NASH EQUILIBRIUM AND THE INDUSTRIAL ORGANIZATION OF MARKETS WITH LARGE FIXED COSTS

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Cournot-Nash models of free entry into industries with large fixed costs yield equilibria with only a few operating firms, and each firm has some monopoly power. I consider a model where each firm's strategy is a function $q(P)$ which specifies how much it will supply at each price. Unlike in Cournot models, the competitive equilibrium (where it exists) is always a Nash equilibrium in supply function strategies, and under weak assumptions it is the only equilibrium. This permits a Nash equilibrium model of the threat of entry as a deterrent to the exercise of monopoly power by operating firms.

A MARSHALLIAN EQUILIBRIUM for an industry with $U$-shaped average cost curves involves a finite number of firms, each operating where marginal cost equals average cost equals market price. It is often argued that if fixed costs are large, then there can only be a few firms in the industry, and therefore firms will not act as price-takers. Current anti-trust laws and court judgements prevent mergers or acquisitions which would reduce the number of operating firms in an industry with large fixed costs. As is well-known, large fixed costs can be formalized as providing a barrier to entry by the use of Cournot-Nash equilibrium models. In this paper, we introduce a new strategy space so that in Nash equilibrium large fixed costs need not be a barrier to the threat of entry.

Throughout the paper we consider an industry producing a homogenous commodity. We assume that the commodity is produced with a technology which anyone can acquire (free entry). However, production costs involve a large fixed cost plus (convex) variable costs. Fixed costs can be so large relative to demand that only a few firms could possibly cover their costs of production.\(^3\)

Section 1 analyzes the way by which Cournot strategies implicitly deter entry. It introduces, in an informal way, the idea that firms can make contracts with customers. This is used, informally, to motivate the assumption that a firm's strategy is a supply function. Section 2 proves the main theorems, namely that under certain assumptions the Marshallian competitive equilibrium is the same as the Nash equilibrium in supply function strategies. Section 3 analyzes Nash equilibrium if there are sunk costs (e.g. due to irreversible investments). It is shown that the results of Section 2 do not hold, and sunk costs (rather than merely fixed costs) can be a deterrent to entry. Section 4 shows that Nash equilibrium in supply functions can exist even when, due to the integer problem, a competitive equilibrium fails to exist. Section 5 (along with parts of Section 1) compares Bertrand equilibria with supply function equilibria which arise in the case when marginal cost is not everywhere constant.

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3 This is in contrast to the recent work of [4, 7, and 8]. They analyzed Cournot-Nash equilibrium with free entry for economies where each firm's fixed cost is small relative to industry demand.
1. INTRODUCTION

Consider an industry with a \( U \)-shaped average cost curve and where industry demand is small relative to the efficient scale of a single firm. Let the cost of production be

\[
C(q) = \begin{cases} 
0 & \text{if } q = 0, \\
K + V(q) & \text{if } q > 0,
\end{cases}
\]

where \( V(\cdot) \) is convex and \( K \geq 0 \). Let the industry demand curve be given by

\[
Q = D(P),
\]

where \( D(\cdot) \) is differentiable and downward sloping. Assume that there are \( n > 1 \) firms with access to the technology in (1).

Figure 1 is drawn for the case where fixed costs are so large relative to demand that the competitive equilibrium would involve exactly one firm producing \( q_c \) and a price of \( P_c \). Can this be sustained by market forces if there is free access to the technology in (1) (i.e., if the industry is subject to free entry)? We would like to argue informally that the answer to this question depends upon the kind of contracts sellers can make with buyers. To show this, we consider the meaning of the Cournot and Bertrand equilibrium concepts. The essential difference between Cournot and Bertrand models is the type of conjecture each firm makes about how the price and quantity sold by other firms will change in response to its own strategy. We will argue that conjectures have an interpretation as contracts.

A Cournot-Nash equilibrium occurs when each of the potential firms must choose a quantity \( q_i \in [0, \infty) \) to maximize its profit, taking the output of all other
firms as given. That is, firm $i$ acts as a monopolist with respect to its residual demand curve

$$Q_i^d(P) = D(P) - \sum_{j \neq i} q_j.$$  

Firm $i$ conjectures that it cannot reduce the amount actually sold by its competitors. So it must share the market with them. Formally, $(q_i^0)_{i=1}^n$ is a Cournot-Nash equilibrium if, for each $i$, $q_i^0$ is a maximizer of profit given by

$$\Pi(q_i^0/q_i^0) \equiv q_iD^{-1}\left(q_i + \sum_{i \neq 1} q_i^0 \right) - C(q_i),$$

where throughout the paper, if $x^0$ is a vector, then $x^0/x_i$ will denote the vector $x^0$ with its $i$th component replaced by $x_i$.

Figure 1 is drawn to have the property that if one firm chooses the monopoly output $q_M$, then any other firm’s residual demand curve $Q_i^d(P) = D(P) - q_M$ is everywhere to the left of the average cost curve. Thus, for the demand and technology in Figure 1, it is Cournot-Nash for exactly one firm to choose $q_M$ and all other firms to choose outputs of zero. Thus even though there is free entry a monopolist is able to deter actual entry by the fact that no other firm can fit in a market with it. No other firm can fit because each potential entrant assumes that the monopolist will keep all of its customers if entry occurs. Why should the entrant assume this; i.e. what is the economic justification for the Cournot-Nash conjecture that a firm’s demand is given by (3)? In particular, why doesn’t an entrant go to buyers and say “sign a contract committing yourselves to purchasing from me and I will enter at $P_M - \epsilon$ and serve you?”

There are at least two answers to this question: (a) the entrant may think that the existing firm will also lower its price in response to the offer or (b) consumers may have signed a legal contract committing themselves to buying from the existing firm. These contracts do not permit them to switch suppliers just because a better offer appears. Of course, (a) and (b) may be false, in which case the entrant would get all of the existing firm’s customers at $P_M - \epsilon$.

If the entrant can make contracts with customers and the existing firm cannot, then it is clear that the entrant will get all of market (as opposed to residual) demand by offering to supply customers at a price of $P_M - \epsilon$. This is implicitly what Bertrand assumes. Under Bertrand, a firm’s strategy is its price $P_i$. Given

\[4\] Throughout the paper $P_M,q_M$ refers to a solution to: $\max_{P,q} PD(q) - C(q)$, subject to eq. (2), where $D(\cdot)$ is the industry demand curve.

\[5\] The term “potential entrant” simply refers to those firms who in Nash equilibrium produce nothing, while “monopolist” refers to a firm who is the only one to produce something in Nash equilibrium. No firm moves first.

\[6\] The idea that agents can contractually commit themselves to a particular type of strategy is a general method for justifying a Nash game form in a particular strategy space. It certainly makes sense for agent $i$ to conjecture that other agents will keep their strategies fixed if they are legally bound by contract to do so.
the prices charged by all firms $P \equiv (P_j^i)_{j=1}^n$, firm $i$'s residual demand function is

\begin{equation}
Q^d_i(P) = \begin{cases} 
0 & \text{if } P_i > P_j \text{ for some } j, \\
D(P)/k & \text{if } P_i \leq P_j \text{ for some } j \text{ and there are } k \\
firms charging } P_i. 
\end{cases}
\end{equation}

Thus $P^0 = (P_j^0)_{j=1}^n$ is a Bertrand equilibrium if, for each $i$, $P_i^0$ solves $\max_{P_i, q_i} P_i q_i - C(q_i)$ subject to $q_i \leq Q^d_i(P^0_i/P_i)$. It is easy to see that monopoly is not a Bertrand equilibrium, since if $P_M$ is the lowest price being charged, then a firm can enter and charge $P - \epsilon$ and get all of the monopolist's customers by (5).\textsuperscript{7} Bertrand conjectures are as if the existing firm is not able to make binding contracts with customers (and thus loses them to an entrant), while Cournot conjectures are as if the existing firm can make contracts with customers and the entrant cannot bid them away.

In many situations, it is reasonable to assume that all firms have the same ability to make contracts with customers. That is, we would like the existing firm to have the same tools for keeping customers as possessed by an entrant for bidding them away. Recall that under Bertrand the entrant undercuts the extant firm and gets all of his customers. Suppose that the extant firm had offered customers contracts which specified his willingness to meet competing offers. He could make contracts stating that he will supply $q_M$ at $P_M$ if there are no competing offers and if a competitor offers customers $P < P_M$ the extant firm agrees to lower his price to $P$. The model is easier to understand if we imagine that each customer wants at most one unit and $D(P)$ is the number of customers willing to pay $P$ or more. Then consumers who sign the extant firm’s contract are assured of getting the lowest price in the market and are committed to buy from him. For reasons to be explained below, it may not be possible for the extant firm to contract to supply $q_M$ at all possible prices (which he may be forced to do by his contract with consumers). To permit output flexibility we define a contract to be a function $q(P)$ which specifies some supply at $P_M$, say $q(P_M)$, and a commitment to supply $q(P)$ customers, charging each $P$, if $P$ is the lowest price offered by competitors. In this case an entrant, taking as given the extant firm’s contract with customers, faces a residual demand curve

\begin{equation}
Q^d(P) = D(P) - q(P),
\end{equation}

which has as special cases the Cournot and Bertrand residual demands (3) and

\textsuperscript{7}If $n = 2$, no Bertrand equilibrium exists for the technology and demand in Figure 1. This is because at $P_1 = P_2 = P_c$, both firms get half of demand which is too small to cover fixed costs. If $P_M = P_1 > P_2$, then firm 2 can raise price a little and increase its profit. If $P_1 = P_2 > P_c$, then either firm will do better by lowering its price just a little. Further, since $P_1 > P_M > P_2$ would cause firm 1 to lower $P_1$, there is no Bertrand equilibrium. (This could be repaired if $Q^d_i(P)$ was made a correspondence at price vectors with more than one lowest price. See Section 5 for further remarks on Bertrand equilibria.)
(5). It is important for a strategy to be a mapping $q(P)$, rather than simply a pair $(P, q)$. We show that if there are fixed costs, then Nash equilibria will not exist in many important cases under the latter strategies.

It remains to specify a Nash equilibrium concept which formalizes the above idea. We wish to avoid a reaction function definition since this will create too many equilibria. Yet we wish to maintain the idea that firms offer customers contracts and these contracts limit the ability of entrants to bid away customers. To achieve this goal, we assume that each firm’s strategy is a supply function $q_i(P)$. Let $\bar{q} \equiv (\bar{q}_i, \bar{\bar{q}})$ be the list of firms’ strategies, where throughout the paper a “−” will be used to denote a mapping. We are faced with the problem of defining an actual price given that all firms make offers to match each other’s prices. This problem can be resolved by the following definition. Given firms’ supply functions and given the market demand function, we can define a number $P = P(\bar{q})$ as the price such that supply equals demand; i.e., $P$ satisfies $\sum_{i=1}^{n} q_i(P) = D(P)$ (assuming for the moment that such a number exists and is unique). Define $\bar{q}_i^0$ to be a Nash equilibrium in supply functions if, for each $i$, $q_i^0(\cdot)$ maximizes

$$\Pi_i(\bar{q}_i^0/\bar{q}_i) = P(\bar{q}_i^0/\bar{q}_i)q_i(P(\bar{q}_i^0/\bar{q}_i)) - C(q_i(P(\bar{q}_i^0/\bar{q}_i)))$$

with respect to all nonnegative supply functions $q_i(\cdot)$.

To see how this works, assume that $n = 2$ and we have the technology pictured in Figure 1. Let firm 2 choose a supply schedule $q_2^0(P)$. Consider firm 1’s attempt to maximize (7). Let firm 1 choose a supply curve $q_1(\cdot)$ which leads to a price of $P$, i.e., $q_1(P) + q_2^0(P) = D(P)$. Its profit is:

$$Pq_1(P) - C(q_1(P)) = P\left[D(P) - q_2^0(P)\right] - C(D(P) - q_2^0(P)).$$

Suppose that $P^*$ is the value for $P$ which maximizes the right-hand side of (8). If $q_1^* = D(P^*) - q_2^0(P^*)$, then the maximal value of (7) is clearly no larger than $\Pi_1^* = P^* q_1^* - C(q_1^*)$. But if firm 1 chooses a supply curve $q_1(P)$ which satisfies $q_1(P^*) = q_1^*$, then $\Pi_1(\bar{q}_i^0/\bar{q}_i) = \Pi_1^*$, since by construction supply equals demand at $P^*$.

That is, to find a maximizer of (7) for firm 1, the firm acts as a monopolist with respect to the residual demand $D(P) - q_2^0(P)$ and finds an optimal price and quantity $P^*, q_1^*$. This can be achieved by firm 1 since any point $(\bar{P}, \bar{q}_1)$ on the residual demand curve (i.e. satisfying $\bar{q}_1 = D(\bar{P}) - q_2^0(\bar{P})$) can be induced by choosing a supply curve which goes through $(\bar{P}, \bar{q}_1)$.

If firm 1 takes as given that firm 2 will match its price and keep $q_2^0(P)$ customers, then it is optimal for firm 1 to set a price of $P^*$. Alternatively, let firm 1 choose any supply schedule $q_1^0(\cdot)$ such that $q_1^0(P^*) = D(P^*) - q_2^0(P^*)$; then if

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8It is not unusual for firms to advertise that they have say 100 units to sell at a price of $5 but that they guarantee to meet any bonafide price offer. These advertisements are legally binding commitments, i.e. contracts. They seem more prevalent than the Cournot commitment which would involve a firm advertising only its output.
customers are offered contracts $q_i^0(P)$, $q_2^0(P)$ supply will equal demand at $P^*$ and firm 1 can do no better by offering to charge a lower price than firm 2. Suppose, further, that if firm 2 takes $q_i^0(\cdot)$ as given, then $q_2^0(\cdot)$ is an optimal response (which we could check via an argument similar to that which we provided for firm 1); then $(\tilde{q}_1^0, \tilde{q}_2^0)$ is a Nash equilibrium in supply functions.

Unlike Cournot and Bertrand where the slope of a firm’s residual demand is exogenous, our model permits the slope of the residual demand to be endogenous. This creates more equilibria than exist under either Cournot or Bertrand. For example, in Figure 1 (and in general with $K > 0$), the competitive equilibrium is never a Cournot equilibrium. This is because the residual demand curve facing any producing firm is never perfectly elastic at $P = P_c$ so that it chooses to produce where price is above marginal cost. On the other hand, the competitive equilibrium (when it exists!) is always a Nash equilibrium in supply functions. To see this, let firms 1 and 2 choose

$$q_1^0(P) = \begin{cases} 0, & P \leq P_c, \\ q_c, & P > P_c, \end{cases} \quad q_2^0(P) = \begin{cases} 0, & P < P_c, \\ q_c, & P \geq P_c, \end{cases}$$

respectively. In this case, firm 2 faces a residual demand curve $D(P) - q_1^0(P)$ which is perfectly elastic at $P = P_c$. Hence it can achieve no higher profit than by choosing a supply curve with $q(P_c) = q_c$, since there is no supply curve $\tilde{q}_2$ which will get $P(\tilde{q}_1^0, \tilde{q}_2) > P_c$. Similarly, firm 1 can never make positive profit if it faces a residual demand $D(P) - q_2^0(P)$ (since $P(\tilde{q}_1^0, \tilde{q}_2^0) \leq P_c$ for all $\tilde{q}_1$; therefore $q_1^0$ is an optimal choice for firm 1).

The monopoly solution (which is the same as the Cournot equilibrium) is also a supply function equilibrium for the technology in Figure 1. That is, let firms 1 and 2 choose

$$q_1^0(P) \equiv q_M \quad \text{and} \quad q_2^0(P) \equiv 0.$$

Clearly $q_1^0(P)$ is best for firm 1, given that firm 2 chooses $q_2^0(P) \equiv 0$. Similarly firm 2 faces a residual demand $D(P) - q_1^0(P) \equiv D(P) - q_M$ which in Figure 1 is everywhere to the left of average cost. So, for all $\tilde{q}_2 \equiv q_2^0(\cdot)$, with associated $P = P(\tilde{q}_1^0, \tilde{q}_2^0)$, it is true that $P \leq C_2(q_2(P)) + q_2(P)$, so it is optimal for firm 2 to set $q_2^0(P) \equiv 0$.

There are also other Nash equilibria in supply functions. The multiplicity of equilibria is due to the fact that firms are offering commitments to customers which have the effect of threats to other firms. With the strategy in (10), firm 1 is stating that it will meet any price offered by a competitor even if that price has firm 1 producing at below its average cost. We can reduce the number of equilibria drastically by eliminating threats to produce at below cost. That is, we will now require that $\tilde{q}_i \equiv q_i(\cdot)$ satisfy

$$Pq_i(P) - C_i(q_i(P)) \geq 0 \quad \text{for all} \quad P \geq 0$$

in order for $\tilde{q}_i$ to be a feasible strategy for firm $i$. There are a number of
justifications for (11): suppose that firm i’s only source of money is from the sale of \( q_i(P) \). Thus it would be impossible for firm i to produce at below average cost. If consumers know this, then they would not sign a contract committing themselves to purchase from firm i because they correctly do not believe i’s commitment to meet competing offers. Thus in Figure 2, if firm 1 chooses \( q_1(P) = q_M \), then firm 2 can offer consumers a contract \( q_2(P) \). Firm 2 offers consumers a price of \( P_2 \). Since firm 1 cannot produce \( q_M \) at price \( P_2 \), firm 2 gets all of its customers. That is, consumers faced with the two contracts assume that \( q_1(P) \) is really the contract \( q_1(P) = q_M \) for \( P \geq P_1 \) and \( q_1(P) = 0 \) for \( P < P_1 \) (equivalently, consumers correctly perceive that firm 1 will only meet offers down to \( P_1 \)). Even if firm 1 does have the cash to finance losses, it may be difficult for the firm to convince consumers that it will do so.\(^9\) The main reason for choosing (11) is, as is shown in the next section, that it restricts threats in a way that leads to equilibria that seem reasonable.

Figure 3 illustrates the mechanism by which Nash equilibria in feasible supply function strategies eliminate noncompetitive outcomes. In the figure, firm 1

\(^9\)This line of argument may seem to contradict our fundamental assumption that firms can make any legally binding contract. Note, however, that if firm 1 violates its contract to produce \( q_M \) at \( P = P_1 \), then a court could surely force the firm to carry out the contract, since the contract is self-financing. However, if firm 1 should fail to produce \( q_M \) at \( P = P_2 \), then the court could not necessarily force the firm to produce \( q_M \) at \( P_2 \) since this would require a subsidy from somewhere. If firm 1 is a limited liability corporation, then any excess cash could be paid out to shareholders just before consumers try to enforce the contract to produce at \( P_2 \). *Ex ante*, the knowledge that the firm could always undo a contract in the above way will make potential entrants and consumers skeptical about non-self financing contracts.
chooses a supply curve $q_1(P)$ which intersects the demand curve at $(q_M, P_M)$, the monopoly point. Note that the only feasible $q_1(P)$ has either $q_1(P_c) = 0$ or $q_1(P_c) = q_c$. It is the former case that is plotted in Figure 3, and we show that it cannot be a Nash equilibrium for all other firms to do nothing. Let firm 2 choose $q_2(P)$ which is underneath $q_1(P)$. If the smallest price at which $q_1(P)$ becomes positive is $P_1$ and $P_1 > P_c$, then $q_2(P)$ can always be chosen so that $q_2(P)$ intersects the demand curve at a price $P_2$ such that $P_c < P_2 < P_1$. Thus firm 2 can get the whole market and make a profit. It is not optimal for firm 2 to set $q_2(P) \equiv 0$ unless $q_1(P_c) = q_c$, for strictly U-shaped average cost. Note that feasibility is essential. If firm 1 could use a Cournot strategy like $q_1(P) = q_M$ for all $P \geq 0$, then firm 2 could not get underneath such a supply curve.

2. CHARACTERIZATION OF SUPPLY FUNCTION EQUILIBRIA

The fact that supply functions must be feasible (i.e., satisfy (11)) means that they will be discontinuous when $K > 0$. To avoid the consequent technical difficulties, we will permit the supply function to be multivalued at points where the supplier is indifferent about which value is chosen.\(^\text{10}\) Thus we say that the

\(^{10}\text{In ordinary production theory a firm is necessarily indifferent between two outputs on its supply correspondence. This is because each firm is choosing a number } q \text{ which maximizes profit. In this paper a firm is choosing a supply correspondence, so we must explicitly add the assumption that the firm is indifferent.} \)
correspondence $q_i(\cdot): R_+ \to \text{subsets of } R_+$ is feasible for firm $i$ if

(12a) \[ q_i(\cdot) \text{ is upper semicontinuous (i.e., has a closed graph); and for} \]

all $P \geq 0$ $q_i(P)$ is non-empty and

(12b) \[ Pq_i - C_i(q_i) \geq 0 \text{ for all } q_i \in q_i(P); \]

(12c) \[ Pq - C(q) = Pq' - C(q') \text{ for all } q, q' \in q_i(P). \]

We denote the strategy set for firm 1 by $S_i$ and let $S = \prod_{i=1}^n S_i$.

Before we can define payoff functions corresponding to supply function strategies, it is necessary to define the "price such that supply equals demand." For the first two theorems it will not matter if there are many prices such that supply equals demand. Given $\bar{q} \in S$, we would like to define $P(\bar{q})$ to be any function from $S$ to $R_+$ such that $\sum q_i(P(\bar{q})) = D(P(\bar{q}))$. However, since supply correspondences need not be convex valued, there may not exist any price such that supply equals demand. If $\bar{q}$ is such that there is no market clearing price, then we associate a price $P(\bar{q})$ as a price where the supply curve jumps from above the demand curve to below the demand curve.

Formally we define a market clearing price function as any function $P(\cdot)$ from $S$ to $R_+$ such that, for each $\bar{q} \in S$, either

(13a) \[ P(\bar{q}) = \inf \left\{ P \mid \text{there exists } Q(P) \in \sum q_i(P) \text{ satisfying } Q(P) \geq D(P) \right\}, \]

or

(13b) \[ P(\bar{q}) = \sup \left\{ P \mid \text{there exists } Q(P) \in \sum q_i(P) \text{ satisfying } Q(P) \leq D(P) \right\}. \]

In (13a) we take the smallest price such that supply is larger than demand, while in (13b) we take the largest price such that supply is less than demand. If supply curves are upward sloping, then (13a) is equivalent to (13b). Given a market clearing price function $P(\cdot)$, we define payoff functions as in (7). A $\bar{q}^0 \in S$ is defined to be a Nash equilibrium in supply functions if, for each $i$, $q_i^0$ is a maximizer of (7) over all $\bar{q}_i \in S_i$.

For the rest of this section we assume that all firms have identical cost functions given by (1). In this case a competitive equilibrium is a price $P_c > 0$, and integer $n_c > 0$, and an output per firm $q_c > 0$ such that

(14) \[ n_c q_c = D(P_c), \quad P_c = C'(q_c), \quad P_c = \frac{C(q_c)}{q_c}. \]

If fixed costs are positive, then in general there will not be an integer $n_c$ such that $n_c q_c = D(C'(q_c))$. However, this is not the reason why firms have monopoly power in a model with large fixed costs and free entry. As was pointed out in the last section, even if a competitive equilibrium exists, the Cournot equilibrium will not be competitive. Modigliani [6] provides a clear statement of the Bain,
Sylos-Labini hypothesis that fixed costs are a barrier to entry and a source of monopoly power. He notes that with large fixed costs, there are few operating firms, so that a firm’s residual demand is not perfectly elastic. In this section, we are trying to determine whether having a small number of operating firms leads to monopoly power. For this reason, it is convenient to ignore the integer problem by concentrating on those cases where a competitive equilibrium does exist. The next theorem shows that, contrary to [6], a firm’s residual demand can be perfectly elastic even if there are few operating firms.

We now prove that the competitive equilibrium is a Nash equilibrium.

**Theorem 1:** Let the number of potential firms \( n \) satisfy \( n \geq n_c + 1 \). Then there exists a Nash equilibrium in supply functions \( \bar{q}^0 \) such that \( P(\bar{q}^0) = P_c \), \( n_c q_c \in \sum q^0_i(P_c) \).

**Proof:** Let firms \( i = 1, 2, \ldots, n_c + 1 \) choose the strategy

\[
q^0_i(P) = \begin{cases} 
(0), & P < P_c, \\
(0, q_c), & P = P_c, \\
(q_c), & P > P_c.
\end{cases}
\]

Let all other firms \( i > n_c + 1 \) set

\[
q^0_i(P) = \{0\}.
\]

Note that \( P(\bar{q}^0) = P_c \), whether (13a) or (13b) is used. Clearly \( n_c q_c \in \sum q^0_i(P_c) \). It remains to show that (15) is a Nash equilibrium. Consider the residual demand curve facing any firm \( j \):

\[
Q^d_j(P) = D(P) - \sum_{i \neq j} q^0_i(P),
\]

where we note that by (13) firm \( j \) can take any selection it wants from the residual demand correspondence. If \( P > P_c \), then \( Q^d_j(P) = \{0\} \); therefore, for any supply function \( \bar{q}_j \), \( P(\bar{q}^0_0 / \bar{q}_j) \leq P_c \) (whether (13a) or (13b) is used). Hence there is no strategy \( \bar{q}_j \) which gives firm \( j \) positive profit when other firms choose \( q_i^0 \). Since (15) gives \( j \) zero profit, it is an optimal strategy. \( Q.E.D. \)

The idea of the proof is that a firm’s residual demand is perfectly elastic at \( P = P_c \) when other firms state that they will enter should \( P \) rise above \( P_c \).

We now prove a partial converse to Theorem 1: if \( \bar{q}^0 \) is Nash in supply functions and there are a large number of potential entrants, then supply equals demand at \( P_c \). The idea of the proof is that if supply is less than demand at \( P_c \) a new firm can fit in and make a profit by choosing an appropriate supply. This can be seen in Figure 2 of the last section. If firm 2 chooses \( q_2(P) \) and all other firms do nothing (i.e., set \( q_i(P) \equiv 0 \)), then this cannot be a Nash equilibrium because one of the firms which does nothing can get higher profit by switching to
a supply curve which is underneath $q_2(\cdot)$ and intersects the demand curve $D$ at a
price below the lowest price at which $q_2(\cdot)$ becomes positive.

Showing that a firm can actually enter and make strictly positive profit, when
price is above the competitive level, is slightly delicate. First, even though
individual supply functions are upper semi-continuous, the aggregate supply
function $\sum_{i=1}^{n} q_i(P)$ need not be, when $n = \infty$. This has the undesirable conse-
quence that in any small neighborhood of $P_c$ aggregate output can be enormous,
while aggregate output is zero at $P_c$. Thus an entrant may not make a profit by
entering. To remove this difficulty, we assume that the number of potential firms
is finite. Once the number of potential firms is finite, it is important that it be
sufficiently large that there is always an entrant around (i.e., a firm that does not
produce a positive amount in equilibrium). To insure this, we make the following
three assumptions:

ASSUMPTION 1: For $P$ large enough, say $\bar{P}$, $D(\bar{P}) = 0$.

ASSUMPTION 2: $\lim_{Q \to \infty} D^{-1}(Q) = 0$.

ASSUMPTION 3: $K > 0$, $V''(Q) > 0$.

These three assumptions imply that there can only be a finite number of
operating firms (i.e. firms with $q_i > 0$) in the industry. This is because with
$U$-shaped average cost Assumption 1 implies that $q_i$ must be bounded away from
zero, while Assumption 2, with Assumption 1, implies that aggregate output must
be finite. Thus given $K$, $V(\cdot)$, and $D(\cdot)$, there is an integer $N < \infty$ which
represents the maximum number of operating firms. We assume that the number
of potential entrants is finite and larger than $N$:

ASSUMPTION 4: $n > N$.

There is one further difficulty. Consider an industry where $n_c = 2$. Suppose
firm 1 chooses a supply curve as in Figure 4, with the property that $D(P) - q_1(P)$
is to the left of the average cost curve for $P > P_c$. If $\lim_{P \to P_c} q_1(P) = \infty$,
then if say firm 2 enters with a feasible $q_2(P)$ underneath $q_1(P)$, then $q_1(P) + q_2(P)$
will be larger than demand for all $P > P_c$. Note, however, that since
$n_c = 2$, there exists a feasible $\tilde{q}_2$ such that $q_1(P_c) + q_2(P_c)$ would equal demand at
$P_c$. That is, firm 2 could drive price to $P_c$ if it entered, and it would make exactly
zero profit. There is no way that firm 2 could make a positive profit against $q_1(P)$
in the figure. Thus it would be a Nash equilibrium for $q_2(P) \equiv 0$ and $q_1(P)$ as in
the figure ($P_M$ is the monopoly price). We could eliminate this case by assuming
that if a firm is indifferent between producing and not producing, then it produces if this doesn’t cause excess supply at all prices $P \geq P_c$. However, we
can also rule out the difficulty by eliminating supply schedules which rise
infinitely fast (as Theorem 2 shows). Thus we restrict the definition of the
strategy sets $S_i$ as follows: let $S_i$ be the set of $\tilde{q}_i$ satisfying (12) such that there
exists a function $M_i(P)$ from $R_+ \rightarrow R_+$ satisfying $PM_i(P) - C_i(M_i(P))$ $\geq 0$ for all $P \geq P_c$, and such that

$$(17a) \quad q_i(P) \leq M_i(P) \quad \text{for all} \quad P \geq P_c;$$

$$(17b) \quad \lim_{\epsilon \downarrow 0} \frac{M_i(P_c + \epsilon) - M_i(P_c)}{\epsilon} \quad \text{exists and is finite.}$$

**Theorem 2:** Let $n_c, P_c, q_c$ be a competitive equilibrium. Assume Assumptions 1–4, and let $q^0_\epsilon$ be a Nash equilibrium when supply functions are required to satisfy (12) and (17); then

$$n_c q_c \in \sum_{i=1}^n q_i^0(P_c).$$

**Proof:** Since for $P$ large enough $D(P) = 0$, there can only be a finite number of firms making positive profit in the Nash equilibrium $q^0_\epsilon$. Let firm $e$ be making zero profit under $q^0_\epsilon$. We show that if $n_c q_c \notin \sum q_i^0(P_c)$, then there is a strategy for firm $e$ strictly better than $q^0_e$. Consider the residual demand faced by $e$, $Q^e_\epsilon(P)$, as defined in (16). Since $q_i^0(\cdot)$ is feasible for each $i$, and $V'' > 0$, there is exactly one positive output level feasible for each firm at $P = P_c$, namely $q_c$. Thus it must be the case that either $q_i^0(P_c) = \{0\}$ or $q_i^0(P_c) = \{0, q_c\}$. Thus if $x = \sum_{i \neq e} q_i^0(P_c)$, then $x = rq_c$ for some integer $r < n_c$ (for if $r \geq n_c$ then $n_c q_c \in \sum q_i^0(P_c)$, since by (12a) $0 \in q_i^0(P_c)$). There are two cases to consider: (a) $r < n_c - 1$, and (b) $r = n_c - 1$.

If $r < n_c - 1$, then it is clear that firm $e$ can find a strategy which yields it positive profit. To see this, note that $K > 0$, (12b), and the upper semicontinuity of $q_i^0(P)$ imply that if $q_i^0(P_c) = \{0\}$, then there is a nbhd of $P_c$ such that $0 \in q_i^0(P)$ for all $P$ in the nbhd. Hence since $n < \infty$ there is a nbhd of $P_c$ such that firm $e$’s residual demand is $Q^e_\epsilon(P) \geq D(P) - (n_c - 2)q_c$. Note that $Q^e_\epsilon(P) > q_c + \epsilon$, for some $\epsilon > 0$, in a neighborhood of $P_c$, because $D(P_c) = n_c q_c$, and $D(\cdot)$ is continu-
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ous. Thus if firm \( e \) chooses \( q_e(P) = (q_e) \) for all \( P > P_c \), \( q_e(P_c) = (0, q_e) \), and zero elsewhere, then \( \inf \{ P \geq 0 | \sum q_e(P) \equiv D(P) > P_c \} \). Thus if \( P(\cdot) \) is defined by (13a), firm \( e \) makes strictly positive profit under \( q_e(P) \). Similarly for \( P(\cdot) \) defined by (13b), since when firm \( e \) chooses \( q_e(P) = q_e \), supply is surely less than demand at prices close to and above \( P_c \).

The case \( r = n_e - 1 \) is slightly more difficult. In this case, in a positive nbhd of \( P_c \), there may be at most \( n_e - 1 \) firms choosing a positive output. By (17) there exists a function \( M(P) \), differentiable from the right at \( P_c \), such that \( q_e^0(P) \equiv M(P) \) for the \( n_e - 1 \) producing firms. Hence in a positive nbhd of \( P_c \), the most that output can be is \((n_e - 1)M(P)\). So firm \( e \)'s residual demand is at least \( D(P) - (n_e - 1)M(P) \). We now show that firm \( e \) can make a strictly positive profit if its residual demand is

\[
Q_e^d(P) \equiv D(P) - (n_e - 1)M(P),
\]

and this will show that it could make a strictly positive profit facing \( \bar{q}^0/\bar{q}_e \).

Consider the problem \( \max_{P \geq 0} \Pi(P) \), where

\[
\Pi(P) \equiv PQ_e^d(P) - C(Q_e^d(P)).
\]

We will show that there is a price \( P^* \) which makes profit in (19) positive. This implies that if firm \( e \) sets \( q_e(P^*) = Q_e^d(P^*) \), then it can achieve a positive payoff contradicting the assertion that \( q_e^0(P) \) was Nash (as \( q_e^0 \) yields zero profit by assumption). To see that such a \( P^* \) exists, note that (19) is differentiable from the right at \( P_e \), so

\[
\Pi'(P_c) = [P_c - C'(Q_e)] Q_e^d(P_c) + Q_e^d(P_c),
\]

(20)

\[
\Pi'(P_c) = [P_c - C'(Q_e^d(P_c))]D'_e(P_c) - (n_e - 1) \times [P_c - C'(Q_e^d(P_c))]M'(P_c) + Q_e^d(P_c),
\]

where all derivatives are from the right.

Since \( M(P) \) must lie above average cost, \( M(P_c) = q_e \), so \( Q_e^d(P_c) = D(P_c) - (n_e - 1)q_e = q_e \). Since \( P_c = C'(q_e) \), (20) becomes \( \Pi'(P_c) = Q_e^d(P_c) = q_e > 0 \). This shows that there is a \( P^* \) and a \( q^* \) given by \( Q_e^d(P^*) \) such that if \( q_e(\cdot) \) satisfies \( q_e(P) = (0) \) for \( \bar{P} \leq P^* \) and \( q_e(P) = (q^*) \) for \( \bar{P} \geq P^* \), then the payoff \( \Pi_e(\bar{q}/\bar{q}_e) \) will be positive when \( \bar{P}(\bar{q}) \) is given by (13a). This is because by the construction of \( \bar{q}_e \), there could not possibly be a price lower than \( P^* \) such that supply is bigger than demand. Similarly, since supply at \( P^* \) is less than or equal to demand under \( \bar{q}^0/\bar{q}_e \), price given by (13b) can be no lower than \( P^* \). \( \square \)

REMARK 1: Theorem 2 makes heavy use of the assumption that there is a unique output level that achieves minimum average cost. If the average cost curve is "flat-bottomed," as in Figure 5, then the theorem is false.\(^{11}\) The figure is

\(^{11}\)I am grateful to an anonymous referee and Glenn Loury for pointing this out to me.
drawn for \( n_c = 1 \). Firm 1 chooses a supply curve which produces \( q_1 \) at \( P_c \) and the monopoly output at \( P_M \) which is the monopoly price. The Figure is drawn so that it is a Nash equilibrium for all other firms to set \( q_i(P) \equiv 0 \). This is because the Figure is drawn such that if any firm chooses a feasible \( q(P) \), then \( q(P) + q_i(P) > D(P) \) for all \( P \geq P_c \). Clearly \( q_i(P) \) is optimal for firm 1 when all other firms set \( q(P) \equiv 0 \). Thus Figure 5 represents a Nash equilibrium where \( n_c q_c \notin D(P_c) \). If the average cost curve is strictly \( U \)-shaped so that there is a unique feasible output at \( P_c \), that output must be \( q_c \). In this case, if firm 1 produced anything at \( P_c \), it would be \( q_c \), so the theorem is correct. If firm 1 produced nothing at \( P_c \), then firm 2 could choose a supply curve underneath that of firm 1 and make a profit. This is what makes the theorem true. This was illustrated in Figure 3 of the last section. The theorem works because it is always possible for an entrant to get underneath the supply curve of extant firms. In the next section, we will see that if one firm has sunk costs, then it can be impossible for an entrant to get underneath the supply curve of the firm with sunk costs.

**Remark 2:** Theorem 2 assumed \( K > 0 \). If \( K = 0 \) and \( V'' > 0 \) and \( n = \infty \), then the theorem is meaningless because a competitive equilibrium does not exist. This is because marginal cost is always larger than average cost at any positive output level. Note that Assumption 4 is violated for finite \( n \) because the maximum number of operating firms, \( N_t \), is infinite. One approach to modelling the economy when \( K = 0 \), and \( V'' > 0 \) is to take a limit of economies as \( K \downarrow 0 \). If this view is taken, then we can define \( n_c = \infty \), \( P_c = \lim_{q \downarrow 0} C'(q) \) and \( n_c q_c = D(P_c) \). If we take the limit as \( K \downarrow 0 \) over values of \( K \) such that a competitive equilibrium exists, then Theorem 2 assures us that \( n_c q_c \) will be a limit of Nash equilibria in supply functions. It should be noted, however, that if the number of potential entrants \( n \) is finite and \( K = 0 \), \( V'' > 0 \), then an argument like that of Theorem 2 cannot be used to show that the Nash equilibrium will occur at price equals
marginal cost (i.e., when $D(C'(q)) = nq$). This is because it is no longer the case that in any Nash equilibrium there must be one firm getting zero profit which can enter and undercut other firms. Indeed all potential firms may be operating where price is above marginal cost. This shows the importance of the condition that $n > N$.

**Remark 3:** If $\nu'' = 0$ and $K = 0$, then Theorem 2 is false. Let marginal cost equal average cost $\equiv c$. Figure 6 illustrates a Nash equilibrium where firm 1 chooses a supply curve which is negatively sloped. If all other firms set $q(P) \equiv 0$, then firm 1 gets the monopoly price $P_M$ at the monopoly output $q_M$. None of the firms setting $q(P) \equiv 0$ can make a positive profit by switching to another strategy because the residual demand facing any firm $D(P) - q_1(P)$ is less than or equal to zero for all $P > c$.

Figure 6 models the idea that one firm can threaten to produce an enormous amount of output if any other firm enters the industry. Permitting downward sloping supply curves leads to threats which are “unrealistic.” Most economists would agree that in an industry with free entry, constant marginal costs and no fixed costs, the outcome would be competitive. It is cases like that of Figure 6 that make one want to require that $q(P)$ be upward sloping, for $q(\cdot)$ to be a feasible strategy. The reader can verify immediately that if $q(\cdot)$ is required to be upward sloping, then the only Nash equilibrium is the competitive equilibrium in the case $\nu'' = K = 0$.

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12 Clearly strategies such as $q_1(\cdot)$ in the figure are not “Nash perfect” because no firm would carry out the threat ex post if it could avoid doing so.

13 The correspondence $q(\cdot)$ is upward sloping if $P_1 > P_2$; then $q_1 \equiv q_2$ for all $q_1 \in q(P_1)$ and $q_2 \in q(P_2)$. 
There is another unrealistic feature about \(q_1(P)\). Given the demand curve in the figure, the firm could never sell \(q_1(P)\) at \(P \neq P_M\). It would always be supplying more than is demanded. Note that it is feasible for the firm to sell more than \(q_c\) at \(P = c\). However, there would never be buyers for the output since demand is only \(q_c\) at \(P = c\). The relevant supply curve is really \(q_1^*(P) = \min(q_1(P), D(P))\) since the supplier could never sell more than \(D(P)\). If this is the supply curve used, then Theorem 2 is true (since \(q_1^*(P)\) coincides with the demand function), but not very strong. This is because there are multiple price equilibria.

Remark 4: The existence of multiple equilibria provides another reason for requiring that supply curves be upward sloping. Figure 7 illustrates a case where \(n_c = 1\) and there are two prices where supply equals demand when firm 1 chooses the indicated \(\bar{q}_1\) and all other firms set \(q_i(P) \equiv 0\). Firm 1’s supply curve goes through the demand curve at both the monopoly and competitive prices. It is Nash for all other firms to do nothing since their residual demand is too small to cover fixed costs. Theorem 2 is not contradicted by this figure since \(n_c q_c \in \sum q_i(P_i)\). However, supply also equals demand at \(P_M\) if \((13a)\) is used to define \(P(\bar{q})\), then \(P(\bar{q})\) for the \(\bar{q}\) indicated in this figure is \(P_c\), while if \((13b)\) is used then \(P(\bar{q}) = P_M\). Thus Theorem 2 simply states that the competitive price is one of the prices such that supply equals demand in Nash equilibrium. If we add the requirement that feasible supply curves are upward sloping, then we have as an immediate consequence of Theorem 2 that \(P(\bar{q}^\Phi) = P_c\). Note that with \((12a)\), \((13a)\) and \((13b)\) are the same if \(q(\cdot)\) is upward sloping.

Remark 5: One problem with downward sloping supply schedules is that it can be quite profitable to misrepresent demand. If we accept \(q_1(P)\) in Figure 7,
then each consumer could get the price down by claiming to demand more than he actually demands. If this effect is strong enough the \( D(P) \) curve can shift to the right through the point \( P_c, q_c \) until there is a unique intersection with \( q_i(P) \) at \( P_c, q_c \). Alternatively, an entrepreneur can announce to consumers that he will supply at a price below \( P_M \). He could then present a very large demand at \( P_M \) so that supply equals demand at a price below \( P_M \) at which he could then serve the market. These remarks are quite informal and require a great deal more analysis before they can be used to justify the elimination of downward sloping supply curves. As will be seen in the next section, the model is rich enough to capture entry deterrence without introducing downward sloping supply. Permitting downward sloping supply introduces stronger threats than I feel are necessary to capture the strategic aspects of entry deterrence. As can be seen from Figures 6 and 7, it permits “overkill”—entry deterrence is possible in situations like constant marginal cost, \( K = 0 \), when I don’t feel it is empirically warranted. Nevertheless, it may be important; e.g. Williamson [13] argues that the threat of an extant firm to increase output should entry occur, is at the essence of predatory competition.

3. NASH EQUILIBRIUM WITH SUNK COSTS

In the previous section we analyzed a model of Nash equilibrium with free entry where firms have large fixed costs. Unlike what occurs with Cournot strategies, the Nash equilibrium in supply functions can be the competitive equilibrium. In this case, even if fixed costs are so large that only one firm can fit in the industry, that firm will act like a perfect competitor since other firms threaten to supply if price rises above the competitive level.

It is important to distinguish large fixed costs from large sunk costs. Suppose firms require a plant which costs \( SK \) to produce the commodity in question. If the plant can be used in other industries, and the plant can be sold for \( SK \), then none of the costs are sunk. An example of such a commodity is “a seat on a plane from Philadelphia to New Haven.” The fixed cost is the cost of the plane. If the plane is not used on that route, then it can be used on some other route. So no sunk costs seem to be involved. It's as if the plant is rented. For this type of technology, easy entry and exit is to be expected.

There are other technologies, where the plant has no use other than in the production of one type of commodity. Consider a commodity like telephone service between New York and London. This can be produced by laying a cable beneath the ocean. Even though the \( ex \ ante \) cost of the cable is \( SK \), the value of the cable in its next best use is zero. The cable represents an irreversible investment. If \( K_0 \) is the \( ex \ ante \) cost of the plant and \( K_1 \) is the \( ex post \) value of the plant (i.e. what it can be sold for), then \( K_0 - K_1 \) represents the amount of the sunk cost. Further, \( K_1 \) is the relevant cost to use in (1) and (12).

Spence [11] and others have emphasized the importance of sunk costs in entry deterring behavior. He used a Stakelberg model (where the entrant is the follower) to illustrate his ideas. The model of the previous section can clarify the role of sunk costs from a Nash point of view. Suppose that firm 1 sinks a cost first. Its relevant cost of production is the \( ex post \) value of the plant \( K_1 \). Assume
that only firm 1 has sunk the cost, while all other firms have not and have costs $K_0$. Figure 8 gives the \textit{ex ante} and \textit{ex post} average cost curves.

In the Figure, firm 1 has lower cost curves than all other firms. The Figure is drawn so that, if firm 1 sets $q_1(P) = q_M$ (the monopoly output) for $P \geq P_1$, then the residual demand curve facing any firm $D(P) - q_M$ is to the left of the \textit{ex ante} average cost curve. No firm can get underneath firm 1’s supply curve because it is feasible for firm 1 to supply $q_M$ at prices below $P_c$. Thus it is a Nash equilibrium to get the monopoly outcome.\footnote{The above argument shows that there is a Nash equilibrium which creates large returns to being the first firm to sink a cost. Which firm is first? This could be modelled in the same way that [5] and [1] have modelled Research and Development. There the firm that is first to discover a new product gets excess returns due to patent protection.}

Figure 8 may seem peculiar because strategies like the indicated $\tilde{q}_2$ cannot make positive profit against $\tilde{q}_1$. When firm 2 chooses $\tilde{q}_2$, it cannot get enough customers to make a positive profit. All the customers stay with firm 1. Since entry is deterred, these customers end up paying $P_M$. If all the customers of firm 1 could get together and contract with firm 2, they would be better off and get a price of $P_2$.\footnote{Customers could also be better off if they acted like a monopsonist, rather than competitively, against each firm.} The existence of fixed costs creates an interesting externality across customers. If a single customer switches from firm 1 to firm 2, then firm 2 cannot cover its fixed costs. If many customers switch, then they each can be served by firm 2. As a consequence, there are multiple Nash equilibria with very different properties. For the technology in Figure 8, the competitive equilibrium can be achieved as a Nash equilibrium. To see this, let firms 1 and 2 choose the strategies in (15a), where $P_c$ is the minimum average cost of the \textit{ex ante} cost function. To see that this is Nash, note that each firm faces a residual demand

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure8.png}
\caption{Figure 8}
\end{figure}
curve like $D(P) - q_c$ which is nonpositive for $P \geq P_c$. Thus, for any $\tilde{q}_1$, $P(q^0_1/\tilde{q}_1) \leq P_c$. Note that the convexity of $V(\cdot)$, and $V'(q_c) = P_c$, implies that $P_c, q_c$ solves: max$_{P, q}[Pq - C(q)]$ subject to $P \leq P_c$ and $q \leq D(P)$. Hence firm 1's payoff is maximized by choosing a supply curve which goes through $P_c, q_c$. If firm 1 chooses $\tilde{q}^0_1$ in (15a), then firm 2 cannot make positive profit since $P(q^0_1/\tilde{q}_2) \leq P_c$ for all $\tilde{q}_2$. Thus $q^0_2$ is an optimal choice. The reader may verify that this argument is true for any $n_c$, so that Theorem 1 remains true if one firm has sunk costs.

4. WHAT HAPPENS WHEN A COMPETITIVE EQUILIBRIUM DOESN'T EXIST?

In this section, we return to the assumption that all firms have identical costs. It will not generally be the case that there will exist a competitive equilibrium when $K > 0$. However, there may be many Nash equilibria in supply functions.

In order to analyze this case, it is convenient to define the least a firm can produce at price $P$:

\begin{equation}
(21) \quad m(P) \equiv \min \left\{ q \mid \frac{C(q)}{q} \leq P \right\},
\end{equation}

and the most a firm can produce at $P$:

\begin{equation}
(22) \quad M(P) \equiv \max \left\{ q \mid \frac{C(q)}{q} \geq P \right\}.
\end{equation}

If we denote the point of minimum average cost by $P_m$, then $M(P_m) = m(P_m)$. We can now define an "approximate competitive equilibrium." Let $n_c$ be an integer which satisfies $n_c m(P_m) \leq D(P_m)$ and $(n_c + 1)m(P_m) > D(P_m)$. Let $q_c > 0$ be defined by $D(C'(q_c)) = n_c q_c$, and let $P_c \equiv C'(q_c)$. If $n_c$ firms choose $q_c$, they will be making strictly positive profit, while if $n_c + 1$ firms enter and price at marginal cost, they will make negative profit. The fact that firms make positive profit in the competitive equilibrium means that there will be some cases where a Nash equilibrium doesn't exist. There are a reasonable set of circumstances where $n_c, q_c, P_c$ can be supported as a Nash equilibrium in supply functions as indicated by the following theorem.

**Theorem 3:** Let $n \geq n_c + 1$, and suppose $m(P) \equiv D(P) - n_c \min(q_c, M(P))$ for all $P \geq P_m$; then there is a Nash equilibrium $q^0$ such that $n_c q_c \in \sum q^0_i(P_c)$.

**Proof:** Let

\begin{align}
(23a) \quad q^0_i(P) &= \begin{cases} 
\min(q_c, M(P)) & \text{for } P \geq P_m, \quad i = 1, 2, \ldots, n_c, \\
0 & \text{for } P \leq P_m,
\end{cases} \\
(23b) \quad q^0_i(P) &= \begin{cases} 
q_c & \text{for } P \geq P_c, \quad i = n_c + 1, \ldots, \\
0 & \text{for } P \leq P_c.
\end{cases}
\end{align}
To see that this is Nash, note that the residual demand facing any firm is perfectly elastic at $P_c$. Hence, for any $\tilde{q}_i$, $P(\tilde{q}_i^0/\tilde{q}_i) \equiv P_c$. Since $V(\cdot)$ is convex, this implies that operating where $V'(q) = P_c$ is best for firms $i = 1, 2, \ldots, n_c$. To see this, note that $P_c, q_c$ solves: max$_{q_i}$ $pq - C(q)$ subject to $P \leq P_c$ and $q \leq D(p) - \sum_{j \neq i} q_j^0(p)$. Since (23a) satisfies $q_i^0(P_c) = q_c$, (23a) is an optimal strategy. Consider firms $n_c + 1, n_c + 2, \ldots$. These firms face a residual demand $D(P) - n_c \min(q_c, M(P))$. By assumption this residual demand curve is to the left of their average cost curve. Hence there is no way these firms can make positive profit. So (23b) is an optimal strategy.

Q.E.D.

To understand the above theorem, let $n_c = 1$. For it to be Nash to have firm 1 set $q(P_c) = q_c$, and all other firms producing nothing at lower prices, it must be the case that no other firm can fit in (i.e., that residual demand is too small to cover fixed costs). A firm that attempts to undercut $P_c$ will have the most difficulty undercutting $P_c$ if firm 1 produces the maximum feasible amount, namely $M(P)$. We set $q_i(P) = \min(q_c, M(P))$ to keep the supply curve upward sloping. If $m(P) < D(P) - n_c \min(q_c, M(P))$ for some $P < P_c$, then $q_c$ need not be sustainable, because small firms can undercut.$^{16}$

It should be pointed out that the Nash equilibrium is not unique under the assumptions of the theorem. For example, if $n_c = 1$ and $m(P) \equiv D(P) - \min(q_M, M(P))$ for all $P \equiv P_m$, then the monopoly outcome $q_M$ will be a Nash equilibrium. (Let firm 1 set $q_i(P) = \min(q_M, M(P))$ for $P \equiv P_m$.)

5. BERTRAND EQUILIBRIUM VS. SUPPLY FUNCTION EQUILIBRIUM

There is a sense in which my equilibrium concept is similar to Bertrand: in both cases a firm can undercut its competitors and get some of their customers. The reader may wonder why I didn’t use Bertrand strategies, namely: let firm $i$’s strategy be a pair $P_i, q_i$ such that $P_i q_i - C_i(q_i) \equiv 0$. Let $P = (P_i)_{i=1}^n, q = (q_i)_{i=1}^n$ be chosen by firms. To define firm $i$’s residual demand, let $I_i(P, q) = \{j \mid P_j \leq P_i\}$ be the set of firms charging no more than firm $i$. Then residual demand is

$$Q_i(P, q) = \max \left(0, D(P) - \sum_{j \in I_i(P, q)} q_j \right).$$

Note that the residual demand in (24) differs from that in (5). If marginal cost is everywhere constant and $K = 0$, then (5) is quite appropriate. If a firm charges more than other firms, then that firm will get no customers. However, when cost curves are U-shaped, it is necessary to keep track of the output of all firms. If, say, firm 1 charges more than firm 2, then it will not, in general, be feasible for firm 2 to supply the whole market, so firm 1 will have some sales. This is what (24) captures.

Given residual demand in (24), each firm chooses $P_i, q_i$ to maximize profit subject to $q_i \leq Q_i(P, q)$. A Nash equilibrium is then defined in the standard way.

$^{16}$See [9] for a discussion of a similar problem pertaining to the sustainability of monopoly.
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The problem with this "Bertrand" Nash equilibrium is that it often fails to exist. The following example, which is similar to that of Edgeworth [3, pp. 111-140], shows the problem with (24) and compares Bertrand equilibrium to equilibrium in supply functions.\(^{17}\)

Unlike the previous sections where we assumed positive fixed costs, we now follow Edgeworth and assume no fixed costs, but instead assume that there is a capacity constraint. Thus let

\[
C(q) = \begin{cases} 
ceq & \text{for } 0 \leq q \leq 1, \\
\infty & \text{for } q > 1, 
\end{cases}
\]

where 1 is the capacity constraint. Let market demand be such that \(1 < D(c) < 2\), so that one firm cannot serve the whole market. Note that the competitive equilibrium is \(P = c\).

Assume that the number of potential firms is 2. Edgeworth noted that no Bertrand equilibrium exists. For let both firms charge \(c\); then if firm 1 raises its price to \(P > c\), its residual demand would be \(D(P) - q_2 \geq D(P) - 1\), since firm 2 cannot sell more than one unit. Hence firm 2 could make positive profit by raising its price above \(c\). Similarly, if both firms set the same price above \(c\), then one firm can increase its profit discontinuously by lowering its price a small amount. So it is not Nash for both firms to charge the same. It's not Nash to have different prices because the lower price firm will be better off by raising its price a little. This shows that there is no Bertrand equilibrium.

To find a Nash equilibrium in supply functions, consider a symmetric Cournot equilibrium \(q^*\). (If demand is linear there will always be a Cournot equilibrium.) Let each firm choose the supply function

\[
q(P) = \begin{cases} 
0, & P \leq c, \\
q^*, & P \geq c. 
\end{cases}
\]

This is a Nash equilibrium in supply functions because each firm's residual demand is the Cournot residual demand \(D(P) - q^*\) for \(P \geq c\).

It is interesting to note that the competitive equilibrium is not a Nash equilibrium in supply functions. This is because the residual demand facing firm 1 is \(D(P) - q_2(P) \geq D(P) - 1\) in a positive neighborhood of \(P = c\). Therefore firm 1 will always choose a supply curve which goes through its residual demand at a price \(P > c\). This shows that Nash equilibrium in supply functions can be quite different than the Bertrand equilibrium and the competitive equilibrium. The above example assumes \(K = 0\). It is easy to construct similar examples when \(K > 0\) and \(V'' > 0\). For example, the reader may verify that if \(n = n_c\) and (17) is satisfied, then a Bertrand equilibrium never exists. This is because if all firms charge \(P_c\) and produce \(q_c\), then one firm can raise its price a little and residual demand will be large enough for it to make a positive profit. (Of course, it cannot be a Bertrand equilibrium for firms to charge above \(P_c\).) The problem is

\(^{17}\)I am grateful to Andreu Mas-Colell for referring me to Edgeworth's example.
that, because of $U$-shaped costs, it is not feasible for the lowest priced firms to serve enough of the market to deter other firms from raising price.

6. CONCLUSIONS

The idea that fixed costs provide a barrier to entry is older than Modigliani's survey suggests. Though recent criticisms of this view have appeared (see, e.g., von Weizsacker [12] for some criticisms and a survey of entry barriers), it is still the generally accepted view. For example, Section 7 of the Clayton Act prohibits mergers or other forms of acquisition when the effect of the merger "may be substantially to lessen competition, or to tend to create monopoly." Enforcement and successful prevention of mergers under Section 7 have involved showing that the acquisition will lead a firm to have a market share above about 15%.\textsuperscript{18} In particular, as Scherer notes, the courts have simply used a market share cut-off to determine when a horizontal merger will lessen competition. This approach can be justified by Cournot-Nash models which show that market share (i.e., the inverse of the number of operating firms) is inversely proportional to the deviation of price from marginal cost.\textsuperscript{19}

Under Cournot-Nash strategies, a firm's residual demand is never perfectly elastic (unless it is infinitesimal), and it is for this reason that the competitive equilibrium is never a Cournot-Nash equilibrium. We have introduced supply function strategies to model the idea that a firm's residual demand can be infinitely elastic due to the threat of entry. We have shown that when a competitive equilibrium exists and there is free entry to a technology with strictly convex variable costs, then the Nash equilibrium in (upward sloping) supply functions will be the competitive equilibrium. Thus fixed costs need not be a barrier to entry. Further, the industry concentration ratio (or the inverse of the number of operating firms) may not be a good measure of monopoly power.

Supply function equilibria model threats in a manageable way (i.e., fewer equilibria are generated than in reaction function strategies). The threat of entry decreases the monopoly power of firms. However, there are situations where threats made by extant firms can deter entry. In particular, if one firm can profitably produce more than its pro rata share of the competitive output at the competitive price, then it can deter entry, and Nash equilibrium can consist of a price higher than the competitive price. In particular, the implicit threats carried by supply function strategies can deter entry if there are sunk costs. Thus our model suggests that it is sunk costs rather than fixed costs which may deter entry.

\textsuperscript{18}See [10, pp. 475–485] for a survey. Scherer points out that the Justice Department has rules calling for prosecution in industries where the top four firms produce 75 per cent of the output, whenever a merger involves the acquiring firm having more than 4 per cent and the acquired firm more than 4 per cent of the market share.

\textsuperscript{19}It is easy to show that under Cournot-Nash with free entry and fixed costs that $(P - C')/P = (1/n) \times (1/\eta)$, where $C'$ is marginal cost, $\eta$ is the elasticity of demand, and $n$ is the number of operating firms (determined by price = average cost); see, e.g., [2].
This is because if one firm "sinks" some costs, then its (ex post) average cost of production is lower than firms which have not yet sunk costs. Hence it can be feasible for it to produce more than its pro rata share of output at a price equal to ex ante minimum average cost.

We have also suggested that the mechanism by which entry can occur is via an entrant offering contracts to the customers of extant firms. If enough of the extant firm's customers agree to leave the extant firm, then the entrant can cover its fixed costs. However, the existence of fixed costs can create an "externality" across the potential customers of the entrant. The entrant can enter only if enough customers jointly agree to leave the existing firm.

The conclusion that entry deterrence is possible in industries with sunken costs (or flat-bottomed cost curves) derives from the assumption that firms which charge the same price at the competitive output level must share customers. An alternative assumption is that if, say, firm 1 has a supply curve everywhere to the right of firm 2 (i.e., firm 1 supplies q at a lower price than firm 2) for all prices above the competitive price, then firm 1 does not have to share its customers with firm 2. Under this alternative assumption, even sunk costs will not deter entry. This assumption was analyzed in an earlier version of this paper which is available on request from the author.

Industrial organization theory contains too many models of imperfect competition. Some method must be found to determine what is the right model for a particular situation. We have tried to suggest that customer behavior can provide a clue to the correct model. We noted that in a Cournot model the entrant is assumed to be unable to get any of the extant firm's customers, while in a Bertrand model it can get all of them. Much further research is needed to develop an explicit model of customer choice in the face of firms competing for their patronage. Such a model could be used to determine the ability of an extant firm to keep customers in the face of an entrant who has not yet sunk costs.

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