Decomposing Long Bond Returns

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Derivatives and Volatility
April 27-28, 2017

Objective dictates modeling framework

Different modeling frameworks are built for different purposes:

- Expectation hypothesis (EH): *Predict short rate move* with yield curve slope.
  - Yield curve shape combines information from expectation, risk premium, and convexity, but expectation dominates the short term.
- Dynamic Term Structure Models (DTSM): Value the whole yield curve based on assumptions on the *full* risk-neutral dynamics of the *short rate*.
  - Uses one yardstick (the short rate) to measure everything else for cross-sectional consistency.
  - Deviations from DTSM valuation can be used to construct statistical arbitrage trading on the yield curve.
- HJM-type models: Price interest rate options based on the current forward curve and views of *forward rate volatility*.
  - Highlight the volatility contribution for option valuation while delta-hedge the yield curve exposure.

Our objective: Analyzing returns on long-dated bonds

- EH uses long rates to predict short rate move, not the other away around.
  - How to predict long rate movements based on the yield curve shape, while accounting for risk premium and convexity?
  - More importantly, how to predict excess returns on long bonds?
- Modeling long rates with DTSM stretches the modeler's imagination on how short rate should move in the next 30-60 years...
  - Mean reversion calibrated to time series or short end of the yield curve implies much smaller movements than observed from long rates.
  - Long rates are neither (easily) predictable, nor converging to a constant. — They move randomly, and with *substantial volatility*.
  - Can we say something useful about a 50-year bond *without* making a 50-year projection?

The distinct behaviors of long bonds ask for a distinct modeling approach.

A new modeling approach

We propose a new modeling framework that is particularly suited for analyzing long bond returns:

- Link pricing directly to P&L attribution of bond investments.
    - The attribution makes it clear on what to bet/hedge
- Price each rate based on its *own* behavior, not that of the short rate.
    - Localization allows one to make less ambitious but more confident statements.
    - The model can say/do something useful about a 50-yr bond investment without making a 50-year projection, especially if one just wants to hold the bond for short term (say a year).
- Generate predictions on bond *returns*, even with no prediction on *rates*.
    - via a separation of expectation/risk premium from convexity.

Notation and the classic setting

- Let B_t be the time-t price of a default-free coupon bond (portfolio) with fixed future cash flows $\{C_j\}$ at times $\{t+\tau_j\} \geq t$ for $j = 1, 2, \cdots, N$.
- The classic valuation of this coupon bond can be represented as

$$\begin{aligned} B_t &= \sum_j C_j \mathbb{E}_t^{\mathbb{P}} \left[M_{t,t+\tau_j} \right] = \sum_j C_j \mathbb{E}_t^{\mathbb{P}} \left[\left(\frac{d\mathbb{Q}}{d\mathbb{P}} \right) e^{-\int_t^{t+\tau_j} r_u du} \right] \\ &= \sum_j C_j \mathbb{E}_t^{\mathbb{Q}} \left[e^{-\int_t^T r_u du} \right]. \end{aligned}$$

  - $\mathbb{E}_t[\cdot]$ — expectation under time-t filtration,
  - $M_{t,T}$ — the pricing kernel linking value at time t to value at time T
  - $\mathbb{P}$ — the real world probability measure,
  - $\mathbb{Q}$ — the so-called risk-neutral measure,
  - r_t — instantaneous short rate
  - $\frac{d\mathbb{Q}}{d\mathbb{P}}$ defines the measure change from $\mathbb{P}$ to $\mathbb{Q}$. It is the martingale component of the pricing kernel that defines the pricing of various risks.
- The *yield-to-maturity* of the bond is defined via the following transformation:

$$B_t \equiv \sum_j C_j \exp(-y_t \tau_j).$$

P&L attribution of a bond investment

- Decompose the ex post bond return wrt its own yield movement:

$$dB_t = \frac{\partial B_t}{\partial t}dt + \frac{\partial B_t}{\partial dy}dy + \frac{1}{2}\frac{\partial^2 B_t}{\partial y^2}(dy)^2 + o(dt)$$

  - $o(dt)$ denotes higher-order terms of dt when yield moves diffusively.
  - This decomposition is *local/particular* to the bond and is tied to the movement of the *yield to maturity* on this particular bond.
- The ex ante expected return from the bond investment is

$$\mathbb{E}_t^{\mathbb{P}}\left[\frac{dB_t}{B_t dt}\right] = y_t - \mu_{t,y}\tau + \frac{1}{2}\sigma_{t,y}^2\tau^2$$

  - $\mu_{t,y}$ — the time-t level of the drift/direction of the yield.
  - $\sigma_{t,y}^2$ — the time-t level of its variance rate.
  - τ and τ^2 — value-weighted maturity (duration) and maturity squared:

$$\tau = \sum_j \frac{C_j e^{-y_t\tau_j}}{B_t}\tau_j, \quad \tau^2 = \sum_j \frac{C_j e^{-y_t\tau_j}}{B_t}\tau_j^2.$$

Decomposing expected return on bond investments

$$\mathbb{E}_t^{\mathbb{P}}\left[\frac{dB_t}{B_t dt}\right] \quad = \quad y_t - \mu_{t,y}\tau + \frac{1}{2}\sigma_{t,y}^2\tau^2$$

- Decomposes expected bond return into three sources:
  1. *Carry*: Bonds with a higher yield have higher returns due to carry.
  2. *Prediction*: Expected rate hike reduces expected return.
  3. *Convexity*: Since bond price and yield exhibit a convex relation, random shaking of yield (without direction) leads to a positive return.
     - A duration neutral portfolio that is long longer-term bonds (convexity) is analogous to a delta-neutral long options position.

- Implications
  - If one has no view on direction, form duration-neutral portfolios (to neutral out the second term).
  - Long/short convexity based on view on volatility estimates
  - Adjust carry trades for convexity.

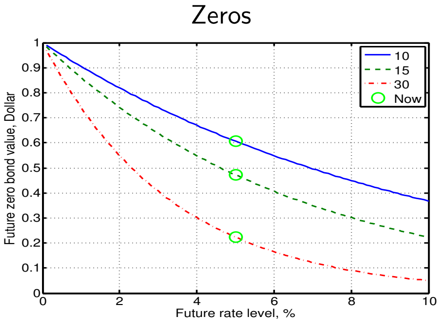
Example: Arbitrage a parallel-shifting flat yield curve

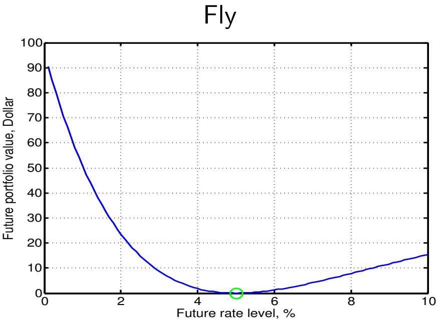
$$\mathbb{E}_t^{\mathbb{P}}\left[\frac{dB_t}{B_t dt}\right] = y_t - \mu_{t,y}\tau + \frac{1}{2}\sigma_{t,y}^2\tau^2$$

- Imagine a situation where
  - zero-coupon yields at long maturities (e.g., at 10, 15, 30 years) are flat and move in parallel: $y_t(\tau) = y_t$.
  - The yields move by substantial amounts, $\sigma_t^2(\tau) = \sigma_t^2 \gg 0$.
- One can form a self-financing, riskless portfolio that makes money:
  - Make the portfolio dollar neutral — Since the yield level is the same. Dollar-neutral leads to zero carry and hence self-financing.
  - Make the portfolio duration neutral — Since the three rates move in parallel driven by the same risk source, duration-neutral cancels out the risk and hence makes the portfolio riskless.
  - Make the portfolio long convexity — positive expected profits.
- Example: Long \$300 10-yr and \$100 30-yr zeros, short \$400 15-yr zero.
  - Dollar neutral: $\$300 + \$100 - \$400 = 0$
  - Duration neutral: $\frac{3}{4}10 + \frac{1}{4}30 - 15 = 0$.
  - Long convexity: $\frac{3}{4}10^2 + \frac{1}{4}30^2 - 15^2 = 300 - 225 = 75$.

The payoff of a fly

$$
\begin{aligned}
dB_t &= \frac{\partial B_t}{\partial t}dt + \frac{\partial B_t}{\partial y}dy + \frac{1}{2}\frac{\partial^2 B_t}{\partial y^2}(dy)^2 + o(dt) \\
&= y_t B_t dt - \tau B_t dy + \frac{1}{2}\tau^2 B_t (dy)^2 + o(dt) \\
dFly_t &= 0 + 0 + \frac{1}{2}\left[\sum_j (w_j \tau_j^2)\right](dy)^2 + o(dt)
\end{aligned}
$$





No dynamic arbitrage pricing on bond investments

- Given the P&L attribution on bond investment,

$$dB_t \quad = \quad \frac{\partial B_t}{\partial t} dt + \frac{\partial B_t}{\partial y} dy + \frac{1}{2} \frac{\partial^2 B_t}{\partial y^2} (dy)^2 + o(dt)$$

- Take expectation under $\mathbb{Q}$, and set the instantaneous expected return to r_t by *no dynamic arbitrage* (NDA):

$$\mathbb{E}_t^{\mathbb{Q}} \left[\frac{dB_t}{B_t dt} \right] = r_t = \frac{\partial B_t}{B_t \partial t} + \frac{\partial B_t}{B_t \partial y} \mu_{t,y}^{\mathbb{Q}} + \frac{1}{2} \frac{\partial^2 B_t}{B_t \partial y^2} \sigma_{t,y}^2$$

- NDA leads to a simple pricing relation for the long bond yield:

$$y_t = r_t + \mu_{t,y}^{\mathbb{Q}} \tau - \frac{1}{2} \sigma_{t,y}^2 \tau^2 .$$

- The fair value of the yield spread $(y_t - r_t)$ on the bond investment is determined by its *current* risk-neutral drift ($\mu_{t,y}^{\mathbb{Q}}$) and volatility ($\sigma_{t,y}$) estimates.

Bond pricing based on *local, near-term dynamics*

$$y_t = r_t + \mu^{\mathbb{Q}}_{t,y}\tau - \frac{1}{2}\sigma^2_{t,y}\tau^2 \quad (1)$$

- *Local*: The fair valuation of the bond investment in (1) does not depend on short-rate dynamics, but only depend on the *behavior of its own yield*.
- *Near-term*: The pricing of the yield does not even depend on its own full dynamics, but only depends on the current level of the drift and volatility.
  - The drift $\mu^{\mathbb{Q}}_{t,y}$ and volatility $\sigma_{t,y}$ can each follow some stochastic process, and/or depend on other rates/economic state variables ...
  - None of these dynamics specifications enter into the pricing relation
- *Views, not (much) dynamics*: One can bring in forecasts/estimates/opinions on volatility, risk premium, & rate prediction, and examine their implications on the yield (curve).
  - The estimates can come from any (other) model assumptions, algorithms, or information sources.

Different frameworks serve different purposes

Classic DTSM

- *Full* short rate dynamics prices bond of *all* maturities.
- Maintain cross-sectional consistency across the whole curve.
- Hard to reconcile long rates with *actual* short rate dynamics.
- Better suited to construct smooth curves with cross-sectional consistency.

New framework

- Each yield is priced according to *its own near-term* predictions.
- Ask for the most relevant predictions for the pricing.
- Hard to maintain cross-sectional consistency across all bonds.
- Better suited to analyze specific bond (portfolios) and connect to (views on) their own, current behaviors.

Pricing the yield curve based on common factors

$$y_t = r_t + \mu^{\mathbb{Q}}_{t,y}\tau - \frac{1}{2}\sigma^2_{t,y}\tau^2 \quad (2)$$

- The stand-alone fair valuation of a particular bond investment depends on views/estimates of its own conditional drift and volatility.
- One can perform joint valuation on the yield curve via a common factor approach:

$$y^i_t = \beta^\top_{i,t} X_t + \omega_{i,t} u_{i,t}, \quad X_t \sim \mathcal{N}(\mu_t - \lambda_t, \Sigma_t)$$

  - X_t can be principal components and/or macroeconomic factors
  - Can be used for both P&L analysis and pricing
  - Analsis/pricing depends only on (empirical) conditional forecasts and views, not on particular dynamics specification.

Decomposing long-bond returns with no rate predictions

- It is difficult to predict long rate movements. So we start by assuming random walk on floating (constant maturity) long rates:

$$dy_t(\tau) = \sigma_t(\tau) dW_t^{\mathbb{P}}.$$

- A risk-neutral drift is induced by market pricing of bond risk ($-dW_t$):

$$dy_t(\tau) = \lambda_t \sigma_t dt + \sigma_t(\tau) dW_t^{\mathbb{Q}}.$$

  - Market price of interest rate tends to be negative, leading to positive market price of bond risk (λ_t) on average.

- The risk-neutral drift of the *fixed-expiry* rates is further adjusted by the local shape of the yield curve ("sliding"):

$$\mu_t^{\mathbb{Q}} = \lambda_t \sigma_t - y_t'(\tau).$$

  - The pricing is based on the yield dynamics of a fixed contract, but it is easier to model/estimate floating rate dynamics (e.g., 30-yr rate).

Decomposing long-bond returns

- Plugging the no-prediction assumption into the pricing relation leads to
$$\frac{\partial\left[y_t\tau\right]}{\partial\tau} = r + \lambda_t\sigma_t(\tau)\tau - \frac{1}{2}\sigma_t^2(\tau)\tau^2.$$
  - For zeros, $\frac{\partial[y_t\tau]}{\partial\tau} = f(\tau)$ is the instantaneous forward rate.
  - Define instantaneous volatility weighted duration and convexity as
$$d_t = \sigma_t(\tau)\tau, \quad c_t = \sigma_t^2(\tau)\tau^2.$$
- Integrate
$$y_t = r + \lambda_t D_t - \frac{1}{2}C_t,$$
with D and C denoting the **integrated duration and convexity**:
$$D_t \equiv \left[\frac{1}{\tau}\int_0^\tau d_t(s)ds\right], \quad C_t \equiv \left[\frac{1}{\tau}\int_0^\tau c_t(s)ds\right]$$
What matters is not just sensitivity (τ), but also volatility.
- In absence of prediction, risk premium drives long rates up, convexity drives long rates down.

Extract bond risk premium from long rates

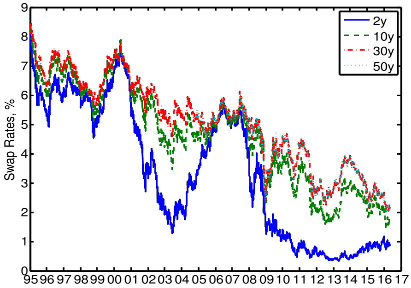
- We can extract bond risk premium from long yield and yield volatility:

$$\lambda_t = \frac{y_t - r_t + \frac{1}{2} C_t}{D_t}$$

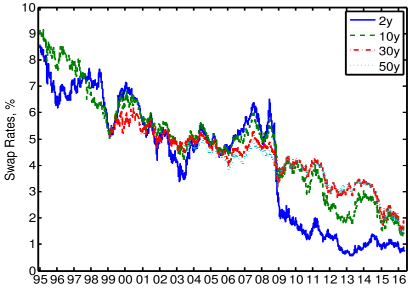
  - Long rates (y_t) and financing cost (r_t) are directly observed.
  - Variance term structure $\sigma_t(\tau)$ can be estimated using recent history (e.g., via GARCH, from options, curve)
- We can then examine whether the ex-ante risk premium predicts ex post bond excess return, without ever the need to fit a predictive regression.

- Data: US and UK swap rates 1995.1.3-2016.5.11, 5378 business days
  - Based on 6-month LIBOR Maturity, 2,3,4,5,7,10,15,20,30
  - Extended maturity since
    US: 2004/11/12 for 40 & 50 years
    UK: 1999/1/19 for 20 & 30 years, 2003/08/08 for 40 & 50 years

US



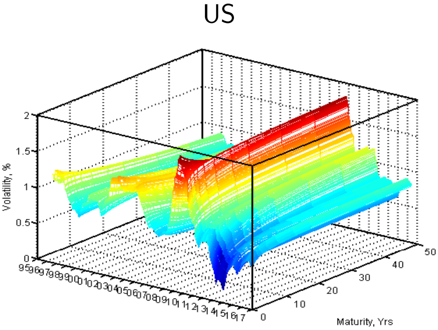
UK

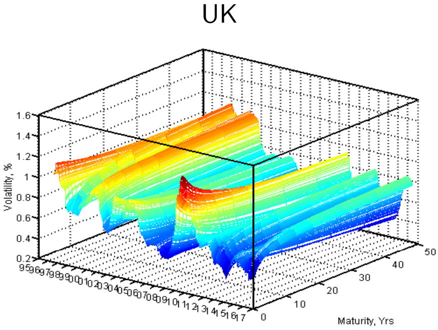


- Stripped Treasury zero rates for robustness check

Swap rate variance term structure estimators

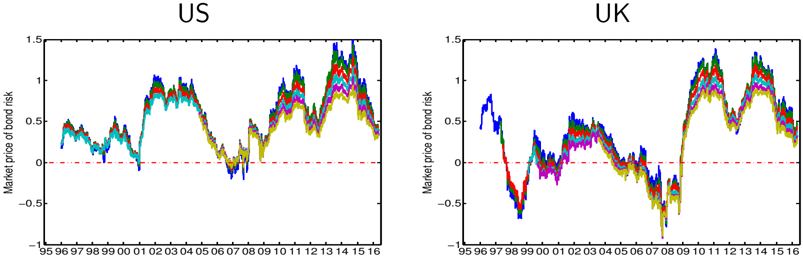
Estimate variance σ_t^2 on each floating swap rate series with a 1y rolling window.





- *Long rates vary as much as, if not more than, short rates.*

Market price of bond risk



- The market price of bond risk extracted from different rates are similar in magnitude and move together:
    - Over the common sample, the cross-correlation estimates among the different λ_t series average 99.67% for US, and 98.76% for UK.
    - The evidence supports a one-factor structure for the bond risk premium, as in Cochrane & Piazessi.
- In the US, market price of risk approached zero in late 1998, 2000, and 2007, but tended to be high during recessions.
- In the UK, the market price became quite negative during 1998 and 2007-2008.

Predicting ex post excess returns with ex ante risk premium

Correlation between ex ante risk premium ($\gamma_t \sigma_t^m$) and ex post excess returns on each par bond, with the average denoting the correlation between the average risk premium and the average bond excess return over the common sample period

Maturity	10	15	20	30	40	50	Average
				Horizon: 6-month			
US	0.31	0.28	0.26	0.24	0.27	0.26	0.29
UK	0.18	0.22	0.22	0.18	0.19	0.22	0.23
				Horizon: One year			
US	0.36	0.36	0.34	0.31	0.31	0.30	0.34
UK	0.36	0.42	0.40	0.32	0.33	0.37	0.39

- The assumption of no prediction on long-dated swap rates leads to significant prediction on bond excess returns.
- The predictors (risk premium) are generated based purely on a variance estimator and the current slope of the yield curve, without estimating predictive regressions.

Formulating rate expectation

If we can predict rate movement, we can further enhance bond return prediction:

- EH uses the yield curve slope to predict short-rate movements.
  - We can at least perform convexity adjustment to generate a more informative slope:

$$AS_t = y_t^L - r_t + \frac{1}{2} C_t^L$$

- EH does not say anything about long-rate prediction, we propose to predict long-rate movements based on anticipated central bank action, which we capture using the yield curve slope at the short end.

$$CB_t = y_t^2 - y_t^1$$

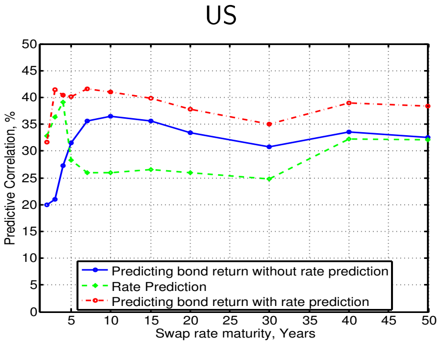
  - Anticipated monetary tightening reins in future inflation, ultimately bringing down long-dated rates (Rotemberg&Woodford (1997)).
- Rate prediction at each maturity reflects the combined effects of the two:

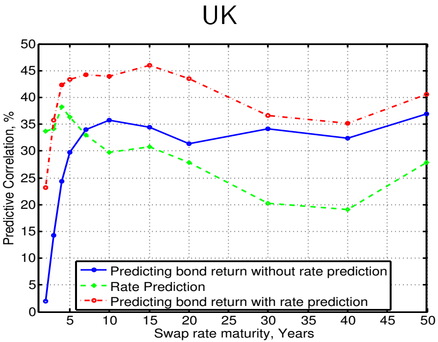
$$\mu_t(\tau) = \frac{1 - e^{-\kappa\tau}}{\kappa\tau} (AS_t + CB_t) - CB_t, \quad \kappa = 1$$

This is just a starting point...

Predictive correlations on rates and excess returns

Universal rate and excess return predictive power across short and long maturities

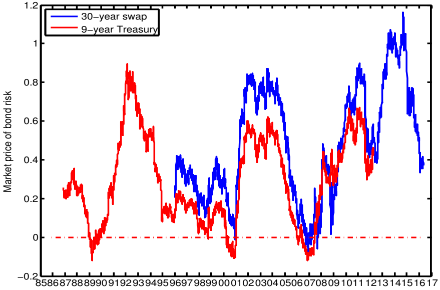




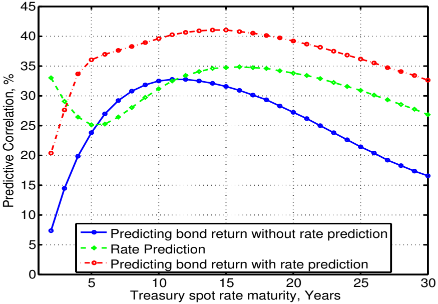
Similar results from stripped Treasury spot rates

over a longer period (1986-

Market price of risk



Predictive correlation



Concluding remarks

- We propose a new modeling framework that is particularly suited for analyzing returns on a particular bond or bond portfolio.
- The framework does not try to model the full dynamics of an instantaneous short rate, but focus squarely on the behavior of the bond yield in question.
- It does not even ask for the full dynamics specification of this bond yield, but only needs estimates of its current expectation, risk premium, and volatility.
  - It can readily accommodate findings from other models, algorithms, information sources.
- The model framework decomposes each yield into three components: expectation, risk premium, and volatility.
  - One can estimate the volatility from historical time series, or infer it from the curvature of the yield curve, or interest rate options.
  - Separating risk premium from expectation can be a very challenging, but very fruitful endeavor.
- We show that we can predict bond excess returns, without running predictive regressions, even by assuming no prediction on interest rates.