Notes on Market Structure

1. **Taxonomy of objects of study in a market**

   - Basic conditions
   - Structure
   - Conduct
   - Performance
   - Government Policy

   **Basic conditions**: technology, economies of scale, economies of scope, location, unionization, raw materials, substitutability of the product, own elasticity, cross elasticities, complementary goods, location, demand growth.

   **Structure**: number and size of buyers and sellers, barriers to entry, product differentiation, horizontal integration, vertical integration, diversification.

   **Conduct**: pricing behavior, research and development, advertising, product choice, collusion, legal tactics, long term contracts, mergers.

   **Performance**: profits, productive efficiency, allocative efficiency, equity, product quality, technical progress.

   **Government Policy**: antitrust, regulation, taxes, investment incentives, employment incentives, macro policies.

2. **The Structure-Conduct-Performance Model**

   Basic Conditions  
   Structure  
   Government Policy  
   Conduct  
   Performance
Costs

<table>
<thead>
<tr>
<th>Costs</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total costs:</td>
<td>( C(q) ) or ( TC(q) )</td>
</tr>
<tr>
<td>Variable costs:</td>
<td>( V(q) )</td>
</tr>
<tr>
<td>Fixed costs:</td>
<td>( F(q) = F, \text{ constant} )</td>
</tr>
<tr>
<td>( C(q) )</td>
<td>( F + V(q) )</td>
</tr>
<tr>
<td>Average total cost:</td>
<td>( ATC(q) = \frac{C(q)}{q} )</td>
</tr>
<tr>
<td>Average variable cost:</td>
<td>( AVC(q) = \frac{V(q)}{q} )</td>
</tr>
<tr>
<td>Average fixed cost:</td>
<td>( ATC(q) = \frac{F}{q} )</td>
</tr>
<tr>
<td>( ATC(q) )</td>
<td>( F/q + AVC(q) )</td>
</tr>
<tr>
<td>Marginal cost:</td>
<td>( MC(q) = C'(q) = \frac{dC}{dq} = V'(q) = \frac{dV}{dq} )</td>
</tr>
</tbody>
</table>

The marginal cost curve \( MC \) intersects the average cost curves \( ATC \) and \( AVC \) at their minimums. See Figure 1. Remember, \( MC \) is the cost of the last unit. When \( MC \) is below average cost (\( ATC \) or \( AVC \)), it tends to drive the average cost down, i.e. the slope of \( ATC \) (or \( AVC \)) is negative. See point A. When \( MC \) is above average cost (\( ATC \) or \( AVC \)), it tends to drive the average cost up, i.e. the slope of \( ATC \) (or \( AVC \)) is positive. See point B. Thus, \( AC \) and \( MC \) are equal at the minimum of \( AC \). The corresponding quantity is called the minimum efficient scale, MES.
3. Perfect Competition

In perfect competition, each firm takes price as given. Its profits are

$$\Pi = pq - C(q).$$

They are maximized at $q^*$ that is found from

$$p = MC(q^*).$$

This means that the quantity-price combinations that the firm offers in the market (its supply curve) follow the marginal cost line. Note that the firm loses money when it charges a price below min ATC. Therefore, for any price below min ATC, the firm shuts down and produces nothing. For any price above min ATC, the firm produces according to curve MC.
4. **Economies of Scale and Scope**

Let the quantity at minimum efficient scale be \( q_1 = \text{MES} \), and the corresponding average cost be \( p_1 = \text{min ATC} \). See Figure 2. Consider the ratio \( n_1 = D(p_1)/q_1 \). It shows how many firms can coexist if each one of them produces the minimum efficient scale quantity and they all charge minimum average cost. Since no firm can charge a lower price than \( \text{min(ATC)} \), \( D(p_1)/q_1 \) defines the maximum possible number of firms in the market. It defines an upper limit on the number of firms in the industry directly from structural conditions. If \( n_1 \) is large (above 20) there is a real possibility for perfect competition. If the number is small, the industry will be oligopolistic or a monopoly.

**Economies of scope** are exhibited when the existence of one line of production (and the extent of its use) creates savings in the costs of another line of production undertaken by the same firm.

5. **Monopoly**

To maximize profits, every firm tries to choose the quantity of output that gives zero marginal profits, which is equivalent to marginal revenue equals marginal cost.

\[
\Pi(q) = R(q) - C(q)
\]

\[
d\Pi/dq = MR(q) - MC(q) = 0.
\]

In general, revenue is

\[
R(q) = qp(q),
\]

where \( p(q) \) is the willingness to pay for quantity \( q \). Marginal revenue is

\[
MR(q) = p + q(dp/dq).
\]
In perfect competition, because a firm cannot influence the prevailing market price (and therefore \( dp/dq = 0 \)), its marginal revenue is exactly equal to price, \( MR(q) = p \). However, in monopoly \( dp/dq < 0 \) (because the demand is downward sloping), and therefore, for every quantity, marginal revenue is below the corresponding price on the demand curve, \( MR(q) < p(q) \). Therefore, the intersection of MR and MC under monopoly is at a lower quantity \( q_m \) than the intersection of MR (=\( p \)) and MC under perfect competition, \( q_c \). It follows that price is higher under monopoly, \( p_m > p_c \). Since surplus is maximized at \( q_c \), monopoly is inefficient. The degree of inefficiency is measured by the triangle of the dead weight loss (DWL). It measures the surplus difference between perfect competition and monopoly.
6. **Elementary Game Theory**

6.1 **Games in Extensive and in Normal (Strategic) Form**

Games describe situations where there is potential for conflict and for cooperation. Many business situations, as well as many other social interactions have both of these such features.

Example 1: Company X would like to be the only seller of a product (a monopolist). The existence of competing firm Y hurts the profits of firm X. Firms X and Y could cooperate, reduce total production, and increase profits. Or they could compete, produce a high quantity and realize
small profits. What will they do?

**Example 2:** Bank 1 competes with bank 2 for customers. Many of their customers use Automated Teller Machines (ATMs). Suppose that each bank has a network of its own ATM machines which are currently available only to its customers. Should bank 1 allow the customers of the other bank to use its ATMs? Should bank 1 ask for reciprocity?

**Example 3:** Computer manufacturer 1 has a cost advantage in the production of network "cards" (interfaces) of type 1. Similarly manufacturer 2 has an advantage in network "cards" of type 2. If they end up producing cards of different types, their profits will be low. However, each firm makes higher profits when it produces the "card" on which it has a cost advantage. Will they produce "cards" of different types? Of the same type? Which type?

A **game in extensive form** is defined by a set of players, i = 1, ..., n, a **game tree**, **information sets**, **outcomes**, and **payoffs**. The game tree defines the sequence and availability of moves in every decision node. Each decision node is identified with the player that decides at that point. We assume there is only a finite number of possible moves at every node. Each branch of the tree ends at an event that we call an **outcome**. The utility associated with the outcome for every player we call his **payoff**. Information sets contain one or more nodes. They show the extent of knowledge of a player about his position in the tree. A player only knows that he is in an information set, which may contain more than one nodes. Information sets allow a game of simultaneous moves to be described by a game tree, despite the sequential nature of game trees. A game where each information set contains only one point is called a game of **perfect information**. (Otherwise it is of **imperfect information**.) For example, in the "Incumbent-Entrant" game, at every point, each player knows all the moves that have happened up to that point. All the information sets contain only a single decision node, and the game is of perfect information. In the "Simultaneous Incumbent-Entrant" game, player I is not sure of player E's decision. He only knows that he is at one of the two positions included in his information set. It is as if players I and E
move simultaneously. This is a game of imperfect information. Note that this small change in the
information sets of player I makes a huge difference in what the game represents -- a simultaneous
or a sequential decision process.

When the utility functions associated with the outcomes are known to both players, the
game is of complete information. Otherwise it is of incomplete information. You may not know
the opponent’s utility function. For example, in a price war game, you may not know the value to
the opponent firm (or the opponent manager) of a certain loss that you can inflict on them.

A game in normal form is a summary of the game in extensive form. This is facilitated by
the use of strategies. A strategy for player \( i \) defines a move for this player for every situation
where player \( i \) might have to make a move in the game. A strategy of player \( i \) is denoted by \( s_i \),
and it belongs in the set of available strategies of player \( i \), \( S_i \). By its nature, a strategy can be very
complicated and long. For example, a strategy for white in chess would have to specify the opening
move, the second move conditional on the 20 alternative first moves of the black, the third move
conditional on the many (at least 20) alternative second moves of the black, and so on. The
advantage of using strategies is that, once each player has chosen a strategy, the outcome (and the
corresponding payoffs) are immediately specified. Thus, the analysis of the game becomes quicker.
Sequential Entrant-Incumbent Game

Enter: E

Stay out: I

Low Q: (8, 8)

High Q: (0, 18)

High Q: (-3, 6)

Simultaneous Entrant-Incumbent Game

Enter: E

Stay out: I

Low Q: (8, 8)

High Q: (0, 18)

High Q: (0, 9)

Low Q: (0, 9)
Example 1:  

**“Simultaneous Incumbent-Entrant”**

<table>
<thead>
<tr>
<th></th>
<th>High Q</th>
<th>Low Q</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Player 2</strong> (Incumbent)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Enter</td>
<td>(-3, 6)</td>
<td>(8, 8)</td>
</tr>
<tr>
<td><strong>Player 1</strong> (Entrant)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stay out</td>
<td>(0, 18)</td>
<td>(0, 9)</td>
</tr>
</tbody>
</table>

Strategies for Player 1: Enter, Stay out. Strategies for Player 2: High Q, Low Q

Example 2: **Prisoners’ Dilemma**

<table>
<thead>
<tr>
<th></th>
<th>silence</th>
<th>talk</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Player 2</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Silence</td>
<td>(5, 5)</td>
<td>(0, 6)</td>
</tr>
<tr>
<td><strong>Player 1</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Talk</td>
<td>(6, 0)</td>
<td>(2, 2)</td>
</tr>
</tbody>
</table>


6.2 **Non-Cooperative Equilibrium**

A pair of strategies \((s, s)\) is a **non-cooperative equilibrium** if and only if each player has no incentive to change his strategy provided that the opponent does not change his strategy. No player has an incentive to unilaterally deviate from an equilibrium position. This means that

\[
\Pi_1(s, s) \geq \Pi_1(s_1, s), \text{ for all } s_1 \in S_1, \text{ and }
\]

\[
\Pi_2(s, s) \geq \Pi_2(s, s_2), \text{ for all } s_2 \in S_2.
\]

Looking at the Prisoners' Dilemma, we see that if player 2 plays "silence" and is expected to
continue playing this strategy, player 1 prefers to play "Talk", since he makes 6 instead of 5 in the payoff. Since player 1 wants to deviate from it, (Silence, silence) is not a non-cooperative equilibrium. If player 2 plays "talk" and is expected to continue playing it, player 1 prefers to play "Talk", since he makes 2 instead of 0 in the payoff. Therefore (Talking, silence) is not a non-cooperative equilibrium. Finally, given that player 1 plays "Talk", player 2 prefers to play "talk" since he gets 2 instead of 0. Since both players prefer not to deviate from the strategy they play in (Talk, talk) if the opponent does not deviate from his strategy in (Talk, talk), this is a non-cooperative equilibrium.

In the "simultaneous Incumbent-Entrant" game, if E enters, the incumbent prefers to play L because his payoff is 8 rather than 6. If the incumbent plays L, the entrant chooses to play Enter, because he prefers 8 to 0. Therefore no player has an incentive to deviate from (Enter, Low Q), and it is a non-cooperative equilibrium. In the same game, if the entrant chooses to stay out, the incumbent replies by producing a high quantity (H) because 18 is better than 9. And, if the incumbent produces a high Q, the entrant prefers to stay out because 0 is better than -3. Therefore no player has an incentive to deviate from (Stay out, High Q), and it is also a non-cooperative equilibrium. Note that there are two equilibria in this game.

To find the equilibrium in the original sequential "Incumbent-Entrant" game of Figure 35, note that if player E enters, player I prefers Low Q, and ends up at (8, 8). If E stays out, I prefers High Q, and ends up at (0, 18). Seeing this, E chooses to enter because in this way he realizes a profit of 8 rather than of 0. Therefore the non-cooperative equilibrium is at (Enter, Low Q), and both firms realize a profit of 8. Note that the other equilibrium of the simultaneous game was eliminated.

6.3 Dominant Strategies

In some games, no matter what strategy player 1 plays, there is a single strategy that
maximizes the payoff of player 2. For example, in the Prisoners' Dilemma if player 1 plays "Silence", it is better for player 2 to play "talk"; and, if player 1 plays "Talk", it is better for player 2 to play "talk" again. Then "talk" is a dominant strategy for player 2. In the same game, note that "Talk" is a dominant strategy for player 1, because he prefers it no matter what player 2 plays. In a game such as this, where both players have a dominant strategy, there is an equilibrium in dominant strategies, where each player plays his dominant strategy. An equilibrium in dominant strategies is necessarily a non-cooperative equilibrium. (Why? Make sure you understand that no player wants to unilaterally deviate from a dominant strategy equilibrium.)

There are games with no equilibrium in dominant strategies. For example, in the simultaneous incumbent-entrant game, the entrant prefers to stay out if the incumbent plays "H". However, the entrant prefers to enter if the incumbent plays "L". Since the entrant would choose a different strategy depending on what the incumbent does, the entrant does not have a dominant strategy. (Similarly, check that the incumbent does not have a dominant strategy.) Therefore in the simultaneous incumbent-entrant game there is no equilibrium in dominant strategies.

6.4 Best Replies

Player 1's best reply to strategy \( s_2 \) of player 2 is defined as the best strategy that player 1 can play against strategy \( s_2 \) of player 2. For example, in the simultaneous incumbent-entrant game, the best reply of the entrant to the incumbent playing "High Q" is "Stay out". Similarly, the best reply of the entrant to the incumbent playing "Low Q" is "Enter". From the point of view of the incumbent, his best reply to the entrant's choice of "Enter" is "Low Q", and his best reply to the entrant's choice of "Stay out" is "High Q". Notice that at a non-cooperative equilibrium both players play their best replies to the strategy of the opponent. For example, at (Stay out, High Q), as we saw, "Stay out" is a best reply to "High Q", and "High Q" is a best reply to "Stay out". This is no coincidence. At the non-cooperative equilibrium no player has an incentive to deviate from the
strategy he plays. This means that his strategy is the best among all the available ones, as a reply to the choice of the opponent. This is just another way of saying that the player chooses the best reply strategy. Therefore at equilibrium each player plays a best reply strategy. This suggests that we can find equilibria by finding first the first reply strategies. We do this in the oligopoly games that we discuss next.

7. Oligopoly

7.1 Cournot Duopoly

When few firms are interacting in a market, they have to take into account the effects of each one’s actions on the others. We use game theory to analyze these strategic situations.

The simplest oligopoly model is due to Augustin Cournot (1838). There is one homogeneous good with demand function \( p(Q) \). Two competing firms, \( i = 1, 2 \), produce \( q_i \) each. \( Q = q_1 + q_2 \). The profit function of firm 1 is

\[
\Pi_1 = q_1 p(q_1 + q_2) - C_1(q_1).
\]

Cournot assumed that each firm assumes that (at equilibrium) its actions have no influence on the actions of the opponent. In game theoretic terms, his equilibrium was the non-cooperative equilibrium of a game where each player uses quantity as his strategy. By assuming that firm 1 has no influence on the output of firm 2, Cournot assumed that the residual
demand facing firm 1 is just a leftward shift by $q_2$ of the industry demand $p(Q)$. Firm 1 is a monopolist on this residual demand.

To find the non-cooperative equilibrium we first define the best reply functions. Maximizing $\Pi_1$ with respect to $q_1$ we get the best reply (or reaction function $R_1$) of player 1,

$$q = b_1(q_2).$$

Similarly, maximizing $\Pi_2$ with respect to $q_2$ we get the best reply (or reaction function) of player 2,

$$q = b_1(q_1).$$

The intersection of the two best reply functions is the non-cooperative equilibrium, $(q_1^*, q_2^*)$.

![Best Reply Functions and Cournot Equilibrium](image)
### 7.2 Cournot Oligopoly With n Firms

Let market demand be \( p(Q) \), where \( Q = q_1 + q_2 + \ldots + q_n \). Costs are \( C_i(q_i) \). Then profits of firm \( i \) are

\[
\Pi_i(q_1, q_2, \ldots, q_n) = q_ip(q_1 + q_2 + \ldots + q_n) - C_i(q_i).
\]

At the non-cooperative equilibrium, marginal profits for all firms are zero,

\[
\frac{\partial \Pi_i}{\partial q_i} = p(Q) + q_ip'(Q) - C_i'(q_i) = 0,
\]

\( i = 1, \ldots, n \).

We define the **market share** of firm \( i \) as

\[
s_i = \frac{q_i}{Q}.
\]

Remembering that the **market elasticity of demand** is

\[
\varepsilon = \frac{(dQ/Q)/(dp/p)}{(dQ/dp)(p/Q)},
\]

we can rewrite (1) as

\[
C_i'(q_i) = p[1 + \frac{q_i}{Q}(Q/p)(dp/dQ)], \text{ i.e., } C_i'(q_i) = p[1 + s_i/\varepsilon], \text{ i.e., }
\]

\[
(p - C_i'(q_i))/p = -s_i/\varepsilon = s_i/|\varepsilon|.
\]

This says that the relative price to marginal cost markup for firm \( i \) is proportional to the market share of firm \( i \), and is also inversely proportional to the market elasticity of demand.

### 7.3 Collusion

If firms were to collude, they would maximize total industry profits,
\[ \Pi = \sum_i \Pi_i = \sum_i [q_ip(q_1 + q_2 + \ldots + q_n) - C_i(q_i)] = Qp(Q) - \sum_i C_i(q_i). \]

To maximize these, each firm would set

\[ \frac{\partial \Pi}{\partial q_i} = 0 \Leftrightarrow p(Q) + q_i p'(Q) = C_i'(q_i), \]

i.e., market-wide marginal revenue equal to individual firm marginal cost.

At the collusive equilibrium there are incentives to cheat and produce more. To see that, consider the marginal profit of firm i if it produces an extra unit of output, starting at the collusive solution. It is

\[ \frac{\partial \Pi_i}{\partial q_i} = p(Q) + q_ip'(Q) - C_i'(q_i) = -(Q - q_i)p'(Q) > 0, \]

by substituting from (2'), and noting that \( p' < 0 \). Thus, firm i has an incentive to violate the collusive arrangement. The essential reason for this is that firm i's marginal revenue is higher than market-wide marginal revenue, i.e., \( p(Q) + q_i p'(Q) > p(Q) + Qp'(Q) \). Since firm i does not take into account the repercussions of its actions for the rest of the industry, starting from the collusive outcome, it has an incentive to increase output.

7.4 Cournot Oligopoly with Constant Marginal Costs

If marginal costs are constant, \( C_i'(q_i) = c_i \), then the equilibrium profits of firm i can be written as

\[ \Pi_i^* = q_i(p(Q) - c_i) = (p(Q) - c_i)Qs_i = pQ(s_i)^2/|\varepsilon|. \]

Then the total industry profits are

\[ \Pi^* = \sum_i \Pi_i^* = pQ(\sum_i (s_i)^2)/|\varepsilon|. \]
We define the **Herfindahl-Hirschman index of concentration** as the sum of the squares of the market shares,

\[
H = \sum_i (s_i)^2.
\]

Since market shares sum to one, \(\sum_i s_i = 1\), the \(H\) index is smaller for more egalitarian distribution of shares, and always lies between 0 and 1. Monopoly results in \(H = 1\), and \(n = 4\) results in \(H = 0\). For any fixed \(n\), \(H\) decreases as the distribution of shares becomes more egalitarian.

For Cournot oligopoly with constant marginal costs, we have from (4) and (5) that

\[
\Pi^* = HpQ/|\xi|,
\]

i.e., that the **industry equilibrium profits are proportional to the \(H\) index and to the total market sales, and inversely proportional to the market elasticity.**

For a symmetric equilibrium all firms have the same cost function (which do not necessarily have constant marginal costs), \(C_i(q_i) = C(q_i)\). Then \(q_i = Q/n\), so that \(s_i = 1/n\). From (2) we have

\[
(p - C')/p = -1/(n \xi) = 1/(n |\xi|).
\]

This says that if firms have the same marginal costs, the relative price to marginal cost markup is inversely proportional to the market elasticity of demand and to the number of competitors in Cournot oligopoly.

### 7.5 Cournot Oligopoly with Constant Marginal Costs and Linear Demand

For linear demand,
\[ p = a - bQ \]

and constant marginal costs, \( C'(q) = c \), optimization by firm i implies from (1),
\[ a - bQ - bQ/n = c, \text{ i.e., } a - c = bQ(n+1)/n, \text{ i.e.,} \]
\[ Q^* = [(a - c)/b][n/(n + 1)], \]
\[ q^* = [(a - c)/b]/(n + 1). \]

The price to marginal cost margin is
\[ p^* - c = a - c - bQ^* = (a - c)/(n + 1), \]
and the equilibrium profits are
\[ \Pi_i^* = (p^* - c)q^* = [(a - c)^2/b]/(n + 1)^2. \]

Therefore price and individual firm's production is inversely proportional to \( n + 1 \), but profits are inversely proportional to \( (n + 1)^2 \). Also note that total (industry) production \( Q^* \) increases in \( n \).

7.6 Leadership; Quantity Leadership

Firm 1, the leader, takes into account the response by firm 2, the follower, to its production decision. The follower acts as a typical Cournot oligopolist, assuming that changes in his production have no influence over the production of his opponent. Thus, the follower assumes, \( dq_1/dq_2 = 0 \). The leader calculates correctly the influence he has on the production of the follower,

\[ \frac{dq_2}{dq_1} = \frac{d(b_2(q_1))}{dq_1} \neq 0. \]

This means that for the follower
\[ \frac{\partial \Pi_2}{\partial q_2} = p(Q) + q_2p'(Q) - C_2'(q_2) = 0. \]
from which $b_2(q_1)$ can be calculated. The leader solves

\[ \frac{\partial \Pi}{\partial q_1} = p(Q) + q_1 p'(Q)(1 + dq_2/dq_1) - C_1'(q_1) = 0. \]

The term $dq_2/dq_1$ is negative, because firm 2 reduces output in response to an increase in output by firm 1. This reduces the weight that firm 1 puts on the negative term $p'$. Therefore firm 1 chooses a higher output in quantity leadership than in Cournot duopoly. In response, firm 2 chooses a lower output in quantity leadership than in Cournot duopoly.

### 7.7 Leadership; Price Leadership

In price leadership, the leader sets the price. The follower(s) act as a competitive firm(s). This means that a follower takes the price as given, and just chooses the output to maximize profits. The leader anticipates the actions of the follower. Since the follower(s) is a competitive firm, it can be described by its supply function $S_f(p)$. The leader is a monopolist on the residual demand $D_R$, which is the difference between the industry demand and the supply of the follower(s), $D_R(p) = D(p) - S_f(p)$. He derives the marginal revenue function $MR_R$, and finds the quantity that solves

\[ MR_R(q_{L}) = MC(q_{L}). \]

The corresponding price on $D_R$ is $p$, and for that price the follower(s) produce $q_{L}$. Note that if the leader and the follower have the same marginal costs, since the leader has to restrict his output (to achieve a high price) and the follower doesn't, the leader will have lower profits than the follower.
7.8 Cartels

A cartel is a collection of firms that coordinate output and price decisions. It is in the common interest of cartel members to restrict output and keep price high. There are two problems with the implementation of this intention. First, firms have to be allocated amounts (quotas) to produce. There can be a significant amount of conflict in the allocation of quotas, since, given a fixed price, each firm wants to produce more at the expense of others. Second, the collusive output is typically small, and the price high in relation to the non-cooperative equilibrium outcome. Thus, firms typically have incentives to break the quota agreement and produce higher quantities.

Consider a cartel that acts as a price leader. It sets the price (like OPEC), while the rest of
the industry accepts the price as given. Then each firm in the cartel produces less than an outsider. This explains the incentive of a cartel member to leave the cartel. While inside the cartel, each member has to restrict production so that the cartel is able set a high price. If a firm moved out of the cartel, and the price remained the same, then the defector would produce more and make higher profits. The catch is that usually, when a firm leaves the cartel, the ability of the cartel to set the price is diminished. Therefore higher profits for the defector are not guaranteed, since he can sell more, but at a lower price. It is possible that for some cartel size, the two opposite incentives balance each other. Thus, there exists the possibility that some cartels will be stable, with no firm wanting to enter, and no firm wanting to leave. Then in a non-cooperative game where firms take into account the effects of their moves (in or out) on the market price, we can have a positive equilibrium size of a cartel.

When it is not in the equilibrium situation discussed above, a cartel can use punishments to keep firms from deviating from the announced quotas. Typically, a violation of the quotas will appear as a reduction of the market price, without it being apparent who was the culprit who sold more than his quota. Faced with this situation, the cartel is unable to punish anyone in particular. It thus resorts to a collective punishment scheme. If the market price falls below a certain threshold level $p_1$, the cartel drops the price (or alternatively increases the quantity to a high level), and keeps it there for a number of time periods, say $t_1$. This is the collective punishment period. Note that all firms in the industry suffer, irrespective of if they participated in the cheating. After this period, the cartel goes back to the original high price $p_h$ and the associated quotas.

### 7.9 Cartel Stability

Consider a cartel as a price leader in an industry of $N$ firms. Let there be $K$ firms in the cartel. The remaining $N-K$ firms in the competitive fringe are price-takers. Let $a = K/N$ be the relative size of the cartel. Since the cartel is a price-leader, it sets the price for all the firms in the
industry. The firms in the competitive fringe take the price as given, and produce their optimal output.

We can solve for the price equilibrium, and write the resulting equilibrium profits for a firm in the cartel as $\Pi_c(a)$, and the profits of a fringe firm as $\Pi_f(a)$. Because the cartel has to restrict output to keep the price high, while the fringe is unrestricted, for any size $a$ of the cartel, a fringe firm will make higher profits than a cartel firm,

$$\Pi_f(a) > \Pi_c(a).$$

However, when a firm changes affiliation, the relative size of the cartel changes. For example, when a firm leaves the cartel, the size of the cartel changes from $a$ to $a - 1/N$. A firm will stay in the cartel if

$$\Pi_c(a) \geq \Pi_f(a - 1/N). \tag{10}$$

Similarly, a firm does not want to leave the fringe and join the cartel if

$$\Pi_f(a) \geq \Pi_c(a + 1/N). \tag{11}$$

If both (10) and (11) hold, then the cartel is called stable at size $a$. It can be shown that a stable cartel exists. If the horizontal distance between $\Pi_c(a)$ and $\Pi_f(a)$ increases, then the stable cartel size is unique.

### 7.10 Bertrand Competition

Bertrand (1881) proposed that a firm use prices as strategies in a game of simultaneous choice for a homogeneous good of demand $D(p)$. Then each firm has a strong incentive to undercut its opponent. Suppose fixed costs are zero, and both firms have the same marginal cost $c$. 
Let firm 2 play price $p_2$. What is the best reply of firm 1? If firm 1 plays $p_1 > p_2$, firm 1 gets zero demand, and zero profits, which doesn't look promising. If firm 1 plays $p_1 = p_2$, it gets half the industry demand and makes profits $\Pi_1(p_2, p_2) = (p_2 - c)D(p_2)/2$. If instead, firm 1 plays $p_1 = p_2 - \varepsilon$ it gets the whole industry demand, and makes profits

$$\Pi_1(p_2 - \varepsilon, p_2) = (p_2 - \varepsilon - c)D(p_2 - \varepsilon).$$

These are higher than $\Pi_1(p_2, p_2)$ for small $\varepsilon$. Why? Because the firm is doubling sales by charging very little less.

Formally, note that

$D(p_2 - \varepsilon) > D(p_2) = 2[D(p_2)/2].$ 

Therefore

$$\Pi_1(p_2 - \varepsilon, p_2) = (p_2 - \varepsilon - c)D(p_2 - \varepsilon) > 2(p_2 - \varepsilon - c)[D(p_2)/2].$$

Since the last term of the RHS and of $\Pi_1(p_2, p_2) = (p_2 - c)[D(p_2)/2]$ are the same, for undercutting to be profitable, it is enough to have

$$2(p_2 - \varepsilon - c) > p_2 - c \iff p_2 - c > 2\varepsilon.$$ 

As long as $p_2 > c$, firm 1 can choose an $\varepsilon > 0$ that fulfills (12). Therefore, if $p_2 > c$, firm 1 has an incentive to undercut firm 2. Therefore, there can be no non-cooperative equilibrium with $p_2 > c$. Is $p_1 = p_2 = c$ an equilibrium? Yes, because the undercutting argument above does not apply.
8. **Cournot's Duopoly of Complementary Goods**

Cournot's (1838) model of two firms selling complementary products provides the clearest introduction to markets of complementary products. In Cournot's model, firms A and B produce components A and B at zero marginal cost. Consumers combine these two components in fixed proportions (i.e. one unit of each) to form a composite product AB. Firms A and B sell these components at prices p and q respectively. See Figure 1a. Demand for the composite product is denoted by D(s) and depends on the sum s of the two component prices, s = p + q. We assume D(s) is continuous with a declining marginal revenue function. We follow Cournot by assuming that each firm chooses price to maximize profits, taking the price of the complementary product as given. Thus, in modern terminology, we solve for the non-cooperative equilibrium (i.e., the Nash equilibrium in prices).

\[
\begin{align*}
&\text{A} \quad p^I \\
&\quad s^I = p^I + q^I \\
&\text{B} \quad q^I
\end{align*}
\]

**Independent Ownership**

\[
\begin{align*}
&\text{A} \\
&\quad s' \\
&\text{B}
\end{align*}
\]

**Joint Ownership**

Figure 9a          Figure 9b

The profits of firms A and B can be written respectively as follows.

\begin{align*}
(13) & \quad \Pi_A = pD(s) = pD(p + q), \\
(14) & \quad \Pi_B = qD(s) = qD(p + q).
\end{align*}

Noting that s = p + q, and differentiating with respect to the own price, we have the first order conditions,
\begin{align*}
\partial \Pi_A / \partial p &= p' D(s) + D(s) = 0, \\
\partial \Pi_B / \partial q &= q' D(s) + D(s) = 0.
\end{align*}

These two equations define best-response functions $p = R_A(q)$ and $q = R_B(p)$ that can be solved for the Nash equilibrium. The equilibrium is easily characterized by summing equations (15) and (16) to define the equilibrium price $s^1$ of the composite good $AB$ as the solution of

\begin{equation}
\begin{aligned}
s^1 D'(s^1) + 2D(s^1) = 0.
\end{aligned}
\end{equation}

Equation (17) can be written in terms of elasticity as

\begin{equation}
\begin{aligned}
\varepsilon(s^1) &\equiv s^1 D'(s^1)/D(s^1) = -2.
\end{aligned}
\end{equation}

A vertically integrated monopolist has profits

\begin{equation}
\Pi(s) = sD(s),
\end{equation}

and upon maximization he will solve

\begin{equation}
\begin{aligned}
\partial \Pi / \partial s &= s' D'(s^1) + D(s^1) = 0.
\end{aligned}
\end{equation}

Equation (19) can be written in terms of elasticity as

\begin{equation}
\begin{aligned}
\varepsilon(s^1) &\equiv s^1 D'(s^1)/D(s^1) = -1.1 \quad 1
\end{aligned}
\end{equation}

\footnote{Note that for positive marginal costs $MC$, equations (17a) and (19a) can be written in terms of the Lerner index markups as}

\begin{align*}
\text{(17b)} \quad & L^1 = (s^1 - MC)/s^1 = 1/\varepsilon \\
\text{(19b)} \quad & L^1 = (s^1 - MC)/s^1 = 2/\varepsilon.
\end{align*}
See Figure 9b. Comparing equations (17) and (19), Cournot observed that the price for the composite good is lower under vertical integration,

\[(20) \quad s^1 > s^J,\]

and therefore \textit{vertical integration is socially beneficial}. Thus, we have the now standard intuition that vertical integration by the producers of complementary products is socially beneficial. Of course, it should be noted that this is a second-best result. The price charged by the integrated monopolist exceeds the perfectly competitive price \(s^O\) given by sum of marginal costs, that is, \(s^O = 0\). Thus,

\[s^O < s^J < s^1.\]