A General Approach to Recovering Market Expectations from Futures Prices (with an Application to Crude Oil)

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The Traditional Consensus

There is a long tradition of using the oil futures prices as a proxy for the market expectation of energy prices in empirical microeconomics.

<u>Examples:</u>

1. Models of purchases of energy-using durables
2. Models of the effect of uncertainty on investment decisions
3. Models of the impact of automotive fuel standards and gasoline taxes.

This practice amounts to treating the risk premium as zero (or at least negligible).

The Emerging New Consensus

<u>Singleton (MSci 2014):</u>
"The evidence for time-varying risk premiums in oil markets … seems compelling".

The presence of such a risk premium can be inferred from evidence of predictable variation in month-to-month returns on oil futures, typically defined as $\left(F_{t+1}^{h-1} - F_t^{h}\right) / F_t^{h}$, where F_t^{h} denotes the price of a futures contract with a maturity of *h* months entered into in month *t*.

Standard no arbitrage arguments imply that

$$F_t^h = E_t\left[S_{t+h}\right] + \operatorname{cov}\left(S_{t+h}, Q_{t+h}\right) / E_t\left[Q_{t+h}\right]$$

where $\operatorname{cov}\left(S_{t+h}, Q_{t+h}\right) / E_t\left[Q_{t+h}\right]$ refers to the risk premium.

⇨ In the absence of a risk premium, $E_t\left[S_{t+h} - F_t^h\right] = 0$, where the prediction error $S_{t+h} - F_t^h$ equals the payoff on an oil futures contract held to maturity.

⇨ Evidence of a predictable component in this payoff such that $E_t\left[S_{t+h} - F_t^h\right] \neq 0$ is consistent with the presence of a time-varying risk premium.

The prediction error $S_{t+h} - F_t^h$ is not stationary and must be transformed in order to estimate the predictable component by regression methods.

The risk premium can be estimated from the regression:

$$\frac{S_{t+h} - F_t^h}{F_t^h} = \alpha_h + \beta_h x_t + \nu_{t+h}, \tag{1}$$

Solving equation (1) for S_{t+h} yields:

$$S_{t+h} = \left(1 + \alpha_h + \beta_h x_t + \nu_{t+h}\right) F_t^h.$$

Hence,

$$E_t\left(S_{t+h}\right) = F_t^h\left(1 + \alpha_h + \beta_h x_t\right) = F_t^h - RP_t^h, \tag{2}$$

where the dollar risk premium is $RP_t^h = F_t^h - E_t(S_{t+h})$.

Full-sample estimation of model (1) under the maintained assumption of stationarity will result in optimal estimates of the risk premium at date t and hence of the oil price expectation prevailing in the market at that point in time.

Empirical Models of Time-Varying Risk Premia

There are three approaches to modelling predictable variation in futures payoffs:

1. Basis regressions

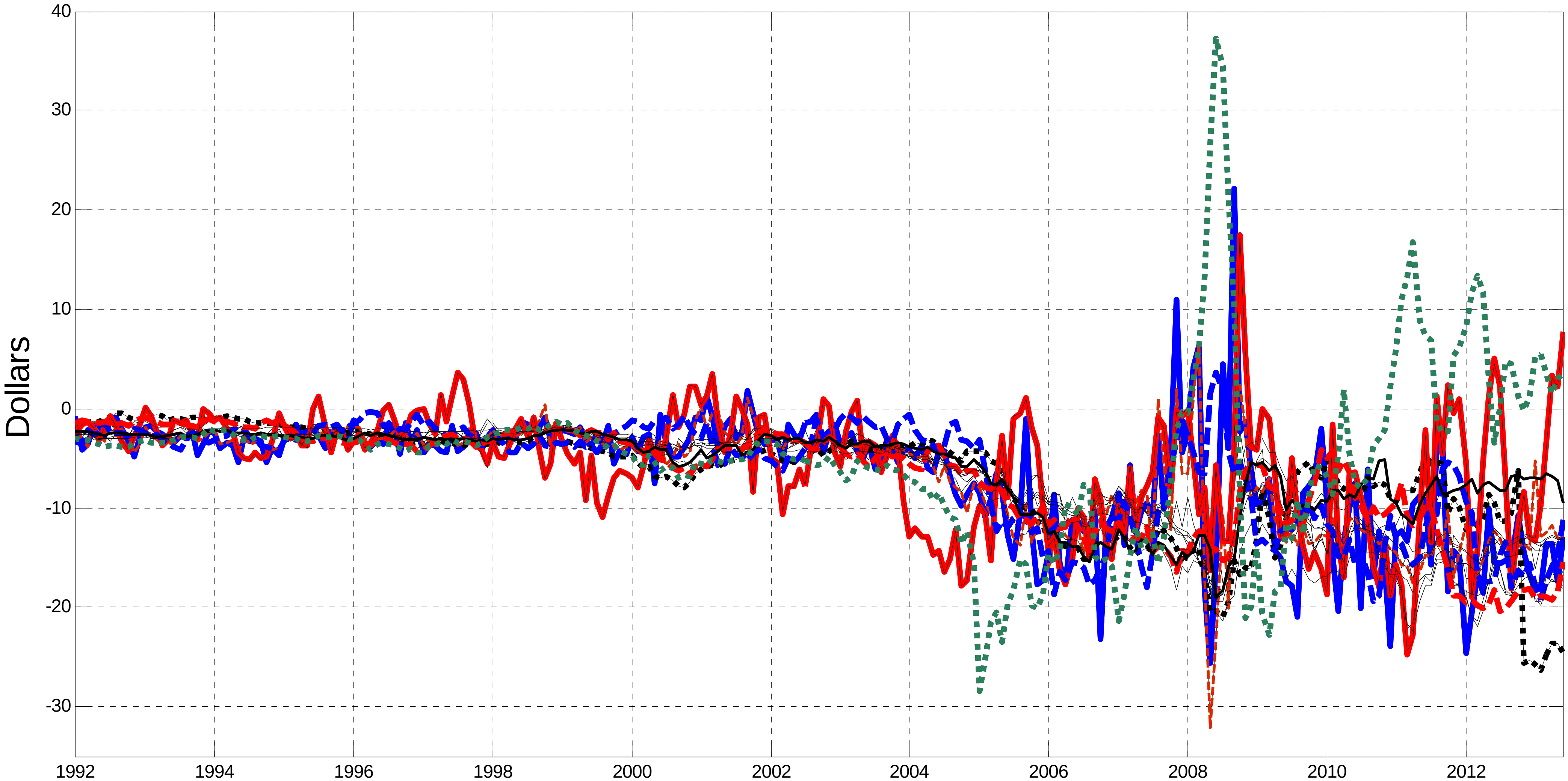
$$\frac{S_{t+h} - F_t^h}{F_t^h} = \alpha_h + \beta_h \left(\frac{F_t^h - S_t}{S_t} \right) + \nu_{t+h},$$

where $\left(F_t^h - S_t\right) / S_t$ is the basis (see Fama and French 1987).

2. Regressions of futures payoffs on financial, macroeconomic, and commodity market predictors, building on earlier studies.

3. Term structure models based on oil futures prices only: RP_t^h is defined as the difference between the rational expectations solution of the model and the actual futures price.

Alternative Monthly Estimates of the Time-Varying Risk Premium in the Oil Futures Market at 1-Year Horizon



A Selection Criterion for Risk Premium Estimates

1. The conventional metric in assessing the accuracy of oil price expectations measures is their MSPE, defined as $E[S_{t+h} - E_t(S_{t+h})]^2$.
2. Standard arbitrage arguments imply that the conditional expectation of the price of oil, $E_t[S_{t+h}] = F_t^h - RP_t^h$.
3. The conditional expectation minimizes the MSPE (see Granger 1969; Granger and Newbold 1986).

Hence, $F_t^h - RP_t^h$ should minimize the MSPE.

⇨ If $F_t^h - \widehat{RP}_t^h$ has higher MSPE than F_t^h, the estimate of the time-varying risk premium is not admissible.

⇨ The most plausible risk premium model delivers the largest MSPE reduction.

Evaluation and Inference

All results are based on the WTI price of crude oil.

WLOG all MSPE estimates have been expressed as ratios relative to the MSPE of the monthly no-change forecast of the WTI spot price of oil.

A ratio below 1 denotes improved accuracy.

Predictive Accuracy of Risk-Adjusted Futures Prices Based on Full-Sample Estimates during 1992.1-2014.6

Horizon h	No Risk Premium F_t^h	Time-Varying Risk Premium $F_t^h\left(1+\hat{\alpha}+\hat{\beta}\left(F_t^h-S_t\right)/S_t\right)$	Constant Risk Premium $F_t^h\left(1+\hat{\alpha}\right)$
3	**0.987**	1.035	1.035
6	**0.982**	1.073	1.082
9	**0.949**	1.074	1.087
12	**0.882***	1.041	1.043

Predictive Accuracy of Risk-Adjusted Futures Prices Based on Full-Sample Estimates evaluated on 1992.1-2014.6

Horizon h	F_t^h	B1	B2	BC	S	DNV1	DNV2	DNV3
3	**0.987**	**0.972***	**0.880****	1.022	**0.992***	**0.927***	1.043	**0.927***
6	**0.982**	1.054	**0.964***	1.073	1.063	1.005	1.095	1.005
9	**0.949**	1.063	1.002	1.078	1.068	1.040	1.122	1.041
12	**0.882***	1.013	**0.901***	1.044	1.004	**0.923***	1.082	**0.923***

Horizon h	GHR1	GHR2	HY1	HY2	PP1	PP2	PP3	BS	HW
3	**0.991***	1.044	1.010	1.007	1.027	1.013	**0.964***	1.027	**0.794***
6	1.015	1.102	1.046	**0.988****	1.089	1.053	1.051	1.073	**0.667***
9	**0.997***	1.118	1.075	**0.986****	1.117	1.038	1.080	1.080	**0.592***
12	**0.831***	1.088	1.071	1.022	1.084	**0.987**	1.045	1.045	**0.535***

Generalized Payoff Regressions

A potential concern is that there is little agreement on the appropriate set of predictors. This suggests forming a payoff regression (labelled “All Predictors”) that includes all 30 return predictors considered in the literature (except for BS because of data limitations).

Based on the unrestricted payoff regression, the statistical significance of each predictor is assessed based on a two-sided t-test of the null of no predictability at the 10% level. Only the statistically significant predictors are retained in the payoff regression labeled “After Pre-testing”.

We also consider equal-weighted model averaging as third option.

Predictive Accuracy of Risk-Adjusted Futures Prices Based on Full-Sample Estimates Evaluated on 1992.1-2014.6

Horizon h	F_t^h	All predictors	After pre-testing	Model Averaging	HW
3	**0.987**	**0.711***	**0.796***	**0.976***	**0.794***
6	**0.982**	**0.738***	**0.885***	1.035	**0.667***
9	**0.949**	**0.764***	**0.862***	1.045	**0.592***
12	**0.882***	**0.568***	**0.667***	**0.980****	**0.535***

Does the all-predictor regression overfit?

Simulations for $h = 12$ show:

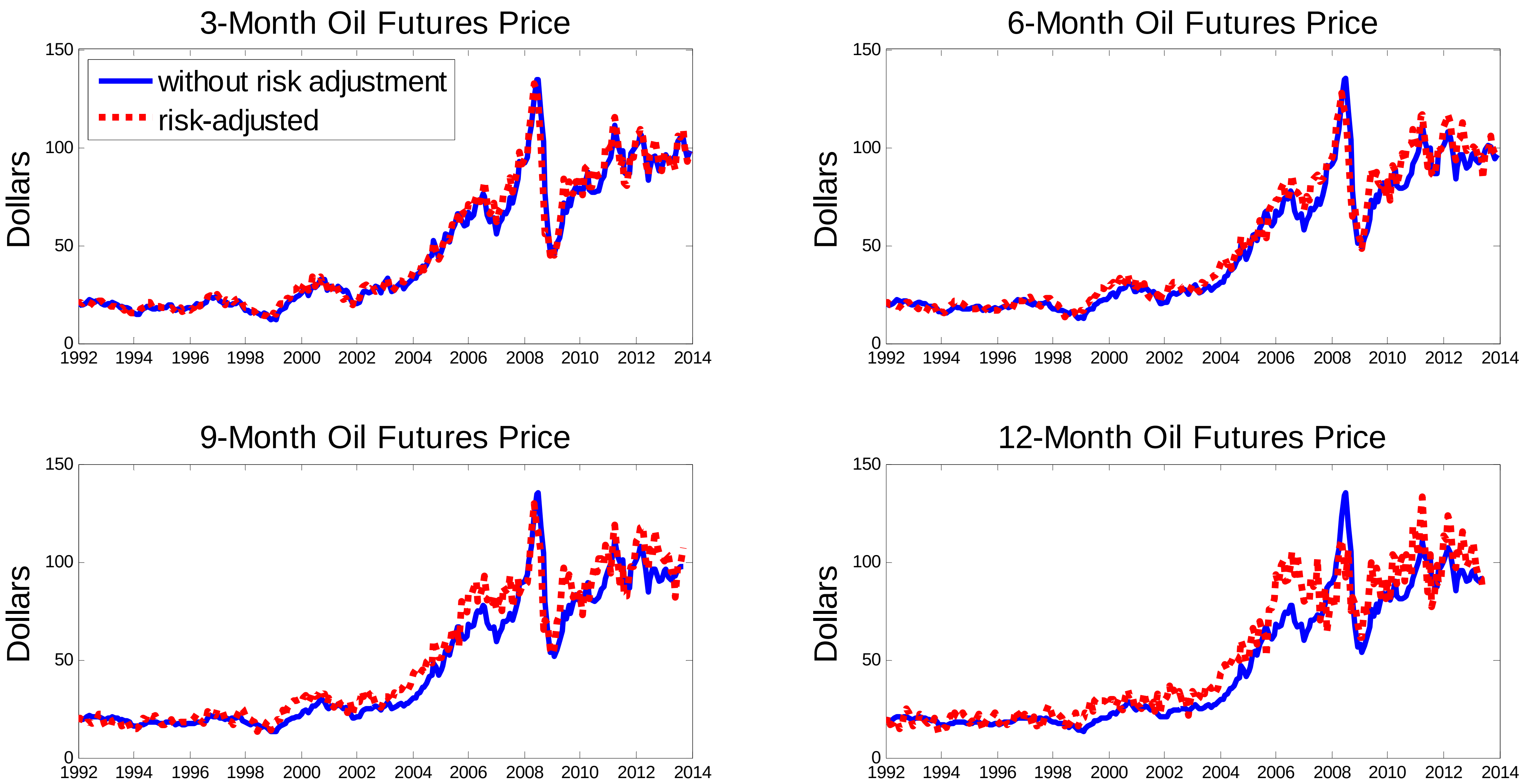
30 redundant white noise predictors lower MSPE ratio by 0.07

30 redundant persistent predictors lower MSPE ratio by 0.38

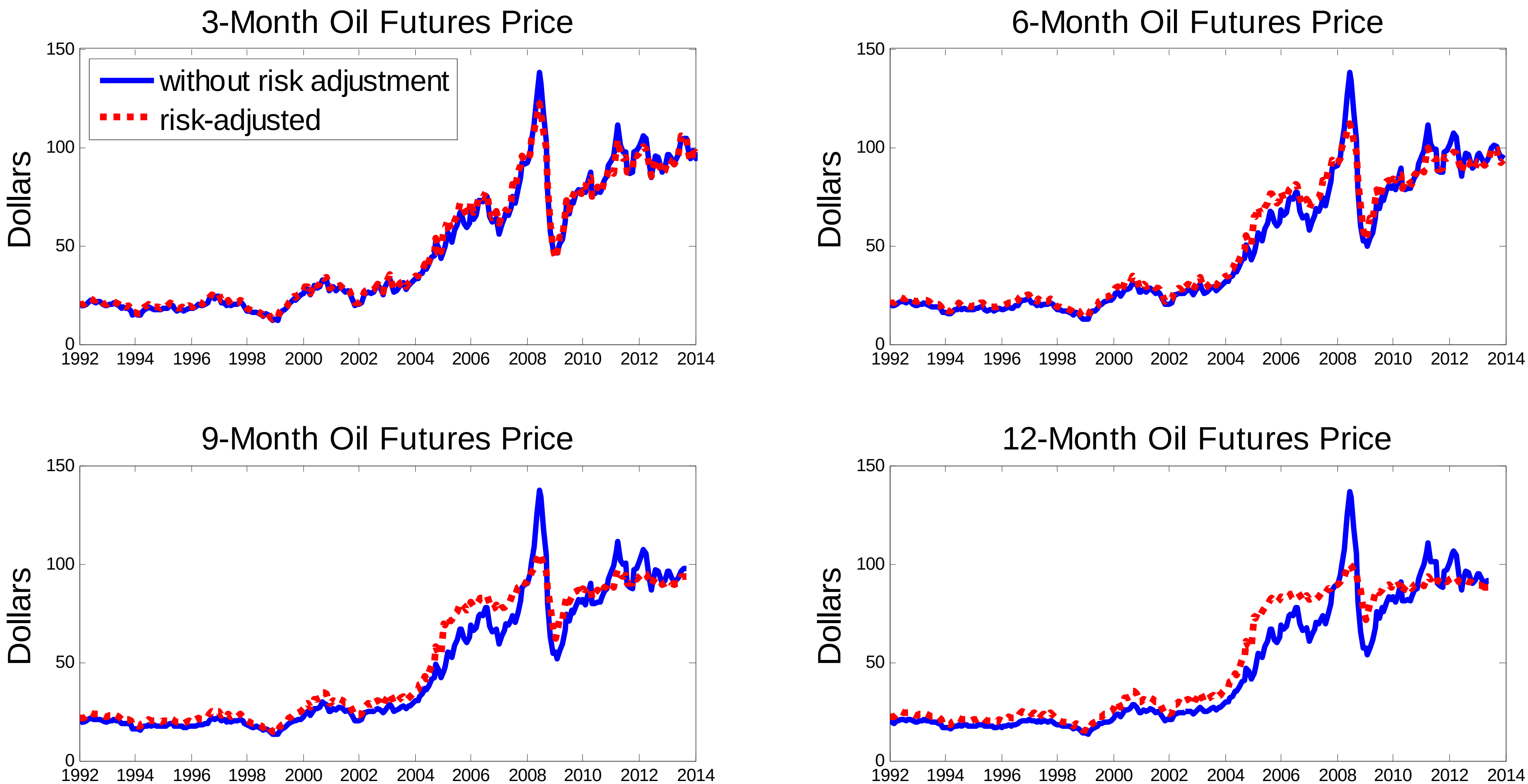
Other Criteria for Evaluating the Estimates

1. One would not expect longer-term oil price expectations to be highly volatile. Except during times of major events affecting the market for oil, they should evolve smoothly over time.
2. In fact, one would expect longer-horizon oil price expectations to evolve more smoothly than the underlying oil futures price.

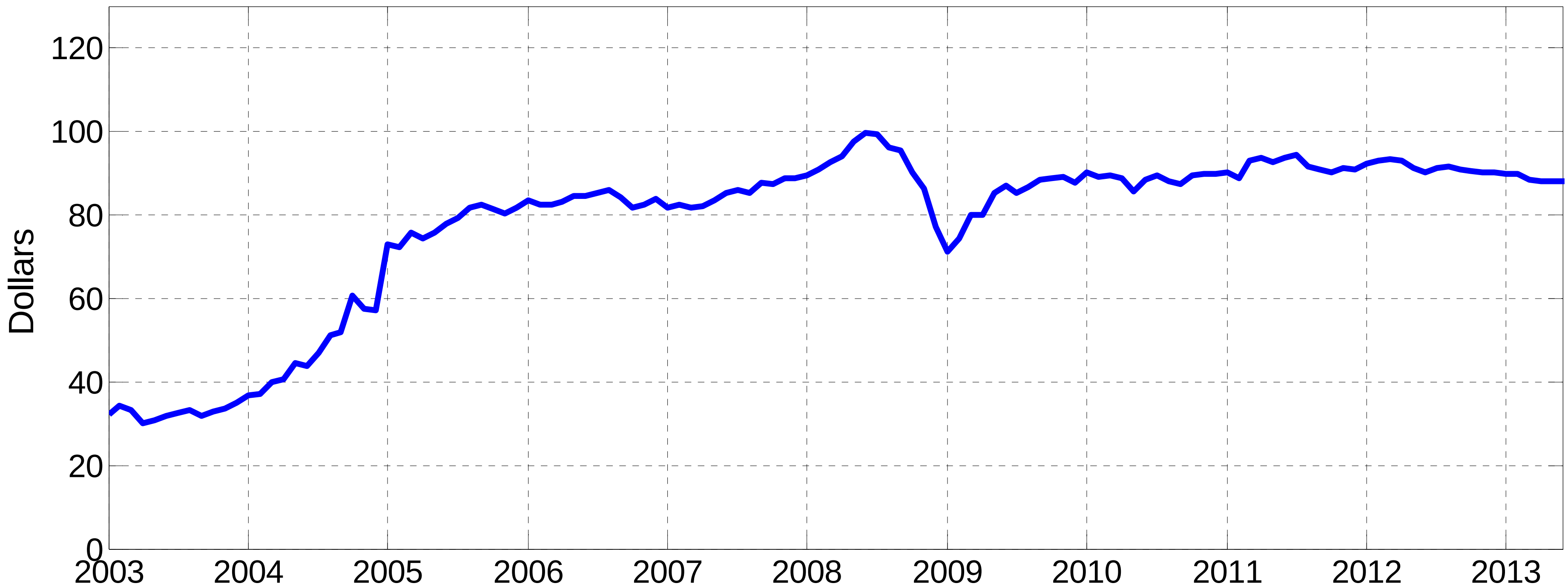
Oil Price Expectations Based on All-Predictor Regression



Oil Price Expectations based on the Hamilton-Wu Model



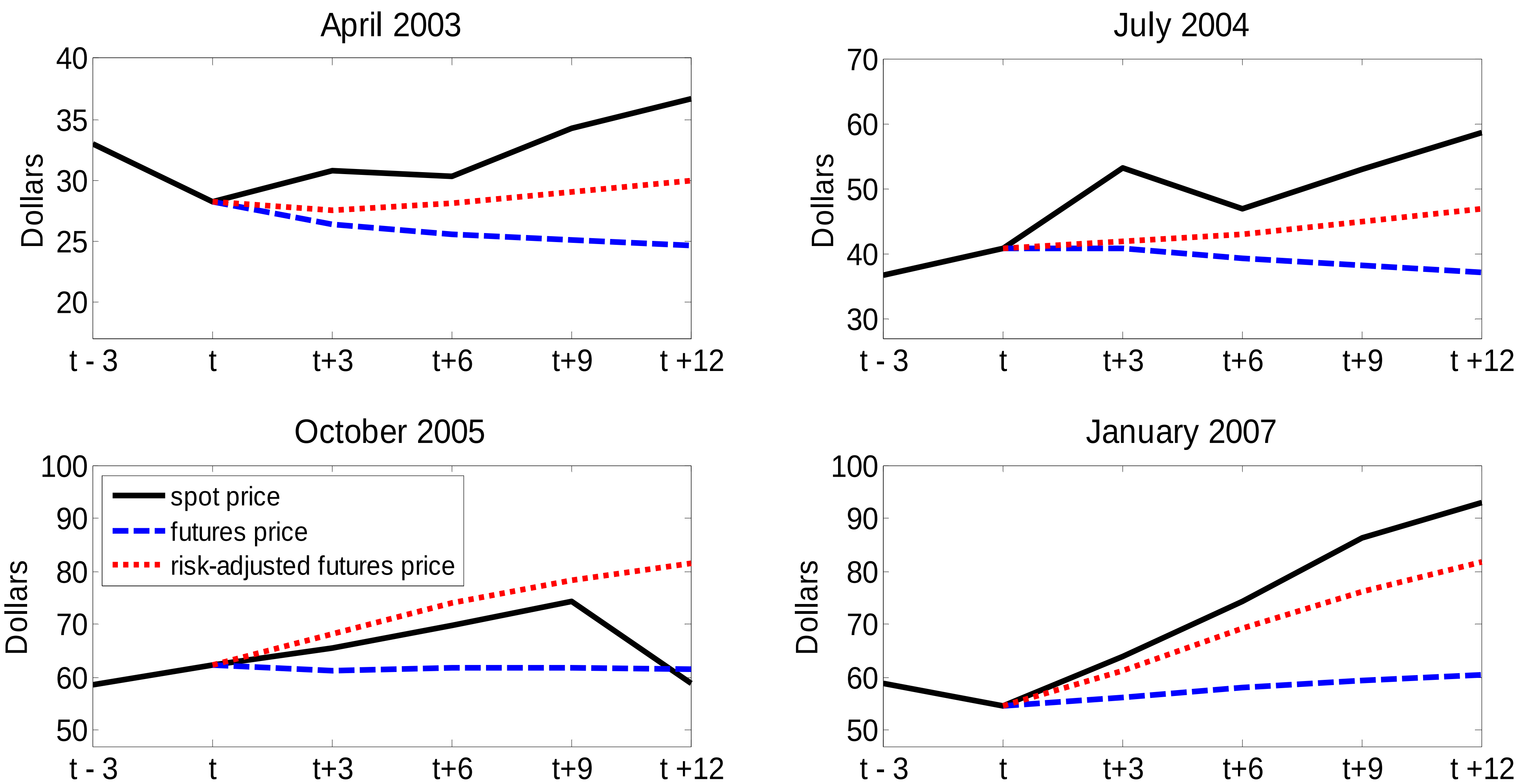
What Did the Market Think in Retrospection?
12-Month Financial Market Oil Price Expectation



NOTES: Risk-adjusted futures price based on Hamilton-Wu model.

Selected trajectories of

F_t^h, the Realized Spot Price S_t, and $F_t^h - \widehat{RP}_t^h$, from the HW Model



MSPE Ratio of Risk-Adjusted Out-of-Sample Forecasts of the Spot Price Based on HW Term-Structure Model

		Evaluation period						
		1992.1:2014.6				2009.1-2014.6		
		Recursive Window			Rolling Window (60 months)	Recursive Window		
Horizon h	F_t^h	Baseline	Alternative 1	Alternative 2	Baseline	Post-break	Alternative 1: Post-break	F_t^h
3	**0.987**	1.083	1.001	**0.987**	1.160	**0.871***	**0.750***	**0.853***
6	**0.982**	1.206	1.158	**0.681**	1.242	**0.676***	**0.628***	**0.743***
9	**0.949**	1.365	1.318	**0.601****	1.275	**0.596***	**0.539***	**0.628***
12	**0.882***	1.511	1.481	**0.539***	1.227	**0.629***	**0.584***	**0.549***

NOTE: Alternative 1 refers to the same model, except we add to the HW forecast the change in the daily oil futures price of maturity h between the day on which the forecast is generated by the HW model and the last trading day of that month. Alternative 2 refers to the recursively evaluated HW model without breaks with the full-sample parameter estimates of the same model imposed. The post-break HW model estimates are based on data starting in 2005.1. None of the other risk premium models succeed out of sample.

Conclusions

- Extensions to quarterly data yield qualitatively similar results

- Implications for measuring oil price shocks are discussed in Baumeister and Kilian (JEP 2016)

Our approach to recovering the market expectation of the spot price can be applied, whenever there is disagreement between alternative models of the time-varying risk premium.

<u>Examples:</u>
Futures markets for foreign exchange, interest rates and other commodities.