Personalized Pricing and Competition*

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Abstract

We study personalized pricing in a general oligopoly model. When the market structure is fixed, the impact of personalized pricing relative to uniform pricing hinges on the degree of market coverage. If market conditions are such that coverage is high, personalized pricing harms firms and benefits consumers, whereas the opposite is true if coverage is low. However, when the market structure is endogenous, personalized pricing benefits consumers because it induces socially optimal firm entry. Finally, when only some firms have data to personalize prices, consumers can be worse off compared to when either all or no firms personalize prices.

Keywords: personalized pricing, competition, price discrimination, consumer data

JEL classification: D43, D82, L13

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1 Introduction

Thanks to advances in information technology, many firms now have access to very rich consumer-level data which they have either collected themselves or acquired from data brokers. With the help of Artificial Intelligence (AI), firms are increasingly able to glean from this data information about individual consumers' preferences. This in turn makes it easier and easier for firms to do personalized pricing, i.e., charge different consumers different prices based on their preferences. Indeed, personalized pricing has already been documented in a wide range of industries including retail, travel, personal finance, mobile gaming and dating.[1] Even as far back as 2012, Staples and Home Depot were found to be using a consumer's browsing history and distance from a competitor's store to offer personalized prices on their websites.[2] In the last decade, firms' ability to personalize prices has likely grown significantly—as has their ability to do it surreptitiously, so as to avoid a potential consumer backlash. As a result, nowadays personalized prices are often concealed, e.g., as personalized discounts which are sent by email or smartphone app.[3]

Personalized pricing has attracted a lot of interest from policymakers in recent years, and its use by firms raises a number of important questions.[4] For example, it is natural to expect that personalized pricing will cause some consumers to pay more and others to pay less—but which consumers gain and which ones lose? And what is the impact on aggregate consumer surplus and firm profit? Taking a longer-term perspective, personalized pricing may also affect firms' incentives to enter a market, and hence impact the market structure. What are the implications of this for total welfare and consumer surplus? Moreover, in some industries, only certain firms have access to data that is needed to personalize prices. How does this data asymmetry affect market performance, and what would be the effect of policies that force data-rich firms to share data with their rivals?

In this paper we develop a general oligopoly model to assess the impact of personalized pricing. Personalized pricing is a type of very fine-tuned price discrimination, where small numbers of consumers with very similar preferences are grouped together and offered the

[1] See https://bit.ly/3A4Rk10 and https://bit.ly/38Ygzq6 for a history of personalized pricing, and OECD (2018), Which? (2018) and http://bit.ly/3E4nDBT for more details of the examples.

[2] See https://on.wsj.com/39sHIFf for further details.

[3] As an example, see https://bit.ly/37OftAc for how Kroger uses its mobile app to offer personalized coupons. There are several other ways that firms can disguise their use of personalized pricing. Certain products, such as financial products, are already somewhat personalized, so it is easy for a firm to also personalize their prices without consumers realizing (see, e.g., FCA, 2019). Even for more standardized products, firms can use "sticky targeting" whereby prices are fixed for all consumers (including the one being targeted) for a short period (Shiller, 2021), or can personalize rankings and search results in order to steer a consumer towards products with a certain price point (see, e.g., Hannak et al., 2014).

[4] See, e.g., reports by OECD (2018), European Commission (2018), Ofcom (2020) and BEIS (2021).

same price. In this paper we focus on the limit case, i.e., first-degree price discrimination, where firms can perfectly infer individual consumers' preferences (and so effectively each consumer has their own group). Although firms may never fully reach this limit case, they are likely to get closer and closer to it as they gain access to increasingly rich data and powerful AI algorithms.[5]

To set the scene, Section 2 reviews two well-known benchmarks from the literature. The first benchmark is monopoly: here personalized pricing allows the firm to extract all the social surplus, and so is good for the firm but bad for consumers. The second benchmark is the classic linear Hotelling model: in a seminal paper, Thisse and Vives (1988) show that personalized pricing (i.e., offering each consumer a different price based on their location on the Hotelling line) leads to a reduction in the price paid by *every* consumer. (Intuitively, each firm tries to poach consumers on its rival's "turf" with low prices, which then forces the rival to charge less to its customer base.) Therefore going from monopoly to duopoly completely reverses the impact of personalized pricing—it now harms firms but benefits consumers. This insight that personalized pricing intensifies competition has been very influential: as we discuss further in the literature review, the model of Thisse and Vives (1988) is an important building block for many subsequent papers, including the burgeoning literature on data privacy and data brokers.

The first contribution of this paper is to reconcile the opposing impacts of personalized pricing under monopoly and Hotelling duopoly. In Section 3 we introduce a discrete-choice model which nests both monopoly and Hotelling as special cases. There is an arbitrary number of (single-product) firms, and consumers' valuations for their products are drawn from a joint distribution. Consumers either buy one of the products or take an outside option. Our model is based on the classic random-utility model developed in Perloff and Salop (1985), but is more general because it allows for correlated product valuations and partial market coverage (i.e., some consumers may take the outside option). Under uniform pricing firms do not have or are banned from using information about individual consumers' preferences, and so offer all consumers the same price. Under personalized pricing firms know each consumer's valuations for all the products, and make personalized offers accordingly.

Section 4 compares market performance in these two regimes. We first study the short-run case where the market structure is fixed. Different from Thisse and Vives (1988), we show under a mild regularity condition that personalized pricing benefits some

[5] In this spirit, Council of Economic Advisers (2015) points out that the "increased availability of behavioral data has also encouraged a shift from third-degree price discrimination based on broad demographic categories towards personalized pricing." Similarly, Varian (2018) writes that "Fully personalized pricing is unrealistic, but prices based on fine grained features of consumers may well be feasible, so the line between third degree and first degree is becoming somewhat blurred."

consumers but *harms* others: consumers who regard their two best products as close substitutes pay less under personalized pricing, but consumers with a strong preference for one product end up paying more. Therefore, what determines which consumers suffer and which consumers benefit from personalized pricing is their *relative* preferences across products, which may not be correlated in a simple way with demographics such as income. Nevertheless, if the market is fully covered (i.e., if all consumers buy) under uniform pricing, competitive personalized pricing does lower industry profit and increase aggregate consumer surplus under a log-concavity condition.[6] We are therefore able to significantly generalize the aggregate welfare results from the classic Thisse and Vives (1988) paper.

We then show, however, that if the market is not fully covered under uniform pricing (which is arguably the more realistic case), the welfare impact of personalized pricing can be completely reversed: competitive personalized pricing can now increase industry profit and lower consumer surplus. For example, this always happens when valuations are independent across firms and exponentially distributed. For a more general valuation distribution, we prove that it happens when the market coverage is sufficiently low due to a very high production cost or a very good outside option. Furthermore, numerical examples suggest that the impact of personalized pricing follows a cut-off rule for common distributions such as the Extreme value (which gives the logit model) and the Normal (which gives the probit model). Specifically, when market conditions (e.g., production cost, outside option, or the number of firms) are such that coverage is sufficiently high, as expected the impact is similar to Thisse and Vives (1988). However when market conditions are such that coverage is sufficiently low the impact is reversed, and when coverage is intermediate personalized pricing benefits both consumers and firms.[7] These examples also suggest that we do not need a very low level of coverage to reverse the results from the full-coverage case: for instance, with the Extreme value distribution, when we use production cost to vary the level of coverage, personalized pricing benefits firms and harms consumers whenever less than about 80% of consumers buy.[8]

The intuition for why competitive personalized pricing can benefit firms and harm consumers when the market is partially covered is as follows. Partial coverage arises when the uniform price excludes some low-valuation consumers from the market. Personalized

[6] The log-concavity condition is needed for the existence of a pure-strategy pricing equilibrium under uniform pricing, and it also ensures that there are relatively few consumers with strong preferences compared to those with weak preferences.

[7] Note that if uniform pricing leads to only partial coverage, personalized pricing increases total surplus by expanding the market. This explains why profit and consumer surplus can both increase.

[8] Different from Thisse and Vives (1988), in our numerical examples the impact of personalized pricing is often qualitatively the same under both monopoly and duopoly; more broadly, it can raise industry profit and harm consumers even when the number of firms is relatively large.

pricing brings at least some of these consumers into the market, because firms can offer them low prices. However, since these consumers have low valuations the positive effect on their surplus is relatively small. On the other hand, consumers who bought under uniform pricing have a high valuation for at least one product, and so relatively many of these consumers have a strong preference for one product over another—meaning that personalized pricing can raise the average price they pay. When this happens, personalized pricing can make consumers worse off overall, even though it expands demand.[9]

We also study the long-run case where the market structure is endogenous. Specifically, we consider a free-entry game where firms choose whether or not to pay a fixed cost to enter the market, and then engage in price competition. We show that if the entry of a new product does not change consumers' preferences over existing products, then with personalized pricing the new entrant fully extracts the increase in match efficiency caused by its entry. Consequently, in the long run, personalized pricing leads to the socially optimal market structure. If we ignore integer constraints, this implies that personalized pricing must benefit consumers in the long run relative to uniform pricing.

In Section 5 we consider the case of asymmetrically informed firms, where only some firms have consumer data to price discriminate. We show that this "mixed" case can be worse for consumers than the symmetric cases where either all or no firms personalize prices. Intuitively, when a firm with consumer data competes with other firms that can only do uniform pricing, it is able to "poach" some consumers for whom it is not their favorite product via a low personalized price. This results in match inefficiency compared to the symmetric cases and can make consumers worse off in aggregate. This suggests that it is sometimes desirable (from consumers' point of view) to force a seller with superior information to share its data with its competitors or prevent it from personalizing prices.

Section 6 considers an alternative information structure under which each firm in the regime of personalized pricing only observes a consumer's valuation for its *own* product. We show that this case resembles a first-price auction, while the case discussed above (where firms observe a consumer's valuation for each product) resembles a second-price auction. Hence, if valuations are IID across products, the well-known revenue equivalence theorem from auction theory implies that these two information structures lead to the same market outcome, and consequently the impact of personalized pricing under the alternative information structure remains unchanged. (We note, however, that uniform pricing has no counterpart in the auctions literature.) Section 7 concludes.

[9] If marginal cost is high enough, each firm will also face a "monopoly segment" of consumers who value only its product above cost. This gives firms some monopoly power and is an additional (familiar) force for personalized pricing to benefit firms and harm consumers. However, this additional monopoly force is not necessary for reversing the impact of personalized pricing relative to the full-coverage case.

Related literature The literature on price discrimination is extensive, but it mainly focuses on imperfect price discrimination. (See the survey papers by Varian, 1989; Armstrong, 2007; Fudenberg and Villas-Boas, 2007; and Stole, 2007.)[10] One exception is the study of spatial price discrimination, where firms can charge customers in different locations different prices. An influential paper within this literature is Thisse and Vives (1988), which can also be reinterpreted as a model of competitive personalized pricing. They consider a two-stage game where firms first choose whether or not to price discriminate and then compete in prices. Using a Hotelling model with uniformly distributed consumers, they show that discriminatory pricing is a dominant strategy for each firm, and so the unique equilibrium features price discrimination. When firms have the same cost, as discussed earlier, they are trapped in a Prisoner's dilemma because every personalized price is below the uniform price.[11]

The Thisse and Vives (1988) framework has been widely used in the subsequent literature. For example, Shaffer and Zhang (2002) use it to study personalized pricing when one firm has a brand advantage over the other, while Chen and Iyer (2002) use it to study personalized pricing when firms first need to advertise to reach consumers. Montes, Sand-Zantman, and Valletti (2019) use it to study whether a monopolistic data intermediary should sell data to one or both competing firms who can use the data to conduct personalized pricing. Chen, Choe, and Matsushima (2020) use it to study consumer identity management which helps consumers avoid being exploited by firms via personalized pricing. In all these studies, an implicit underlying assumption is that competitive personalized pricing in the benchmark case intensifies competition, harms firms and benefits consumers. Our paper shows that this is not necessarily true in a more general model which allows for partial market coverage.[12]

Our paper is closely related to Anderson, Baik, and Larson (2021) (ABL henceforth), who also use a general discrete-choice framework to study competitive personalized pric-

[10] Since perfect price discrimination is the limit case of third-degree price discrimination, our paper is more related to competitive third-degree price discrimination. However, the approaches in that literature (e.g., the idea of best-response asymmetry in Corts, 1998 or the indirect utility approach in Armstrong and Vickers, 2001) are not directly useful for studying our problem.

[11] When firms have different costs, the low-cost firm can earn more than under uniform pricing, but within the parameter range in Thisse and Vives (1988) industry profit is still lower and consumer surplus is still higher under discriminatory pricing.

[12] Jullien, Reisinger, and Rey (2022) develop a generalized Hotelling setup, and show for example that personalized pricing can raise profit when a manufacturer competes with a retailer that sells its product. Lu and Matsushima (2022) consider a Hotelling setup where consumers can buy from both firms. When the additional utility gain from buying a second product is sufficiently high, firms are close to being monopolists, and hence personalized pricing benefits firms and harms consumers.

ing.[13] One important difference is that they have full market coverage—whereas our paper allows for partial market coverage, and emphasizes that this can qualitatively change the impact of personalized pricing. Another important difference is that in our paper firms can freely offer personalized prices, leading to a relatively simple pure-strategy pricing equilibrium; ABL, by contrast, assume that it is costly for firms to send targeted discounts, which leads to a mixed-strategy equilibrium in both pricing and advertising.[14] Our modeling choice captures the idea that the cost of making personalized offers is mainly a fixed one, due to investments in buying consumer data and developing AI tools. Moreover ABL do not consider the long-run impact of personalized pricing, or the case with asymmetrically informed firms.

There is also growing empirical research on personalized pricing. One strand looks for evidence of personalized pricing. As discussed earlier, detecting personalized pricing is hard because sellers have incentives to disguise personalized offers. Nevertheless Hannak et al. (2014) find evidence of some form of personalization on 9 out of 16 e-commerce sites in their study, while Aparicio, Metzman, and Rigobon (2021) document evidence that increasing use of algorithmic pricing is associated with increasing price differentiation (for the same product at the same time but across different delivery zipcodes). The other strand of the empirical literature assesses the impact of personalized pricing (see, e.g., Waldfogel, 2015; Shiller, 2020; Kehoe, Larsen, and Pastorino, 2020; and Dube and Misra, 2021). For instance, Shiller (2020) shows that if Netflix could use rich consumer-level web-browsing data to implement price discrimination, its profit could increase by about 13%, while the profit improvement would be tiny if it only relied on demographic information.

2 Two Benchmarks

In this section we briefly recap two well-known benchmarks from the existing literature.

Monopoly The impact of personalized pricing (or first-degree price discrimination) under monopoly is straightforward. Suppose consumers wish to buy at most one unit of a product, and have heterogeneous valuations for it. Under uniform pricing, the firm sets a standard monopoly price: consumers who value the product more than this price

[13] See also Section 4 of Ali, Lewis, and Vasserman (2021) which uses a similar oligopoly discrete-choice model with full market coverage to study endogenous consumer information disclosure. They show that the Thisse and Vives (1988) welfare result continues to hold in a partial revelation equilibrium.

[14] Such randomized offers cause consumers to sometimes buy the wrong product, which harms match efficiency. As a result, in ABL personalized pricing can make both firms and consumers worse off compared to uniform pricing. In contrast, in our model it is possible that both firms and consumers are better off under personalized pricing as it can expand demand.

buy and obtain positive surplus, while all other consumers are excluded from the market. Under personalized pricing, each consumer with a valuation above marginal cost is offered a personalized price equal to their valuation, and they all buy. As a result, total surplus is maximized but it is fully extracted by the monopolist. Personalized pricing therefore increases total welfare and firm profit but reduces consumer surplus.

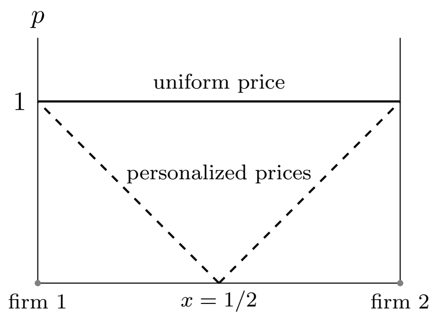
Hotelling duopoly The other well-known case is the linear Hotelling model studied by Thisse and Vives (1988). Suppose consumers are uniformly distributed along a unit-length Hotelling line. The two firms have cost normalized to zero, with firm 1 located at the leftmost point on the line and firm 2 located at the rightmost point. A consumer with location x values firm 1's product at $v_1 = V - x$ and firm 2's product at $v_2 = V - (1-x)$, where V is large enough that the market is fully covered in equilibrium. Under uniform pricing firms set the standard Hotelling price of 1. Under personalized pricing the firms compete for each consumer individually. Consumers with location $x < 1/2$ prefer product 1, while consumers with location $x > 1/2$ prefer product 2. Firms therefore engage in asymmetric Bertrand competition, resulting in the following equilibrium price schedules:

$$\begin{array}{l} p_1(x) = v_1 - v_2 = 1 - 2x \;\text{ and }\; p_2(x) = 0 \;\text{ for }\; x \in [0, \frac{1}{2}] \\ p_1(x) = 0 \;\text{ and }\; p_2(x) = v_2 - v_1 = 2x - 1 \;\text{ for }\; x \in [\frac{1}{2}, 1] \end{array}, \tag{1}$$

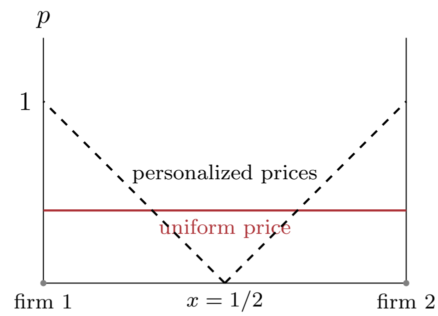
where $p_i(x)$ is the price offered by firm $i = 1, 2$ to the consumer at x. Each consumer buys her preferred product, and those with stronger preferences pay more. Figure 1a depicts the uniform price (the solid curve) and personalized prices paid by different consumers at different locations (the dashed curve). Note that each consumer pays (weakly) less under personalized pricing because $p_i(x) \leq 1$. Personalized pricing therefore harms firms and benefits consumers—the opposite of what happens under monopoly. (Under both uniform and personalized pricing the market is fully covered and consumers buy their preferred product, so personalized pricing has no impact on total welfare.)

However, the result that each personalized price is lower than the uniform price can easily be overturned. To see this, suppose instead that consumers are distributed along the Hotelling line according to a symmetric and strictly log-concave (so single-peaked) density. Personalized prices are *independent* of the distribution and so are the same as in (1), but the uniform price, which equals 1 over the density of consumers at $x = 1/2$, is now strictly below 1. Hence, as depicted in Figure 1b, consumers near the middle of the line (with relatively weak preferences) still pay less under personalized pricing, but those near the two ends of the line (with relatively strong preferences) now pay more.[15] The impact of personalized pricing on industry profit and aggregate consumer surplus is then less clear, once we move to this more general Hotelling model.

[15] Armstrong (2007) makes the same point by considering a specific non-uniform distribution.



(a) Uniform distribution



(b) Single-peaked distribution

Figure 1: The impact of personalized pricing in the Hotelling model

Moreover, even this more general Hotelling model is somewhat special: for example, the market is fully covered, and there are only two firms. Therefore in the next section we develop a more general model, which includes Hotelling as a special case, and use it to study the welfare impact of personalized pricing. We will show that by maintaining full market coverage, we can significantly generalize the aggregate welfare results in Thisse and Vives; by relaxing full market coverage, however, we can obtain very different conclusions—competitive personalized pricing can benefit firms and harm consumers in aggregate. Moreover, because we allow for more than two firms, we will be able to endogenize the number of firms in the market and thus study the long-run impact of personalized pricing.

3 A General Oligopoly Model

There are n competing firms in a market, each supplying a differentiated product at constant marginal cost c. There is also a unit mass of consumers, each wishing to buy at most one unit of one of the products. If a consumer buys nothing she obtains an outside option with zero surplus. Consumers perfectly know their own valuations for the n products, which are denoted by $\mathbf{v} = (v_1, ..., v_n) \in \mathbf{R}^n$. In the population $\mathbf{v}$ is distributed according to an *exchangeable* joint cumulative distribution function (CDF) $\tilde{F}(\mathbf{v})$, with corresponding density function $\tilde{f}(\mathbf{v})$. (The exchangeability means that any permutation of $(v_1, ..., v_n)$ has the same joint CDF; it implies that there are no systematic quality differences across products.) We assume that the density function $\tilde{f}(\mathbf{v})$ is everywhere finite and differentiable. Let F and f be respectively the common marginal CDF and density function of each v_i, and let $[\underline{v}, \overline{v}]$ be its support, where infinite valuation bounds are allowed.[16] To ease the exposition, we assume that $\tilde{F}$ has full support on $[\underline{v}, \overline{v}]^n$, but this is not crucial for the main results. To ensure an active market, we assume $c < \overline{v}$.

We consider two different pricing regimes. Under uniform pricing, firms set the same price for every consumer (e.g., because they do not have access to data on consumer preferences). Under personalized pricing, firms perfectly observe each consumer's vector of valuations $\mathbf{v} = (v_1, ..., v_n)$ and offer them a personalized price. In either regime, after seeing the prices consumers choose the product with the highest surplus (if positive); if consumers are indifferent between several offers they choose the product with the highest valuation (so that total welfare is maximized). Both firms and consumers are risk neutral.

Remarks. We will see that the degree of market coverage (i.e., how many consumers buy) affects the impact of personalized pricing. We have chosen to normalize the outside option, and to vary the degree of coverage via the marginal cost c. However the same qualitative insights obtain if instead we normalize the production cost and vary the outside option.

We assume that either all firms do uniform pricing, or all firms do personalized pricing. In the latter case we also assume that firms observe each consumer's vector of valuations $\mathbf{v} = (v_1, ..., v_n)$. However in Section 5 we will study the case where only some firms can do personalized pricing, and in Section 6 we will consider an alternative information structure where under personalized pricing firm i observes only v_i.

We allow a consumer's valuations for different products to be correlated, but sometimes we focus on the IID case where the v_i's are independent across products (which is the leading case in the literature on random-utility oligopoly models). Our analysis below does not explore how the degree of correlation affects the impact of personalized pricing, but we provide some discussion about this in the Online Appendix.

Notation. It will be convenient to introduce the following notation. Let $G(\cdot|v_i)$ and $g(\cdot|v_i)$ be respectively the CDF and density function of $\max_{j \neq i}\{v_j\}$, the valuation for firm i's best competing product, conditional on v_i. Let $v_{n:n}$ and $v_{n-1:n}$ denote the highest and second-highest order statistics of $(v_1, ..., v_n)$, and let $F_{(n)}(v)$ and $F_{(n-1)}(v)$ be their respective CDFs. Then

$$F_{(n)}(v) = \tilde{F}(v, ..., v) = \int_{\underline{v}}^{v} G(v|v_i) dF(v_i) \ , \tag{2}$$

and

$$F_{(n-1)}(v) = F_{(n)}(v) + n \int_{v}^{\overline{v}} G(v|v_i) dF(v_i) \ . \tag{3}$$

[16]In the duopoly case, our set-up nests Hotelling with a symmetric location distribution if v_1 and v_2 are large enough (to cover the market) and we treat $v_1 - v_2$ as a consumer's location. For any location distribution, there is at least one correlation structure over (v_1, v_2) that generates it.

To understand $F_{(n-1)}(v)$, notice that for the second-highest valuation to be below v, either all the v_i's must be less than v, or exactly one of them must be above v and the others be below v. Let $f_{(n)}(v)$ and $f_{(n-1)}(v)$ be the associated density functions. In the IID case we have $G(v|v_i) \overset{\text{IID}}{=} F(v)^{n-1}$, $F_{(n)}(v) \overset{\text{IID}}{=} F(v)^n$ and

$$F_{(n-1)}(v) \overset{\text{IID}}{=} F(v)^n + n(1-F(v))F(v)^{n-1} \ .$$

In order to solve the uniform pricing game, it is useful to define the random variable

$$x_z \equiv v_i - \max_{j \neq i}\{z, v_j\} \ , \tag{4}$$

where z is a constant. Since $x_z = v_i - z - \max_{j \neq i}\{0, v_j - z\}$, one can interpret it as a consumer's preference for product i relative to the best alternative (including the outside option) when all products are sold at price z. Let $H_z(x)$ and $h_z(x)$ be respectively the CDF and density function of x_z. More explicitly,

$$1 - H_z(x) = \Pr[v_i - x > \max_{j \neq i}\{z, v_j\}] = \int_{z+x}^{\overline{v}} G(v_i - x|v_i) dF(v_i) \ , \tag{5}$$

and

$$h_z(x) = G(z|z+x)f(z+x) + \int_{z+x}^{\overline{v}} g(v_i - x|v_i) dF(v_i) \ . \tag{6}$$

When z is irrelevant (i.e., when $z \leq \underline{v}$), let $H(x)$ and $h(x)$ be respectively the CDF and density function of $x \equiv v_i - \max_{j \neq i}\{v_j\}$; we use them for the case of full market coverage.[17]

3.1 Uniform pricing

We first study the regime of uniform pricing, where firms are unable to price discriminate. We focus on a symmetric pure-strategy pricing equilibrium, and let p denote the equilibrium uniform price.[18] Using the definition of x_z and $H_z(x)$ in equations (4) and (5), when firm i unilaterally deviates to a price p_i its deviation demand is

$$\Pr[v_i - p_i > \max_{j \neq i}\{0, v_j - p\}] = \Pr[v_i - \max_{j \neq i}\{p, v_j\} > p_i - p] = 1 - H_p(p_i - p) \ ,$$

and its deviation profit is $(p_i - c)[1 - H_p(p_i - p)]$. It is clear that a firm will never set a price below marginal cost c or above the maximum valuation $\overline{v}$.

To ensure that the uniform pricing equilibrium is uniquely determined by the first-order condition, we make the following assumption:

[17] Anderson, Baik, and Larson (2021) also use such notation to simplify demand expressions when the market is assumed to be fully covered.

[18] If the joint density $\tilde{f}$ is log-concave, the pricing equilibrium is unique and symmetric in the duopoly case (Caplin and Nalebuff, 1991) and in the IID case (Quint, 2014).

Assumption 1. $1 - H_z(x)$ *is log-concave in* x *and* $\frac{1-H_z(0)}{h_z(0)}$ *is non-increasing in* z.

In the Online Appendix we report some primitive conditions under which this assumption holds. For example, the first condition holds if the joint density $\tilde{f}$ is log-concave (Caplin and Nalebuff, 1991), and both conditions hold in the IID case with a log-concave f. Assumption 1 also holds in the Hotelling case (see footnote 16) provided that $v_1 - v_2$ has a log-concave density. In the rest of the paper, we suppose that Assumption 1 holds whenever uniform pricing is involved.

Given the first condition in Assumption 1, firm i's deviation profit is log-concave in p_i, and so the equilibrium price p must solve the first-order condition

$$p - c = \phi(p) \ , \tag{7}$$

where

$$\phi(p) \equiv \frac{1 - H_p(0)}{h_p(0)} = \frac{\int_p^{\overline{v}} G(v|v)dF(v)}{G(p|p)f(p) + \int_p^{\overline{v}} g(v|v)dF(v)} \ . \tag{8}$$

To interpret this, note that $1 - H_p(0)$ is each firm's equilibrium demand, while $h_p(0)$ is the absolute value of the equilibrium demand slope and it measures how many consumers are marginal for each firm. (The first term in $h_p(0)$ captures the extensive margin when a firm raises its price, and the second term captures the intensive margin due to competition.)

Given the second condition in Assumption 1, $\phi(p)$ is non-increasing (and constant for $p \le \underline{v}$), so the equilibrium price is unique. We can then show the following result. (All omitted proofs in this section and Section 4 are available in the Appendix.)

Lemma 1. *(i) If* $c \le \underline{v} - \phi(\underline{v})$, *the equilibrium uniform price satisfies*

$$p - c = \phi(\underline{v}) = \frac{1/n}{\int_{\underline{v}}^{\overline{v}} g(v|v)dF(v)} \tag{9}$$

and $p \le \underline{v}$, *such that the market is fully covered in equilibrium.*
(ii) Otherwise, the equilibrium uniform price uniquely solves (7) and $p > \underline{v}$, *such that the market is not fully covered in equilibrium.*

Lemma 1 shows that the market is partially covered under uniform pricing whenever the marginal cost c is sufficiently high. Note that a sufficient (but by no means necessary) condition is $c \ge \underline{v}$, i.e., some consumers value a product less than marginal cost.

The literature on random-utility oligopoly models usually studies the IID case, such that $G(v|v) \overset{\text{IID}}{=} F(v)^{n-1}$ and $g(v|v) \overset{\text{IID}}{=} (n-1)F(v)^{n-2}f(v)$, and so

$$\phi(p) \overset{\text{IID}}{=} \frac{[1 - F(p)^n]/n}{F(p)^{n-1}f(p) + \int_p^{\overline{v}} f(v)dF(v)^{n-1}} \ . \tag{10}$$

Most papers further assume that the market is fully covered (e.g., Perloff and Salop, 1985; Gabaix et al., 2016; Anderson, Baik, and Larson, 2021), in which case $p \leq \underline{v}$ and therefore $\phi(p)$ simplifies to $1/[n \int_{\underline{v}}^{\overline{v}} f(v) dF(v)^{n-1}]$.[19]

Industry profit under uniform pricing is

$$\Pi_U \equiv n(p-c)[1-H_p(0)] = n\frac{[1-H_p(0)]^2}{h_p(0)} \ , \tag{11}$$

where we have used the equilibrium price condition (7). (Note that the equilibrium demand $1 - H_p(0)$ can be also written as $\frac{1}{n}[1 - F_{(n)}(p)]$ due to firm symmetry.) Since all consumers buy their favorite product as long as it gives them a positive surplus, (aggregate) consumer surplus is

$$V_U \equiv \mathbb{E}[\max\{0, v_{n:n} - p\}] = \int_p^{\overline{v}} (v-p) dF_{(n)}(v) = \int_p^{\overline{v}} [1 - F_{(n)}(v)] dv \ , \tag{12}$$

where the last equality uses integration by parts. Note that the expressions for Π_U and V_U are valid regardless of whether or not the market is fully covered.

3.2 Personalized Pricing

Now consider the regime where firms perfectly observe each consumer's vector of valuations $\mathbf{v} = (v_1, ..., v_n)$ and set personalized prices accordingly. In this case, firms engage in asymmetric Bertrand competition for each consumer. To rule out uninteresting equilibria, we assume that firms do not price below cost (which is a weakly dominated strategy).

To solve for equilibrium prices, consider a consumer who values, say, firm 1's product the highest and firm 2's product the second highest. There are three cases. First, if $v_1 \leq c$, the consumer takes the outside option and so without loss we can set all prices to c. Second, if $v_1 > c \geq v_2$, firm 1 is effectively a monopolist and so charges v_1, and without loss all other prices can be set to c. Third, if $v_1 > v_2 > c$ then firms 1 and 2 compete for the consumer. Firm 2 Bertrand competes down to c, while firm 1 charges $c + v_2 - v_1$ and wins the consumer. The prices of the other $n - 2$ firms can be set to c without loss.

Therefore, we can focus on the following pricing equilibrium:

Lemma 2. *Under personalized pricing, firm i's equilibrium pricing schedule is:*

$$p(v_i, \mathbf{v}_{-i}) = \begin{cases} c + \underbrace{v_i - \max_{j \neq i}\{c, v_j\}}_{x_c} & \text{if } v_i \geq \max_{j \neq i}\{c, v_j\} \\ c & \text{otherwise} \ , \end{cases} \tag{13}$$

[19] An exception is Section 4.2 of Zhou (2017), which shows that in the IID case $\phi(p)$ in (10) is decreasing and the equilibrium price decreases in n if f is log-concave.

where $\mathbf{v}_{-i}$ denotes a consumer's valuations for all products other than i. The market is fully covered if and only if $c \leq \underline{v}$.

Intuitively, if a firm's product is a consumer's favorite and she values it above cost, the firm charges the consumer a price equal to the difference between her valuation for its product and that of the best alternative (which is either the outside option, or the second-best product sold at marginal cost). Since all consumers buy their favorite product or take the outside option in equilibrium, the market is fully covered whenever $c \leq \underline{v}$. This condition for full coverage is weaker than the one under uniform pricing. Therefore, if the market is fully covered under uniform pricing, it is also fully covered under personalized pricing.

To calculate profit, notice from (13) that when $x_c \geq 0$ firm i sells to the consumer and earns margin $p(v_i, \mathbf{v}_{-i}) - c = x_c$, whereas when $x_c < 0$ firm i does not sell to the consumer. Hence firm i's equilibrium profit is $\int_0^\infty x dH_c(x)$, and industry profit is

$$\Pi_D = n \int_0^\infty x dH_c(x) = n \int_0^\infty [1 - H_c(x)] dx \ . \tag{14}$$

Consumers always buy their favorite product provided they value it above c. Given the equilibrium pricing schedule in (13), it is clear that a consumer's favorite firm, say firm i, sets its price such that $v_i - p(v_i, \mathbf{v}_{-i}) = \max_{j \neq i}\{0, v_j - c\}$, i.e., the consumer is indifferent between its product and the next best option. Therefore, consumer surplus under personalized pricing is

$$V_D \equiv \mathbb{E}[\max\{0, v_{n-1:n} - c\}] = \int_c^{\overline{v}} (v - c) dF_{(n-1)}(v) = \int_c^{\overline{v}} [1 - F_{(n-1)}(v)] dv \ . \tag{15}$$

Note that the expressions for Π_D and V_D are valid regardless of whether or not the market is fully covered. Comparing V_D and V_U, there is a trade-off: in the personalized-pricing equilibrium it is *as if* consumers buy the second-best product at price c, while in the uniform-pricing equilibrium consumers buy the best product at the uniform price $p > c$.

4 The Impact of Personalized Pricing

We now examine how a shift from uniform to personalized pricing affects market performance. We first study the short-run impact, when the number of firms n is taken as given. (Since the monopoly case is trivial, our analysis focuses on $n \geq 2$.) We then study the long-run impact, when n is determined by firms' free-entry decisions.

4.1 The short-run impact with a fixed market structure

Suppose the number of firms n is fixed. Our first result shows that, under a mild regularity condition, the highest personalized price exceeds the uniform price, and hence personalized pricing harms some consumers. Recall that $h(x)$ is the density of $v_i - \max_{j \neq i}\{v_j\}$.

Lemma 3. *Suppose $h(x) < h(0)$ for $x > 0$. Then the highest personalized price exceeds the uniform price.*

The condition in Lemma 3 holds, for example, in the IID case with a log-concave f. However it fails in the linear Hotelling model studied earlier in Section 2, because there $h(x)$ is constant in $x \geq 0$; this explains why, in that case, the highest personalized price exactly equals the uniform price (as shown in Thisse and Vives, 1988).

Figure 2 illustrates the duopoly case with full market coverage, i.e., $p < \underline{v}$. Under personalized pricing, consumers with $v_1 > v_2$ buy from firm 1 and pay $v_1 - v_2 + c$, while consumers with $v_1 < v_2$ buy from firm 2 and pay $v_2 - v_1 + c$. Compared to the uniform-pricing regime with price p, consumers with strong relative preferences (high $|v_1 - v_2|$) in the northwest and southeast corners pay more, while those with weaker relative preferences (low $|v_1 - v_2|$) in the middle region pay less. The importance of *relative* rather than *absolute* valuations implies that the common wisdom that richer consumers pay more under personalized pricing may fail under competition. In particular, rich consumers with a high valuation for both products (the northeast corner) pay less under personalized pricing. Similarly poor consumers, who tend to have a low valuation for both products (the southwest corner), also pay less. Only rich and choosy consumers (the northwest and southeast corners) pay more under personalized pricing.

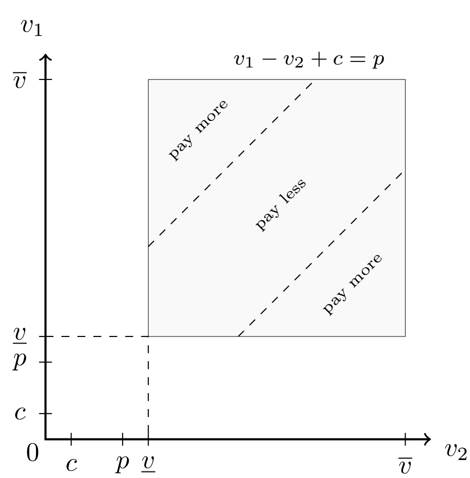


Figure 2: The impact of personalized pricing with full market coverage

The remainder of this subsection addresses the subtler question of how personalized pricing affects industry profit and *aggregate* consumer surplus.

4.1.1 The case of full market coverage

We first study the case where the market is fully covered under uniform pricing (and so, as explained earlier, is also fully covered under personalized pricing). Total welfare is therefore the same under either pricing regime, because in both cases every consumer buys her preferred product. The following result reports the impact of personalized pricing on profit and consumer surplus.

Proposition 1. *Suppose $c \leq \underline{v} - \phi(\underline{v})$ (in which case the market is fully covered under both pricing regimes). Then relative to uniform pricing, personalized pricing harms firms and benefits consumers.*

Proof. Under the stated full-coverage condition, $x_z = v_i - \max_{j \neq i}\{z, v_j\}$ simplifies to $x = v_i - \max_{j \neq i}\{v_j\}$ for both $z = p$ and $z = c$ as $c < p \leq \underline{v}$. Recall that H and h are respectively the CDF and density function of x. Then from (11) and $1 - H(0) = \frac{1}{n}$, we can see that industry profit under uniform pricing is

$$\Pi_U = p - c = \frac{1}{nh(0)} \ ,$$

while from (14) we can see that industry profit under personalized pricing is

$$\Pi_D = n \int_0^{\infty} [1 - H(x)]dx = n \int_0^{\infty} \frac{1 - H(x)}{h(x)} dH(x) \leq n \frac{[1 - H(0)]^2}{h(0)} = \frac{1}{nh(0)} \ .$$

The inequality follows because, under Assumption 1, $1 - H$ is log-concave and therefore $\frac{1-H}{h}$ is decreasing. Therefore, firms suffer from personalized pricing. Since total welfare is unchanged, consumers benefit from personalized pricing. $\square$

Recalling our discussion of Figure 2, the log-concavity condition in Assumption 1 ensures that there are relatively more consumers with weak preferences who pay less under personalized pricing. Hence personalized pricing harms firms but benefits consumers in aggregate. Note that since our set-up includes Hotelling as a special case (see footnote 16), Proposition 1 significantly generalizes the result in Thisse and Vives (1988).[20]

[20] Anderson, Baik, and Larson (2021) do not highlight it, but this generalization of Thisse-Vives is also implied by their Proposition 6 which does comparative statics in their advertising cost parameter. Our proof is similar to that of their Proposition 7 which shows the opposite result when $1 - H$ is log-convex.

4.1.2 The case of partial market coverage

We now turn to the case where the market is not fully covered under uniform pricing. From Lemma 1, we know this happens when $c > \underline{v} - \phi(\underline{v})$. Personalized pricing now expands total demand and strictly increases total welfare. The reason is that under uniform pricing a consumer buys if her best match is above the uniform price $p > c$, whereas under personalized pricing she buys if her best match is above c.

Before investigating the impact of personalized pricing on firms and consumers, we offer an alternative formula to calculate industry profit under personalized pricing:

$$\Pi_D = n \int_c^{\overline{v}} \int_c^{v} G(x|v) dx dF(v) \ , \tag{16}$$

which is more convenient to use in some of the subsequent analysis.[21] In the IID case, by integration by parts and using $G(x|v) = F(x)^{n-1}$, it simplifies to

$$\Pi_D \overset{\text{IID}}{=} \int_c^{\overline{v}} \frac{1 - F(v)}{f(v)} dF(v)^n \ . \tag{17}$$

We will now show that when the market is only partially covered, competitive personalized pricing can raise profit and lower aggregate consumer surplus. To understand why, it is useful to first investigate why the simple proof in Proposition 1 breaks down with partial coverage. Under Assumption 1, we still have that

$$\Pi_D = n \int_0^{\infty} [1 - H_c(x)] dx \leq n \frac{[1 - H_c(0)]^2}{h_c(0)} \ , \tag{18}$$

but now the last term is greater than

$$\Pi_U = n \frac{[1 - H_p(0)]^2}{h_p(0)} \ ,$$

because $p > c$ and both $1 - H_z(0)$ and $\frac{1 - H_z(0)}{h_z(0)}$ decrease in z. (In the full-coverage case, $c < p \leq \underline{v}$ and so $H_c = H_p = H$.) This observation also suggests that if $1 - H_z(x)$ is log-linear in x, then the inequality in (18) binds and so we have $\Pi_D > \Pi_U$ whenever the market is not fully covered. That is indeed what we show in the following example.

[21] To understand this alternative formula, notice that conditional on firm i winning a consumer and its product being valued at v_i, its expected profit margin is

$$m(v_i) \equiv v_i - \int_{\underline{v}}^{v_i} \max\{c, x\} d \frac{G(x|v_i)}{G(v_i|v_i)} = \frac{\int_c^{v_i} G(x|v_i) dx}{G(v_i|v_i)} \ ,$$

where we have used (13) and integration by parts. Then industry profit under personalized pricing is $\Pi_D = n \int_c^{\overline{v}} m(v_i) G(v_i|v_i) dF(v_i)$, which is equal to (16).

An exponential distribution example It is illuminating to first consider an exponential distribution example. Suppose the v_i's are independent and exponentially distributed with $F(v) = 1 - e^{-(v-\underline{v})}$ on $[\underline{v}, \infty)$. Then $\phi(p)$ defined in (10) equals 1,[22] and so the equilibrium price is $p = 1 + c$ regardless of whether or not the market is covered (and irrespective of the number of firms). This means that under uniform pricing a fraction $F(1+c)^n$ of consumers are excluded from the market—and so industry profit is $\Pi_U = 1 - F(1+c)^n$. Meanwhile under personalized pricing, using (17) and the fact that $1 - F(v) = f(v)$ in this exponential example, we immediately have $\Pi_D = 1 - F(c)^n$.

We then have the following observation:

Proposition 2. *Suppose valuations are IID exponential. Relative to uniform pricing, personalized pricing has no impact on firms or consumers if the market is fully covered under uniform pricing, but it benefits firms and harms consumers whenever the market is not fully covered.*

Proof. With full market coverage (which requires $1 + c \leq \underline{v}$), it is immediate that $\Pi_D = \Pi_U = 1$, and since welfare is the same in both regimes, so must be consumer surplus.[23]

With partial coverage (meaning that $1 + c > \underline{v}$), the profit result is also immediate, because $\Pi_D = 1 - F(c)^n > \Pi_U = 1 - F(1+c)^n$. To prove the consumer surplus result, note that

$$V_U = \int_{1+c}^{\infty} (v - c) dF(v)^n - \Pi_U \text{ and } V_D = \int_{c}^{\infty} (v - c) dF(v)^n - \Pi_D \ ,$$

where the integral term in each expression is the total welfare in each regime. The former is greater than the latter if and only if

$$F(1+c)^n - F(c)^n > \int_{c}^{1+c} (v - c) dF(v)^n \ ,$$

i.e., if the increase of profit under personalized pricing exceeds the welfare improvement. This condition must be true as $v - c < 1$ for $v \in (c, 1+c)$. $\square$

With partial market coverage, personalized pricing increases welfare by expanding demand, but it boosts profit so much that in aggregate consumers *suffer* from it. One way to see the intuition of this proposition is as follows. Notice that under personalized pricing total demand is $1 - F(c)^n$, and so the *average* price that consumers pay is $1 + c$, which is exactly equal to the uniform price. Personalized pricing therefore raises profit,

[22] Using integration by parts, the denominator in (10) can be rewritten as $f(\overline{v}) - \int_p^{\overline{v}} F(v)^{n-1} f'(v) dv$. For the exponential distribution $f(\overline{v}) = 0$ and $f(v) = -f'(v)$, so this equals the numerator of (10).

[23] One can show that $1 - H(x)$ is log-linear in this exponential example, which explains why it is an edge case of Proposition 1.

because it expands the size of the market. At the same time, this market expansion is from consumers whose highest valuation is between c and $1+c$—and since this is below the average price, personalized pricing lowers aggregate consumer surplus.

The production cost and market coverage Given the full-coverage result in Proposition 1, it is clear that for a more general (regular) distribution, the impact of personalized pricing can only be reversed when the market is sufficiently far away from being fully covered. As we saw earlier, by changing the marginal cost c we can change the degree of market coverage. In particular, when c is sufficiently close to the valuation upper bound $\overline{v}$, most consumers are excluded from the market. In that case we can show that the impact of personalized pricing is completely different from the full coverage case.

Proposition 3. *If $f(\overline{v}) > 0$ or in the IID case with a log-concave $f(v)$, there exists a $\hat{c}$ such that when $c > \hat{c}$, personalized pricing benefits firms and harms consumers. More precisely,*

$$2 \leq \lim_{c \to \overline{v}} \frac{\Pi_D}{\Pi_U} \leq e \quad \textit{and} \quad \lim_{c \to \overline{v}} \frac{V_D}{V_U} = 0 \ .$$

Notice that, in the limit case with large c, profit and consumer surplus in both regimes tend to zero. At the same time, Proposition 3 implies that for this limit case (i) profit in the discriminatory regime is at least twice as large as under uniform pricing, and (ii) consumer surplus tends to zero faster with discriminatory pricing.[24]

Before discussing the intuition for Proposition 3, we first report some numerical results on how the impact of personalized pricing varies with c. The contrast between Propositions 1 and 3 suggests a possible cutoff result, whereby personalized pricing benefits firms if and only if c exceeds a threshold c', and harms consumers if and only if c exceeds a threshold c'' (with $c'' > c'$ because personalized pricing raises total welfare). Although it appears hard to formally prove such a cutoff result for a general distribution, numerical simulations suggest it is true. This is illustrated by Figure 3, which plots the impact of personalized pricing on profit ($\Pi_D - \Pi_U$) and consumer surplus ($V_D - V_U$) for four common distributions (in the IID case) and for different values of c.

Figure 3a considers the exponential case with $F(v) = 1 - e^{-(v-1)}$ on $[1, \infty)$. At $c = 0$, we have $p = 1$ and the market is (just) covered under uniform pricing and so the impact is zero, but for higher values of c the market is only partly covered, so as explained before personalized pricing benefits firms and harms consumers. Figures 3b and 3c consider, respectively, the Extreme value distribution with $F(v) = e^{-e^{-(v-2)}}$ (which leads to a logit model), and the Normal distribution with mean 2 and variance 1 (which leads to a probit model). In both cases, for low values of c (when coverage is high) personalized pricing

[24] As detailed in the proof, the precise value of $\lim_{c \to \overline{v}} \frac{\Pi_D}{\Pi_U}$ depends on the tail behavior of $f(v)$.

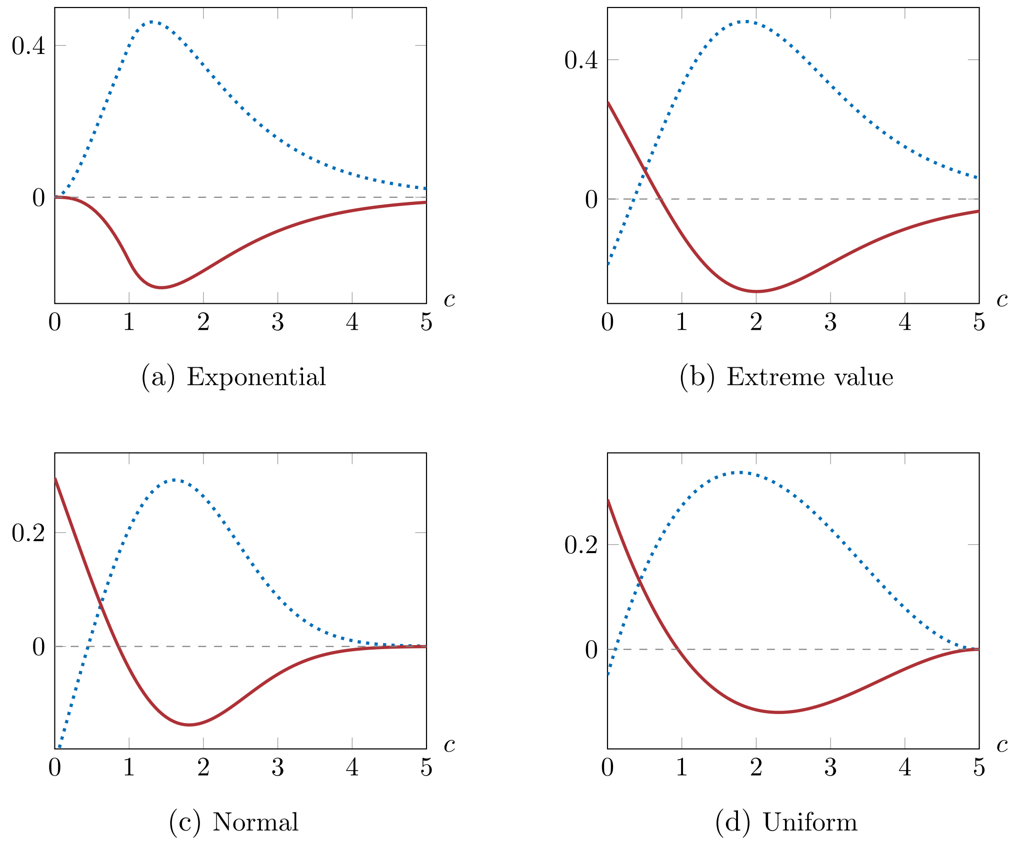


Figure 3: The impact of personalized pricing when $n = 2$, for different values of c
(The dotted and solid lines represent, respectively, the change in industry profit and consumer surplus.)

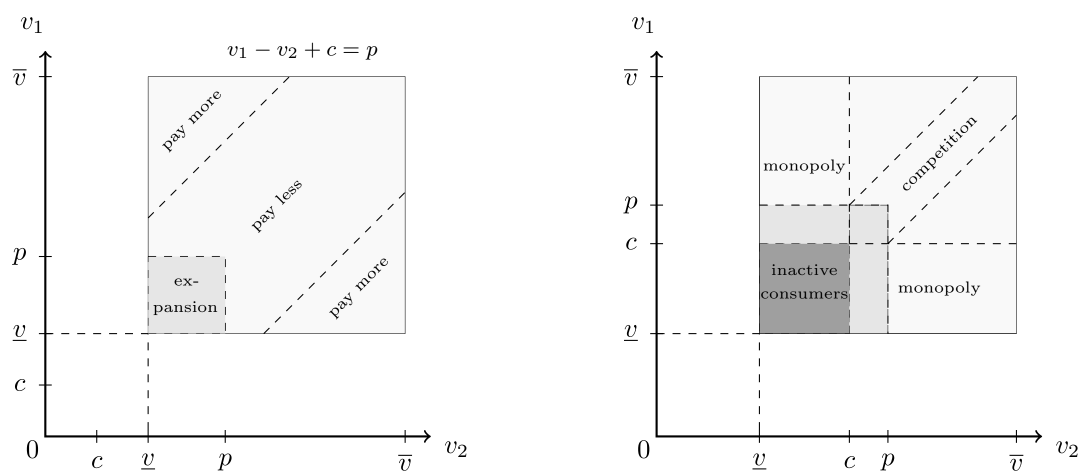
harms firms and benefits consumers as in the full-coverage case, for high values of c (when coverage is low) personalized pricing has the opposite impact, while for intermediate c both consumers and firms benefit from personalized pricing. Finally, the same pattern is also observed in Figure 3d, which considers the case where valuations are uniformly distributed on $[0, 5]$.[25]

We emphasize that although Proposition 3 is proved in the limit when market coverage is close to zero, in our examples we do not need a very low degree of market coverage to reverse the impact of personalized pricing. In the Extreme value example, we find that personalized pricing starts to benefit firms provided less than about 89% of consumers buy under uniform pricing, and to harm consumers provided less than about 82% of them buy under uniform pricing. For the Normal example these thresholds are about 85% and 76% respectively, while for the uniform example they are about 82% and 71%.

We now discuss the intuition of how the impact of personalized pricing depends on the marginal cost c by using Figure 4. Start with the case depicted in Figure 4a, where

[25]Note that as c tends to the maximum valuation $\overline{v} = 5$, the impact of personalized pricing on profit and consumer surplus vanishes because in both regimes the number of consumers who buy tends to zero.

$c < \underline{v} < p$ (so the market is fully covered under personalized pricing, but only partially covered under uniform pricing). Following our earlier discussion of Figure 2, a move from uniform to personalized pricing causes consumers in the northwest and southeast corners to pay more, and causes consumers in the middle region to pay less. One important difference with Figure 2 is the new “expansion” region—some consumers are excluded from the market under uniform pricing, but buy under personalized pricing. Now consider the impact of an increase in c. First, the expansion region grows, which is a positive effect for consumers. However this positive effect is relatively small, because the consumers in the expansion region have low valuations. Second, under personalized pricing firms fully pass cost increases through to consumers, whereas under uniform pricing firms share some of the burden (as can be seen from equation (7)). One effect of this is that those consumers who gain from personalized pricing now gain less, and those who lose from personalized pricing now lose more. Another effect is that the two corner regions become larger; this implies that relatively more of the consumers who bought under uniform pricing are now harmed by personalized pricing. Consequently, as c increases, personalized pricing can harm consumers even though it expands demand.



(a) The case of $c < \underline{v}$

(b) The case of $c > \underline{v}$

Figure 4: The impact of personalized pricing with partial market coverage

When $c > \underline{v}$, as depicted in Figure 4b, a new effect emerges. Now some consumers value only one product above cost. Under personalized pricing, each firm acts as a true monopolist over these “captive” consumers and extracts all their surplus; these consumers lie in the “monopoly” regions in the figure. (Consumers in the dark region value both products below cost and so are inactive in both regimes.) For the consumers in the “competition” region, who value both products above cost, the situation is the same as in Figure 4a. As c increases, both the monopoly and competition regions shrink, but the monopoly region becomes relatively larger. When c is close to $\overline{v}$ the monopoly region dominates, so the impact of personalized pricing becomes qualitatively the same as in the monopoly case, as proved in Proposition 3. (Intuitively, in this case, conditional on a consumer valuing one product above cost, it is very unlikely that she values another product above cost. Hence each firm is essentially a monopolist competing only against the outside option.) Nevertheless, we emphasize that, as already indicated in the exponential example (for $\underline{v} - 1 < c < \underline{v}$), under partial coverage personalized pricing can benefit firms and harm consumers even when there are no monopoly regions (as is the case in Figure 4a).

Remark. As noted earlier, in order to control the degree of market coverage, we could instead normalize the marginal cost and then vary consumers' outside option. Since the outside option plays a very similar role to the cost, all the above discussion would carry over. For example, when the outside option is sufficiently good, each firm acts almost like a monopolist (which competes only against the outside option) and therefore personalized pricing benefits firms and harms consumers.

The number of firms and market coverage Another parameter which influences market coverage is the number of firms. When $n = 1$ we have the standard monopoly case; when n is large the best match should be relatively high, and so intuitively the impact of personalized pricing should be similar to the full-coverage case. (More rigorously, since the profit in both regimes often goes to zero as $n \to \infty$, it also matters how fast they converge to the full-coverage outcome as $n \to \infty$.) In the following, we first provide an analytical result regarding the case of large n, and then demonstrate by numerical examples that with partial coverage the impact of competitive personalized pricing can remain qualitatively similar as in the monopoly case for a relatively large range of n.

We deal with the case where n is large by approximating the equilibrium outcome. However, the approximation of the uniform equilibrium price when n is large is technically difficult. We rely on the approximation results for the IID and full-coverage case developed in Gabaix et al. (2016), and extend them to the case with partial coverage.

Proposition 4. *Consider the IID case with a log-concave $f(v)$, and let*

$$\gamma = \lim_{v \to \overline{v}} \frac{d}{dv} \left(\frac{1 - F(v)}{f(v)} \right) \tag{19}$$

denote the tail index of the valuation distribution of each product. If $\gamma \in (-1, 0)$, there exists $\hat{n}$ such that when $n > \hat{n}$ personalized pricing harms firms and benefits consumers.

When f is log-concave, we must have $\gamma \in [-1, 0]$.[26] Unfortunately, our approximation in the proof is not precise enough for a meaningful comparison if $\gamma = -1$ or 0. This rules out many common distributions such as the uniform, exponential, extreme value, and normal (see Table 1 in Gabaix et al., 2016). However, the numerical examples below demonstrate that our comparison results when n is large continue to hold in those examples.

Given that the impact of personalized pricing in Proposition 4 is the opposite of that under monopoly, this suggests the possibility of a cutoff result in terms of n. Since an analytic result seems hard to obtain, we instead report some numerical examples in Figure 5 below (the IID case with $c = 2$ and the same distributions used in Figure 3). Figure 5a is for the exponential distribution, and confirms our earlier analytic result that personalized pricing always benefits firms and harms consumers when the market is not fully covered. Figure 5b shows that for the Extreme value distribution, personalized pricing benefits firms if and only if $n < 10$, and harms consumers if and only if $n < 7$. A qualitatively similar pattern emerges in Figure 5c for the Normal distribution. Figure 5d shows that for the uniform distribution personalized pricing benefits firms for $n < 4$ and harms consumers for $n < 3$. Although the impact of personalized pricing can be non-monotonic in n, it goes to zero as n becomes large.

4.2 The long-run impact in a free-entry market

Since the ability to do personalized pricing affects firm profits, in the long-run it may influence firms' entry decisions and hence also the market structure. To investigate this we now consider a standard free-entry game, where firms first decide whether or not to enter by paying a fixed cost, and then after entering they compete in prices. (As often assumed in the literature, we consider a large number of potential entrants, and use a sequential entry game to avoid coordination problems.)

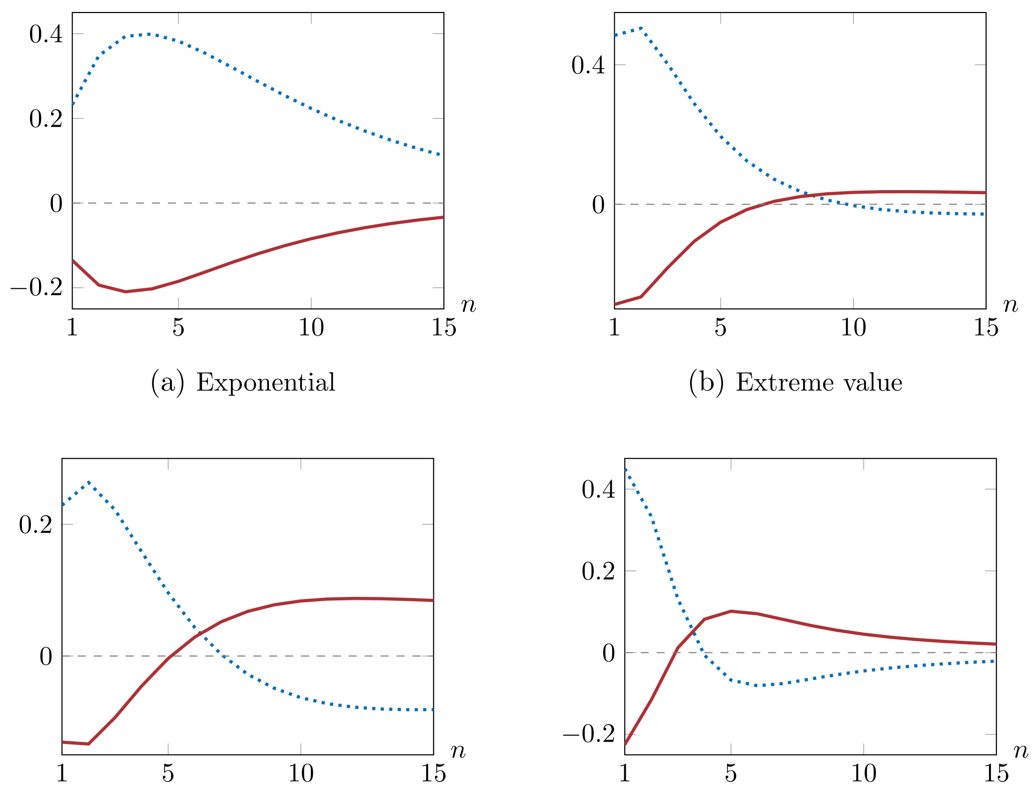
Let us first study the case with personalized pricing. As explained earlier, firms engage in asymmetric Bertrand competition. Therefore the profit on each consumer is simply the difference between her best and second-best product valuations, adjusted for the marginal cost. Hence, with n firms in the market, each firm's profit can be expressed as

$$\tfrac{1}{n}\Pi_D = \tfrac{1}{n}\mathbb{E}[\max\{c, v_{n:n}\} - \max\{c, v_{n-1:n}\}] \ . \tag{20}$$

[26] When f is log-concave, so is $1 - F$. Then $\frac{1-F}{f}$ is decreasing, so $\gamma \le 0$. To see $\gamma \ge -1$, notice that

$$\frac{d}{dv}\left(\frac{1 - F(v)}{f(v)}\right) = -1 - \frac{1 - F(v)}{f(v)}\frac{f'(v)}{f(v)} \ .$$

If $\lim_{v\to\overline{v}} f'(v) \le 0$, the claim is obvious. If $\lim_{v\to\overline{v}} f'(v) > 0$, then we must have $\overline{v} < \infty$ and $f(\overline{v}) > 0$, in which case $\frac{1-F(\overline{v})}{f(\overline{v})} = 0$ and given the log-concavity of f we also have $\frac{f'(\overline{v})}{f(\overline{v})} < \infty$. Then $\gamma = -1$.



(a) Exponential

(b) Extreme value

(c) Normal

(d) Uniform

Figure 5: The impact of personalized pricing when $c = 2$, for different values of n
(The dotted and solid lines represent, respectively, the change in industry profit and consumer surplus.)

On the other hand, the increase in match efficiency when the number of firms goes from $n - 1$ to n is

$$\mathbb{E}[\max\{c, v_{n:n}\}] - \mathbb{E}[\max\{c, \hat{v}_{n-1:n-1}\}] \ , \tag{21}$$

where $\hat{v}_{n-1:n-1}$ denotes the best match among the original $n-1$ products. (We use $\hat{v}_i$ to denote the valuation for product $i \le n-1$ when there are only $n-1$ firms in the market.) To determine whether there is too much or too little entry relative to the social optimum (under the usual assumption that the social planner is unable to control firms' pricing behavior once they enter the market), it then suffices to compare (20) and (21). It turns out that they are actually equal to each other under the following assumption.

Assumption 2. *Entry of a new firm does not affect consumers' valuations for existing products. That is, the distribution of* $(\hat{v}_1, ..., \hat{v}_{n-1})$ *when there are* $n-1$ *firms in the market is the marginal distribution of* $(v_1, ..., v_{n-1})$ *when there are* n *firms in the market.*

This assumption is trivially satisfied in the IID case, and can also hold with correlated valuations (e.g., if we fix a grand set of firms with a certain joint valuation distribution).[27]

[27] Assumption 2 fails if entry of a new firm induces existing firms to reposition their products. This

Lemma 4. *Under Assumption 2, the free-entry equilibrium under personalized pricing is unique and it is also socially optimal.*

The intuition for this result is straightforward. Suppose $n-1$ firms are already in the market, and consider the entry of an n^{th} firm. Amongst consumers with $v_n \leq \max_{j\leq n-1}\{c, v_j\}$, this additional firm creates no social surplus and earns zero profit. However, amongst consumers with $v_n > \max_{j\leq n-1}\{c, v_j\}$, this new firm raises total surplus by $v_n - \max_{j\leq n-1}\{c, v_j\}$, and fully extracts it via Bertrand competition. As a result, the incentives of the social planner and this new firm are perfectly aligned. (Using the terminology introduced by Mankiw and Whinston, 1986, here the business-stealing effect and the product-diversity effect exactly cancel out each other.)

This result is in the same spirit as Spence (1976). He shows that in a competitive market with perfect discrimination, each firm's choice of quantity or product characteristic is socially optimal because its profit is equal to its marginal contribution to total surplus. Building on this observation, Bhaskar and To (2004) study an extended game in which firms enter, then choose product characteristics, then set prices. They show that under perfect discrimination there is weakly too much entry from a social welfare point of view. However, they also show that if entry of a new firm does not change existing firms' choice of product characteristics, then entry is socially efficient as in our Lemma 4.[28]

Now consider the case with uniform pricing. Let n^* denote the socially optimal number of firms. A simple corollary of Lemma 4 is the following:

Corollary 1. *Suppose Assumption 2 holds and each firm's profit under uniform pricing decreases in n.*[29]
(i) Entry under uniform pricing is excessive if $\Pi_U > \Pi_D$ at $n = n^*$, but insufficient if $\Pi_U < \Pi_D$ at $n = n^*$.
(ii) Uniform pricing thus leads to excessive entry if the market is fully covered at $n = n^*$.

Part (i) of the corollary is simply because, as we have just shown, a new entrant's profit under personalized pricing is equal to the (expected) increase in match efficiency due to its entry. To understand part (ii), recall from Proposition 1 that with full coverage

happens, for example, in the Salop circle model. (Contrary to our Lemma 4, Stole, 2007 shows that entry is socially excessive in the Salop model under perfect price discrimination. See also Abrardi et al., 2022.) However Assumption 2 can hold in other spatial models, such as Chen and Riordan (2007) where entry of a new firm does not cause existing ones to reposition.

[28] We thank John Vickers for drawing our attention to these papers. A similar idea is also explored in the auction literature with endogenous bidder participation (see, e.g., Levin and Smith, 1994).

[29] The assumption that a firm's profit under uniform pricing decreases in n ensures uniqueness of the free-entry equilibrium. It is easy to see that this assumption must hold if the equilibrium uniform price decreases in n (which, as shown in Zhou, 2017, is true at least in the IID case with a log-concave f).

it is always the case that $\Pi_U > \Pi_D$. Anderson, de Palma, and Nesterov (1995) and Tan and Zhou (2021) also prove this excessive entry result in the IID case; our proof is much simpler than theirs, and our result is more general since it also potentially allows for correlated valuations.

Finally, if we make the usual assumption of no integer constraint, such that firm entry is pinned down by the zero-profit condition, Lemma 4 also implies the following:

Proposition 5. *Compared to uniform pricing, personalized pricing benefits consumers in the long run.*

In the long-run firms earn zero profit (after accounting for the fixed entry cost) in both pricing regimes. Therefore since total welfare is maximized under personalized pricing, so is aggregate consumer surplus.[30]

5 Asymmetrically Informed Firms

So far we have assumed that either all firms do uniform pricing or all firms do personalized pricing. We have therefore implicitly assumed that all firms have access to the same data and technology. However, in certain markets, some firms may have more data and better technology than others. For example, Amazon possesses lots of information about customer shopping behavior, and in principle can use this information to offer personalized prices for its own products—whereas third-party sellers of similar products on Amazon are often smaller retailers who lack such information. Similarly, in music streaming, large players like Amazon and Apple may have more data, and also the ability to make more refined personalized offers, compared to smaller players like Spotify and Pandora. This section investigates market performance in such a "mixed" case, where some firms can price discriminate while others cannot. By comparing with the two symmetric regimes studied earlier, we can then evaluate the impact of policies that prevent firms with more data from price discriminating or that force them to share their data so that all firms can price discriminate.

In order to capture this "mixed" case, suppose that firms 1 to k have consumer data and can price discriminate, while firms $k+1$ to n have no consumer data and therefore have to offer a uniform price. When $k = 0$ all firms do uniform pricing as in Section 3.1, whereas when $k = n$ all firms do personalized pricing as in Section 3.2. When $0 < k < n$

[30] If we consider the integer constraint, this result might break down. This can happen when firms earn more profit (after accounting for the fixed cost) under personalized pricing than under uniform pricing; for example, it can happen if the entry cost is so high that only one firm enters under either regime.

a subtle technical issue arises: if all firms set prices simultaneously, there is no pure-strategy pricing equilibrium,[31] and the mixed-strategy equilibrium is rather complicated to characterize. To avoid this problem, we assume that the $n-k$ firms with no consumer data simultaneously set their uniform prices first, and after seeing their prices the other k firms use their data to simultaneously offer personalized prices. This timing seeks to capture the idea that firms with lots of data often also have more advanced pricing technologies and so can adjust their prices more frequently.

The equilibrium analysis of the "mixed" case is more involved than the two symmetric cases studied earlier, but the overall logic is similar. We therefore relegate the details to the Online Appendix. Here we report some results on how the mixed regime compares with the uniform and personalized pricing regimes from earlier. Let Π_M, V_M and W_M be respectively industry profit, consumer surplus, and total welfare in the mixed regime. We begin with two analytical results, and then show that qualitatively similar insights emerge in numerical simulations for the Extreme Value and Normal distributions.

When the production cost c is sufficiently high we can compare the three regimes analytically. In particular, we can show that if $f(\overline{v}) > 0$, there exists a $\hat{c}$ such that when $c > \hat{c}$ we have $\Pi_U < \Pi_M < \Pi_D$, $V_D < V_M < V_U$, and $W_U < W_M < W_D$ i.e., for each of the three welfare measures the mixed regime is ranked in the middle.[32] Intuitively, recall that when c is sufficiently large, each firm approximately acts like a monopolist. As a result, as more firms are able to personalize prices, profit and welfare increase but consumers become worse off.

When valuations are IID exponential, it turns out that any firm that cannot price discriminate charges $1+c$ regardless of k. This simple pricing result enables us to compare the three regimes analytically for any level of c.

Proposition 6. *Suppose valuations are IID exponential. Then:*
(i) There exists a $\widetilde{c}_\Pi$ such that for $c < \widetilde{c}_\Pi$ we have $\Pi_U \leq \Pi_M = \Pi_D$, and for $c > \widetilde{c}_\Pi$ we have $\Pi_U < \Pi_M < \Pi_D$.
(ii) There exists a $\widetilde{c}_V$ such that for $c < \widetilde{c}_V$ we have $V_M < V_D \leq V_U$, and for $c > \widetilde{c}_V$ we

[31] In any hypothetical pure-strategy equilibrium, each uniform-pricing firm has a positive measure of consumers who are indifferent between it and some personalized-pricing firm but buy from the latter. If the uniform-pricing firm slightly reduces its price, it wins these consumers and so its demand increases discontinuously. This discontinuity in demand leads to non-existence of pure-strategy pricing equilibrium. This issue is well known in the literature, and the usual approach to avoid it is to consider a sequential pricing game as we do here. See, e.g., Thisse and Vives (1988) for a similar treatment in their duopoly model when only one firm can price discriminate.

[32] More precisely, for c close to $\overline{v}$, slightly abusing notation, for any $0 \leq k \leq n$ we can show $\Pi_M \approx kf(\overline{v})\frac{\varepsilon^2}{2} + (n-k)f(\overline{v})\frac{\varepsilon^2}{4}$ and $V_M \approx (n-k)f(\overline{v})\frac{\varepsilon^2}{8}$. (When $k=n$, $V_M = V_D$ is of less than second order.) It is then clear that profit and total welfare increase in k while consumer surplus decreases in k.

have $V_D < V_M < V_U$.
(iii) There exists a $\widetilde{c}_W$ *such that for* $c < \widetilde{c}_W$ *we have* $W_M < W_U \leq W_D$*, and for* $c > \widetilde{c}_W$ *we have* $W_U < W_M < W_D$.

When c is relatively high the mixed case is intermediate for profit, welfare, and consumer surplus. When c is relatively low, however, the mixed case is the worst for both total welfare and consumer surplus. Intuitively, in the mixed case firms that are able to personalize can "poach" some consumers for whom they are not the consumer's favorite product via a low personalized price. This harms match efficiency and can also make consumers worse off in aggregate.

Numerical simulations suggest that qualitatively similar patterns emerge when product valuations are IID and drawn from other distributions. Figure 6 below depicts industry profit, consumer surplus, and total welfare, for respectively the Extreme value and Normal distributions used before (for $n = 2$, and in the mixed case $k = 1$). At high values of c the mixed regime is again intermediate for all three measures of market performance. At low values of c the mixed regime is again worst for both consumers and total welfare. One interesting difference with the exponential example, though, is that for low c industry profit is strictly highest in the mixed regime. Intuitively, the asymmetry in information and timing allows the firms to better segment the market, increasing industry profit.

Overall, our results suggest that when coverage is relatively high (i.e., c is relatively low), policies which force large firms to share their data, or which prevent those firms from personalizing their prices, tend to benefit consumers and overall welfare.

6 An alternative information structure

So far we have assumed that in the regime of personalized pricing firms observe a consumer's valuation for *every* product. Here we consider a natural alternative, where firms observe only a consumer's valuation for *their own* product, i.e., each firm i observes only v_i. For convenience, we refer to these two cases as "full" and "partial" discrimination respectively. For simplicity, we return to the case with symmetrically informed firms.

Under partial discrimination a firm offers a price $p(v)$ to a consumer who has valuation v for its product. It turns out that $p(v)$ can be derived in the same way as a bid in a standard first-price auction. This is because we can interpret a firm as "bidding" surplus of $v - p(v)$ to a consumer who has valuation v for its product, and then the consumer picking the best (non-negative) "bid."

Formally, recall that $G(\cdot|v_i)$ and $g(\cdot|v_i)$ are respectively the CDF and density function

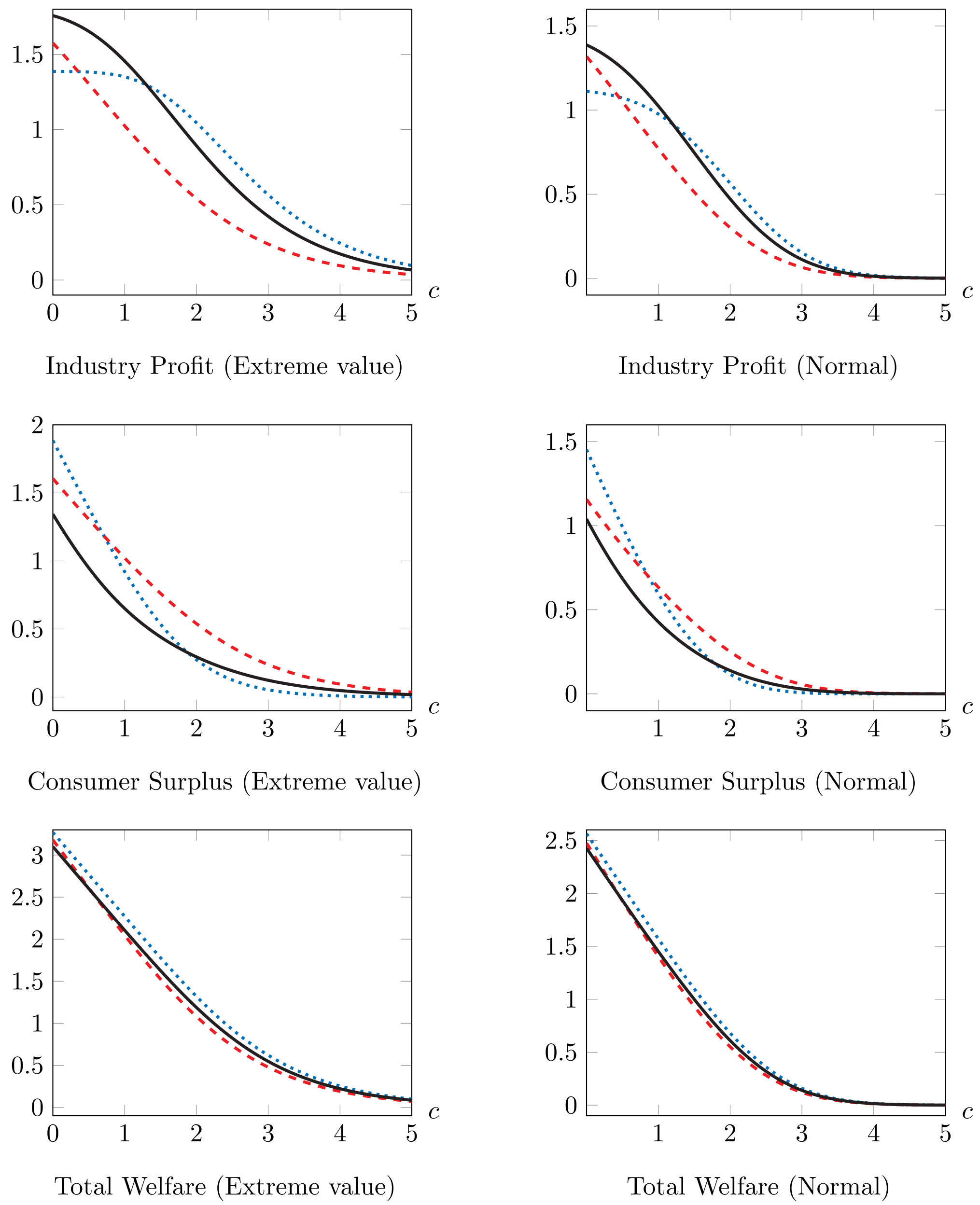


Figure 6: Asymmetric case versus symmetric cases for different values of c
(The solid, dashed, and dotted curves are respectively the mixed, uniform, and discriminatory cases.)

of $\max_{j \neq i}\{v_j\}$ conditional on v_i, and define

$$b(z) \equiv \int_c^z [1 - L(x|z)]dx \quad \text{with} \quad L(x|z) = \exp\left(-\int_x^z \frac{g(t|t)}{G(t|t)}dt\right). \tag{22}$$

(Here $b(\cdot)$ is the equilibrium bidding function in a standard first-price auction with interdependent values; see, e.g., Milgrom and Weber, 1982.) We impose the following regularity condition to ensure that $b(v)$ is monotonically increasing:

Assumption 3. *For all $z \in [\underline{v}, \overline{v}]$, $[v - b(z) - c]\frac{g(z|v)}{G(z|v)}$ increases in v whenever it is positive.*

This assumption holds if $\frac{g(z|v)}{G(z|v)}$ increases in v, which is satisfied when product valuations are IID or positively affiliated in the sense of Milgrom and Weber (1982). The assumption can also hold when $\frac{g(z|v)}{G(z|v)}$ decreases in v, provided it does not decrease too quickly.

Lemma 5. *Suppose Assumption 3 holds. Under partial discrimination there exists a symmetric equilibrium in which each firm uses the price schedule $p(v) = v - b(v)$, where $b(v)$ is defined in equation (22) and it is strictly increasing in v.*

This lemma implies that a consumer buys her best product provided its valuation exceeds marginal cost. This is because the surplus $b(v) = v - p(v)$ offered by a firm with valuation v is positive if and only if $v > c$, and is also strictly increasing in v.

Proposition 7. *Suppose Assumption 3 holds. Comparing partial and full discrimination:*
(i) Total welfare is the same under both regimes.
(ii) When valuations are IID, profit and consumer surplus are the same in both regimes.
(iii) Otherwise firms earn more (less) under partial discrimination if $\frac{g(z|v)}{G(z|v)}$ increases (decreases) in v, and the opposite is true for consumer surplus.

The intuition for these results is as follows. For part (i), total welfare is the same under both partial and full discrimination because in both regimes consumers buy the best product conditional on its valuation exceeding marginal cost. Parts (ii) and (iii) exploit the connection with auction theory. In particular, notice that under *full* discrimination competition is the same as in a second-price auction—because the winning firm earns a profit equal to the difference between the highest and second-highest valuations (including the outside option). The well-known revenue equivalence theorem then implies that, when valuations are IID, firm and consumer payoffs are the same under both full and partial discrimination.[33] Meanwhile the theory of auctions with interdependent valuations (e.g., Milgrom and Weber, 1982) implies that bidders (i.e., firms) are better off and the auctioneer (i.e., consumers) are worse off with partial information if valuations are positively affiliated, while the reverse is true if they are negatively affiliated.[34]

Proposition 7 implies that with IID valuations, all our earlier results about the (short- and long-run) impact of full discrimination carry over to partial discrimination. However,

[33]Note, however, that partial and full discrimination affect different consumers differently. A consumer's payment is determined by her valuation for the best product under partial discrimination, but by the gap between the highest two valuations under full discrimination. Hence consumers with very weak (respectively, strong) preferences tend to prefer full (respectively, partial) discrimination.

[34]We note that while the auctions literature focuses on the case of positive affiliation, negative affiliation may be reasonable in our context, e.g., if products differ in characteristics space, and consumers have different preferences over different characteristics.

outside the IID case, the correlation structure of product valuations matters for whether consumers prefer firms to have more or less information about their tastes.

Although beyond the scope of the current paper, it would be interesting to also consider other information structures, and investigate how the welfare impact of price discrimination changes with the amount of information that firms have access to. This is, however, a challenging question because the space of information structures is large and the pricing equilibrium under some information structures is complicated to characterize. One approach is to consider a special class of information structures. For example, in the linear Hotelling model, some papers (e.g., Liu and Serfes, 2004; Bounie, Dubus, and Waelbroeck, 2021) have studied the class of interval information structures and shown that providing firms with finer information has a non-monotonic impact on profit and consumer surplus. (This non-monotonic relationship is also suggested by our comparison between partial and full discrimination.) Another approach is to explore the welfare limits when arbitrary information structures are feasible. For example, Bergemann, Brooks, and Morris (2015) and Elliott, Galeotti, Koh, and Li (2021) have studied this issue in respectively the monopoly and competition cases.[35] Our approach, by focusing on simpler information structures, is arguably more suitable for evaluating policies which simply either allow or ban the use of consumer data.

7 Conclusion

We have investigated the impact of personalized pricing—a form of price discrimination which is becoming increasingly relevant in the digital economy—in a general oligopoly model. Our paper delivers three main insights: (i) In the short run with a fixed market structure, competitive personalized pricing harms firms and benefits consumers under a standard log-concavity condition if the market is fully covered. However, the impact can be completely reversed in the (arguably more realistic) case without full market coverage; for example, personalized pricing raises industry profit and decreases consumer surplus when the production cost is sufficiently high or the outside option is sufficiently good. (ii) In the long run with an endogenous market structure, personalized pricing induces the socially optimal level of entry and so favors consumers. (iii) When some firms can use consumer data to price discriminate while others cannot, this “mixed” case can be the worst for consumers—and hence policies which prevent data-rich firms from price discriminating, or which force them to share their data, can benefit consumers.

[35] The information structures available to firms might also be constrained by communication incentives between firms and consumers. See, e.g., Ali, Lewis, and Vasserman (2021), and Ichihashi and Smolin (2022) for some recent research in this direction.

There are two important issues that our paper does not address. First, do firms have incentives to adopt personalized pricing? This of course depends on how costly it is for firms to acquire consumer data and develop the technology needed for personalized pricing, as well as the extent to which firms can commit to a pricing strategy before making offers. If it is costless to implement personalized pricing, and if firms simultaneously choose whether to do personalized pricing and what prices to offer, then the only equilibrium is that all firms do personalized pricing because a discriminatory pricing schedule includes uniform pricing as a special case. Second, do consumers have incentives to allow their data to be collected and then used for personalized pricing? For example, privacy policies like GDPR in the EU and CCPA in California give consumers some control over what data is harvested and how it is used. We are investigating this interesting question in some ongoing work.

Finally, throughout the paper we have focused on a standard IO setting. However our model also applies to other markets—such as competing employers that offer personalized wages according to workers' preferences for different job positions, or competing schools that offer personalized scholarships according to students' preferences and family incomes.

Appendix: Omitted Proofs for Sections 3 and 4

Proof of Lemma 1. We prove the existence and uniqueness of the equilibrium uniform price. (The rest of the lemma follows from arguments in the text.) Clearly $p - c < \phi(p)$ at $p = c$. Since $\phi(p)$ is non-increasing due to Assumption 1, it suffices to show that $p - c > \phi(p)$ at $p = \overline{v}$. This must be true if $\overline{v} = \infty$, because $\phi(p)$ is non-increasing and thus finite as $p \to \infty$. It also holds if $\overline{v} < \infty$ and $f(\overline{v}) > 0$, because in that case $\phi(\overline{v}) = 0$. Finally, then, consider $\overline{v} < \infty$ and $f(\overline{v}) = 0$, in which case $f(v)$ must be decreasing for v sufficiently close to $\overline{v}$. Notice that $\phi(p) \le \frac{\int_p^{\overline{v}} f(v)dv}{G(p|p)f(p)}$, which for p close to $\overline{v}$ is itself weakly less than $\frac{(\overline{v}-p)f(p)}{G(p|p)f(p)} = \frac{\overline{v}-p}{G(p|p)}$. This is less than $\overline{v} - c$ when p is close to $\overline{v}$.[36] □

Proof of Lemma 3. Using equation (13) the highest personalized price is $p_{\max} = c + \overline{v} - \max\{c, \underline{v}\}$. If $\underline{v} \le c$ then $p_{\max} = \overline{v} > p$. If $\underline{v} > c$ then $p_{\max} = c + \overline{v} - \underline{v}$, and so $p < p_{\max}$ if and only if $p - c < \overline{v} - \underline{v}$. Under Assumption 1, $\phi(p)$ is constant for $p \le \underline{v}$ and non-increasing for $p > \underline{v}$, and so the uniform price satisfies $p - c = \phi(p) \le \phi(\underline{v}) = \frac{1}{nh(0)}$. Meanwhile,

$$\frac{1}{n} = \int_0^{\overline{v}-\underline{v}} h(x)dx < h(0)(\overline{v} - \underline{v}) \ ,$$

where the equality is from the fact that $\Pr[v_i \ge \max_{j \ne i}\{v_j\}] = \frac{1}{n}$, and the inequality is from the assumption that $h(x) < h(0)$ for $x > 0$. Therefore we have $p - c < \overline{v} - \underline{v}$. □

Proof of Proposition 3. We first prove a few lemmas:

Lemma 6. *Under uniform pricing, the equilibrium pass-through rate $p'(c)$ at $c \to \overline{v}$ is*
(i) $p'(\overline{v}) = \frac{1}{2}$ if $f(\overline{v}) > 0$;
(ii) $p'(\overline{v}) = \frac{2}{3}$ if $f(\overline{v}) = 0$ and $f'(\overline{v}) < 0$;
(iii) $p'(\overline{v}) \in [\frac{1}{2}, 1]$ if $f(\overline{v}) = 0$, $f'(\overline{v}) = 0$, and $f(v)$ is log-concave.

Proof. From the equilibrium price condition $p - c = \phi(p)$, we derive $p'(c) = \frac{1}{1-\phi'(p)}$. Since $\phi'(\cdot) \le 0$ under Assumption 1, we must have $p'(c) \le 1$. Since $p \to \overline{v}$ as $c \to \overline{v}$, it suffices to examine $-\phi'(\overline{v})$.

Recall that

$$\phi(p) = \frac{\int_p^{\overline{v}} G(v|v)dF(v)}{h_p(0)} \ ,$$

[36] To ease the exposition we assumed that the joint distribution of valuations has full support on $[\underline{v}, \overline{v}]^n$. However Lemma 1 also holds under the weaker condition that if $G(\hat{v}|\hat{v}) = 0$ for some $\hat{v}$, then $G(v|v) = 0$ for any $v \le \hat{v}$. (This is true, e.g., when the joint distribution has a convex support.) Define $\underline{v}^* \equiv \max\{v : G(v|v) = 0\}$. Then $\underline{v}$ in the lemma can be replaced by $\underline{v}^*$.

where $h_p(0) = G(p|p)f(p) + \int_p^{\overline{v}} g(v|v)dF(v)$. Then

$$-\phi'(p) = \frac{G(p|p)f(p)}{h_p(0)} + \frac{\int_p^{\overline{v}} G(v|v)dF(v) \times \frac{\partial h_p(0)}{\partial p}}{h_p(0)^2} \ , \tag{23}$$

where $\frac{\partial h_p(0)}{\partial p} = G_2(p|p)f(p) + G(p|p)f'(p)$ with $G_2(x|y) \equiv \frac{\partial G(x|y)}{\partial y}$.

(i) Suppose first $f(\overline{v}) > 0$. Then $\lim_{p\to\overline{v}} h_p(0) = f(\overline{v}) > 0$, and so as $p \to \overline{v}$, the second term in (23) equals 0 and $-\phi'(\overline{v}) = \frac{f(\overline{v})}{f(\overline{v})} = 1$. Therefore, $p'(\overline{v}) = \frac{1}{2}$.

(ii) Suppose $f(\overline{v}) = 0$ and $f'(\overline{v}) < 0$. Then in the limit the first term in (23) equals

$$\lim_{p\to\overline{v}} \frac{G(p|p)f(p)}{h_p(0)} = \frac{1}{1 + \lim_{p\to\overline{v}} \frac{\int_p^{\overline{v}} g(v|v)dF(v)}{G(p|p)f(p)}} = 1 \ . \tag{24}$$

Here the first equality is from dividing both the numerator and denominator by $G(p|p)f(p)$, and the second equality is because L'hopital's rule implies that

$$\lim_{p\to\overline{v}} \frac{\int_p^{\overline{v}} g(v|v)dF(v)}{G(p|p)f(p)} = \lim_{p\to\overline{v}} \frac{-g(p|p)f(p)}{g(p|p)f(p) + G_2(p|p)f(p) + G(p|p)f'(p)} = 0 \ ,$$

where we have used $f(\overline{v}) = 0$, $f'(\overline{v}) < 0$, and $G_2(\overline{v}|\overline{v}) = 0$ (which is from the fact that $G(\overline{v}|y) = 1$ for any y). On the other hand, in the limit the second term in (23) equals $-\frac{1}{2}$. This is because

$$\lim_{p\to\overline{v}} \frac{h_p(0)^2}{\int_p^{\overline{v}} G(v|v)dF(v)} = -2 \lim_{p\to\overline{v}} \frac{h_p(0)\frac{\partial h_p(0)}{\partial p}}{G(p|p)f(p)} = -2 \lim_{p\to\overline{v}} \frac{\partial h_p(0)}{\partial p} \ , \tag{25}$$

where the first step is from L'hopital's rule and the second step uses (24). The claim then follows since $\lim_{p\to\overline{v}} \frac{\partial h_p(0)}{\partial p} \neq 0$ in this case. Therefore, $-\phi'(\overline{v}) = \frac{1}{2}$ and so $p'(\overline{v}) = \frac{2}{3}$.

(iii) The last possibility is $f(\overline{v}) = 0$ and $f'(\overline{v}) = 0$. We focus on the case when $f(v)$ is log-concave. Notice that $\frac{\partial h_p(0)}{\partial p} = f(p)[G_2(p|p) + G(p|p)\frac{f'(p)}{f(p)}]$. Given $f(\overline{v}) = 0$, $f'(v) < 0$ when v is sufficiently large; meanwhile, given f is log-concave, $\frac{f'}{f}$ is decreasing and so $\lim_{p\to\overline{v}} \frac{f'(p)}{f(p)}$ must be strictly negative. This, together with $G_2(\overline{v}|\overline{v}) = 0$, implies that $\frac{\partial h_p(0)}{\partial p}$ must be negative when p is sufficiently large. Therefore, for sufficiently large p, the second term in (23) is negative and so

$$-\phi'(p) \leq \frac{G(p|p)f(p)}{h_p(0)} \leq 1 \ ,$$

where the second inequality uses the expression for $h_p(0)$. This leads to $-\phi'(\overline{v}) \leq 1$ and so $p'(\overline{v}) \geq \frac{1}{2}$. □

Lemma 7. *If $f(\overline{v}) > 0$ or in the IID case,*

$$\lim_{p\to\overline{v}} \frac{\phi(p)f(p)}{1 - F(p)} = 1 \ .$$

Proof. Notice that

$$\frac{\phi(p)f(p)}{1-F(p)} = \frac{\frac{\int_p^{\overline{v}} G(v|v)dF(v)}{1-F(p)}}{G(p|p) + \frac{\int_p^{\overline{v}} g(v|v)dF(v)}{f(p)}} \ .$$

Concerning the numerator, we have

$$\lim_{p\to\overline{v}} \frac{\int_p^{\overline{v}} G(v|v)dF(v)}{1-F(p)} = \lim_{p\to\overline{v}} \frac{G(p|p)f(p)}{f(p)} = 1 \ ,$$

where the first equality is from L'hopital's rule. Concerning the denominator, it obviously converges to 1 if $f(\overline{v}) > 0$. If $f(\overline{v}) = 0$ but we focus on the IID case, $\int_p^{\overline{v}} g(v)dF(v) = -G(p)f(p) - \int_p^{\overline{v}} G(v)f'(v)dv$ by integration by parts. The denominator then becomes $-\int_p^{\overline{v}} G(v)f'(v)dv/f(p)$, which converges to 1 as well by L'hopital's rule. □

Lemma 8. *In the IID case, if $f(v)$ is log-concave and $f(\overline{v}) = 0$, then*

$$e^{2p'(\overline{v})-1} \leq \lim_{c\to\overline{v}} \frac{f(c)}{f(p)} \leq e^{2-\frac{1}{p'(\overline{v})}} \ .$$

Proof. Given $f(v)$ is log-concave, we can rewrite it as $f(v) = e^{\mu(v)}$ with $\mu(v)$ being a concave function. (Given $f(\overline{v}) = 0$, $\mu(v)$ must be decreasing as well when v is sufficiently large and $\lim_{v\to\overline{v}} \mu(v) = -\infty$.) Then $f(c)/f(p) = e^{\mu(c)-\mu(p)}$, and so it suffices to show that

$$2p'(\overline{v}) - 1 \leq \lim_{c\to\overline{v}}[\mu(c) - \mu(p)] \leq 2 - \frac{1}{p'(\overline{v})} \ .$$

First, given $\mu(\cdot)$ is concave, we have

$$\mu(c) - \mu(p) \leq -(p-c)\mu'(p) \ . \tag{26}$$

From the equilibrium price condition in the IID case, we have

$$\frac{(p-c)[F(p)^{n-1}f(p) + \int_p^{\overline{v}} f(v)dF(v)^{n-1}]}{[1-F(p)^n]/n} = 1$$

for any c. In the limit of $c \to \overline{v}$, both the numerator and denominator must go to zero. Therefore, L'hopital's rule implies that

$$\lim_{c\to\overline{v}} \frac{(p-c)F(p)^{n-1}f'(p)p'(c) + [p'(c)-1][F(p)^{n-1}f(p) + \int_p^{\overline{v}} f(v)dF(v)^{n-1}]}{-F(p)^{n-1}f(p)p'(c)} = 1 \ ,$$

or

$$\lim_{c\to\overline{v}} -(p-c)\frac{f'(p)}{f(p)} + \lim_{c\to\overline{v}} \frac{1-p'(c)}{p'(c)}[1 + \frac{\int_p^{\overline{v}} f(v)dF(v)^{n-1}}{F(p)^{n-1}f(p)}] = 1 \ . \tag{27}$$

Notice that

$$\begin{aligned}\lim_{p\to\overline{v}}\frac{\int_p^{\overline{v}} f(v)dF(v)^{n-1}}{F(p)^{n-1}f(p)} &= \lim_{p\to\overline{v}}\frac{-(n-1)F(p)^{n-2}f(p)^2}{(n-1)F(p)^{n-2}f(p)^2+F(p)^{n-1}f'(p)}\\ &= \lim_{p\to\overline{v}}\frac{-(n-1)f(p)}{(n-1)f(p)+F(p)f'(p)/f(p)}\\ &= 0\ ,\end{aligned}$$

where the first equality is from L'hopital's rule and the last equality is from $f(\overline{v})=0$ and the fact that under our conditions, $\lim_{p\to\overline{v}} f'(p)/f(p)<0$. (Given $f(\overline{v})=0$, $f'/f<0$ for sufficiently large p; given$f$ is log-concave, f'/f is decreasing.) Then,

$$\lim_{c\to\overline{v}}-(p-c)\frac{f'(p)}{f(p)}=\lim_{c\to\overline{v}}-(p-c)\mu'(p)=1-\frac{1-p'(\overline{v})}{p'(\overline{v})}=2-\frac{1}{p'(\overline{v})}\ , \tag{28}$$

where the first equality used $f(p)=e^{\mu(p)}$ and the second is from (27). Together with (26), this implies $\lim_{c\to\overline{v}}[\mu(c)-\mu(p)]\le 2-\frac{1}{p'(\overline{v})}$.

Second, given $\mu(\cdot)$ is concave, we also have

$$\mu(c)-\mu(p)\ge -(p-c)\mu'(c)\ . \tag{29}$$

Notice that

$$\lim_{c\to\overline{v}}-(p-c)\mu'(c)=\lim_{c\to\overline{v}}-(p-c)\frac{f'(c)}{f(c)}=\lim_{c\to\overline{v}}-(p-c)\frac{f'(p)}{f(p)}\frac{f(p)}{f'(p)}\frac{f'(c)}{f(c)}\ ,$$

and

$$\lim_{c\to\overline{v}}\frac{f(p)}{f'(p)}\frac{f'(c)}{f(c)}=\lim_{c\to\overline{v}}\frac{f(p)}{f(c)}\times\lim_{c\to\overline{v}}\frac{f'(c)}{f'(p)}=\lim_{c\to\overline{v}}\frac{f(p)}{f(c)}\times p'(\overline{v})\lim_{c\to\overline{v}}\frac{f(c)}{f(p)}=p'(\overline{v})\ ,$$

where the manipulation in the first step is legitimate because $f(p)/f(c)<1$ given $f(v)$ is decreasing at least for large v and $\lim_{c\to\overline{v}}\frac{f'(c)}{f'(p)}=p'(\overline{v})\lim_{c\to\overline{v}}\frac{f(c)}{f(p)}$ is also finite as shown in the first step. Together with (28), these results imply that

$$\lim_{c\to\overline{v}}-(p-c)\mu'(c)=2p'(\overline{v})-1\ .$$

This, together with (29), proves $\lim_{c\to\overline{v}}[\mu(c)-\mu(p)]\ge 2p'(\overline{v})-1$. □

Profit result Recall that $\Pi_U=(p-c)[1-F_{(n)}(p)]=n(p-c)\int_p^{\overline{v}}G(v|v)dF(v)$ and $\Pi_D=n\int_c^{\overline{v}}\int_c^v G(x|v)dxdF(v)$. Both go to zero as $c\to\overline{v}$. Then L'hopital rule implies that

$$\lim_{c\to\overline{v}}\frac{\Pi_D}{\Pi_U}=\lim_{c\to\overline{v}}\frac{\int_c^{\overline{v}}G(c|v)dF(v)}{[1-p'(c)]\int_p^{\overline{v}}G(v|v)dF(v)+\phi(p)G(p|p)f(p)p'(c)}\ , \tag{30}$$

where we have used the equilibrium price condition $p - c = \phi(p)$. Divide both the numerator and denominator by $1 - F(p)$. Notice that

$$\lim_{c \to \overline{v}} \frac{\int_p^{\overline{v}} G(v|v) dF(v)}{1 - F(p)} = \lim_{c \to \overline{v}} \frac{G(p|p) f(p)}{f(p)} = 1 \ .$$

This, together with Lemma 7, implies that the denominator of (30) divided by $1 - F(p)$ converges to 1. Therefore,

$$\lim_{c \to \overline{v}} \frac{\Pi_D}{\Pi_U} = \lim_{c \to \overline{v}} \frac{\int_c^{\overline{v}} G(c|v) dF(v)}{1 - F(p)} \ . \tag{31}$$

Consider first the general case with $f(\overline{v}) > 0$. L'hopital's rule implies that (31) equals

$$\lim_{c \to \overline{v}} \frac{-G(c|c) f(c) + \int_c^{\overline{v}} g(c|v) dF(v)}{-f(p) p'(c)} = \frac{1}{p'(\overline{v})} = 2 \ ,$$

where we have used $p'(\overline{v}) = \frac{1}{2}$ from Lemma 6(i). Therefore, in this case, $\lim_{c \to \overline{v}} \frac{\Pi_D}{\Pi_U} = 2$.

When $f(\overline{v}) = 0$, we focus on the IID case. Then (31) simplifies to

$$\lim_{c \to \overline{v}} \frac{G(c)[1 - F(c)]}{1 - F(p)} = \frac{1}{p'(\overline{v})} \lim_{c \to \overline{v}} \frac{f(c)}{f(p)} \ .$$

If $f(\overline{v}) = 0$ and $f'(\overline{v}) < 0$, L'hopital's rule implies that the above limit equals $\frac{1}{p'(\overline{v})^2}$. Using $p'(\overline{v}) = \frac{2}{3}$ from Lemma 6(ii), we have $\lim_{c \to \overline{v}} \frac{\Pi_D}{\Pi_U} = \frac{9}{4} \in (2, e)$.

If $f(\overline{v}) = 0$, $f'(\overline{v}) = 0$, and f is log-concave, then Lemma 8 implies that

$$\frac{e^{2p'(\overline{v}) - 1}}{p'(\overline{v})} \leq \lim_{c \to \overline{v}} \frac{\Pi_D}{\Pi_U} \leq \frac{e^{2 - \frac{1}{p'(\overline{v})}}}{p'(\overline{v})} \ .$$

From Lemma 6(iii), we know $p'(\overline{v}) \in [\frac{1}{2}, 1]$. One can check that in this range both the upper bound and lower bound are increasing functions of $p'(\overline{v})$. Therefore, in this case $\lim_{c \to \overline{v}} \frac{\Pi_D}{\Pi_U} \in [2, e]$. (The above bounds result also implies that $\lim_{c \to \overline{v}} \frac{\Pi_D}{\Pi_U} = e$ if $p'(\overline{v}) = 1$. This is the case for many distributions such as exponential, extreme value, and normal.)

Consumer surplus result To prove the consumer surplus result, we need one more lemma:

Lemma 9. *Recall that $f_{(n)}(v)$ is the density of $F_{(n)}(v)$. Then $f_{(n)}(\overline{v}) = n f(\overline{v})$.*

Proof. Notice that $F_{(n)}(v)$, the CDF of the valuation for the best product, can also be written as

$$F_{(n)}(v) = \int_{[\underline{v}, v]^n} \tilde{f}(\mathbf{v}) d\mathbf{v} \ .$$

Then

$$f_{(n)}(v) = n \int_{[\underline{v},v]^{n-1}} \tilde{f}(v, \mathbf{v}_{-i}) d\mathbf{v}_{-i} \ ,$$

where we have used the exchangeability of $\tilde{f}$. Therefore, $f_{(n)}(\overline{v}) = nf(\overline{v})$ by using the definition of the marginal density. □

Recall that $V_U = \int_p^{\overline{v}} [1 - F_{(n)}(v)]dv$ and $V_D = \int_c^{\overline{v}} [1 - F_{(n-1)}(v)]dv$. Both go to zero as $c \to \overline{v}$. Therefore, by L'hopital's rule we have

$$\lim_{c \to \overline{v}} \frac{V_D}{V_U} = \frac{1}{p'(\overline{v})} \lim_{c \to \overline{v}} \frac{1 - F_{(n-1)}(c)}{1 - F_{(n)}(p)} = \frac{1}{p'(\overline{v})^2} \lim_{c \to \overline{v}} \frac{f_{(n-1)}(c)}{f_{(n)}(p)} \ . \tag{32}$$

Consider first the general case with $f(\overline{v}) > 0$. From the definition of $F_{(n-1)}(v)$ in (3), we obtain

$$f_{(n-1)}(v) = f_{(n)}(v) - n[G(v|v)f(v) - \int_v^{\overline{v}} g(v|v_i)dF(v_i)] \ .$$

Therefore, $f_{(n-1)}(\overline{v}) = f_{(n)}(\overline{v}) - nf(\overline{v}) = 0$, where the second equality uses Lemma 9. On the other hand, from the definition of $F_{(n)}(v)$ in (2), $f_{(n)}(v) = G(v|v)f(v) + \int_{\underline{v}}^{v} g(v|v_i)dF(v_i)$, and so $f_{(n)}(\overline{v}) > 0$ given $f(\overline{v}) > 0$. Therefore, in this general case (32) equals 0.

When $f(\overline{v}) = 0$, we consider the IID case with a log-concave f. Then $f_{(n-1)}(c) = n(n-1)(1 - F(c))F(c)^{n-2}f(c)$ and $f_{(n)}(p) = nF(p)^{n-1}f(p)$. Therefore,

$$\lim_{c \to \overline{v}} \frac{f_{(n-1)}(c)}{f_{(n)}(p)} = (n-1) \lim_{c \to \overline{v}} [1 - F(c)] \frac{f(c)}{f(p)} \ ,$$

and so $\lim_{c \to \overline{v}} \frac{V_D}{V_U} = 0$ if $\lim_{c \to \overline{v}} \frac{f(c)}{f(p)}$ is finite.

If $f(\overline{v}) = 0$ and $f'(\overline{v}) < 0$, then

$$\lim_{c \to \overline{v}} \frac{f(c)}{f(p)} = \lim_{c \to \overline{v}} \frac{f'(c)}{f'(p)p'(c)} = \frac{1}{p'(\overline{v})} = \frac{3}{2} \ ,$$

where the first equality is from L'hopital's rule, the second uses $f'(\overline{v}) < 0$, and the third uses Lemma 6(ii). This is of course finite.

If $f(\overline{v}) = 0$ and $f'(\overline{v}) = 0$, Lemma 6(iii) and Lemma 8 jointly imply that $\lim_{c \to \overline{v}} \frac{f(c)}{f(p)}$ is finite. This completes the whole proof.

□

Proof of Proposition 4. If the market becomes fully covered under uniform pricing when n exceeds a threshold (which can happen if $c < \underline{v}$), then our result is obvious. In the following, we focus on the case where the market is not fully covered for any n.

In the IID case industry profit under uniform pricing can be written as

$$\Pi_U = \frac{[1-F(p)^n]^2/n}{F(p)^{n-1}f(p) + \int_p^{\overline{v}} f(v)dF(v)^{n-1}} \ .$$

Under the log-concavity condition, the uniform price p is decreasing in n, and so $F(p)^n$ must be of order $o(\frac{1}{n})$, i.e., $\lim_{n\to\infty} \frac{F(p)^n}{1/n} = 0$. Meanwhile, Theorem 1 in Gabaix et al. (2016), which approximates the Perloff-Salop price, shows that as $n \to \infty$,

$$\int_{\underline{v}}^{\overline{v}} f(v)dF(v)^{n-1} \sim f(F^{-1}(1-\frac{1}{n})) \cdot \Gamma(2+\gamma) \ ,$$

where $\Gamma(\cdot)$ is the Gamma function. (Notice that $\Gamma(x)$ decreases first and then increases in $x \in [1,2]$, and it is strictly positive but no greater than 1 in that range, and $\Gamma(1) = \Gamma(2) = 1$.) At the same time, notice that $\int_{\underline{v}}^{p} f(v)dF(v)^{n-1} < F(p)^{n-1} \times \max_{v\in[\underline{v},p]} f(v)$, so it must be of order $o(\frac{1}{n})$ given f is finite. Therefore, as $n \to \infty$, we have

$$\Pi_U \sim \frac{[1-o(\frac{1}{n})]^2}{o(\frac{1}{n})/(\frac{1}{n}) + nf(F^{-1}(1-\frac{1}{n})) \cdot \Gamma(2+\gamma)} \ .$$

Since the price is decreasing in n, Π_U must be finite for any n. This implies that $\lim_{n\to\infty} nf(F^{-1}(1-\frac{1}{n})) > 0$. Therefore, when n is large, those $o(\frac{1}{n})$ terms can be safely ignored. This yields

$$\Pi_U \sim \frac{1}{nf(F^{-1}(1-\frac{1}{n})) \cdot \Gamma(2+\gamma)} \ . \tag{33}$$

Industry profit under personalized pricing is

$$\Pi_D = \int_c^{\overline{v}} \frac{1-F(v)}{f(v)} dF(v)^n = \int_{F(c)}^1 \frac{1-t}{f(F^{-1}(t))} dt^n \ .$$

Proposition 2 in Gabaix et al. (2016) has shown that, as $n \to \infty$,

$$\mathbb{E}[v_{n:n} - v_{n-1:n}] = \int_0^1 \frac{1-t}{f(F^{-1}(t))} dt^n \sim \frac{\Gamma(1-\gamma)}{nf(F^{-1}(1-\frac{1}{n}))} \ .$$

Notice that

$$\Pi_D = \mathbb{E}[v_{n:n} - v_{n-1:n}] - \int_0^{F(c)} \frac{1-t}{f(F^{-1}(t))} dt^n \ .$$

The second term equals $\frac{1-\tilde{t}}{f(F^{-1}(\tilde{t}))} F(c)^n$ for some $\tilde{t} \in (0, F(c))$, and so is of order $o(\frac{1}{n})$ and can be safely ignored when n is large. Therefore, for large n, we have

$$\Pi_D \sim \frac{\Gamma(1-\gamma)}{nf(F^{-1}(1-\frac{1}{n}))} \ . \tag{34}$$

Comparing (33) and (34), we can claim that when n is sufficiently large, personalized pricing reduces profit (and so improves consumer surplus) if

$$\Gamma(1-\gamma)\Gamma(2+\gamma) < 1 \ ,$$

which is true when $\gamma \in (-1, 0)$. (Notice that the equality holds when $\gamma = -1$ or 0. In these cases, unfortunately, the approximations are not precise enough to generate meaningful comparison results in the limit.) $\square$

Proof of Lemma 4. Notice that

$$\begin{aligned}\mathbb{E}[\max\{c, v_{n:n}\}] = \tfrac{1}{n}\mathbb{E}[\max\{c, v_n\}|v_n > \max\{v_1, ..., v_{n-1}\}] \\ + (1-\tfrac{1}{n})\mathbb{E}[\max\{c, v_1, ..., v_{n-1}\}|v_n < \max\{v_1, ..., v_{n-1}\}] \ ,\end{aligned}$$

and with Assumption 2 we also have

$$\begin{aligned}\mathbb{E}[\max\{c, \hat{v}_{n-1:n-1}\}] = \tfrac{1}{n}\mathbb{E}[\max\{c, v_1, ..., v_{n-1}\}|v_n > \max\{v_1, ..., v_{n-1}\}] \\ + (1-\tfrac{1}{n})\mathbb{E}[\max\{c, v_1, ..., v_{n-1}\}|v_n < \max\{v_1, ..., v_{n-1}\}] \ .\end{aligned}$$

Therefore, the match efficiency improvement in (21) is equal to

$$\tfrac{1}{n}\mathbb{E}[\max\{c, v_n\} - \max\{c, v_1, ..., v_{n-1}\}|v_n > \max\{v_1, ..., v_{n-1}\}] \ ,$$

which is just equal to (20).

We need to further show that both the free-entry equilibrium and the socially optimal solution are unique. (Otherwise, a free-entry equilibrium could differ from a socially optimal solution due to a selection issue.) It suffices to show that (20) is decreasing in n. To see that, it is more convenient to use the expression for Π_D in (14). Under Assumption 2, $x_c = v_i - \max_{j\neq i}\{c, v_j\}$ must become smaller in the sense of first-order stochastic dominance as one more firm is added, and so $1 - H_c(x)$ decreases in n for any x. This implies that $\frac{1}{n}\Pi_D$ decreases in n. $\square$

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Online Appendix: Other Omitted Proofs and Details [Not For Publication]

Primitive conditions for Assumption 1

We report some primitive conditions for Assumption 1. We use the following notation

$$G_2(x|y) \equiv \frac{\partial G(x|y)}{\partial y} \ . \tag{35}$$

Lemma 10. *(i) If the joint density $\tilde{f}$ is log-concave, then $1 - H_z(x)$ is log-concave in x. (ii) $\phi(z) = \frac{1-H_z(0)}{h_z(0)}$ is non-increasing in z if (a) $G_2(v|v) \geq 0$ and $f'(v) \geq 0$, or (b) $\tilde{f}$ is log-concave and $\frac{G_2(v|v)}{G(v|v)}$ is non-increasing in v. (In particular, condition (b) holds in the IID case with a log-concave f.)*

Proof. (i) Note that

$$1 - H_z(x) = \int_{A_x} \tilde{f}(\mathbf{v}) d\mathbf{v} \ ,$$

where $A_x = \{\mathbf{v} : v_i - \max_{j \neq i}\{z, v_j\} > x\}$. To prove $1 - H_z(x)$ is log-concave in x, according to the Prékopa-Borell Theorem (see, e.g., Caplin and Nalebuff, 1991), it suffices to show that, for any $\lambda \in [0, 1]$, we have

$$\lambda A_{x_0} + (1-\lambda) A_{x_1} \subset A_{\lambda x_0 + (1-\lambda)x_1} \ , \tag{36}$$

where the former is the Minkowski average of A_{x_0} and A_{x_1}. Let $\mathbf{v}^0 \in A_{x_0}$ and $\mathbf{v}^1 \in A_{x_1}$, i.e.,

$$v_i^0 > z + x_0 \ \text{ and } \ v_i^0 > v_j^0 + x_0 \ \text{ for any } j \neq i \ ,$$

and

$$v_i^1 > z + x_1 \ \text{ and } \ v_i^1 > v_j^1 + x_1 \ \text{ for any } j \neq i \ .$$

These immediately imply that

$$v_i^\lambda > z + \lambda x_0 + (1-\lambda)x_1 \ \text{ and } \ v_i^\lambda > v_j^\lambda + \lambda x_0 + (1-\lambda)x_1 \ \text{ for any } j \neq i \ ,$$

where $v_i^\lambda = \lambda v_i^0 + (1-\lambda)v_i^1$. Hence, we have $\mathbf{v}^\lambda \in A_{\lambda x_0 + (1-\lambda)x_1}$, and so (36) holds.

(ii) Recall that

$$\phi(z) = \frac{\int_z^{\overline{v}} G(v|v) dF(v)}{G(z|z)f(z) + \int_z^{\overline{v}} g(v|v) dF(v)} \ .$$

For $z \leq \underline{v}$, $\phi(z)$ is a constant and so is non-increasing. In the following, we focus on $z > \underline{v}$. Using $\frac{dG(v|v)}{dv} = g(v|v) + G_2(v|v)$, one can check that $\phi'(z) \leq 0$ if and only if

$$G(z|z)f(z) + \int_z^{\overline{v}} g(v|v) dF(v) + \left(\frac{f'(z)}{f(z)} + \frac{G_2(z|z)}{G(z|z)} \right) \int_z^{\overline{v}} G(v|v) dF(v) \geq 0 \ . \tag{37}$$

This must be true if condition (a) holds. To see condition (b), notice that the log-concavity of the joint density $\tilde{f}$ implies log-concavity of the marginal density f and so $\frac{f'(z)}{f(z)} \geq \frac{f'(v)}{f(v)}$ for $v \geq z$. Therefore, a sufficient condition for (37) is

$$G(z|z)f(z) + \int_z^{\overline{v}} g(v|v)dF(v) + \int_z^{\overline{v}} G(v|v)f'(v)dv + \frac{G_2(z|z)}{G(z|z)} \int_z^{\overline{v}} G(v|v)dF(v) \geq 0 \ .$$

Applying integration by parts to the third term, we can rewrite the above condition as

$$f(\overline{v}) - \int_z^{\overline{v}} G_2(v|v)dF(v) + \frac{G_2(z|z)}{G(z|z)} \int_z^{\overline{v}} G(v|v)dF(v) \geq 0 \ .$$

A sufficient condition for this to hold is that

$$\frac{G_2(z|z)}{G(z|z)} \geq \frac{G_2(v|v)}{G(v|v)}$$

for any $v \in [z, \overline{v}]$. This is true if $\frac{G_2(v|v)}{G(v|v)}$ is non-increasing in v. □

Valuation correlation

It is interesting to consider how the impact of personalized pricing depends on the degree of correlation in product valuations, but it appears hard to obtain general results. It is clear that in the limit case with perfectly (positively) correlated valuations the impact of personalized pricing disappears—because firms will price at marginal cost under both the uniform and discriminatory regimes. The following example suggests that, as one might expect, a greater degree of correlation weakens the impact of personalized pricing.

Example: bivariate Normal distribution. Suppose $n = 2$ and valuations are drawn from a bivariate Normal distribution with mean μ, variance σ, and correlation coefficient $\rho \in (-1, 1)$. Suppose the market is fully covered.[37] Then one can show that:[38]

$$\Pi_D = 2\sigma\sqrt{\frac{1-\rho}{\pi}} \quad \text{and} \quad \Pi_U = \sigma\sqrt{\pi(1-\rho)} \ , \quad \text{and hence} \quad \Pi_D - \Pi_U \propto -\sqrt{1-\rho} \ .$$

In this example, as product valuations become more correlated (i.e., as ρ increases), profit falls under both uniform and discriminatory pricing, and the impact of personalized

[37]Strictly speaking, full market coverage is impossible because the lower support of the valuation distribution is unbounded. However the market is almost covered if μ is large enough for a given c.

[38]Since the joint density of the bivariate Normal is log-concave, the first part of Assumption 1 is satisfied (the second part is not needed since the market is covered). Also notice that $v_1 - v_2$ is normally distributed with mean 0, variance $2\sigma^2(1-\rho) \equiv \tau^2$, and density function $h(x) = \frac{1}{\sqrt{2\pi}\tau}e^{-\frac{x^2}{2\tau^2}}$. Hence

$$\Pi_U = \frac{1}{2h(0)} \ , \quad \text{and} \quad \Pi_D = 2\int_0^\infty xh(x)dx = -2\tau^2 \int_0^\infty h'(x)dx = 2\tau^2 h(0) \ ,$$

where we used $xh(x) = -\tau^2 h'(x)$. Substituting in for $h(0)$ and τ gives the stated profit expressions.

pricing on profit also becomes smaller. (Given the assumption of a covered market, the impact on consumer surplus also gets smaller.) Intuitively, higher correlation means that products are less differentiated, and so firms compete more fiercely. In the limit case of perfect positive correlation (i.e., as $\rho \to 1$), personalized pricing has no impact on profit or consumer surplus because firms' products become homogeneous. (Notice, however, that in this example the profit ratio $\frac{\Pi_D}{\Pi_U} = \frac{2}{\pi}$ is *independent* of the degree of correlation.)

Omitted Details and Proofs for Section 5

Equilibrium analysis

We first derive the equilibrium prices in the mixed regime. (All proofs can be found in the next section.) Consider $0 < k < n$ and let p denote the equilibrium price of each firm that cannot price discriminate.

Personalized prices For any given uniform price vector $\mathbf{p}_U = (p_{k+1}, ..., p_n)$ offered by firms $k+1$ to n that move first, firms 1 to k compete as in Section 3.2, except that now consumers have an outside option $v_0 \equiv \max\{0, v_{k+1} - p_{k+1}, ..., v_n - p_n\}$. Let

$$\hat{x}_{\mathbf{p}_U,c} \equiv v_1 - c - \max\{v_0, v_2 - c, ..., v_k - c\}$$

be the advantage of firm 1 relative to a consumer's other alternatives (including the outside option) when all firms that can price discriminate charge a price equal to marginal cost. Therefore in equilibrium firm 1, say, offers personalized prices

$$p_1(v_1, ..., v_n; \mathbf{p}_U) = \begin{cases} c + \hat{x}_{\mathbf{p}_U,c} & \text{if } \hat{x}_{\mathbf{p}_U,c} > 0 \\ c & \text{otherwise} \end{cases},$$

and wins a consumer if and only if $\hat{x}_{\mathbf{p}_U,c} > 0$.

In the equilibrium we are looking for, firms $k+1$ to n offer the same uniform price p, i.e., $\mathbf{p}_U = (p, ...p)$, where p will be solved for later. In that case, we simplify the notation $\hat{x}_{\mathbf{p}_U,c}$ to $\hat{x}_{p,c}$, and let $\hat{H}_{p,c}$ be its CDF. Since firm 1 wins a consumer if and only if $\hat{x}_{p,c} > 0$, its equilibrium profit is

$$\hat{\pi}_D = \int_0^\infty x d\hat{H}_{p,c}(x) = \int_0^\infty [1 - \hat{H}_{p,c}(x)]dx \ . \tag{38}$$

Whenever $p > c$, $\hat{x}_{p,c}$ exceeds $x_c = v_1 - c - \max_{j>1}\{0, v_j - c\}$ (which was introduced in Section 3.2) in the sense of first order stochastic dominance. Therefore comparing with equation (14) from earlier, each personalized-pricing firm here in the mixed regime earns more than in the regime where all firms price discriminate (i.e., $\hat{\pi}_D > \frac{1}{n}\Pi_D$).

The uniform price We now solve for the equilibrium uniform price p charged by firms $k+1$ to n. To do this, it is useful to define the random variable

$$\tilde{x}_{p,c} \equiv v_n - p - \max\{0, v_1 - c, ..., v_k - c, v_{k+1} - p, ..., v_{n-1} - p\} \ , \tag{39}$$

which is the advantage of firm n relative to a consumer's other alternatives (including the outside option) when all firms charge their lowest possible equilibrium price (i.e., c for firms that do personalized pricing, and p for all the others). Let $\tilde{H}_{p,c}$ and $\tilde{h}_{p,c}$ be respectively the CDF and density function of $\tilde{x}_{p,c}$.

Suppose firm n unilaterally deviates to price p_n. Using (39) its deviation demand is

$$\Pr[v_n - p_n > \max\{0, v_1 - c, ..., v_k - c, v_{k+1} - p, ..., v_{n-1} - p\}] = 1 - \tilde{H}_{p,c}(p_n - p) \ , \tag{40}$$

and its deviation profit by $(p_n - c)[1 - \tilde{H}_{p,c}(p_n - p)]$. To understand (40), notice that if, say, $v_n - p_n < v_1 - c$, firm n cannot win the consumer because firm 1 can offer her more surplus with a personalized price close to marginal cost. Therefore, to calculate firm n's deviation demand we should set the price of each firm that can price discriminate to c. This implies that the existence of firms that can do personalized pricing intensifies competition for firms that cannot.

To ensure the firm's problem is well-behaved we make the following assumption:

Assumption 4. *$1 - \tilde{H}_{p,c}(x)$ is log-concave in x, and $\frac{1-\tilde{H}_{p,c}(0)}{\tilde{h}_{p,c}(0)}$ is non-decreasing in c and non-increasing in both p and k.*

In the next section we show that this assumption holds in the IID case with a log-concave f. Under this assumption, the equilibrium uniform price p solves

$$p - c = \frac{1 - \tilde{H}_{p,c}(0)}{\tilde{h}_{p,c}(0)} \ , \tag{41}$$

and each firm that does uniform pricing earns profit

$$\tilde{\pi}_U = (p - c)[1 - \tilde{H}_{p,c}(0)] = \frac{[1 - \tilde{H}_{p,c}(0)]^2}{\tilde{h}_{p,c}(0)} \ . \tag{42}$$

Lemma 11. *Suppose Assumption 4 holds. The equilibrium uniform price p uniquely solves* (41)*, and it is lower than in the uniform pricing regime and decreasing in k.*

Intuitively, as more firms are able to personalize their price, their ability to poach consumers with low personalized offers induces the uniform-pricing firms to cut their price. Since the equilibrium uniform price p is lower than in the case where all firms set a uniform price, for any given deviation price p_n, firm n's deviation demand in this mixed regime is smaller than in the regime of uniform pricing. This implies that each uniform-pricing firm earns less in this mixed regime than in the regime of uniform pricing (i.e., $\tilde{\pi}_U < \frac{1}{n}\Pi_U$).

Welfare measures Industry profit in the mixed regime is

$$\Pi_M = k\hat{\pi}_D + (n-k)\tilde{\pi}_U \ . \tag{43}$$

Expected consumer surplus is

$$V_M = \mathbb{E}[\max\{0, v_{k-1:k} - c, v_{n-k:n-k} - p\}] \ , \tag{44}$$

where $v_{k-1:k}$ denotes the second best among $\{v_1, ..., v_k\}$ and $v_{n-k:n-k}$ denotes the best among $\{v_{k+1}, ..., v_n\}$. (If $k = 1$ the $v_{k-1:k} - c$ term vanishes.) To see this, recall that $v_0 = \max\{0, v_{n-k:n-k} - p\}$ is a consumer's outside option when the first k firms compete for her by offering personalized prices. If $\max\{v_1 - c, ..., v_k - c\} \geq v_0$, the consumer buys from one of the first k firms, in which case her surplus is $\max\{v_0, v_{k-1:k} - c\}$. (If the second best among the k firms is worse than v_0, the consumer's surplus is v_0; otherwise it is $v_{k-1:k} - c$.) Otherwise, she takes the outside option v_0. Therefore, the expected consumer surplus is $\mathbb{E}[\max\{v_0, v_{k-1:k} - c\}]$, which leads to the expression for V_M in equation (44). Finally, total welfare in the mixed case is simply $W_M = \Pi_M + V_M$.

Omitted proofs

All the omitted proofs for the case of asymmetrically informed firms can be found in this section. We begin with some preliminary results.

Lemma 12. *Suppose valuations are IID. Then*

$$\frac{1 - \tilde{H}_{p,c}(0)}{\tilde{h}_{p,c}(0)} = \frac{\int_p^{\overline{v}} F(v-p+c)^k F(v)^{n-k-1} dF(v)}{f(\overline{v})F(\overline{v}-p+c)^k - \int_p^{\overline{v}} F(v-p+c)^k F(v)^{n-k-1} df(v)} \ . \tag{45}$$

Proof. Since valuations are IID we can write

$$1 - \tilde{H}_{p,c}(x) = \int_{p+x}^{\overline{v}} F(v-p+c-x)^k F(v-x)^{n-k-1} dF(v) \ , \tag{46}$$

and hence we can write that

$$\frac{1 - \tilde{H}_{p,c}(0)}{\tilde{h}_{p,c}(0)} = \frac{\int_p^{\overline{v}} F(v-p+c)^k F(v)^{n-k-1} dF(v)}{F(c)^k F(p)^{n-k-1} f(p) + \int_p^{\overline{v}} f(v) d[F(v-p+c)^k F(v)^{n-k-1}]} \ . \tag{47}$$

Integrating the denominator by parts then gives the stated expression. □

Lemma 13. *Suppose valuations are IID and f is log-concave. Then Assumption 4 holds.*

Proof. Firstly, log-concavity of $1 - \tilde{H}_{p,c}(x)$ can be established using very similar steps as in the proof of Lemma 10, and so we omit the details.

Secondly, define $\lambda(v) = F(v-p+c)^k F(v)^{n-k-1} f(v)$. After some manipulations, one can show that (45) is non-decreasing in c if and only if the following is weakly positive:

$$\int_p^{\overline{v}} \lambda(v)dv \int_p^{\overline{v}} \lambda(v)\frac{f(v-p+c)}{F(v-p+c)}\frac{f'(v)}{f(v)}dv - \int_p^{\overline{v}} \lambda(v)\frac{f'(v)}{f(v)}dv \int_p^{\overline{v}} \lambda(v)\frac{f(v-p+c)}{F(v-p+c)}dv$$
$$+\lambda(\overline{v})\int_p^{\overline{v}} \lambda(v)\left[\frac{f(v-p+c)}{F(v-p+c)} - \frac{f(\overline{v}-p+c)}{F(\overline{v}-p+c)}\right] dv \ . \tag{48}$$

The first line of (48) can be written as

$$\Delta = \int_p^{\overline{v}}\int_p^{\overline{v}} \lambda(v)\lambda(w)\frac{f(w-p+c)}{F(w-p+c)}\left[\frac{f'(w)}{f(w)} - \frac{f'(v)}{f(v)}\right] dvdw \ ,$$

or, alternatively, after changing the order of integration, as

$$\Delta = \int_p^{\overline{v}}\int_p^{\overline{v}} \lambda(v)\lambda(w)\frac{f(v-p+c)}{F(v-p+c)}\left[\frac{f'(v)}{f(v)} - \frac{f'(w)}{f(w)}\right] dvdw \ .$$

Summing these last two equations together, we obtain that

$$2\Delta = \int_p^{\overline{v}}\int_p^{\overline{v}} \lambda(v)\lambda(w)\left[\frac{f(w-p+c)}{F(w-p+c)} - \frac{f(v-p+c)}{F(v-p+c)}\right]\left[\frac{f'(w)}{f(w)} - \frac{f'(v)}{f(v)}\right] dvdw \geq 0 \ ,$$

where the inequality follows because f logconcave implies that f/F and f'/f are both decreasing. The second line of (48) is also weakly positive due to f/F being decreasing. Hence (45) is indeed non-decreasing in c.

Thirdly, again using the definition of $\lambda(v) = F(v-p+c)^k F(v)^{n-k-1} f(v)$, one can show that (45) is non-increasing in p if and only if the following is weakly negative:

$$-k\left[\int_p^{\overline{v}} \lambda(v)dv \int_p^{\overline{v}} \lambda(v)\frac{f(v-p+c)}{F(v-p+c)}\frac{f'(v)}{f(v)}dv - \int_p^{\overline{v}} \lambda(v)\frac{f'(v)}{f(v)}dv \int_p^{\overline{v}} \lambda(v)\frac{f(v-p+c)}{F(v-p+c)}dv\right]$$
$$+F(c)^k F(p)^{n-1-k} f(p)\int_p^{\overline{v}} \lambda(v)\left[\frac{f'(v)}{f(v)} - \frac{f'(p)}{f(p)}\right] dv$$
$$+\lambda(\overline{v})\left\{\int_p^{\overline{v}} \lambda(v)k\left[\frac{f(\overline{v}-p+c)}{F(\overline{v}-p+c)} - \frac{f(v-p+c)}{F(v-p+c)}\right] dv - F(c)^k F(p)^{n-k-1} f(p)\right\} \ . \tag{49}$$

Using the previous step of the proof, the first line of (49) is negative. Meanwhile the second and third lines of (49) are also negative because f logconcave implies that f'/f and f/F are decreasing. Hence (45) is indeed non-increasing in p.

Finally, to prove that (45) is non-increasing in k, define

$$\tilde{x}_{p,c,q} \equiv v_n - p - \max\{0, v_1 - q, ..., v_k - c, v_{k+1} - p, ..., v_{n-1} - p\} \ ,$$

as the advantage of product n when product 1 is priced at q, products $2,\ldots,k$ are priced at c, and products $k+1,\ldots,n$ are priced at p. Letting $\tilde{H}_{p,c,q}(x)$ denote its CDF, we have

$$1-\tilde{H}_{p,c,q}(x)=\int_{p+x}^{\overline{v}} F(v-p+c-x)^{k-1}F(v-p+q-x)F(v-x)^{n-k-1}dF(v)\ .$$

After some manipulations we can then write

$$\frac{1-\tilde{H}_{p,c,q}(0)}{\tilde{h}_{p,c,q}(0)} = \frac{\int_p^{\overline{v}} F(v-p+c)^{k-1}F(v-p+q)F(v)^{n-k-1}dF(v)}{f(\overline{v})F(\overline{v}-p+c)^{k-1}F(\overline{v}-p+q)-\int_p^{\overline{v}} F(v-p+c)^{k-1}F(v-p+q)F(v)^{n-k-1}df(v)}\ . \tag{50}$$

Note that when $q=c$ this degenerates to (45) with k firms doing personalized pricing, and when $q=p$ it degenerates to (45) but with $k-1$ firms doing personalized pricing. Therefore to prove that (45) is non-increasing in k, it is sufficient to prove that (50) is increasing in q. One can show that this is true if and only if the following is positive:

$$\begin{aligned}&\int_p^{\overline{v}}\tilde{\lambda}(v)dv\int_p^{\overline{v}}\tilde{\lambda}(v)\frac{f(v-p+q)}{F(v-p+q)}\frac{f'(v)}{f(v)}dv-\int_p^{\overline{v}}\tilde{\lambda}(v)\frac{f'(v)}{f(v)}dv\int_p^{\overline{v}}\tilde{\lambda}(v)\frac{f(v-p+q)}{F(v-p+q)}dv\\&+\tilde{\lambda}(\overline{v})\int_p^{\overline{v}}\tilde{\lambda}(v)\left[\frac{f(v-p+q)}{F(v-p+q)}-\frac{f(\overline{v}-p+q)}{F(\overline{v}-p+q)}\right]dv\ ,\end{aligned} \tag{51}$$

where we define $\tilde{\lambda}(v)=F(v-p+c)^{k-1}F(v-p+q)F(v)^{n-k-1}f(v)$. However notice that (51) is the same as (48), just with c replaced by q and $\lambda(v)$ replaced by $\tilde{\lambda}(v)$. Therefore using the same steps as in the second part of the proof, it is easy to show that (51) is positive. Hence (45) is indeed non-increasing in k as claimed. □

Lemma 14. *Suppose valuations are IID standard exponential. Then*

$$\frac{1-\tilde{H}_{p,c}(0)}{\tilde{h}_{p,c}(0)}=1\quad \textit{and}\quad \frac{1-\hat{H}_{p,c}(0)}{\hat{h}_{p,c}(0)}=1\ . \tag{52}$$

Proof. The first equation follows from equation (45), and the fact that with the standard exponential $f(\overline{v})=0$ and $df(v)=-dF(v)$.

To derive the second equation, first note that

$$1-\hat{H}_{p,c}(x)=\int_{c+x}^{\overline{v}} F(v-x)^{k-1}F(v-x+p-c)^{n-k}dF(v)\ , \tag{53}$$

and hence

$$\begin{aligned}\hat{h}_{p,c}(x)&= F(c)^{k-1}F(p)^{n-k}f(c+x)+\int_{c+x}^{\overline{v}} f(v)d\left[F(v-x)^{k-1}F(v-x+p-c)^{n-k}\right]\\&= F(\overline{v}-x)^{k-1}F(\overline{v}-x+p-c)^{n-k}f(\overline{v})-\int_{c+x}^{\overline{v}} F(v-x)^{k-1}F(v-x+p-c)^{n-k}df(v)\\&= 1-\hat{H}_{p,c}(x)\ ,\end{aligned}$$

where the second line uses integration by parts, and the third line again uses $f(\overline{v}) = 0$ and $df(v) = -dF(v)$. Therefore $[1 - \hat{H}_{p,c}(0)]/\hat{h}_{p,c}(0) = 1$. $\square$

We now prove the remaining results.

Proof of Lemma 11. Since $1 - \tilde{H}_{p,c}(x)$ is log-concave in x, a uniform-pricing firm's profit is quasiconcave in its price. Hence the first-order condition (41) is sufficient to determine the equilibrium uniform price. Equation (41) has a unique solution because its lefthand side is strictly increasing in p while its righthand side is non-increasing in p. Moreover this solution is decreasing in k because the lefthand side of (41) is independent of k while the righthand side is non-increasing in k.

Finally, we prove that for any $0 < k < n$ the uniform price p is lower than in the uniform-pricing regime. To this end, let p_U denote the equilibrium price when all firms do uniform pricing. Towards a contradiction, suppose that $p > p_U$. Notice that

$$\frac{1 - \tilde{H}_{p,c}(0)}{\tilde{h}_{p,c}(0)} \leq \frac{1 - \tilde{H}_{p,p}(0)}{\tilde{h}_{p,p}(0)} = \frac{1 - H_p(0)}{h_p(0)} \leq \frac{1 - H_{p_U}(0)}{h_{p_U}(0)} \ , \tag{54}$$

where the first inequality follows because $p > c$ and $\frac{1-\tilde{H}_{p,c}(0)}{\tilde{h}_{p,c}(0)}$ is non-decreasing in c (from Assumption 4), the equality follows from inspection of (10) and (47), and the second inequality follows from the supposition that $p > p_U$ and because $\frac{1-H_p(0)}{h_p(0)}$ is non-increasing in p (from Assumption 1). However we also know from equations (7) and (41) that

$$p_U - c = \frac{1 - H_{p_U}(0)}{h_{p_U}(0)} \quad \text{and} \quad p - c = \frac{1 - \tilde{H}_{p,c}(0)}{\tilde{h}_{p,c}(0)} \ .$$

Combined with equation (54) this implies that $p \leq p_U$. But this is a contradiction to our original supposition that $p > p_U$. $\square$

Proof of Proposition 6. As a preliminary step, we note that $p = c + 1$. This follows from the first-order condition (41), and Lemma 14 which shows that the righthand side of the first-order condition equals 1.

Consider industry profit. Using equation (42) and $p = c + 1$ we can write

$$\tilde{\pi}_U = (p - c)[1 - \tilde{H}_{p,c}(0)] = 1 - \tilde{H}_{p,c}(0) \ .$$

Using equation (38) and Lemma 14 we can also write

$$\hat{\pi}_D = \int_0^\infty [1 - \hat{H}_{p,c}(x)]dx = \int_0^\infty \left[\frac{1 - \hat{H}_{p,c}(x)}{\hat{h}_{p,c}(x)}\right] d\hat{H}_{p,c}(x) = 1 - \hat{H}_{p,c}(0) \ .$$

Hence using equation (43) we can write industry profit as

$$\begin{aligned}
\Pi_M &= k[1-\hat{H}_{p,c}(0)] + (n-k)[1-\tilde{H}_{p,c}(0)] \\
&= k\int_c^\infty F(v)^{k-1}F(v+1)^{n-k}dF(v) + (n-k)\int_{1+c}^\infty F(v)^{n-k-1}F(v-1)^k dF(v) \\
&= k\int_c^\infty F(v)^{k-1}F(v+1)^{n-k}dF(v) + (n-k)\int_c^\infty F(v+1)^{n-k-1}F(v)^k dF(v+1) \\
&= \int_c^\infty dF(v+1)^{n-k}F(v)^k = 1 - F(c+1)^{n-k}F(c)^k \ ,
\end{aligned}$$

where the second line uses equations (46) and (53) as well as $p = c+1$, and the third line uses a change of variables. Recall from page 18 that $\Pi_D = 1-F(c)^n$ and $\Pi_U = 1-F(c+1)^n$. Hence when $c \leq 0$ then $\Pi_M = \Pi_D$ but otherwise $\Pi_M < \Pi_D$. Similarly when $c \leq -1$ then $\Pi_M = \Pi_U$ but otherwise $\Pi_M > \Pi_U$. Item (i) then follows.

Now consider welfare. Clearly $W_D > W_M$ because under the discriminatory regime each consumer buys the product i with the highest value of $v_i - c$ conditional on it being positive, which is not the case in the mixed regime. Similarly when the market is fully covered in the uniform regime (i.e., when $c \leq -1$) $W_D = W_U$ because every consumer buys the product with the highest v_i, but otherwise $W_D > W_U$ due to some consumers with a valuation above cost being excluded from the market in the uniform regime.

Now compare W_U and W_M. Note that when $c \leq -1$ then under both regimes the uniform price is $1+c \leq 0$ and hence the market is covered; it is immediate then that $W_U > W_M$ because under the uniform pricing regime each consumer buys the product i with the highest value of $v_i - c$, whereas this is not the case in the mixed regime. In the remainder of this part of the proof consider $c > -1$. Letting $F_{j:j}$ denote the CDF of the highest of j random variables, it is convenient to write

$$\begin{aligned}
W_M &= \int_p^\infty (v-c)F_{k:k}(v-p+c)dF_{n-k:n-k}(v) + \int_c^\infty (v-c)F_{n-k:n-k}(v+p-c)dF_{k:k}(v) \\
&= \int_{c+1}^\infty (v-c)F_{k:k}(v-1)dF_{n-k:n-k}(v) + \int_c^\infty (v-c)F_{n-k:n-k}(v+1)dF_{k:k}(v) \\
&= \int_c^\infty (v-c+1)F_{k:k}(v)dF_{n-k:n-k}(v+1) + \int_c^\infty (v-c)F_{n-k:n-k}(v+1)dF_{k:k}(v) \\
&= \int_c^\infty F_{k:k}(v)dF_{n-k:n-k}(v+1) + \int_c^\infty [1-F_{n-k:n-k}(v+1)F_{k:k}(v)]dv \ ,
\end{aligned}$$

where the second line uses $p = c+1$, the third line uses a change of variables, and the fourth line integrates by parts. Also note that $W_U = \int_{1+c}^\infty (v-c)dF(v)^n$. After some simplifications one can then write that

$$\begin{aligned}
\frac{d}{dc}(W_U - W_M) &= -nF(c+1)^{n-1}f(c+1) - F(c+1)^{n-k}F(c)^k \\
&\quad + F(1+c)^n + F(c)^k(n-k)F(c+1)^{n-k-1}f(c+1) \ . \qquad (55)
\end{aligned}$$

When $c \in (-1, 0]$ we have that $F(c) = 0$, and hence using the fact that $f(c+1) = 1 - F(c+1)$, (55) simplifies to

$$\begin{aligned} -nF(c+1)^{n-1}f(c+1) + F(c+1)^n &= -nF(c+1)^{n-1}[1 - F(c+1)] + F(c+1)^n \\ &\propto -n[1 - F(c+1)] + F(c+1) \\ &\leq -2[1 - F(1)] + F(1) < 0 \ , \end{aligned}$$

i.e., for $c \in (-1, 0]$, $W_U - W_M$ is decreasing in c. Otherwise, for $c > 0$, we have that

$$\begin{aligned} \frac{d}{dc}(W_U - W_M) &\propto \frac{-nF(c+1)^k f(c+1)}{F(c)^k} + \frac{F(c+1)^{k+1}}{F(c)^k} + (n-k)f(c+1) - F(c+1) \\ &= \frac{-nX^k(1-X)}{[1 - e(1-X)]^k} + \frac{X^{k+1}}{[1 - e(1-X)]^k} + (n-k)(1-X) - X \ , \end{aligned} \tag{56}$$

where the second line uses $f(c+1) = 1 - F(c+1)$ and $F(c) = 1 - e[1 - F(c+1)]$, and then defines $X \equiv F(c+1)$. Notice that $X \in (1 - e^{-1}, 1)$. It is straightforward (but lengthy) to show that (56) is negative as $X \to 1 - e^{-1}$, zero as $X \to 1$, concave, and decreasing in X around $X = 1$. We can therefore conclude that $W_U - W_M$ is quasiconvex in c, and increasing in c for sufficiently high c. Given that $W_U > W_M$ for $c \leq -1$, and $\lim_{c \to \infty}(W_U - W_M) = 0$. Item (iii) then follows.

Finally, consider consumer surplus. Using equation (44) we can write

$$\begin{aligned} V_M &= \int_p^\infty (v-p)F_{k-1:k}(v - p + c)dF_{n-k:n-k}(v) + \int_c^\infty (v-c)F_{n-k:n-k}(v + p - c)dF_{k-1:k}(v) \\ &= \int_c^\infty (v-c)F_{k-1:k}(v)dF_{n-k:n-k}(v+1) + \int_c^\infty (v-c)F_{n-k:n-k}(v+1)dF_{k-1:k}(v) \\ &= \int_c^\infty (v-c)dF_{k-1:k}(v)F_{n-k:n-k}(v+1) \\ &= \int_c^\infty [1 - F_{k-1:k}(v)F_{n-k:n-k}(v+1)]dv \end{aligned} \tag{57}$$

where the second line uses $p = c + 1$ and changes the variable of integration in the first part, and the fourth line integrates by parts.

We start by proving that $V_M < V_U$. It is straightforward to see that $V_M < V_U$ when $c \leq -1$; this follows because we have just proved that for this range of c, $W_M < W_U$ while $\Pi_M = \Pi_U$. Now consider $c > -1$ and note that $V_U = \int_c^\infty [1 - F(v+1)^n]dv$. Hence

$$\frac{d}{dc}(V_U - V_M) = F(c+1)^n - F_{k-1:k}(c)F_{n-k:n-k}(c+1) \ . \tag{58}$$

When $c \in (-1, 0]$ we have that (58) is strictly positive because $F_{k-1:k}(c) = 0$ for this range of c. When $c > 0$ we can rewrite (58) as

$$\begin{aligned}
\frac{d}{dc}(V_U - V_M) &= F(c+1)^n - \{F(c)^k + k[1-F(c)]F(c)^{k-1}\}F(c+1)^{n-k} \\
&\propto \frac{F(c+1)^k}{F(c)^{k-1}} - F(c) - k[1-F(c)] \\
&= \frac{[1-e^{-1}(1-Y)]^k}{Y^{k-1}} - Y - k(1-Y) \ , \qquad (59)
\end{aligned}$$

where the final line uses $F(c+1) = 1 - e^{-1}[1-F(c)]$ and defines $Y = F(c)$. Note that $Y \in (0,1)$. It is straightforward to show that (59) is positive as $Y \to 0$, is zero at $Y = 1$, is convex in Y, and strictly increasing in Y as $Y \to 1$. We therefore conclude that $V_U - V_M$ is quasiconcave in c. However we also know that $V_U > V_M$ for $c \leq -1$, and we know that $V_U - V_M = 0$ as $c \to \infty$. Hence $V_U > V_M$ for all values of c.

We now prove the relationship between V_M and V_D. It is straightforward to see that $V_M < V_D$ when $c \leq 0$; this follows because we have just proved that for this range of c, $W_M < W_D$ while $\Pi_M = \Pi_D$. Now consider $c > 0$ and note that

$$V_D = \int_c^\infty [1 - F_{(n-1)}(v)]dv = \int_c^\infty [1 - F(v)^n - n[1-F(v)]F(v)^{n-1}]dv \ . \qquad (60)$$

Hence we can write that

$$\begin{aligned}
\frac{d}{dc}(V_D - V_M) &= F(c)^n + n[1-F(c)]F(c)^{n-1} - F_{k-1:k}(c)F_{n-k:n-k}(c+1) \\
&= F(c)^n + n[1-F(c)]F(c)^{n-1} - [F(c)^k + k[1-F(c)]F(c)^{k-1}]F(c+1)^{n-k} \\
&\propto F(c) + n[1-F(c)] - [F(c) + k[1-F(c)]]\left(\frac{F(c+1)}{F(c)}\right)^{n-k} \\
&= Z + n(1-Z) - [Z + k(1-Z)]\left(\frac{1-e^{-1}(1-Z)}{Z}\right)^{n-k} \ , \qquad (61)
\end{aligned}$$

where the final line uses $F(c+1) = 1 - e^{-1}[1-F(c)]$ and defines $Z = F(c)$. Note that $Z \in (0,1)$. It is straightforward to show that (61) is negative as $Z \to 0$, is zero at $Z = 1$, is concave in Z, and strictly decreasing in Z as $Z \to 1$. We therefore conclude that $V_D - V_M$ is quasiconvex in c and increasing in c for large enough c. However we also know that $V_D > V_M$ for $c \leq 0$, and we know that $V_D - V_M = 0$ as $c \to \infty$. Hence there exists a critical $c > 0$ such that $V_D < V_M$ for cost above this critical value. We also know from earlier that $V_U \geq V_D$ with strict inequality for all $c > -1$. Item (ii) then follows. □

Omitted Proofs for Section 6

Proof of Lemma 5. The proof largely follows the literature on auctions with interdependent values. We look for a symmetric equilibrium where $b(v) = v - p(v)$ is the equilibrium surplus bidding function and $b(v)$ increases monotonically in v. When a firm observes consumer valuation v but deviates and bids according to valuation z, its expected profit is

$$[v - b(z) - c]G(z|v) \ . \tag{62}$$

The derivative with respect to z is

$$-b'(z)G(z|v) + [v - b(z) - c]g(z|v) \ . \tag{63}$$

The deviation profit (62) should be maximized at $z = v$ in symmetric equilibrium, and so the first-order condition is

$$-b'(v)G(v|v) + [v - b(v) - c]g(v|v) = 0 \ ,$$

from which we derive a differential equation

$$b'(v) = [v - b(v) - c]\frac{g(v|v)}{G(v|v)} \ . \tag{64}$$

The natural boundary condition is $b(c) = 0$, which allows us to solve for

$$b(v) = \int_c^v (x - c)dL(x|v) = \int_c^v [1 - L(x|v)]dx \ ,$$

where $L(x|v)$ is defined in (22). Notice that $b'(v) = -\int_c^v \frac{\partial L(x|v)}{\partial v}dx > 0$ (where we have used the facts that $L(v|v) = 1$ and $L(x|v)$ decreases in v), so $b(v)$ is indeed increasing. To check that the first-order condition is sufficient, substitute (64) into (63) to get

$$\begin{aligned} G(z|v)&\left(-b'(z) + [v - b(z) - c]\frac{g(z|v)}{G(z|v)}\right) \\ &= G(z|v)\left([v - b(z) - c]\frac{g(z|v)}{G(z|v)} - [z - b(z) - c]\frac{g(z|z)}{G(z|z)}\right) \ . \end{aligned}$$

Under Assumption 3 this is positive for $z < v$ and negative for $z > v$, and hence the first-order condition is indeed sufficient for defining the equilibrium. □

Proof of Proposition 7. For part (i), note that under partial discrimination $b(v)$ is strictly positive and strictly increasing in v whenever $v > c$, and so a consumer buys the best-matched product whenever its valuation exceeds marginal cost. The same is true under full discrimination, so the two regimes yield the same total welfare.

For parts (ii) and (iii) let us compare profit. (The comparison of consumer surplus is just the opposite.) When a firm wins a consumer with valuation v, its profit is

$$p(v) - c = v - b(v) - c = \int_c^v L(x|v)dx \ . \tag{65}$$

Recall that as derived in footnote 21 the counterpart under full discrimination is

$$\frac{\int_c^v G(x|v)dx}{G(v|v)} \ . \tag{66}$$

Suppose first that $\frac{g(z|v)}{G(z|v)}$ increases in v. Then

$$\begin{aligned} L(x|v) = \exp\left(-\int_x^v \frac{g(t|t)}{G(t|t)}dt\right) &\geq \exp\left(-\int_x^v \frac{g(t|v)}{G(t|v)}dt\right) \\ &= \exp\left(-\int_x^v [\ln G(t|v)]'dt\right) = \frac{G(x|v)}{G(v|v)} \ . \end{aligned}$$

Therefore, (65) is greater than (66), i.e., firms earn more under partial discrimination. The opposite is true if $\frac{g(z|v)}{G(z|v)}$ decreases in v. In the IID case, $\frac{g(z|v)}{G(z|v)}$ is independent of v, so the equivalence result follows. $\square$