Time Varying Corporate Capital Stocks and the Cross Section and Intertemporal Variation in Stock Returns

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November 18, 2008

We thank Thomas Chemmanur, session participants at the 2008 UNC-Duke Corporate Finance Conference, and seminar participants at Columbia University and University of South Carolina for helpful comments. This is an extensively revised version of the previous paper circulated under the title "Equity Issuance and Expected Returns: Theory and New Evidence." We also thank Sugato Bhattacharyya, Ines Chaieb, Takao Kobayashi, Nobuya Takezawa, Akiko Watanabe, session participants at the 2008 European Finance Association Meetings, the 2008 Asian/Nippon Finance Association Meetings, and the 2008 Singapore International Conference on Finance, and seminar participants at Hitotsubashi University and Nomura Securities for comments on that version.

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JEL Classification: G12

Keywords: capital investment, productivity of capital, equity issuance, expected return, overlapping generations model, supply shocks, excessive volatility.

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Abstract

This paper uses a general equilibrium model to examine an economy in which firm managers seek to maximize their individual firm's value through the costly adjustment of their capital stock in response to economic shocks. These economic shocks impact both the number of capital units each firm has and how productive each unit is. The ultimate value of these corporate assets is determined by risk averse investors that trade in a competitive multiple security market. Because capital stocks change slowly over time, the relative return to owning them does as well. This generates both cross sectional and intertemporal return patterns in which economic shocks lead to large returns, followed by what appear to be long term abnormal returns in the other direction.

Stock returns appear to display a number of long run cross sectional and intertemporal patterns. One of the most studied is probably the tendency for returns to increase in a firm's book-to-market ratio and decrease in its size (Fama and French (1992)). But there are others. Marsh (1982), Asquith and Mullins (1986), Mikkelson and Partch (1986), Jung, Kim and Stulz (1996), and Baker and Wurgler (2002) all find that following a new equity issue a stock's return is lower than one might otherwise forecast. At the opposite end Ikenberry, Lakonishok, and Vermaelen (1995) find that after a firm engages in share repurchases it tends to have above normal returns. The negative relationship between net equity issuance and subsequent returns is further confirmed in both the U.S. (Fama and French (2007) and Pontiff and Woodgate (2008)) and international (McLean, Pontiff, and Watanabe (2008)) markets. Most importantly for this paper, Titman, Wei and Xie (TWX, 2004) show that the equity issue and repurchase findings are in fact tied to a firm's investments. Firms that invest today tend to have lower returns going forward and visa versa (also see Lyandres, Sun, and Zhang (2007) and Xing (2007)).[1] The goal of this paper is to provide an explanation for this phenomenon in a tractable general equilibrium framework and to generate a number of new testable cross-sectional predictions.

In the model both firms and investors play an active role in the determination of equilibrium prices and thus expected returns. Firms create goods and services across a number of industries by employing industry specific capital that varies over time. One source of this variation comes from employing individuals. These employees sell their human capital to the firm which then converts it to corporate capital. Another source of variation comes through the direct purchase and sale of capital in the financial markets.

[1] Lyandres, Sun, and Zhang (2007) find that an investment factor explains most of the new issues puzzle, reducing 75% to 80% of SEO and IPO underperformance. Motivated by the Q theory, Xing (2007) shows that an investment growth factor does about as well as the value factor, driving out the value effect.

It is assumed that if a firm adds or subtracts from its capital stock in this manner it is relatively less expensive to do so slowly. The demand side of the model comes from risk adverse investors that own shares in the industries and trade them in a competitive market.

By using an overlapping generations framework based on Spiegel (1998) and related to those in Watanabe (2008), and Biais, Bossaerts, and Spatt (2008) the model is not only tractable but can easily be validated against readily available data sources. Another nice feature of the model is that the CAPM holds. However, while it holds period-by-period the model is not static. The return an investor can expect to earn by investing in an industry varies over time as the firms vary their capital levels. In particular, the CAPM beta is a decreasing function of capital investment.

One can think of capital units in this setting somewhat more concretely by considering a firm such as Tyson Industries which is in the poultry business. In their case a capital unit would be a chicken farm and the cash flow would be the profit per farm (or somewhat equivalently the price per pound of chick produced). In the model and reality, the number of farms Tyson has varies over time as their employees are able to build and improve them to a greater or lesser degree each period. At the same time the profits generated by each farm also varies with poultry and feed prices. In reaction to these events Tyson then creates (or sells) additional farms by employing financial capital. High capital values naturally lead them to add to their capital base. Of course, they do so gradually as it typically will not pay to speed up the creation of new farms too much. But, as Tyson and their competitors gradually change their capital base they also undo the economic shocks that lead to the high capital values to begin with.

Notice that the two events that move capital values: supply (number of farms in the above example) and profit per unit (poultry prices) also move stock prices. Since firms adjust their capital bases in response to abnormally high or low capital values stock prices also move both with the initial shock and the subsequent reaction by firms to that shock. This then leads to patterns similar to those in TWX as well as the prior literature on stock sales and repurchases. That is, high stock returns are accompanied by an immediate increase in capital accumulation. Afterwards returns are below normal and capital accumulation in the industry tapers off. But, as in TWX the return phenomena are tied to changes in corporate capital levels and not stock sales and repurchases *per se*.

We provide empirical evidence that is consistent with the above story. Our model implies that the productivity of corporate capital and capital investment, or equivalently the book-to-market ratio, are the key variables to determine the cross-sectional variation in stock returns. We measure productivity by the ratio of earnings per unit capital to the cost of creating unit capital. We find that the zero investment portfolio that goes long high-productivity growth firms and short low-productivity growth firms earns a value-weighted average return of 0.81% per month. The risk-adjusted alpha from the standard four factor model is 0.92% per month. Both of these numbers are not only statistically significant (at the 1% level), but also economically significant.

This paper is not the first to theoretically examine the relationship between stock returns and both real and financial corporate capital adjustments. In response to the findings in the empirical literature on new share issues a number of authors have proposed behavioral explanations in which managers take advantage of overvalued shares to raise capital (Loughran, Ritter, and Rydqvist (1984), Ritter (1991), Loughran and

Ritter (1995), Rajan and Servaes (1997), Pagano, Panetta, and Zingales (1998), Baker and Wurgler (2000) and Lowry (2003)). In contrast, recent models by Pastor and Veronesi (2005) and Dittmar and Thakor (2007) both offer rational explanations.

Like Pastor and Veronesi (2005) and Dittmar and Thakor (2007) this paper also proposes a rational model that generates return series similar to what is seen in the data. In Pastor and Veronesi market conditions change exogenously over time along three dimensions: expected returns, aggregate profitability, and uncertainty regarding future profitability. This leads to a number of phenomena including IPO waves and post-IPO returns that are lower than one might expect in a static model. In Dittmar and Thakor a firm's managers and the investing public may not agree on the value associated with a new investment. When the divergence is large firms finance with debt, and when it is small with equity. What drives their result is that a firm's stock value is likely to be higher when investors and managers share the same beliefs. That occurs because when the beliefs are similar the investors think it is less likely that management will engage in wasteful investment. This in turn increases the appeal of equity financing as well but it also means that going forward shareholder returns are likely to be lower as the level of agreement between them and management has nowhere to go but down.

This paper contributes to the above articles by also seeking to explain the phenomena between investment and returns documented in TWX. Another contribution is to do so within a general equilibrium framework. That allows the model to examine not only time variation in returns, but betas, cross sectional patterns, and the relationship these all bear to variables like industry productivity. In the Pastor and Veronesi (2005) paper market conditions are exogenous and firms react to them, here they are endogenous and influenced by the firms. This interplay allows the model to also make some predictions regarding how overall capital investment impacts the future trajectory of the economy. Also, where Dittmar and Thakor (2007) look at how heterogeneous beliefs influence returns in this article everyone has identical beliefs.

Other related models are those by Berk, Green and Naik (1999) and Carlson, Fisher, and Giammarino (2004, 2006). These authors use real options models to examine how a firm's expected return will vary over time and focus on the relationship between a firm's book-to-market and size that they generate. As they show, it tends to induce patterns that look like those found in Fama and French (1992). The firms in this paper's model have a much simpler investment problem yet generate similar book-to-market return patterns. Another difference is in the data needed to corroborate each model's predictions. Using commonly available data sources it is often difficult to know where and to what degree real option values are influencing a firm's current stock price. In the model developed here one only needs information like the firm's current capital and investment levels. While that does not make the model any more or less likely to be "right" it does make it easier to test and potentially refute. Finally, as both Berk, Green and Naik (1999) and Carlson, Fisher, and Giammarino (2004) acknowledge, their models are set up in a partial equilibrium framework with either the pricing kernel or the demand function exogenously given. In contrast, our model again is a general equilibrium model in which prices equilibrate supply and demand through market clearing.

The paper is structured as follows. Section 1 presents the model. Section 2 contains the analysis, followed by some empirical evidence in Section 3. Section 4 concludes.

1. A Competitive Model with Capital Adjustments

1.1 Setting

There are K production factors which the paper will also refer to as industry sectors. Each production factor is used by a continuum of competitive equity value maximizing price taking firms with mass of unity. There is a single risk free bond that pays r per period and serves as the numeraire with a constant value of 1. The production factors evolve over time via:

$$N_t = N_{t-1} + \eta_t + Y_t \tag{1}$$

where N_t equals the $K\times1$ vector of production factors, t the time period. The η_t represents the influence of human capital on the total supply of corporate capital. In the model people are born with a human capital endowment which in aggregate equals η_t. Through their employment this human capital is then converted into corporate capital and has the impact shown in (1). From the perspective of investors η_t is a normally distributed random vector with mean zero and variance-covariance matrix Σ_η. The Y_t term is a $K\times1$ vector of capital created by firms in addition to what they get from the amount generated by their employees in the normal course of their business.[2]

In each period the production factors pay a $K\times1$ dividend vector D_t that evolves via:

$$D_t = D_{t-1} + G\left(\bar{D} - D_{t-1}\right) + \delta_t. \tag{2}$$

[2] Other functional forms with various interpretations are clearly possible. For example, it is possible to change the assumption that employee capital contributions have a mean of zero by including a depreciation component to (1). In the long run capital stocks will then adjust so that depreciation offsets the average capital added by labor.

Here G is a $K \times K$ matrix of constants representing the speed at which asset payouts mean revert, $\bar{D}$ a $K \times 1$ vector of constants representing the long run payout per asset class, and the term δ_t is a $K \times 1$ normally distributed random vector with zero mean and variance-covariance matrix Σ_δ.

1.2 Firms

Each firm's output comes from a single production factor.[3] Firm fk, (i.e., firm f using factor k) seeks to maximize its current equity value as follows:

$$\max_{y_{fk,t}} \left(n_{fk,t-1} + \eta_{fk,t} + y_{fk,t} \right) p_{k,t} - c_{1k} y_{fk,t} - \frac{1}{2} c_{2k} y_{fk,t}^2 - \eta_{fk,t} p_{k,t}, \tag{3}$$

where $p_{k,t}$ is the period t market price of a unit of capital associated with the k^{th} production factor. The expression $n_{fk,t} = n_{fk,t-1} + \eta_{fk,t} + y_{fk,t}$ is the date-t capital employed by firm fk. The human capital it employs to create additional corporate capital is represented by the $\eta_{fk,t}$ term and $y_{fk,t}$ is the new capital deployed beyond what is created by the employee base in the normal course of business.[4] Thus, the term $\eta_{fk,t} p_{k,t}$ in (3) implies that firms have to pay their employees the full market value of the capital they create. Implicitly, this means both sides of the labor market are competitive. By contrast, $y_{fk,t}$ corresponds to deployed capital that creates positive net present value. The constants c_{1k} and c_{2k} represent capital adjustment costs for the k^{th} production factor. All firms in an industry are assumed to face the same costs c_{1k} and c_{2k}.

[3] In principle, firms can produce more than a single type of capital output. Assuming that the cost of building or liquidating capital is assessed at the firm level for each production factor separately, there is no loss of generality in considering firms that specialize only in a single type of output.

[4] There is no physical limit to the amount of new capital that can be deployed. Also, to maintain tractability new capital is financed only through the issuing (repurchase in the case of negative deployment) of equity. One could also allow for the use of riskless debt without any fundamental change to the model's results.

Each of the $c_{\cdot k}$ terms represent a different aspect of the costs associated with creating productive capital. The c_{1k} parameter captures the base line cost of constructing a unit of production. For example, consider a poultry producer like Tyson. For it c_{1k} equals the cost of building a new chicken farm. This ultimately depends on the price of raw materials like wood, wire, trucks, and the like and not the market value of Tyson's own assets. Thus, the firm can potentially profit by building new farms when their market value exceeds their construction value and by selling them off when the reverse is true. The c_{2k} parameter captures the cost of increasing the speed with which assets are created or sold. Presumably, rushing the construction of a new chicken farm increases its ultimate cost but does allow the firm to generate cash flows from it earlier on. Naturally, whether a firm wishes to rush production of a new facility depends upon how much it expects to earn on it.

Differentiating (3) with respect to $y_{fk,t}$, recalling that the firms take the price vector as given, and then solving for $y_{fk,t}$ yields for each production factor a total capital issuance of

$$y_{fk,t} = \frac{p_{k,t} - c_{1k}}{c_{2k}}. \tag{4}$$

Or, integrating both sides over f and recalling that the total mass is unity,

$$y_{k,t} = \frac{p_{k,t} - c_{1k}}{c_{2k}}, \tag{5}$$

where $y_{k,t} = \int y_{fk,t}\, df$ is the total amount of new capital deployed in factor k. For reference, let $N_{k,t} = \int n_{fk,t}\, df$ and $\eta_{k,t} = \int \eta_{fk,t}\, df$. Writing equation (5) in vector form:

$$y_t = C_{2D}^{-1}(P_t - C_1), \tag{6}$$

where C_1 is the vector of linear costs with the k^{th} element c_{1k}, and C_{2D} is a $K\times K$ matrix with the k^{th} diagonal element equal to c_{2k} and zeros elsewhere thus:

$$C_{2D} = \begin{bmatrix} c_{21} & & & 0 \\ & c_{22} & & \\ & & \ddots & \\ 0 & & & c_{2K} \end{bmatrix}. \tag{7}$$

1.3 Population

Investors, like firms, take prices as given. A continuum of investors with unit mass is born in period t, consume and then die in period t+1. Each investor has a negative exponential utility function with risk aversion parameter θ. The only endowment an investor begins life with is his or her human capital. In their first period of life they sell their human capital to firms (that convert it to corporate capital) and buy and sell securities to fund their retirement.

Let $X_{i,t}$ represent the $K\times 1$ portfolio of share holdings of investor i in period t. Each share is assumed to represent one unit of a production factor. Let $w_{i,t}$ be the wealth with which investor i is born at date t. The assumption that people are born only with human capital implies $w_{i,t}$ equals the market value of that capital. Furthermore, because investors have negative exponential utility functions and all of the random variables are normally distributed the initial allocation of human capital does not impact the model's equilibrium results. Thus, all that is needed to proceed is knowledge that in the aggregate the incoming human capital equals η_t and that those with skills associated with industry k will earn $\eta_{k,t} p_{k,t}$.

Based on the above discussion and letting R=1+r an investor's period t+1 consumption equals:

$$X'_{i,t}\left(P_{t+1}+D_{t+1}-RP_t\right)+Rw_{i,t} \tag{8}$$

because it is assumed that he or she sells the portfolio prior to death. Again using the assumption that all of the random vectors are normally distributed, and that the investors have negative exponential utilities investors maximize their expected utility by solving the following mean-variance problem:

$$\max_{X_{i,t}} E_t\left[X'_{i,t}\left(P_{t+1}+D_{t+1}-RP_t\right)+Rw_{i,t}\right]-\frac{\theta}{2}\operatorname{var}_t\left[X'_{i,t}\left(P_{t+1}+D_{t+1}-RP_t\right)+Rw_{i,t}\right]. \tag{9}$$

This reduces to,

$$\theta\operatorname{var}_t\left[Q_{t+1}\right]X_{i,t}=E_t\left[Q_{t+1}\right], \tag{10}$$

where

$$Q_{t+1}=P_{t+1}+D_{t+1}-RP_t \tag{11}$$

is the excess payoff vector from a unit position in each type of capital, and $\operatorname{var}_t\left[Q_{t+1}\right]$ is its variance-covariance matrix. Integrating over the continuum of investors and setting the market clearing condition $N_t=\int X_{i,t}\,di$ yields,

$$\theta\operatorname{var}_t\left[Q_{t+1}\right]N_t=E_t\left[Q_{t+1}\right]. \tag{12}$$

1.4 Equilibrium

Investors conjecture that prices are determined via the following formula:

$$P_t=A_0+A_1N_t+A_2D_t \tag{13}$$

where A_0 is a K×1 vector, while A_1 and A_2 are K×K matrices. Next, update the time subscripts in (13) to t+1 and then plug equations (1), (2) and (6) into equation (13) in order to solve for P_{t+1} in terms of the parameter values known at time t and the unknown t+1 shocks:

$$P_{t+1} = \left(I - A_1 C_{2D}^{-1}\right)^{-1} \left\{A_0 + A_1\left(N_t + \eta_{t+1} - C_{2D}^{-1} C_1\right) + A_2\left[D_t + G\left(\bar{D} - D_t\right) + \delta_{t+1}\right]\right\}. \quad (14)$$

Using (13), equation (14) can be rewritten as

$$P_{t+1} = (I - A_1 C_{2D}^{-1})^{-1} \left\{P_t + A_1\left(\eta_{t+1} - C_{2D}^{-1} C_1\right) + A_2\left[G\left(\bar{D} - D_t\right) + \delta_{t+1}\right]\right\}, \quad (15)$$

implying that the price vector follows a VAR(1) process. With some algebra, use (11) and (15) to write

$$\begin{aligned} E_t\left[Q_{t+1}\right] &= \left[\left(I - A_1 C_{2D}^{-1}\right)^{-1} - RI\right] P_t \\ &\quad + \left(I - A_1 C_{2D}^{-1}\right)^{-1}\left[A_2 G\left(\bar{D} - D_t\right) - A_1 C_{2D}^{-1} C_1\right] + D_t + G\left(\bar{D} - D_t\right). \end{aligned} \quad (16)$$

Similarly,

$$\begin{aligned} &\operatorname{var}_t\left[Q_{t+1}\right] = \operatorname{var}_t\left[\left(I - A_1 C_{2D}^{-1}\right)^{-1}\left(A_1 \eta_{t+1} + A_2 \delta_{t+1}\right) + \delta_{t+1}\right] \\ &= \left[\left(I - A_1 C_{2D}^{-1}\right)^{-1} A_1 \Sigma_\eta A_1^{'} \left(I - A_1 C_{2D}^{-1}\right)^{-1\prime} + \left(\left(I - A_1 C_{2D}^{-1}\right)^{-1} A_2 + I\right) \Sigma_\delta \left(\left(I - A_1 C_{2D}^{-1}\right)^{-1} A_2 + I\right)^{\prime}\right] \\ &\equiv V. \end{aligned}$$

(17)

To solve for the equilibrium values of the A's, replace $E_t\left[Q_{t+1}\right]$ and $\operatorname{var}_t\left[Q_{t+1}\right]$ in equation (12) with the corresponding terms in equations (16) and (17). The coefficients of N_t and D_t must vanish separately as well as those that do not multiply any time varying parameters. This yields for the terms that do not multiply either N_t or D_t,

$$\left[\left(I - A_1 C_{2D}^{-1}\right)^{-1} - RI\right] A_0 + \left(I - A_1 C_{2D}^{-1}\right)^{-1}\left[A_2 G \bar{D} - A_1 C_{2D}^{-1} C_1\right] + G\bar{D} = 0, \quad (18)$$

while for the terms multiplying N_t,

$$\left[\left(I - A_1 C_{2D}^{-1}\right)^{-1} - RI\right] A_1 -$$

$$\theta\left[\left(I - A_1 C_{2D}^{-1}\right)^{-1} A_1 \Sigma_\eta A_1' \left(I - A_1 C_{2D}^{-1}\right)^{-1\prime} + \left(\left(I - A_1 C_{2D}^{-1}\right)^{-1} A_2 + I\right) \Sigma_\delta \left(\left(I - A_1 C_{2D}^{-1}\right)^{-1} A_2 + I\right)'\right] = 0, \quad (19)$$

and finally for the terms multiplying D_t,

$$\left[\left(I - A_1 C_{2D}^{-1}\right)^{-1} A_2 + I\right]\left(I - G\right) - RA_2 = 0. \quad (20)$$

2. Analysis

2.1 Steady State

The economy is defined to be in a steady state in period t if firms do not actively seek to change their capital stock and if the expected change in the payout per unit of capital is expected to remain unchanged. This is a useful base case as it yields the model's predictions regarding unconditional moments in the data. From there it is then possible to see how various shocks to the system will impact estimated returns, risk factors and other financial and economic variables of interest.

Firm's do not actively change their capital stock in period t if Y_t equals a $K\times 1$ vector of zeros and if dividends are also expected to remain unchanged implying $E[D_{t+1}]=D_t$. From equation (6) the vector Y_t will equal zero if and only if $P_t = C_1$. Similarly, asset payouts are expected to remain unchanged if and only if $D_t = \overline{D}$. The unconditional expected return to an investor from holding a claim in one unit of corporate asset k equals

$$E\left[r_{k,t+1}\right] = \frac{E\left[p_{k,t+1}\right] - p_{k,t} + d_{k,t}}{p_{k,t}}, \quad (21)$$

where $d_{k,t}$ represents the k'th element of the vector D_t. Employing the condition that $p_{k,t} = c_{1k}$ and $d_{k,t} = \bar{d}_k$ in (15) and using the result in (21) leads to the following proposition:

Proposition 1: If the economy is in steady state then

$$E\left[r_{k,t+1}\right] = \frac{\bar{d}_k}{c_{1k}}. \tag{22}$$

Proposition 1 implies that if a firm uses asset class k then its stock's average return will equal the long run ratio of that asset's ability to generate cash flows per unit to its unit creation cost, which we call "productivity." Further note, the right hand side of (22) can (at least in principle) be calculated with data commonly available. For a firm it should equal the long run average earnings divided by the per period change in book value or similar measures of a firm's cash flow and productive assets.

Firm k's steady state expected returns in equation (22) are independent of risk or risk attitudes in the economy. The reason is that the number of capital units deployed adjusts to the point at which investors bear an optimal, or steady state, level of risk. Essentially, N_t in equation (12) adjusts to the point where it offsets the term $\theta \operatorname{var}_t\left[Q_{t+1}\right]$.

2.1.1 Book-to-Market in the Steady State

Since c_{k1} is the cost of replacing a unit of capital, when done as economically as possible, it should correspond somewhat to a firm's per unit of capital book value in industry k as well. The firm's actual book value in this case would be $c_{k1}n_{k,t}$. In the long run the steady state requirement that $p_{k,t} = c_{1k}$ thus implies that the long run book-to-market ratio and thus Tobin's q for an industry should equal 1. This result will come into play in the next section where the impact of deviations from the steady state are examined and will establish a value versus growth "premium" in stock returns.

2.2 Steady State Disrupted by a One Time Shock to Capital

Imagine the economy is in its long run steady state as of period $t-1$ and there is a one time shock to capital (η) or cash flows (D) in period t. To simplify the notation needed for the analyses define the following variables:

$$\begin{aligned}&\hat{P}_t \equiv P_t - C_{1,}\\&\Delta D_t \equiv D_t - D_{t-1} = G\left(\bar{D} - D_{t-1}\right) + \delta_t, \text{ and}\\&F \equiv I - A_1 C_{2D}^{-1}.\end{aligned} \tag{23}$$

Subtracting C_1 from both sides of (15) and making the above substitutions yields:

$$\hat{P}_t = F^{-1}\left(\hat{P}_{t-1} + A_1\eta_t + A_2\Delta D_t\right). \tag{24}$$

Rolling (24) back and then substituting out $\hat{P}_t$ for P_t produces the equilibrium price vector that investors expect to occur going forward:

$$P_t = C_1 + \sum_{s=0}^{\infty} F^{-s-1}(A_1\eta_{t-s} + A_2\Delta D_{t-s}) \tag{25}$$

implying the impulse response τ periods after a time t supply shock is given by $F^{-\tau-1}A_1\eta_t$. Similarly, the impulse response τ periods after a time t dividend change is given by $F^{-\tau-1}A_2\Delta D_t$. Since $F \equiv I - A_1 C_{2D}^{-1}$ as long as A_1 is negative definite equation (25) implies that a capital or cash flow shock decays roughly at the rate of $0 < \left\|F^{-1}\right\| < 1$ (in some matrix norm) per period. The next proposition says that this will always occur in an economy with a large quadratic adjustment cost (C_{2D}).

Proposition 2: As C_{2D}^{-1} approaches zero, A_1 tends to a negative definite matrix in an equilibrium in which A_1 is finite.

Proof. See the Appendix for the proof of this and all other propositions.

In fact, it is straightforward to confirm that the equilibria with finite A_1 converge to those of Spiegel (1998) as $C_{2D}^{-1} \to 0$. Under this assumption C_{2D}^{-1} equals the zero matrix and (20) simplifies to,

$$-A_2(rI+G)+I-G=0 \tag{26}$$

and thus A_2 equals $(I-G)(rI+G)^{-1}$. Next (19) reduces to,

$$rA_1+\theta\left[A_1\Sigma_\eta A_1'+R^2(rI+G)^{-1}\Sigma_\delta(rI+G)^{-1\prime}\right]=0, \tag{27}$$

which can now be solved for A_1, while using (18) then yields $A_0=\frac{R}{r}(rI+G)^{-1}G\bar{D}$.

Assuming A_1 is negative definite, equation (25) provides a number of empirical predictions. At time 0 suppose a shock creates a large positive price move across stocks. Equation (25) shows that this will then be followed by a declining price series. Note, this does not mean returns are negative as investors continue to receive a cash flow stream from the assets. But it does mean returns are lower than they are on average. Looking at returns, the implication is that a large return in one direction will lead to lower future returns in the other. Also, note what this implies about the relationship between capital expenditures and future returns. When an industry capital unit fetches a value above its long run equilibrium value, firms in that industry increase their holdings of it (see equation (6)). Thus, if a shock generates a large price increase that will in turn generate new investment by firms in the industry. This will be followed by lower equilibrium returns for investors, lower capital prices for the industry, and reduced investment. The process continues on like this until the steady state equilibrium is restored.

2.2.1 Book-to-Market and Expected Returns

As discussed in Section 2.1 for industry k the replacement cost for a unit of capital, when done as economically as possible equals c_{1k}. Thus, in steady state since $p_{k,t} = c_{1k}$ one has that the book-to-market ratio should equal one. But short run shocks will change that. For example, if the cash flow (d_k) to a particular type of capital goes up so will the market value of that asset. This will decrease the book-to-market ratio and induce capital accumulation by firms in the industry.

If A_1 is negative definite then the analysis in the prior section implies a cross sectional relationship between book-to-market and expected returns. A shock that decreases the book-to-market ratio today should be followed by future capital accumulation and lower than average expected returns to shareholders. This will continue until the "growth" stock sees its market-to-book (or equivalently Tobin's q) return to 1. The reverse will be true for "value" stocks.

The above analysis provides a rationale for the value-versus-growth return relationship that is both complementary to and separate from that in either Berk, Green and Naik (1999) or Carlson, Fisher and Giammarino (2004). In the prior models the premium results from firms altering their value through the exercise or expiration of growth options. Here the relationship also comes from capital changes in the underlying firms. But the firms in the model presented here do not exercise an option that leads to the price change, but rather react to one by building new capital that actually undoes the price change.[5]

[5] It is worth noting that while there is considerable evidence for a value premium in stock returns there is some question as to whether or not it is concentrated primarily in securities shunned by institutional investors. See Houge and Loughran (2006) and Phalippou (2007) for evidence on this issue.

2.3 Other Limits of Interest

Two other limits also yield simplified equilibrium expressions and will prove useful for developing the model's implications. The first occurs as investors become more and more risk neutral: $\theta \to 0$. From equation (19), there are two possibilities for A_1. Either it also tends to zero or, alternatively, $\left[\left(I - A_1 C_{2D}^{-1}\right)^{-1} - RI\right] \to 0$ meaning that $A_1 \to \frac{r}{1+r} C_{2D} > 0$. The latter has the undesirable equilibrium implication of upward sloping demand curves for employed capital. Thus, the only economically sensible equilibrium is one in which $\lim_{\theta \to 0} A_1 \to 0$. In turn, this implies that $\lim_{\theta \to 0} A_2 \to (I - G)(rI + G)^{-1}$. Notice that near this limit A_1 is negative definite as $-rA_1 \approx \theta V$ from equation (19). Thus, one has yet another set of sufficient conditions for Proposition 2 to hold with regard to cash flow shocks.

A second limit examined in this section that will prove useful occurs as the variance of the cash flow shocks tends to zero: $\Sigma_\delta \to 0$. In this case, equation (19) becomes

$$\left[\left(I - A_1 C_{2D}^{-1}\right)^{-1} - RI\right] A_1 = \theta \left(I - A_1 C_{2D}^{-1}\right)^{-1} A_1 \Sigma_\eta A_1' \left(I - A_1 C_{2D}^{-1}\right)^{-1\prime} . \tag{28}$$

One obvious solution has A_1 approach 0 from below, which in turn implies $\lim_{\Sigma_\delta \to 0} A_2 \to (I - G)(rI + G)^{-1}$. Thus, one has the result that Proposition 2 holds in this limit as well. One way to justify concentrating on this equilibrium is that it holds if A_1 has a power series solution in θ. See the Appendix for a proof.

2.3.1 Price behavior in the limits $\theta\Sigma_\delta \to 0$ and $\theta\Sigma_\delta \to \infty$

As the risk from δ becomes either very small or large relative to the population's risk aversion the equilibrium matrix equations approach relatively simple limits. This additional simplicity then makes it possible to generate more precise implications from the model.

Proposition 3: *As the risk from δ goes to zero, $\theta\Sigma_\delta \to 0$, the equilibrium price goes to*

$$\begin{aligned} P_t \overset{\theta\Sigma_\delta \to 0}{\to} \; & \frac{1+r}{r}(rI+G)^{-1}G\bar{D} + (rI+G)^{-1}(I-G)D_t \\ & + \frac{(1+r)^2}{r^2}Z_0(rI+G)^{-1}G\bar{D} + \frac{(rI+G)^{-2}}{r}Z_0(I-G)^2 G\bar{D} - \frac{1}{r}Z_0(C_1+G\bar{D}) \\ & + (rI+G)^{-2}Z_0(I-G)^2 D_t + Z_0 C_{2D} N_t, \end{aligned} \quad (29)$$

where Z_0 represents: $Z_0 = -\frac{(1+r)^2}{r}(rI+G)^{-1}\theta\Sigma_\delta (rI+G)^{-1\prime}C_{2D}^{-1}$. *In the opposite limit as the risk from δ becomes very large, $\theta\Sigma_\delta \to \infty$, the equilibrium prices goes to*

$$P_t \overset{\theta\Sigma_\delta \to \infty}{\to} \; \frac{1}{1+r}(C_1 - G\bar{D}) - \frac{1}{1+r}\theta\Sigma_\delta\left[\Sigma_\delta^{-1}C_{2D}\Sigma_\eta C_{2D} + I\right]N_t + \frac{1}{1+r}(I-G)D_t. \quad (30)$$

With a little bit of additional work equations (29) and (30) generate a number of comparative statics regarding how prices move in response to various state variables.

Proposition 4: *Assume that all matrices are diagonal and that I-G is strictly positive definite. Then in the limits $\theta\Sigma_\delta \to 0$ and $\theta\Sigma_\delta \to \infty$,* $\partial p_{k,t}/\partial d_{k,t} > 0$ and $\partial p_{k,t}/\partial n_{k,t} < 0$.

Moreover, $\lim_{\theta\Sigma_\delta \to 0}\frac{\partial p_{k,t}}{\partial d_{k,t}} < \lim_{\theta\Sigma_\delta \to \infty}\frac{\partial p_{k,t}}{\partial d_{k,t}}$ *and* $\lim_{\theta\Sigma_\delta \to 0}\frac{\partial p_{k,t}}{\partial n_{k,t}} > \lim_{\theta\Sigma_\delta \to \infty}\frac{\partial p_{k,t}}{\partial n_{k,t}}$.

The comparative statics show that because "the laws of supply and demand" hold with regard to real assets they then become reflected in stock prices as well. The result that

$\partial p_{k,t} / \partial d_{k,t} > 0$ states that the value of an asset increases if the cash flow it generates increases. On the other hand, $\partial p_{k,t} / \partial n_{k,t} < 0$ implies that if the supply of an asset goes up then its price has to come down to clear the market. Returning to the Tyson example the first inequality states that if the price of chickens increases so will the value of each of their farms. In contrast, the second inequality shows that if there is an overall increase in the number of such farms in the economy then the value of each farm will decline. Counter intuitively, as the third and fourth inequalities show, while an economy with a low cash flow risk (δ) shows less price sensitivity to cash flow shocks, it yields higher price sensitivity to asset supplies.

2.3.2 Profits, Sharpe Ratios, and approximate expected returns in the limits $\theta\Sigma_\delta \to 0$ and $\theta\Sigma_\delta \to \infty$

Assume that all matrices are diagonal and that *I-G* is strictly positive definite. In the two limits, we then have that

$$\mathrm{E}_t\left[Q_{t+1}\right] \overset{\theta\Sigma_\delta \to \infty}{\to} \bar{D} - rC_1 - RC_{2D}Y_t + (I-G)(D_t - \bar{D}), \tag{31}$$

and

$$\mathrm{E}_t\left[Q_{t+1}\right] \overset{\theta\Sigma_\delta \to 0}{\to} \bar{D} - rC_1 - rC_{2D}Y_t + r(rI+G)^{-1}(I-G)(D_t - \bar{D}). \tag{32}$$

Thus excess profits are negatively related to capital issuance, Y_t, and positively related to deviations of capital payout from the unconditional. Moreover, both sensitivities are more pronounced when cash flow risk (δ) or risk aversion is high.

The Sharpe Ratio for each industry can be calculated as

$$SR_k = \frac{\mathrm{E}_t[q_{k,t+1}]}{\mathrm{SD}_t[q_{k,t+1}]}. \tag{33}$$

By writing $\mathrm{VAR}_t[Q_{t+1}] = \frac{1}{\theta}(F^{-1} - R)ZC_{2D}$, and taking the limits, one concludes that

$$SR_k \overset{\theta\Sigma_\delta \to \infty}{\to} \frac{\left\{\bar{D} - rC_1 - RC_{2D}Y_t + (I-G)(D_t - \bar{D})\right\}_k}{\sqrt{c_{k2}^2 \sigma_{\eta,k}^2 + \sigma_{\delta,k}^2}}, \tag{34}$$

and

$$SR_k \overset{\theta\Sigma_\delta \to 0}{\to} \frac{\left\{\bar{D} - rC_1 - rC_{2D}Y_t + r(rI+G)^{-1}(I-G)(D_t - \bar{D})\right\}_k}{R\sigma_{\delta,k}(rI+G)_k^{-1}}. \tag{35}$$

As with the excess profits, the Sharpe Ratio is negatively related to capital issuance, Y_t, and positively related to deviations of capital payout from the unconditional. Here too, both sensitivities are more pronounced when risk or risk aversion is high.

By writing $p_{k,t} = c_{2k}y_{k,t} + c_{1k}$, and dividing the excess profit of industry k by the price of a unit of industry k's capital, we can obtain from equations (31) and (32) that the excess return on industry k's shares is

$$\mathrm{E}_t\left[r_{k,t+1}^e\right] \overset{\theta\Sigma_\delta \to \infty}{\to} \frac{\bar{d}_k - c_{2k}y_{k,t} + (1-g_k)(d_{k,t} - \bar{d}_k)}{c_{2k}y_{k,t} + c_{1k}} - r, \tag{36}$$

and

$$\mathrm{E}_t\left[r_{k,t+1}^e\right] \overset{\theta\Sigma_\delta \to 0}{\to} \frac{\bar{d}_k + \frac{r}{r+g_k}(1-g_k)(d_{k,t} - \bar{d}_k)}{c_{2k}y_{k,t} + c_{1k}} - r, \tag{37}$$

For small shocks away from the steady state, one can linearly approximate these two equations in the shocks as

$$\mathrm{E}_t\left[r^e_{k,t+1}\right] \overset{\theta\Sigma_\delta\to\infty}{\to} \approx \underbrace{\frac{\bar{d}_k}{c_{1k}} - r}_{\text{steady state part}} - (1+\frac{\bar{d}_k}{c_{k1}})\frac{c_{2k}}{c_{1k}} y_{t,k} + \frac{(1-g_k)}{c_{1k}}(d_{k,t} - \bar{d}_k), \tag{38}$$

and

$$\mathrm{E}_t\left[r^e_{k,t+1}\right] \overset{\theta\Sigma_\delta\to 0}{\to} \approx \underbrace{\frac{\bar{d}_k}{c_{1k}} - r}_{\text{steady state part}} - \frac{\bar{d}_k}{c_{1k}}\frac{c_{2k}}{c_{1k}} y_{k,t} + \frac{r}{r+g_k}\frac{(1-g_k)}{c_{1k}}(d_{k,t} - \bar{d}_k), \tag{39}$$

In each case, the deviation from steady state expected returns are negatively related to capital issuance, Y_t, and positively related to deviations of capital payout from the unconditional mean. As with the excess profits and Sharpe Ratios, the sensitivities are more pronounced when risk or risk aversion is high.

2.4 Cross-sectional returns

2.4.1 The book-to-market effect

Equation (6) states that the market value of a unit of capital is increasing in Y_t. Holding the payout per unit of capital constant, equations (38) and (39) indicate that expected returns are decreasing in the per-unit market price of capital. Because a firm in our model is an aggregate of units of capital, a high book-to-market ratio for firm k in our model corresponds to a (relatively) low value of $p_{k,t}$, and therefore (relatively) *high* expected returns. For instance, the cross-sectional dispersion corresponding to this book-to-market effect from equation (38) will be roughly of the order of $\frac{\bar{c}_2}{\bar{c}_1}\bar{\sigma}_\eta$, where the 'bar' signifies a cross-sectional average. This allows one to estimate the order of magnitude of the capital adjustment costs, C_{2D}, to the observed book-to-market effect.

2.4.2 Earnings momentum

Bernard and Thomas (1989) demonstrate the presence of earnings momentum: firms that announce high (lower) earnings exhibit positive (negative) 'abnormal' returns relative to their pre-announcement risk-adjustment. This can be interpreted as a change in 'risk' subsequent to the announcement or that markets inadequately adjust for the impact of earnings announcements. Equations (38) and (39) indicate that firms whose payouts increase will have an ex-post higher expected return, consistent with the post-earnings drift of Bernard and Thomas (1989).

2.5 Capital Investment and Expected Return

Generally in a model with a downward sloping demand curve a negative supply shock increases the current price. In the current model, this is additionally associated with an observable change in capital investment in the same direction as the price change. When an asset's value is high companies can profit by creating more of it. However, intuitively, this should then lead to lower future stock returns as capital accumulation by the industry results in a gradual reduction of that particular asset's market value. Therefore, one expects to see a negative relation between future expected stock returns and capital investment.

To formally analyze the above scenario define firm *k*'s excess return as

$$r^e_{k,t+1} = \frac{q_{k,t+1}}{p_{k,t}}, \tag{40}$$

where $q_{k,t+1}$ is the k^{th} element of Q_{t+1}. Throughout the rest of the paper assume that the price and supply of capital are positive.[6] The next proposition asserts that the expected excess return decreases with capital investment as long as the quadratic adjustment cost is sufficiently large, as assumed in Proposition 2.

Proposition 5: As $C_{2D}^{-1} \to 0$ a firm's expected excess return decreases with capital investment caused by supply shocks:

$$\lim_{C_{2D}^{-1} \to 0} \frac{\partial E\,[r_{k,t+1}^{e}]}{\partial y_{k,t}} < 0. \tag{41}$$

Notice that the result in Proposition 5 looks like that found in the data by TWX and the related literature on new stock issuances and repurchases. Firms issue or retire securities to buy or sell capital in response to shocks in the real economy that also impact stock prices. But, as industries alter their capital stocks they also cause the value of their capital to move in the opposite direction which also impacts their stock's value. As Proposition 5 shows the result is a negative relationship between capital changes and stock returns.

2.6 Capital Investment and CAPM Beta

Since the model's random variables are normally distributed and since the stock market is assumed to be competitive and frictionless the CAPM must hold. To verify this rewrite the equilibrium condition in equation (12) as

$$E_t[Q_{t+1}] = \theta \operatorname{cov}_t(Q_{t+1}, Q_{M,t+1}), \tag{42}$$

[6] While both the price and supply are normally distributed in our model, one can arbitrarily reduce the probability of their assuming negative values. The distribution of the ratio of normals is called the Fieller distribution and its application is abundantly found in the statistics literature.

where $Q_{M,t+1} \equiv Q'_{t+1}N_{t+1}$ is the excess payoff on the market portfolio. Pre-multiply the N_t' to obtain

$$E_t[Q_{M,t+1}] = \theta \operatorname{var}_t(Q_{M,t+1}). \tag{43}$$

Dividing these two expressions side by side and rearranging, we have

$$E_t[Q_{t+1}] = \frac{\operatorname{cov}_t(Q_{t+1}, Q_{M,t+1})}{\operatorname{var}_t(Q_{M,t+1})} E_t[Q_{M,t+1}]. \tag{44}$$

Define the vector of excess returns and the excess market return as

$$\begin{aligned} r_{t+1}^e &= Q_{t+1} \div P_t, \\ r_{M,t+1}^e &\equiv \frac{Q_{M,t+1}}{P_{M,t+1}} = \frac{Q_{M,t+1}}{P_{t+1}^{'} N_{t+1}}, \end{aligned} \tag{45}$$

where ÷ denotes the elementwise division operator, and rewrite equation (44) in terms of excess returns:

$$E_t[r_{t+1}^e] = \frac{\operatorname{cov}_t(r_{t+1}^e, r_{M,t+1}^e)}{\operatorname{var}_t(r_{M,t+1}^e)} E_t[r_{M,t+1}^e] \equiv \beta_t E_t[r_{M,t+1}^e]. \tag{46}$$

Here, the vector of betas can be written as

$$\beta_t = \frac{\operatorname{cov}_t(r_{t+1}^e, r_{M,t+1}^e)}{\operatorname{var}_t(r_{M,t+1}^e)} = (VN_t) \div P_t \bullet \frac{N_t' P_t}{N_t V N_t}. \tag{47}$$

Its k'th element is

$$\beta_{k,t} = \frac{e_k' V N_t}{p_{k,t}} \bullet \frac{N_t' P_t}{N_t V N_t} = \frac{E_t[r_{k,t+1}^e]}{E_t[r_{M,t+1}^e]}, \tag{48}$$

where e_k is the choice vector with 1 in its k'th element and 0 elsewhere. Since a firm's expected return decreases with supply-induced capital investment as long as the quadratic adjustment cost is sufficiently large (see Proposition 5), we expect that the CAPM beta will also decrease. The next proposition shows that this is true in a large economy with independent industries:

Proposition 6: Consider a large economy with independent industries (i.e. V and A_1 are diagonal). As $C_{2D}^{-1} \to 0$, a firm's CAPM beta decreases with capital investment caused by a supply shock:

$$\lim_{\substack{C_{2D}^{-1} \to 0, \\ K \to \infty}} \frac{\partial \beta_{k,t}}{\partial y_{k,t}} < 0. \tag{49}$$

Intuitively, the assumption of cross-sectional independence ensures that the supply shock does not cause market wide price movement. Therefore, in a large economy the firm k shock only affects its own price and has a negligible effect on the expected market return. Thus, the result on the expected return in Proposition 5 translates into the beta. Importantly, this result suggests modeling the CAPM beta as a (decreasing) function of capital investment in an empirical asset pricing test.

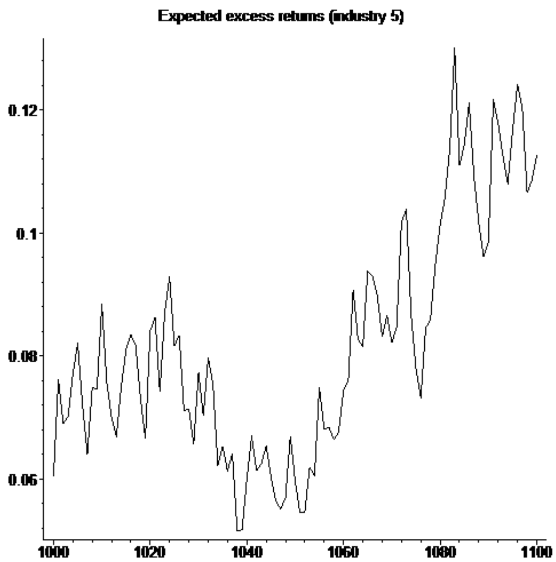
2.7 Numerical illustration

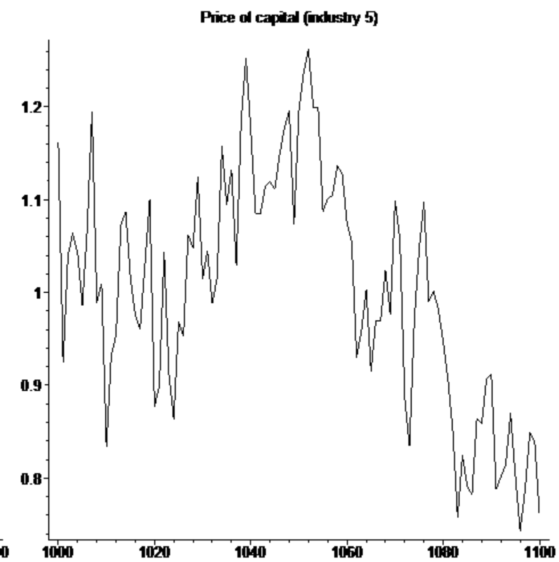
The analytic results in the preceding sections provide a qualitative sense that the model can be consistent with a number of stylized facts in the literature. In order to illustrate this better we choose a set of parameters highlighting the cross-sectional effects. Our intention is not to perform a full scale calibration to industry data and cross-sectional moments at this stage (as is done, for instance, in Carlson, Fisher, and Giammarino, 2004).

We consider the case of ten iid industries, where Σ_δ, Σ_η, and G are proportional to the identity matrix. We focus on ten industries so that we can focus on the equivalent of cross-sectional deciles when calculating return moments. Each period corresponds to a year. Without loss of generality, we normalize the steady-state book value of capital to be 1. The risk free rate is chosen to be R = 1.01, consistent with the realized real rate of return over the past half century, while $\overline{D}$ is chosen to be 0.09, so that the steady state rate of return is 8% per year. We set the volatility of payoffs and 'productivity shocks' to be $\sigma_\delta = \sigma_\eta$ = 8% and the rate of payoff mean-reversion is g = 0.5 (where $G = gI$). The remaining parameters are chosen to help arrive at 'reasonable' magnitudes for the stylized cross-sectional moments. Specifically, we set θ = 2 and C_2 = 6. These parameters completely determine the model.

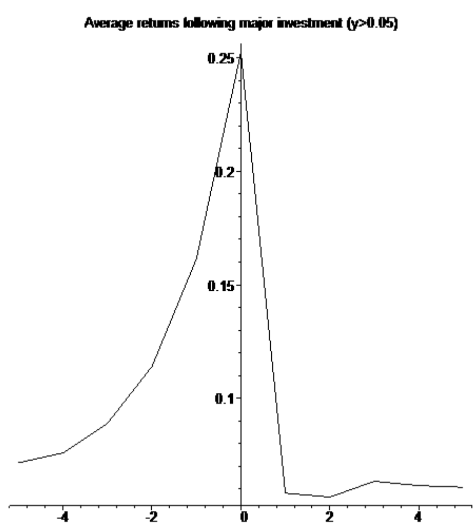
The coefficient, a_1, solves a degree-five polynomial, which under our parameter specification has a unique negative real root at -0.5166; given that a negative real value for a_1 is the only sensible economic solution, this means that this particular parameter specification is not complicated by the presence of multiple equilibria. The remaining coefficients in the equation relating price to quantity and payoffs are a_0 = 1.814, and a_2 = 0.910. In the steady state, the size of an industry is solved by setting the price to C_1 and the payoff to $\overline{D}$, yielding 1.73 units of capital. The standard deviation of the price of a unit of capital is 0.152. The rate of mean-reversion of the price, is F^{-1} = 0.921.

We simulate 10,000 years of the economy assuming that it is initially at the steady state. There was no instance in which the price fell below 0.5 or rose above 1.6. The average difference between the industry with the highest price per unit capital and that with the lowest price per unit capital is 0.38, implying a difference of about 6% in the amount of equity issued between the industry with the highest growth (i.e., positive y_t) and that with the lowest growth. Below is a plot of the price per unit capital in industry 5 after a ‘burn-in’ period of 1000 years; this is beside a plot of the corresponding expected excess returns for the same industry over the same period.





As explained earlier, when the price of capital is high expected returns are low, and vice versa. The model also produces the familiar run-ups preceding major issuance events, and which are subsequently followed by declining returns. An instance of this is plotted below.



The figure illustrates the average returns 5 years before and five years after a 'major investment' made by the leading investing industry (i.e., the event is said to take place whenever the leading industry makes an investment of y>5%). While the graph plots actual returns (and not cumulative abnormal returns, CARs), it should be clear that the expect returns prior to the event are higher than the expected returns following the event, thus using a market model to adjust for risk will result in the observed patterns in CARs.

We calculate the market capital of each industry by multiplying its date *t* price per unit capital by the size of the industry. We calculate the market-to-book ratio of an industry to be its price per unit capital. By sorting the industries with respect to size, market-to-book, investment, and earnings, we can calculate the various asset pricing moments, reported below:

Portfolio	Average Excess Returns
SMB	1.0%
HML	5.4%
Low Investment	3.3%
High Investment	-2.1%
High Payoffs	0.8%
Low Payoffs	-0.4%

The SMB returns are the average difference between the annual *expected* returns of the smallest industry (in market value) and the largest industry at date *t*. We use expected returns rather than realized returns to improve the power of the test (and because we can calculate these in our model). We only use the latter half of the simulated sample (using the first half makes a negligible difference given the number of significant digits we keep). The HML returns are the average difference between the annual expected returns of the highest book-to-market industry and the lowest book-to-market industry at date *t*. Both the SMB and HML returns are consistent with stylized cross-sectional evidence in magnitude and sign. The 'Low Investment' portfolio returns report the difference between the lowest *y* industry expected returns at date *t* and the unconditional expected returns (iid in our parameterization, and equal to 9%). The 'High Investment' portfolio returns are similarly calculated. Both are consistent with the observed issuance puzzle. The payoff portfolios report a similar average for the industry that happens to post the highest (resp. lowest) change in payoffs between dates *t-1* and *t*. These later results are consistent with the earnings momentum phenomenon observed by Bernard and Thomas (1989).

3. Evidence

The results in Section 2.2.1 and Proposition 1 imply that the book-to-market ratio in the short run and the productivity in the steady state are the key variables to determine the cross-sectional variation in expected returns. We now examine this point empirically. Consistent with the model's implication, we will find that average returns increase with the proxies of these two quantities.

3.1 Data and Methodology

We obtain accounting variables from the Compustat annual file. $\bar{d}_k$ in Equation (22) can be measured by the long run average earnings per unit capital. We compute this as the ratio of "Operating Income Before Depreciation" (Compustat Xpressfeed data item *OIBDP*, FTP data item 13) to lagged "Property Plant and Equipment - Total (Gross)"

(*PPEGT*, data 7). c_{1k} in the denominator of that equation measures the cost of creating unit capital, or the per period change in the book value of a firm's productive assets. To avoid division by zero, we employ the gross (rather than net) growth rate of *PPEGT* (data 7). Given by the ratio of these two quantities, our proxy for a firm's productivity, *PROD*, at the end of fiscal year t effectively equals

$$PROD_t = \frac{OIBDP_t / PPEGT_{t-1}}{PPEGT_t / PPEGT_{t-1}} = \frac{OIBDP_t}{PPEGT_t}. \tag{50}$$

The construction of the book-to-market ratio (*BM*) follows Fama and French (1993). Based on the firm characteristics at the end of fiscal year $t - 1$, we form portfolios in June of calendar year t and measure value-weighted monthly returns from July through next June. The conservative six-month lag accounts for possible delay in the dissemination of accounting information and follows the usual practice. The monthly returns and variables necessary to compute market capitalization are from the Center for Research in Security Prices (CRSP), which are matched to the Compustat data by the CRSP-Compustat Merged Database. We use only ordinary common shares (CRSP Share Code 10 or 11) of firms in non-financial industries (one digit SIC code not equal to 6), because investment of financial firms may be very different in nature from that of non-financial firms. We use only NYSE firms (CRSP Exchange Code 1) to compute breakpoints for ranking, but include NYSE, AMEX, and NASDAQ firms (CRSP Exchange Code 1, 2, and 3) in portfolio formation. Our final sample runs from July 1968 through December 2006.

3.2 Result

3.2.1 One dimensional sort on *PROD*

Table 1 shows the characteristics, excess returns, and risk-adjusted alphas of decile portfolios sorted by *PROD*. The second column tells us that firms in the lowest *PROD* decile incur losses on average. The market capitalization (*SIZE*) tends to increase, and *BM* to decrease, with *PROD*, but the relations are not monotone. In fact, we will see variations in *BM* within a given *PROD* quintile, and vice versa, when portfolios are double sorted by these quantities in the next subsection. The table also indicates that there are relatively a large number of firms (*N*) in the top and bottom deciles; this implies that many NASDAQ firms fall in these two extreme *PROD* deciles, and that the point estimates of *SIZE* and *BM* may not properly represent the characteristics of firms in those deciles.

To the extent of such variation in characteristics, the excess return (*EXRET*) may not exhibit a linear relationship with *PROD*. This appears to be the case in the column for *EXRET*, from which one might incorrectly conclude that the underperformance of low *PROD* firms primarily comes from the lowest *PROD* decile only. To account for the potential loadings on risk factors, we compute alpha from a time series regression of each excess portfolio return on the excess market return and the size, value, and momentum factors.[7] The estimated four-factor alpha (*ALPHA*) increases with *PROD* more monotonically, and tends to be negative for low productivity firms and positive for high productivity firms. While the zero-investment portfolio that goes long the highest

[7] The four factors are *MKTRF*, *SMB*, *HML*, and *MOM*, respectively, downloaded from Kenneth French's web site. We thank him for making these series available.

productivity firms and short the lowest productivity firms earns a significant, albeit only moderate, average return of 0.36% per month, its risk-adjusted alpha is 0.56% per month and is significant at the 1% level. This demonstrates that, consistent with Proposition 1, firms with higher productivity earn higher expected returns and that this productivity premium cannot be explained by existing risk factors. Another way to control for existing priced factors is to further sort firms by the characteristics to which the risk factors are correlated. This is the subject of the next two subsections.

3.2.2 Two dimensional sort on *BM* and *PROD*

Table 2 presents the characteristics of 25 portfolios formed as the cross section of *PROD* and *BM* quintiles. Panel A indicates that average firms in the lowest *PROD* quintile again incur losses. Except for this quintile (and perhaps the highest-*PROD* fourth-largest *BM* portfolio), the level of productivity is controlled fairly well by the independent double sort. Panel B reports the average size in million dollars. Firms in the lowest *PROD* quintile tend to be small, especially for growth firms. If this has any implication on our result, the size effect will work against us; if high productivity firms tend to be large in size, we would expect them to earn low average returns, rather than high returns implied by Proposition 1. Panel C demonstrates that the independent double sort controls for the book-to-market ratio quite well, as there is little variation in *BM* along the columns. The number of stocks in Panel D assures us that each portfolio is well populated on average.

Panel E deserves attention. Excess return generally increases in *PROD* controlling for *BM*. The productivity spread, given by the return on a zero-investment portfolio that goes long the highest productivity firms and short the lowest productivity firms within a *BM* quintile, monotonically decreases with the level of *BM*. The long-short portfolio yields 0.81% per month among the growth firms, which is significant at the 1% level. On the other hand, the value spread is strongest among low productivity firms, yielding 1.05% per month. Interestingly, the value spread monotonically decreases with the level of *PROD*. The two numbers shown above are quite high. A legitimate concern is that these spreads may partially reflect the reward for bearing known risks. The four-factor alphas in Panel F control for this possibility. As anticipated, the value spread is significantly reduced after taking into account the loadings on the value and other factors. However, the productivity spread barely changes or even increases for growth stocks upon risk adjustment; the four-factor alpha of the zero-investment productivity portfolio is 0.92% among growth firms. This magnitude of alpha is not only statistically significant (at the 1% level), but also economically significant. The alpha decreases monotonically with *BM*. For concreteness, the next subsection further controls for size.

3.2.3 Three dimensional sort on Size, *BM* and *PROD*

Table 3 reports the characteristics of 27 portfolios formed as the cross section of *SIZE*, *BM*, and *PROD* terciles. For simplicity, we focus on the lowest and highest productivity terciles as we are interested in the productivity spread. Panel A shows the market capitalization of the nine *SIZE-BM* terciles at the lowest and highest productivity levels. Again, if there is any bias resulting from size, it will work against us because the highest productivity firms tend to be larger than lowest productivity firms, thereby reducing the productivity spread. Similarly, the book-to-market ratio in Panel B appears to be well controlled. Panel C confirms that the productivity spread is highest among small to mid growth firms, yielding 0.72% to 0.76% per month, both significant at 1%. These spreads barely change with risk adjustment; the four-factor alphas in Panel D for the corresponding portfolios are 0.68% and 0.61% per month, respectively. Indeed, the alpha for the largest growth portfolio is also significant at the 5% level, yielding 0.49% per month.

Overall, the empirical result presented in this section is consistent with Proposition 1, which says that high productivity firms should earn high returns. This productivity effect cannot be explained by existing risk factors.

4. Conclusion

Traditionally the asset pricing literature has taken the set of corporate assets as given and then asked what the equilibrium returns should be to those that hold them. Recently a number of papers have begun to look at the problem when corporate assets change over time. Articles by Spiegel (1998), Watanabe (2008), Biais, Bossaerts, and Spatt (2008), Pastor and Veronesi (2005), Dittmar and Thakor (2007), Berk, Green and Naik (1999), and Carlson, Fisher, and Giammarino (2004, 2006) all fall into this category. This paper seeks to add to this literature a general equilibrium view of the problem. Rather than take the pricing kernel as given or the movement in asset supplies both are under the population's control to at least some degree.

In this paper asset prices are endogenously determined period by period via market clearing conditions. At the same time corporate capital stocks are impacted by both random fluctuations and firms as they seek to add and subtract from their capital base in response to market conditions. The end result is a tractable model that yields a number of empirical predictions many of which are consistent with the data. Among these are the following:

- Stock returns should be positively correlated with a proxy for productivity of capital, such as the earnings yield on a firm's capital stock.
- Large returns (price moves) in one direction will be followed by a decaying series in the opposite direction.
- Capital expenditures will be negatively correlated with future returns.
- Since the CAPM holds, period-by-period in the model, the above relationships regarding returns also hold for period-by-period betas. This, however, also implies that empirical models that do not allow betas with time trends will be incorrectly specified. In particular, the CAPM beta should be modeled as a decreasing function of capital investment.

We plan to calibrate our model and empirically examine these predictions in future work.

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6. Appendix

6.1 Proofs

Proposition 2: As C_{2D}^{-1} approaches zero, A_1 tends to a negative definite matrix in an equilibrium in which it is finite.

Proof. Rewrite equation (19) as

$$\left[\left(I - A_1 C_{2D}^{-1}\right)^{-1} - RI\right] A_1 = \theta V\ , \tag{51}$$

where V is the covariance matrix of excess payoffs defined in equation (17). If A_1 is finite, $A_1 C_{2D}^{-1}$ in the left hand side approaches zero as $C_{2D}^{-1} \to 0$. In the limit, we have

$$-r \lim_{C_{2D}^{-1} \to 0} A_1 = \theta V\ . \tag{52}$$

Since the right hand side of this equation is positive definite by construction, A_1 must converge to a negative definite matrix. ■

Proposition 3: As the risk from δ goes to zero, $\theta\Sigma_\delta \to 0$, the equilibrium price goes to

$$\begin{aligned} P_t \overset{\theta\Sigma_\delta \to 0}{\to} \quad & \frac{1+r}{r}(rI+G)^{-1} G\bar{D} + (rI+G)^{-1}(I-G)D_t \\ & + \frac{(1+r)^2}{r^2} Z_0 (rI+G)^{-1} G\bar{D} + \frac{(rI+G)^{-2}}{r} Z_0 (I-G)^2 G\bar{D} - \frac{1}{r} Z_0 (C_1 + G\bar{D}) \\ & + (rI+G)^{-2} Z_0 (I-G)^2 D_t + Z_0 C_{2D} N_t , \end{aligned} \tag{29}$$

where Z_0 represents: $Z_0 = -\frac{(1+r)^2}{r}(rI+G)^{-1}\theta\Sigma_\delta (rI+G)^{-1\prime} C_{2D}^{-1}$. In the opposite limit as the risk from δ becomes very large, $\theta\Sigma_\delta \to \infty$, the equilibrium prices goes to

$$P_t \overset{\theta\Sigma_\delta \to \infty}{\to} \quad \frac{1}{1+r}(C_1 - G\bar{D}) - \frac{1}{1+r}\theta\Sigma_\delta \left[\Sigma_\delta^{-1} C_{2D} \Sigma_\eta C_{2D} + I\right] N_t + \frac{1}{1+r}(I-G)D_t . \tag{30}$$

Proof: It is straightforward to show the following limits for the matrix coefficients, A_0, A_1, and A_2.

$$A_1 \overset{\theta\Sigma_\delta \to 0}{\to} -\frac{(1+r)^2}{r}(rI+G)^{-1}\theta\Sigma_\delta (rI+G)^{-1\prime} \tag{53}$$

and

$$A_1 \overset{\theta\Sigma_\delta \to \infty}{\to} -\frac{1}{1+r}\theta\Sigma_\delta\left[\Sigma_\delta^{-1}C_{2D}\Sigma_\eta C_{2D}+I\right]. \tag{54}$$

Notice that A_1 is negative definite in both limits. Likewise, assuming the commutability of matrices and using a first-order Taylor approximation,

$$A_2 \overset{\theta\Sigma_\delta \to 0}{\to} (rI+G)^{-1}(I-G)+(rI+G)^{-2}Z_0(I-G)^2 \tag{55}$$

where $Z_0 = -\frac{(1+r)^2}{r}(rI+G)^{-1}\theta\Sigma_\delta(rI+G)^{-1\prime}C_{2D}^{-1}$, and

$$A_2 \overset{\theta\Sigma_\delta \to \infty}{\to} \frac{1}{1+r}(I-G). \tag{56}$$

Finally,

$$\begin{aligned} A_0 \overset{\theta\Sigma_\delta \to 0}{\to} & \frac{1+r}{r}(rI+G)^{-1}G\bar{D}+\frac{(1+r)^2}{r^2}Z_0(rI+G)^{-1}G\bar{D} \\ & +\frac{(rI+G)^{-2}}{r}Z_0(I-G)^2G\bar{D}-\frac{1}{r}Z_0(C_1+G\bar{D}) \end{aligned} \tag{57}$$

where, again, $Z_0 = -\frac{(1+r)^2}{r}(rI+G)^{-1}\theta\Sigma_\delta(rI+G)^{-1\prime}C_{2D}^{-1}$, and

$$A_0 \overset{\theta\Sigma_\delta \to \infty}{\to} \frac{1}{1+r}(C_1-G\bar{D}). \tag{58}$$

Summarizing,

$$P_t \overset{\theta\Sigma_\delta \to 0}{\to} \frac{1+r}{r}(rI+G)^{-1}G\bar{D} + (rI+G)^{-1}(I-G)D_t$$
$$+\frac{(1+r)^2}{r^2}Z_0(rI+G)^{-1}G\bar{D} + \frac{(rI+G)^{-2}}{r}Z_0(I-G)^2G\bar{D} - \frac{1}{r}Z_0(C_1+G\bar{D}) \quad (59)$$
$$+(rI+G)^{-2}Z_0(I-G)^2D_t + Z_0C_{2D}N_t,$$

and

$$P_t \overset{\theta\Sigma_\delta \to \infty}{\to} \frac{1}{1+r}(C_1-G\bar{D}) - \frac{1}{1+r}\theta\Sigma_\delta\left[\Sigma_\delta^{-1}C_{2D}\Sigma_\eta C_{2D}+I\right]N_t + \frac{1}{1+r}(I-G)D_t. \quad (60)$$

■

Proposition 5: In the limit of Proposition 2, a firm's expected excess return decreases with capital investment caused by supply shocks:

$$\lim_{C_{2D}^{-1}\to 0}\frac{\partial E\,[r_{k,t+1}^e]}{\partial y_{k,t}} < 0.$$

Proof. From equation (5), there is a positive relation between the price and capital investment of each firm. That is, the denominator of equation (40) increases with capital investment. Thus, it suffices to show that its numerator decreases with capital investment and equivalently with the price. Invert the price conjecture in (13) for N_t,

$$N_t = A_1^{-1}(P_t - A_0 - A_2D_t) \quad (61)$$

and rewrite the market-clearing condition in (12) as

$$\begin{aligned} E_t\left[Q_{t+1}\right] &= \theta V N_t \\ &= \theta V A_1^{-1}(P_t - A_0 - A_2D_t) \\ &= [(I - A_1C_{2D}^{-1})^{-1} - RI](P_t - A_0 - A_2D_t), \end{aligned} \quad (62)$$

where we have used equation (51) in the last line. For a change in P_t caused by supply shocks (and not by dividend shocks), equation (62) implies that

$$\frac{\partial E_t\left[Q_{t+1}\right]}{\partial P_t'} = (I - A_1C_{2D}^{-1})^{-1} - RI \xrightarrow{C_{2D}^{-1}\to 0} -rI\ . \quad (63)$$

The k'th diagonal element of this derivative shows that $\lim_{C_{2D}^{-1}\to 0} \frac{\partial E\,[Q_{k,t+1}]}{\partial p_{k,t}} = -r < 0.$ This completes the proof. ■

Proposition 6: Consider a large economy with independent industries (V and A_1 are diagonal). In the limit of Proposition 2, a firm's CAPM beta decreases with capital investment caused by a supply shock:

$$\lim_{\substack{C_{2D}^{-1}\to 0,\\ K\to\infty}} \frac{\partial \beta_{k,t}}{\partial y_{k,t}} < 0.$$

Proof. Since all elements of V are nonnegative, each term in equation (48) is strictly positive as long as prices and supply are, and one can take its logarithm:

$$\log \beta_{k,t} = \log e_k' V N_t - \log P_{k,t} + \log N_t' P_t - \log N_t V N_t. \tag{64}$$

Again, due to the positive relationship between the price and capital investment of each firm (see equation (5)), it suffices to show that this quantity decreases with an increase in firm k'th price caused by supply shocks. Noting that N_t is a fuction of P_t (see equation (61)), differentiate the above expression with respect to P_t:

$$\frac{\partial \log \beta_{k,t}}{\partial P_t} = \frac{A_1'^{-1} V e_k}{e_k' V N_t} - \frac{e_k}{p_{k,t}} + \frac{A_1'^{-1} P_t + N_t}{N_t' P_t} - \frac{2A_1'^{-1} V N_t}{N_t V N_t}. \tag{65}$$

The k'th element is

$$\frac{\partial \log \beta_{k,t}}{\partial p_{k,t}} = \frac{e_k' A_1'^{-1} V e_k}{e_k' V N_t} - \frac{1}{p_{k,t}} + \frac{e_k' A_1'^{-1} P_t + n_{k,t}}{N_t' P_t} - \frac{2e_k' A_1'^{-1} V N_t}{N_t V N_t}. \tag{66}$$

The first two terms are the derivative of the log expected firm return and the last two terms the derivative of the log expected market return. If the last two terms vanish in a large economy, we are left with the first two terms, which we expect to be negative given the result in Proposition 5. This indeed happens when both V and A_1 are diagonal, which allows us to write:

$$\frac{\partial \log \beta_{k,t}}{\partial p_{k,t}} = \frac{a_{1kk}^{-1}}{n_{k,t}} - \frac{1}{p_{k,t}} + \frac{a_{1kk}^{-1} p_{k,t} + n_{k,t}}{\sum_{j=1}^{K} n_{j,t} p_{j,t}} - \frac{2a_{1kk}^{-1} v_{kk} n_{k,t}}{\sum_{j=1}^{K} v_{jj} n_{j,t}^2} \xrightarrow[C_{2D}^{-1} \to 0]{K \to \infty} \frac{1}{a_{1kk} n_{k,t}} - \frac{1}{p_{k,t}} < 0. \quad (67)$$

where a_{1kk} and v_{kk} are the k'th diagonal element of A_1 and V, respectively, and we have used the fact that the two summations in the denominator are sums of positive terms and therefore diverge to infinity in the limit. The condition that $C_{2D}^{-1} \to 0$ ensures the negative definiteness of A_1 and hence $a_{1kk} < 0$ as presented in Proposition 2. ■

6.2 Series Solution to the Equilibrium Matrices

Assume, for simplicity, that $G = 0$. Conjecture that

$$Z(\theta) \equiv A_1 C_{2D}^{-1} = \sum_{n=1}^{\infty} z_n \frac{\theta^n}{n!}. \quad (68)$$

Henceforth, suppress the θ-dependence of Z. Let $F \equiv \left(I - Z\right)$, use equation (20) to write $F^{-1}A_2 + I = R\left(F^{-1} - RI\right)^{-1}$, and plug this into (19) to get

$$Z = \theta\left[(F^{-1} - RI)^{-1} F^{-1} Z C_{2D} \Sigma_\eta C_{2D} Z' F^{-1\prime} + R^2 (F^{-1} - RI)^{-2} \Sigma_\delta (F^{-1} - RI)^{-1\prime}\right] C_{2D}^{-1}. \quad (69)$$

The idea is now to apply the differential operator, $\frac{\partial^n}{\partial \theta^n}$ to each side of equation (69), set θ to zero, and solve for z_n. This yields a *unique* solution for Z because the right side of equation (69) is multiplied by θ and thus the application of this solution procedure yields an equation of the form,

$$z_n = f(R, \Sigma_\eta, \Sigma_\delta, C_{2D}, \{z_i\}_{i=1}^{n-1}). \tag{70}$$

It is straight forward to solve for the first few coefficients. For the fourth order coefficients or higher, the procedure becomes tedious. It should be immediately obvious that the first and second order coefficients of Z do not depend on Σ_η.

Solving for $\mathbf{z_1}$: Writing,

$$z_1 = \left[R^2 (I - RI)^{-2} \Sigma_\delta (I - RI)^{-1\prime} \right] C_{2D}^{-1}, \tag{71}$$

thus

$$z_1 = -\frac{(1+r)^2}{r^3} \Sigma_\delta C_{2D}^{-1}. \tag{72}$$

In particular, the coefficient is negative, implying downward sloping demand curves, as desired.

Solving for $\mathbf{z_2}$: Write,

$$z_2 = \frac{\partial}{\partial \theta} \left[R^2 (F^{-1} - RI)^{-2} \Sigma_\delta (F^{-1} - RI)^{-1\prime} \right]_{\theta=0} C_{2D}^{-1}, \tag{73}$$

and employ the identity $\frac{\partial}{\partial \theta} G^{-1}(\theta) = -G^{-1}(\theta) \left(\frac{\partial}{\partial \theta} G^{-1}(\theta) \right) G^{-1}(\theta)$ to eventually yield

$$z_2 = 2\frac{(1+r)^4}{r^7} (\Sigma_\delta C_{2D}^{-1})^2. \tag{74}$$

It would probably be worthwhile to work out the next term (later). What should be clear, is that z_3 and, in fact, all higher order coefficients vanish if z_1 and z_2 vanish. In other words, as conjectured earlier, this solution has the property that $\lim_{\Sigma_\delta \to \infty} Z \to 0$, and thus

$\lim_{\Sigma_\delta \to \infty} A_2 \to \frac{1}{r}$.

Table 1: Portfolios sorted on productivity. This table shows the characteristics of decile portfolios sorted on productivity. The productivity measure, *PROD*, is the ratio of Compustat annual item "Operating Income Before Depreciation" to "Property Plant and Equipment - Total (Gross)." *SIZE* is the average market capitalization of member firms in millions of dollars. *BM* is the average book-to-market ratio, constructed as in Fama and French (1993). *N* is the average number of firms. *EXRET* is the excess value-weighted return with the t-statistic in parentheses. *ALPHA* is the intercept from the time-series regression of the excess portfolio return on the excess market return and the size, value, and momentum factors, with the t-statistic in parentheses. *, **, and *** represent significance at 10, 5, and 1%, respectively. Based on the firm characteristics at the end of fiscal year $t-1$, we form portfolios in June of calendar year t and measure value-weighted monthly returns from July through next June. We use only ordinary common shares on NYSE, AMEX, and NASDAQ of firms in non-financial industries. Only NYSE firms are used to compute breakpoints for ranking. The final sample runs from July 1968 through December 2006.

PROD rank	*PROD*	*SIZE*	*BM*	*N*	*EXRET*	(t-stat)	*ALPHA*	(t-stat)
1	-1.104	195	1.16	784	0.0013	(0.44)	-0.0025	(-1.56)
2	0.099	957	1.27	221	0.0048 **	(2.42)	-0.0017 *	(-1.75)
3	0.130	1,113	1.18	237	0.0054 ***	(2.70)	-0.0005	(-0.54)
4	0.164	1,263	1.11	263	0.0050 **	(2.30)	-0.0012	(-1.32)
5	0.205	1,090	1.00	280	0.0059 **	(2.55)	0.0001	(0.05)
6	0.254	1,099	0.93	273	0.0054 **	(2.41)	0.0004	(0.52)
7	0.311	1,376	0.84	282	0.0031	(1.34)	-0.0003	(-0.39)
8	0.388	1,770	0.77	290	0.0050 **	(2.22)	0.0019 **	(2.31)
9	0.522	1,623	0.69	330	0.0056 **	(2.42)	0.0034 ***	(4.09)
10	1.877	1,189	0.61	511	0.0049 *	(1.74)	0.0031 ***	(3.43)
10-1					0.0036 *	(1.91)	0.0056 ***	(3.02)

Table 2: Portfolios sorted on the book-to-market ratio and productivity. This table shows the characteristics of 25 portfolios formed as the cross section of the book-to-market ratio and productivity quintiles. The panels report the following quantities: Panel A: Productivity, the ratio of Compustat annual item "Operating Income Before Depreciation" to "Property Plant and Equipment - Total (Gross)"; Panel B: Size, the average market capitalization in millions of dollars; Panel C: The book-to-market ratio, as described in Fama and French (1993); Panel D: The average number of firms; Panel E: The excess value-weighted return; Panel F: The four-factor alpha, computed as the intercept from the time-series regression of the excess portfolio return on the excess market return and the size, value, and momentum factors. *, **, and *** represent significance at 10, 5, and 1%, respectively. Based on the firm characteristics at the end of fiscal year $t-1$, we form portfolios in June of calendar year t and measure value-weighted monthly returns from July through next June. We use only ordinary common shares on NYSE, AMEX, and NASDAQ of firms in non-financial industries. Only NYSE firms are used to compute breakpoints for ranking. The final sample runs from July 1968 through December 2006.

Panel A: Productivity

		BM				
		1	2	3	4	5
	1	-1.89	-0.96	-0.44	-0.33	-0.27
	2	0.15	0.15	0.15	0.15	0.14
PROD	3	0.23	0.23	0.23	0.23	0.22
	4	0.36	0.35	0.35	0.34	0.34
	5	1.26	1.17	1.31	2.03	1.26

Panel B: Size ($ million)

		BM				
		1	2	3	4	5
	1	190	374	477	531	284
	2	1,630	2,408	1,605	990	452
PROD	3	2,630	1,465	861	566	367
	4	3,774	1,426	682	395	355
	5	2,600	790	394	343	326

Panel C: Book-to-market ratio

		BM				
		1	2	3	4	5
	1	0.22	0.55	0.79	1.07	2.20
	2	0.29	0.56	0.79	1.07	1.93
PROD	3	0.29	0.55	0.79	1.06	1.86
	4	0.29	0.55	0.78	1.06	1.86
	5	0.26	0.55	0.78	1.06	1.87

Panel D: Number of stocks

		BM 1	2	3	4	5
	1	276	124	124	164	318
	2	55	67	93	123	162
PROD	3	89	113	114	113	124
	4	146	142	116	89	79
	5	362	190	122	90	76

Panel E: Excess returns

		BM 1	2	3	4	5	5-1
	1	-0.0033	0.0024	0.0028	0.0040 **	0.0072 ***	0.0105 ***
	2	0.0024	0.0050 **	0.0055 **	0.0071 ***	0.0089 ***	0.0065 **
PROD	3	0.0023	0.0066 ***	0.0069 ***	0.0087 ***	0.0093 ***	0.0070 ***
	4	0.0037	0.0043 *	0.0076 ***	0.0083 ***	0.0090 ***	0.0053 **
	5	0.0048 *	0.0080 ***	0.0080 ***	0.0068 **	0.0082 ***	0.0034
	5-1	0.0081 ***	0.0056 ***	0.0052 ***	0.0028	0.0010	

Panel F: Four-factor alphas

		BM 1	2	3	4	5	5-1
	1	-0.0056 **	-0.0035 *	-0.0029 *	-0.0021 **	-0.0011	0.0044 *
	2	-0.0025	-0.0010	-0.0005	-0.0005	-0.0002	0.0023
PROD	3	-0.0013	0.0007	0.0014	0.0007	0.0028	0.0041 *
	4	0.0013	0.0004	0.0012	0.0015	0.0011	-0.0002
	5	0.0036 ***	0.0026 **	0.0017	-0.0002	-0.0003	-0.0039 **
	5-1	0.0092 ***	0.0061 ***	0.0046 **	0.0019	0.0008	

Table 3: Portfolios sorted on size, the book-to-market ratio, and productivity. This table shows the characteristics of 27 portfolios formed as the cross section of the size, book-to-market ratio, and productivity terciles. Productivity is measured by the ratio of Compustat annual item "Operating Income Before Depreciation" to "Property Plant and Equipment - Total (Gross)." The panels report the following quantities: Panel A: Size, the average market capitalization in millions of dollars; Panel B: The book-to-market ratio, as described in Fama and French (1993); Panel C: The value-weighted return on a zero-investment portfolio that goes long highest productivity firms and short lowest productivity firms; Penal D: The four-factor alpha, computed as the intercept from the time-series regression of the zero-investment portfolio return on the excess market return and the size, value, and momentum factors. *, **, and *** represent significance at 10, 5, and 1%, respectively. Based on the firm characteristics at the end of fiscal year t – 1, we form portfolios in June of calendar year t and measure value-weighted monthly returns from July through next June. We use only ordinary common shares on NYSE, AMEX, and NASDAQ of firms in non-financial industries. Only NYSE firms are used to compute breakpoints for ranking. The final sample runs from July 1968 through December 2006.

Panel A: Size ($ million)

(i) Low productivity portfolios

		SIZE		
		1	2	3
	1	57	515	5,062
BM	2	59	608	5,356
	3	44	623	3,728

(ii) High productivity portfolios

		SIZE		
		1	2	3
	1	96	604	9,558
BM	2	80	561	4,421
	3	54	520	7,216

Panel B: Book-to-market ratio

(i) Low productivity portfolios

		SIZE		
		1	2	3
	1	0.28	0.32	0.39
BM	2	0.80	0.83	0.83
	3	2.04	1.55	1.43

(ii) High productivity portfolios

		SIZE		
		1	2	3
	1	0.35	0.32	0.29
BM	2	0.78	0.73	0.71
	3	1.63	1.41	1.37

Panel C: Zero-investment portfolio returns (high - low productivity)

		SIZE		
		1	2	3
	1	0.0072 ***	0.0076 ***	0.0034
BM	2	0.0017	0.0030 **	0.0024
	3	-0.0003	-0.0002	0.0001

Panel D: Four-factor alphas on zero-investment portfolios (high - low productivity)

		SIZE		
		1	2	3
	1	0.0068 ***	0.0061 ***	0.0049 **
BM	2	0.0012	0.0024	0.0019
	3	-0.0005	0.0009	0.0017