Tenuous Beliefs and the Price of Uncertainty

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Outline

I. Success of Recursive Utility Models?
II. Shock Exposure and Price Elasticities
III. Distinguishing Risk Aversion from Belief Distortions?
IV. Uncertainty Aversion as a Model of Belief Distortions
    a. Model Ambiguity
    b. Model Misspecification
V. Growth-rate Uncertainty and Valuation

Recursive Valuation

- Use a recursive utility model (see Koopmans, Kreps & Porteus, Epstein & Zin, …) to highlight how uncertainty about future events affects asset valuation.
- Explore ways in which expectations and uncertainty about future growth rates influence risky claims to consumption.

Investigate how beliefs about the future are reflected in current-period assessments through continuation values. The *forward-looking* nature of the recursive utility model provides an additional channel through which *perceptions* about the future matter. (Bansal-Yaron and many others.)

Recursive Utility

Consider the aggregator specified in terms of C_t the current period consumption and V_t the continuation value :

$$V_t = \left[(C_t)^{1-\rho} + \exp(-\delta\epsilon) \left[\mathcal{R}_t(V_{t+\epsilon}) \right]^{1-\rho} \right]^{\frac{1}{1-\rho}}$$

where

$$\mathcal{R}_t \left(V_{t+\epsilon} \right) = \left(E \left[(V_{t+\epsilon})^{1-\gamma} | \mathcal{F}_t \right] \right)^{\frac{1}{1-\gamma}}$$

adjusts the continuation value $V_{t+\epsilon}$ for risk. $\frac{1}{\rho}$ is the elasticity of intertemporal substitution and δ is a subjective discount rate. The parameter ϵ is the decision interval.

Stochastic Discount Factor

$$\frac{S_{t+\epsilon}}{S_t} = \exp(-\delta\epsilon)\left(\frac{C_{t+\epsilon}}{C_t}\right)^{-\rho}\left[\frac{V_{t+\epsilon}}{\mathcal{R}_t(V_{t+\epsilon})}\right]^{\rho-\gamma}$$

- ▷ Continuation value gives a structured way to enhance the impact of the perceptions about the future.
- ▷ Special case: Power utility sets $\rho = \gamma$.
- ▷ Multiply to compound over multiple periods.
- ▷ When $\rho = 1$

$$\left[\frac{V_{t+\epsilon}}{\mathcal{R}_t(V_{t+\epsilon})}\right]^{1-\gamma} = \frac{(V_{t+\epsilon})^{1-\gamma}}{E\left[(V_{t+\epsilon})^{1-\gamma}|\mathcal{F}_t\right]}$$

has conditional expectation equal to unity. Equivalent interpretation as distorted beliefs.

An Asset Pricing Perspective on Impulses and Propagation

Imagine an impulse or shock W_ϵ that happens tomorrow.

- ▷ This shock has an impact on a macro time series or a cash flow at future times ϵ, 2ϵ, ...
- ▷ Exposure in the future of the underlying time series to this shock requires compensation today, say time zero. The magnitude of the compensation or price depends on the date of the cash flow.
- ▷ Alternative shocks require different compensations or “prices”.
- ▷ Exposures to the uncertainty are state dependent.

Dynamic macroeconomic models imply impulse responses

Dynamic models of asset prices imply compensations to shock exposures.

Elasticities

Counterparts to impulse response functions pertinent to valuation:

- shock-exposure elasticities
- shock-price elasticities

These are the ingredients to risk premia, and they have a term structure induced by the changes in the investment horizons and the state of macroeconomy.

Diffusion models use Malliavin derivatives as inputs.

Hansen-Scheinkman (*Finance and Stochastics*), Borovička-Hansen-Hendricks-Scheinkman (*Journal of Financial Econometrics*), Hansen (Fisher-Schultz, *Econometrica*), Borovička and Hansen ((*Journal of Econometrics*), Borovička-Hansen-Scheinkman (*Mathematical and Financial Economics*)

Quantitative Example

Long-run risk (Bansal-Yaron) reinterpreted as an AK model with adjustment costs.

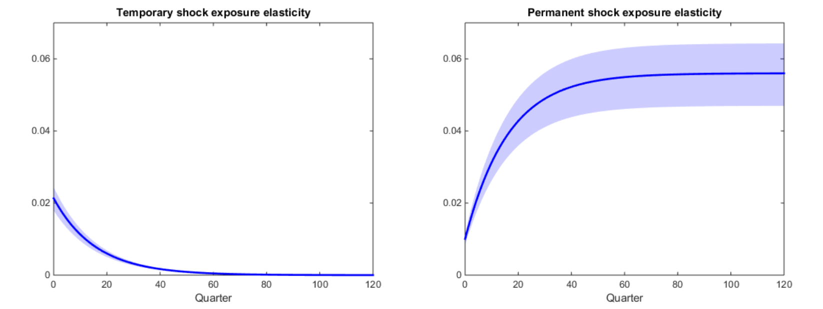
$$dZ_t^{[1]} = -.021 \left(Z_t^{[1]}\right) dt + \sqrt{Z_t^{[2]}} \begin{bmatrix} .031 & -.015 & 0 \end{bmatrix} dW_t$$

$$dZ_t^{[2]} = -.013 \left(Z_t^{[2]} - 1\right) dt + \sqrt{Z_t^{[2]}} \begin{bmatrix} 0 & 0 & -.038 \end{bmatrix} dW_t$$

$$dY_t = .01(.15 + Z_t^{[1]})dt + .01\sqrt{Z_t^{[2]}} \begin{bmatrix} .34 & .7 & 0 \end{bmatrix} dW_t$$

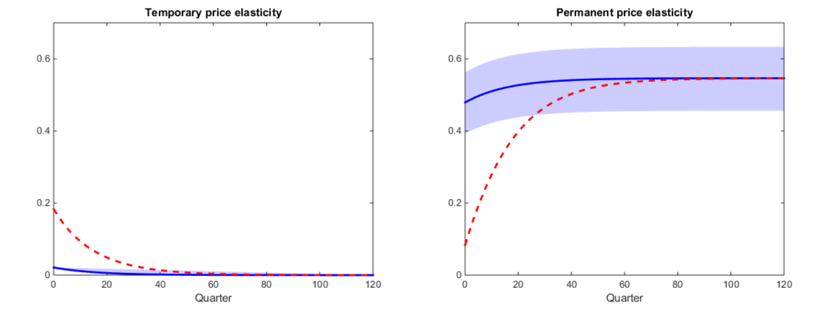
- Y_t is the logarithm of consumption;
- Process $Z^{[1]}$ captures predictability in growth rates;
- Process $Z^{[2]}$ captures stochastic volatility;
- Components of dW_t:
  - Permanent shock;
  - Transitory shock;
  - Stochastic volatility shock.

Impulse Responses



Bands depict .1 and .9 deciles.

Shock-Price Elasticities



Recursive utility and Power utility. Bands depict .1 and .9 deciles.

Success?

- The mechanism relies on endowing investors with knowledge of statistically subtle components of the macro time series. Where does this confidence come from?
- Imposes stochastic volatility exogenously.
- Imposes large risk aversion.

Risk Aversion or Subjective Belief Distortion

- Introduce a positive martingale process $\widetilde{M}$ with a unit expectation.
- Form $\widetilde{S} = \frac{S}{\widetilde{M}}$.
- Then by construction:

$$S = \widetilde{S} \times \widetilde{M}$$

- Use $\widetilde{M}$ to distort investor beliefs and use $\widetilde{S}$ as an alternative stochastic discount factor.

Cannot distinguish belief distortions from stochastic discount factors without further restrictions!

Disentangling belief distortions from stochastic discount factors

$$S = \widetilde{S} \times \widetilde{M}$$

where $\widetilde{M}$ is a positive martingale with a unit expectation.

- Impose parametric restrictions on $\widetilde{S}$ and explore bounds and partial identification
- Preclude martingale components in $\widetilde{S}$ as in Ross recovery
- Impose parametric restrictions on $\widetilde{S}$ and $\widetilde{M}$ as in behavioral finance and research on ambiguity aversion.

Beyond Risk Aversion

Multiple components

- Model risk – what probabilities does a model assign to events in the future?
- Model ambiguity – how much confidence do we place in each model?
- Model misspecification – how do we use models that are not perfect?

Subjective Probability?

- De Finetti:
  "Subjectivists should feel obligated to recognize that any opinion (so much more the initial one) is only vaguely acceptable…So it is important not only to know the exact answer for an exactly specified initial problem, but what happens changing in a reasonable neighbourhood the assumed opinion."
- Savage:
  "No matter how neat modern operational definitions of personal probability may look, it is usually possible to determine the personal probabilities of events only very crudely."

Making Ambiguity Aversion and a Preference for Robustness Operational

- Explore a family of “posteriors/priors” used to weight models possibly relative to a benchmark specification. Dynamic learning plays a central role. (Robust Bayesian analysis)
- Explore a family of alternative potential models or a class of perturbations to a benchmark model subject to constraints or penalization. Future perturbations may not be tied to the past making learning about them impossible. (Control theory and statistical origins.)

Use the decision problem to target the member of the family that has the largest utility consequences.

Long-term Macroeconomic Uncertainty

Joel Mokyr

"There are a myriad of reasons why the future should bring more technological progress than ever before – perhaps the most important being that technological innovation itself creates questions and problems that need to be fixed through further technological progress." (2013)

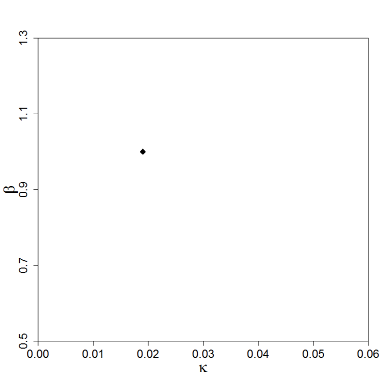
Robert Gordon

"…the rise and fall of growth are inevitable when we recognize that progress occurs more rapidly in some time periods than others…The 1870-1970 century was unique: Many of these inventions could only happen once, and others reached natural limits." (2016)

Slope Uncertainty

$$dY_t = \alpha_y + \beta Z_t + \sigma_y \cdot dW_t \qquad \text{macro evolution}$$

$$dZ_t = \alpha_z - \kappa Z_t + \sigma_z \cdot dW_t \qquad \text{growth evolution}$$



Sets of parameter values (β, κ) constrained by relative entropy.

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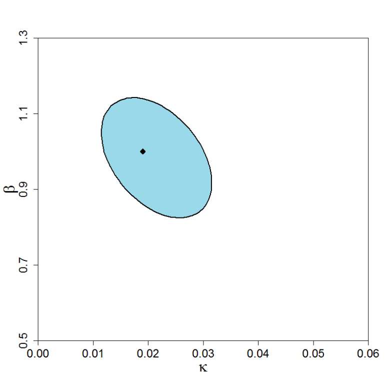

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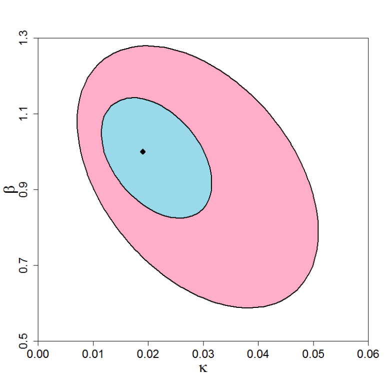

Sets of parameter values (β, κ) constrained by relative entropy.

Model Ambiguity

Specify alternative models as in Chen and Epstein (Econometrica) with axiomatic support from Epstein and Schneider (JET)

- Introduce a family of alternative structured models
- Impose “rectangularity” by possibly augmenting the set of models to achieve a form of dynamic consistency

Restrict conditional means instant-by-instant. We motivate this by considering time-varying parameter models for the slope coefficients (β, κ).

Reference: Hansen and Sargent paper on “Tenuous Beliefs”

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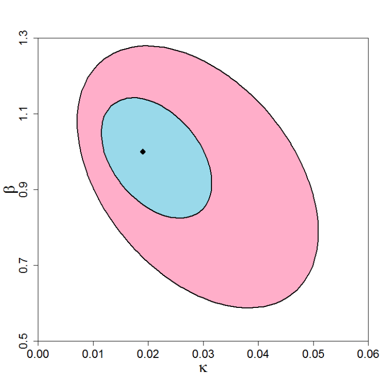


Sets of parameter values (β, κ) constrained by relative entropy.

Parameterizing alternative models

- Change the evolution of W:

$$dW_t = S_t dt + dW_t^S$$

where W^S is a Brownian motion and S_t is a history dependent drift distortion used to represent an alternative structured model.

- Associated martingale (likelihood ratio) M^S

$$dM_t^S = M_t^S S_t \cdot dW_t$$

and expectations:

$$E^S\left[B_t | \mathcal{F}_0\right] = E\left[M_t^S B_t | \mathcal{F}_0\right]$$

- Relative entropy:

$$\lim_{t\to\infty} \frac{1}{t} E\left[M_t^S \left(\log M_t^S\right) \middle| \mathcal{F}_0\right] = \lim_{t\to\infty} \frac{1}{2t} E\left(\int_0^t M_\tau^S |S_\tau|^2 d\tau \middle| \mathcal{F}_0\right)$$

$$= \lim_{\delta\downarrow 0} \frac{\delta}{2} E\left(\int_0^\infty \exp(-\delta\tau) M_\tau^S |S_\tau|^2 d\tau \middle| \mathcal{F}_0\right).$$

Restricting alternative models

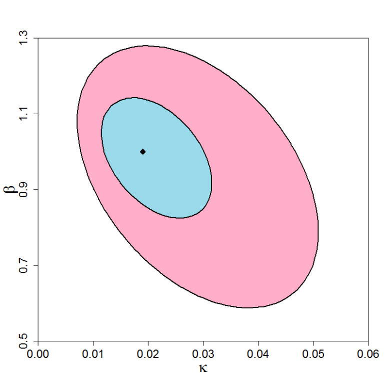
When S is an alternative Markov process,

- compute relative entropy magnitude
- compute corresponding value function

We restrict the family models holding fixed relative entropy and the corresponding value function. Holding both fixed implies a rectangular and convex set of alternative structured models.

Slope Uncertainty

$$dY_t = \alpha_y + \beta Z_t + \sigma_y \cdot W_{t+1} \qquad \text{macro evolution}$$
$$dZ_{t+1} = \alpha_z - \kappa Z_t + \sigma_z \cdot W_{t+1} \qquad \text{growth evolution}$$



Sets of parameter values (β, κ) constrained by relative entropy.

Model Misspecification

Statistical models we use in practice are misspecified. Aim of robust approaches:

- use models in sensible ways rather than discard them
- use probability and statistics to provide tools for assessing sensitivity to potential misspecification

Construct a specification of preferences as in Hansen-Sargent(AER) and Maccheroni-Marinacci-Rustichini (*Econometrica*, *JET*)

Potential Misspecification

- ▷ Change the evolution of W:

$$dW_t = U_t dt + dW_t^U$$

where W^U is a Brownian motion and U_t is a history dependent drift distortion.

- ▷ Associated martingale (likelihood ratio) M^U

$$dM_t^U = M_t^U U_t \cdot dW_t$$

- ▷ Discounted relative entropy relative to structured model:

$$E\left[\exp(-\delta t) M_t^U \left(\log M_t^U - \log M_t^S\right) \Big| \mathcal{F}_0\right]$$
$$= \lim_{\delta \downarrow 0} \frac{\delta}{2} E\left(\int_0^\infty \exp(-\delta\tau) M_\tau^U |U_\tau - S_\tau|^2 d\tau \Big| \mathcal{F}_0\right).$$

Combine Both Approaches

- Model ambiguity over a family of models as a special case of Chen and Epstein
- Model misspecification relative to the family model models by extending Hansen and Sargent

Penalize using:

$$E\left(\int_0^\infty \exp(-\delta\tau) M_\tau^U |U_\tau - S_\tau|^2 d\tau \middle| \mathcal{F}_0\right)$$

where S captures "structured" model ambiguity and U captures potential model misspecification by including "unstructured" alternatives.

Uncertainty and Financial Markets

Bear Bull Rumble, Adrian deRooy

Worst-case Model of a Belief Distortion

- The analysis yields a (constrained) worst-case probability.
- Apply the theory of two-person games. The decision maker optimizes taking as given the worst-case probability.
- Decentralize with worst-case probability.

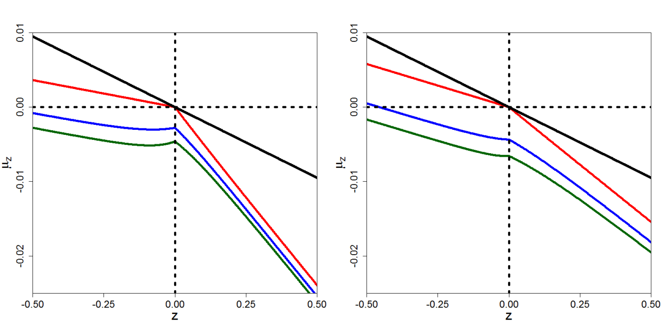
Concerns about model misspecification look like belief distortions.

Market Adjustments for Uncertainty

Suppose the private sector is uncertain about future macroeconomic growth rates

- Investors fear persistence in bad times and fear the lack of persistence in good times
- Induces fluctuations in the market price of uncertainty

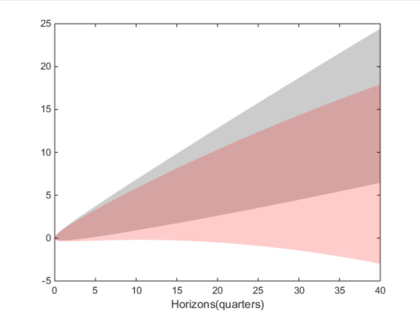
Local Growth Rate Uncertainty



Growth rate drift functions. Left panels: larger structured entropy. Right panels: smaller benchmark entropy. **Black**: baseline model; **red**: worst-case benchmark model; **blue**: Chernoff half life 120; and **green**: Chernoff half life 60.

Tilting Probabilities

Market Adjustments for Uncertainty



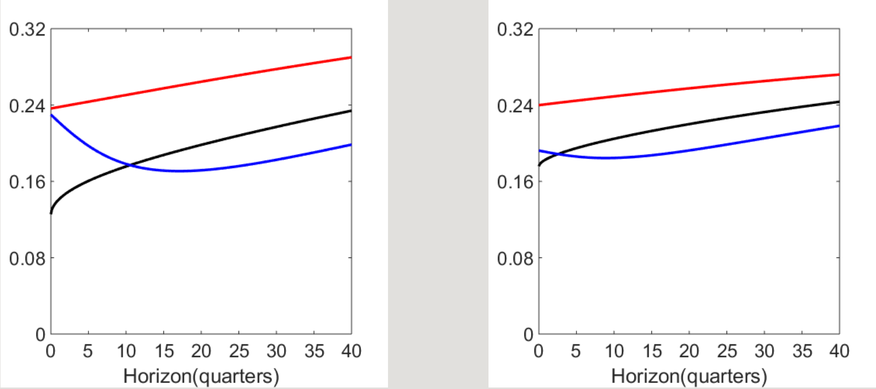
The **black** solid line depicts the median under the baseline model and the shaded region gives the .1 and .9 deciles. The red dashed line is the median under the worst-case model and the red shaded region gives the .1 and .9 deciles.

An Asset Pricing Perspective on Impulses

Imagine an impulse or shock that happens at time $t + \epsilon$.

- This shock has an impact on a macro time series or a cash flow at future times $t + \epsilon$ and beyond, ...
- Exposure in the future of the underlying time series to this shock requires compensation today, say time zero. The magnitude of the compensation or price depends on the date of the cash flow and shock.
- Compensations are state dependent.

Uncertainty Prices



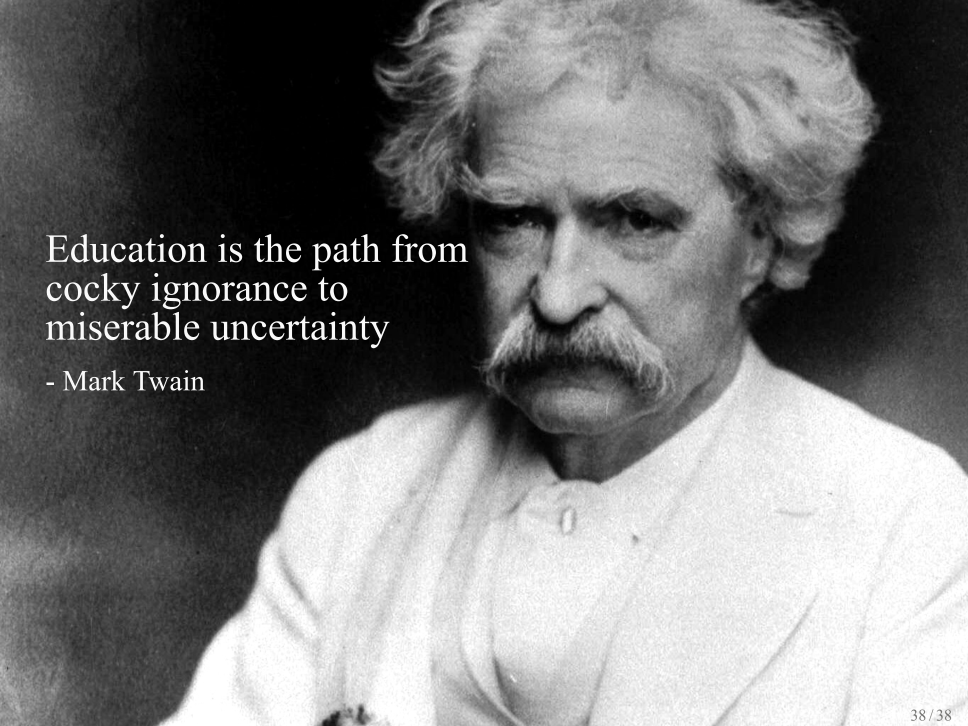
Shock-price elasticities for alternative horizons for the second shock. Left panel: larger structured entropy. **Black**: median of the Z stationary distribution **red**: .1 decile; and **blue**: .9 decile.

What We Have Achieved

- tractable approach for confronting uncertainty
- a mechanism for inducing fluctuations in asset values
- investors fear persistence in bad times and fear the lack of persistence in good times

Broader Perspective

- difficult to disentangle risk aversion from belief distortions
- belief distortions are more compelling in environments in which uncertainty is complex
- statistical tools provide valuable ways to assess environmental complexity
- value to pushing beyond the risk model commonly embraced in economics and finance


Education is the path from cocky ignorance to miserable uncertainty
- Mark Twain