Forecasting the Distribution of Option Returns

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Some Important Questions Relating to Options Markets

1. What is the expected return of an option?

2. What is the riskiness of an option?

Economic importance of these questions:
- Option returns reveal compensation investors demand for taking on state-dependent exposures
- Needed for portfolio choice problems involving options
Motivating Example: No-arbitrage SVJ Model
Option Risk and Return in Affine No-Arbitrage Models

SVJ model under \( \mathbb{P} \) measure

\[
\begin{align*}
    dS_t &= (r + \mu)S_t dt + S_t \sqrt{V_t} dW_t^{s}(\mathbb{P}) + J_t(\lambda^P, \mu^P, \sigma^P) \\
    dV_t &= \kappa(\theta^P - V_t) dt + \sigma_v \sqrt{V_t} dW_t^v(\mathbb{P})
\end{align*}
\]

\( \mathbb{Q} \) measure shifts parameters \( \theta^Q, \lambda^Q, \mu^Q, \sigma^Q \)

Fix parameters (from Broadie et al. 2009). For each contract,

1. Retrieve latent state \( V_t \) (invert each \( t \) from \( \sim 90 \) day ATM call)
2. \( \mathbb{P} \) spec: Simulate \( S_{t+1}^b, V_{t+1}^b, b = 1 : B \)
3. \( \mathbb{Q} \) spec: Evaluate option price \( (P_{i,t+1}^b) \) at each simulated \( S_{t+1}^b, V_{t+1}^b \)
4. Result: \( \{P_{i,t+1}^b\}_{b=1}^B \)

Model-based conditional distribution of next-period prices for each contract-day
Option Risk and Return in Affine No-Arbitrage Models

- How frequently does realized option price fall below the $x^{th}$ percentile of the simulated next-day price distribution?

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<tr>
<th>Target</th>
<th>SVJ</th>
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<tr>
<td>99.0</td>
<td>60.8</td>
</tr>
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</table>

- Model-forecasted distributions starkly inconsistent with data
  - Simulated distribution too narrow: Actuals frequently below low quantiles and above high quantiles
Overview of Method
Overview of Method

Uncertainty about future option price comes in two layers

1. Where on the surface will the contract migrate to at $t + 1$?
   ▶ Deterministic roll-down in time-to-maturity dimension
   ▶ Stochastic change in moneyness dimension due to underlying return
   ▶ Option return uncertainty due to “shocks to contract moneyness”

2. What will the shape of the IV surface look like at $t + 1$?
   ▶ Option return uncertainty due to “shocks to surface shape”
   ▶ Time series factor models describe surface variation with $R^2$ approaching 100%

▶ Characterize distribution of future option returns by characterizing distribution of $r_{t+1}$ (1) and $IV_{t+1}$ (2)
Step 1: Define the IV Surface

- S&P 500 data from 1996-2015, OTM contracts only
- Define dimensions of the surface
  1. Time-to-maturity, $\tau$, on grid [30, 60, 91, 122, 152, 182, 273, 365]
  2. Moneyness of contract, $m = \frac{\log(K/S)}{\text{VIX} \sqrt{\tau}}$, on grid $[-3:0.25:1.5]$
- Grid point interpolation of contract-level IV’s each day
Step 2: Model of the Surface, Underlying

System backbone: Factor vector $X_t = (r_t, \log VIX_t, \text{PC's})$, evolves as

$$X_t = \mu + \rho X_{t-1} + \Sigma_{t-1} \epsilon_t$$  \hspace{1cm} (1)

IV at grid point $(m, \tau)$ obeys factor model:

$$\log IV_{m, \tau, t} = \beta_{m, \tau} (1, X_t')' + \gamma_{m, \tau, t-1} u_{m, \tau, t}$$  \hspace{1cm} (2)

Comments:

▶ Important to specify (2) in logs
▶ Panel $R^2$ in (2) is 99%, minimum $R^2$ at any grid point is 96%
▶ Co-variance matrices are GARCH models
▶ $\hat{\epsilon}_t = \hat{\Sigma}_t^{-\frac{1}{2}} e_t$, where $e_t$ are VAR errors (likewise for $\hat{u}_t$)
Step 3: Bootstrap Conditional Price Distribution \((t + 1|t)\)

1. Fix day \(t\) conditioning information. This includes
   - State of the system: \(S_t, X_t, \hat{\Sigma}_t, \hat{\gamma}_t\) and model parameters \((\hat{\mu}, \hat{\rho}, \hat{\beta}, ... )\)
   - Fix contract: \(\tau, K, m\)

2. Draw sample, \(b = 1 : B\), from the empirical distribution of \(\hat{\epsilon}_t\) and \(\hat{u}_t\)

3. For each draw \(b\), feed shocks through estimated system
   - \(X_{t+1}^b = \hat{\mu} + \hat{\rho}X_t + \hat{\Sigma}_t\epsilon_{t+1}^b \Rightarrow VIX_{t+1}^b, S_{t+1}^b = S_t \exp(r_{t+1}^b)\)
   - \(\tau^b = \tau - 1/365\) and \(m^b = \log(K/S_{t+1}^b) / VIX_{t+1}^b \sqrt{\tau_{t+1}}\)
   - \(\log IV_{m^b, \tau^b, t+1} = \hat{\beta}_{m^b, \tau^b} X_{t+1}^b + \hat{\gamma}_{m^b, \tau^b, t} \hat{u}_{t+1}^b\)
   - Finally, \(P_{t+1}^b = BS(S_{t+1}^b, IV_{m^b, \tau^b, t+1}, K, \tau^b)\)

\(\{P_{t+1}^b\}_{b=1}^B\) is conditional forecast of price distribution for individual contract
Step 3: Bootstrap Conditional Price Distribution \( (t + 1|t) \)

1. Fix day \( t \) conditioning information. This includes
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   - \( \log \text{IV}^b_{m^b,\tau^b,t+1} = \hat{\beta}^b_{m^b,\tau^b}X^b_{t+1} + \hat{\gamma}^b_{m^b,\tau^b,t} \hat{u}^b_{t+1} \)
   - Finally, \( P^b_{t+1} = \text{BS}(S^b_{t+1}, \text{IV}^b_{m^b,\tau^b,t+1}, K, \tau^b) \)

\( \{P^b_{t+1}\}_{b=1}^B \) is conditional forecast of price distribution for *individual contract*
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\(\{P_{t+1}^b\}_{b=1}^B\) is conditional forecast of price distribution for individual contract
Case Study: Aug. 28, 2015

S&P at 1989, VIX at 26
Case Study: Aug. 28, 2015

S&P at 1989, VIX at 26
Case Study: Aug. 28, 2015
S&P at 1989, VIX at 26

Put: TTM=21, K=1930

Option Payoff
Spot + IV Surface Bootstrap
Spot Bootstrap
Current Price
Realized Future Option Price

Call: TTM=21, K=2050

Option Payoff
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S&P at 1989, VIX at 26

△-Hedged P&L to Short Option Position

Put: TTM=21, K=1930
Vol: 5.3
Skew: -2.62
Kurt: 15.1

Call: TTM=21, K=2050
Vol: 4.2
Skew: -1.97
Kurt: 10.3
Case Study: Aug. 28, 2015
S&P at 1989, VIX at 26

△-Hedged P&L to Short Option Position

Short Put P&L with Call Hedge, Delta Hedge
Vol: 2.8
Skew: 0.01
Kurt: 11.1
Portfolio Management: Bootstrap Hedge Ratios
Delta-Neutral Risk Reversal

- Evaluating accuracy of portfolio forecast distribution

### Panel A: Mean Regressions

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### Panel B: Percentiles

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<tr>
<td>99.0</td>
<td>99.4</td>
<td>98.9</td>
</tr>
</tbody>
</table>

- Dynamic bootstrap hedge ratio: Out-of-sample Sharpe 1.2 (p.a.)
- Static hedge ratio: Sharpe 0.6 (p.a.)
Case Study: Aug. 28, 2015
S&P at 1989, VIX at 26
Empirical Analysis: S&P 500 Index Options
Assessing Accuracy of Distribution Forecasts

- Target distributions
  - Delta-hedged P&L of short option position
    \[ PL_{t+1}^b = \frac{1}{S_t} \left[ P_t - P_{t+1}^b + \Delta_t (S_{t+1}^b - S_t) \right] \]

- Test accuracy of forecasted conditional distributions at the contract-level
  - Mean, variance, and quantile forecasts
  - Hedge ratios, optimized portfolios

- Baseline for comparison: SVJ model-based forecasts
Out-of-Sample Performance Evaluation

For each day $t$

- Use 1,000-day estimation sample ending at $t$
- Estimate model, form forecast $t + 1$ price distribution
- Form mean, std. dev., percentiles, etc., of bootstrap distribution
### Forecasting P&L

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**Date FE** 'No' 'No' 'No' 'Yes' 'Yes' 'Yes' 'Yes' 'Yes'

**$R^2$** 0.1 4.5 0.2 4.8 0.7 8.4 0.2 8.7

**N** 1157660 1157660 1157660 1157660 1157660 1157660 1157660 1157660

**Note:** Standard errors clustered by date. All Greek coefficients except those on Gamma are multiplied by 100.
Forecasting Quantiles

- For each contract, we forecast quantiles of next-period price from the bootstrap distribution \( \{P^b\}_{b=1}^B \)

- \( Q_x (\{P^b\}_{b=1}^B) \) is the price below which \( x\% \) of the bootstrapped prices lie

- Assess accuracy of quantile forecast with

  \[
  \frac{1}{N} \sum_{i=1}^{N} \mathbf{1} [P_i \leq Q_x (\{P_i^b\}_{b=1}^B)]
  \]

  where \( i \) is a contract-day and \( N \) is number of observations

- The quantile exceedence frequency should be close to \( x\% \) for a good forecast
Forecasting Quantiles

- Exceedence frequency at various target quantiles
- Pooling all contract-days
- Bootstrapped statistical model, simulated SVJ model

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<th>Target</th>
<th>Out-of-sample Bootstrap</th>
<th>SVJ</th>
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<tr>
<td>99.0</td>
<td>98.7</td>
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Forecasting Option Return Quantiles

- Exceedence frequency in moneyness/maturity bins

![Graphs showing exceedence frequency in moneyness/maturity bins for Bootstrap and SVJ models.](image)

- **Bootstrap**
  - Moneyness vs. Percent Exceedences for different maturities (60 days, 90 days, 180 days, 365 days).
  - Each maturity level is represented by a different line color.

- **SVJ**
  - Similar to Bootstrap, but with a different scale and additional lines for 365 days and 180 days.
Hedging Application

Delta Hedge

- Regress $P_{i,t+1} - P_{i,t}$ on $\Delta_{i,t}(S_{t+1} - S_t)$, where $\Delta$ comes from
  1. Bootstrap
  2. Simulated SVJ
  3. Black-Scholes

<table>
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<tr>
<th>Bootstrap</th>
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<th>2</th>
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<td>0.99</td>
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Portfolio Choice Application
Conclusions

We propose a simple, statistically-driven means of measuring risk and return to options positions

I showed you

- Risk/return of state-dependent market exposures
- Hedging
- Portfolio optimization

We also study

- Empirical “Sharpe ratio surface”
  - Comparison with no-arb models
- Multi-horizon forecasts