{k,o,t}\widehat{d}_{k,oi,t}\right]^{-\lambda}\right]^{-1/\lambda}.$$

Compute $\widehat{\pi}_{k,oi,t} = \widehat{A}_{k,o,t}\left(\frac{\widehat{c}_{k,o,t}\widehat{d}_{k,oi,t}}{\widehat{P}^{I}_{k,i,t}}\right)^{-\lambda}$.

And finally $\pi_{k,oi,t} = \pi_{k,oi,0} \times \widehat{\pi}_{k,oi,t}$.

Step 5: Compute $\left\{e^{K}_{k,i,t}\right\}_{t=1}^{T_{SS}}$, $\left\{e^{U}_{k,i,t}\right\}_{t=1}^{T_{SS}}$, $\left\{e^{S}_{k,i,t}\right\}_{t=1}^{T_{SS}}$. For each $t = 1, ..., T_{SS}-1$ compute:

$$\widehat{e}^{K}_{k,i,t} = \frac{\left[\widehat{p}^{h}_{k,i,t}\right]^{\rho-\sigma}\left[\widehat{P}^{K}_{i,t}\right]^{1-\rho}}{\left[\widehat{p}^{v}_{k,i,t}\right]^{1-\sigma}},$$

$$\widehat{e}^{S}_{k,i,t} = \frac{\left[\widehat{p}^{h}_{k,i,t}\right]^{\rho-\sigma}\left[\widehat{w}^{S}_{k,i,t}\right]^{1-\rho}}{\left[\widehat{p}^{v}_{k,i,t}\right]^{1-\sigma}},$$

$$\widehat{e}^{U}_{k,i,t} = \frac{\left[\widehat{w}^{U}_{k,i,t}\right]^{1-\sigma}}{\left[\widehat{p}^{v}_{k,i,t}\right]^{1-\sigma}},$$

$$e^{K}_{k,i,t} = e^{K}_{k,i,0} \times \widehat{e}^{K}_{k,i,t},$$

$$e^{S}_{k,i,t} = e^{S}_{k,i,0} \times \widehat{e}^{S}_{k,i,t},$$

$$e^{U}_{k,i,t} = e^{U}_{k,i,0} \times \widehat{e}^{U}_{k,i,t}.$$

Step 6: Solve for $\left\{V^{s}_{k,i,t}\right\}_{t=1}^{T_{SS}}$ for $s = U, S$.

Step 6a: Solve for Steady-State Bellman for $s = U, S$:

$$\left(V^{s}_{k,i,T_{SS}}\right)^{g+1} = \widetilde{\lambda}^{s}_{i,t} w^{s}_{k,i,T_{SS}} + \eta^{s}_{k,i} + \zeta_i \log\left(\sum_{l=0}^{K}\exp\left(\frac{-C^{s}_{kl,i} + \delta\left(V^{s}_{l,i,T_{SS}}\right)^{g} - \delta\left(V^{s}_{k,i,T_{SS}}\right)^{g}}{\zeta_i}\right)\right) + \delta\left(V^{s}_{k,i,T_{SS}}\right)^{g}$$

Step 6b: Solve Bellman Equations backwards for $t = T_{SS} - 1, ..., 1 \Rightarrow \left\{V_{k,i,t}^{s}\right\}_{t=1}^{T_{SS}-1}$:

$$V_{k,i,t}^{s} = \widetilde{\lambda}_{i,t}^{s} w_{k,i,t}^{s} + \eta_{k,i}^{s} + \zeta_i \log\left(\sum_{l=0}^{K} \exp\left(\frac{-C_{kl,i}^{s} + \delta \widehat{\phi}_{i,t+1}^{s} V_{l,i,t+1}^{s}}{\zeta_i}\right)\right)$$

Step 6c: Obtain transition rates for $t = 1, ..., T_{SS} - 1$:

$$s_{kl,i,t,t+1}^{s} = \frac{\exp\left(\frac{-C_{kl,i}^{s} + \delta \widehat{\phi}_{i,t+1}^{s} V_{l,i,t+1}^{s}}{\zeta_i}\right)}{\sum_{k'=0}^{K} \exp\left(\frac{-C_{kk',i}^{s} + \delta \widehat{\phi}_{i,t+1}^{s} V_{k',i,t+1}^{s}}{\zeta_i}\right)} \text{ for } s = U, S.$$

Step 7: Compute allocation paths going forward ($t = 0$ to $t = T_{SS} - 1$):

$$L_{k,i,t+1}^{s} = \sum_{l=1}^{K} L_{l,i,t}^{s} s_{lk,i,t,t+1}^{s}.$$

Step 8: Solve for $\{Y_{k,i,t}\}_{t=1}^{T_{SS}}$:

$$\begin{aligned} Y_{k,o,t} = & \sum_{i=1}^{N} \pi_{k,oi,t} \mu_{k,i,t} \left(\sum_{s=U,S} E_{i,t}^{C,s}\right) \\ & + \sum_{i=1}^{N} \sum_{l=1}^{K} \pi_{k,oi,t} \left(\alpha_{k,i,t} \gamma_{l,i,t} e_{l,i,t}^{K} + (1 - \gamma_{l,i,t}) \nu_{lk,i,t}\right) Y_{l,i,t}. \end{aligned}$$

Step 9: Compute $\left(w_{k,i,t+1}^{s}\right)' = \frac{e_{k,i,t+1}^{s} \gamma_{k,i,t} Y_{k,i,t+1}}{L_{k,i,t+1}^{s}}$.

Step 10: Compute $dist\left(\left\{w_{k,i,t+1}^{s}\right\}, \left\{\left(w_{k,i,t+1}^{s}\right)'\right\}\right)$ and update $w_{k,i,t+1}^{s} = (1 - \lambda_w) w_{k,i,t+1}^{s} + \lambda_w \left(w_{k,i,t+1}^{s}\right)'$ until convergence.

Step 11: Compute disposable income (to be used in Step 10 of Outer Loop):

$$I_{i,t}^{s} = \sum_{k=1}^{K} e_{k,i,t}^{s} \gamma_{k,i,t} Y_{k,i,t} \text{ for } s = U, S.$$

Algorithm for New Steady State

We will refer to $t = 0$ as the initial steady state and $t = 1$ as the final steady state.

This algorithm normalizes world output to 1: $\sum_{k=1}^{K}\sum_{i=1}^{N} Y_{k,i,1} = 1$.

Step 0: We start with an initial steady state characterized by $\{\pi_{k,oi,0}\}$, $\left\{e^U_{k,i,0}\right\}$, $\left\{e^S_{k,i,0}\right\}$, $\left\{e^K_{k,i,0}\right\}$, $\left\{NX^U_{i,0}\right\}$, $\left\{NX^S_{i,0}\right\}$. Consider a change in (long-run) productivity $\widehat{A}_{k,i} \equiv \frac{A_{k,i,1}}{A_{k,i,0}}$ and trade costs $\widehat{d}_{k,oi} \equiv \frac{d_{k,oi,1}}{d_{k,oi,0}}$.

Step 1: Guess new steady-state allocations $\left\{L^U_{k,i,1}\right\}$ and $\left\{L^S_{k,i,1}\right\}$.

Step 2: Guess new steady-state wages $\left\{w^U_{k,i,1}\right\}$ and $\left\{w^S_{k,i,1}\right\}$.

Step 3: Compute wage changes $\widehat{w}^U_{k,i} = \frac{w^U_{k,i,1}}{w^U_{k,i,0}}$, $\widehat{w}^S_{k,i} = \frac{w^S_{k,i,1}}{w^S_{k,i,0}}$.

Step 4: Obtain trade shares $\{\pi_{k,oi,1}\}$.
First, iteratively solve the system below, conditional on $\widehat{w}^U_{k,i}$ and $\widehat{w}^S_{k,i}$:

$$\widehat{P}^K_i = \prod_{k=1}^{K}\left(\widehat{P}^I_{k,i}\right)^{\alpha_{k,i}},$$

$$\widehat{p}^h_{k,i} = \left[\frac{e^K_{k,i,0}}{1-e^U_{k,i,0}}\left(\widehat{P}^K_i\right)^{1-\rho} + \frac{e^S_{k,i,0}}{1-e^U_{k,i,0}}\left(\widehat{w}^S_{k,i}\right)^{1-\rho}\right]^{\frac{1}{1-\rho}},$$

$$\widehat{p}^v_{k,i} = \left[e^U_{k,i,0}\left(\widehat{w}^U_{k,i}\right)^{1-\sigma} + \left(1-e^U_{k,i,0}\right)\left(\widehat{p}^h_{k,i}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}},$$

$$\widehat{c}_{k,i} = \left(\widehat{p}^v_{k,i}\right)^{\gamma_{k,i}}\prod_{l=1}^{K}\left(\widehat{P}^I_{l,i}\right)^{(1-\gamma_{k,i})\nu_{kl,i}},$$

$$\widehat{P}^I_{k,i} = \left[\sum_{o=1}^{N}\pi_{k,oi,0}\widehat{A}_{k,o}\left[\widehat{c}_{k,o}\widehat{d}_{k,oi}\right]^{-\lambda}\right]^{-1/\lambda}.$$

Compute $\widehat{\pi}_{k,oi} = \widehat{A}_{k,o}\left(\frac{\widehat{c}_{k,o}\widehat{d}_{k,oi}}{\widehat{P}^I_{k,i}}\right)^{-\lambda}$.

And finally, $\pi_{k,oi,1} = \pi_{k,oi,0} \times \widehat{\pi}_{k,oi}$.

Step 5: Compute $\left\{e^K_{k,i,1}\right\}$, $\left\{e^U_{k,i,1}\right\}$, $\left\{e^S_{k,i,1}\right\}$.

First, obtain changes:

$$\widehat{e}_{k,i}^{K}=\frac{\left[\widehat{p}_{k,i}^{h}\right]^{\rho-\sigma}\left[\widehat{P}_{i}^{K}\right]^{1-\rho}}{\left[\widehat{p}_{k,i}^{v}\right]^{1-\sigma}},$$

$$\widehat{e}_{k,i}^{S}=\frac{\left[\widehat{p}_{k,i}^{h}\right]^{\rho-\sigma}\left[\widehat{w}_{k,i}^{S}\right]^{1-\rho}}{\left[\widehat{p}_{k,i}^{v}\right]^{1-\sigma}},$$

$$\widehat{e}_{k,i}^{U}=\frac{\left[\widehat{w}_{k,i}^{U}\right]^{1-\sigma}}{\left[\widehat{p}_{k,i}^{v}\right]^{1-\sigma}}.$$

Then obtain final levels.

$$e_{k,i,1}^{K}=e_{k,i,0}^{K}\times\widehat{e}_{k,i}^{K},$$

$$e_{k,i,1}^{S}=e_{k,i,0}^{S}\times\widehat{e}_{k,i}^{S},$$

$$e_{k,i,1}^{U}=e_{k,i,0}^{U}\times\widehat{e}_{k,i}^{U}.$$

Step 6: Solve for $\{Y_{k,i,1}\}$.

$$\begin{aligned}Y_{k,o,1}=&\sum_{i=1}^{N}\sum_{l=1}^{K}\pi_{k,oi,1}\left(\mu_{k,i}\gamma_{l,i}\left(1-e_{l,i,1}^{K}\right)+\alpha_{k,i}\gamma_{l,i}e_{l,i,1}^{K}+\left(1-\gamma_{l,i}\right)\nu_{lk,i}\right)Y_{l,i,1}\\&-\sum_{i=1}^{N}\pi_{k,oi,1}\mu_{k,i}NX_{i,1},\end{aligned}$$

$$\sum_{k=1}^{K}\sum_{i=1}^{N}Y_{k,i,1}=1.$$

Step 7: Update wages:

$$\left(w_{k,i,1}^{U}\right)^{new}=\frac{e_{k,i,1}^{U}\gamma_{k,i}Y_{k,i,1}}{L_{k,i,1}^{U}},$$

$$\left(w_{k,i,1}^{S}\right)^{new}=\frac{e_{k,i,1}^{S}\gamma_{k,i}Y_{k,i,1}}{L_{k,i,1}^{S}}.$$

Go back to Step 3 until convergence of wages.

Step 8: Compute aggregate expendtures.

$$E_{i,1}^{C,U}=\sum_{k=1}^{K}w_{k,i,1}^{U}L_{k,i,1}^{U}-NX_{i,1}^{U},$$

$$E_{i,1}^{C,S} = \sum_{k=1}^{K} w_{k,i,1}^{S} L_{k,i,1}^{S} - NX_{i,1}^{S}.$$

Step 9: Compute Lagrange multipliers $\widetilde{\lambda}_{i,1}^{U} = \frac{\overline{L}_i^U}{E_{i,1}^{C,U}}$, $\widetilde{\lambda}_{i,1}^{S} = \frac{\overline{L}_i^S}{E_{i,1}^{C,S}}$.

Step 10: Solve the Bellman Equations $\left\{V_{k,i,1}^{s}\right\}$:

$$\left(V_{k,i,1}^{s}\right)^{g+1} = \widetilde{\lambda}_{i,t}^{s} w_{k,i,T_{SS}}^{s} + \eta_{k,i}^{s} + \zeta_i \log\left(\sum_{l=0}^{K} \exp\left(\frac{-C_{kl,i}^{s} + \delta\left(V_{l,i,1}^{s}\right)^{g} - \delta\left(V_{k,i,1}^{s}\right)^{g}}{\zeta_i}\right)\right) + \delta\left(V_{k,i,1}^{s}\right)^{g}.$$

Step 11: Update allocations $L_{k,i,1}$.

Compute transition rates for $s = U, S$:

$$s_{kl,i,1}^{s} = \frac{\exp\left(\frac{-C_{kl,i}^{s} + \delta V_{l,i,1}^{s}}{\zeta_i}\right)}{\sum_{k'=0}^{K} \exp\left(\frac{-C_{kk',i}^{s} + \delta V_{k',i,1}^{s}}{\zeta_i}\right)}.$$

Obtain their implied steady-state allocations by computing $\left(s_{kl,i,1}^{s}\right)^{\infty}$. Multiply these shares by $\overline{L}_i^s$ to obtain $\left(L_{k,i,1}^{s}\right)'$, and update:

$$\left(L_{k,i,1}^{s}\right)^{new} = \left(1 - \lambda_L\right) L_{k,i,1}^{s} + \lambda_L \left(L_{k,i,1}^{s}\right)'.$$

Step 12: Armed with $\left(L_{k,i,1}^{s}\right)^{new}$ go to Step 2 until $dist\left(L_{k,i,1}^{s}, \left(L_{k,i,1}^{s}\right)'\right)$ is very small.

Algorithm for the Transition, Exogenous Trade Imbalances

Consider paths $\{A_{k,i,t}\}$, $\{d_{k,oi,t}\}$. Also, consider paths $\left\{\widehat{\phi}_{i,t}\right\}$ with $\widehat{\phi}_{i,t} = 1$ for all $t \geq T$ for some $T << T_{SS}$. Condition on an exogenous paths $\left\{NX^s_{i,t}\right\}$ across countries and skill levels.

Define $\widehat{x}_t \equiv \frac{x_t}{x_0}$.

Step 1: Guess paths for multipliers $\left\{\widetilde{\lambda}^s_{i,t}\right\}_{t=1}^{T_{SS}}$ for for each country i and $s = U, S$.

Step 2: Compute $E^{C,s}_{i,t} = \frac{\overline{L}^s_i}{\widetilde{\lambda}^s_{i,t}}$

Step 3: Guess wage paths $\left\{w^s_{k,i,t}\right\}_{t=1}^{T_{SS}}$ for $k = 1, ..., K$; $i = 1, ..., N$; and $s = U, S$.

Step 4: Obtain series $\{\pi_{k,oi,t}\}_{t=1}^{T_{SS}}$.

For each $t = 1, ..., T_{SS}$, compute $\widehat{w}^U_{k,i,t} = \frac{w^U_{k,i,t}}{w^U_{k,i,0}}$, $\widehat{w}^S_{k,i,t} = \frac{w^S_{k,i,t}}{w^S_{k,i,0}}$ and iteratively solve the system:

$$\widehat{P}^K_{i,t} = \prod_{k=1}^{K} \left(\widehat{P}^I_{k,i,t}\right)^{\alpha_{k,i}},$$

$$\widehat{p}^h_{k,i,t} = \left[\frac{e^K_{k,i,0}}{1 - e^U_{k,i,0}} \left(\widehat{P}^K_{i,t}\right)^{1-\rho} + \frac{e^S_{k,i,0}}{1 - e^U_{k,i,0}} \left(\widehat{w}^S_{k,i,t}\right)^{1-\rho}\right]^{\frac{1}{1-\rho}},$$

$$\widehat{p}^v_{k,i,t} = \left[e^U_{k,i,0} \left(\widehat{w}^U_{k,i,t}\right)^{1-\sigma} + \left(1 - e^U_{k,i,0}\right) \left(\widehat{p}^h_{k,i,t}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}},$$

$$\widehat{c}_{k,i,t} = \left(\widehat{p}^v_{k,i,t}\right)^{\gamma_{k,i}} \prod_{l=1}^{K} \left(\widehat{P}^I_{l,i,t}\right)^{(1-\gamma_{k,i})\nu_{kl,i}},$$

$$\widehat{P}^I_{k,i,t} = \left[\sum_{o=1}^{N} \pi_{k,oi,0} \widehat{A}_{k,o,t} \left[\widehat{c}_{k,o,t}\widehat{d}_{k,oi,t}\right]^{-\lambda}\right]^{-1/\lambda}.$$

Compute $\widehat{\pi}_{k,oi,t} = \widehat{A}_{k,o,t} \left(\frac{\widehat{c}_{k,o,t}\widehat{d}_{k,oi,t}}{\widehat{P}^I_{k,i,t}}\right)^{-\lambda}$.

And finally $\pi_{k,oi,t} = \pi_{k,oi,0} \times \widehat{\pi}_{k,oi,t}$.

Step 5: Compute $\left\{e^K_{k,i,t}\right\}_{t=1}^{T_{SS}}$, $\left\{e^U_{k,i,t}\right\}_{t=1}^{T_{SS}}$, $\left\{e^S_{k,i,t}\right\}_{t=1}^{T_{SS}}$. For each $t = 1, ..., T_{SS} - 1$ compute:

$$\widehat{e}^K_{k,i,t} = \frac{\left[\widehat{p}^h_{k,i,t}\right]^{\rho-\sigma} \left[\widehat{P}^K_{i,t}\right]^{1-\rho}}{\left[\widehat{p}^v_{k,i,t}\right]^{1-\sigma}},$$

$$\widehat{e}^{S}_{k,i,t} = \frac{\left[\widehat{p}^{h}_{k,i,t}\right]^{\rho-\sigma}\left[\widehat{w}^{S}_{k,i,t}\right]^{1-\rho}}{\left[\widehat{p}^{v}_{k,i,t}\right]^{1-\sigma}},$$

$$\widehat{e}^{U}_{k,i,t} = \frac{\left[\widehat{w}^{U}_{k,i,t}\right]^{1-\sigma}}{\left[\widehat{p}^{v}_{k,i,t}\right]^{1-\sigma}},$$

$$e^{K}_{k,i,t} = e^{K}_{k,i,0} \times \widehat{e}^{K}_{k,i,t},$$

$$e^{S}_{k,i,t} = e^{S}_{k,i,0} \times \widehat{e}^{S}_{k,i,t},$$

$$e^{U}_{k,i,t} = e^{U}_{k,i,0} \times \widehat{e}^{U}_{k,i,t}.$$

Step 6: Solve for $\left\{V^{s}_{k,i,t}\right\}_{t=1}^{T_{SS}}$ for $s = U, S$.

Step 6a: Solve for Steady-State Bellman for $s = U, S$:

$$\left(V^{s}_{k,i,T_{SS}}\right)^{g+1} = \widetilde{\lambda}^{s}_{i,t} w^{s}_{k,i,T_{SS}} + \eta^{s}_{k,i} + \zeta_i \log\left(\sum_{l=0}^{K} \exp\left(\frac{-C^{s}_{kl,i} + \delta\left(V^{s}_{l,i,T_{SS}}\right)^{g} - \delta\left(V^{s}_{k,i,T_{SS}}\right)^{g}}{\zeta_i}\right)\right) + \delta\left(V^{s}_{k,i,T_{SS}}\right)^{g}$$

Step 6b: Solve Bellman Equations backwards for $t = T_{SS} - 1, ..., 1 \Rightarrow \left\{V^{s}_{k,i,t}\right\}_{t=1}^{T_{SS}-1}$:

$$V^{s}_{k,i,t} = \widetilde{\lambda}^{s}_{i,t} w^{s}_{k,i,t} + \eta^{s}_{k,i} + \zeta_i \log\left(\sum_{l=0}^{K} \exp\left(\frac{-C^{s}_{kl,i} + \delta\widehat{\phi}^{s}_{i,t+1} V^{s}_{l,i,t+1}}{\zeta_i}\right)\right)$$

Step 6c: Obtain transition rates for $t = 1, ..., T_{SS} - 1$:

$$s^{s}_{kl,i,t,t+1} = \frac{\exp\left(\frac{-C^{s}_{kl,i} + \delta\widehat{\phi}^{s}_{i,t+1} V^{s}_{l,i,t+1}}{\zeta_i}\right)}{\sum_{k'=0}^{K} \exp\left(\frac{-C^{s}_{kk',i} + \delta\widehat{\phi}^{s}_{i,t+1} V^{s}_{k',i,t+1}}{\zeta_i}\right)} \text{ for } s = U, S.$$

Step 7: Compute allocation paths going forward ($t = 0$ to $t = T_{SS} - 1$):

$$L^{s}_{k,i,t+1} = \sum_{l=1}^{K} L^{s}_{l,i,t} s^{s}_{lk,i,t,t+1}.$$

Step 8: Solve for $\{Y_{k,i,t}\}_{t=1}^{T_{SS}}$:

$$Y_{k,o,t} = \sum_{i=1}^{N} \pi_{k,oi,t} \mu_{k,i} \left(\sum_{s=U,S} E_{i,t}^{C,s} \right) + \sum_{i=1}^{N} \sum_{l=1}^{K} \pi_{k,oi,t} \left(\alpha_{k,i} \gamma_{l,i} e_{l,i,t}^{K} + (1 - \gamma_{l,i}) \nu_{lk,i} \right) Y_{l,i,t}.$$

$$\sum_{o=1}^{N} \sum_{k=1}^{K} Y_{k,o,t} = 1$$

Step 9: Compute $\left(w_{k,i,t}^{s}\right)' = \frac{e_{k,i,t}^{s} \gamma_{k,i} Y_{k,i,t}}{L_{k,i,t}^{s}}$.

Step 10: Compute $dist\left(\left\{w_{k,i,t+1}^{s}\right\}, \left\{\left(w_{k,i,t+1}^{s}\right)'\right\}\right)$ and update $w_{k,i,t+1}^{s} = (1 - \lambda_w) w_{k,i,t+1}^{s} + \lambda_w \left(w_{k,i,t+1}^{s}\right)'$ until convergence.

Step 11: Compute disposable income:

$$I_{i,t}^{s} = \sum_{k=1}^{K} e_{k,i,t}^{s} \gamma_{k,i} Y_{k,i,t} \text{ for } s = U, S.$$

Step 12: Update $\left\{E_{i,t}^{C,s}\right\}$:

$$E_{i,t}^{C,s} = I_{i,t}^{s} - NX_{i,t}^{s}.$$

Step 13a: Compute $\left(\widetilde{\lambda}_{i,t}^{s}\right)' = \frac{\overline{L}_{i}^{s}}{E_{i,t}^{C,s}}$for all t.

- Step 13b: Compute $dist\left(\left\{\widetilde{\lambda}_{i,t}^{s}\right\}, \left\{\left(\widetilde{\lambda}_{i,t}^{s}\right)'\right\}\right)$
- Step 13c: Update $\widetilde{\lambda}_{i,t}^{s} \leftarrow (1 - \alpha_\lambda) \left(\widetilde{\lambda}_{i,t}^{s}\right)' + \alpha_\lambda \left(\widetilde{\lambda}_{i,t}^{s}\right)'$ for a small step size α_λ and go back to Step 2 until convergence.