Stable Value Funds: Performance to Date

David F. Babbel
The Wharton School, University of Pennsylvania and Charles River Associates

Miguel A. Herce
Charles River Associates

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Abstract:
Little in the scholarly economics literature is directed specifically to stable value funds, although they occupy a leading place among retirement investment vehicles. They are offered in almost half of all defined contribution plans in the USA, with more than $800 billion dollars worth of assets under management. This paper rigorously examines their performance throughout the entire period since their inception in 1973. We produce a composite index of stable value returns. We conduct mean-variance analysis, Sharpe and Sortino ratio analysis, stochastic dominance analysis, and optimal multi-period portfolio composition analysis. Our evidence suggests that stable value funds dominate two (and nearly three) major asset classes based on a historical analysis, and that they often occupy a significant position in optimal portfolios across a broad range of risk aversion levels. We discuss the factors that contributed to stable value's remarkable performance and whether it can continue to maintain it into the future. In our paper, innovations are achieved in constructing efficient stochastic dominance algorithms, incorporating return expectations in multi-period portfolio construction, and in examining the multi-relations among competing stable value funds.

Key words: Stable value, defined contribution, optimal asset allocation, stochastic dominance

JEL Classifications: G11, G22, G23, J26

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1. INTRODUCTION

In this updated study[1] we reexamine the past history of stable value (SV) funds and their performance to date. A growing volume of industry and practitioner research has provided a detailed look at how the funds are managed and the stable values secured.[2] Additionally, academic articles increasingly are including SV funds within the purview of their studies,[3] although it is difficult to find in-depth scholarly treatments of SV funds performance. The lack of rigorous performance studies is rather surprising because SV funds occupy such a prominent place among retirement investment vehicles, with over $800 billion of assets under management.[4] They are offered as an investment option in almost half of all defined contribution (DC) plans, including 457, 403(b) and 401(k) plans, and by February 2009 reached 36.7 percent of their assets.[5] They are also available to participants in some Section 529 Tuition Assistance Plans.

* David F. Babbel is Professor of The Wharton School, University of Pennsylvania, and Senior Advisor to Charles River Associates. Babbel@Wharton.Upenn.edu

** Miguel Herce, Ph.D., is a Principal of Charles River Associates in Boston. MHerce@Crai.com

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[1] Our previous studies (Babbel and Herce, 2007, 2009) did not include the most recent data, and did not include some improved estimation techniques and sensitivity analysis that are used in this updated study.

[2] See, for example, *Stable Times,* a quarterly publication of the Stable Value Investment Association, *MetLife Stable Value Study* (2010), Paradis (2001), and Fabozzi (1998).

[3] See, for example, Elton, Gruber and Blake (2007), Redding (2009), and Tang et al (2009).

[4] Stable Value Investment Association (SVIA) members had $561 billion under management as of December 31 2009, which covered 173,050 plans. The SVIA estimates that over $250 billion in additional funds are managed by nonmembers.

[5] Allocations are more typically 15-25 percent, but due to stock market declines, the percentage of assets in SV funds was unusually high. Current statistics are available at www.stablevalue.org

In this updated study, we provide a rigorous analysis of the performance of SV funds, enlisting an extended data set that goes from 1973 through January 1, 2010. We compare their performance to that of basic asset classes such as U.S. large and small stocks, long-term government and corporate bonds, intermediate-term government and corporate bonds, and money market funds, using three methods: mean-variance analysis, stochastic dominance analysis, and an enhanced multiperiod utility analysis. Our study shows that since the inception of stable funds in late 1988, and their precursors in 1973, none of these other asset classes have dominated them; on the contrary, SV funds have dominated money market and intermediate-term government/credit bond funds (and nearly dominated long-term corporate bonds as well) over a wide range of risk aversion levels and, when combined with small stocks and long-term government bonds, they occupy a prominent and often dominant part in optimal portfolios.

Before concluding our study we explain the value proposition for SV funds – how they have been able to generate notable returns for their investors – and whether the ingredients to their past performance can be expected to continue into the future. We consider the recent financial crisis and revisit how the funds have weathered the crisis.

2. BACKGROUND

Stable value funds are labeled in various ways, including Capital Accumulation, Principal Protection, Guaranteed, Preservation, Income funds, as well as Guaranteed Investment Contracts (GICs), among others. Early forms of SV funds have been around since the 1970s, coinciding with the development of U.S. defined contribution (DC) plans. From their outset, these funds consisted largely of laddered maturity GICs[6] issued by insurance companies. The returns, which were guaranteed regardless of the performance of the underlying assets, were fully backed by the GIC issuer's general account, but some plan sponsors sought an alternative structure that would provide them with greater flexibility and control, as the assets backing the GIC contracts were owned and managed by the insurer and the underlying investment strategy was that of the insurer's general account.

Such concerns were addressed, in part, with the creation in late 1988 of separate account GICs, where the assets underlying the contracts, although still owned by the insurance company, were held in separate accounts for the exclusive benefit of the plan(s) participating in the separate account and could not be used to discharge claims against the general account of the insurer. In this structure, the guaranties against any investment shortfalls stemming from poor performance of the separate account assets were secured by assets in the general account and surplus. This innovation provided additional flexibility to fund managers, who could pursue a customized investment strategy rather than a general account investment strategy.

Then, in mid-1994, the SV funds market broadened when synthetic GICs (also called Trust GICs) were created in an effort to allow plans to retain legal title to plan assets, and to pro-

[6] Traditional GICs are issued by an insurance company that guarantees the principal invested and pays a periodically-reset interest rate for a certain period of time.

vide additional flexibility in terms of investment strategy and asset selection. This structure enabled non-insurers to manage the plan assets, which are directly owned by the participating plan(s), while the financial protections were secured by banks, insurers, and other financial institutions. Today synthetic GICs occupy a prominent position in stable value, and coexist with traditional and separate account GICs.[7] Each structure of stable value carries features that are preferable to different plan sponsors. Some prefer the yields that may be available through general account GIC-based plans; others prefer the investment flexibility of separate account GICs, while still others seek the advantages of synthetic GIC-based funds.

A stable value fund offers principal protection and liquidity to individual investors, and steady returns that are competitive with intermediate-term bond yields. However, over ensuing one-to-three-month intervals, the guaranteed rate of return moves more slowly than intermediate-term bond yields. This is achieved by having a process and investment contract, discussed in the following section, which allows the provider to smooth market volatility through annuitizing gains and losses over the duration of the portfolio. This smoothing is effected through the rate reset mechanism and insulates against day-to-day volatility. As a consequence of these features and the underlying intermediate-term bond investments, SV funds provide investors with returns of very low volatility. This combination of bond-yield-like returns and low volatility generates contract or book value accounting of the investment.[8]

The underlying investment portfolios of all three forms of stable value funds are typically comprised of high quality, short maturity (usually well under five years) corporate and government bonds, mortgage-backed securities, and asset-backed securities. In the case of synthetics, the portfolio is protected against interest rate risk through an investment contract or "wrap" obtained from a high quality bank, insurer or other financial institution. This means that in all but a few prespecified circumstances, investors in an SV fund are able to transact (make deposits, withdrawals, transfers) at book or contract value, which is deposits plus accrued interest, less any past withdrawals.

Stable value funds do not require a set holding period but provide full access to the participant's principal and accumulated interest without a penalty. However, they are subject to the general restrictions within the overall plan. For example, many plans restrict participants from the direct transfer to a competing short-duration bond or money market fund by requiring that money transferred out of SV be first invested in a non-competing (e.g., stock or long-term bond) fund for a short period such as 30-90 days to eliminate arbitrage. This rule, together with the fact that plan participants do not act in concert regarding the allocation of their funds, allow the investment contract protections ("wrappers") to be purchased for a fraction of what it would cost if interest arbitrageurs dominated the pool and were revising their allocations aggressively.

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[7] See 2010 MetLife Stable Value Study, pp. 38-9.

[8] See SVIA (2005).

3. CREDITING RATE FORMULA

From an investor's viewpoint, SV funds operate like a passbook savings account. They accrue interest at a prespecified crediting rate that is generally updated every one to three months to incorporate changing market conditions. Their principal is secure and grows over time by the amounts of interest credited to their account. Crediting rates on SV funds change more slowly than bond yields and are computed according to a formula which basically produces an internal rate of return for the investment by requiring that the contract (or book) value of the portfolio converge to its market value by the end of the assumed duration. We define the variables in the formula as follows:

- *CR*: crediting rate applied to the accounts of investors in the SV funds
- *MV*: market value of the underlying portfolio
- *CV*: contract value of the underlying portfolio
- *D*: duration of the underlying portfolio or duration of a benchmark portfolio
- *Y*: yield of the underlying portfolio, as described further below.

Given these variables, the future market value (*FMV*) of the portfolio is given by

$$FMV = MV(1+Y)^D, \tag{1}$$

and the crediting rate that guarantees this value, given the current contract value, is the solution to

$$FMV = CV(1+CR)^D. \tag{2}$$

Therefore, the crediting rate formula is given by equating the right-hand-side of expressions (1) and (2), and solving for CR,

$$CR = (1+Y)\left(\frac{MV}{CV}\right)^{1/D} - 1. \tag{3}$$

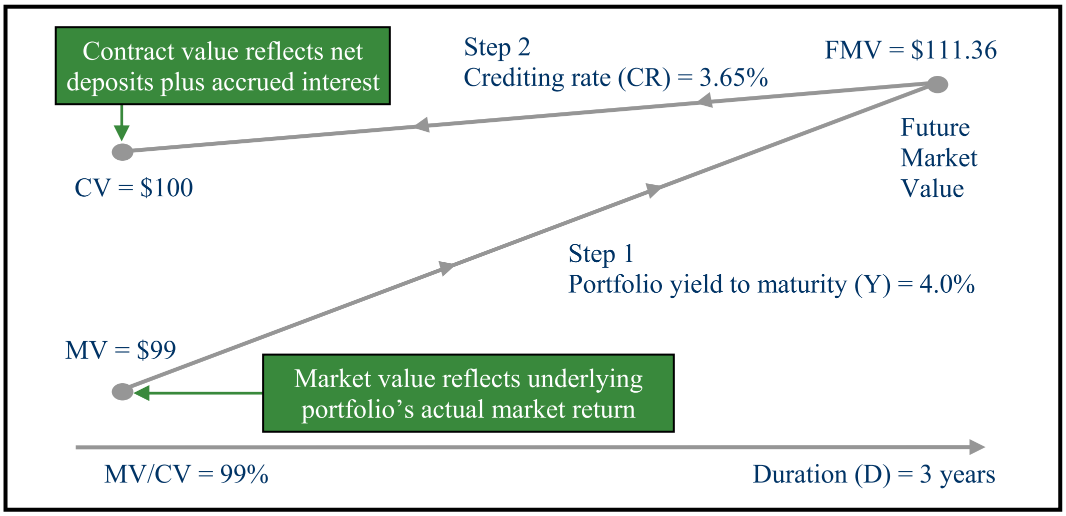
Although other variations of the crediting rate formula are also used, expression (3) is the one most generally used.[9] In addition to small differences in the crediting rate formulae, managers do not always calculate the inputs to the formulae in the same way. For example, with respect to the measure of duration *D*, some fund managers use the duration of a benchmark portfolio, while others use the duration of the underlying securities. The yield measure *Y* is most often a duration market-weighted bond equivalent yield, although some variations have occurred.

Illustration 1 details how the crediting rate is designed to make the contract value of the portfolio converge to its market value over the duration of the portfolio, assuming market conditions do not change in the interim. The case illustrated is where the contract value exceeds the market value, but the same procedure is used in the opposite case.

[9] See, SVIA (2005, p. 4).

As indicated below, the crediting rate effectively smoothes returns by distributing gains and losses over a period of time related to the duration of the portfolio. The crediting rate formula below implies that the crediting rate is between the portfolio's return and its yield, *Y*, and closer to the portfolio's yield the longer the duration, *D*, is. The important thing to remember is that individual investors receive the same rate of return as the stated crediting rate, since principal is protected.

Illustration 1: Setting Synthetic GIC Crediting Rates



Step 1
Calculate estimated future market value by compounding the current market value at the portfolio's current yield to maturity for a period equal to the portfolio's duration.

Step 2
Determine new crediting rate as the discount rate that equates the estimated future market value with the current contract value.

Source: SVIA

4. PERFORMANCE MEASUREMENT

In this study we will measure the performance of SV funds vis-à-vis money market investments, intermediate-term government/credit bonds, long-term government bonds, corporate bonds, and small and large stocks using three methods of analysis: mean-variance analysis, stochastic dominance analysis, and enhanced multiperiod utility analysis. Each method has its advantages and drawbacks, but together we get a fairly clear picture of how well SV funds have performed. We conduct our analyses over the 21-year period beginning in 1989 when synthetics became in important component in the stable value market and, as a robustness test, over the extended 37-year period starting in 1973 that was dominated by general account and separate account GICs.

We begin with a mean-variance analysis, more because of its simplicity and ubiquitous use in practice than its theoretical properties.[10] Strictly speaking, the validity of this approach hinges upon whether investors consider variance to be an adequate measure of investment risk. In other words, investor preferences must be satisfactorily modeled using quadratic utility.

Beginning as early as 1967, Arditti determined that investors considered measures of downside risk beyond variance, and countless additional studies along similar lines have continued until now to demonstrate that variance is an inadequate measure of either security or portfolio risk.[11] However, if the distribution of market returns can be fully described by its first two moments, then restricting one's performance analysis to a mean-variance analysis can be justified, even if investors would otherwise be concerned about higher (and non-existent) moments of the return distribution. But all tests with which we are familiar demonstrate that return distributions for stocks, bonds, and money market instruments cannot adequately be characterized by their means and variances, nor does modified Brownian motion fully capture the movement in these asset returns.

Accordingly, we next measure investment performance using stochastic dominance analysis. Introduced in 1969 by Hanoch and Levy and by Hadar and Russell to remedy the shortcomings of mean-variance analysis, stochastic dominance approaches have the clear advantage of taking into account all moments and other characteristics of the return distributions, and providing investment dominance analyses that do not depend upon knowing the exact shapes of investor preference functions. This has another distinct advantage over the mean-variance approach, which cannot be valid for various horizons simultaneously because it relies on log-normally distributed returns, which if valid (under certain conditions) for single-period returns is not valid for multiperiod returns. By contrast, the stochastic dominance approach remains valid because it is distribution-free. The limitations and additional virtues of this approach are discussed at length in the authoritative treatise by Levy (2006). While some of the limitations have been overcome by a plethora of research, dating from the 1970s to the present, there remain two:

1) Stochastic dominance methods do not provide guidance into the construction of a portfolio from various individual securities.
2) Stochastic dominance methods do not provide an equilibrium price for securities.

Our third approach is an indirect approach to performance measurement, based on the discrete-time multiperiod investment theory of Mossin (1968), Hakansson (1971, 1974), Leland (1972), Ross (1974), and Huberman and Ross (1983). It remedies certain failings of mean-variance analysis as well as the limitations of stochastic dominance analysis at the high cost

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[10] An excellent treatise on this approach is provided by Markowitz (1987).

[11] Indeed, as reported by Douglas (1969), John Lintner's initial cross sectional tests conducted in 1965 found that residual risk, which according to the Capital Asset Pricing Theory's version of mean-variance analysis is not supposed to be priced by the marketplace, was indeed important to investors. More rigorous studies since then have reconfirmed these early findings. Recently, Cvitanic, Polimenis, and Zapatero (2008) have found that ignoring higher moments can lead to significant overinvestment in risky securities, especially when volatility is high.

of specifying the form of the intertemporal preference function. To mitigate this high cost, in part, we will conduct our analysis across a wide range or risk aversion levels. Grauer and Hakansson (1982, 1985, 1986, 1987, 1993, and 1995) and others, applied this theory to the asset allocation problem with some success, where an empirical probability assessment approach was used to implement a set of investment strategies. We will not rehearse the details of the methodology here, as they are well documented in Grauer and Hakansson (1986). The approach is indirect in the sense that we will determine whether, based on past asset return patterns and a range of current expectations, SV assets would enter the optimal portfolio in any significant way.

Our calibration periods were comprised of 80 consecutive quarters, at year-ends 2006, 2007, and 2008, which are twice as long as the 40-quarter periods relied upon by Grauer and Hakansson in most of their studies.[12] We undertook an enhancement to their approach by inserting for each quarter the then current expected returns (or a range thereof) for each asset class rather than using their historical mean returns and spreads. This is especially important for interest-bearing securities, because the lagging 80-quarter average returns may not reflect yield conditions and expected returns for the current quarters. Also, for stocks, rather than use historical average excess returns, we instead conducted a sensitivity analysis using a range of spreads above Treasury bills and bonds to test for the effect of different expected returns.[13] Then we adjusted the rolling time series of lagging quarters by using upper moment and co-moment preserving spreads, while substituting expected returns for only the first moment. Clearly this is more realistic and considers the available information on expected returns for each quarter across all asset classes, while taking full advantage of the relative stability of upper moments and co-moments of their distributions. We then derived optimal asset allocations for a wide range of risk aversion levels.

5. DATA

We begin the first phase of our analysis in January 1989, just a few months beyond the inception of SV funds, and continue through December, 2009. Later we extend the starting date back to 1973 to include the precursors of SV funds. These analyses are performed separately for reasons explained later. For non-SV investments, we use total monthly returns on the S&P 500 index, Ibbotson's small stocks index, long-term government bonds index and corporate bonds index, the Barclays-Lehman Intermediate Government/Credit index, and the Merrill Lynch 3-month Treasury bill index. For SV funds, we use total net monthly returns and asset values (or quarterly where monthly data are not available) on various SV fund families managed by members of the Stable Value Investment Association (SVIA.) From the stable value data we produce a composite index of stable value returns, including comingled

[12] This was done to ensure that we captured multiple full business cycles in our estimations, as recently cycles have lengthened markedly. (See, for example, http://www.nber.org/cycles.html) Because correlations across asset classes are sometimes quite different during the expansion portion of a cycle than during the contraction phase, robust estimations are best made across both phases of several full cycles.

[13] See Merton (1980) for a discussion of expected market returns.

funds, externally managed separate accounts, internally managed separate accounts, and life insurance general account stable value.[14] This index of returns is provided in Appendix B.

We note that except for small stocks and SV funds, returns are gross returns and need to be adjusted for management fees and transaction costs to be comparable to returns on small stocks and SV investments. We do this by subtracting average fees and expenses reported by the Investment Company Institute (ICI) for stock, bond and money market funds over the period of our analysis from the corresponding large stocks, bond or money market returns.[15] Figure 1 below shows the evolution of mutual fund fees and expenses over the period of our analysis.

We have net return data on a subset of up to nine major SV fund complexes, although some of these funds began operations after January of 1989.[16] Figure 2 shows the number of major SV fund complexes for which return data exist over each month in the period of interest. The number of fund complexes reporting their return data over time and the number of funds within each complex mirror the growth in the number of funds in the entire market and are considered representative of the overall population of SV funds. We note that, as the industry expands, more and more plan sponsors offer SV investment options.

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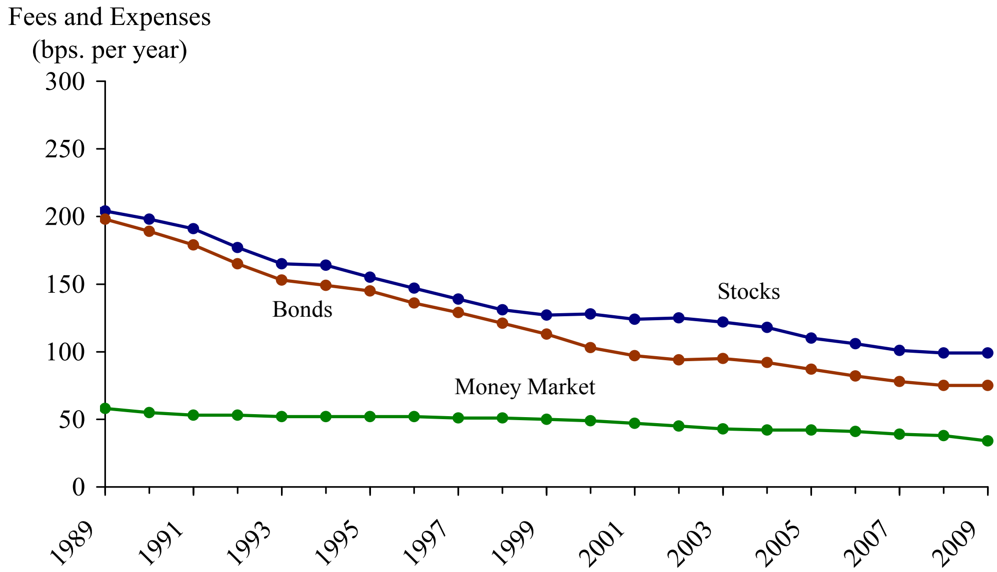
[14] Our two composite indices were based on assets under management of $236.45 billion as of the beginning of 2010. Our composite indices, which covered approximately 75,000 plans, included general account funds, expernally managed separate account funds, and commingled funds.

An alternative index available from Hueler Companies focuses on commingled funds. In 2010, this index tracks approximately 85% of the 40,000 commingled funds under management, which totaled $100 billion. See K. Hueler testimony before the U.S. Senate, June 16, 2010. http://aging.senate.gov/events/hr222kh.pdf Hueler Companies also construct indices on sub-samples of externally managed separate account funds (which in aggregate constituted about 1/3 of all stable value funds). See http://www.hueler.com/ We found the Hueler index of commingled fund monthly returns to be 98.9% correlated with monthly returns on our equally-weighted composite index and 98.6% correlated with monthly returns on our value-weighted composite index over the period from 9/1988 – 12/2009, and less than a single basis point apart.

[15] Annual fund fees and expenses from 1980 through 1989 are from ICI's Research Fundamentals, Vol. 14, No. 6, October 2005, pp. 3, 6, and 7. Annual fund fees and expenses from 1999 through 2009 are from ICI's Research Fundamentals, Vol. 19, No. 2, Figure 2, p. 3.

[16] Each major fund complex contains numerous individual funds, whose returns are closely linked. For years prior to 2009, two of the fund complexes reported quarterly net returns. For 2009, all of the returns we obtained are quarterly. (Over the past few years there has been a move from away from quarterly to monthly guaranty periods among several funds, so our assumption of quarterly guaranty periods across all funds will result in minor estimation errors.) As the return on an SV fund is generally a quarterly crediting rate, set prior to the quarter to which it applies, for quarterly return series we use this quarterly rate to estimate a monthly return. For each month in a given quarter, the monthly return may be calculated as follows: $R_{Monthly} = (1 + R_{Quarterly})^{(\text{Days in Month} \div \text{Days in Quarter})} - 1$. This method results in a days-in-month pattern that very closely follows the pattern observed in the other SV monthly return series we obtained.

Figure 1. Annual Fees and Expenses for Stock, Bond, and Money Market Funds



Source: ICI

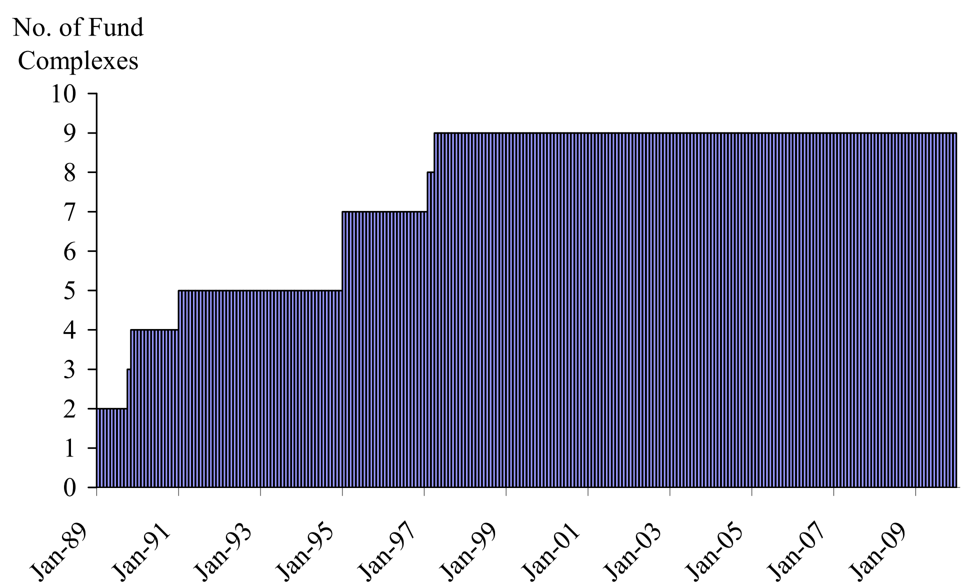
Using these data as well as asset values for each fund complex, we construct a value-weighted average return series. Figure 3 plots this series together with net monthly return series for the intermediate-term government/credit and money market fund series.

The return series of individual funds in the SV average are highly correlated among themselves. Both the average of pairwise correlation coefficients and an efficient measure of multiple correlation — the multirelation coefficient — indicate that the SV return series is highly representative of the individual fund returns.[17] In Table 1 we report these coefficients for a set of five major SV fund complexes with data from January 1991 through 2009, and for a set of nine major fund complexes with data from April 1997 through 2009.

We observe that the return series for the fund complexes comprising the index are highly correlated, even when first differences are used to eliminate the downward trend in the data.

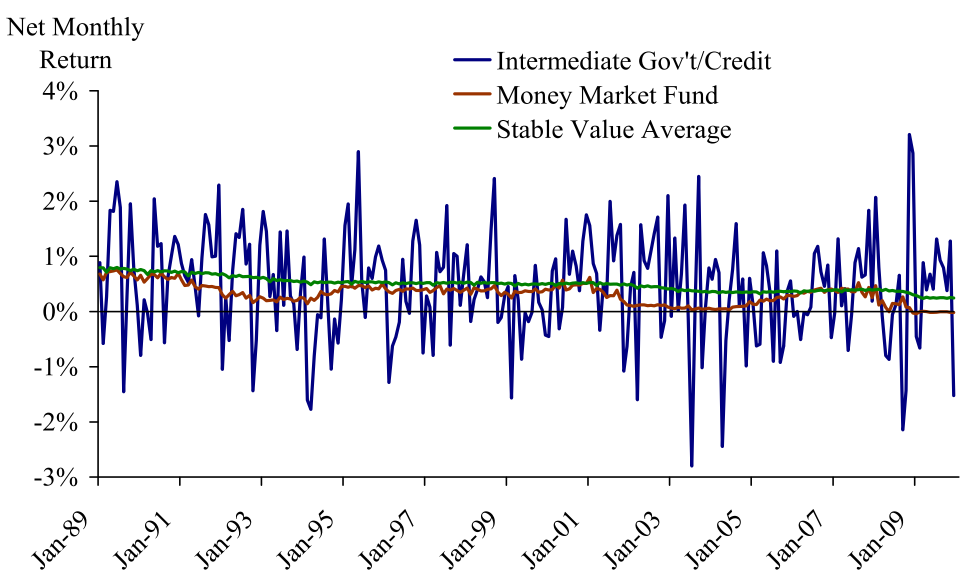
[17] The multirelation coefficient and its calculation are introduced and described in Zvi Drezner (1995). A significance level test of the coefficient is provided by Dear and Drezner (1997).

Figure 2. Number of Fund Complexes by Month in the Stable Value Average Return Index



Source: SVIA

Figure 3. Net Monthly Returns



Source: Bloomberg, ICI, SVIA.

Table 1. Correlation Among Return Series in the Stable Value Average

	5 Fund Complexes Jan-91 to Dec-09		9 Fund Complexes Apr-97 to Dec-09	
	Levels	Differences	Levels	Differences
Multirelation Coefficient	99.6%	95.4%	99.5%	95.1%
Average of Pairwise Correlation Coefficients	94.2%	78.7%	88.7%	72.0%

Note: The multirelation coefficient is one minus the smallest eigenvalue of the correlation matrix of the data. See, Zvi Drezner, Multirelation - a correlation among more than two variables, *Computational Statistics & Data Analysis* 19 (1995) 283-292.

Table 2 below presents summary statistics for the seven asset classes we study. It shows that over the period of January 1989 through December 2009, SV investments have had, on average, a higher net monthly return and a lower return volatility than either money market or intermediate-term government/credit funds. As expected, when compared to stocks or long-term bonds, SV funds have exhibited both lower average returns and volatility. These facts lie behind the results that we present in the next section.

Table 2. Summary Statistics — Net Monthly Returns, Jan-89 to Dec-09

	Large Stocks	Small Stocks	Long-Term Gov't Bonds	Long-Term Corp. Bonds	Intermediate Gov't/Credit	Money Market	Stable Value
No. of Months	252	252	252	252	252	252	252
Mean	0.72%	1.02%	0.64%	0.60%	0.46%	0.32%	0.49%
Standard Deviation	4.31%	5.89%	2.85%	2.54%	0.96%	0.18%	0.13%
Minimum	-16.88%	-20.71%	-11.31%	-9.55%	-2.80%	-0.04%	0.24%
Maximum	11.28%	23.58%	14.36%	15.54%	3.21%	0.76%	0.80%
Jarque-Bera Test	35.36	33.66	122.65	539.61	4.62	N/A	N/A
JB p-value	0.0000	0.0000	0.0000	0.0000	0.0992	N/A	N/A

Note 1: The Jarque-Bera test is a test of the null hypothesis of normality. When the null hypothesis is true, the JB statistic has an approximate Chi-square distribution with two degrees of freedom. 5%, 1% and 0.1% critical values are, respectively, 5.99, 9.21, and 13.82.

Note 2: The Jarque-Bera test assumes independent returns. This is a reasonable assumption for all but money market and stable value returns and we do not apply the JB test to these series.

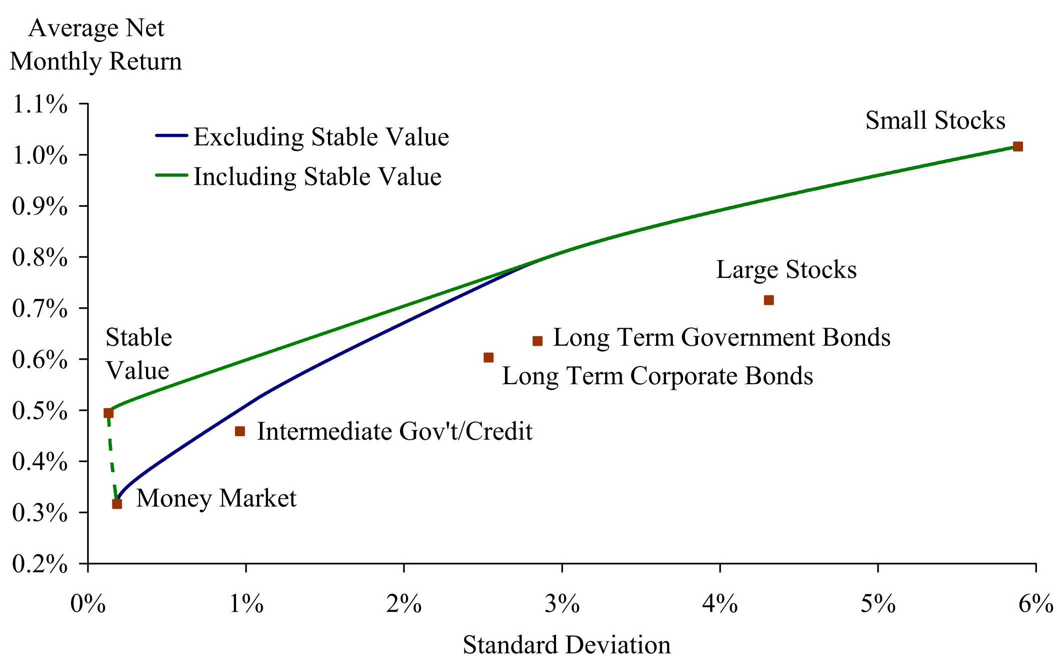
6. RESULTS

6.1 Mean-variance analysis. As indicated earlier, mean-variance analysis provides a characterization of the trade-off between risk and return that is neither supported by the statistical properties of the return data, nor by the theoretical logic of risk aversion. Despite these shortcomings, the mean-variance approach provides useful insights into the ability of SV investments to dominate other asset classes.

In this section we present evidence supporting the conclusion that, even as stand-alone investment, SV funds have been superior in the mean-variance sense to money market and intermediate-term government/credit bond funds. We also show, based solely on historical returns that, when included in optimal mean-variance portfolios, SV funds contribute significantly to the portfolio, to the exclusion of money market, intermediate-term government/credit bonds, long-term corporate bonds and even large stocks. In other words, optimal mean-variance portfolios contain only SV funds, long-term government bonds and small stocks in proportions that naturally vary with the expected return (or, alternatively, the expected volatility) of the optimal portfolio.

When discussing summary statistics for our net monthly return data in Table 2, we observed that, over the period of our study from January 1989 through December 2009, SV returns exhibited both a higher mean and lower volatility than either money market or intermediate-term government/credit bond returns. This feature can be seen in Figure 4 below, where we plot two efficient frontiers, one including all seven asset classes in our study and one that excludes SV funds.

Figure 4. Efficient Frontiers for Alternative Asset Classes



It is interesting to note the potentially large scope for improvement that inclusion of SV investments brings to an optimal mean-variance portfolio for more than two thirds of the expected return range. As revealing as Figure 4 is, it does not show the full extent to which SV investments contribute to an optimal portfolio since it says nothing about the relative allocations of wealth among SV funds and other investments at different points along the efficient frontier. Table 3-A reports these optimal weights (again based solely on historical returns) for selected expected monthly returns ranging from 0.49%, the historical average net return for SV funds, to 1.02%, the historical small stocks net return.

We observe that no optimal mean-variance portfolio along the efficient frontier includes money market instruments, intermediate-term bonds or long-term corporate bonds. Not even large US stocks are included. We also observe that SV funds predominate in the lower portion of the expected return range, where one would conventionally anticipate seeing money market and intermediate-term bond investments.

Table 3-A. Optimal Weights for Mean-Variance Efficient Portfolios (Jan-89 to Dec-09)

		Asset Class						
Expected Return	Standard Deviation	Large Stocks	Small Stocks	Long-Term Gov't Bonds	Long-Term Corp. Bonds	Intermediate Gov't/Credit	Money Market	Stable Value
0.49%	0.13%	0	0	0	0	0	0	99.9%
0.52%	0.28%	0	3.8%	4.6%	0	0	0	91.6%
0.55%	0.52%	0	7.5%	9.3%	0	0	0	83.1%
0.57%	0.76%	0	11.2%	14.0%	0	0	0	74.8%
0.60%	1.01%	0	15.0%	18.7%	0	0	0	66.3%
0.62%	1.25%	0	18.7%	23.4%	0	0	0	57.9%
0.65%	1.50%	0	22.4%	28.1%	0	0	0	49.4%
0.68%	1.75%	0	26.2%	32.8%	0	0	0	41.1%
0.70%	1.99%	0	29.9%	37.5%	0	0	0	32.6%
0.73%	2.24%	0	33.6%	42.2%	0	0	0	24.2%
0.76%	2.49%	0	37.4%	46.9%	0	0	0	15.7%
0.78%	2.74%	0	41.1%	51.6%	0	0	0	7.3%
0.81%	2.99%	0	45.2%	54.8%	0	0	0	0
0.83%	3.26%	0	52.0%	48.0%	0	0	0	0
0.86%	3.58%	0	58.9%	41.1%	0	0	0	0
0.89%	3.92%	0	65.7%	34.3%	0	0	0	0
0.91%	4.29%	0	72.6%	27.4%	0	0	0	0
0.94%	4.67%	0	79.4%	20.6%	0	0	0	0
0.96%	5.07%	0	86.3%	13.7%	0	0	0	0
0.99%	5.47%	0	93.1%	6.9%	0	0	0	0
1.02%	5.89%	0	100.0%	0	0	0	0	0

Note: Weights may not add up to 100% across a given row due to rounding.

Table 3-B repeats the optimal portfolio analysis but where SV is not included as an available asset class. Two observations are important here. First, money market instruments, intermediate-term bonds, and long-term corporate bonds now enter the optimal portfolios at various levels of portfolio risk, along with small stocks and long-term government bonds. Second, large stocks never enter mean-variance-efficient portfolios model when calibrations are based on the past 21 years of historical returns and correlations.

Another interesting observation can be made if the first two columns of Table 3-B are compared to those of Table 3-A. Noting that Table 3-B begins at expected returns below and standard deviations above those of Table 3-A, owing to the preclusion of SV from Table 3-B, it is clear that portfolio risk levels are substantially higher for all levels of return (except for the highest returns, with 100% allocation in small stock) across all levels of portfolio returns.

Table 3-B. Optimal Weights for Mean-Variance Efficient Portfolios Excluding SV (Jan-89 to Dec-09)

		Asset Class					
Expected Return	Standard Deviation	Large Stocks	Small Stocks	Long-Term Gov't Bonds	Long-Term Corp. Bonds	Intermediate Gov't/Credit	Money Market
0.32%	0.18%	0	0.3%	0	0.1%	0	99.6%
0.35%	0.26%	0	2.2%	0	0	15.0%	82.8%
0.39%	0.41%	0	4.1%	0	0	30.1%	65.8%
0.42%	0.57%	0	6.0%	0	0	45.3%	48.7%
0.46%	0.74%	0	7.9%	0	0	60.2%	31.8%
0.49%	0.92%	0	9.8%	0	0	75.4%	14.8%
0.53%	1.10%	0	12.3%	0	0	87.7%	0
0.56%	1.30%	0	15.8%	8.6%	0	75.5%	0
0.60%	1.51%	0	19.2%	17.7%	0	63.1%	0
0.63%	1.74%	0	22.6%	26.8%	0	50.6%	0
0.67%	1.97%	0	26.0%	35.9%	0	38.1%	0
0.70%	2.21%	0	29.4%	44.9%	0	25.7%	0
0.74%	2.45%	0	32.8%	53.9%	0	13.2%	0
0.77%	2.69%	0	36.2%	63.0%	0	0.8%	0
0.81%	2.98%	0	45.0%	55.0%	0	0	0
0.84%	3.36%	0	54.1%	45.9%	0	0	0
0.88%	3.80%	0	63.3%	36.7%	0	0	0
0.91%	4.28%	0	72.4%	27.6%	0	0	0
0.95%	4.80%	0	81.6%	18.4%	0	0	0
0.98%	5.33%	0	90.8%	9.2%	0	0	0
1.02%	5.89%	0	100.0%	0	0	0	0

Note: Weights may not add up to 100% across a given row due to rounding.

Continuing in the spirit of mean-variance analysis, we turn to the Sharpe ratio so commonly used in asset allocation and performance measurement.[18] The Sharpe ratio measures excess return per unit of risk according to the formula:

$$\text{Sharpe Ratio} = \frac{E[R - R_f]}{\sqrt{Var[R - R_f]}}, \tag{4}$$

where R is the asset return, R_f is the return on the risk-free rate of return, and $E[R - R_f]$ is the expected value of the excess of the asset return over the risk-free rate. This ratio is used as a measure of how well an investor is compensated per unit of risk taken. Higher ratios denote greater return for the same level of risk.

We also use the Sortino ratio to focus more on the downside risk.[19] The Sortino ratio is based on the Sharpe ratio, but penalizes for only those returns that fall below the target return, which in our case will be the average riskless rate of return over the period of analysis. The ratio gives the actual rate of return in excess of the risk-free rate per unit of downside risk, and is as calculated below:

[18] The original "Reward-to-Variability" performance ratio, better known as simply the "Sharpe ratio" of William Sharpe was modified by him in 1994. The modified version of his ratio is used in this analysis. See Sharpe (1994).

[19] See Sortino and Price (1994) and Sortino and Van der Meer (1991) for a description of the Sortino Ratio. The theoretical foundations for the Sortino Ratio are provided in Pedersen and Satchell (2004).

$$\text{Sortino Ratio} = \frac{E[R - R_f]}{\sqrt{\left(\int_{-\infty}^{R_f} \left(R_f - R\right)^2 f(R) dR\right)}}. \qquad (5)$$

The Sharpe and Sortino ratios for monthly net return data are reported in Table 4-A.

Table 4-A. Sharpe and Sortino Ratios (Monthly Data, Jan-89 to Dec-09)

	Large Stocks	Small Stocks	Long-Term Gov't Bonds	Long-Term Corp. Bonds	Intermediate Gov't/Credit	Stable Value
Mean of Excess Returns	0.40%	0.70%	0.32%	0.29%	0.14%	0.18%
STDEV of Excess Returns	4.30%	5.90%	2.83%	2.53%	0.94%	0.12%
Target Semi-Deviation	3.08%	4.03%	1.91%	1.64%	0.63%	0.01%
Sharpe Ratio	0.093	0.119	0.113	0.113	0.152	1.527
Sortino Ratio	0.130	0.174	0.167	0.175	0.228	12.002

Note: The target rate for the Sortino ratio is the average Money Market fund rate. Therefore, the numerator is the same in both ratios.

We note that the Sharpe ratio values for five of the asset classes are mostly clustered together, but that for SV the ratio is about ten times greater than the highest of the other asset classes. This pattern is even more pronounced for the Sortino ratio. The extremely high Sortino ratio assigned to SV funds, relative to those assigned to other asset classes, results from the fact that throughout the entire 252-month period under consideration, the risk-free rate exceeded the SV credited rate only for thirteen months, and by small amounts. Hence, there were only a few, small observations that factored into the denominator.

What is most noteworthy about these ratios is how much higher they are for SV funds than for the other asset classes. Although our calculations are based primarily on monthly data, we also provide the analogous ratios based on annual returns in Table 4-B. Again we observe very large Sharpe and Sortino ratios for SV funds relative to other investment classes.

Table 4-B. Sharpe and Sortino Ratios (Annual Data, 1989 to 2009)

	Large Stocks	Small Stocks	Long-Term Gov't Bonds	Long-Term Corp. Bonds	Intermediate Gov't/Credit	Stable Value
Mean of Excess Returns	5.71%	9.02%	4.11%	3.47%	1.79%	2.23%
STDEV of Excess Returns	19.07%	23.01%	11.23%	7.68%	3.82%	1.31%
Target Semi-Deviation	12.02%	11.66%	6.19%	3.78%	2.07%	0.19%
Sharpe Ratio	0.299	0.392	0.366	0.451	0.468	1.696
Sortino Ratio	0.475	0.774	0.664	0.918	0.864	11.980

Note: The target rate for the Sortino ratio is the average Money Market fund rate. Therefore, the numerator is the same in both ratios.

What we can say from this ratio analysis is that the structure of SV returns is very different from that of other asset classes, and that its structure does not lend itself well to traditional mean-variance metrics for comparison. Moreover, these mean-variance findings are derived from return distributions that, for most investment classes, are decidedly not normal, as evidenced by the bottom row of Table 2 displayed earlier. Accordingly, we now turn to present alternative and more powerful analyses that buttress the implications our mean-variance analyses.

6.2 Stochastic dominance analysis. We next discuss the ability of SV funds to dominate alternative asset classes in the sense of stochastic dominance (SD) which, as we indicated earlier, provides dominance criteria under very general conditions with respect to an investor's attitudes toward risk and considers higher moments of the asset return distributions.

First-degree stochastic dominance (FSD) imposes only one preference restriction – investors prefer more wealth to less wealth. In addition to this requirement, second-degree dominance (SSD) requires investors to be risk averse, i.e., to dislike a drop in wealth more than they like a wealth increase of the same magnitude. The development of third-degree stochastic dominance (TSD) was motivated by a long observed preference among investors for positively skewed returns. A subset of the class of investors that prefer returns exhibiting third-degree stochastic dominance is the important group whose preferences are characterized by decreasing absolute risk aversion (DARA). Such investors are willing to pay less for insuring against a given sized risk, on average, as they accumulate greater wealth, which appears to accord with observed behavior toward risk. Fourth-degree stochastic dominance (4SD) was developed to capture investors' aversion toward kurtosis, where returns are characterized by peaked distributions and fat tails, such that losses can be extreme. Of course kurtosis can favor investors who have asymmetric claims toward returns, such as investors in call options, but for investors who have equal claims to both tails of a distribution, the fatter tails cause a disproportionate loss in utility.[20]

Table 5 presents the SD results among the seven asset classes in our study, based solely on historical returns.[21] Only the SV fund historical returns distribution dominates two asset classes in the stochastic dominance sense up to the fourth degree; none of the other asset classes dominates SV funds.

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[20] See the detailed exposition in Levy (2006) for a complete characterization of the necessary and sufficient conditions for SD.

[21] Stochastic dominance analysis is not restricted to basing return and risk expectations solely on historical returns, but that is the approach we take in this study.

Table 5. Stochastic Dominance Among Alternative Asset Classes
Net Monthly Returns Jan-89 — Dec-09

Does the Investment in Column Stochastically Dominate the Investments in Rows?

	Stable Value	Money Market	Intermediate Gov't/Credit	Long-Term Corporate Bonds	Long-Term Government Bonds	Large Stocks	Small Stocks
Stable Value		NO	NO	NO	NO	NO	NO
Money Market	**YES: FSD, SSD, TSD, 4SD**		NO	NO	NO	NO	NO
Intermediate Gov't/Credit	NO: FSD **YES: SSD, TSD, 4SD**	NO		NO	NO	NO	NO
Long-Term Corporate Bonds	NO	NO	NO		NO	NO	NO
Long-Term Government Bonds	NO	NO	NO	NO		NO	NO
Large Stocks	NO	NO	NO	NO	NO		NO
Small Stocks	NO	NO	NO	NO	NO	NO	

Note: A cell with a single NO result indicates that the column investment does not SD the row investment for any of the first four degrees.

Turning to SV funds, they stochastically dominated money market investments by the first-degree and, as a corollary, by any higher degree as well. This is a direct consequence of the fact that when sorted returns for SV and money market funds are compared in a pairwise fashion over the past two decades, the SV return was always greater than the corresponding money market return. In other words, the empirical cumulative distribution function (CDF) of the money market returns was strictly above and to the left of the empirical CDF of the SV returns, meaning that for any given return the probability of obtaining a lower return with a money market fund is greater than with an SV fund. Consequently, any investor who preferred more wealth to less wealth should have avoided investing in money market funds when SV funds were available, irrespective of risk preferences.

Although SV funds failed to stochastically dominate intermediate-term government/credit bonds by the first degree, they dominated by the second and higher degrees. This result is a direct consequence of the fact that while the empirical CDFs of these two asset classes cross (thus preventing first-degree stochastic dominance), positive intermediate-term bond returns during the period of our study were never large enough, relative to corresponding SV returns, to make at least some risk-averse investors prefer the riskier intermediate-term bond investment. Technically, the integral of the difference between the intermediate-term bond return distribution and the SV return distribution is positive for any return.

The results in this sub-section are remarkable. Not surprisingly, there is no stochastic dominance of any one traditional class over another; indeed dominance is rarely encountered. Accordingly, it was surprising to find that SV investments dominated two of the major traditional investment classes.

6.3 Intertemporal optimization analysis. The intertemporal investment model of Grauer and Hakansson (1982, 1985, 1986, 1987, 1993, and 1995) considers an investor who, at the beginning of each period, allocates wealth among various investment alternatives so as to maximize expected utility of wealth. The investment alternatives we consider are the same as in our previous analysis, but the investment horizon is assumed to be a quarter. Therefore, the return data we use are net quarterly returns for the period Q1-1989 through Q4-2008.

At the beginning of the optimization quarter t, the investor chooses a portfolio that maximizes expected utility of wealth,

$$E[U(1+r_t)] = E\left[\frac{1}{a}(1+\sum_i w_{it}r_{it})^a\right], \qquad (6)$$

subject to $w_{i,t} \geq 0$ with $\sum_i w_{i,t} = 1$ for all t. $E[\cdot]$ is the expectations operator, w_{it} is the fraction of wealth allocated to investment i in period t, r_{it} is the investment i return that will obtain at the end of quarter t, and $a \leq 1$ is a risk parameter. The function $U(\cdot)$ is the familiar constant relative risk aversion (CRRA) utility function with the coefficient of relative risk aversion given by $\rho = 1 - a$.

In order to evaluate the expectation in (6) we need the unknown joint distribution of quarterly returns. We thus estimate this expectation using observed return data for the 80 quarters prior to a decision quarter t.[22] We also use yield data on money market funds known at the beginning of quarter t and historical equity and bond premiums, as explained below.

Note that SV funds offer a crediting rate which is generally reset at the beginning of each quarter. This means that for each decision period, the SV return is known at the time the investor solves the optimization problem. Therefore, the SV return is not random.[23] However, the remaining investments, large and small stock funds, long-term government and corporate

[22] We explain the use of an 80-quarter estimation window in footnotes 12 and 23.

[23] We have ignored in our modeling here that a tiny part of the monthly return reflects return on cash and is therefore not entirely known at the beginning of the quarter. Moreover, large funds may have several overlapping cohort segments that constitute a given quarterly segment, and the cohorts may mature at different times during the quarter. As they roll off, they are substituted by a new cohort segment. In the past, this substitution has given rise to as much as an 8 basis points change in the overall returns of a given quarter. These changes are too small to affect the results of this section, so they are not considered here.

bond funds, intermediate-term government/credit bond funds and money market funds have random decision period returns at the beginning of the period and thus the expected utility of wealth needs to be estimated. Grauer and Hakansson (1982, 1985, 1986, 1987, 1993, and 1995) use the realized returns over the 40 quarters prior to each decision period in order to estimate the expected utility of wealth and solve the investor's optimization problem. We follow a slightly different approach, to avoid the difficulties involved in estimating expected returns using only historical data (Merton, 1980).[24]

For a given optimization quarter, the expected return on money market funds is taken to be the yield available on the day prior to the optimization quarter, expressed as a quarterly rate.

In the case of bond funds (composed of either long-term government bonds, or long-term corporate bonds, or intermediate-term government/credit bonds) we calculate for each quarterly return series the difference between actual returns and the sample mean. We then take this residual series and add to it an expected return that is constructed as the money market expected return described above (expressed as a quarterly rate) augmented by an appropriate historical bond premium calculated on the basis of quarterly return data from 1926 through the quarter. For example, in the case of long-term government bonds the historical quarterly average excess return over 3-month Treasury bills from 1926 through 2009 is 0.5136% or 2.07% per year. We therefore would estimate the expected return on long-term government bonds, to be used in the optimization problem for the first quarter of 2010, as the sum of the money market yield at the beginning of 2010, expressed as a quarterly rate, and the 0.5136% historical quarterly bond premium. We incorporate this quarterly rate into the quarterly residual series for the sample of interest (in this case, the 80 quarters in the 1990-2009 period) to obtain a return series with the desired expected return. This procedure is also applied to our long-term corporate and intermediate-term bond series, and to our large and small stock series (with the exception mentioned below).[25] The procedure we use to estimate expected returns gives us 80-quarter series of money market, bond, and stock returns that preserve the same second and higher moments and co-moments as the original quarterly return data.

Instead of calculating historical small and large stocks equity premiums, we consider alternative values for the large stocks equity premium, ranging from 2.5%-5.5% per year over the money market expected return calculated as described above. These values reflect the recent and current discussion on the equity premium that is likely to obtain over the foreseeable future (Siegel, 2005; Welch, 2009; Fernandez and del Campo, 2010). Concerning the small stocks premium, we assume that it will exceed the large stock equity premium by the

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[24] Properly speaking, solving the investor's maximization problem does not require direct estimation of expected returns. However, the sample mean of expected utility in expression (6), based on data observed over the 80 quarters prior to the decision period, will indirectly inherit the problems associated with estimation of expected returns, especially during a period where expected asset returns may have been time-varying. In our 2007 study, we experimented with estimation windows extending from 40 quarters to 111 quarters, and generally found that beyond 80 quarters, the estimation windows were quite stable.

[25] In order to use data from 1926, we estimate the intermediate-term historical premium using the Ibbotson's intermediate government bond data since the Barclays-Lehman intermediate government/credit index is only available beginning in 1973.

difference between the historical average small stocks return and the historical large stocks return over the period 1926 through the end of each study period.

The discussion in the two preceding paragraphs can be summarized in the following expression for the asset returns we use in this analysis:

$$\tilde{r}_{i,t} = \mu_{MM} + \pi_i + \varepsilon_{i,t}, t = 1, \ldots 80, \qquad (7)$$

where μ_{MM} is the money market yield on the day prior to the beginning of the optimization quarter, π_i is the risk premium of asset class i (small and large stocks, long-term government and corporate bonds, and intermediate-term bonds), constructed as explained above, and $\varepsilon_{i,t} = r_{i,t} - \bar{r}_i$ is the excess return for asset class i over the sample return over the period of interest.

We now proceed to examine three interesting recent periods whose diverse market conditions will give rise to representative results.

6.3.1 Optimal Asset Allocation for Q1-10. We first find optimal weights for all seven investment alternatives, by solving the optimization problem in expression (6) for the first quarter of 2010, using the empirical distribution of returns, as described above, for the 80 quarters from 1990 through 2009 in order to calculate expected utility of wealth. Figure 5 shows optimal portfolio weights for selected values of the risk aversion coefficient ρ, and selected values of the large stock equity premium.

Figure 5 reveals that, with the exception of a very small proportion of long-term government bonds for moderate risk aversion levels when the large stocks premium is 5.5% per year, only small stocks and SV funds are relevant in the optimal portfolios, irrespective of the level of risk aversion. As expected, SV funds represent a larger fraction of the optimal portfolio as risk aversion increases. Also as expected, a larger equity premium increases the proportion of small stocks in the optimal portfolio for any given risk aversion level.

Table 6 reports the premiums and corresponding expected returns that underlie the optimal weights in Figure 5. Premiums and expected returns for bonds, and expected returns used for money market and SV investments do not depend on the large stocks premium used and are only reported once.

The extreme results obtained in this case, where only various combinations of small stocks and stable value investments are considered optimal regardless of the degree of risk aversion, are due to the very low money market yields observed at year-end 2009, with short-term interest rates at near-zero levels. (Recall that the spreads used to form expected returns for other asset classes in the model are tied to money market rates; accordingly, these expected returns are also low.) Because of upheaval in financial markets due to the recent crisis, this is a situation that, while optimal for an investor making decisions at the beginning of 2010, may not be representative.

Figure 5. Optimal Portfolio Weights - Optimization Quarter Is Q1 - 10

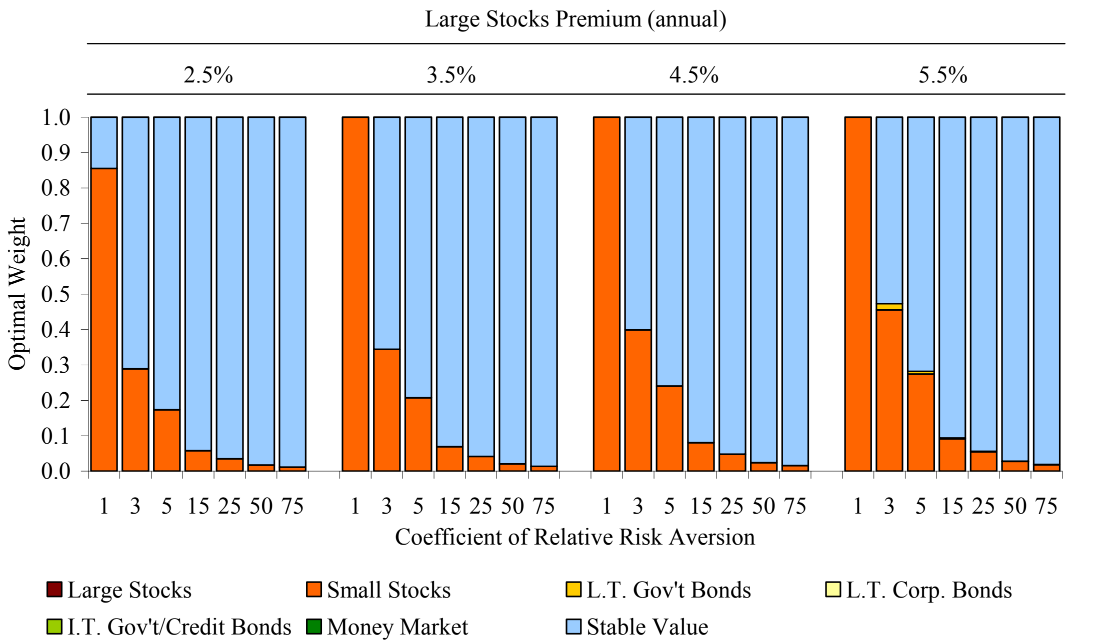


Table 6. Premiums and Expected Returns for Optimization Quarter Q1-10

	Large Stocks Premium							
	2.5%		3.5%		4.5%		5.5%	
Asset	Premium	Expected Return	Premium	Expected Return	Premium	Expected Return	Premium	Expected Return
Large Stocks	2.50%	2.56%	3.50%	3.56%	4.50%	4.56%	5.50%	5.56%
Small Stocks	8.04%	8.10%	9.04%	9.10%	10.04%	10.10%	11.04%	11.10%
L.T. Gov't Bonds	2.08%	2.14%	------------------------- same pattern repeats -----------------------------------					
L.T. Corp. Bonds	2.46%	2.52%	------------------------- same pattern repeats -----------------------------------					
I.T. Gov't/Credit	1.75%	1.81%	------------------------- same pattern repeats -----------------------------------					
Money Market		0.06%	--					
Stable Value		3.10%	--					

Nevertheless, this extreme situation allows for some interesting sensitivity analysis. In order to quantify what expected return would be necessary for one of the excluded asset classes to enter into the optimal portfolio, we conduct an experiment where expected returns, as reported in the panel corresponding to a large stock premium of 3.5% in Figure 5 and Table 6, are increased, for each value of the coefficient of relative risk aversion, until a given asset class enters the optimal portfolio with either a 1% or a 10% weight.[26]

[26] We do not consider money market funds in this experiment.

Table 7 reports the results of this analysis.

Table 7. Additional Basis Points Required for Asset Class to Enter Optimal Portfolio

	Large Stocks		L.T Gov't Bonds		L.T. Corp. Bonds		I.T. Gov't/Credit	
ρ	1%	10%	1%	10%	1%	10%	1%	10%
1	299	317	36	118	49	121	120	183
3	293	320	25	53	41	69	106	110
5	294	338	28	73	44	89	107	112
15	303	436	38	174	54	188	108	126
25	313	664*	49	273	65	284	110	139
50	337	1383*	74	518	89	519	113	172
75	362	2128*	99	757	114	753	116	205

Notes:

[1] For a given risk coefficient and asset class, an entry in the table is the number of basis points that the premium over money market funds needs to increase by, above the premium shown in Table 6, for the asset class to enter the optimal Q1-09 portfolio with a weight of either 1% or 10%, as indicated in column headings. Remember that the necessary increase in basis points shown here are in addition to the 350 basis points of equity premium already assumed in our base case being examined.

[*] In these cases, large stocks enter the optimal portfolio to the exclusion of small stocks.

Table 7 displays the increase in expected returns (in basis points) for the different cases considered, across a range of risk coefficients similar to that applied by Grauer and Hakansson.[27] For instance, at a risk aversion index of 5, the expected excess returns (above the risk-free interest rate) on large stocks would need to be increased above the assumed expected excess return of 3.5% by an additional 294 basis points before they would enter the optimal portfolio with a 1% weight, and by 338 basis points before they would constitute as much as 10% of the portfolio, assuming that the expected return on all other asset classes remains unchanged.

An examination of the calculations shows that, with the exception of the row corresponding to $\rho = 1$, the increments to expected return required for an asset class to enter the optimal portfolio grow as risk aversion increases. For example, in the case of large stocks with an assumed equity risk premium of 350 basis points, an additional 436 basis points of expected return would be required for them to reach a 10% portfolio weight when the risk aversion coefficient is 15, compared to only 338 more basis points at a risk aversion coefficient of 5. This general pattern can also be seen when long-term or intermediate-term bonds are considered. In the former case, however, the amount of additional basis points required to achieved optimal weights of either 1% or 10% is less than that for large stocks due to the smaller volatility of long-term bonds, compared to large stocks, in spite of the smaller expected returns that long-term bonds have, compared to SV funds (see Table 6). For intermediate-term bonds, we observe that the amount of additional basis points required to get them into the

[27] Grauer and Hakansson use values of the coefficient a, and we use values of the coefficient of relative risk aversion $\rho = 1 - a$. A recent study confirms that the broad range of risk aversion levels used by Grauer and Hakansson has strong empirical support. See Malloy, Moskowitz, and Vissing-Jørgensen (2009).

optimal portfolio is not as sensitive to risk aversion or to the desired weight (1% or 10%) as is the case for long-term bonds. This is because in this analysis, intermediate-term bonds have an expected return that is significantly smaller than that of SV investments (1.81% v. 3.10% per year) so that the number of basis points required is larger than in the case of long-term bonds to reach a 1% weight, and also because their volatility is smaller than that of long-term bonds so that the optimal weight is not as dependent on risk aversion.

The exception to the general pattern just described is that the amount of basis points required to make large stocks, long-term bonds, and intermediate-term bonds enter the optimal portfolio is *larger* for a risk aversion coefficient of $\rho = 1$ than for the next value we consider, $\rho = 3$. This is the case because for $\rho = 1$ the only investment class in the optimal portfolio is small stocks so that large stocks, long-term bonds, and intermediate-term bonds can become part of the optimal portfolio only when their expected return is high enough to partially crowd out small stocks. For values of ρ greater that one, SV funds are a significant part of the optimal portfolio so that large stocks and, especially, long-term and intermediate-term bonds are able to partially crowd small stocks and/or SV funds out of the optimal portfolio with a smaller amount of basis point increase for moderate risk aversion levels (and even for high risk aversion levels in the case of intermediate-term bonds).

6.3.2 Optimal Asset Allocation for Q3-08. Next we calculate optimal portfolios for the third quarter of 2008, using data for the prior 80 quarters. Figure 6 shows the optimal portfolio weights for selected values of the risk parameter ρ, and selected large stocks annual risk premiums.

The underlying premiums and expected returns are presented in Table 8.

The annual expected money market return and SV return are, respectively, 1.90% and 4.53%, compared to 0.06% and 3.10% for year-end 2009 (see Table 6), and the expected returns for other asset categories are significantly higher as well, yet we observe basically the same optimal portfolios as in the previous case, with small stocks and SV funds being the only significant investment classes, except for a small fraction of long-term government bonds at higher large stocks premiums. This pattern of optimal portfolios being comprised largely of SV and small stocks is not that uncommon, it turns out, when expected returns are based on historic spreads and current yields. In fact, in our 2007 and 2009 papers, we found these portfolios to be the norm.

Figure 6. Optimal Portfolio Weights - Optimization Quarter Is Q3 - 08

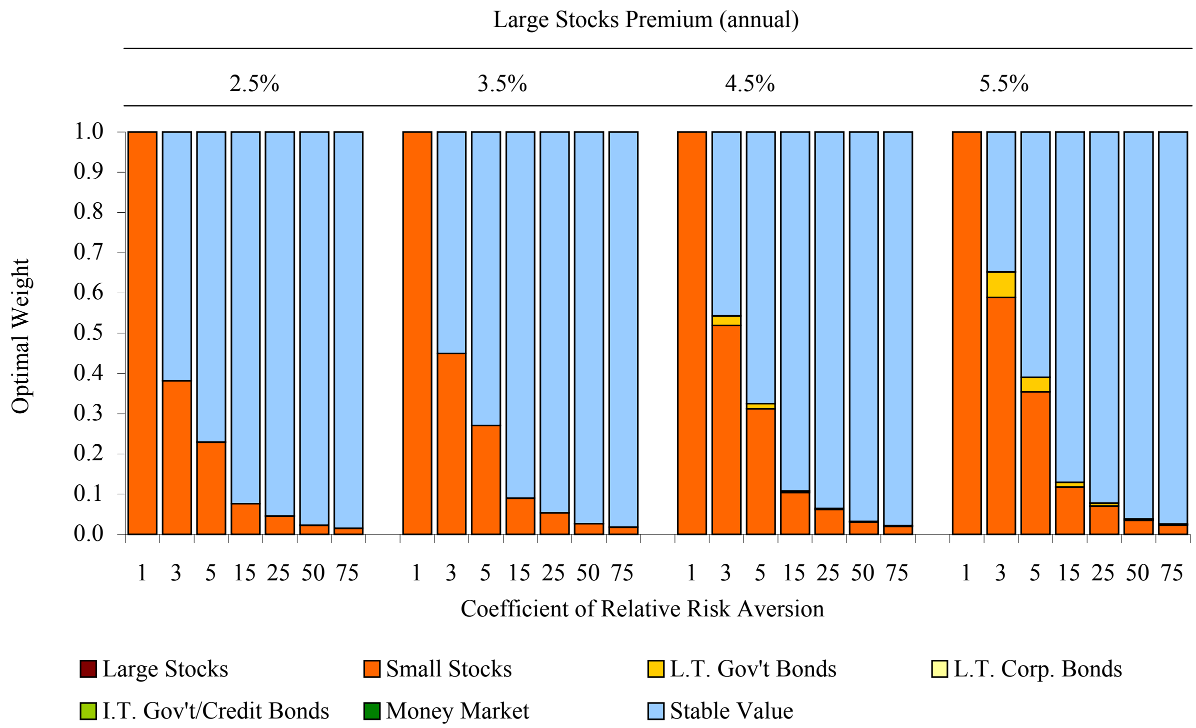


Table 8. Premiums and Expected Returns for Optimization Quarter Q3-08

	Large Stocks Premium							
	2.5%		3.5%		4.5%		5.5%	
Asset	Premium	Expected Return	Premium	Expected Return	Premium	Expected Return	Premium	Expected Return
Large Stocks	2.50%	4.40%	3.50%	5.40%	4.50%	6.40%	5.50%	7.40%
Small Stocks	8.05%	9.95%	9.05%	10.95%	10.05%	11.95%	11.05%	12.95%
L.T. Gov't Bonds	2.02%	3.92%	------------------------- same pattern repeats ------------------------------------					
L.T. Corp. Bonds	2.27%	4.17%	------------------------- same pattern repeats ------------------------------------					
I.T. Gov't/Credit	1.70%	3.60%	------------------------- same pattern repeats ------------------------------------					
Money Market		1.90%	---					
Stable Value		4.53%	---					

6.3.3 Optimal Asset Allocation for Q1-07. We finally calculate optimal portfolios for the first quarter of 2007, again using data for the prior 80 quarters. Figure 7 shows the optimal portfolio weights for selected values of the large stocks risk premium and of the risk parameter ρ, and Table 9 shows the underlying premiums and derived expected returns.

Figure 7. Optimal Portfolio Weights - Optimization Quarter Is Q1 - 07

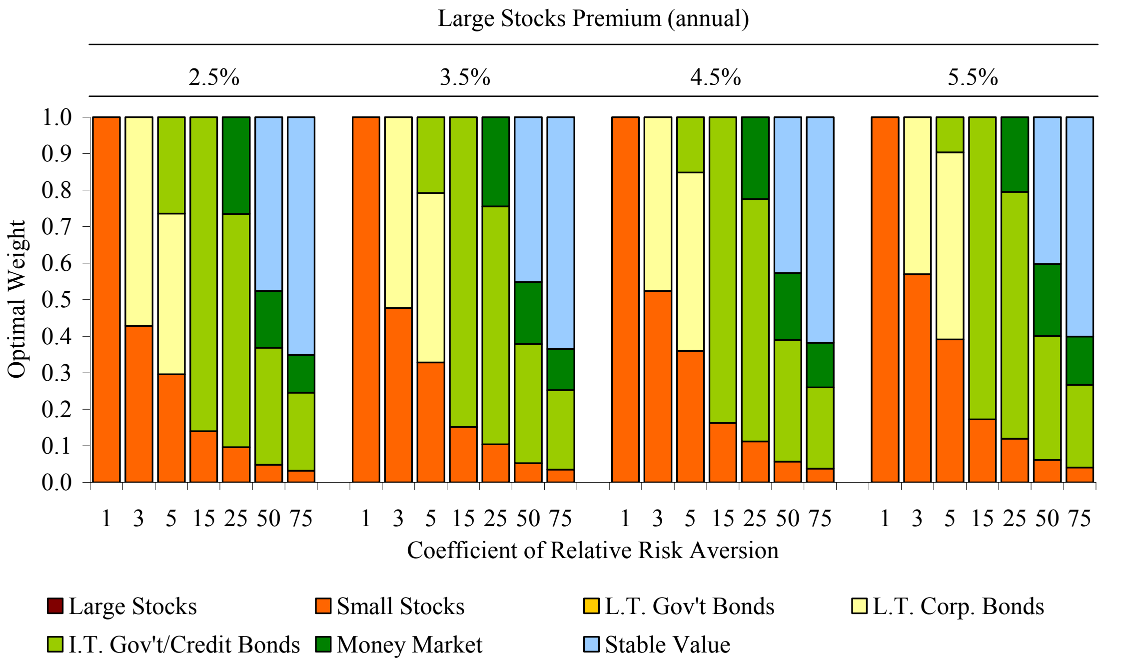


Table 9. Premiums and Expected Returns for Optimization Quarter Q1-07

	Large Stocks Premium							
	2.5%		3.5%		4.5%		5.5%	
Asset	Premium	Expected Return	Premium	Expected Return	Premium	Expected Return	Premium	Expected Return
Large Stocks	2.50%	7.52%	3.50%	8.52%	4.50%	9.52%	5.50%	10.52%
Small Stocks	8.31%	13.33%	9.31%	14.33%	10.31%	15.33%	11.31%	16.33%
L.T. Gov't Bonds	2.00%	7.02%	------------------------- same pattern repeats -----------------------------------					
L.T. Corp. Bonds	2.40%	7.42%	------------------------- same pattern repeats -----------------------------------					
I.T. Gov't/Credit	1.63%	6.65%	------------------------- same pattern repeats -----------------------------------					
Money Market		5.02%	--					
Stable Value		4.87%	--					

This is a case where both money market and intermediate-term bond funds have expected returns above the SV return of 4.87%, which is contrary to the historical experience. For this reason, money market and intermediate-term bond funds play a significant role in the optimal portfolios for larger risk aversion levels. In effect, they largely substitute for SV holdings at moderate levels of risk aversion.

The yield curve at year-end 2006 was fairly flat with relatively high money market yields that exceeded SV rates. This has happened only rarely since 1989. Also, interest rates had been increasing steadily for about three years, a fact that disadvantages SV returns that tend to adjust to intermediate-term bond returns with a lag. This helps explain the limited presence of SV funds in the optimal portfolios for the first quarter of 2007. Notwithstanding these findings, we encountered no evidence of significant withdrawals from SV funds during this quarter. As stated earlier, the provisions governing SV funds generally do not foster yield chasing behavior.

It is also interesting to note that long-term corporate bonds at modest degrees of risk aversion and intermediate-term bonds at somewhat higher risk aversion degrees, have a significant presence in the optimal portfolios, and that money market funds also enter. This is a consequence of the relatively high expected returns for these asset classes, given their respective volatilities.

7. Robustness Analysis

While important, the implications of our three-fold analysis should not be regarded as dispositive. It should be recalled that over the 21-year period of the first phase of our analysis, which began with the inception of modern SV funds in 1989, interest rates exhibited a general decline to half their initial level, albeit with occasional and protracted periods of rising interest rates interspersed. Such a period of decline would tend to favor longer duration fixed income investments, including stable value, over money market funds. We sought to remedy this by examining the precursors of SV funds – traditional -GICs – to see how they fared during the period of rapidly rising interest rates that characterized the late 1970s and early 1980s.

Table 10 reports summary statistics for the extended 37-year period beginning in February of 1973 and ending in December of 2009.[28]

Comparing Tables 2, and 10, we observe that average returns for a particular asset vary considerably, especially for small stocks, money market and SV returns. Small stocks have an average net monthly return of 1.25% for the extended period, compared with 1.02% for the period 1989 through 2009. This difference is essentially due to the better performance of small stocks over the period 1973 – 1988 and the smaller impact of the negative returns in 2008 on an average calculated over 443 months instead of 252. The 14 basis points jump in money market average return of 0.46% for the extended period is due to the higher yields on debt instruments observed during the late seventies and early eighties. This is also the reason that the SV average return increased by 9 basis points to 0.58% for the extended period compared to the average of 0.49% over the period 1989 – 2009.

[28] Many SV funds use the Lehman Intermediate Government/Credit Bond yield series as a benchmark. We use the wrapped index based on this series as our proxy for SV returns prior to the beginning of our SV average return series in September of 1988. For the period of time during which the Lehman intermediate government/credit wrapped series and our SV return series overlap, September of 1988 through February of 2008, the correlation coefficient between the series is 98.5% and the average difference between them is only 4.955 basis points. In order to use a consistent proxy for net SV returns, we subtract this average difference from the Lehman intermediate government/credit wrapped monthly return for the period from February 1973 to August 1988.

Table 10. Summary Statistics — Net Monthly Returns, Feb-73 to Dec-09

	Large Stocks	Small Stocks	Long-Term Gov't Bonds	Long-Term Corp. Bonds	Intermediate Gov't/Credit	Money Market	Stable Value
No. of Months	443	443	443	443	443	443	443
Mean	0.74%	1.25%	0.61%	0.60%	0.52%	0.46%	0.58%
Standard Deviation	4.55%	6.29%	3.14%	2.86%	1.26%	0.29%	0.17%
Minimum	-21.71%	-29.19%	-11.31%	-9.55%	-4.79%	-0.04%	0.24%
Maximum	16.62%	27.67%	15.06%	15.54%	8.81%	1.80%	0.92%
Jarque-Bera Test	51.53	115.50	87.24	181.19	370.51	N/A	N/A
JB p-value	0.0000	0.0000	0.0000	0.0000	0.0000	N/A	N/A

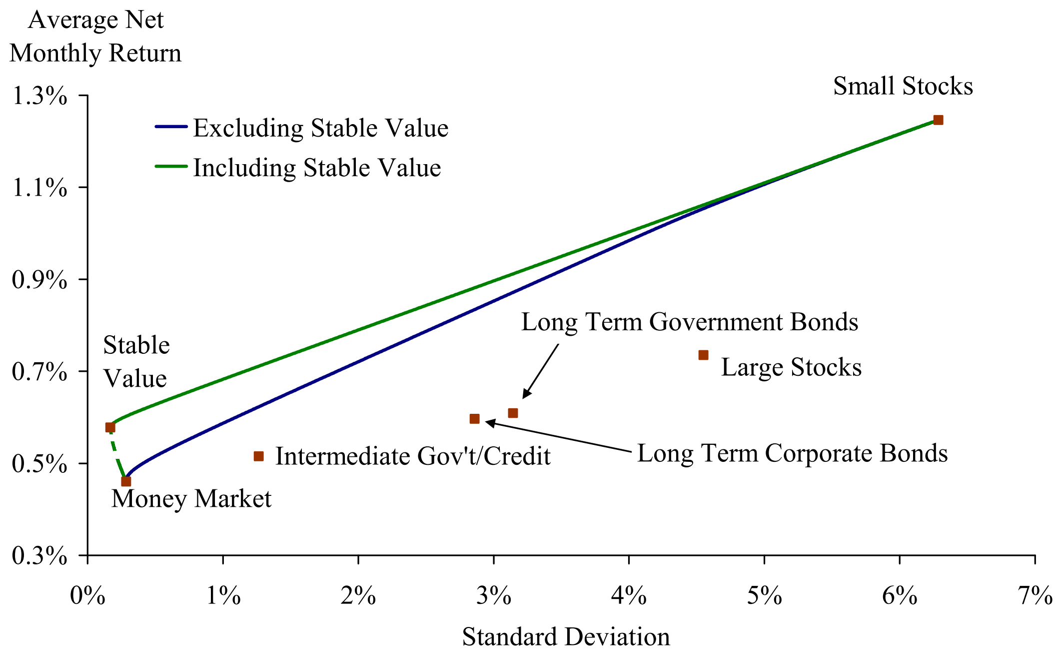
Note 1: The Jarque-Bera test is a test of the null hypothesis of normality. When the null hypothesis is true, the JB statistic has an approximate Chi-square distribution with two degrees of freedom. 5%, 1% and 0.1% critical values are, respectively, 5.99, 9.21, and 13.82.

Note 2: The Jarque-Bera test assumes independent returns. This is a reasonable assumption for all but money market and stable value returns and we do not apply the JB test to these series.

Evidence of non-normality is even stronger for the extended sample period, across all asset classes to which the Jarque-Bera test applies.

The differences we observe in mean returns across the two sample periods considered are reflected in the efficient frontier corresponding to the February 1973 to December 2009 data, shown in Figure 8.

Figure 8. Efficient Frontiers for Alternative Asset Classes (Feb-73 to Dec-09)



We observe a much flatter efficient frontier for the extended sample period. This is primarily the result of the higher small stocks average return and the lower long-term government bond returns, compared to the corresponding values for the 1989 – 2009 sample period.[29]

The composition of the efficient portfolios reported earlier in Table 3-A changes in some respects when the extended sample is considered, but it also remains the same in other respects.

Table 11-A reports these efficient portfolios for selected points along the efficient frontier. It shows that, as was the case for the 1989 to 2009 sample, large stocks, long-term corporate bonds, intermediate-term bonds and money market assets are excluded from the efficient frontier. We also observe that the weights of SV funds are markedly larger than for the portfolios shown in Table 3-A.

[29] An important factor in the degree of curvature of the efficient frontier is the correlation among asset classes. The correlation between small stocks returns and long-term government returns is −7.2% for the 1989 – 2009 period and 6.6% for the period 1973 – 2009.

Table 11-A. Optimal Weights for Mean-Variance Efficient Portfolios (Feb-73 to Dec-09)

		Asset Class						
Expected Return	Standard Deviation	Large Stocks	Small Stocks	Long-Term Gov't Bonds	Long-Term Corp. Bonds	Intermediate Gov't/Credit	Money Market	Stable Value
0.58%	0.17%	0	0.1%	0	0	0	0	99.9%
0.61%	0.36%	0	5.1%	0.1%	0	0	0	94.8%
0.65%	0.65%	0	10.1%	0.4%	0	0	0	89.6%
0.68%	0.96%	0	15.0%	0.7%	0	0	0	84.3%
0.71%	1.27%	0	20.0%	1.0%	0	0	0	79.0%
0.75%	1.58%	0	25.0%	1.3%	0	0	0	73.7%
0.78%	1.90%	0	30.0%	1.6%	0	0	0	68.5%
0.81%	2.21%	0	35.0%	1.9%	0	0	0	63.2%
0.85%	2.52%	0	39.9%	2.2%	0	0	0	57.9%
0.88%	2.83%	0	44.9%	2.4%	0	0	0	52.6%
0.91%	3.15%	0	49.9%	2.7%	0	0	0	47.3%
0.95%	3.46%	0	54.9%	3.0%	0	0	0	42.1%
0.98%	3.77%	0	59.9%	3.3%	0	0	0	36.8%
1.01%	4.09%	0	64.9%	3.6%	0	0	0	31.5%
1.05%	4.40%	0	69.8%	3.9%	0	0	0	26.2%
1.08%	4.72%	0	74.8%	4.2%	0	0	0	21.0%
1.11%	5.03%	0	79.8%	4.5%	0	0	0	15.7%
1.15%	5.34%	0	84.8%	4.8%	0	0	0	10.4%
1.18%	5.66%	0	89.8%	5.1%	0	0	0	5.1%
1.21%	5.97%	0	94.8%	5.2%	0	0	0	0
1.25%	6.29%	0	100.0%	0	0	0	0	0

Note: Weights may not add up to 100% across a given row due to rounding.

Another important difference with Table 3-A is that the weight of long-term government bonds is now much decreased across all efficient portfolios and the weight of small stocks is correspondingly increased.

Table 11-B repeats the optimal portfolio analysis but where SV is not included as an available asset class. We observe that money market instruments enter the optimal portfolios at various levels of portfolio risk, along with small stocks and long-term government bonds. However, unlike the results of the shorter period shown in Table 3-B, intermediate-term bonds and long-term corporate bonds no longer enter the optimal portfolios at any levels of portfolio risk. We also observe that once again, large stocks never enter mean-variance-efficient portfolios model when calibrations are based on the past 36 years of historical returns and correlations. Finally, in comparing the first two columns of Table 11-A with the corresponding columns in Table 11-B, we see that for most given expected returns in a portfolio precluded from SV assets, the risk is higher than for the case when SV is included.

Table 11-B. Optimal Weights for Mean-Variance Efficient Portfolios Excluding SV (Feb-73 to Dec-09)

		Asset Class					
Expected Return	Standard Deviation	Large Stocks	Small Stocks	Long-Term Gov't Bonds	Long-Term Corp. Bonds	Intermediate Gov't/Credit	Money Market
0.46%	0.28%	0	0.3%	0	0	0	99.8%
0.50%	0.42%	0	4.8%	2.3%	0	0	93.0%
0.54%	0.67%	0	9.2%	5.2%	0	0	85.6%
0.58%	0.95%	0	13.7%	8.1%	0	0	78.2%
0.62%	1.24%	0	18.1%	11.1%	0	0	70.8%
0.66%	1.53%	0	22.5%	14.0%	0	0	63.5%
0.70%	1.82%	0	27.0%	16.9%	0	0	56.1%
0.74%	2.12%	0	31.4%	19.9%	0	0	48.7%
0.78%	2.42%	0	35.8%	22.8%	0	0	41.4%
0.81%	2.71%	0	40.2%	25.7%	0	0	34.0%
0.85%	3.01%	0	44.7%	28.7%	0	0	26.7%
0.89%	3.31%	0	49.1%	31.6%	0	0	19.3%
0.93%	3.61%	0	53.6%	34.5%	0	0	11.9%
0.97%	3.90%	0	58.0%	37.4%	0	0	4.6%
1.01%	4.20%	0	63.1%	36.9%	0	0	0
1.05%	4.52%	0	69.2%	30.8%	0	0	0
1.09%	4.85%	0	75.4%	24.6%	0	0	0
1.13%	5.20%	0	81.5%	18.5%	0	0	0
1.17%	5.55%	0	87.7%	12.3%	0	0	0
1.21%	5.92%	0	93.8%	6.2%	0	0	0
1.25%	6.29%	0	100.0%	0	0	0	0

Note: Weights may not add up to 100% across a given row due to rounding.

Comparable Sharpe and Sortino ratios are calculated for the extended sample and reported in Tables 12-A and 12-B below.

We note that unlike the shorter period (1989-2009), during which the money market rate exceeded the SV rate during only approximately five percent of the 252 months, the extended period of 443 months contained 76 months (about seventeen percent of the sample) during which money market yields exceeded the SV rates. It is therefore interesting to observe that extending the sample to include a period of significantly higher interest rates does not affect the relative performance of SV funds.[30] Again, the Sharpe and Sortino ratios were spread across a relatively narrow range for the other asset classes, but the SV ratios soared far above the rest for both monthly and annual data, as shown in Tables 12-A and 12-B.

[30] The shorter period featured the higher money market rates at the end of 2006 and beginning of 2007. The extended period also featured many months during the late 1970s and early1980s when this happened.

Table 12-A. Sharpe and Sortino Ratios (Monthly Data, Feb-73 to Dec-09)

	Large Stocks	Small Stocks	Long-Term Gov't Bonds	Long-Term Corp. Bonds	Intermediate Gov't/Credit	Stable Value
Mean of Excess Returns	0.27%	0.79%	0.15%	0.14%	0.05%	0.12%
STDEV of Excess Returns	4.55%	6.30%	3.11%	2.84%	1.21%	0.21%
Target Semi-Deviation	3.23%	4.22%	2.09%	1.89%	0.83%	0.05%
Sharpe Ratio	0.060	0.125	0.048	0.048	0.045	0.563
Sortino Ratio	0.085	0.186	0.071	0.072	0.066	2.260

Note: The target rate for the Sortino ratio is the average Money Market fund rate. Therefore, the numerator is the same in both ratios.

Table 12-B. Sharpe and Sortino Ratios (Annual Data, 1974 to 2009)

	Large Stocks	Small Stocks	Long-Term Gov't Bonds	Long-Term Corp. Bonds	Intermediate Gov't/Credit	Stable Value
Mean of Excess Returns	4.49%	11.34%	2.07%	1.85%	0.79%	1.51%
STDEV of Excess Returns	18.15%	22.94%	12.28%	10.73%	4.87%	2.33%
Target Semi-Deviation	11.81%	10.95%	6.70%	5.39%	2.72%	0.66%
Sharpe Ratio	0.247	0.494	0.168	0.173	0.161	0.651
Sortino Ratio	0.380	1.036	0.309	0.343	0.290	2.289

Note: The target rate for the Sortino ratio is the average Money Market fund rate. Therefore, the numerator is the same in both ratios.

Turning to the issue of stochastic dominance, we find that SV no longer dominates money market investments by the first degree, as was the case for the 1989 to 2009 period. This can be easily deduced from Table 10 above since the maximum money market return exceeds the maximum SV return, thereby precluding the possibility of SV exhibiting FSD over money market returns. This lack of first-degree dominance is a consequence of the high interest rates observed in the late seventies and early eighties, compared to the smaller SV returns which behave like smoothed intermediate-term bond yields. However, over the extended period SV returns continue to exhibit second-, third-, and fourth-degree stochastic dominance over money market funds, as well as over intermediate-term government/credit bonds.

It is also interesting to note that the distribution of SV returns *almost* dominates the distribution of long-term corporate bond returns by the second and higher degrees and is only prevented of doing so by the largest four monthly long-term corporate bond returns in a set of 443 monthly returns. Table 10 illustrates this result since we observe that the average returns of these two series are very similar while the standard deviation of SV returns (0.17%) is much smaller than that of long-term corporate bonds (2.86%).

The analysis in this section shows that the relatively good performance of SV investments is robust (one is tempted to say stable) to the sample period considered in the sense that the extended period we use includes extreme interest rate movements as well as extreme stock return fluctuations.

8. The Value Proposition, Challenges and Future Prospects

The positive evidence thus far, which covers the entire period of existence for traditional and synthetic forms of stable value funds, is intriguing and raises two questions: (1) What has been the value proposition that allowed these returns to be achieved? and (2) Should we expect this kind of performance over the future?

The value proposition[31] appears to have two facets. "On the asset side, some of the return above money market yields comes from investing at durations sufficient to capture the term premium that has been traditionally available in most markets. The funds also are able to take on a very small amount of credit and convexity risk and thereby gain some additional spread." They also sometimes invest in less liquid securities and occasionally GICs, which provide higher spreads than the most liquid assets. The first two of these factors help explain why SV returns have generally outpaced money market yields, but do not explain why they also have outpaced intermediate-term government/credit returns. For that we must turn to the liability side of fund management.

On the liability side, the contribution to performance derives from behavioral finance factors. Stable value funds have contingent liquidity that FASB and GASB define as "benefit responsive."[32] From the point of view of participants, SV has the same liquidity as money market funds. Banks, insurers or other financial institutions that issue the investment contracts take on the transfer/liquidity risk as well as the investment/market risk that everyone will not withdraw at the same time, since the underlying portfolio has to be less than contract value to cause the investment contract to make up the difference between market and contract value.

"Stable value providers mitigate their risk largely by being astute at predicting and underwriting participant behavior. Providers know that people tend to use less liquidity than they think they need. They know that the vast majority of participants, because of tax penalties, equity wash requirements that inhibit interest rate arbitrage, and simple investor inertia, tend to leave their money in stable value options for a fairly long time. Providers are willing, therefore, to guarantee a fixed rate for a traditional GIC or a minimum zero percent crediting rate each quarter for a synthetic GIC. Defined contribution participants want mostly princi-

[31] This section benefitted from our discussions with industry experts listed in the initial footnote.

[32] "Benefit responsiveness" means that plan participants transact at book value (principal plus accrued interest), and is provided through one or more different types of investment contracts. Benefit responsiveness criteria are established by the Financial Accounting Standards Board (FASB) in FSP AAG INV-a and all five requirements must be met. The requirements are: (1) the investment contract is directly between the fund and the issuer and prohibits the fund from assigning or selling the contract or its proceeds to another party without the consent of the issuer; (2) repayment of principal and interest credited to participants in the fund is a financial obligation of the issuer or the contract is a synthetic GIC guaranteed by a financially responsible third party (if realization of full contract value for a particular investment contract is not or no longer probable, the investment contract is not considered fully benefit-responsive); (3) the terms of the investment contract require all permitted participant-initiated transactions, such as withdrawals for benefits, loans, or transfers to other funds within the plan, with the fund to occur at contract value with no conditions, limits, or restrictions; (4) it is not probable that an event will occur that limits the ability of the fund to transact at contract value with the issuer; and (5) the fund has to allow participants reasonable access to their funds.

pal stability and do not need daily liquidity for all balances." This allows the wrap providers to offer their guaranties at a lower price than otherwise would be the case. Any disintermediation that has occurred prior to the recent financial crisis has been occasioned by a run-up in equity prices, and not associated with a rising interest rate environment. This means that the wrap providers historically have not suffered massive withdrawals when costly interest rate arbitrage typically occurs. Because defined contribution participants tend to care more about principal stability than daily liquidity for all balances, this allows fund managers to take advantage of the steepness of the yield curve and seek credit and liquidity spreads. By contrast, money market funds must provide both principal stability and a lot more liquidity, which participants pay for in the form of a lower return.

To be able to maintain the attractiveness of their funds, SV fund managers will need to continue facing yield curves that generally have positive steepness. High quality assets that offer attractive yields due to lower liquidity, negative convexity (which does little harm to stable yields), and adequate credit spreads must continue to be available at suitable prices. Furthermore, for SV funds to seek such yields will require that they be offered only through vehicles such as defined contribution programs which cater to investors willing to be patient, while eschewing day traders and interest arbitragers. Stable value funds would probably not survive outside of that kind of protected environment. (We note that hedge funds often provide a different kind of "stickiness" with respect to exit and cash-out provisions.) In the past stable value has incorporated relatively low management fees into its funds (averaging 60% of the fees charged by the average bond fund), which have included a low wrap cost of around 4-8 b.p. Recently that wrap fee reached levels as high as 12-20 b.p., creating upward pressure on total fees subtracted from the returns offered to investors. However, as additional wrap providers entered and reentered the market, these fees have fallen. In any case, aggregate fees have remained low and this has contributed to SV's continuing good performance.

Since their inception in 1973, SV funds have undergone several severe tests. They survived the market fallout from the OPEC cartels, the severe stock market declines of 1973-4, 2000-2002, the stock market crash of 1987, the interest rate spikes of the early 80's, the liquidity and credit spread crisis of the late 90s, the plummeting interest rates of the 21st century, economic booms and deep recessions, the terrorist challenges to financial markets in 2001, and countless other daunting circumstances. With very few exceptions,[33] they appear to have weathered the recent financial crisis of 2007-8, even while the number of wrap providers dwindled briefly from 25 to less than a dozen, precipitating steep increases in wrap fees among those remaining. By 2010 the number of wrap providers has increased again, includ-

[33] The most notable exception of which we are aware was linked to the sudden Lehman Brothers bankruptcy. This resulted in a small monthly decline in fund value of 1.7% in December, albeit an overall positive return over the year of 2%. In another instance, a fund manager (State Street) infused an additional $610 million into its SV fund to offset some investment losses that it felt should not accrue to plan participants. One supplemental savings fund associated with Chrysler was not subject to the Employee Retirement Income Security Act (ERISA), and the protective umbrella from creditors that it affords. Therefore, Chrysler liquidated the fund and realized the loss for participants in advance of any potential claim by other creditors. There have been other funds that experienced severe stress, yet they weathered the worst part of the storm and have managed their market-to-book asset ratios back into the traditional ranges. Of course, there is always the peril that just because an investment fund has "stable value" in its name or otherwise masquerades as such, it may not conform to the essential indicia of a stable value fund. For example, http://www.sec.gov/litigation/complaints/2009/comp21010.pdf

ing some new entrants. At the end of 2009, their average market value of assets was 101% of the book value of assets, although the ratio had dipped to as low as 95% during the height of the crisis. As of June 30, 2010, SVIA members reported average market value ratios of 102.94%, and SV average yields were in excess of 3% per annum, while intermediate-term government bond yields were at less than half of that level, at 1.4%, 6-month bank CD's averaged 0.3% and money market rates continued to hover near zero.

Five factors have fostered the sort of disciplined investment behavior that has helped SV funds to ride out the recent crisis, as well as previous challenging market conditions over the past 36 years.

1) FASB's accounting standard, which grants contract value accounting protocol to SV funds rather than mark-to-market accounting, requires the overall portfolio to be of overall very high credit quality. If it is not, a fund cannot use contract value accounting.
2) SV fund investment guidelines set the overall direction of the portfolio and usually have stringent quality standards, limits in allocations to specific asset classes, and duration limits. Since SV is a conservative investment, the guidelines reflect this. The key principles of the Stable Value Investment Association reinforce the plan rules.
3) Wrap providers require specific asset credit quality standards in order to obtain their wrap. They also have set a limit that is generally between one and five percent for a credit bucket in which any 'troubled' assets are put. The SV fund manager usually has a set time period to rehabilitate the assets in the credit bucket. If the manager violates the terms of the wrap contract, the wrap is potentially void.
4) In the event that a wrap provider exits the business, other wrap providers have historically divided up the vacant share. For example, a typical structure had six wrap providers each underwriting 16.5% of the coverage. If one provider exited the business, the other five had contractually agreed to increase their coverage such that each provider would cover 20%. Today this arrangement is disappearing and the step-up provision is typically for only 90 days. If an existing wrap provider exits the business, the exit could transpire over (typically) a 3 ½ year time frame, and if another provider is not found, the funds associated with that segment would go into cash, lowering the overall fund yields but maintaining their character as SV assets.
5) The Stable Value Investment Association's membership reports no use of leverage through mid-2010. None of their annual surveys have reported the use of any leverage. Even if the plan investment guidelines permitted leverage, it is doubtful that a wrap provider would ever allow it and to date none have undertaken it. In light of the recent financial crisis, which is precipitated and exacerbated, in part, by high leverage, SV funds are in a relatively advantageous position.

As long as these stringent standards are adhered to by fund managers, and individual plans are well designed, we do not anticipate a systemic failure to occur.

Stable value (SV) funds have been in the news in connection with recent financial market reform initiatives. There was concern over language in the original bills, which defined a swap/derivative so broadly that it could have swept in some stable value wraps and classified them as swaps. These investment contracts are critical to certain SV funds, yet their treatment as swaps would impose various roadblocks to their future usage. The finalized Act directed that within 15 months of enactment, the Commodity Futures Trading Commission and the Securities Exchange Commission, in consultation with the Department of Labor, Treasury, and applicable state entities conduct a study to determine whether stable value wraps fall within the definition of swaps. If they are deemed swaps, then it will be determined whether an exemption for stable value funds is warranted. Providers of SV investment contracts have indicated that if the wraps are classified as swaps, and an exemption from the law is not granted, they will have sufficient difficulty imposed upon them that some wraps and certain forms of funds may be discontinued, although currently existing funds will be grandfathered at the end of the carve-out period. The stakes are high, as SV funds have been among the most popular of all defined contribution plan asset classes.

As President Barack Obama signed the "Dodd-Frank Wall Street Reform and Consumer Protection Act" (H.R. 4173) into law on July 21, 2010, we would expect the indicated study on the classification of SV funds to occur late in 2011 and the associated regulatory decision to be taken shortly thereafter.

9. Concluding Remarks

In this paper we use mean-variance (including Sharpe and Sortino ratios), stochastic dominance and intertemporal optimization analyses to explore the performance of SV funds vis-à-vis U.S. large and small cap stocks, long-term government and corporate bonds, intermediate-term government/credit bonds and money market funds, during the period February 1973 through December 2009. Despite the different focus of the three methodologies used, the results we obtain under each analysis reinforce each other in the sense that SV funds outperform some of the alternative investments we consider in one or more dimensions.

In the mean-variance sense, including SV funds in efficient portfolios considerably increases expected net return, and SV even predominates over long-term bonds, for levels of risk in the lower two-thirds of the observed monthly return volatility range. In general, if the historical returns and volatility can serve as proxies for future expectations, efficient portfolios would not include large stocks, long-term corporate bonds, intermediate-term government/credit bonds or money market investments. Rather, efficient portfolios would be mostly comprised of long-term government bonds, small stocks, and SV in proportions that depend on risk tolerance.

Stochastic dominance (SD) analysis provides preference orderings among competing investment alternatives for all investors within a certain class of utility functions. The strength of SD analysis resides in the minimal requirements imposed on preferences but at the same time this may result in an inability to rank alternative investments that could be easily ranked according to more restrictive approaches such as mean-variance. It is therefore quite remarkable to have found that SV investments stochastically dominate money market funds by the first and higher order degrees. They also dominate intermediate-term government/credit funds by the second-order and higher degrees, including dominance for the important class of investors characterized by DARA preferences. No other asset class was found to dominate SV funds.[34]

Intertemporal optimization methods allow us to use the full joint empirical return distribution of alternative investments in order to determine optimal wealth allocations that depend on the risk aversion parameter of the investor. This analysis concludes that, for moderately and highly risk-averse investors, SV funds are, under reasonable yield curve assumptions, a major component of an optimal portfolio, to the exclusion of money market funds and the near exclusion of intermediate-term bonds.

[34] In an earlier version of this study, http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1465755, we found that long-term government bonds stochastically dominated, in the second and higher degrees, large stocks over the period January, 1989 – December, 2008. This very interesting result was largely due to poor stock performance during the crash of 2007 – 2008. Relatively high large stock returns during 2009 have made this dominance disappear for the period 1989 – 2009.

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Appendix A. An Alternative Stochastic Dominance Algorithm

In this appendix we present a recursive algorithm that can be easily applied to test stochastic dominance of an arbitrary degree (up to fourth degree in most practical applications).

The theory and practice of stochastic dominance (SD) is discussed in great detail in Levy (2006). In Chapter 5, Levy describes algorithms used to test for first, second, and third degree stochastic dominance (FSD, SSD and TSD, respectively). The inputs for these algorithms are the sorted realized rates of return, for the period of interest, of the assets for which SD is to be tested. For a given asset, the sorted rates of return can be used to construct an empirical cumulative distribution function (CDF), assuming that each return has a probability $1 \div n$ of occurring, where n is the number of observations and where repeated returns have probability $h \div n$, and h is the number of repeated values.

Levy (2006) presents separate algorithms for FSD, SSD and TSD. The algorithms for FSD and SSD are based on pairwise comparison of sorted returns and cumulative sorted returns, respectively, for the two asset classes being checked.

For FSD, this approach amounts to computing the difference of the empirical CDFs of the two asset classes for each point $i \div n$, with $i = 1, 2, \ldots, n$. For SSD, the approach amounts to computing the integral of the CDFs difference, again for each point $i \div n$, with $i = 1, 2, \ldots, n$. Note that the difference calculation for FSD and the integral of these differences for SSD are in effect calculated over the y-axis. Because of the linear functions involved, this approach yields the same test results as if the calculations had been done over the horizontal axis.

In the case of third or higher degree SD, however, this equivalence no longer holds, since the integrals now become non-linear functions of the values in the respective axis and, furthermore, using the same approach as in the FSD or SSD algorithms may lead to wrong conclusions. For this reason, Levy (2006, Section 5.4) presents algorithms for FSD, SSD and TSD based on integrals of the CDFs of the two asset classes. However, since the set of realized returns is different for each investment, these algorithms first have to be applied separately to the individual CDF of each investment and then combined in order to compare the resulting integrals at each point of interest.

We use an alternative approach that allows the derivation of a simple, unified and recursive algorithm, directly applied to the difference between the CDFs of the two asset classes being compared and having the same structure, independent of the degree of stochastic dominance for which one wants to test.

Our algorithm can be derived by (1) considering the union of realized returns for both asset classes over the period of interest as the set of all possible values under the distribution of a given asset class and (2) assigning zero probability to those return values that are not realizations of that asset class. For realized values, the empirical probability of a return value is taken to be the relative frequency with which that value is observed, i.e. $h \div n$ for values that are repeated *h* times in a sample of *n* observations. We let *m* denote the number of unique realized returns across both asset classes.[35]

Table A-1 illustrates how hypothetical realized return data on two investments, *F* and *G*, observed over 20 periods are used to construct the set of unique realized returns and their empirical probability density functions, *f*(*r*) and *g*(*r*).

The first panel of Table A-1 shows the sorted hypothetical returns for the two investments. The second panel shows their empirical probability density functions (pdf). For instance, a −4% return has frequency 0.20 for the first asset and 0 for the second.

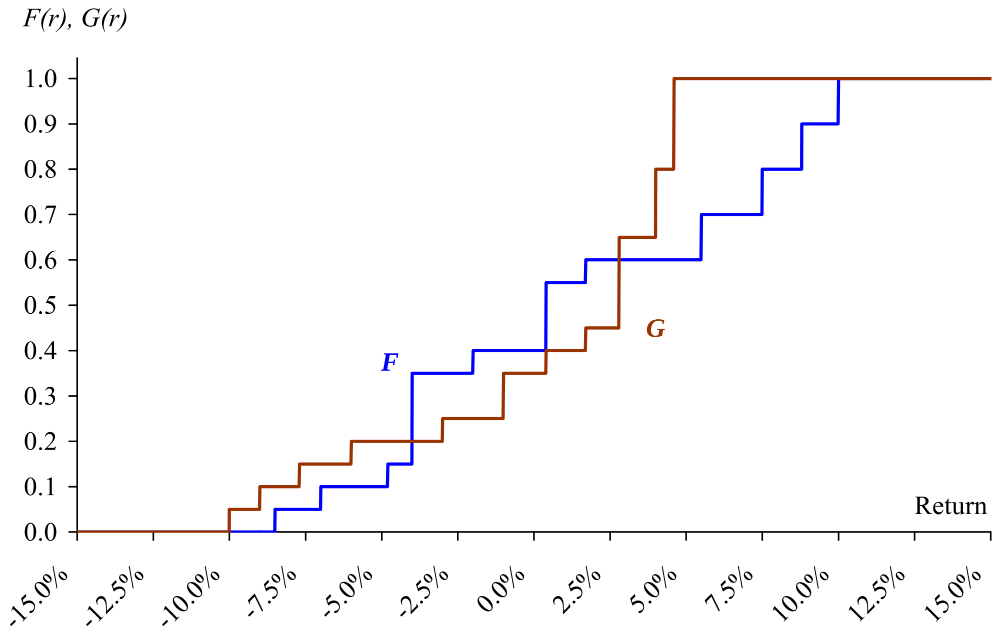
Figure A-1 plots the empirical cumulative distribution functions (CDF) of the two investments.

Table A-1. Distribution of Returns for Two Asset Classes

	Panel 1		Panel 2		
	Sorted Returns		Empirical pdf of Union of Returns		
Obs.	***F***	***G***	Return	***f(r)***	***g(r)***
1	-8.5%	-10.0%	-10.0%	0	0.05
2	-7.0%	-9.0%	-9.0%	0	0.05
3	-4.8%	-7.7%	-8.5%	0.05	0
4	-4.0%	-6.0%	-7.7%	0	0.05
5	-4.0%	-3.0%	-7.0%	0.05	0
6	-4.0%	-1.0%	-6.0%	0	0.05
7	-4.0%	-1.0%	-4.8%	0.05	0
8	-2.0%	0.4%	-4.0%	0.20	0
9	0.4%	1.7%	-3.0%	0	0.05
10	0.4%	2.8%	-2.0%	0.05	0
11	0.4%	2.8%	-1.0%	0	0.10
12	1.7%	2.8%	0.4%	0.15	0.05
13	5.5%	2.8%	1.7%	0.05	0.05
14	5.5%	4.0%	2.8%	0	0.20
15	7.5%	4.0%	4.0%	0	0.15
16	7.5%	4.0%	4.6%	0	0.20
17	8.8%	4.6%	5.5%	0.10	0
18	8.8%	4.6%	7.5%	0.10	0
19	10.0%	4.6%	8.8%	0.10	0
20	10.0%	4.6%	10.0%	0.10	0

[35] For a large sample of realized returns, *m* will generally be greater than the sample size *n*, but there is no theoretical reason why this should be the case.

Figure A-1. Empirical CDFs of Two Asset Classes



Visual inspection of Figure A-1 reveals that no return distribution exhibits FSD over the other since the vertical distance $G(r) - F(r)$ changes sign for different values of r. More precisely, define

$$I_1(r) = G(r) - F(r) = G(r_i) - F(r_i) \equiv p_i, \quad r_i \le r < r_{i+1}, \quad i = 1, \cdots, m. \tag{A1}$$

In particular, we want to evaluate $I_1(r)$ at the joint unique realized return values, $r_1, r_2, \cdots, r_m$. The definition for theoretical k^{th}-degree stochastic dominance requires

$$I_k(r) = \int_{-\infty}^{r} I_{k-1}(z)dz \ge 0, \text{ with } I_1 \text{ given by (A1) above, } k = 1, 2, \cdots, \tag{A2a}$$

$$I_j(r_m) \ge 0, j = 2, 3, \cdots, k-1, \tag{A2b}$$

for all r, with at least one strict inequality.

The arrangement of the observed returns for investments F and G illustrated in Table A-1 allows us to calculate the integrals in (A2a) and (A2b) directly from the data. Specifically,

$$I_2(r) = I_2(r_{i-1}) + I_1(r_{i-1})(r - r_{i-1}), \quad \text{for} \quad r_{i-1} \le r < r_i, \quad i = 2, \cdots, m, \quad \text{with} \quad I_2(r_1) = 0. \tag{A3}$$

This expression can be easily evaluated at the observed return values r_1, r_2, $\cdots$, r_m. In general, and corresponding exactly to the theoretical conditions in (A2a) and (A2b), it can be shown that

$$I_k(r) = \sum_{j=0}^{k-1} \frac{1}{j!} I_{k-j}(r_{i-1})(r - r_{i-1})^j, \quad \text{for} \quad r_{i-1} \le r < r_i, \tag{A4}$$

with $I_k(r_1) = 0, \quad i = 2, 3, \cdots, m; \quad k = 2, 3, \cdots.$

Note that the expression given in (A4) allows us to recursively calculate the test integrals for any desired degree of stochastic dominance at points on interest, once the CDF difference in (A1), based on the union of unique realized returns for the two asset classes of interest, is obtained. The conditions in (A2a) and (A2b) can then be checked in order to determine stochastic dominance.

A conclusive determination of third- and higher-degree SD, however, cannot be based on checking conditions (A2a) and (A2b) at the observed return points alone. The reason is that the integrals $I_3(r)$, $I_4(r)$, … are non-linear functions of r within a given interval $[r_{i-1}, r_i)$. We next discuss the additional conditions for interior points in the context of our example.

Table A-2 shows the results of the dominance tests (up to fourth degree) applied to the hypothetical return data of Table A-1.

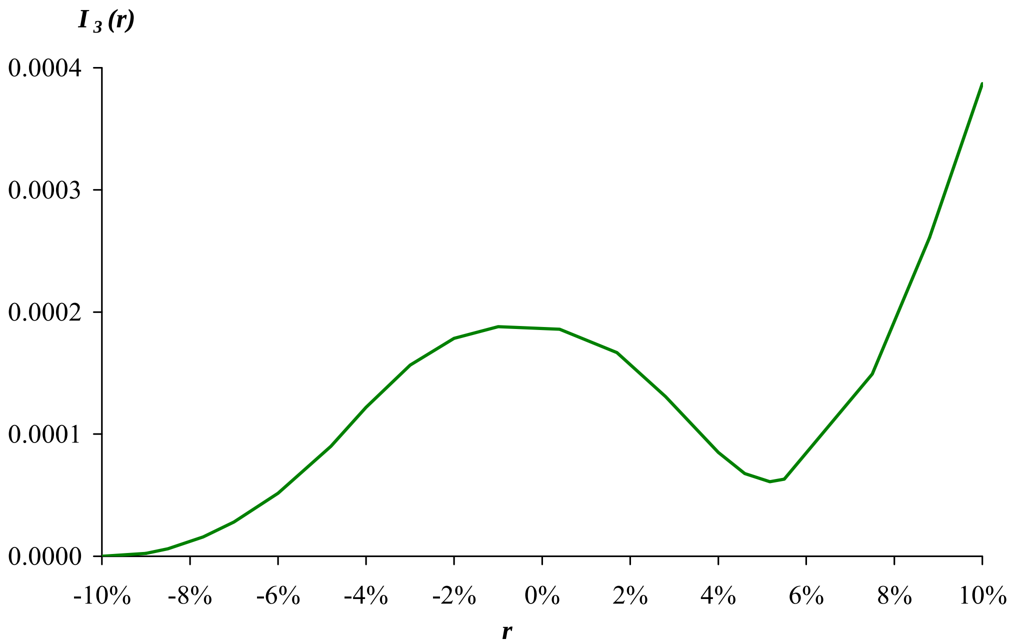
Table A-2. Stochastic Dominance Tests

	pdf		***CDF***		***FSD***	***SSD***	***TSD***	***4SD***
Return	***f(r)***	***g(r)***	***F(r)***	***G(r)***	I_1	I_2	I_3	I_4
-10.0%		0.05	0.00	0.05	0.050	0.0000	0.00000	0.000000
-9.0%		0.05	0.00	0.10	0.100	0.0005	0.00000	0.000000
-8.5%	0.05		0.05	0.10	0.050	0.0010	0.00001	0.000000
-7.7%		0.05	0.05	0.15	0.100	0.0014	0.00002	0.000000
-7.0%	0.05		0.10	0.15	0.050	0.0021	0.00003	0.000000
-6.0%		0.05	0.10	0.20	0.100	0.0026	0.00005	0.000001
-4.8%	0.05		0.15	0.20	0.050	0.0038	0.00009	0.000001
-4.0%	0.20		0.35	0.20	-0.150	0.0042	0.00012	0.000002
-3.0%		0.05	0.35	0.25	-0.100	0.0027	0.00016	0.000004
-2.0%	0.05		0.40	0.25	-0.150	0.0017	0.00018	0.000005
-1.0%		0.10	0.40	0.35	-0.050	0.0002	0.00019	0.000007
0.4%	0.15	0.05	0.55	0.40	-0.150	-0.0005	0.00019	0.000010
1.7%	0.05	0.05	0.60	0.45	-0.150	-0.0025	0.00017	0.000012
2.8%		0.20	0.60	0.65	0.050	-0.0041	0.00013	0.000014
4.0%		0.15	0.60	0.80	0.200	-0.0035	0.00009	0.000015
4.6%		0.20	0.60	1.00	0.400	-0.0023	0.00007	0.000016
5.5%	0.10		0.70	1.00	0.300	0.0013	0.00006	0.000016
7.5%	0.10		0.80	1.00	0.200	0.0073	0.00015	0.000018
8.8%	0.10		0.90	1.00	0.100	0.0099	0.00026	0.000021
10.0%	0.10		1.00	1.00	0.000	0.0111	0.00039	0.000025

Table A-2 shows that $I_1(r)$ (defined here as $G - F$) changes sign and so no distribution stochastically dominates the other in the first-degree sense. Similarly, $I_2(r)$ alternates sign, which means that neither F SSD G nor G SSD F.

By contrast, we see that $I_3(r)$ is positive for all relevant points and $I_2(10.0\%) > 0$. This allows us to conclude that *F* TSD *G*. Figure A-2 plots the function $I_3(r)$ for the relevant range of observed returns in the example of Table A-1.

Figure A-2. Third Degree Stochastic Dominance Test



As mentioned above, since the $I_3(r)$ function is non-linear between any two contiguous observed return points, one should also check interior points other than the observed returns in Table A-2. As Levy (2006) points out, this needs only be checked at interior points between two observed return points for which $I_3(r)$ turns from a decreasing function to an increasing function or, equivalently, when $I_2(r)$, which is the first derivative of $I_3(r)$, changes from negative to positive (between 4.6% and 5.5%, as shown in Table A-2).

With no loss of generality, let the two contiguous observed return points for which $I_3(r)$ turns from a decreasing function to an increasing function be r_{i-1} and r_i and note that the function $I_3(r)$ is a second degree polynomial over $[r_{i-1}, r_i)$:

$$y = \alpha + \beta(r - r_{i-1}) + \chi(r - r_{i-1})^2 \qquad \text{(A5)}$$

where $\alpha = I_3(r_{i-1})$, $\beta = I_2(r_{i-1})$, and $\chi = \frac{1}{2} I_1(r_{i-1})$ are known constants The minimum value of expression (A5) is attained at

$$r_{\min} = r_{i-1} - \frac{\beta}{2\chi} = r_{i-1} - \frac{I_2(r_{i-1})}{I_1(r_{i-1})}. \qquad \text{(A6)}$$

For our example, this value is $r_{min} = 5.175\%$ with $I_3(r_{min}) = 0.000061$. The positive value of $I_3(r_{min})$, together with the information from Table A-2, leads to the conclusion that *F* TSD *G*. From this result, it follows as a corollary that *F* also dominates *G* stochastically for any degree higher than third.[36]

[36] Levy (2006) has a discussion on how existing TSD algorithms fail to check for interior points, but errors crept in apparently during the editorial process such that his example in Section 5.5, pp. 189 and 190, appears to be flawed. First, there seems to be missing data for an additional period in Levy's table on p. 189. This missing period needs to have observed returns of 10% for distribution *F* and 5% for distribution *G* in order to be able to exactly match the figures reported by Levy on p. 189. But even when this is taken care of, and Levy's calculations are reproduced, it is the case that the interior point he correctly identifies, at a return of 25%, gives a theoretical minimum value for $I_3(25\%)$ of exactly zero. What Levy considers a negative minimum value of $-3.3\text{x}10^{-7}$ (incorrectly reported as positive $3.3\text{x}10^{-7}$ in Levy, p. 189) is simply due to rounding error and so his conclusion that "… *F* does not dominate *G* by the TSD …" is not valid. Indeed, for Levy's example, straightforward application of the compact formulae and interior points check developed in this paper would conclude that *F* dominates *G* by the TSD.

Appendix B: Average Stable Value Return Data

Month	Stable Value Average Return	Lehman Intermediate Gov't/Credit Wrapped return	Month	Stable Value Average Return	Lehman Intermediate Gov't/Credit Wrapped return
Jan-73			Sep-76		0.587228%
Feb-73		0.476873%	Oct-76		0.607204%
Mar-73		0.532366%	Nov-76		0.585282%
Apr-73		0.520602%	Dec-76		0.602456%
May-73		0.535653%	Jan-77		0.604089%
Jun-73		0.520858%	Feb-77		0.547520%
Jul-73		0.540396%	Mar-77		0.605571%
Aug-73		0.547295%	Apr-77		0.586718%
Sep-73		0.531252%	May-77		0.606120%
Oct-73		0.567163%	Jun-77		0.590393%
Nov-73		0.541847%	Jul-77		0.611673%
Dec-73		0.544935%	Aug-77		0.615583%
Jan-74		0.545416%	Sep-77		0.597374%
Feb-74		0.492252%	Oct-77		0.619964%
Mar-74		0.550047%	Nov-77		0.600700%
Apr-74		0.536322%	Dec-77		0.620142%
May-74		0.555730%	Jan-78		0.619475%
Jun-74		0.534842%	Feb-78		0.558943%
Jul-74		0.553138%	Mar-78		0.622245%
Aug-74		0.556077%	Apr-78		0.603850%
Sep-74		0.542132%	May-78		0.627543%
Oct-74		0.560393%	Jun-78		0.607610%
Nov-74		0.547704%	Jul-78		0.633252%
Dec-74		0.568963%	Aug-78		0.635698%
Jan-75		0.572286%	Sep-78		0.619403%
Feb-75		0.514434%	Oct-78		0.644699%
Mar-75		0.572266%	Nov-78		0.628413%
Apr-75		0.556498%	Dec-78		0.657583%
May-75		0.582873%	Jan-79		0.660768%
Jun-75		0.553244%	Feb-79		0.596608%
Jul-75		0.582859%	Mar-79		0.662341%
Aug-75		0.589965%	Apr-79		0.641325%
Sep-75		0.572216%	May-79		0.665216%
Oct-75		0.598113%	Jun-79		0.645807%
Nov-75		0.571699%	Jul-79		0.666375%
Dec-75		0.595119%	Aug-79		0.670995%
Jan-76		0.594530%	Sep-79		0.655738%
Feb-76		0.550772%	Oct-79		0.682432%
Mar-76		0.599371%	Nov-79		0.661676%
Apr-76		0.580603%	Dec-79		0.682912%
May-76		0.600712%	Jan-80		0.689376%
Jun-76		0.587276%	Feb-80		0.638020%
Jul-76		0.608672%	Mar-80		0.698262%
Aug-76		0.607409%	Apr-80		0.694225%

Appendix B: Average Stable Value Return Data

Month	Stable Value Average Return	Lehman Intermediate Gov't/Credit Wrapped return	Month	Stable Value Average Return	Lehman Intermediate Gov't/Credit Wrapped return
May-80		0.699675%	Jan-84		0.928960%
Jun-80		0.670174%	Feb-84		0.869910%
Jul-80		0.702349%	Mar-84		0.931516%
Aug-80		0.703168%	Apr-84		0.904589%
Sep-80		0.693170%	May-84		0.935604%
Oct-80		0.730102%	Jun-84		0.907200%
Nov-80		0.712558%	Jul-84		0.947171%
Dec-80		0.755742%	Aug-84		0.961176%
Jan-81		0.760647%	Sep-84		0.934894%
Feb-81		0.693914%	Oct-84		0.969278%
Mar-81		0.766001%	Nov-84		0.937828%
Apr-81		0.749855%	Dec-84		0.963678%
May-81		0.781967%	Jan-85		0.959816%
Jun-81		0.784064%	Feb-85		0.868398%
Jul-81		0.814333%	Mar-85		0.960690%
Aug-81		0.824476%	Apr-85		0.930249%
Sep-81		0.803677%	May-85		0.958899%
Oct-81		0.835528%	Jun-85		0.930067%
Nov-81		0.815020%	Jul-85		0.953216%
Dec-81		0.856702%	Aug-85		0.951312%
Jan-82		0.872496%	Sep-85		0.918202%
Feb-82		0.797215%	Oct-85		0.947212%
Mar-82		0.895838%	Nov-85		0.916210%
Apr-82		0.879889%	Dec-85		0.942873%
May-82		0.919710%	Jan-86		0.942280%
Jun-82		0.888273%	Feb-86		0.845340%
Jul-82		0.932645%	Mar-86		0.938250%
Aug-82		0.930370%	Apr-86		0.896636%
Sep-82		0.905025%	May-86		0.915793%
Oct-82		0.941025%	Jun-86		0.876157%
Nov-82		0.908366%	Jul-86		0.897944%
Dec-82		0.938150%	Aug-86		0.886649%
Jan-83		0.934918%	Sep-86		0.845016%
Feb-83		0.839803%	Oct-86		0.864422%
Mar-83		0.932777%	Nov-86		0.833344%
Apr-83		0.905671%	Dec-86		0.852197%
May-83		0.931476%	Jan-87		0.848532%
Jun-83		0.902330%	Feb-87		0.759056%
Jul-83		0.933494%	Mar-87		0.835885%
Aug-83		0.931343%	Apr-87		0.801289%
Sep-83		0.901141%	May-87		0.823605%
Oct-83		0.931737%	Jun-87		0.792147%
Nov-83		0.898305%	Jul-87		0.813850%
Dec-83		0.931283%	Aug-87		0.809516%

Appendix B: Average Stable Value Return Data

Month	Stable Value Average Return	Lehman Intermediate Gov't/Credit Wrapped return	Month	Stable Value Average Return	Lehman Intermediate Gov't/Credit Wrapped return
Sep-87		0.781124%	May-91	0.709755%	0.743619%
Oct-87		0.804027%	Jun-91	0.686080%	0.715392%
Nov-87		0.775856%	Jul-91	0.701838%	0.737414%
Dec-87		0.800238%	Aug-91	0.699539%	0.736270%
Jan-88		0.800212%	Sep-91	0.678267%	0.708837%
Feb-88		0.747408%	Oct-91	0.686607%	0.733613%
Mar-88		0.797150%	Nov-91	0.662595%	0.701714%
Apr-88		0.766211%	Dec-91	0.684831%	0.716947%
May-88		0.790575%	Jan-92	0.655344%	0.710609%
Jun-88		0.763380%	Feb-92	0.611027%	0.657770%
Jul-88		0.791503%	Mar-92	0.653681%	0.701018%
Aug-88		0.790381%	Apr-92	0.635367%	0.676936%
Sep-88	0.769651%	0.769053%	May-92	0.656478%	0.693201%
Oct-88	0.804741%	0.796728%	Jun-92	0.632012%	0.668966%
Nov-88	0.776769%	0.769545%	Jul-92	0.635361%	0.688890%
Dec-88	0.795738%	0.795144%	Aug-92	0.636738%	0.685180%
Jan-89	0.788179%	0.799345%	Sep-92	0.614607%	0.655225%
Feb-89	0.715121%	0.723608%	Oct-92	0.620533%	0.672093%
Mar-89	0.791954%	0.801100%	Nov-92	0.600696%	0.647569%
Apr-89	0.775100%	0.778665%	Dec-92	0.619491%	0.666757%
May-89	0.798607%	0.803733%	Jan-93	0.594040%	0.667210%
Jun-89	0.770608%	0.778039%	Feb-93	0.538639%	0.600146%
Jul-89	0.783897%	0.802874%	Mar-93	0.593701%	0.661488%
Aug-89	0.769876%	0.799275%	Apr-93	0.561676%	0.636471%
Sep-89	0.744265%	0.769729%	May-93	0.578067%	0.653185%
Oct-89	0.767897%	0.793219%	Jun-93	0.558972%	0.628908%
Nov-89	0.739290%	0.764517%	Jul-93	0.563053%	0.650165%
Dec-89	0.759240%	0.784203%	Aug-93	0.561312%	0.647009%
Jan-90	0.743047%	0.782279%	Sep-93	0.544257%	0.621285%
Feb-90	0.669335%	0.699685%	Oct-93	0.552008%	0.639366%
Mar-90	0.743649%	0.773000%	Nov-93	0.535450%	0.614972%
Apr-90	0.718367%	0.748588%	Dec-93	0.552268%	0.626385%
May-90	0.743670%	0.769617%	Jan-94	0.535196%	0.624795%
Jun-90	0.716718%	0.745928%	Feb-94	0.486976%	0.561834%
Jul-90	0.737348%	0.771735%	Mar-94	0.538856%	0.617995%
Aug-90	0.735966%	0.768956%	Apr-94	0.520922%	0.591786%
Sep-90	0.711843%	0.735294%	May-94	0.538892%	0.613934%
Oct-90	0.730279%	0.759891%	Jun-94	0.523039%	0.595437%
Nov-90	0.704188%	0.732417%	Jul-94	0.532889%	0.613384%
Dec-90	0.723528%	0.758674%	Aug-94	0.533223%	0.616396%
Jan-91	0.718837%	0.755445%	Sep-94	0.517652%	0.596666%
Feb-91	0.646674%	0.679382%	Oct-94	0.534669%	0.614247%
Mar-91	0.715330%	0.748982%	Nov-94	0.520065%	0.594695%
Apr-91	0.688488%	0.722300%	Dec-94	0.539302%	0.622929%

Appendix B: Average Stable Value Return Data

Month	Stable Value Average Return	Lehman Intermediate Gov't/Credit Wrapped return	Month	Stable Value Average Return	Lehman Intermediate Gov't/Credit Wrapped return
Jan-95	0.546563%	0.628277%	Sep-98	0.507133%	0.546557%
Feb-95	0.498032%	0.566009%	Oct-98	0.517269%	0.564135%
Mar-95	0.548904%	0.629344%	Nov-98	0.498425%	0.537602%
Apr-95	0.527618%	0.609250%	Dec-98	0.515102%	0.554361%
May-95	0.550268%	0.630830%	Jan-99	0.509748%	0.552500%
Jun-95	0.528162%	0.610668%	Feb-99	0.460965%	0.496711%
Jul-95	0.538536%	0.630048%	Mar-99	0.508929%	0.544948%
Aug-95	0.538824%	0.624509%	Apr-99	0.484005%	0.525601%
Sep-95	0.521230%	0.602555%	May-99	0.499492%	0.542810%
Oct-95	0.538904%	0.623132%	Jun-99	0.485003%	0.522125%
Nov-95	0.521917%	0.600992%	Jul-99	0.499810%	0.535507%
Dec-95	0.538687%	0.616475%	Aug-99	0.501594%	0.533796%
Jan-96	0.522641%	0.611875%	Sep-99	0.492017%	0.516232%
Feb-96	0.483688%	0.565209%	Oct-99	0.505092%	0.535852%
Mar-96	0.521322%	0.598964%	Nov-99	0.489591%	0.519595%
Apr-96	0.501492%	0.578068%	Dec-99	0.507980%	0.539284%
May-96	0.520319%	0.594383%	Jan-00	0.507482%	0.537835%
Jun-96	0.504551%	0.573747%	Feb-00	0.472132%	0.503673%
Jul-96	0.519697%	0.594820%	Mar-00	0.510146%	0.541613%
Aug-96	0.520219%	0.593777%	Apr-00	0.498462%	0.530301%
Sep-96	0.504607%	0.572627%	May-00	0.516192%	0.546783%
Oct-96	0.523366%	0.593527%	Jun-00	0.502250%	0.530007%
Nov-96	0.507596%	0.574724%	Jul-00	0.516478%	0.553297%
Dec-96	0.522122%	0.594751%	Aug-00	0.518876%	0.554288%
Jan-97	0.523831%	0.592231%	Sep-00	0.504101%	0.537820%
Feb-97	0.476161%	0.533457%	Oct-00	0.524536%	0.555464%
Mar-97	0.523924%	0.591095%	Nov-00	0.507711%	0.537466%
Apr-97	0.511667%	0.568774%	Dec-00	0.526027%	0.555365%
May-97	0.528591%	0.590180%	Jan-01	0.523350%	0.555913%
Jun-97	0.512519%	0.569972%	Feb-01	0.473556%	0.494207%
Jul-97	0.531515%	0.589724%	Mar-01	0.524601%	0.545819%
Aug-97	0.529384%	0.592852%	Apr-01	0.492705%	0.521564%
Sep-97	0.515575%	0.568485%	May-01	0.507732%	0.533179%
Oct-97	0.533361%	0.588827%	Jun-01	0.493289%	0.513249%
Nov-97	0.514197%	0.569048%	Jul-01	0.498631%	0.531785%
Dec-97	0.533357%	0.587784%	Aug-01	0.498958%	0.531087%
Jan-98	0.528631%	0.584556%	Sep-01	0.476119%	0.511626%
Feb-98	0.477590%	0.524787%	Oct-01	0.500865%	0.516109%
Mar-98	0.531793%	0.578403%	Nov-01	0.481673%	0.495939%
Apr-98	0.511342%	0.557562%	Dec-01	0.494484%	0.508123%
May-98	0.528227%	0.574947%	Jan-02	0.469541%	0.502786%
Jun-98	0.512820%	0.554920%	Feb-02	0.422024%	0.455203%
Jul-98	0.523491%	0.572419%	Mar-02	0.466895%	0.503608%
Aug-98	0.523853%	0.570549%	Apr-02	0.448308%	0.489212%

Appendix B: Average Stable Value Return Data

Month	Stable Value Average Return	Lehman Intermediate Gov't/Credit Wrapped return	Month	Stable Value Average Return	Lehman Intermediate Gov't/Credit Wrapped return
May-02	0.464094%	0.500062%	Jan-06	0.370371%	0.433592%
Jun-02	0.445143%	0.478730%	Feb-06	0.335824%	0.388691%
Jul-02	0.452020%	0.488833%	Mar-06	0.374618%	0.430926%
Aug-02	0.450031%	0.473877%	Apr-06	0.365431%	0.412282%
Sep-02	0.428207%	0.461185%	May-06	0.378981%	0.428993%
Oct-02	0.427316%	0.474012%	Jun-06	0.369846%	0.412204%
Nov-02	0.409289%	0.449792%	Jul-06	0.382862%	0.425892%
Dec-02	0.419417%	0.462904%	Aug-06	0.386266%	0.434106%
Jan-03	0.402327%	0.461204%	Sep-06	0.375652%	0.419894%
Feb-03	0.360587%	0.413510%	Oct-06	0.389207%	0.435504%
Mar-03	0.396274%	0.456425%	Nov-06	0.380389%	0.421100%
Apr-03	0.381171%	0.436410%	Dec-06	0.397295%	0.435073%
May-03	0.390806%	0.451808%	Jan-07	0.390722%	0.432395%
Jun-03	0.376325%	0.436740%	Feb-07	0.355538%	0.389474%
Jul-03	0.373253%	0.444972%	Mar-07	0.396348%	0.434612%
Aug-03	0.369509%	0.433032%	Apr-07	0.383573%	0.418638%
Sep-03	0.353931%	0.419354%	May-07	0.398854%	0.432751%
Oct-03	0.363775%	0.432732%	Jun-07	0.386131%	0.417193%
Nov-03	0.351753%	0.417707%	Jul-07	0.397457%	0.431665%
Dec-03	0.366312%	0.432535%	Aug-07	0.403885%	0.432560%
Jan-04	0.361412%	0.428422%	Sep-07	0.382532%	0.417361%
Feb-04	0.334667%	0.400975%	Oct-07	0.401909%	0.432019%
Mar-04	0.361942%	0.426905%	Nov-07	0.390080%	0.418925%
Apr-04	0.348828%	0.412278%	Dec-07	0.399324%	0.432583%
May-04	0.356152%	0.423777%	Jan-08	0.406287%	0.430785%
Jun-04	0.344029%	0.410078%	Feb-08	0.378175%	0.400857%
Jul-04	0.354412%	0.425892%	Mar-08	0.400156%	
Aug-04	0.352582%	0.426940%	Apr-08	0.372115%	
Sep-04	0.341845%	0.414085%	May-08	0.383518%	
Oct-04	0.354091%	0.430749%	Jun-08	0.369662%	
Nov-04	0.343386%	0.417120%	Jul-08	0.370473%	
Dec-04	0.357151%	0.431714%	Aug-08	0.363500%	
Jan-05	0.356057%	0.435219%	Sep-08	0.351483%	
Feb-05	0.323502%	0.414544%	Oct-08	0.336656%	
Mar-05	0.358910%	0.458680%	Nov-08	0.303537%	
Apr-05	0.350696%	0.439539%	Dec-08	0.293346%	
May-05	0.361519%	0.449475%	Jan-09	0.261809%	
Jun-05	0.352008%	0.443744%	Feb-09	0.236442%	
Jul-05	0.365916%	0.446187%	Mar-09	0.261809%	
Aug-05	0.370136%	0.440450%	Apr-09	0.241456%	
Sep-05	0.358234%	0.430575%	May-09	0.249516%	
Oct-05	0.368584%	0.437039%	Jun-09	0.241456%	
Nov-05	0.358640%	0.420587%	Jul-09	0.249136%	
Dec-05	0.373290%	0.436035%	Aug-09	0.249136%	

Appendix B: Average Stable Value Return Data

Month	Stable Value Average Return	Lehman Intermediate Gov't/Credit Wrapped return	Month	Stable Value Average Return	Lehman Intermediate Gov't/Credit Wrapped return
Sep-09	0.241088%				
Oct-09	0.254833%				
Nov-09	0.246601%				
Dec-09	0.254833%				