Mitigating Disaster Risks in the Age of Climate Change*

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April 2, 2023

Abstract

Emissions abatement alone cannot address the consequences of global warming for weather disasters. We model adaptation to mitigate disaster risks to capital stock. Optimal adaptation — a mix of firm-level efforts and public spending funded by a tax on capital — depends on learning regarding the adverse consequences of global warming for disaster arrivals. We apply our model to country-level control of flooding from tropical cyclones. Learning is needed to rationalize empirical findings, including the response of growth rates and asset prices to disaster arrivals. Adaptation is more valuable under a learning than a counterfactual no-learning environment. Moreover, optimal carbon taxes over time depend on adaptation and properties of the learning process.

JEL Classification: H56; G01; E20

Keywords: Climate change, weather disasters, adaptation, capital tax, learning, asset prices, tropical cyclones, social cost of carbon

*We thank Lars Peter Hansen, Patrick Bolton, Jose Scheinkman, Esteban Rossi-Hansberg, Julian Koelbel, Jeffrey Kubik, Markus Leippold, Toan Phan, Bernard Salanié, Tano Santos, Stijn Van Nieuwerburgh, Laura Veldkamp, and seminar participants at University of Chicago-Institute for Mathematical and Statistical Innovation, Columbia University, Imperial College London, INQUIRE, SHOF-ECGI Sustainability Conference, and University of Zurich for helpful comments. We especially thank Jiangmin Xu for his help with the paper.

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1 Introduction

Global costs of weather-related disasters have increased sharply in recent decades. While this trend increase is partly due to economic growth and exposure of physical capital (Pielke et al., 2008), recent climate research links climate change to more frequent disasters (National Academy of Sciences, 2016). Emissions abatement will only impact such losses decades down the road and might not fully address the consequences for weather disasters. Hence, adaptations to mitigate natural disaster risks, be it flooding from tropical cyclones or damage from wildfires, need to play a major role going forward.

In contrast to emissions abatement, which have been the main focus of research using integrated assessment models (Nordhaus, 2017; Golosov, Hassler, Krusell, and Tsyvinski, 2014), such adaptation strategies have thus far been relatively under-emphasized both in climate change research and practice (Bouwer et al., 2007). Since there is uncertainty on the impact of global warming for the frequency of disasters, adaptation naturally depends on households learning about these consequences.[1]

To address these issues, we introduce learning and adaptation into a continuous-time stochastic general-equilibrium model with disasters along the lines emphasized by Rietz (1988), Barro (2006), and especially Pindyck and Wang (2013). Output is determined by an AK growth function augmented with capital adjustment costs (e.g., Hayashi, 1982) that give rise to rents for installed capital and the value of capital (Tobin's average q). Disaster shocks following a Poisson process destroy capital stock, affect equilibrium asset prices, and reduce the welfare of households endowed with recursive utility (Epstein and Zin, 1989).

Mitigation of these disaster shocks is modeled via a combination of two adaptation technologies: (1.) adaptation spending at the firm level that reduces the exposure of a firm's capital to the disaster shock (e.g. sandbags and other temporary barriers to protect buildings) and (2.) spending at the aggregate level that requires collective action which reduces the conditional damage of a disaster arrival and tail risk for all agents in the economy (e.g., an early warning system, infrastructure maintenance and preparedness, and other government funded programs.)[2]

[1] For instance, scientific consensus on the impact of global warming on the frequency of hurricanes changed markedly in 2005, when a record number of hurricanes including Katrina made landfall (Emanuel, 2005). Recent weather disasters have moved public opinion on the consequences of climate change (see, e.g., the Yale Climate Opinion Maps website at https://climatecommunication.yale.edu/visualizations-data/ycom-us/.)

[2] See Bennetton et al. (2022) and Fried (2023) for evidence on the value of flood control adaptations.

Our model generates the following key properties and predictions. First, while the planner's first-best solution features an optimal mix of spending on both adaptation technologies, firms do not internalize the benefits of aggregate risk mitigation and underspend on total risk mitigation in market economies. We prove that an optimal tax on capital to fund government spending on reducing aggregate tail risks restores the first-best solution while still maintaining a balanced budget.

Second, belief that the economy is in the bad state (B) is a key state variable driving equilibrium outcomes. "Bad" news (an unexpected arrival) leads to a discontinuous jump (worsening) of belief, as a disaster arrival is a discrete event also serving as a discrete signal.[3] Absent any arrivals, belief drifts gradually towards the good (G) state, as no news is good news when it comes to arrival of disasters in our model.

Third, unexpected disaster arrivals have not only direct effects of capital destruction (Pindyck and Wang, 2013) but also indirect effects due to learning that the world is riskier than anticipated. As a result, the effects of disaster arrivals on economic growth are also time-varying and persistent. Additionally, Tobin's q falls and the stock market risk premium rises upon a disaster arrival. Without the learning channel in our model, asset valuation multiples, e.g., Tobin's q, would not move upon disaster arrivals as predicted by Pindyck and Wang (2013). The disaster arrival effects on growth, valuation, and risk premium are a major difference between our model and the literature.[4]

We then quantify the importance of learning and adaptation for disaster risk mitigation in the context of tropical cyclones, which include hurricanes, typhoons, cyclones, and tropical storms,[5] that are estimated to affect nearly 35% of the global population. Using panel data covering 109 countries over the period of 1950-2010, we calibrate our model via simulation to target moments pertaining to the macroeconomy (aggregate consumption, investment, and output), to financial markets (the risk-free rate, equity risk premium and Tobin's q), and to the arrivals of tropical cyclones and adaptation (e.g., government flood control budgets). We confirm findings in the literature that a typical disaster leads to 1% reduction in GDP growth (Hsiang and Jina, 2014). We also present new findings that country-level asset prices (the

[3]Our model generates time-varying disaster arrival rates via learning (also see e.g., Wachter and Zhu, 2019, Colin-Dufresne, Johannes, and Lochstoer, 2016).

[4]See Hong, Karolyi, and Scheinkman (2020) for a review of recent findings on weather disasters and climate risks including the impact of sea-level rise on coastal property prices. Beliefs of the risks are shown to play a role (Bakkensen and Barrage, 2017).

[5]They are referred to as tropical storms or hurricanes in Atlantic, typhoons in the Pacific, and cyclones in Indian Ocean.

risk-free rate, Tobin's q, and equity risk premium) also respond strongly to disaster arrivals, thus allowing us to internally calibrate parameters governing the learning process.

The first finding of our quantitative analysis is that large learning effects are needed to rationalize the data. The second finding is that the value of adaptation is much higher than under the counterfactual no-learning environment. That is, a large part of the value of optimal adaptation derives from uncertainty associated with learning about the climate state. The third finding is that there is a significant gap between welfare in a competitive economy (with only private adaptation) and welfare in the first-best economy, which is implementable in a market economy with optimal capital taxes. Our quantitative conclusions are generally robust to two changes to the model: (1) a generalized belief updating process that allows the underlying state to switch between the good and bad states, and (2) different risk preferences.

Having established the importance of adaptation for mitigating disaster risks in a learning environment, we then explain how learning and adaptation influence the social cost of carbon. We consider a tractable extension, incorporating features from the social cost of carbon model of Van den Bremer and Van der Ploeg (2021). Output depends on both capital and fossil fuels. Using fossil fuels increases the stock of carbon in the atmosphere, which leads to larger expected damages from disasters. But there is still uncertainty about damages, which our society learns about from disaster arrivals, and then makes adaptation decisions in response. Equilibrium outcomes depend on both the belief (about how likely the economy is in the bad state) and the carbon stock. We show that both a fossil-fuel tax (set at the equilibrium social cost of carbon) and an adaptation (capital) tax are needed for the society to attain the first-best solution.

We use our calibration of the planner's first-best solution to highlight the role of learning. In the no-learning first-best benchmark, recent integrated assessment models (with Epstein-Zin risk preferences and productivity shocks) project the social cost of carbon to increase over time as carbon stock gradually rises (Jensen and Traeger, 2014; Cai and Lontzek, 2019). Learning and adaptation affect the social cost of carbon in several ways. First, overall levels are naturally lower due to adaptation. Moreover, the slopes and variances of projections over time depend the society's prior belief and the speed of convergence of beliefs to a steady state. Posterior beliefs have a bi-modal distribution, which widens the inter-quartile range of social cost of carbon projections over time.

Our model differs in three key respects from Brestchler and Vinograd (2014), who also model how optimal emissions abatement mitigates disaster risks. First we incorporate learning as our asset-pricing and other key implications would fail badly absent learning. Second we allow for an optimal mix of fossil fuel taxes and adaptation. Finally, our quantitative analysis also differs in that we infer the latent learning process from asset prices. Our learning findings complement the analysis of model uncertainty for climate policy (Barnett, Brock, and Hansen, 2020).

2 Model

In this section, we develop a model in which there is an externality when it comes to the mitigation of disaster risks in a market economy. Time is continuous and the horizon is infinite. There is a continuum of identical firms and households, both with a unit measure.

2.1 Firms' and Households' Optimization Problems

Firm production. A firm produces output, Y_t, using its capital stock, K_t, the sole factor of production. Specifically, Y_t is proportional to its contemporaneous capital stock K_t:

$$Y_t = AK_t \,, \tag{1}$$

where $A > 0$ is a constant that defines productivity. This is a version of the AK model but importantly generalized with capital adjustment costs as we show later.

Firm investment, capital accumulation, and arrival of jumps (disasters). Let I_t denote firm investment. The firm's capital stock K_t evolves as:

$$dK_t = (I_{t-} - \delta_K K_{t-})\, dt + \sigma_K K_{t-} d\mathcal{W}_t^K - N_{t-} K_{t-} (1 - Z) d\mathcal{J}_t \ , \tag{2}$$

where δ_K is the depreciation rate of capital. The second term captures continuous diffusive shocks to capital, where $\mathcal{W}_t^K$ is a standard Brownian motion and the parameter σ_K is the diffusion volatility. This term is the standard source of shocks for AK models in macroeconomics and sometimes is interpreted as stochastic depreciation shocks. The last term in (2) captures the loss to the firm's capital from a stochastic arrival of a disaster.

The process $\mathcal{J}_t$ in (2) is a Poisson process where each jump arrives at a constant but unobservable rate, which we denote by λ. We will return to discuss the details for the arrival rate λ. There is no limit to the number of these jump shocks. If a jump does not arrive at t, i.e., $d\mathcal{J}_t = 0$, the third term disappears. To emphasize the timing of potential jumps, we use $t-$ to denote the pre-jump time so that a discrete jump may or may not arrive at t. The N_{t-} process is chosen by the firm to mitigate its exposures to disasters, which we introduce later.

Without reducing disaster exposures (which implies $N_{t-} = 1$), upon a disaster arrival at t ($d\mathcal{J}_t = 1$), a stochastic fraction $(1 - Z) \in (0, 1)$ of the firm's capital stock K_{t-} is permanently destroyed at t and hence the surviving capital stock is $K_t = ZK_{t-}$. (For example, if the firm incurred no disaster exposure reduction spending at $t-$ and a shock arrived at t destroying 15 percent of capital stock, we would have $Z = 0.85$.) Naturally, anticipating damages caused by these disasters, the firm has incentives to *ex-ante* reduce its exposures to disaster shocks by spending resources (e.g., sandbags to keep a building from flooding during a tropical cyclone.)

Let $\Xi(Z)$ and $\xi(Z)$ denote the cumulative distribution function (cdf) and probability density function (pdf) for the stochastic fraction of capital recovery Z, respectively, conditional on a jump arrival. While the firm takes the distribution of Z as given, the society as a whole can spend resources to influence the distribution of Z by making disasters less damaging to the economy. We introduce the determinants of $\Xi(Z)$ at the aggregate level in Section 2.4.

Reducing a firm's disaster exposure (firm-level adaptation). Let X^e_{t-} denote the firm's adaptation spending to reduces its exposure to a disaster, where the superscript e refers to *exposure* at $t-$. With this spending at $t-$, should a disaster arrive at t, the firm decreases its capital loss from $(1 - Z)K_{t-}$ to $N_{t-}(1 - Z)K_{t-}$, where $N_{t-} \in [0, 1]$ depends on X^e_{t-}. The effect of this spending on capital stock dynamics is captured by the N_{t-} term in (2). Let $x^e_{t-} = X^e_{t-}/K_{t-}$ denote the firm's scaled disaster exposure reduction spending.

To preserve our model's homogeneity property, we assume that N_{t-} is a function of x^e_{t-}:

$$N_{t-} = N(x^e_{t-})\,. \tag{3}$$

Equations (2) and (3) imply that if we double X^e_{t-} and capital stock K_{t-} simultaneously, the benefit from reducing disaster damages (in units of goods) also doubles. To see why, observe that $N_{t-} = N(x^e_{t-})$ is unchanged with the simultaneous doubling of X^e_{t-} and K_{t-} but the amount of loss reduced by adaptation, is doubled since K_{t-} has doubled.

We require $N'(x^e) \leq 0$ as adaptation spending reduces damages. Additionally, the marginal effect of spending on reducing damages is decreasing in x^e, which implies $N''(x^e) \geq 0$. Finally, by definition, $N(0) = 1$, as no adaptation spending ($x^e = 0$) no damage reduction.

Capital adjustment costs and firm's objective. Following the q theory of investment (Hayashi, 1982; Abel and Eberly, 1994), we assume that when investing $I_t dt$, the firm incurs additional capital adjustment costs, which we denote by $\Phi_t dt$. That is, the total cost of investment per unit of time is $(I_t + \Phi_t)$ including both capital purchase and adjustment costs. Let CF_t denote the firm's cash flow/dividend payout:

$$CF_t = Y_t - (I_t + \Phi_t) - X_t^e \,. \tag{4}$$

Let $cf_t = CF_t/K_t$ denote the scaled cash flow and $i_t = I_t/K_t$ denote the investment-capital ratio. Next, we specify the capital adjustment cost function. Following Hayashi (1982), we assume that $\Phi(I, K)$ is homogeneous with degree one in I and K by writing:

$$\Phi(I, K) = \phi(i)K \ , \tag{5}$$

where $\phi(i)$ is increasing and convex.

The representative firm chooses investment I and the adaptation spending X^e to maximize its risk-adjusted present value of future cash flows by solving:[6]

$$\max_{I,\, X^e} \ \mathbb{E}\left[\int_0^\infty \frac{\mathbb{M}_t}{\mathbb{M}_0}\left(Y_t - (I_t + \Phi_t) - X_t^e\right) dt\right] \ , \tag{6}$$

where $\mathbb{M}$ is the equilibrium stochastic discount factor (SDF) to be determined later. Note that the firm takes $\mathbb{M}$ as given when solving its problem. Let Q_0 denote the firm's value at $t = 0$, the solution for (6). Because installing capital is costly, installed capital earns rents in equilibrium so that Tobin's average q, the ratio between the firm's value (Q_0) and the replacement cost of capital (K_0), exceeds one.

Households' preferences. We work with the recursive utility developed by Epstein and Zin (1989) and formulated in continuous time by Duffie and Epstein (1992). The life-time utility of our representative consumer's recursive preferences is given by:

$$V_0 = \mathbb{E}\left[\int_0^\infty f(C_t, V_t) dt\right] \ , \tag{7}$$

where $f(C, V)$ known as the normalized aggregator is given by

$$f(C, V) = \frac{\rho}{1-\psi^{-1}} \frac{C^{1-\psi^{-1}} - ((1-\gamma)V)^{\omega}}{((1-\gamma)V)^{\omega-1}} \tag{8}$$

[6]Financial markets are perfectly competitive and complete. While the firm can hold financial positions (e.g., DIS contracts in net zero supply), these financial hedging transactions generate zero NPV for the firm. Therefore, financial hedging policies are indeterminate, a version of the Modigliani-Miller financing irrelevant result. The firm can thus ignore financial contracts without loss of generality.

and $\omega = (1 - \psi^{-1})/(1 - \gamma)$. Here ρ is the rate of time preference, ψ is the elasticity of intertemporal substitution (EIS), γ is the coefficient of relative risk aversion. Unlike expected utility, recursive preferences as defined by (7) and (8) disentangle risk aversion from the EIS.[7] To check the robustness of our analysis, we also analyze our model with external habit formation proposed by Campbell and Cochrane (1999) in Subsection 7.7.

2.2 Bayesian Belief Updating about the Disaster Arrival Frequency

Next, we turn to the disaster arrival process. The arrival rate λ while constant is unobservable to the agent.[8] Therefore, an arrival of a disaster not only destroys capital stock, but also serves as a signal from which households and firms update their beliefs about λ.

While the true disaster arrival rate λ is constant by assumption, households and firms do not have complete information about the value of λ. What the households and firms know at time 0 is that the true value of λ is either λ_G or λ_B with $\lambda_B > \lambda_G$. If the true value of λ is λ_B rather than λ_G, capital stock is more likely to be hit by a disaster (i.e., a negative jump). We refer to the low-arrival-rate and high-arrival-rate scenarios as the good (G) state and the bad (B) state, respectively. Additionally, all agents are endowed with the same prior belief π_{0-} that the true value of λ is λ_B. In sum, all agents in our model have the same information sets, share the same prior, and use the same Bayes rule to update beliefs.

Let π_t denote the time-t posterior belief that $\lambda = \lambda_B$:

$$\pi_t = \mathbb{P}_t(\lambda = \lambda_B)\,, \tag{10}$$

where $\mathbb{P}_t(\,\cdot\,)$ is the conditional probability at t. The expected disaster arrival rate at t, λ_t, is:

$$\lambda_t = \mathbb{E}_t(\lambda) = \lambda(\pi_t) = \lambda_B \pi_t + \lambda_G(1 - \pi_t)\,, \tag{11}$$

which is a weighted average of λ_B and λ_G. A higher value of π_t corresponds to a belief that the economy is more likely in State B where the jump arrival rate is $\lambda_B > \lambda_G$.

What leads the agent's belief to worsen (increasing π) is jump arrivals. What leads the belief to revise favorably is no jump arrivals. In this sense, no-jump news is good news.

[7] If $\gamma = \psi^{-1}$ so that $\omega = 1$, we have the standard constant-relative-risk-aversion (CRRA) expected utility, represented by the additively separable aggregator:

$$f(C, V) = \frac{\rho\, C^{1-\gamma}}{1-\gamma} - \rho\, V. \tag{9}$$

[8] In Section OA of the Online Appendix, we generalize our model to a setting where the unobservable disaster arrival rate λ is stochastic and follows a two-state continuous-time Markov chain.

Mathematically, the agent updates his belief using the Bayes rule:[9]

$$d\pi_t = \sigma_\pi(\pi_{t-})\left(d\mathcal{J}_t - \lambda_{t-}dt\right) , \tag{12}$$

where

$$\sigma_\pi(\pi) = \frac{\pi(1-\pi)(\lambda_B - \lambda_G)}{\lambda(\pi)} = \frac{\pi(1-\pi)(\lambda_B - \lambda_G)}{\lambda_B\pi + \lambda_G(1-\pi)} > 0 \,. \tag{13}$$

Here, signals come from $\mathcal{J}_t$. Note that π_t and λ_t are both martingales which can be seen from (12) as $\mathbb{E}_{t-}[d\mathcal{J}_t] = \lambda_{t-}dt$. When a disaster strikes at t, the belief immediately increases from the pre-jump level π_{t-} to $\pi_t^{\mathcal{J}}$ by $\sigma_\pi(\pi_{t-})$, where

$$\pi_t^{\mathcal{J}} = \pi_{t-} + \sigma_\pi(\pi_{t-}) = \frac{\pi_{t-}\,\lambda_B}{\lambda(\pi_{t-})} > \pi_{t-} \,. \tag{14}$$

If there is no arrival ($d\mathcal{J}_t = 0$) over dt, the household becomes more optimistic. In this case,

$$\frac{d\pi_t}{dt} = \mu_\pi(\pi_{t-}) = \pi_{t-}(1-\pi_{t-})(\lambda_G - \lambda_B) \,, \tag{15}$$

using $\mu_\pi(\pi_{t-}) = -\sigma_\pi(\pi_{t-})\lambda(\pi_{t-})$. Equation (15) is a logistic differential equation. Conditional on no jump ($d\mathcal{J}_v = 0$) for $v \in (s,t)$, we obtain the closed-form logistic function for π_t:

$$\pi_t = \frac{\pi_s e^{-(\lambda_B - \lambda_G)(t-s)}}{1 + \pi_s\left(e^{-(\lambda_B-\lambda_G)(t-s)} - 1\right)} \,. \tag{16}$$

In Figure 1, we plot a simulated path for π starting from $\pi_{0-} = 0.1$. It shows that absent a jump arrival, belief becomes more optimistic and π_t decreases deterministically between two consecutive jumps following the logistic function given in (16). Once a jump arrives at t, the belief worsens moving upward to $\pi_t^{\mathcal{J}}$ given in (14) by a discrete amount $\sigma_\pi(\pi_{t-})$ given in (13).

2.3 Competitive Market Structure and Equilibrium

Next, we turn to the competitive market structure and define market equilibrium. Financial markets are dynamically complete. Without loss of generality, it is sufficient to assume that the following financial securities exist at all time t: (i) a risk-free asset thats pays interest at the equilibrium rate of r_t and (ii) the aggregate equity market.[10]

[9] See Theorem 19.6 in Liptser and Shiryaev (2001).

[10] For markets to be dynamically complete, we also need actuarially fair diffusion and jump hedging contracts (for each possible jump contingency) as in Pindyck and Wang (2013). The net demand is zero for all hedging contracts. For expositional simplicity, we omit these hedging contracts and refer readers to Pindyck and Wang (2013) for related detailed analysis.

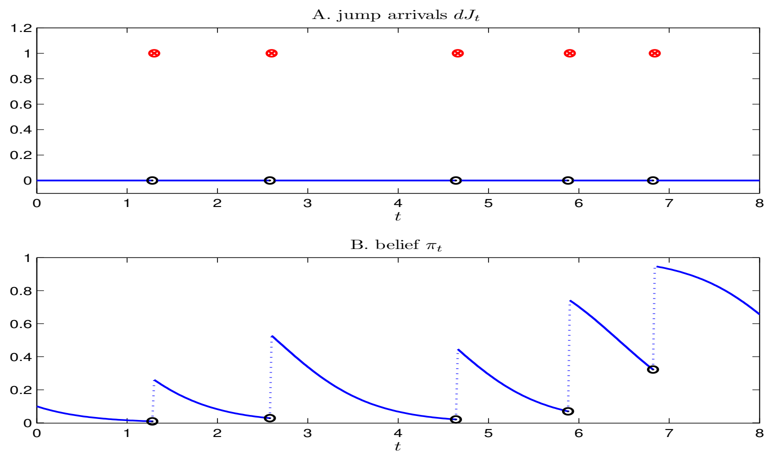

Figure 1: This figure simulates a path for disaster arrival times in Panel A and plots the corresponding belief updating process in Panel B starting with $\pi_0 = 0.08$. The belief decreases deterministically in the absence of jumps but discretely increases upward upon a jump arrival.

To ease exposition, we use **boldfaced** letters to refer to aggregate variables so as to differentiate from the corresponding micro-level variables.

Let $\{\mathbf{Q}_t\}$ denote the equilibrium ex-dividend aggregate stock market value and $\{\mathbf{D}_t\}$ denote the aggregate dividends, respectively. The cum-dividend return is then given by

$$\frac{d\mathbf{Q}_t + \mathbf{D}_{t-}dt}{\mathbf{Q}_{t-}} = \mu_{\mathbf{Q}}(\pi_{t-})dt + \sigma_K d\mathcal{W}_t^K + \left(\frac{\mathbf{Q}_t^{\mathcal{J}}}{\mathbf{Q}_{t-}} - 1\right)d\mathcal{J}_t \,, \tag{17}$$

where $\mu_{\mathbf{Q}}(\pi)$ is the expected stock market return (leaving aside the jump effect). We later verify that the diffusion volatility of the stock market return equals σ_K, the same as the diffusion volatility given in (2). Finally, the last term captures the effect of jumps on returns.

Competitive equilibrium. We define the recursive competitive equilibrium as follows: (a.) Taking the equilibrium risk-free rate r and the equilibrium aggregate stock market return process (17) as given, the representative household chooses consumption C and allocation to the aggregate stock market Γ to maximize lifetime utility given by (7)-(8);[11] (b.) Taking the equilibrium SDF $\{\mathbb{M}_t; t \geq 0\}$ as given, the representative firm chooses investment I and the disaster exposure mitigation spending X^e to maximize its market value given in (6); (c.) The interest rate r, the stock market return process (17), and the SDF $\{\mathbb{M}_t; t \geq 0\}$ are consistent with the households' and firms' optimal decisions and all markets clear in equilibrium.

[11]Since each household is infinitesimally small and has no impact on any aggregate variables, there is no incentive to spend on mitigation. We provide additional discussions later in the paper.

2.4 Source of Externality: Technology Reducing Tail Risk of the Damage Distribution $\Xi(Z)$ for All Firms

Next, we introduce another adaptation technology, which reduces the tail risk of the aggregate disaster distribution $\Xi(Z)$. In contrast to the first type of adaptation technology, which operated at the firm level, this second type of adaptation technology operates at the aggregate level and features an externality (a realistic aspect of adaptation) as its effectiveness depends on collective contributions of all firms in aggregate.

We assume that the *aggregate* spending made at $t-$ can curtail left-tail disaster (jump) risks at t if a jump arrives at t. The idea is that changing the distribution of Z for all firms is very costly and requires a spending that is at the order of a fraction of the aggregate capital stock $\mathbf{K}$. Let $\mathbf{X}^d_{t-}$ denote the *aggregate* spending on this distribution-tail-curtailing technology, where the superscript d refers to the notion that this spending is to make the *distribution* of fractional loss $(1-Z)$ less damaging. Let $\mathbf{x}^d_{t-} = \mathbf{X}^d_{t-}/\mathbf{K}_{t-}$ denote this scaled aggregate adaptation spending. Since aggregate risk reduction is a public good, no firm has incentives to spend on this new technology. This is the reason why markets fail.

Specifically, by spending on aggregate tail risk reduction, we change the distribution of the post-jump fractional recovery Z from $\Xi(Z)$ to $\Xi(Z;\mathbf{x}^d_{t-})$. While simultaneously doubling this type of aggregate adaptation spending $\mathbf{X}^d_{t-}$ and the aggregate capital stock $\mathbf{K}_{t-}$ does not change the distribution $\Xi(Z;\mathbf{x}^d_{t-})$, as the ratio $\mathbf{x}^d_{t-} = \mathbf{X}^d_{t-}/\mathbf{K}_{t-}$ remains unchanged, doing so doubles the benefit of this public spending (i.e., the total reduction of damages) *in levels* as the benefit is proportional to $\mathbf{K}_{t-}(1-Z)$ at the aggregate level.[12]

We have completed the description of our market economy model. Before solving it in Section 4, we first analyze the planner's problem. The first-best solution for the planner's model serves as an important benchmark for our analysis of the market economy.

3 Planner's Problem and its First-Best Solution

The social planner chooses consumption $\mathbf{C}$, investment $\mathbf{I}$, and adaptation spendings $\mathbf{X}^d$ and $\mathbf{X}^e$ to maximize the representative household's utility given in (7)-(8) subject to the representative firm's production/capital accumulation technology, the adaptation technologies, and the aggregate resource constraint: $\mathbf{C}+\mathbf{I}+\mathbf{\Phi}+\mathbf{X}^d+\mathbf{X}^e=\mathbf{Y}=A\mathbf{K}$.

[12]This is similar to the homogeneity assumption for disaster distribution (private adaptation) mitigation spending X^e_{t-}.

To save on notation, we drop the subscript fb in this section until the end of this section where we summarize the first-best solution.

Dynamic programming. Let $V(\mathbf{K},\pi)$ denote the representative household's value function. The Hamilton-Jacobi-Bellman (HJB) equation for the planner is:

$$
\begin{aligned}
0 \;=\; \max_{\mathbf{C},\mathbf{I},\mathbf{x}^e\,\mathbf{x}^d} \; & f(\mathbf{C},V) + (\mathbf{I}-\delta_K\mathbf{K})V_{\mathbf{K}}(\mathbf{K},\pi) + \mu_\pi(\pi)V_\pi(\mathbf{K},\pi) + \frac{1}{2}\sigma_K^2\mathbf{K}^2V_{\mathbf{KK}}(\mathbf{K},\pi) \\
& +\lambda(\pi)\mathbb{E}^{\mathbf{x}^d}\left[V\left(\mathbf{K}^{\mathcal{J}},\pi^{\mathcal{J}}\right) - V(\mathbf{K},\pi)\right] \;, \qquad (18)
\end{aligned}
$$

where $\pi^{\mathcal{J}}$ is the post-jump belief given in (14), $\mathbf{K}^{\mathcal{J}}$ is the post-jump capital stock given by

$$
\mathbf{K}^{\mathcal{J}} = \left(1 - N(\mathbf{x}^e)(1-Z)\right)\mathbf{K}\,, \qquad (19)
$$

$\mu_\pi(\pi)$ is the expected change of belief absent jumps given in (15), $\lambda(\pi)$ is the jump arrival rate given in (11), and $\mathbb{E}^{\mathbf{x}^d}[\,\cdot\,]$ is the expectation operator with respect to the pdf $\xi(Z;\mathbf{x}^d)$ for the recovery fraction Z for a given level adaptation spending $\mathbf{x}^d$ to reduce aggregate risk.

The first term on the right side of (18) is the household's normalized aggregator (Duffie and Epstein, 1992); the second term captures how investment $\mathbf{I}$ affects $V(\mathbf{K},\pi)$; the third term reflects how belief updating (in the absence of jumps) impacts $V(\mathbf{K},\pi)$; and the fourth term captures the effect of capital-stock diffusion shocks on $V(\mathbf{K},\pi)$. It is worth noting that as the signals in our learning model are discrete (jump arrivals), there is no diffusion-induced quadratic-variation term involving $V_{\pi\pi}$ in the HJB equation (18).

Direct (value destroying) versus learning effects. Finally, the last term (on the second line) of (18) captures the effect of jumps on the expected change in $V(\mathbf{K},\pi)$. This term captures rich economic forces and warrants additional explanations. When a jump arrives at t ($d\mathcal{J}_t = 1$), capital falls from $\mathbf{K}_{t-}$ to $(1-Z)\mathbf{K}_{t-}$ absent exposure mitigation spending. By spending $\mathbf{x}^e_{t-}$ to reduce the exposure, the planner reduces the capital loss from $(1-Z)\mathbf{K}_{t-}$ by $N(\mathbf{x}^e_{t-})(1-Z)\mathbf{K}_{t-}$, so that the post-jump capital is $\mathbf{K}^{\mathcal{J}}_t = (1-N(\mathbf{x}^e_{t-})(1-Z))\mathbf{K}_{t-}$ at t.

In sum, a jump triggers two effects on $V(\mathbf{K},\pi)$. First, there is a direct capital destruction effect. As a jump arrival lowers capital stock from $\mathbf{K}_{t-}$ to $\mathbf{K}^{\mathcal{J}}_t = (1-N(\mathbf{x}^e_{t-})(1-Z))\mathbf{K}_{t-}$, the value function decreases from $V(\mathbf{K}_{t-},\pi_{t-})$ to $V(\mathbf{K}^{\mathcal{J}}_t,\pi_{t-})$ even if we ignore the agent's belief updating due to learning. Second, there is a learning (belief-updating) effect. As a jump arrival also cause the belief to increase from π_{t-} to $\pi^{\mathcal{J}}_t$ given in (14), the agent becomes more pessimistic causing the value function to further decrease from $V(\mathbf{K}_t^{\mathcal{J}}, \pi_{t-})$ to $V(\mathbf{K}_t^{\mathcal{J}}, \pi_t^{\mathcal{J}})$. These two effects reinforce each other over time leading to potentially significant losses.

The planner chooses consumption $\mathbf{C}$, investment $\mathbf{I}$, two types of adaptation spendings, $\mathbf{X}^d$ and $\mathbf{X}^e$, to maximize recursive utility given in (7)-(8) by setting the sum of the five terms on the right side of (18) to zero, implied by the optimality argument underpinning the HJB equation for recursive utility (see Duffie and Epstein, 1992). Because of the resource constraint, it is sufficient to focus on $\mathbf{I}$, $\mathbf{X}^d$ and $\mathbf{X}^e$ as control variables.

First-order conditions for investment and two types of adaptation spendings. The first-order condition (FOC) for investment $\mathbf{I}$ is

$$(1 + \Phi_{\mathbf{I}}(\mathbf{I}, \mathbf{K})) f_{\mathbf{C}}(\mathbf{C}, V) = V_{\mathbf{K}}(\mathbf{K}, \pi) \ . \tag{20}$$

The right side of (20), $V_{\mathbf{K}}(\mathbf{K}, \pi)$, is the marginal (utility) benefit of accumulating capital stock. The left side of (20) is the marginal cost of accumulating capital, which is given by the product of forgone marginal utility of consumption $f_{\mathbf{C}}(\mathbf{C}, V)$ and the marginal cost of accumulating capital, $(1 + \Phi_{\mathbf{I}}(\mathbf{I}, \mathbf{K}))$. Because of capital adjustment costs, increasing $\mathbf{K}$ by one unit requires incurring investment costs more than one unit, which explains the marginal adjustment cost $\Phi_{\mathbf{I}}(\mathbf{I}, \mathbf{K})$. Because of non-separability of preferences, $f_{\mathbf{C}}(\mathbf{C}, V)$ depends on not just consumption $\mathbf{C}$ but also the continuation utility V.

The FOC for the scaled aggregate tail risk reduction spending, $\mathbf{x}^d \geq 0$, is

$$f_{\mathbf{C}}(\mathbf{C}, V) = \frac{1}{\mathbf{K}} \lambda(\pi) \int_0^1 \left[\frac{\partial \xi(Z; \mathbf{x}^d)}{\partial \mathbf{x}^d} V\left(\mathbf{K}^{\mathcal{J}}, \pi^{\mathcal{J}}\right) \right] dZ \ , \tag{21}$$

if the solution is positive, $\mathbf{x}^d > 0$.[13] The planner chooses $\mathbf{x}^d$ to equate the marginal cost of adaptation, which is the forgone marginal (utility) benefit of consumption $f_{\mathbf{C}}(\mathbf{C}, V)$ given on the left side of (21), with the marginal benefit of adaptation given on the right side of (21).[14] By spending $\mathbf{x}^d$ per unit of capital to make the distribution of Z less damaging, the planner changes the pdf $\xi(Z; \mathbf{x}^d)$ for the fractional capital recovery, Z, from $\xi(Z; 0)$ to $\xi(Z; \mathbf{x}^d)$.

Similarly, the FOC for the scaled aggregate disaster exposure reduction spending $\mathbf{x}^e$ is

$$f_{\mathbf{C}}(\mathbf{C}, V) = -\lambda(\pi) N'(\mathbf{x}^e) \mathbb{E}^{\mathbf{x}^d} \left[(1 - Z) V_{\mathbf{K}}\left(\mathbf{K}^{\mathcal{J}}, \pi^{\mathcal{J}}\right) \right] \ , \tag{22}$$

[13] Otherwise, $\mathbf{x}^d = 0$ as adaptation in reality cannot be negative. When do we see $\mathbf{x}^d = 0$? One scenario is when the technology is very inefficient. In this case, the marginal benefit of spending on disaster distribution mitigation spending is less than one, causing the planner to set $\mathbf{x}^d = 0$.

[14] The second-order condition (SOC) $\lambda(\pi) \int_0^1 \left[\frac{\partial^2 \xi(Z; \mathbf{x}^d)}{\partial (\mathbf{x}^d)^2} V\left(\mathbf{K}^{\mathcal{J}}, \pi^{\mathcal{J}}\right) \right] dZ < 0$ is satisfied.

if the solution is strictly positive, $\mathbf{x}^e > 0$.[15] That is, the planner optimally chooses $\mathbf{x}^e$ to equate the marginal benefit of reducing the disaster exposure with the marginal cost of doing so. By spending $\mathbf{x}^e_{t-}$ per unit of capital, the planner reduces the post-jump fractional capital loss from $(1-Z)\mathbf{K}_{t-}$ to $\mathbf{K}_{t-} - \mathbf{K}_t^{\mathcal{J}} = N(\mathbf{x}^e_{t-})(1-Z)\mathbf{K}_{t-}$.

Using the homogeneity property to simplify the solution. Our model has the following homogeneity property. If we double capital stock $\mathbf{K}$, it is optimal for the planner to simultaneously double its quantity choices: the two types of adaptation spendings $\mathbf{X}^d$ and $\mathbf{X}^e$, investment $\mathbf{I}$, and consumption $\mathbf{C}$ at all time. As a result, the value function $V(\mathbf{K},\pi)$ is homogeneous with degree $(1-\gamma)$ in $\mathbf{K}$. We can write $V_{fb}(\mathbf{K},\pi)$ as follows:

$$V_{fb}(\mathbf{K},\pi) = \frac{1}{1-\gamma}\left(b_{fb}(\pi)\mathbf{K}\right)^{1-\gamma}, \tag{23}$$

where $b_{fb}(\pi)$ is a welfare measure proportional to certainty equivalent wealth under first best to be determined as part of the solution. Using the FOCs (20), (21), (22), substituting the value function $V(\mathbf{K},\pi)$ given in (23) into the HJB equation (18), and simplifying these equations, we obtain the following four-equation ODE system for $b(\pi)$, $\mathbf{i}(\pi)$, $\mathbf{x}^d(\pi)$, and $\mathbf{x}^e(\pi)$:

$$\begin{aligned}0 = {} & \frac{\rho}{1-\psi^{-1}}\left[\left(\frac{b(\pi)}{\rho(1+\phi'(\mathbf{i}(\pi)))}\right)^{1-\psi} - 1\right] + \mathbf{i}(\pi) - \delta_K - \frac{\gamma\sigma_K^2}{2} + \mu_\pi(\pi)\frac{b'(\pi)}{b(\pi)} \\ & + \frac{\lambda(\pi)}{1-\gamma}\left[\left(\frac{b\left(\pi^{\mathcal{J}}\right)}{b(\pi)}\right)^{1-\gamma}\mathbb{E}^{\mathbf{x}^d(\pi)}\left((1-N(\mathbf{x}^e(\pi))(1-Z))^{1-\gamma}\right) - 1\right],\end{aligned} \tag{24}$$

$$b(\pi) = \left[A - \mathbf{i}(\pi) - \phi(\mathbf{i}(\pi)) - \mathbf{x}^d(\pi) - \mathbf{x}^e(\pi)\right]^{1/(1-\psi)}\left[\rho(1+\phi'(\mathbf{i}(\pi)))\right]^{-\psi/(1-\psi)}, \tag{25}$$

$$\frac{1}{1+\phi'(\mathbf{i}(\pi))} = \lambda(\pi)\left[\frac{b\left(\pi^{\mathcal{J}}\right)}{b(\pi)}\right]^{1-\gamma} N'(\mathbf{x}^e(\pi))\mathbb{E}^{\mathbf{x}^d(\pi)}\left[(Z-1)(1-N(\mathbf{x}^e(\pi))(1-Z))^{-\gamma}\right], \tag{26}$$

$$\frac{1}{1+\phi'(\mathbf{i}(\pi))} = \frac{\lambda(\pi)}{1-\gamma}\left[\frac{b\left(\pi^{\mathcal{J}}\right)}{b(\pi)}\right]^{1-\gamma}\int_0^1\left[\frac{\partial\xi(Z;\mathbf{x}^d(\pi))}{\partial\mathbf{x}^d}(1-N(\mathbf{x}^e(\pi))(1-Z))^{1-\gamma}\right]dZ. \tag{27}$$

We derive the system of ODEs (24)-(27) in Appendix A.1.

Next, we provide the boundary conditions at $\pi = 0$ and $\pi = 1$ and discuss the intuition. As we show, the model at the two boundaries map to the model in Pindyck and Wang (2013) with a generalization of allowing for the two types of adaptation spendings. When $\pi = 0$,

[15] Otherwise, $\mathbf{x}^e = 0$ since adaptation cannot be negative.

the economy is permanently in state G as there is no learning and the solution boils down to solving the four unknowns, $b(0)$, $\mathbf{i}(0)$, $\mathbf{x}^d(0)$, and $\mathbf{x}^e(0)$, via the following four-equation system:

$$-\frac{\left[\frac{b(0)}{\rho(1+\phi'(\mathbf{i}(0)))}\right]^{1-\psi}-1}{1-\psi^{-1}}\rho = \mathbf{i}(0)-\delta_K-\frac{\gamma\sigma_K^2}{2}+\frac{\lambda_G\left[\mathbb{E}^{\mathbf{x}^d(0)}((1-N(\mathbf{x}^e(0))(1-Z))^{1-\gamma})-1\right]}{1-\gamma}\,, \quad (28)$$

$$b(0)\left[\rho(1+\phi'(\mathbf{i}(0)))\right]^{\psi/(1-\psi)} = \left[A-\mathbf{i}(0)-\phi(\mathbf{i}(0))-\mathbf{x}^d(0)-\mathbf{x}^e(0)\right]^{1/(1-\psi)}\,, \quad (29)$$

$$\frac{1}{1+\phi'(\mathbf{i}(0))} = \lambda_G N'(\mathbf{x}^e(0))\mathbb{E}^{\mathbf{x}^d(0)}\left[(Z-1)(1-N(\mathbf{x}^e(0))(1-Z))^{-\gamma}\right]\,, \quad (30)$$

$$\frac{1}{1+\phi'(\mathbf{i}(0))} = \frac{\lambda_G}{1-\gamma}\int_0^1\left[\frac{\partial\xi(Z;\mathbf{x}^d(0))}{\partial\mathbf{x}^d}(1-N(\mathbf{x}^e(0))(1-Z))^{1-\gamma}\right]dZ\,. \quad (31)$$

Once π reaches zero at time t (i.e., $\pi_t=0$), $\mathbf{i}$, $\mathbf{x}^d$, $\mathbf{x}^e$, $\mathbf{c}$, and welfare measure b all remain constant at all time $s\geq t$. By applying essentially the same analysis to the other boundary at $\pi=1$, i.e., when the economy reaches state B, we solve for the four unknowns, $b(1)$, $\mathbf{i}(1)$, $\mathbf{x}^d(1)$ and $\mathbf{x}^e(1)$, via (A.4)-(A.7), another four-equation system in Appendix A.1.

Next, we summarize our model's solution for the entire belief region $\pi\in[0,1]$.

Proposition 1 *The first-best solution is given by the value function (23), where the welfare measure $b_{fb}(\pi)$, $\mathbf{i}_{fb}(\pi)$ and the policy rules, $\mathbf{x}^d_{fb}(\pi)$, and $\mathbf{x}^e_{fb}(\pi)$, solve the four-equation ODE system (24)-(27) in $0\leq\pi\leq 1$ region subject to the boundary conditions (28)-(31) for $\pi=0$ and (A.4)-(A.7) for $\pi=1$.*

See Appendix A.1 for a proof.

4 Competitive Markets Solution

While the planner's (first-best) public adaptation spending is strictly positive, no firms have incentives to reduce the aggregate risk distribution in a market economy. We show that the market solution is equivalent to the planner's solution for the case where only the disaster *exposure* reduction technology is available.

4.1 Firm Adaptation and Investment

At the micro level, the firm maximizes its market value given by (6) taking the following (endogenously determined equilibrium) SDF $\mathbb{M}_t$ as given:

$$\frac{d\mathbb{M}_t}{\mathbb{M}_{t-}} = -r_{t-}dt-\gamma\sigma_K d\mathcal{W}_t^K+(\eta_t-1)\left(d\mathcal{J}_t-\lambda(\pi_{t-})dt\right)\,. \quad (32)$$

The first term on the right side of (32) states the equilibrium restriction that the drift of $d\mathbb{M}_t/\mathbb{M}_{t-}$ equals $-r_{t-}dt$ (Duffie, 2001), where the equilibrium risk-free rate r_{t-} is a function of (π_{t-}), $r_{t-} = r(\pi_{t-})$. The second term on the right side of (32) is the diffusion martingale and $\gamma\sigma$ is the equilibrium market price of diffusion risk as in Pindyck and Wang (2013), which we verify later. As $\lambda(\pi_{t-})dt = \mathbb{E}_{t-}(d\mathcal{J}_t)$, the last term in (32) is a jump martingale under the physical measure. This implies that when a jump arrives at t, the SDF changes discretely from $\mathbb{M}_{t-}$ to $\mathbb{M}_t^{\mathcal{J}}$ by a multiple of endogenously determined market price of jump risk η_t:

$$\frac{\mathbb{M}_t^{\mathcal{J}}}{\mathbb{M}_{t-}} = \eta_t \,, \tag{33}$$

which is a function of belief π_{t-} and the realized value of Z: $\eta_t = \eta(\pi_{t-}; Z)$.[16]

Applying the Ito's Lemma to firm value $Q(K_t, \pi_t) = q(\pi_t)K_t$ given in (6) and using (32), we obtain the following HJB equation for Tobin's q, $q(\pi)$, (see Appendix B.1):

$$\begin{aligned} r(\pi)q(\pi) = \max_{i,\, x^e,\, x^d} \quad & A - i - \phi(i) - x^e - x^d + (i - \delta_K)q(\pi) + \mu_\pi(\pi)q'(\pi) - \gamma\sigma_K^2 q(\pi) \\ & + \lambda(\pi)\mathbb{E}^{\mathbf{x}^d}\left[\eta(\pi; Z)\left(q(\pi^{\mathcal{J}})(1 - N(x^e)(1 - Z)) - q(\pi)\right)\right] \,. \end{aligned} \tag{34}$$

The expectation operator in the last (jump) term (34) takes the aggregate disaster mitigation spending in the economy, $\mathbf{x}^d$, as given. Additionally, there are three optimality conditions.

First, (34) implies that $x^d = 0$, as a firm is infinitesimal and hence reducing aggregate disaster risk brings no benefit but only cost to itself.[17] Second, unlike x^d, (34) implies a rather different FOC for the firm's exposure reduction spending x^e:

$$1 = -\lambda(\pi)q(\pi^{\mathcal{J}})N'(x^e)\mathbb{E}^{\mathbf{x}^d}\left[(1 - Z)\eta(\pi; Z)\right] \,. \tag{35}$$

By spending a dollar at the margin on exposure risk mitigation, the firm reduces the destruction of its capital stock by $-(1 - Z)N'(x^e) > 0$ units should a jump arrive. Upon a jump arrival, the gross percentage change of SDF is $\mathbb{M}_t^{\mathcal{J}}/\mathbb{M}_{t-} = \eta(\pi_{t-}; Z)$ and the Tobin's q jumps from $q(\pi)$ to $q(\pi^{\mathcal{J}})$. To obtain the marginal benefit of spending on exposure mitigation X^e, we multiply the marginal reduction of capital stock destruction caused by a jump arrival, $-(1 - Z)N'(x^e) > 0$, by $\lambda(\pi)q(\pi^{\mathcal{J}})\eta(\pi; Z)$, and then integrate over all possible values of Z. The resulting expected marginal value of mitigating the disaster exposure, given on the right side of (35), equals one, the marginal cost of mitigating the exposure on the left side of (35).

[16]We provide equilibrium solutions for $r(\pi_{t-})$ and $\eta(\pi_{t-}; Z)$ in Section 5.3 and Subsections 4.3, respectively.

[17]To be precise, since the firm's adaptation spending x^d has positive marginal cost but zero marginal benefit, the FOC cannot hold with equality and the corner solution $x^d = 0$ is optimal.

The FOC for investment implied by (34) is:

$$q(\pi) = 1 + \phi'(i(\pi))\,, \tag{36}$$

which is the standard investment optimality condition that equates the marginal q to the marginal cost of investing $1+\phi'(i(\pi))$. The homogeneity property implies that the average q equals the marginal q as in Hayashi (1982).

4.2 Household Optimization

We show that the household's value function, $J_t = J(W_t, \pi_t)$, is homogeneous with degree $1-\gamma$ in wealth W. That is, $J_t = J(W_t, \pi_t)$ takes the form of:

$$J(W,\pi) = \frac{1}{1-\gamma}(u(\pi)W)^{1-\gamma}\,, \tag{37}$$

where $u(\pi)$ is a welfare measure that will be endogenously determined.

First, note that no household spends on disaster exposure or disaster distribution mitigation spendings: $X^d = 0$ and $X^e = 0$, as each household is infinitesimally small and has impact on neither the aggregate disaster distribution nor the aggregate disaster exposure. Second, we solve for the household's optimal consumption C and allocation to the risky asset Γ using the following HJB equation:

$$\begin{aligned} 0 = \max_{C,\Gamma} \quad & f(C,J) + \mu_\pi(\pi)J_\pi + \lambda(\pi)\int_0^1 \left[J\left(W^{\mathcal{J}},\pi^{\mathcal{J}}\right) - J(W,\pi)\right]\xi(Z;\mathbf{x}^d)dZ \\ & + \left[r(\pi)W + (\mu_{\mathbf{Q}}(\pi) - r(\pi))\Gamma - C\right]J_W + \frac{\sigma_K^2\Gamma^2 J_{WW}}{2}\,, \end{aligned} \tag{38}$$

where $\mu_{\mathbf{Q}}(\pi)$ is defined in (17), $\pi^{\mathcal{J}}$ is the post-jump belief given in (14), and $W^{\mathcal{J}}$ is the post-jump wealth given by

$$W_t^{\mathcal{J}} = W_{t-} + \left(\frac{\mathbf{Q}_t^{\mathcal{J}}}{\mathbf{Q}_{t-}} - 1\right)\Gamma_{t-}\,. \tag{39}$$

The aggregate stock market valuation $\mathbf{Q}_t$ is proportional to the aggregate capital stock $\mathbf{K}$: $\mathbf{Q}_t = \mathbf{q}(\pi_t)\mathbf{K}_t$ where $\mathbf{q}(\pi_t)$ is the Tobin's q for $\mathbf{K}$ in equilibrium. When a jump arrives,

$$\frac{\mathbf{Q}_t^{\mathcal{J}}}{\mathbf{Q}_{t-}} = \frac{\mathbf{q}(\pi_t^{\mathcal{J}})\mathbf{K}_t^{\mathcal{J}}}{\mathbf{q}(\pi_{t-})\mathbf{K}_{t-}} = \frac{\mathbf{q}(\pi_t^{\mathcal{J}})}{\mathbf{q}(\pi_{t-})}(1 - N(\mathbf{x}_{t-}^e)(1-Z))\,. \tag{40}$$

Equation (40) states that aggregate stock market value changes from $\mathbf{Q}_{t-} = \mathbf{q}(\pi_{t-})\mathbf{K}_{t-}$ to $\mathbf{Q}_t^{\mathcal{J}} = \mathbf{q}(\pi_t^{\mathcal{J}})\mathbf{K}_t^{\mathcal{J}}$ as a jump arrives for two reasons: 1.) capital stock decreases from $\mathbf{K}_{t-}$ to $\mathbf{K}_t^{\mathcal{J}} = [1 - N(\mathbf{x}_{t-}^e)(1-Z)]\mathbf{K}_{t-}$ by a fraction of $N(\mathbf{x}_{t-}^e)(1-Z)$ and 2.) the aggregate Tobin's q

changes from $\mathbf{q}(\pi_{t-})$ to $\mathbf{q}(\pi_t^{\mathcal{J}})$, where $\pi_t^{\mathcal{J}} = \pi_{t-}\lambda_B/\lambda(\pi_{t-})$ is given in (14). For brevity, we drop the time subscripts when it does not cause confusion. That is, we write $\mathbf{Q}^{\mathcal{J}}/\mathbf{Q} = \mathbf{Q}_t^{\mathcal{J}}/\mathbf{Q}_{t-}$.

Substituting (37) into the consumption FOC $f_C(C, J) = J_W(W, \pi)$ and simplifying the expression, we obtain the following consumption rule:

$$C(\pi) = \rho^{\psi} u(\pi)^{1-\psi} W\,. \tag{41}$$

Consumption is linear in wealth with a π-dependent marginal propensity to consume. Simplifying the household's FOC for the market portfolio allocation Γ, we obtain:

$$\Gamma = -\frac{\mu_{\mathbf{Q}}(\pi) - r(\pi)}{\sigma_K^2}\frac{J_W(W,\pi)}{J_{WW}(W,\pi)} + \frac{\lambda(\pi)}{\sigma_K^2}\mathbb{E}^{\mathbf{x}^d}\left[\left(1 - \frac{\mathbf{Q}^{\mathcal{J}}}{\mathbf{Q}}\right)\frac{J_W\left(W^{\mathcal{J}}, \pi^{\mathcal{J}}\right)}{J_{WW}(W,\pi)}\right]. \tag{42}$$

The first term in (42) is the standard Merton's mean-variance demand (absent jumps) and the second term in (42) captures the intertemporal hedging demand as a jump arrival causes both the household's belief π and wealth W as well as the stock market $\mathbf{Q}$ to jump discretely.

4.3 Market Equilibrium

In equilibrium, the household invests all wealth in the stock market, $W_t = \Gamma_t = \mathbf{Q}_t$. We can show that the ratio of the pre-jump and the post-jump SDF $\mathbb{M}_t$ in equilibrium, η_t, is given by

$$\eta_t = \frac{\mathbb{M}_t^{\mathcal{J}}}{\mathbb{M}_{t-}} = \frac{J_W(\mathbf{Q}_t^{\mathcal{J}}, \pi_t^{\mathcal{J}})}{J_W(\mathbf{Q}_{t-}, \pi_{t-})}\,. \tag{43}$$

The second equality in (43) states that η_t equals the ratio of the household's post-jump marginal value of wealth $J_W(\mathbf{Q}_t^{\mathcal{J}}, \pi_t^{\mathcal{J}})$ and the pre-jump marginal value of wealth $J_W(\mathbf{Q}_{t-}, \pi_{t-})$. This is because in equilibrium both the household's pre-jump and post-jump wealth are in the stock market: $W_{t-} = \mathbf{Q}_{t-}$ and $W_t^{\mathcal{J}} = \mathbf{Q}_t^{\mathcal{J}}$. Using the homogeneity property, we write η_t as:

$$\eta_t = \eta(\pi_t; Z, \mathbf{x}_{t-}^e) = \left(\frac{u(\pi_t^{\mathcal{J}})}{u(\pi_{t-})}\right)^{1-\gamma}\left(\frac{\mathbf{q}(\pi_t^{\mathcal{J}})}{\mathbf{q}(\pi_{t-})}(1 - N(\mathbf{x}_{t-}^e)(1-Z))\right)^{-\gamma}. \tag{44}$$

We can further simplify the household's HJB equation (38) as:

$$0 = \frac{\psi^{-1}\rho^{\psi}u(\pi_{t-})^{1-\psi} - \rho}{1-\psi^{-1}} + \mu_{\mathbf{Q}}(\pi_{t-}) + \mu_{\pi}(\pi_{t-})\frac{u'(\pi_{t-})}{u(\pi_{t-})} - \frac{\gamma\sigma_K^2}{2} + \frac{\lambda(\pi_{t-})}{1-\gamma}\left[\mathbb{E}^{\mathbf{x}^d}\left(\eta_t\,\frac{\mathbf{Q}_t^{\mathcal{J}}}{\mathbf{Q}_{t-}}\right) - 1\right], \tag{45}$$

where η_t is given in (44) and $\mu_{\mathbf{Q}}(\pi_{t-})$ defined in (17) is given by[18]

$$\begin{aligned}\mu_{\mathbf{Q}}(\pi_{t-}) &= r(\pi_{t-}) + \gamma\sigma_K^2 + \lambda(\pi_{t-})\mathbb{E}_{t-}^{\mathbf{x}^d}\left[\eta_t\left(1 - \frac{\mathbf{Q}_t^{\mathcal{J}}}{\mathbf{Q}_{t-}}\right)\right] &(46)\\ &= \frac{\mathbf{c}(\pi_{t-})}{\mathbf{q}(\pi_{t-})} + \mathbf{i}(\pi_{t-}) - \delta_K + \mu_{\pi}(\pi_{t-})\frac{\mathbf{q}'(\pi_{t-})}{\mathbf{q}(\pi_{t-})}\,. &(47)\end{aligned}$$

[18]We use the FOC given in (42) and the equilibrium condition $\Gamma_t = W_t$ to obtain (46). Substituting the resource constraint $c(\pi) = A - i(\pi) - \phi(i(\pi)) - \mathbf{x}^e(\pi)$ into the ODE (34) for $q(\pi)$, we obtain (47).

In equilibrium, the household invests all wealth in the stock market, $W_t = \Gamma_t = \mathbf{Q}_t$. Additionally, both the aggregate disaster exposure and distribution mitigation spendings in a laissez-faire economy equal zero: $\mathbf{X}^e = \mathbf{X}^d = 0$.[19] In sum, the model solution is given by 1.) the ODE (45) for $u(\pi)$ and the FOCs (41)-(42) for households and 2.) the ODE (34) for $q(\pi)$ and the FOCs (35)-(36) for firms. We can also show that this solution of our market model is the same as that of a planner's problem, where the planner has no access to the adaptation technology that curtails tail risk ($\mathbf{x}^d(\pi) = 0$). This planner's problem is easier to solve. Rather than solving for $u(\pi)$ and $q(\pi)$ in our market economy, it is equivalent to solve for $b(\pi)$ and optimal policies in the planner's economy. Next, we summarize this equivalence result.

Proposition 2 *The market solution is the same as the planner's solution where there is no adaptation technology to change the distribution of the recovery fraction Z ($\mathbf{x}^d(\pi) = 0$).*

See Appendix B.3 for proof. Note that this proposition states that the Welfare Theorem applies when there is no such adaptation technology.

5 Taxation and Asset Prices

In this section, we show that introducing optimal capital taxation into our competitive market economy of Section 2 changes the market-economy solution given in Section 4 to the one implied by the planner's first-best solution given in Section 3. We then derive the asset prices that would hold under a given economy type.

5.1 Firm and Household Optimization under Capital Taxation

The government taxes the firm's capital stock K_t at a rate of $\tau_t = \mathbf{x}^d_{fb,t}$, where $\mathbf{x}^d_{fb,t}$ is the first-best mitigation spending to change the distribution of Z, obtained in Section 3. Then, the government spends $\mathbf{X}^d_t = \tau_t \mathbf{K}_t$ to reduce the tail risk of the disaster distribution.[20] We write the tax rate τ_t as a function of π_t: $\tau_t = \tau(\pi_t) = \mathbf{x}^d_{fb,t} = \mathbf{x}^d_{fb}(\pi_t)$.

Facing a capital tax rate of $\tau(\pi_t)$ and taking the equilibrium SDF $\mathbb{M}_t$ as given, each firm solves the following problem:

$$\max_{I,\, X^e,\, X^d} \mathbb{E}\left[\int_0^\infty \left(\frac{\mathbb{M}_t}{\mathbb{M}_0}\left[(A - \tau(\pi_t))\, K_t - I_t - \Phi_t - X^e_t - X^d_t\right]\right) dt\right] . \tag{48}$$

[19]Since households contribute nothing to disaster exposure and distribution mitigation spendings, using the law of large numbers, the aggregate exposure and distribution mitigation spendings are also zero.

[20]Equivalently the government can impose via a tax on sales $Y_t = AK_t$ at the firm level.

First, the firm does not spend on disaster distribution mitigation ($X^d = 0$), as there is no benefit for the firm. In effect, the tax lowers the firm's productivity from A to $A - \tau(\pi_t)$. Applying the Ito's Lemma to firm value $Q(K_t, \pi_t) = q(\pi_t)K_t$ given in (6), and using (32), we obtain the following HJB equation for $q(\pi_t)$:

$$r(\pi)q(\pi) = \max_{i,\, x^e} \; A - \tau(\pi) - i - \phi(i) - x^e + (i(\pi) - \delta_K)q(\pi) + \mu_\pi(\pi)q'(\pi) - \gamma\sigma_K^2 q(\pi) + \lambda(\pi)\mathbb{E}^{\mathbf{x}^d}\left[\eta(\pi; Z, \mathbf{x}^e)\left(q(\pi^{\mathcal{J}})(1 - N(x^e)(1 - Z)) - q(\pi)\right)\right] . \quad (49)$$

Note that the tax rate $\tau(\pi)$ appears in (49). The FOCs for i and x^e are given by (35) and (36), respectively, the same as in the no-tax competitive-market economy model of Section 4.

For brevity, we refer readers to Section 4 for the household's problem, as it is in effect the same as in the previous section. Next, we prove that incorporating optimal capital taxation into the competitive-market economy yields the first-best solution.

5.2 Optimal Capital Taxation Restores First-Best

In this section, we show that the household's value function in the competitive economy with optimal taxes is the same as the value function under the first-best. As the household's value function in a market economy depends on wealth W while the planner's value function depends on $\mathbf{K}$, we use the equilibrium result $W_t = \mathbf{q}(\pi_t)\mathbf{K}_t$ in the market economy with taxation to write the household's value function as $J(W_t, \pi_t) = J(\mathbf{q}(\pi_t)\mathbf{K}_t, \pi_t)$. The value functions in the two economies are equal, $V(\mathbf{K}_t, \pi_t) = J(W_t, \pi_t)$, if and only if $b(\pi)$ in the first-best economy equals the product $u(\pi)\mathbf{q}(\pi)$ in the competitive economy with taxes.

Specifically, we show the following results: (1.) the first-order conditions for $\mathbf{i}(\pi)$ and $\mathbf{x}^e(\pi)$ in the competitive economy with an optimal tax rate set at the $\mathbf{x}^d_{fb,t}$ are the same as those in the planner's economy; (2.) the implied ODE for $u(\pi)\mathbf{q}(\pi)$ in the competitive market economy is the same as the ODE (24) for $b(\pi)$ in the planner's economy; (3.) all the boundary conditions at $\pi = 0$ and $\pi = 1$ in the two economies are the same. Below is a proof.

First, combining the equilibrium aggregate investment FOC, $\mathbf{q}(\pi) = 1+\phi'(\mathbf{i}(\pi))$, implied by (36) with the optimal scaled consumption rule $\mathbf{c}(\pi) = \rho^\psi u(\pi)^{1-\psi}\mathbf{q}(\pi) = (\rho\mathbf{q}(\pi))^\psi \left[u(\pi)\mathbf{q}(\pi)\right]^{1-\psi}$, implied by (41) and $W = \mathbf{q}(\pi)\mathbf{K}$, we obtain the following expression for consumption:

$$\mathbf{c}(\pi) = \left[\rho(1 + \phi'(\mathbf{i}(\pi)))\right]^\psi \left[u(\pi)\mathbf{q}(\pi)\right]^{1-\psi} . \quad (50)$$

Using the goods-market clearing condition $\mathbf{c}(\pi) = A - \tau(\pi) - \mathbf{i}(\pi) - \phi(\mathbf{i}(\pi)) - \mathbf{x}^e(\pi)$ and

$b(\pi) = u(\pi)\mathbf{q}(\pi)$, we obtain the following expression:

$$b(\pi) = [A - \tau(\pi) - \mathbf{i}(\pi) - \phi(\mathbf{i}(\pi)) - \mathbf{x}^e(\pi)]^{1/(1-\psi)} \left[\rho(1 + \phi'(\mathbf{i}(\pi)))\right]^{-\psi/(1-\psi)} , \tag{51}$$

which is the same as the investment FOC, given in (25), for the planner's problem, provided that the capital tax rate equals $\mathbf{x}^d_{fb}(\pi)$: $\tau(\pi) = \mathbf{x}^d_{fb}(\pi)$. Note that (51) summarizes both the consumer's and the firm's optimization FOCs in the market economy with optimal taxes.

Second, substituting (44) for η into the FOC (35) for disaster exposure mitigation x^e in the competitive market economy, we obtain

$$1 = -\lambda(\pi) q(\pi^{\mathcal{J}}) N'(x^e) \mathbb{E}^{\mathbf{x}^d} \left[(1 - Z) \left(\frac{u(\pi^{\mathcal{J}})}{u(\pi)} \right)^{1-\gamma} \left(\frac{\mathbf{q}(\pi^{\mathcal{J}})}{\mathbf{q}(\pi)} (1 - N(\mathbf{x}^e)(1 - Z)) \right)^{-\gamma} \right] . \tag{52}$$

Using the investment FOC $q(\pi) = 1 + \phi'(i(\pi))$, the equilibrium conditions, $q(\pi) = \mathbf{q}(\pi)$, $i(\pi) = \mathbf{i}(\pi)$, and the $b(\pi) = u(\pi)\mathbf{q}(\pi)$ result for the two economies, we obtain

$$1 = -N'(\mathbf{x}^e(\pi)) \lambda(\pi) (1 + \phi'(\mathbf{i}(\pi))) \left[\frac{b(\pi^{\mathcal{J}})}{b(\pi)} \right]^{1-\gamma} \mathbb{E}^{\mathbf{x}^d(\pi)} \left[(1 - Z)(1 - N(\mathbf{x}^e(\pi))(1 - Z))^{-\gamma} \right] , \tag{53}$$

which is the same as the planner's FOC (26) for $\mathbf{x}^e$. So far, we have verified that the FOCs for investment and exposure mitigation spending in the two economies are the same.

Third, substituting (47) into (45) and using the consumption rule $c(\pi) = \rho^\psi u(\pi)^{1-\psi} q(\pi)$ implied by the FOC (41), we may rewrite the ODE (45) for the household's $u(\pi)$ as

$$\begin{aligned} 0 = \; & \frac{\rho^\psi u(\pi)^{1-\psi} - \rho}{1 - \psi^{-1}} + i(\pi) - \delta_K + \mu_\pi(\pi) \frac{q'(\pi)}{q(\pi)} + \mu_\pi(\pi) \frac{u'(\pi)}{u(\pi)} - \frac{\gamma\sigma^2}{2} \\ & + \frac{\lambda(\pi)}{1 - \gamma} \left[\mathbb{E}^{\mathbf{x}^d} \left(\eta(\pi; Z, \mathbf{x}^e) \, \frac{\mathbf{Q}^{\mathcal{J}}}{\mathbf{Q}} \right) - 1 \right] \\ = \; & \frac{\rho^\psi u(\pi)^{1-\psi} - \rho}{1 - \psi^{-1}} + i(\pi) - \delta + \mu_\pi(\pi) \left(\frac{u'(\pi)}{u(\pi)} + \frac{q'(\pi)}{q(\pi)} \right) - \frac{\gamma\sigma^2}{2} \\ & + \frac{\lambda(\pi)}{1 - \gamma} \left[\left(\frac{u(\pi^{\mathcal{J}}) \, \mathbf{q}(\pi^{\mathcal{J}})}{u(\pi)\mathbf{q}(\pi)} \right)^{1-\gamma} \mathbb{E}^{\mathbf{x}^d} ((1 - N(\mathbf{x}^e)(1 - Z))^{1-\gamma}) - 1 \right] . \end{aligned} \tag{54}$$

We obtain (54) by using $\eta(\pi; Z, \mathbf{x}^e)$ given in (44) and $\mathbf{Q}^{\mathcal{J}}/\mathbf{Q}$ given in (40).

Fourth, using the conjecture $b(\pi) = u(\pi)\mathbf{q}(\pi) = u(\pi)(1 + \phi'(\mathbf{i}(\pi)))$, we may simplify the ODE (54) and obtain the following ODE for $b(\pi) = u(\pi)\mathbf{q}(\pi)$:

$$\begin{aligned} 0 = \; & \frac{\rho}{1 - \psi^{-1}} \left[\left[\frac{b(\pi)}{\rho(1 + \phi'(\mathbf{i}(\pi)))} \right]^{1-\psi} - 1 \right] + \mathbf{i}(\pi) - \delta_K + \mu_\pi(\pi) \frac{b'(\pi)}{b(\pi)} - \frac{\gamma\sigma^2}{2} \\ & + \frac{\lambda(\pi)}{1 - \gamma} \left[\left(\frac{b(\pi^{\mathcal{J}})}{b(\pi)} \right)^{1-\gamma} \mathbb{E}^{\mathbf{x}^d} ((1 - N(\mathbf{x}^e)(1 - Z))^{1-\gamma}) - 1 \right] , \end{aligned} \tag{55}$$

which is the same as the ODE (24) for $b(\pi)$ in the first-best economy. Finally, applying the same arguments as the above to the boundaries at $\pi = 0$ and $\pi = 1$, we can show that the two economies have the same FOCs and, moreover, $b(0) = u(0)\mathbf{q}(0)$ and $b(1) = u(1)\mathbf{q}(1)$. In sum, we have verified that setting the capital tax at $\tau(\pi) = \mathbf{x}^d_{fb}(\pi)$ in the market economy yields the same allocation as in the first-best economy. Next we summarize this result.

Proposition 3 *Setting the capital tax rate $\tau(\pi_t)$ to $\mathbf{x}^d_{fb}(\pi_t)$ for all firms and then spending all tax proceeds each period to mitigate the tail risk of the disaster distributio: $\tau(\pi_t) = \mathbf{x}^d_{fb}(\pi_t)$, the competitive-market economy attains the first-best resource allocation.*

5.3 Asset Prices

Next, we report and discuss the equilibrium asset pricing implications.

Proposition 4 *Tobin's average q for the aggregate capital stock is $\mathbf{q}(\pi) = 1 + \phi'(\mathbf{i}(\pi))$, where $\mathbf{i}(\pi)$ is the optimal investment-capital ratio. The equilibrium risk-free rate, $r(\pi)$, is given by*

$$\begin{aligned} r(\pi) \ &= \rho + \psi^{-1}(\mathbf{i}(\pi) - \delta_K) - \frac{\gamma(\psi^{-1}+1)\sigma_K^2}{2} - \left[(1-\psi^{-1})\left(\frac{u'(\pi)}{u(\pi)} + \frac{\mathbf{q}'(\pi)}{\mathbf{q}(\pi)}\right) - \frac{\mathbf{q}'(\pi)}{\mathbf{q}(\pi)}\right]\mu_\pi(\pi) \\ &\quad - \lambda(\pi)\left[\mathbb{E}^{\mathbf{x}^d}(\eta(\pi; Z, \mathbf{x}^e)) - 1\right] - \lambda(\pi)\frac{\psi^{-1}-\gamma}{1-\gamma}\left[1 - \mathbb{E}^{\mathbf{x}^d}\left(\frac{\mathbf{Q}^{\mathcal{J}}}{\mathbf{Q}}\eta(\pi; Z, \mathbf{x}^e)\right)\right], \end{aligned} \tag{56}$$

where $\eta(\pi; Z, \mathbf{x}^e)$ is given in (44) and $\mathbf{Q}^{\mathcal{J}}/\mathbf{Q}$ is the jump-triggered (gross) percentage change of the stock market value given in (40). The stock market risk premium, $rp(\pi)$, is

$$rp(\pi) = \gamma\sigma_K^2 - \lambda(\pi)\mathbb{E}^{\mathbf{x}^d}\left[(\eta(\pi; Z, \mathbf{x}^e) - 1)\left(\frac{\mathbf{Q}^{\mathcal{J}}}{\mathbf{Q}} - 1\right)\right]. \tag{57}$$

These results apply to both the market economy with taxation and the one without.

Out of the six terms in (56), the first three terms are the contributing factors to the equilibrium interest rate in AK models with diffusion shocks. The fourth term captures the effect of belief updating. The fifth term describes how the jump-induced expected change of the marginal value of wealth ($\mathbb{M}^{\mathcal{J}}/\mathbb{M}$) contributes to the risk-free rate. The sixth term captures the additional effect of jumps on the equilibrium risk-free rate due to the household's recursive (non-separable) Epstein-Zin preferences rather than expected utility.[21]

There are two terms for the market risk premium rp given in (57). In addition to the diffusion risk premium (the first term), there is a jump risk premium (the second term),

[21] To be precise, for recursive utility, $f_{CV} \neq 0$ and therefore the SDF $\mathbb{M}_t$ is not additively separable, which makes jumps to have an additional intertemporal effect. For expected utility ($\gamma = \psi^{-1}$), this term disappears.

which equals the expectation over the product of the (net) percentage change of marginal value of wealth ($\mathbb{M}$), $(\eta(\pi; Z, \mathbf{x}^e) - 1)$, and the (net) percentage change of the stock market value given in (40), both of which are caused by jump arrivals. A downward jump causes the household's marginal utility to increase $(\eta(\pi; Z, \mathbf{x}^e) = \mathbb{M}^{\mathcal{J}}/\mathbb{M} \geq 1)$. As the stock market valuation decreases upon a jump arrival, $(\mathbf{Q}^{\mathcal{J}} < \mathbf{Q})$, the jump risk premium is positive.

6 Application to Tropical Cyclones

We apply our model of learning and adaptation for weather disasters to tropical cyclones and leverage our asset pricing results to highlight the learning channel. Beyond the fact that one-third of the global population is potentially affected by cyclones, there is also a great deal of uncertainty on the consequences of global warming for their frequency. According to the most recent authoritative survey of 50 climate model projections (Knutson et al., 2020), there are considerable disagreements across these models on the frequencies of major tropical cyclones in a world that is $2^o c$ higher than in the pre-industrial era. The most pessimistic model projects 2.25 times of pre-industrial levels, whereas the most optimistic model projects a slight decrease relative to pre-industrial levels. The median model projects a modest 13% increase relative to pre-industrial.

We provide a list of moments on the frequency of cyclones alongside macroeconomic and financial aggregates for a panel of countries that we use to show the importance of financial markets learning from cyclone arrivals. Our largest sample contains annual observations for the real GDP per capita growth rate and cyclone landfalls across 109 countries from 1960 to 2010 with 5,410 county-year observations in total.[22]

6.1 Frequencies of Landfalls and Spendings on Flood Control

Let $\text{Landfall}_{i,t}$ be an indicator variable that equals one if and only if country i experienced at least one cyclone landfall that is "tropical storm" or higher in year t. Table 1 reports the sample statistics of cyclone landfalls for each of the four regions.[23] Globally, a country on average experiences a tropical cyclone landfall once every 7.4 years, as the disaster arrival rate is 0.135 per annum (in Table 1.)

[22]These are the same set of countries as in Hsiang and Jina (2014) excluding Taiwan for which there is no GDP data from the World Bank Development Indicator.

[23]We assign the 109 countries into four regions: North Atlantic (including North America, the Caribbean, and West Europe) West Pacific (including Oceania), North India (including North India, Middle East, North

Table 1: Summary statistics of cyclone landfalls

Region	(1) Total # of country-year obs.	(2) Total # of cyclone landfall obs.	(3) Freq. of landfalls = (2)/(1): Disaster arrival rate estimate
North Atlantic	1,587	229	0.144
West Pacific	638	326	0.511
North India	719	75	0.104
South Atlantic	2,466	99	0.040
Global	5,410	729	0.135

The primary adaptation for countries in our sample is government flood control budgets. Unlike the landfall data, such data is not readily available. We hand collected data on government flood control budgets based on public sources by focusing on countries in the West Pacific (including Oceania), which according to Table 1 faces the most frequent tropical cyclone landfalls. We are able to obtain through various sources 72 country-year observations of government flood control budgets for a cross section of eight countries.[24] For this cross section, the average annual government flood control budget is around 0.1% (10 basis points) of the country's capital stock with a standard deviation of 0.05% across country-years observations. There are also private spendings as well on flood control according to field studies, which typically place these private spendings somewhat around 0.03% – 0.05% of capital stock, below the 0.1% of capital stock for public spendings (Genovese and Thaler, 2020).

To provide some perspectives on these small expenditures on flood control, over this sample period, the output-to-capital ratio is about 30% (with a standard deviation of 17%). The investment-capital ratio is 7% (with a standard deviation of 4%) and the consumption-capital ratio is 22% (with a standard deviation of 13%). The small expenditures on adaptation presumably reflect a belief that the consequences of global warming are relatively mild but they may significantly increase should the frequencies of arrivals increase and the society quickly updates beliefs towards the most pessimistic model projections.

6.2 Damage from Landfalls and Asset Market Reactions

Importantly, we retrieve two key panel regression estimates on the response of growth and asset prices to the arrival of cyclones that highlight the importance of learning in financial markets. According to Proposition 1 and Proposition 4, how policies (e.g., investment and

Africa, and Central Europe), and South Atlantic (including Latin America and Sub-Saharan Africa).

[24]West Pacific countries include China, Japan, Korea, and the Philippines. Oceania countries include Australia, Indonesia, New Zealand and Papua New Guinea.

consumption) and asset prices respond to a cyclone arrival depend on beliefs π which change over time. A landfall is bad news and leads to more pessimistic beliefs for growth going forward. Asset prices also fall in anticipation of more frequent disasters in the future. In the model of Pindyck and Wang (2013), which is a special case of our no-learning model, disasters lead to a destruction in capital stock $\mathbf{K}$ but the growth rate is identically and independently distributed at all time. That is, even after a disaster arrival destroys a fraction of the country's capital stock, there is no impact at all on either growth projection or asset prices (e.g., Tobin's q, the risk-free rate and the risk premium) going forward in Pindyck and Wang (2013). This is because there is no learning in their model.

Table 2: Baseline model estimation results

	Dependent variable: Growth rate of real GDP per capita				
	(1) North Atlantic	(2) West Pacific	(3) North India	(4) South Atlantic	(5) Global
Landfall	-0.0061* (-2.01)	-0.0029* (-1.94)	-0.0088*** (-3.35)	-0.0275*** (-3.69)	-0.0077*** (-4.29)
Country FE	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes
Time trends	Yes	Yes	Yes	Yes	Yes

We now show that landfalls damage growth and asset prices respond adversely to news of cyclone arrivals, consistent with our model and the importance of learning. Table 2 reports the estimates of the impact of a major cyclone making landfall on GDP growth for each region and also for the world. The dependent variable is the per capita GDP growth rate. The independent variable is the Landfall indicator. The Panel regression has country fixed effects, year fixed effects, and country-specific quadratic time trends. A landfall disaster reduces the (expected) annual growth rate by 0.61%, 0.29%, 0.88%, and 2.75% in North Atlantic, West Pacific, North India, and South Atlantic respectively, and by 0.77% in the global sample. Since the average annual growth rate in our sample is 1.95% (with a standard deviation of 5.09%), a landfall, which lowers the annual growth rate by 0.77% on average, is quite economically damaging.[25]

Since the data availability for financial variables is quite limited before 1990, and to be consistent with our samples using real GDP growth data, the sample period of macro-financial variables for the cyclone landfall analysis is from 1990 to 2010. Even then, we only have a

[25]Our estimates are consistent with those reported in Hsiang and Jina (2014), who estimate the marginal effect of windspeed on GDP growth damage.

subset of countries that have the relevant financial variables. Panel A of Table 3 reports the unconditional moments for asset prices pooling all these remaining countries. These moments including a risk-free rate of 1.43% and an equity risk premium of 5.26%, a volatility of equity market returns of 26.57%,, and a Tobin's average q of 2.49.

Table 3: Summary statistics of asset prices

Panel A provides the summary statistics of the financial variables used in our study. RealRF is real interest rate (nominal interest rate minus inflation rate). ERP is equity risk premium (stock market return net of nominal interest rate). TobinQ is Tobin's average q. VolRET is volatility of annual stock market return. Annual risk-free nominal interest rate, inflation rate, and stock market return data at the country level are from the IMF and the World Bank. Panel B reports regression of these asset-pricing moments on cyclone landfalls. Estimates for RealRF and ERP are in percentages. t-statistics with clustered robust standard errors are shown in parentheses below the estimates. The sample period is 1990 to 2010 for the cyclone sample.

Panel A: Summary Statistics

	Mean	Standard deviation	Median	10 percentile	90 percentile
RealRF (%)	1.43	4.32	1.32	-4.40	6.91
ERP (%)	5.26	24.26	5.61	-27.47	37.36
TobinQ	2.49	4.84	1.51	0.60	3.65
VolRET (%)	26.57	8.26	26.38	15.24	37.52

Panel B: Asset Market Reaction to Landfalls

	RealRF	ERP	TobinQ
Landfall	-0.090**	0.307**	-0.101**
	(-2.34)	(2.48)	(-2.11)

As before with real GDP growth in Table 2, we use a Panel regression model in Panel B of Table 3 to measure the impact of a cyclone landfall on a country's real interest rate (RealRF), equity risk premium (ERP), or Tobin's average q (TobinQ) by using country and time fixed effects. The panel regression model regresses financial variables on an indicator for cyclone landfall (Landfall) for the whole sample. A cyclone landfall on average reduces Tobin's average q by 0.10, lower the real interest rate by 0.09%, and increases equity risk premium by 0.31% per annum. These estimates are inconsistent with models of disasters absent learning, e.g., Pindyck and Wang (2013), as we discussed earlier.

7 Quantitative Analysis

We now use the moments in Section 6 to calibrate our model.

7.1 Distributional and Functional Form Specifications

As in Barro (2006) and Pindyck and Wang (2013), we assume that the distribution function of the recovery fraction Z upon a cyclone arrival is given by a power law over $Z \in (0, 1)$:

$$\Xi(Z; \mathbf{x}^d) = Z^{\beta(\mathbf{x}^d)} \,, \tag{58}$$

where $\beta(\mathbf{x}^d)$ is the exponent function that depends on scaled disaster distribution mitigation $\mathbf{x}^d$. To ensure that our model is well defined, we require $\beta(\mathbf{x}^d) > \gamma - 1$.

Conditional on a jump arrival, the expected fractional capital loss for a firm is given by

$$\ell(\pi) = N(x^e)(1 - \mathbb{E}^{\mathbf{x}^d}(Z)) = \frac{N(x^e)}{\beta(\mathbf{x}^d) + 1} \,. \tag{59}$$

The larger the value of $\beta(\,\cdot\,)$, the smaller the expected fractional loss $\mathbb{E}^{\mathbf{x}^d}(1-Z)$ even absent the firm's disaster exposure mitigation x^e. To capture the benefit of public mitigation, we assume that $\beta(\mathbf{x}^d)$ is increasing in $\mathbf{x}^d$: $\beta'(\mathbf{x}^d) > 0$. The benefit of public disaster distribution mitigation $\mathbf{x}^d$ is to increase the capital stock recovery (upon the arrival of a disaster) in the sense of first-order stochastic dominance in that $\Xi(Z; \mathbf{x}^d)$ decreases with $\mathbf{x}^d$.

Let $g_t = g(\pi_t)$ denote a firm's expected growth rate including the jump effect. The homogeneity property implies that growth is independent of the aggregate capital $\mathbf{K}$ and

$$g(\pi) = i(\pi) - \delta_K - \lambda(\pi)\ell(\pi) = i(\pi) - \delta_K - \frac{\lambda(\pi)N(x^e)}{\beta(\mathbf{x}^d) + 1} \,. \tag{60}$$

We specify the firm's exposure mitigation technology $N(x^e)$ as follows:

$$N(x^e) = 1 - (x^e)^\zeta \,, \tag{61}$$

where $0 < \zeta < 1$. That is, the more exposure mitigation spending x^e the smaller the (fractional) damage, i.e., the lower the level of $N(x^e)$. Additionally, the marginal benefit of x^e on reducing damages diminishes. We use the following linear specification for $\beta(\mathbf{x}^d)$ which governs the public disaster distribution mitigation technology:

$$\beta(\mathbf{x}^d) = \beta_0 + \beta_x \mathbf{x}^d \,, \tag{62}$$

with $\beta_0 \geq \max\{\gamma - 1, 0\}$ and $\beta_x > 0$. The coefficient β_0 is the exponent for the distribution function of the fractional recovery Z in the absence of mitigation. The coefficient β_x is a key parameter in our model and measures the efficiency of the aggregate disaster distribution mitigation technology.

Finally, we use the widely used quadratic adjustment cost function (e.g., Hayashi, 1982):

$$\phi(i) = \frac{\theta i^2}{2} \,, \tag{63}$$

where the parameter θ measures how costly it is to adjust capital.

7.2 Calibration and Parameter Choices

Table 4: PARAMETER VALUES

Parameters	Symbol	Value
disaster jump arrival rate in State G	λ_G	0.1
disaster (jump) arrival rate in State B	λ_B	0.8
prior of being in State B	π_0	0.08
power law exponent absent adaptation	β_0	39
distribution adaptation technology parameter	β_x	$1,800$
exposure adaptation technology parameter	ζ	0.4
elasticity of intertemporal substitution	ψ	1.5
time rate of preference	ρ	5%
productivity parameter	A	27%
quadratic adjustment cost parameter	θ	17
coefficient of relative risk aversion	γ	8
capital diffusion volatility	σ_K	8%
depreciation rate of capital	δ_K	6%

All parameter values, whenever applicable, are continuously compounded and annualized.

Our model has 13 parameters. We calibrate these parameters by targeting 13 moments described in Section 6. The calibrated values of these parameters are given in Table 4.

The new parameters in our analyses are the three for the learning process (λ_G, λ_B, and π_0) and the other three for the adaptation technologies (β_0, β_x and ζ). In order to determine these six parameters, we use six moments from our panel data on the frequencies of tropical cyclone landfalls, their impact on GDP growth and asset prices (risk-free rate, equity risk premium, and Tobin's average q), and the levels of private and public adaptation spendings that we obtained and reported in Section 6, i.e., around 0.1% and 0.04% of capital stock, respectively.

A number of the macro-finance moments we are targeting, such as the risk-free rate rate and equity risk premium (Panel A of Table 3), are similar to those targeted in the asset pricing literature. Hence, our preference parameters, e.g., the EIS ψ and coefficient of relative risk aversion γ, are similar to those used in this literature. For instance, Bansal and Yaron (2004) show that setting the coefficient of relative risk aversion γ to a value between 7 to 10 and an EIS ψ to be larger than one is necessary to match the equity risk premium and the risk-free rate. Similarly, the parameters for the production part of our model, e.g., productivity, capital adjustment costs, and the capital depreciation rate, are chosen to match the aggregate output and production targets discussed in Section 6.1. The calibrated values turn out to be close to those in the literature (e.g., Eberly, Rebelo, and Vincent, 2012), suggesting that our calibration strategy yields sensibly robust parameter values for our quantitative analysis.

Next, we use these parameters to analyze a few economies. In Figure 2, we plot and compare the solutions for three economies: 1.) the planner's first-best solution (solid blue lines), 2.) the market economy (dashed red lines) and 3.) the planner's solution with no learning (dotted black lines). In the next two subsections, we do two pair-wise comparisons.

7.3 Comparing First-Best with Competitive-Market Solutions

In this subsection, we compare the first-best with competitive-market solutions. A key feature that both economies share is learning.

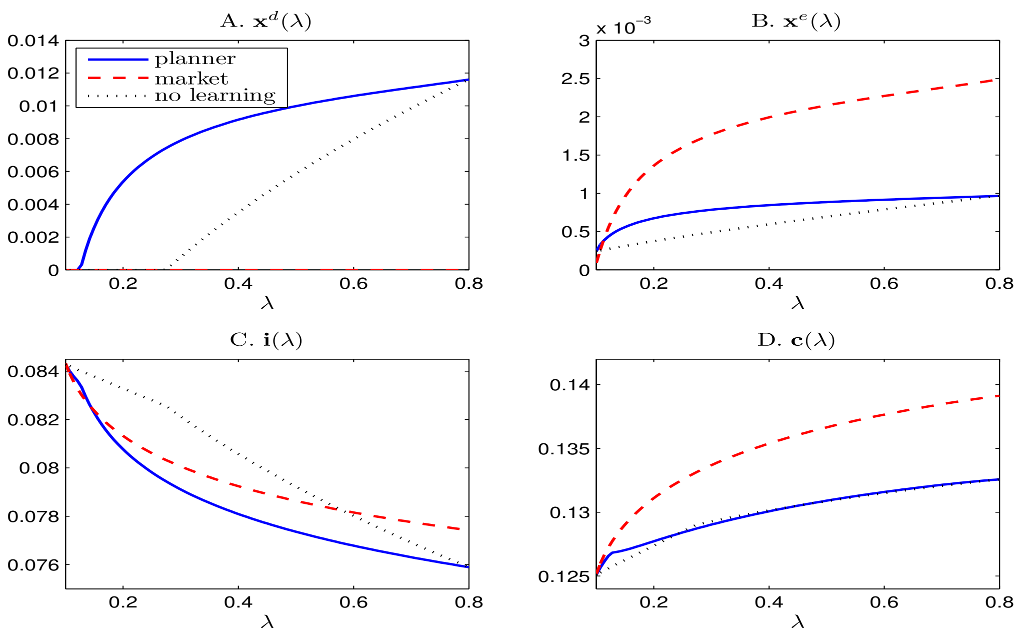

Figure 2: This figure compare the solutions for three economies: 1.) the planner's first-best solution (solid blue lines), 2.) the market economy (dashed red lines), and 3.) the planner's solution with no learning (dotted black lines). The first two economies feature Bayesian learning. The parameters values are given in Table 4.

In Panel A of Figure 2, we see that public mitigation $\mathbf{x}^d$ (solid blue line) rapidly increases with the expected disaster arrival rate λ to 1.18% per annum in the first-best economy. In contrast, the market solution features no public mitigation spending (dashed red line) regardless of beliefs due to externalities.

Panel B shows that private mitigation $\mathbf{x}^e$ in both economies increases with λ. Moreover, $\mathbf{x}^e$ is higher in the market economy than in the first-best economy as the marginal benefit of private mitigation is higher in the market economy than in the planner's economy as it is the only measure to mitigate disaster risk in the market economy (for almost all levels of λ except at the very low levels of λ). The difference of $\mathbf{x}^e$ increases with λ but the total mitigation spendings given by the sum, $\mathbf{x}^e + \mathbf{x}^d$, are lower in the market economy than in the first-best economy, meaning that the combined risk mitigation in the economy is still under-provided in the laissez faire market economy.

Panel C shows that for almost all levels of λ, investment $\mathbf{i}$ is lower in the first-best economy than in the market economy. Panel D shows that for almost all levels of λ, consumption $\mathbf{c}$ is lower in the first-best economy than in the market economy.

Now we turn to Figure 3. We define WTP $\zeta_p(\pi)$ and $\zeta_m(\pi)$ as the fraction of capital the market economy with no adaptation is willing to give up to transition to the planner's first-best economy and the market economy with just private adaptation, respectively.[26] Panel A shows that both WTPs increase with belief π.[27] The WTP wedge $\zeta_p(\pi) - \zeta_m(\pi)$ measures the additional welfare gain of having access to the tail-risk public adaptation technology in a market economy that already has access to the firm-level exposure adaptation technology. This additional welfare gain increases with λ and is quite substantial for the real-world relevant range of values for λ.

In Panel B of Figure 3, we show that the conditional damage $\ell(\lambda)$ in both the first-best and

[26] To calculate the WTP measures, $\zeta_p(\pi)$ and $\zeta_m(\pi)$, we use the representative household's value functions (welfare measures proportional to the certainty equivalent wealth) for the three economies. Formally, we use

$$\zeta_p(\pi) = 1 - \frac{\underline{b}(\pi)}{b_{fb}(\pi)} \quad \text{and} \quad \zeta_m(\pi) = 1 - \frac{\underline{b}(\pi)}{\widehat{b}(\pi)} > 0 \ ,$$

where b_{fb}, $\widehat{b}$, and $\underline{b}$ are the welfare measures (proportional to certainty equivalent wealth) in the (planner's) first-best economy, the market economy (with access to both adaptation technologies but only private mitigation technology will be adapted in equilibrium), and the market economy (with access to neither adaptation technology), respectively.

[27] We can decompose the WTPs into the risk premium and timing premium components by building on the idea and extending the procedure proposed in Epstein, Farhi, and Strzalecki (2014). We show that for our calibrated baseline, while the timing premium is also important, the risk premium component is the major contributor to the total WTP.

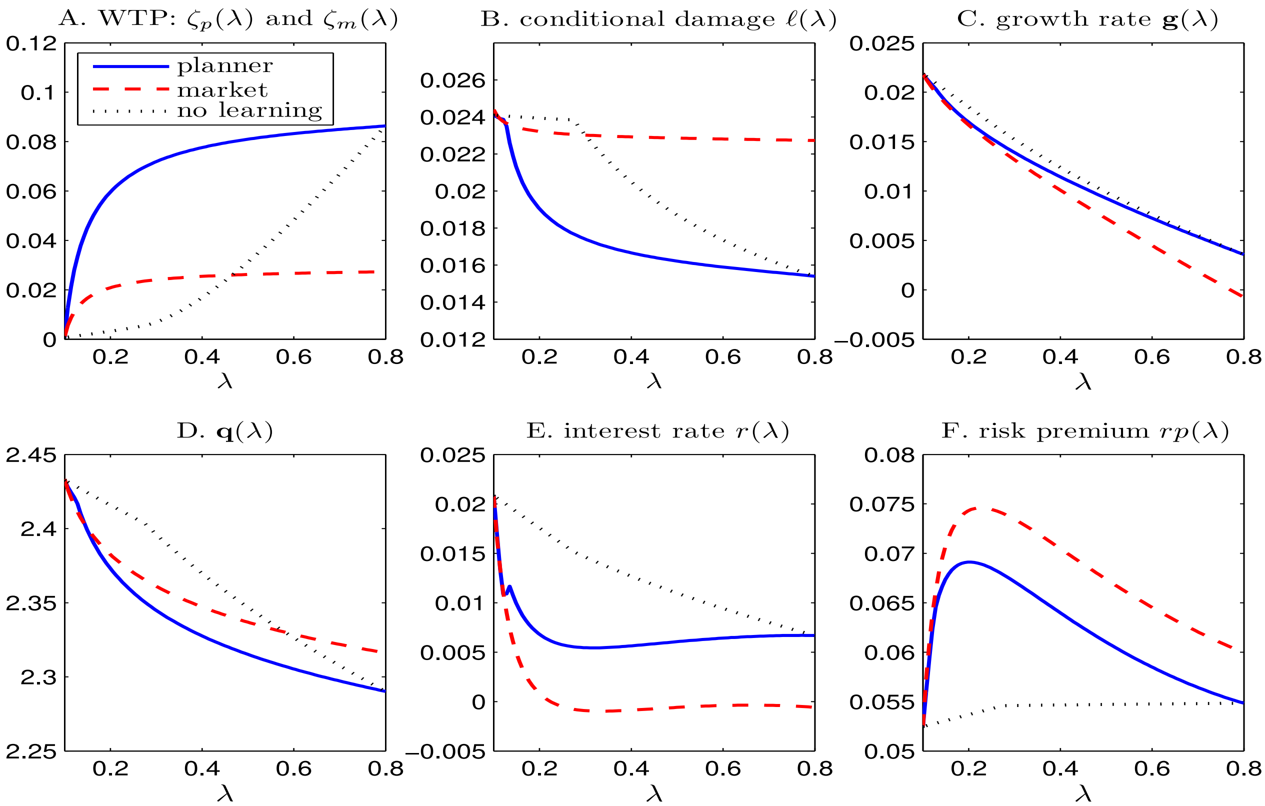

Figure 3: This figure compare the solutions for three economies: 1.) the planner's first-best economy (solid blue lines), 2.) the market economy (dashed red lines), and 3.) the planner's economy with no learning (dotted black lines). The first two economies feature Bayesian learning. The parameters values are given in Table 4.

market economies decrease with λ. Additionally, the conditional damage $\ell(\lambda)$ in the first-best economy is lower than in the market economy. Moreover, as λ increases, the wedge between $\ell(\lambda)$ in the two economies widens. Because of larger risk mitigation and smaller conditional damage $\ell(\lambda)$ in the first-best economy than in the market economy, the expected growth rate $\mathbf{g}(\lambda)$ is higher in the first-best economy than the market economy (Panel C). This is because the society is more prepared in the first-best economy than in the market economy. The growth-rate difference in the two economies increases with λ and is quantitatively large for the real-world relevant range of λ.

In Panels D, E, and F of Figure 3, we show that in both the first-best and market economies, Tobin's average q decreases as belief worsens, however, the interest rate and risk premium are nonlinear and non-monotonic in λ. This is because while the mean growth prospect gets worse as λ increases, uncertainty is the highest for the intermediate range of λ. As both the first-moment and higher-order-moment effects are important in the first-best and market economies, the impact of λ on the interest rate and risk premium are nonlinear and non-monotonic.

7.4 Learning versus No-Learning Counterfactual

In this subsection, we assess the value of learning by comparing the solution of our first-best model with learning (of Section 3) with the solution of a counterfactual planner's model with no learning. In the counterfactual no-learning model, we assume that the disaster arrival rate is fixed at a given value of λ and then solve the model. We find that adaptation spendings (in Panels A and B of Figure 2) for $\pi \in (0, 1)$ are larger in our learning model (solid blue lines) than in our counterfactual no-learning model (dotted black lines). As the $\pi = 0$ state (where $\lambda = \lambda_G = 0.1$) and the $\pi = 1$ state (where $\lambda = \lambda_B = 0.8$) are absorbing, the solutions for the first-best learning model (solid blue lines) and the planner's no-learning model (dotted black lines) are the same at $\pi = 0$ and $\pi = 1$ states.That is, adaptation spendings are the highest in the learning model where there is uncertainty over climate states (intermediate values of λ). Investment in our learning model is also lower than in our no-learning counterfactual model (Panel C), but consumption differences in the two economies are limited (Panel D).

7.5 Comparative Statics

In Online Appendix OD, we conduct comparative static analyses with respect to four key parameters: the EIS (ψ), the disaster arrival rate in state B (λ_B), the time rate of preference (ρ), and the coefficient of relative risk aversion (γ). Our main mitigation findings are robust across these three parameter values. The main difference lies in valuation ratios, e.g., the price-dividend ratio. When EIS $\psi = 1$, the price-dividend ratio, $\mathbf{q}/\mathbf{c}$, equals $1/\rho$, the inverse of the time rate of preference, for all levels of π, which is known in the asset-pricing literature, e.g., Wachter (2013). When ψ is greater (less) than one, this $\mathbf{q}/\mathbf{c}$ ratio decreases (increases) with π. That is, equity valuation ratios react negatively to bad (e.g., disaster arrival) news consistent with the reason why the long-run risk literature chooses $\psi > 1$.

7.6 Generalized Learning Model with Stochastic Arrival Rate λ_t

The disaster arrival rate in our baseline model of Section 2, while unobservable, is constant. In Appendix OA, we generalize our baseline model to allow for the unobservable disaster arrival rate to be stochastic, by using a two-state Markov Chain (see, e.g., Wachter and Zhu, 2019). We show that our main quantitative results and conclusions continue to hold in the generalized model of Appendix OA where the transition rates between states G and B are small.

7.7 External Habit Model

In Appendix OB, we replace the Epstein-Zin recursive utility used in our baseline model of Section 2 with another widely-used risk preference—the external habit model proposed by Campbell and Cochrane (1999). For brevity, we focus on the planner's solution. We calibrate our external habit model by targeting the same moments as we do for our baseline model whenever feasible. The quantitative implications on mitigation spendings and welfare in our external habit model are similar to those in our baseline model with Epstein-Zin preferences. However, the two models generate opposite predictions on how investment $\mathbf{i}$ and Tobin's average $\mathbf{q}$ respond as belief becomes more pessimistic (π increases). While both $\mathbf{i}$ and $\mathbf{q}$ increase with π in our habit model, the opposite holds in our baseline Epstein-Zin model. The intuition follows from our discussion regarding comparative statics with respect to ψ.[28]

8 Implications for the Social Cost of Carbon

In this section, we propose an extension of our model that allows us to tractably draw out the implications of learning and adaptation to weather disasters for the social cost of carbon.

8.1 Fossil Fuels, Carbon Stock, and Disasters

First, we introduce fossil-fuel-usage caused emissions, H_t, as an additional factor of production at the micro level, so that firm production Y_t is given by:

$$Y_t = AK_t^{\alpha} H_t^{1-\alpha}\,, \tag{64}$$

with $0 < \alpha < 1$, as in Golosov et al. (2014) and Van den Bremer and Van der Ploeg (2021). The stock of (aggregate) atmospheric carbon that exceeds the pre-industrial atmospheric carbon stock associated with man-made emissions, which we denote by $\mathbf{S}_t$, evolves:

$$d\mathbf{S}_t = (\mathbf{H}_{t-} - \delta_S \mathbf{S}_{t-})dt + \sigma_S \mathbf{S}_{t-} d\mathcal{W}_t^S\,, \tag{65}$$

[28] From the long-run risk literature and the comparative static analysis for our baseline Epstein-Zin model with respect to ψ in Section OD of the Online Appendix, we know that an EIS (lower than one) causes the valuation ratios, e.g., the price-dividend ratio, to go up in response to bad news. Our habit model inherits this property, which explains the key differences between the two utility models.

where $\mathbf{H}_t$ is the aggregate fossil fuel emissions by all firms: $\mathbf{H}_t = \int H_t^\nu d\nu$, δ_S is the decaying rate of the atmospheric carbon stock, $\mathcal{W}_t^{\mathbf{S}}$ is a standard Brownian motion, and the parameter σ_S is the volatility of atmospheric carbon. Let ϑ denote the correlation coefficient between $\mathcal{W}_t^{\mathbf{S}}$ and the standard Brownian motion $\mathcal{W}_t^K$. We measure both the firm-level H and the aggregate $\mathbf{H}$ in units of carbon and therefore also measure $\mathbf{S}$ in units of carbon (e.g., tons of carbon). Let p_H denote the price of carbon in units of consumption good, the numeraire.

To model the damage of the aggregate carbon stock $\mathbf{S}_t$, we assume that the distribution of the post-jump fractional recovery Z depends on $\mathbf{S}_t$. That is, the damage of weather disaster shocks is still uncertain and increases in expectation with $\mathbf{S}_t$. As in our baseline model, firms and households learn from disaster arrivals over time regarding the severity of climate risk. To maintain the homogeneity structure of our model, we assume that the distribution function of the post-jump fractional recovery Z, Ξ, depends on both aggregate adaptation spending, $\mathbf{x}_{t-}$, and the scaled carbon stock, $\mathbf{s}_{t-} = \mathbf{S}_{t-}/\mathbf{K}_{t-}$, i.e., $\Xi(Z; \mathbf{x}_{t-}^d, \mathbf{s}_{t-})$.

The carbon-to-productive-capital ratio $\mathbf{s} = \mathbf{S}/\mathbf{K}$ evolves as follows:

$$d\mathbf{s}_t = \mu_s(\pi_{t-}, \mathbf{s}_{t-})dt + \mathbf{s}_{t-}\left[\sigma_S d\mathcal{W}_t^S - \sigma_K d\mathcal{W}_t^K + N_{t-}(1-Z)d\mathcal{J}_t\right], \tag{66}$$

where $\mu_s(\pi_{t-}, \mathbf{s}_{t-})$ is given by

$$\mu_s(\pi_{t-}, \mathbf{s}_{t-}) = \mathbf{h}_{t-} - \left(\mathbf{i}_{t-} - \delta_K + \delta_S - \sigma_K^2 + \vartheta\sigma_K\sigma_S\right)\mathbf{s}_{t-}. \tag{67}$$

8.2 Planner's Solution

Let $V(\mathbf{K}, \mathbf{S}, \pi)$ denote the representative household's value function. The following HJB equation characterizes the planner's problem:

$$\begin{aligned} 0 = \max_{\mathbf{C}, \mathbf{I}, \mathbf{x}^e \mathbf{x}^d, \mathbf{H}} \quad & f(\mathbf{C}, V) + (\mathbf{I} - \delta_K \mathbf{K}) V_{\mathbf{K}} + \mu_\pi(\pi) V_\pi + (\mathbf{H} - \delta_S \mathbf{S}) V_{\mathbf{S}} + \frac{\sigma_K^2 \mathbf{K}^2 V_{\mathbf{KK}}}{2} + \frac{\sigma_S^2 \mathbf{S}^2 V_{\mathbf{SS}}}{2} \\ & + \vartheta\sigma_K\sigma_S \mathbf{KS} V_{\mathbf{KS}} + \lambda(\pi)\mathbb{E}^{\mathbf{x}^d}\left[V\left((1 - N(\mathbf{x}^e)(1-Z))\mathbf{K}, \mathbf{S}, \pi^{\mathcal{J}}\right) - V(\mathbf{K}, \mathbf{S}, \pi)\right], \end{aligned} \tag{68}$$

subject to the following resource constraints:

$$A\mathbf{K}_t^\alpha \mathbf{H}_t^{1-\alpha} = \mathbf{Y}_t = \mathbf{C}_t + \mathbf{I}_t + \Phi(\mathbf{I}_t, \mathbf{K}_t) + \mathbf{X}_t^d + \mathbf{X}_t^e + p_H \mathbf{H}_t. \tag{69}$$

Recall that p_H is the price of carbon in units of the (numeraire) consumption good.

Unlike in our baseline model without $\mathbf{S}$, the household not only takes into account the evolution of $\mathbf{S}$ (via the drift term involving $V_{\mathbf{S}}$ and the quadratic-variation term involving

$V_{\mathbf{SS}}$), but also has incentives to hedge against shocks to carbon stock $\mathbf{S}$ (via the quadratic-covariation term involving $V_{\mathbf{KS}}$).

The first-order condition (FOC) for fossil fuel $\mathbf{H}$ is

$$\left(p_H - (1-\alpha)A\mathbf{K}^{\alpha}\mathbf{H}^{-\alpha}\right) f_{\mathbf{C}}(\mathbf{C}, V) = V_{\mathbf{S}}(\mathbf{K}, \mathbf{S}, \pi) \ . \tag{70}$$

The right side of (70), $V_S(\mathbf{K}, \mathbf{S}, \pi)$, is the marginal (utility) benefit of using fossil fuel. The left side of (70) is the marginal (utility) cost of fossil fuel, given by the product of forgone marginal utility of consumption $f_{\mathbf{C}}(\mathbf{C}, V)$ and $(p_H - (1-\alpha)A\mathbf{K}^{\alpha}\mathbf{H}^{-\alpha})$, the latter of which has two terms: the relative price of fossil fuel p_H and the reduction of marginal product of carbon stock.)

As in Golosov et al. (2014) and Van den Bremer and Van der Ploeg (2021), we use

$$m_t \equiv -\frac{V_{\mathbf{S}}(\mathbf{K}_t, \mathbf{S}_t, \pi_t)}{f_{\mathbf{C}}(\mathbf{C}_t, V_t)} \tag{71}$$

to denote the social cost of carbon (SCC), marginal utility cost of emitting an additional ton of carbon divided by the marginal utility of consumption. Rewriting the FOC for fossil fuel, (70) and using SCC defined in (71), we obtain:

$$\frac{(1-\alpha)\mathbf{Y}_t}{\mathbf{H}_t} = p_H + m_t \ . \tag{72}$$

We show that the value function V is homogeneous with degree $(1-\gamma)$ in $\mathbf{K}$ and $\mathbf{S}$:

$$V(\mathbf{K}, \mathbf{S}, \pi) = \frac{1}{1-\gamma}\left(b(\pi, \mathbf{s})\mathbf{K}\right)^{1-\gamma}, \tag{73}$$

where $\mathbf{s} = \mathbf{S}/\mathbf{K}$ and $b(\pi, \mathbf{s})$ is a measure of welfare proportional to the household's certainty equivalent wealth under optimality.

Using the FOCs and substituting the value function $V(\mathbf{K}, \mathbf{S}, \pi)$ given in (73) into the HJB equation (68), and simplifying the equations, we obtain the following five-equation PDE system for $b(\pi, \mathbf{s})$, $\mathbf{i}(\pi, \mathbf{s})$, $\mathbf{x}^d(\pi, \mathbf{s})$, $\mathbf{x}^e(\pi, \mathbf{s})$ and $\mathbf{h}(\pi, \mathbf{s})$:

$$\begin{aligned} 0 = {} & \frac{\rho}{1-\psi^{-1}}\left[\left[\frac{b(\pi, \mathbf{s})}{\rho(1+\phi'(\mathbf{i}(\pi, \mathbf{s})))}\right]^{1-\psi} - 1\right] + \mathbf{i}(\pi, \mathbf{s}) - \delta_K - \frac{\gamma\sigma_K^2}{2} + \mu_\pi(\pi)\frac{b_\pi(\pi, \mathbf{s})}{b(\pi, \mathbf{s})} \\ & + (\mathbf{h}(\pi, \mathbf{s}) - \delta_S\mathbf{s})\frac{b_s(\pi, \mathbf{s})}{b(\pi, \mathbf{s})} + \frac{\sigma_S^2\mathbf{s}^2}{2}\left(\frac{b_{\mathbf{ss}}(\pi, \mathbf{s})}{b(\pi, \mathbf{s})} - \gamma\frac{(b_{\mathbf{s}}(\pi, \mathbf{s}))^2}{b(\pi, \mathbf{s})^2}\right) + (1-\gamma)\vartheta\sigma_K\sigma_S\mathbf{s}\frac{b_{\mathbf{s}}(\pi, \mathbf{s})}{b(\pi, \mathbf{s})} \\ & + \frac{\lambda(\pi)}{1-\gamma}\left[\mathbb{E}^{\mathbf{x}^d(\pi, \mathbf{s})}\left(\frac{(1-N(\mathbf{x}^e(\pi, \mathbf{s}))(1-Z))b\left(\pi^{\mathcal{J}}, \mathbf{s}^{\mathcal{J}}\right)}{b(\pi, \mathbf{s})}\right)^{1-\gamma} - 1\right], \end{aligned} \tag{74}$$

$$b(\pi,\mathbf{s}) = [A\mathbf{h}(\pi,\mathbf{s})^{1-\alpha} - \mathbf{i}(\pi,\mathbf{s}) - \phi(\mathbf{i}(\pi,\mathbf{s})) - \mathbf{x}^d(\pi,\mathbf{s}) - \mathbf{x}^e(\pi,\mathbf{s}) - p_H\mathbf{h}(\pi,\mathbf{s})]^{1/(1-\psi)} \left[\rho(1+\phi'(\mathbf{i}(\pi,\mathbf{s})))\right]^{-\psi/(1-\psi)}, \tag{75}$$

$$\frac{b_{\mathbf{s}}(\pi,\mathbf{s})}{b(\pi,\mathbf{s}) - \mathbf{s}b_{\mathbf{s}}(\pi,\mathbf{s})} = \frac{p_H - (1-\alpha)A\mathbf{h}(\pi,\mathbf{s})^{-\alpha}}{1+\phi'(\mathbf{i}(\pi,\mathbf{s}))}, \tag{76}$$

$$\frac{1}{1+\phi'(\mathbf{i}(\pi,\mathbf{s}))} = \lambda(\pi)\mathbb{E}^{\mathbf{x}^d(\pi,\mathbf{s})}\left[\frac{(Z-1)N'(\mathbf{x}^e(\pi,\mathbf{s}))\left(b\left(\pi^{\mathcal{J}},\mathbf{s}^{\mathcal{J}}\right) - \mathbf{s}^{\mathcal{J}}b_{\mathbf{s}}\left(\pi^{\mathcal{J}},\mathbf{s}^{\mathcal{J}}\right)\right)}{b(\pi,\mathbf{s})} \times \left(\frac{(1-N(\mathbf{x}^e(\pi,\mathbf{s}))(1-Z))b\left(\pi^{\mathcal{J}},\mathbf{s}^{\mathcal{J}}\right)}{b(\pi,\mathbf{s})}\right)^{-\gamma}\right], \tag{77}$$

$$\frac{1}{1+\phi'(\mathbf{i}(\pi,\mathbf{s}))} = \frac{\lambda(\pi)}{1-\gamma}\int_0^1 \left[\frac{\partial\xi(Z;\mathbf{x}^d(\pi,\mathbf{s}))}{\partial\mathbf{x}^d}\left(\frac{(1-N(\mathbf{x}^e(\pi,\mathbf{s}))(1-Z))b\left(\pi^{\mathcal{J}},\mathbf{s}^{\mathcal{J}}\right)}{b(\pi,\mathbf{s})}\right)^{1-\gamma}\right] dZ\,. \tag{78}$$

where $\mathbf{s}^{\mathcal{J}} = \frac{\mathbf{s}}{1-N(\mathbf{x}^e(\pi,\mathbf{s}))(1-Z)}$ is the post-jump ratio carbon-productive-capital ratio $\mathbf{s}$.

Recall that $\mathbf{s}$ is a mean-reverting process. Because $\pi = 0$ and $\pi = 1$ are absorbing states, we can obtain the boundary conditions at $\pi = 0$ and $\pi = 1$ by substituting $\pi = 0$ and $\pi = 1$ into (74)-(78).

Proposition 5 *In a competitive market economy, household consumption, corporate investment, and disaster risk exposure mitigation attain the first-best levels provided that the government chooses the following policies. First, the government sets its fossil fuel tax at:*

$$m \equiv -\frac{V_S(\mathbf{K},\mathbf{S},\pi)}{f_{\mathbf{C}}(\mathbf{C},V)} = -\frac{b_{\mathbf{s}}(\pi,\mathbf{s})}{\rho}\left(\frac{\mathbf{c}(\pi,\mathbf{s})}{b(\pi,\mathbf{s})}\right)^{\psi^{-1}}, \tag{79}$$

where

$$\mathbf{c}(\pi,\mathbf{s}) = A\mathbf{h}(\pi,\mathbf{s})^{1-\alpha} - \mathbf{i}(\pi,\mathbf{s}) - \phi(\mathbf{i}(\pi,\mathbf{s})) - \mathbf{x}^d(\pi,\mathbf{s}) - \mathbf{x}^e(\pi,\mathbf{s}) - p_H\mathbf{h}(\pi,\mathbf{s})\,. \tag{80}$$

Second, the government selects the capital tax rate $\tau(\pi_t)$ at the first-best level $\mathbf{x}^d_{fb}(\pi_t)$ for all firms and then spends 100% of the capital tax proceeds each period to mitigate the tail risk of the disaster distribution, balancing its budget period by period.

That is, the government uses fossil fuel taxes to address the tackle externality and levy carbon taxes to address the lack of mitigation spending. By combining these two policies, the government implements the first-best outcome.

8.3 Calibration

Next, we calibrate our generalized model with carbon stock. The solution of this generalized model boils down to a PDE system even after we use the homogeneity property to simplify our analysis. After incorporating fossil fuel and carbon dynamics, our model has 20 parameters (Table 5) while our baseline model of Section 7 has 13 parameters (Table 4).

First, we assume that the β function that describes the disaster damage distribution depends on not only $\mathbf{X}^d$ and $\mathbf{K}$ but also $\mathbf{S}$ in our carbon model as follows:

$$\beta(\mathbf{x}^d, \mathbf{s}) = \beta_0 + \beta_x \mathbf{x}^d - \beta_s \mathbf{s}\,, \tag{81}$$

where $\mathbf{x}^d = \mathbf{X}^d/\mathbf{K}$, $\mathbf{s} = \mathbf{S}/\mathbf{K}$, and β_x and β_s are positive parameters. Compared with (62), we now incorporate the effect of carbon stock $\mathbf{s}$ on disaster damages, which is captured by a key new parameter β_s.

Generalizing our calibration procedure for the baseline model, we determine the eight parameters in the first Panel of Table 5 (λ_G, β_0, β_x, β_s, ζ, λ_B, δ_S, and π_0) by targeting the GDP growth, asset prices (risk-free rate, equity risk premium, and Tobin's average q), the levels of private and public adaptation spendings (reported in Section 6), the steady state of $\mathbf{s}$ at 0.25, and also importantly the level of social cost of carbon, which is around \$40. as in the literature, e.g., Cai and Lontzek (2019).

The four parameters in the second Panel of Table 5 (α, σ_S, p_H, and $\mathbf{s}_0$) are related to fossil fuels and carbon stock dynamics in our model. We use the parameter values from the carbon economics literature for these parameters (see e.g., Van den Bremer and Van der Ploeg, 2021).

For the three preference parameters (the EIS ψ, risk aversion γ, and the time rate of preference ρ) and the four production parameters (productivity A, the quadratic adjustment cost θ, capital diffusion volatility σ, and capital depreciation rate δ_K) reported in the last Panel of Table 5, we use the same values as those in Table 4 for our baseline model without carbon (of Section 2). Finally, we set the correlation coefficient between capital shocks and carbon stock shocks to zero: $\vartheta = 0$.

8.4 Social Cost of Carbon (SCC) Projections

We discuss the economics of SCC projections in two steps using the planner's solution. To ease our exposition, we first consider the counterfactual by shutting down the learning channel and then incorporate Bayesian learning and analyze its effect on SCC.

Table 5: Parameter Values for Generalized Model with Carbon

Parameters	Symbol	Value
disaster (jump) arrival rate in State G	λ_G	0.1
disaster (jump) arrival rate in State B	λ_B	0.8
prior of being in State B	π_0	0.1
power law exponent absent adaptation	β_0	39
distribution adaptation technology parameter	β_x	$3,500$
exposure adaptation technology parameter	ζ	0.2
damage parameter from atmospheric carbon	β_s	$14,000$
carbon decaying rate	δ_S	3%
return-to-scale parameter	α	0.96
volatility of carbon stock growth	σ_S	7.5%
price of carbon input	p_H	540
initial value of $\mathbf{s}$	$\mathbf{s}_0$	$0.13/p_H$
elasticity of intertemporal substitution	ψ	1.5
time rate of preference	ρ	4.8%
productivity parameter	A	27%
quadratic adjustment cost parameter	θ	18
coefficient of relative risk aversion	γ	8
capital diffusion volatility	σ_K	8%
depreciation rate of capital	δ_K	6%
correlation between capital and carbon stocks	ϑ	0

All parameter values, whenever applicable, are continuously compounded and annualized.

In Figure 4, we report the mean and quantiles of SCC projections under the counterfactual without learning. First, the mean of time-t conditional distribution of SCC increases over time (see Panel A). This is because the stock of carbon accumulates over time in expectation and SCC increases with carbon stock. This effect is similar to the one emphasized in recent integrated assessment models with Epstein-Zin risk preferences, e.g., Cai and Lontzek (2019). Second, the quantiles of time-t conditional distribution of SCC also increase over time (see Panel B). Moreover, the wedge for any pair of SCC quantiles widens in the first twenty years because the expected change (drift) of carbon stock is positive before reaching the stochastic steady state, which takes about twenty years for our counterfactual no-learning case. After the stochastic steady state is reached in about twenty years, the wedge between a pair of SCC quantiles barely changes over time.

In Figure 5, we report the mean and quantiles of SCC projections over fifty years under the planner's first-best solution with learning. First, we note that the SCC projections are

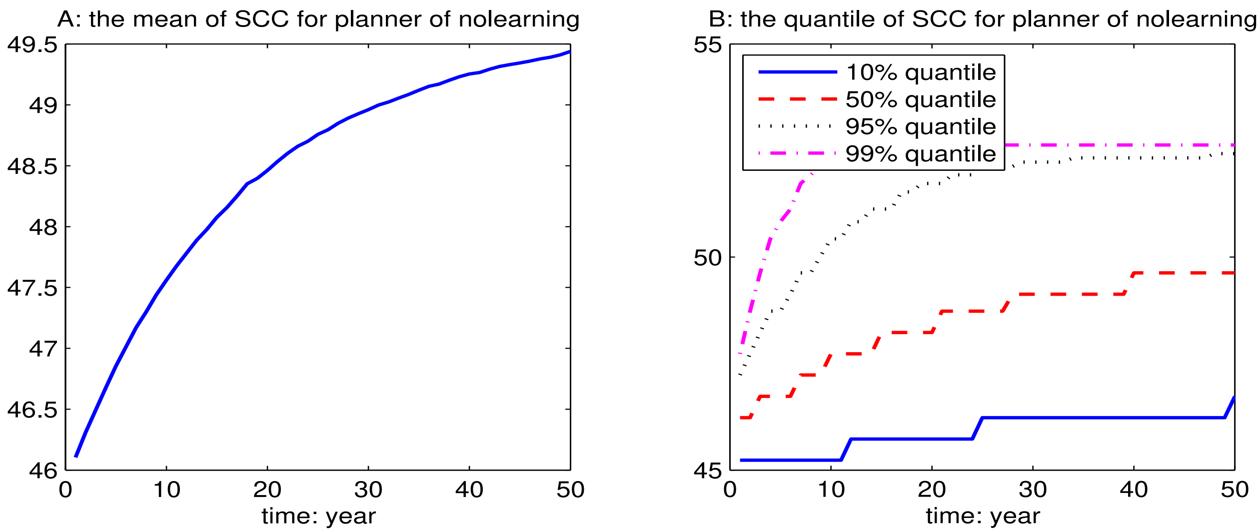

Figure 4: Mean (Panel A) and quantiles (Panel B) of the conditional distribution of the social cost of carbon (SCC) in the counterfactual model without learning. The parameters values are reported in Table 5.

lower in our learning model than in the no-learning counterfactual analyzed earlier. This is consistent with our model's prediction that it is more valuable to adapt when the representative household can learn from disaster arrivals than cannot. As a result, the planner adapts more in our learning model that in the counterfactual no-learning model (see Section 7.4).

Second, unlike in the no-learning counterfactual where the mean of SCC is increasing over time, the mean of SCC projections in our learning model first decreases over time, bottoms out around ten years, and then increases over time (see Panel A of figure 5). Why does the mean of SCC increase over the first ten years? This is because belief is a martingale and SCC is concave in belief.[29] After the household's belief converges to either G or B in about ten years (which occurs with a very high probability), the mean of SCC projections increases over time because upon reaching the stochastic steady state, the carbon stock increases in expectation over time and SCC increases with carbon stock. This latter force operating after the stochastic steady state is similar to that in the no-learning model that we analyzed earlier.

Note that the SCC quantiles in our learning model are also non-monotonic over time tracking the shape of the mean of SCC over time (see Panel B of figure 5). This is because the same two forces (for the first ten years and after reaching the stochastic steady state) are

[29] The martingale convergence theorem implies that belief eventually settles either at state G with probability one or at state B with probability one. Because SCC is concave in belief, using Jensen's inequality, we conclude that the mean of SCC first decreases over time.

at work.

Finally, the wedge between a pair of SCC projections, e.g., at 10% and 99% quantiles, in our learning model is much larger than the wedge for the same pair of SCC projections in the no-learning counterfactual. This is again because in our learning model, belief eventually settles either at state G with probability one or at state B with probability one, in effect increasing the dispersion of SCC projections.

In sum, contrasting Figures 4 and 5, we bring out the key roles of learning and adaptation in our generalized Bayesian model with carbon stock and adaption.

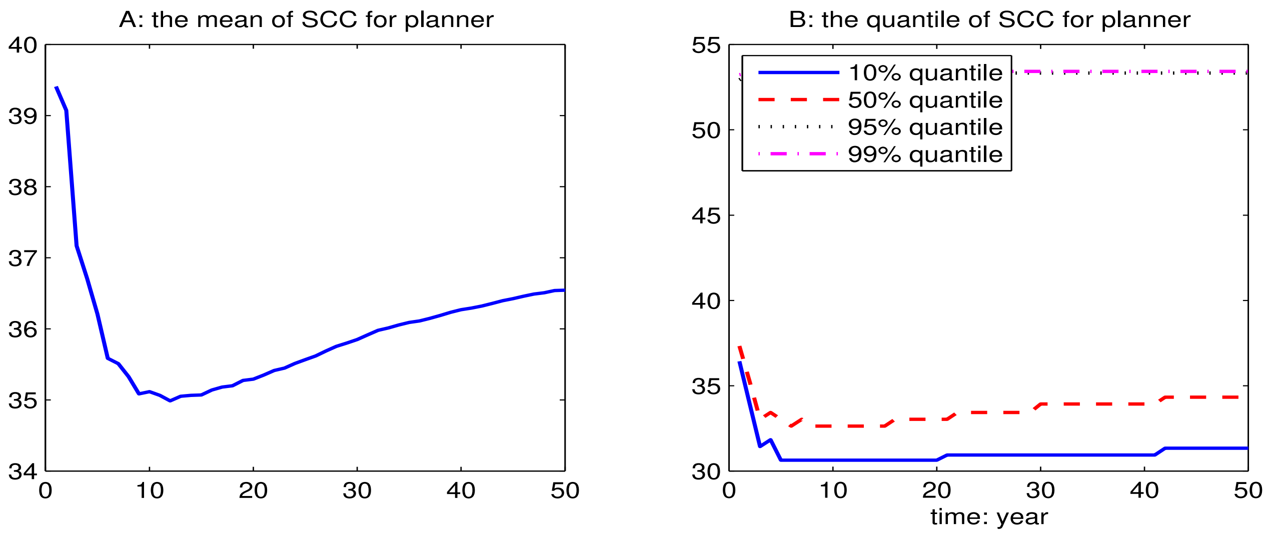

Figure 5: The mean (Panel A) and quantiles (Panel B) of the social cost of carbon (SCC) projections for the generalized learning model with carbon of Section 7.

9 Conclusion

We develop a model where households and firms learn from exogenous natural disaster arrivals about arrival rates and adapt to mitigate potential future damages. Adaptation spending—by curtailing aggregate risk and insuring sustainable growth—is undersupplied relative to the first-best planner's solution in competitive markets due to externalities. The planner's solution can be implemented via a capital tax and subsidy scheme. We apply our model to country-level control of flooding from major tropical cyclones and calibrate the latent learning process using new empirical findings on the response of asset prices to disaster arrivals. Adaptation with learning is higher than under the counterfactual without learning due to uncertainty.

Projections for social costs of carbon over time also depend on the resolution of uncertainty and properties of the underlying learning process.

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Appendices

A Proof for Planner's First-Best Economy in Section 3

A.1 Planner's Resource Allocation

Substituting the value function (23) into the FOC (20) for investment, the FOC (21) for the aggregate disaster distribution adaptation spending, and the FOC (22) for the aggregate disaster exposure adaptation spending, we obtain:

$$b(\pi) = \mathbf{c}(\pi)^{1/(1-\psi)} \left[\rho(1+\phi'(\mathbf{i}(\pi)))\right]^{-\psi/(1-\psi)} \,, \tag{A.1}$$

$$\rho \mathbf{c}(\pi)^{-\psi^{-1}} b(\pi)^{\psi^{-1}-1} = \frac{\lambda(\pi)}{1-\gamma}\left(\frac{b(\pi^{\mathcal{J}})}{b(\pi)}\right)^{1-\gamma} \int_0^1 \left[\frac{\partial \xi(Z;\mathbf{x}^d)}{\partial \mathbf{x}^d}(1-N(\mathbf{x}^e(\pi))(1-Z))^{1-\gamma}\right] dZ \,, \tag{A.2}$$

$$\rho \mathbf{c}(\pi)^{-\psi^{-1}} b(\pi)^{\psi^{-1}-1} = \lambda(\pi)\left[\frac{b\left(\pi^{\mathcal{J}}\right)}{b(\pi)}\right]^{1-\gamma} N'(\mathbf{x}^e(\pi))\mathbb{E}^{\mathbf{x}^d(\pi)}\left[(Z-1)(1-N(\mathbf{x}^e(\pi))(1-Z))^{-\gamma}\right] \,, \tag{A.3}$$

where the post-jump $\pi^{\mathcal{J}}$ is given in (14) as a function of the pre-jump π. Substituting the resource constraint, $\mathbf{c}(\pi) = A - \mathbf{i}(\pi) - \phi(\mathbf{i}(\pi)) - \mathbf{x}^d(\pi) - \mathbf{x}^e(\pi)$, into (A.1), we obtain (25). Substituting (A.1) into (A.3), we obtain (26) and substituting (A.1) into (A.2), we obtain (27). Finally, substituting the value function (23) and the FOC (25) into the HJB equation (18), we obtain the ODE (24).

At $\pi = 1$, we have the following four equations that characterize $b(1)$, $\mathbf{i}(1)$, $\mathbf{x}^d(1)$, and $\mathbf{x}^e(1)$:

$$0 = \frac{\left(\frac{b(1)}{\rho(1+\phi'(\mathbf{i}(1)))}\right)^{1-\psi} - 1}{1-\psi^{-1}}\rho + \mathbf{i}(1) - \delta_K - \frac{\gamma\sigma_K^2}{2} + \frac{\lambda_B\left[\mathbb{E}^{\mathbf{x}^d(1)}((1-N(\mathbf{x}^e(1))(1-Z))^{1-\gamma}) - 1\right]}{1-\gamma} \,, \tag{A.4}$$

$$b(1) = [A - \mathbf{i}(1) - \phi(\mathbf{i}(1)) - \mathbf{x}^d(1) - \mathbf{x}^e(1)]^{1/(1-\psi)} \left[\rho(1+\phi'(\mathbf{i}(1)))\right]^{-\psi/(1-\psi)} \,, \tag{A.5}$$

$$1 = \lambda_B(1+\phi'(\mathbf{i}(1)))N'(\mathbf{x}^e(1))\mathbb{E}^{\mathbf{x}^d(1)}\left[(Z-1)(1-N(\mathbf{x}^e(1))(1-Z))^{-\gamma}\right] \,, \tag{A.6}$$

$$1 = \frac{\lambda_B(1+\phi'(\mathbf{i}(1)))}{1-\gamma}\int_0^1 \left[\frac{\partial \xi(Z;\mathbf{x}^d(1))}{\partial \mathbf{x}^d}(1-N(\mathbf{x}^e(1))(1-Z))^{1-\gamma}\right] dZ \,. \tag{A.7}$$

Solving (A.4)-(A.7) yields $b(1)$, $\mathbf{i}(1)$, $\mathbf{x}^d(1)$, and $\mathbf{x}^e(1)$. Similarly, we obtain the four equations, (28)-(31), for the left boundary, $\pi = 0$. Solving these four equations yields $b(0)$, $\mathbf{i}(0)$, $\mathbf{x}^d(0)$, and $\mathbf{x}^e(0)$. In sum, we now have fully characterized the model solution summarized in Proposition 1.

A.2 Asset Pricing Implications of the Planner's Problem

Duffie and Epstein (1992) show that the SDF $\{\mathbb{M}_t : t \geq 0\}$ implied by the planner's solution is:

$$\mathbb{M}_t = \exp\left[\int_0^t f_V(\mathbf{C}_s, V_s)\, ds\right] f_{\mathbf{C}}(\mathbf{C}_t, V_t) \,. \tag{A.8}$$

Using the FOC for investment (20), the value function (23), and the resource constraint, we obtain:

$$f_{\mathbf{C}}(\mathbf{C}, V) = \frac{1}{1+\phi'(\mathbf{i}(\pi))} b(\pi)^{1-\gamma}\mathbf{K}^{-\gamma} \tag{A.9}$$

and

$$f_V(\mathbf{C},V)=\frac{\rho}{1-\psi^{-1}}\left[\frac{(1-\omega)\mathbf{C}^{1-\psi^{-1}}}{((1-\gamma))^{\omega-1}}V^{-\omega}-(1-\gamma)\right]=-\epsilon(\pi)\,, \tag{A.10}$$

where

$$\epsilon(\pi)=-\frac{\rho(1-\gamma)}{1-\psi^{-1}}\left[\left(\frac{\mathbf{c}(\pi)}{b(\pi)}\right)^{1-\psi^{-1}}\left(\frac{\psi^{-1}-\gamma}{1-\gamma}\right)-1\right]. \tag{A.11}$$

Using the equilibrium relation between $b(\pi)$ and $\mathbf{c}(\pi)$, we simplify (A.11) as:

$$\begin{aligned}\epsilon(\pi)=\rho+\left(\psi^{-1}-\gamma\right)&\left[\mathbf{i}(\pi)-\delta_K-\frac{\gamma\sigma_K^2}{2}+\mu_\pi(\pi)\frac{b'(\pi)}{b(\pi)}\right]\\ +\left(\psi^{-1}-\gamma\right)&\left[\frac{\lambda(\pi)}{1-\gamma}\left(\left(\frac{b\left(\pi^{\mathcal{J}}\right)}{b(\pi)}\right)^{1-\gamma}\mathbb{E}^{\mathbf{x}^d}\left[(1-N(\mathbf{x}^e)(1-Z))^{1-\gamma}\right]-1\right)\right],\end{aligned} \tag{A.12}$$

where the post-jump belief $\pi^{\mathcal{J}}$ given in (14) is a function of the pre-jump belief π. For expected utility where $\psi=1/\gamma$, we have $\epsilon(\pi)=\rho$. Using Ito's Lemma and the optimal allocation, we obtain

$$\begin{aligned}\frac{d\mathbb{M}_t}{\mathbb{M}_{t-}}&=-\epsilon(\pi)dt-\gamma\left[(\mathbf{i}(\pi)-\delta_K)dt+\sigma_K d\mathcal{W}_t^K\right]+\frac{\gamma(\gamma+1)}{2}\sigma_K^2dt+\left((1-\gamma)\frac{b'(\pi)}{b(\pi)}-\frac{\mathbf{i}'(\pi)\phi''(\mathbf{i}(\pi))}{1+\phi'(\mathbf{i}(\pi))}\right)\mu_\pi(\pi)dt\\ &+\left[\frac{1+\phi'\left(\mathbf{i}(\pi)\right)}{1+\phi'\left(\mathbf{i}(\pi^{\mathcal{J}})\right)}\left(\frac{b\left(\pi^{\mathcal{J}}\right)}{b(\pi)}\right)^{1-\gamma}(1-N(\mathbf{x}^e)(1-Z))^{-\gamma}-1\right]d\mathcal{J}_t\,.\end{aligned} \tag{A.13}$$

As the expected percentage change of $\mathbb{M}_t$ equals $-r_t$ per unit of time (Duffie, 2001), we obtain the following expression for the equilibrium interest rate:

$$\begin{aligned}r(\pi)\;&=\rho+\psi^{-1}(\mathbf{i}(\pi)-\delta_K)-\frac{\gamma(\psi^{-1}+1)\sigma_K^2}{2}-\left[(1-\psi^{-1})\frac{b'(\pi)}{b(\pi)}-\frac{\mathbf{i}'(\pi)\phi''(\mathbf{i}(\pi))}{1+\phi'(\mathbf{i}(\pi))}\right]\mu_\pi(\pi)\\ &-\lambda(\pi)\left[\frac{1+\phi'\left(\mathbf{i}(\pi)\right)}{1+\phi'\left(\mathbf{i}(\pi^{\mathcal{J}})\right)}\left(\frac{b\left(\pi^{\mathcal{J}}\right)}{b(\pi)}\right)^{1-\gamma}\mathbb{E}^{\mathbf{x}^d}((1-N(\mathbf{x}^e)(1-Z))^{-\gamma})-1\right]\\ &-\lambda(\pi)\left[\frac{\psi^{-1}-\gamma}{1-\gamma}\left(1-\left(\frac{b\left(\pi^{\mathcal{J}}\right)}{b(\pi)}\right)^{1-\gamma}\mathbb{E}^{\mathbf{x}^d}((1-N(\mathbf{x}^e)(1-Z))^{1-\gamma})\right)\right].\end{aligned} \tag{A.14}$$

Since $\mathbf{D}_t=\mathbf{C}_t$ and $\mathbb{M}_{t-}\mathbf{D}_{t-}dt+d(\mathbb{M}_t\mathbf{Q}_t)$ is a martingale under the physical measure (Duffie, 2001), Applying Ito's Lemma to $\mathbb{M}_{t-}\mathbf{D}_{t-}dt+d(\mathbb{M}_t\mathbf{Q}_t)$ and setting its drift to zero, we obtain

$$\begin{aligned}\frac{\mathbf{c}(\pi)}{\mathbf{q}(\pi)}&=\rho-(1-\psi^{-1})\left[\mathbf{i}(\pi)-\delta_K-\frac{\gamma\sigma_K^2}{2}+\mu_\pi(\pi)\frac{b'(\pi)}{b(\pi)}\right]\\ &+\lambda(\pi)\frac{1-\psi^{-1}}{1-\gamma}\left[1-\left(\frac{b\left(\pi^{\mathcal{J}}\right)}{b(\pi)}\right)^{1-\gamma}\mathbb{E}^{\mathbf{x}^d}\left[1-N(\mathbf{x}^e)(1-Z)\right]^{1-\gamma}\right].\end{aligned} \tag{A.15}$$

We obtain the aggregate Tobin's average $\mathbf{q}$ from (A.15). For the special case with $\psi=1$ and any risk aversion $\gamma>0$, the dividend yield (and equivalently the consumption-wealth ratio) is $\mathbf{c}(\pi)/\mathbf{q}(\pi)=\rho$.

B Proof for Market Equilibrium Solution in Section 4

B.1 Firm Value Maximization

First, using Ito's Lemma, we obtain the following dynamics for $Q_t = Q(K_t, \pi_t)$:

$$\begin{aligned} dQ_t &= \left((I - \delta_K K) Q_K + \frac{1}{2}\sigma_K^2 K^2 Q_{KK} + \mu_\pi(\pi) Q_\pi \right) dt + \sigma_K K Q_K d\mathcal{W}_t^K \\ &\quad + \left(Q((1 - N(x^e)(1 - Z))K, \pi^{\mathcal{J}}) - Q(K, \pi) \right) d\mathcal{J}_t \,. \end{aligned} \tag{B.16}$$

No arbitrage implies that the drift of $\mathbb{M}_{t-}(AK_{t-} - I_{t-} - \Phi(I_{t-}, K_{t-}) - X_{t-}^e - X_{t-}^d)dt + d\left(\mathbb{M}_t Q_t\right)$ is zero under the physical measure (Duffie, 2001). Applying Ito's Lemma to this martingale, we obtain

$$\begin{aligned} 0 = \max_{I, x^e, x^d} \ & \mathbb{M}(AK - I - \Phi(I, K) - x^e K - x^d K) + \mathbb{M}\left((I - \delta_K K) Q_K + \frac{1}{2}\sigma_K^2 K^2 Q_{KK} + \mu_\pi(\pi) Q_\pi \right) \\ & + Q\left[-r(\pi) - \lambda(\pi)\left(\mathbb{E}^{\mathbf{x}^d}(\eta(\pi; Z, \mathbf{x}^e)) - 1 \right) \right] \mathbb{M} - \mathbb{M}\gamma\sigma_K^2 K Q_K \\ & + \lambda(\pi)\mathbb{E}^{\mathbf{x}^d}\left[\eta(\pi; Z, \mathbf{x}^e) Q((1 - N(x^e)(1 - Z))K, \pi^{\mathcal{J}}) - Q(K, \pi) \right] \mathbb{M} \,. \end{aligned} \tag{B.17}$$

And then by using the homogeneity property $Q(K, \pi) = q(\pi)K$, we obtain the simplified HJB equation (34). Simplifying the FOC for the exposure mitigation spending implied by (B.17), we obtain (35). Similarly, simplifying the investment FOC implied by (B.17), we obtain (36).

B.2 Household's Optimization Problem

We conjecture and verify that the cum-dividend return of the aggregate asset market is given by

$$\frac{d\mathbf{Q}_t + \mathbf{D}_{t-} dt}{\mathbf{Q}_{t-}} = \mu_{\mathbf{Q}}(\pi_{t-})dt + \sigma_K d\mathcal{W}_t^K + \left(\frac{\mathbf{Q}_t^{\mathcal{J}}}{\mathbf{Q}_{t-}} - 1 \right) d\mathcal{J}_t \,, \tag{B.18}$$

where $\mu_{\mathbf{Q}}(\pi)$ is the expected cum-dividend return (leaving aside the jump effect), defined in (17), to be determined in equilibrium. In (B.18), the diffusion volatility in equilibrium equals σ_K, the same parameter for the capital accumulation process given in (2).

The representative household accumulates wealth as:[30]

$$dW_t = r(\pi_{t-}) W_{t-} dt + (\mu_{\mathbf{Q}}(\pi_{t-}) - r)\Gamma_{t-} dt + \sigma_K \Gamma_{t-} d\mathcal{W}_t^K - C_{t-} dt + \left(\frac{\mathbf{Q}_t^{\mathcal{J}}}{\mathbf{Q}_{t-}} - 1 \right) \Gamma_{t-} d\mathcal{J}_t \,. \tag{B.19}$$

By using the W process given in (B.19), we obtain the HJB equation (38) for the household's value function. The FOCs for consumption C and the market portfolio allocation Γ are given by

$$f_C(C, J) = J_W(W, \pi) \tag{B.20}$$

$$\sigma_K^2 \Gamma J_{WW}(W, \pi) = -(\mu_{\mathbf{Q}}(\pi) - r(\pi)) J_W(W, \pi) + \lambda(\pi)\mathbb{E}^{\mathbf{x}^d}\left[\left(1 - \frac{\mathbf{Q}^{\mathcal{J}}}{\mathbf{Q}} \right) J_W\left(W^{\mathcal{J}}, \pi^{\mathcal{J}} \right) \right] . \tag{B.21}$$

Subsituting (37) into (B.20), we obtain the optimal consumption rule given by (41). Simplifying the FOC for Γ given by (B.21), we obtain (42).

[30]The first four terms in (B.19) are standard as in the classic portfolio-choice problem with no insurance or disasters (Merton, 1971). The last term is the loss of the household's wealth from her portfolio's exposure to the market portfolio. (We leave out the disaster insurance demand as they net out to zero in equilibrium and do not change the equilibrium analysis.) Pindyck and Wang (2013) provide a detailed description of their dynamically complete markets setting (with various diffusion and stage-contingent actuarially fair jump hedging contracts.). Our dynamically complete markets setting builds on Pindyck and Wang (2013).

B.3 Market Equilibrium

First, the firm's (scaled) disaster exposure adaptation spending is positive and equals the aggregate exposure mitigation spending: $x^e = \mathbf{x}^e > 0$. Second, in equilibrium, the household invests all wealth in the market portfolio and holds no risk-free asset, $\Gamma = W$ and $W = \mathbf{Q}$. Simplifying the FOCs, (41) and (42), and using the value function (37), we obtain:

$$c(\pi) = \rho^{\psi} u(\pi)^{1-\psi} \mathbf{q}(\pi)\,, \tag{B.22}$$

$$\mu_{\mathbf{Q}}(\pi) = r(\pi) + \gamma\sigma_K^2 + \lambda(\pi)\left[\mathbb{E}^{\mathbf{x}^d}(\eta(\pi; Z, \mathbf{x}^e)) - \frac{\mathbf{q}(\pi^{\mathcal{J}})}{\mathbf{q}(\pi)}\mathbb{E}^{\mathbf{x}^d}((1 - N(\mathbf{x}^e)(1-Z))\eta(\pi; Z, \mathbf{x}^e))\right]. \tag{B.23}$$

Then substituting (37) into the HJB equation (38), we obtain (45). Using these equilibrium conditions, we simplify the HJB equation (38) as follows:

$$0 = \frac{1}{1-\psi^{-1}}\left(\frac{\mathbf{c}(\pi)}{\mathbf{q}(\pi)} - \rho\right) + \left(\mu_{\mathbf{Q}}(\pi) - \frac{\mathbf{c}(\pi)}{\mathbf{q}(\pi)}\right) - \frac{\gamma\sigma_K^2}{2} + \mu_\pi(\pi)\frac{\mathbf{u}'(\pi)}{\mathbf{u}(\pi)} + \frac{\lambda(\pi)}{1-\gamma}\left[\frac{\mathbf{q}(\pi^{\mathcal{J}})}{\mathbf{q}(\pi)}\mathbb{E}^{\mathbf{x}^d}((1 - N(\mathbf{x}^e)(1-Z)\eta(\pi; Z, \mathbf{x}^e))) - 1\right]. \tag{B.24}$$

Third, by substituting $\mathbf{c}(\pi) = A - \mathbf{i}(\pi) - \phi(\mathbf{i}(\pi)) - \mathbf{x}^e$ into (34), we obtain

$$0 = \frac{\mathbf{c}(\pi)}{\mathbf{q}(\pi)} - r(\pi) + \mathbf{i}(\pi) - \delta_K + \mu_\pi(\pi)\frac{\mathbf{q}'(\pi)}{\mathbf{q}(\pi)} - \gamma\sigma_K^2 - \lambda(\pi)\left[\mathbb{E}^{\mathbf{x}^d}(\eta(\pi; Z, \mathbf{x}^e)) - \frac{\mathbf{q}(\pi^{\mathcal{J}})}{\mathbf{q}(\pi)}\mathbb{E}^{\mathbf{x}^d}((1 - N(\mathbf{x}^e)(1-Z))\eta(\pi; Z, \mathbf{x}^e))\right]. \tag{B.25}$$

By using the homogeneity property and comparing (B.18) and (B.16), we obtain

$$\mu_{\mathbf{Q}}(\pi) = \frac{\mathbf{c}(\pi)}{\mathbf{q}(\pi)} + \mathbf{i}(\pi) - \delta_K + \mu_\pi(\pi)\frac{\mathbf{q}'(\pi)}{\mathbf{q}(\pi)}\,. \tag{B.26}$$

Then substituting (B.26) into (B.24), we obtain

$$\frac{\mathbf{c}(\pi)}{\mathbf{q}(\pi)} = \rho - (1-\psi^{-1})\left[\mathbf{i}(\pi) - \delta_K - \frac{\gamma\sigma_K^2}{2} + \mu_\pi(\pi)\left(\frac{\mathbf{u}'(\pi)}{\mathbf{u}(\pi)} + \frac{\mathbf{q}'(\pi)}{\mathbf{q}(\pi)}\right)\right] + \lambda(\pi)\left(\frac{1-\psi^{-1}}{1-\gamma}\right)\left[1 - \frac{\mathbf{q}(\pi^{\mathcal{J}})}{\mathbf{q}(\pi)}\mathbb{E}^{\mathbf{x}^d}((1 - N(\mathbf{x}^e)(1-Z))\eta(\pi; Z, \mathbf{x}^e))\right]. \tag{B.27}$$

Substituting (B.27) into (B.25), we obtain the following expression for the equilibrium risk-free rate:

$$r(\pi) = \rho + \psi^{-1}(\mathbf{i}(\pi) - \delta_K) - \frac{\gamma(\psi^{-1}+1)\sigma_K^2}{2} - \left[(1-\psi^{-1})\left(\frac{u'(\pi)}{u(\pi)} + \frac{\mathbf{q}'(\pi)}{\mathbf{q}(\pi)}\right) - \frac{\mathbf{q}'(\pi)}{\mathbf{q}(\pi)}\right]\mu_\pi(\pi) - \lambda(\pi)\left[\mathbb{E}^{\mathbf{x}^d}(\eta(\pi; Z, \mathbf{x}^e)) - 1\right] - \lambda(\pi)\left[\frac{\psi^{-1}-\gamma}{1-\gamma}\left(1 - \frac{\mathbf{q}(\pi^{\mathcal{J}})}{\mathbf{q}(\pi)}\mathbb{E}^{\mathbf{x}^d}((1 - N(\mathbf{x}^e)(1-Z))\eta(\pi; Z, \mathbf{x}^e))\right)\right]. \tag{B.28}$$

Using (B.18) and (B.23), we obtain the following expression for the market risk premium $rp(\pi)$:

$$rp(\pi) = \mu_{\mathbf{Q}}(\pi) + \lambda(\pi)\left(\frac{\mathbf{Q}^{\mathcal{J}}}{\mathbf{Q}} - 1\right) - r(\pi) = \gamma\sigma_K^2 - \lambda(\pi)\mathbb{E}^{\mathbf{x}^d}\left[(\eta(\pi; Z, \mathbf{x}^e) - 1)\left(\frac{\mathbf{Q}^{\mathcal{J}}}{\mathbf{Q}} - 1\right)\right], \tag{B.29}$$

which implies (57).

In sum, we have derived the equilibrium resource allocations and the asset pricing implications summarized in Proposition 2 and Proposition 4.

Online Appendices

OA Model with Stochastic Disaster Arrival Rate

The disaster arrival rate in our baseline model of Section 2, while unobservable, is constant. In this section, we generalize the baseline model to allow for the unobservable disaster arrival rate to be stochastic.[1] We assume that the disaster arrival rate follows a two-state continuous-time Markov chain taking two possible values, λ_G in state G and $\lambda_B > \lambda_G$ in state B. Let φ_G denote the transition rate from state G to state B and φ_B denote the transition rate from state B to state G. That is, over a small time period Δt, the transition probability from the G state to the B state is $\varphi_G \Delta t$ and similarly the transition probability from the B state to the G state is $\varphi_B \Delta t$. Our baseline unobservable constant λ model of Section 2 is a special case of this model with $\varphi_G = \varphi_B = 0$.

OA.1 Model

As in our baseline model, let π_t denote the conditional probability that the economy is in state B. The belief process $\{\pi_t\}$ evolves as:

$$d\pi_t = \mathbb{E}_{t-}[d\pi_t] + \sigma_\pi(\pi_{t-})\left(d\mathcal{J}_t - \lambda_{t-}dt\right) , \tag{OA.1}$$

where $\sigma_\pi(\pi)$ is given by (13) and $\lambda_{t-} = \lambda_B \pi_{t-} + \lambda_G(1-\pi_{t-})$ is the expected disaster arrival rate at $t-$ given in (11). Note that the second term is a martingale by construction. Since the economy follows a two-state Markov chain, the expected change of belief is given by

$$\mathbb{E}_{t-}[d\pi_t] = (\varphi_G - (\varphi_B + \varphi_G)\pi_{t-})dt\,. \tag{OA.2}$$

We can thus rewrite (OA.1) as follows:

$$d\pi_t = (\varphi_G - (\varphi_B + \varphi_G)\pi_{t-})dt + \sigma_\pi(\pi_{t-})\left(d\mathcal{J}_t - \lambda_{t-}dt\right) . \tag{OA.3}$$

Equation (OA.3) implies that π_t in our generalized model is no longer a martingale. This is in sharp contrast with our baseline model (with constant arrival rate), where belief π_t given in (12) is a martingale. Rewriting the drift term in (OA.3), we see that the expected change of belief π_t in our generalized learning model is given by the difference between $\varphi_G(1-\pi_{t-})$, which is the transition rate out of state G, φ_G, multiplied by $1-\pi_{t-}$, the conditional probability in state G, and $\varphi_B\pi_t$, which is the transition rate out of state B, φ_B, multiplied by π_{t-}, the conditional probability in state B.[2]

We note that the jump martingale term (the second term in (OA.3)) in our generalized model is the same as in the belief updating process (12) for our baseline model. As a result, when a disaster strikes at t, the belief immediately increases from the pre-jump level π_{t-} to $\pi_t = \pi^{\mathcal{J}}$ by $\sigma_\pi(\pi_{t-})$, where $\pi^{\mathcal{J}}$ is given by (14), the same as in our baseline model with unobservable constant arrival rate λ.

[1] Ghaderi, Kilic, and Seo (2022) also develops a Bayesian learning model that builds on Wachter (2013).

[2] As a result, when $\pi_t = 0$ (in the G state for sure), the drift of belief π_t is exactly φ_G, the arrival rate from the G to the B state. Similarly by symmetry, when $\pi_t = 1$ (in the B state for sure), the drift is exactly $-\varphi_B$.

Taking these results together, absent jump arrivals (i.e., $d\mathcal{J}_t = 0$), we obtain the following expression for the rate at which belief changes, $\widehat{\mu}_\pi(\pi_{t-}) = d\pi_t/dt$:

$$\widehat{\mu}_\pi(\pi) = (\varphi_G - (\varphi_B + \varphi_G)\pi) - \pi(1-\pi)(\lambda_B - \lambda_G)\,. \tag{OA.4}$$

Generalizing the unobservable λ from a constant to a stochastic process (two-state Markov chain) does not change the belief updating upon the immediate arrival of a jump. However, belief updating conditional on no jump arrival is different from the baseline case with unobservable constant arrival rate λ.

Next, we calculate the posterior belief π_t at t conditional on no jump arrival over the time interval (s,t), i.e., $dJ_v = 0$ for $s < v \le t$. Using (OA.3) and integrating $\{\pi_v; v \in (s,t)\}$ from s to t conditional on no jump over the interval (s,t), we obtain the following function:

$$\pi_t \;=\; \pi_s - \frac{2(\delta_0\pi_s^2 + \delta_1\pi_s + \delta_2)(e^{-\sqrt{\delta_1^2 - 4\delta_0\delta_2}(t-s)} - 1)}{(\sqrt{\delta_1^2 - 4\delta_0\delta_2} + \delta_1 + 2\delta_0\pi_s)(e^{-\sqrt{\delta_1^2-4\delta_0\delta_2}(t-s)} - 1) + 2\sqrt{\delta_1^2 - 4\delta_0\delta_2}}\,, \tag{OA.5}$$

where $\delta_0 = -(\lambda_G - \lambda_B)$, $\delta_1 = \lambda_G - \lambda_B - (\varphi_G + \varphi_B)$, and $\delta_2 = \varphi_G$. For our baseline model ($\varphi_G = \varphi_B = 0$), π_t in (OA.5) can be simplified to (16).

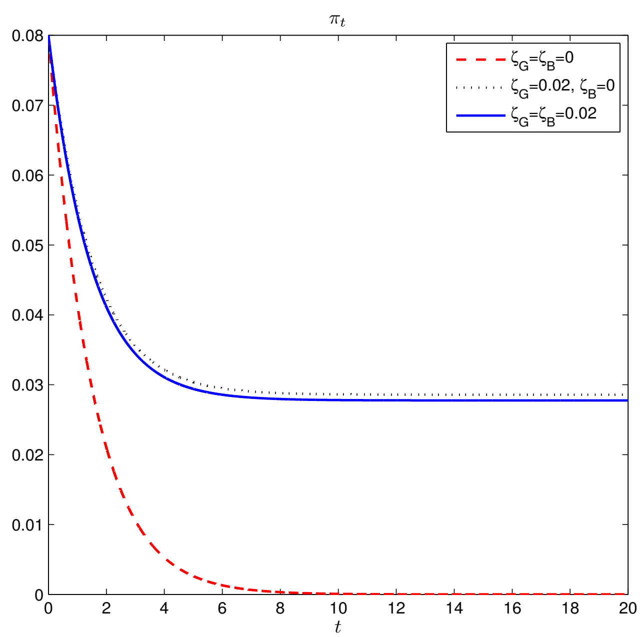

Figure O-1: This figure plots the time series of π_t absent jumps in our generalized model, where the jump arrival rate, λ, is unobservable and follows a two-state Markov chain taking on two possible values ($\lambda_G = 0.1$ and $\lambda_B = 0.8$) with a prior of $\pi_0 = 0.08$ that the current value of λ is λ_B. Our baseline model with constant unobservable λ corresponds to $\varphi_G = \varphi_B = 0$ (the dashed red line).

In Figure O-1, we plot the belief process $\{\pi_t : t \in (0,20)\}$ conditional on no jump arrival, which means $d\mathcal{J}_v = 0$ where $v \in (0,t) = (0,20)$, for three cases: 1.) the stationary case with $\varphi_G = \varphi_B = 2\%$ (the solid blue line); 2.) the case with $\varphi_G = 2\%$ and $\varphi_B = 0$, where the economy is eventually absorbed at the B state, (the dotted black line); and 3.) the baseline constant λ case as $\varphi_G = \varphi_B = 0$ (the dashed red line). The prior for the low value of λ is set at $\pi_0 = 0.08$ for all three cases.

First, for the two cases with stochastic λ, π_t decreases with t even absent jump arrivals. For example, the solid blue line (for the $\varphi_G = \varphi_B = 2\%$ case) shows that π_t slowly decreases to 0.0277 in twenty years absent jump arrivals. For the other case where the B state is absorbing ($\varphi_B = 0$), π_t decreases to 0.0285 at $t = 20$ absent jumps (the dotted black line.) The belief dynamics for these two cases with stochastic λ are similar to the dynamic for our constant unobservable λ model (the dashed red line), which shows that π_t decreases over time to zero and the agent becomes more optimistic (the no-news-is-good-news result), and the only difference is the long-run mean absent jump arrivals. So long as the transition rates φ_G and φ_B are small (which is the practically relevant case), our baseline model (with constant unobservable λ) and the stochastic unobservable λ model generate similar quantitative predictions. For parsimony, we use the constant λ model for our quantitative analysis in the paper.

OA.2 Solution

Using the belief process $\{\pi_t\}$ given in (OA.3), we obtain the following HJB equation for the planner's allocation problem:

$$
\begin{aligned}
0 \;=\; & \max_{\mathbf{C},\mathbf{I},\mathbf{x}^e\,\mathbf{x}^d} f(\mathbf{C},V) + (\mathbf{I}-\delta_K\mathbf{K})V_{\mathbf{K}}(\mathbf{K},\pi) + \widehat{\mu}_\pi(\pi)V_\pi(\mathbf{K},\pi) + \frac{1}{2}\sigma_K^2\mathbf{K}^2V_{\mathbf{KK}}(\mathbf{K},\pi) \\
& +\lambda(\pi)\mathbb{E}^{\mathbf{x}^d}\left[V\left((1-N(\mathbf{x}^e)(1-Z))\mathbf{K},\pi^{\mathcal{J}}\right) - V(\mathbf{K},\pi)\right] \,, \qquad \text{(OA.6)}
\end{aligned}
$$

where $\widehat{\mu}_\pi(\pi)$ is given in (OA.4). The FOCs for aggregate investment $\mathbf{I}$, (scaled) aggregate disaster distribution mitigation spending $\mathbf{x}^d$, and (scaled) aggregate disaster exposure mitigation spending $\mathbf{x}^e$ are the same as those for our baseline model (with constant unobservable λ), which are given in (20), (21), and (22), respectively.

Substituting the value function $V(\mathbf{K},\pi)$ given in (23) and its derivatives into the HJB equation (OA.6), using the three FOCs ((20), (21), and (22)), and simplifying these equations, we obtain the four-equation ODE system for $b(\pi)$, $\mathbf{i}(\pi)$, $\mathbf{x}^d(\pi)$ and $\mathbf{x}^e(\pi)$, given in

$$
\begin{aligned}
0 \;=\; & \frac{\rho}{1-\psi^{-1}}\left[\left[\frac{b(\pi)}{\rho(1+\phi'(\mathbf{i}(\pi)))}\right]^{1-\psi} - 1\right] + \mathbf{i}(\pi) - \delta_K - \frac{\gamma\sigma_K^2}{2} + \widehat{\mu}_\pi(\pi)\frac{b'(\pi)}{b(\pi)} \\
& + \frac{\lambda(\pi)}{1-\gamma}\left[\left(\frac{b\left(\pi^{\mathcal{J}}\right)}{b(\pi)}\right)^{1-\gamma}\mathbb{E}^{\mathbf{x}^d(\pi)}((1-N(\mathbf{x}^e(\pi))(1-Z))^{1-\gamma}) - 1\right]. \qquad \text{(OA.7)}
\end{aligned}
$$

and (25)-(27) for $\pi \in (0,1)$. The key difference between (OA.7) and the ODE (24) for $b(\pi)$ in our baseline model (with constant but unobservable λ) is that the drift of π absent jumps, $\widehat{\mu}_\pi(\pi)$ given in (OA.4), appears in (OA.7) while $\mu_\pi(\pi)$ given in (15) appears in the ODE (24).[3] The other three equations for $\mathbf{i}(\pi)$, $\mathbf{x}^d(\pi)$ and $\mathbf{x}^e(\pi)$ for our stochastic λ model are (25), (26), and (27), the same as those for our baseline model of Section 2.

[3]The wedge $\widehat{\mu}_\pi(\pi) - \mu_\pi(\pi) = (\varphi_G - (\varphi_B + \varphi_G)\pi)$ precisely captures the effect of stochastic transition between the G and B states.

Next, we turn to the boundary conditions at $\pi = 0$ and $\pi = 1$. At $\pi = 0$, we have

$$0 = \frac{\rho}{1-\psi^{-1}}\left[\left[\frac{b(0)}{\rho(1+\phi'(\mathbf{i}(0)))}\right]^{1-\psi} - 1\right] + \mathbf{i}(0) - \delta_K - \frac{\gamma\sigma_K^2}{2} + \frac{\varphi_G b'(0)}{b(0)} + \frac{\lambda_G\left[\mathbb{E}^{\mathbf{x}^d(0)}((1-N(\mathbf{x}^e(0))(1-Z))^{1-\gamma}) - 1\right]}{1-\gamma}. \quad \text{(OA.8)}$$

Compared with the boundary condition (28) for $b(\pi)$ at $\pi = 0$ in our baseline model of Section 2, we have a new term $\frac{\varphi_G b'(0)}{b(0)}$ on the right side of (OA.8). This is because when $\pi_t = 0$, while the state at t is G for sure, it stochastically transitions out of G to B at the rate of φ_G. Our baseline model of Section 2 is a special case with $\varphi_G = 0$. The other three boundary conditions at $\pi = 0$ for $\mathbf{i}(\pi)$, $\mathbf{x}^d(\pi)$ and $\mathbf{x}^e(\pi)$ in our stochastic λ model are (29), (30), and (31), the same as those for our baseline model of Section 2.[4]

Similarly, at $\pi = 1$, we have

$$0 = \frac{\rho}{1-\psi^{-1}}\left[\left[\frac{b(1)}{\rho(1+\phi'(\mathbf{i}(1)))}\right]^{1-\psi} - 1\right] + \mathbf{i}(1) - \delta_K - \frac{\gamma\sigma_K^2}{2} - \frac{\varphi_B b'(1)}{b(1)} + \frac{\lambda_B\left[\mathbb{E}^{\mathbf{x}^d(1)}((1-N(\mathbf{x}^e(1))(1-Z))^{1-\gamma}) - 1\right]}{1-\gamma}, \quad \text{(OA.9)}$$

where the term $-\frac{\varphi_B b'(1)}{b(1)}$ describes the stochastic transition into G from B. All other terms are the same as in (A.4), the corresponding condition for our baseline model of Section 2. The other three boundary conditions at $\pi = 1$ for $\mathbf{i}(\pi)$, $\mathbf{x}^d(\pi)$ and $\mathbf{x}^e(\pi)$ in our stochastic λ model are (A.5), (A.6), and (A.7), the same as those for our baseline model of Section 2.[5]

Next, we summarize the solution for our generalized learning model.

Proposition 6 *The first-best solution for our generalized learning model is given by the value function (23) and the quartet policy rules, $b(\pi)$, $\mathbf{i}(\pi)$, $\mathbf{x}^d(\pi)$, and $\mathbf{x}^e(\pi)$, where $0 \le \pi \le 1$, via the four-equation ODE system ((OA.7), (25), (26), and (27)) with the four conditions ((OA.8), (29), (30), and (31)) for $\pi = 0$, and ((OA.9), (A.5), (A.6), and (A.7)) for $\pi = 1$.*

OA.3 Quantitative Analysis

Next, we analyze the solutions for our generalized model with stochastic unobservable λ. For the stochastic λ model, we set both the transition rate from state G to B (φ_G) and that from state B to G (φ_B) to 2%, i.e., $\varphi_G = \varphi_B = 1/50 = 2\%$, which imply an average duration of 50 years for both G and B states. In the long run, the economy is in either state G or B with equal (50%) probability.

To ease exposition and facilitate comparison with our baseline (constant unobservable λ) model of Section 2, we use the same values for all the other parameters as in our baseline model.

[4] Note that when $\pi = 0$, we also have $\pi^{\mathcal{J}} = 0$. This is why the last term in (OA.8) does not involve $b(\cdot)$ while the last term in (OA.7) for $\pi \in (0,1)$ does.

[5] As for the $\pi = 0$ case, when $\pi = 1$, we also have $\pi^{\mathcal{J}} = 1$. This is why the last term in (OA.9) does not involve $b(\cdot)$ while the last term in (OA.7) for $\pi \in (0,1)$ does.

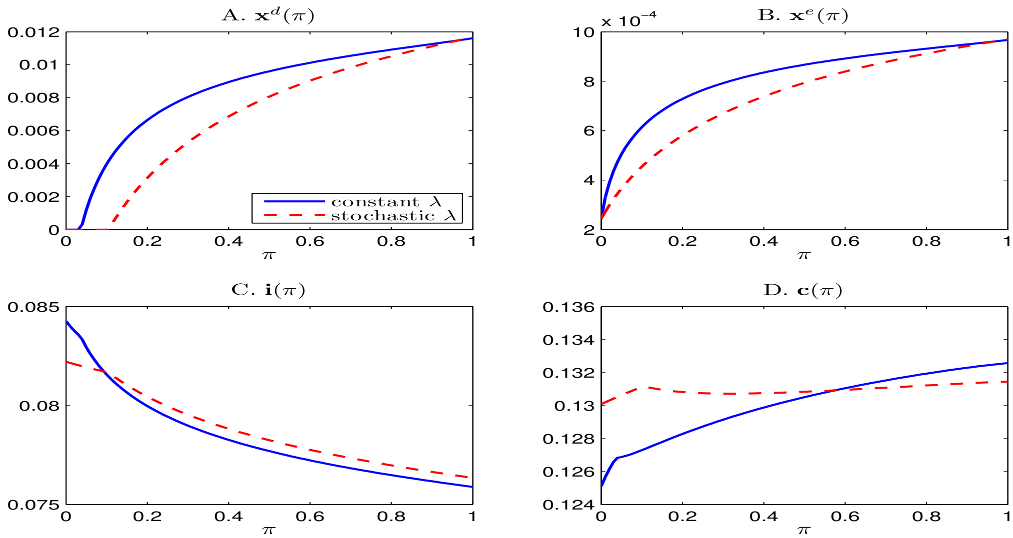

Figure O-2: This figure compares two learning models: the constant λ and the stochastic λ models. The transition rates are $\varphi_G = \varphi_B = 0.02$ for the stochastic λ model (solid blue lines). The transition rates are $\varphi_G = \varphi_B = 0$ for our baseline (constant λ) model (dashed red lines).

In Figure O-2, we plot (scaled) public mitigation $\mathbf{x}^d(\pi)$ (Panel A), (scaled) private mitigation $\mathbf{x}^e(\pi)$ (Panel B), investment-capital ratio $\mathbf{i}(\pi)$ (Panel C), and consumption-capital $\mathbf{c}(\pi)$ (Panel D) as functions of belief π for the planner's first-best solutions: the solid blue lines are for the baseline constant λ model and the dashed red lines are for the stochastic λ model.

Panels A and B show that for both public mitigation $\mathbf{x}^d(\pi)$ and private exposure mitigation $\mathbf{x}^e(\pi)$ are significantly lower for the stochastic λ model, and this is intuitive because the agent is exposure to less uncertainty about the belief due the mean reversion of π in the stochastic λ model, which induces less mitigation motivation. Quantitatively, the differences for investment and consumption are of very small (second- and third-order effects, as we can see from the scale for the vertical axes in Panels C and D.) This is because the transition of λ occurs once every fifty years on average.

Note that investment and consumption are even flatter (less responsive to changes of belief) in the stochastic λ model than in the constant λ model. Figure O-3 corroborates the belief mean reversion effect on welfare, growth, and valuation by showing that the welfare measure, the WTP $\zeta_p(\pi)$ (Panel A), the expected growth rate $\mathbf{g}(\pi)$ (Panel C), Tobin's average q, and the risk-free rate $r(\pi)$ are all smoother (flatter) as functions of π in the stochastic λ model than in the constant λ model.

The intuition is as follows. As belief mean reversion in the stochastic λ model, the agent is less optimistic in the low-π state but also less pessimistic in the high-π state, in the stochastic λ model, i.e., compared with the constant λ model. As a result, the planner reduces both consumption and investment in response to changes of belief (so that the planner better smoothes investment/consumption across states and over time.)

In sum, our analysis shows that for plausible values of slow belief mean reversion, the quantitative results of our learning model (with stochastic λ) are similar to those of our learning model (with constant λ). And we confirm the intuition that belief mean reversion reduces the impact of learning on welfare, valuation and policy rules.

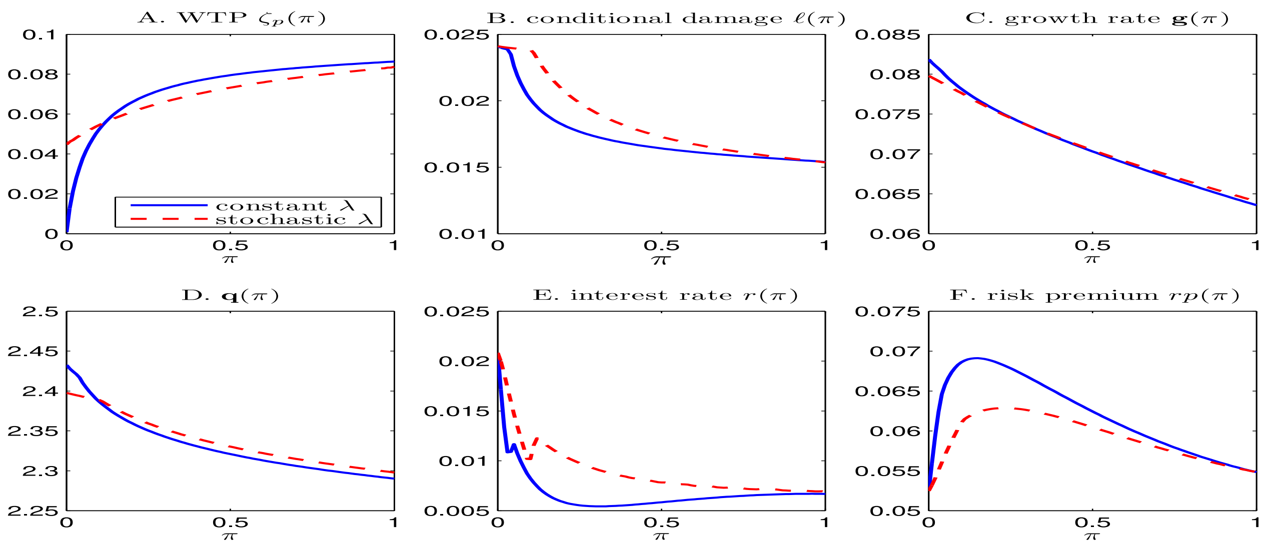

Figure O-3: This figure compares two learning models: the constant λ and the stochastic λ models. The transition rates are $\varphi_G = \varphi_B = 0.02$ for the stochastic λ model (solid blue lines). The transition rates are $\varphi_G = \varphi_B = 0$ for our baseline (constant λ) model (dashed red lines).

OB External Habit Model

We now solve the model with external habit (Campbell-Cochrane) preferences of Section 7.7 and provide a quantitative analysis.[6]

OB.1 Model

The representative agent has a non-expected utility over consumption $\{C_t; t \geq 0\}$ relative to a stochastic habit process $\{\mathcal{H}_t; t \geq 0\}$ (Campbell and Cochrane, 1999) given by:

$$\mathbb{E}\left(\int_0^{\infty} \rho e^{-\rho t} U(C_t, \mathcal{H}_t) dt\right) , \tag{OA.10}$$

where $\rho > 0$ is the time rate of preference, $U(C, \mathcal{H}) = \frac{(C-\mathcal{H})^{1-\gamma}}{1-\gamma}$, and $\gamma > 0$ is a curvature parameter. It is convenient to work with S_t, the surplus consumption ratio at t defined as

$$S_t = \frac{C_t - \mathcal{H}_t}{C_t} . \tag{OA.11}$$

Let s_t be its natural logarithm: $s_t = \ln(S_t)$. As in Campbell and Cochrane (1999) and this literature, we assume that s_t follows a mean-reverting process with stochastic volatility:

$$ds_t = (1 - \kappa_s)(\overline{s} - s_t)dt + \delta(s_t)\sigma_K d\mathcal{W}_t^K , \tag{OA.12}$$

where $\overline{s} > 0$ is the steady-state value of s_t and κ_s measures the degree of persistence.[7] The function $\delta(s_t)$ in (OA.12) is the same sensitivity function as the one in Campbell and Cochrane

[6] An alternative to the external habit model analyzed in this section is to specify an internal habit formation model as in Jermann (1998). Due to space constraints, we leave the internal habit formation model out.

[7] We write $1 - \kappa_s$ as the rate of mean reversion as in Campbell and Cochrane (2015). The higher the value of κ_s, the more persistent the s_t process. The $\kappa_s = 1$ special case corresponds to a unit-root process.

(1999) and stated in Section OB of the Online Appendix. The production side of the economy and the learning model are the same as in our baseline model of Section 2.

Planner's solution. The (log) surplus consumption ratio $\{s_t; t \geq 0\}$ acting as the exogenous preferences shock is the new state variable. Let $V(\mathbf{K}, \pi, s)$ denote the household's value function. The following HJB equation characterizes the planner's optimal resource allocation:

$$\rho V = \max_{\mathbf{C}, \mathbf{I}, \mathbf{x}^e \mathbf{x}^d} \quad \rho \frac{(\mathbf{C}e^s)^{1-\gamma}}{1-\gamma} + (\mathbf{I} - \delta_K \mathbf{K}) V_{\mathbf{K}} + \mu_\pi(\pi) V_\pi + (1-\kappa_s)(\overline{s} - s) V_s + \frac{\sigma_K^2 \mathbf{K}^2 V_{\mathbf{KK}}}{2} + \frac{1}{2}\sigma_K^2 \delta(s)^2 V_{ss}$$
$$+ \sigma_K^2 \delta(s) \mathbf{K} V_{\mathbf{K}s} + \lambda(\pi) \mathbb{E}^{\mathbf{x}^d} \left[V\left((1 - N(\mathbf{x}^e)(1-Z))\mathbf{K}, \pi^{\mathcal{J}}, s \right) - V(\mathbf{K}, \pi, s) \right] . \qquad \text{(OA.13)}$$

Unlike in our baseline model with the Epstein-Zin utility, the agent now not only takes into account the evolution of s (via the drift term involving V_s and the quadratic-variation term involving V_{ss}), but also has incentives to hedge against shocks to the surplus consumption ratio (via the quadratic-covariation term involving $V_{\mathbf{K}s}$).

We show that the value function $V(\mathbf{K}, \pi, s)$ is homogeneous with degree $(1-\gamma)$ in $\mathbf{K}$:

$$V(\mathbf{K}, \pi, s) = \frac{1}{1-\gamma} \left(b(\pi, s) \mathbf{K} \right)^{1-\gamma} , \qquad \text{(OA.14)}$$

where $b(\pi, s)$ is a measure of welfare proportional to the certainty equivalent wealth under optimality. (To ease comparison, we still use b as the function for the welfare measure here but with the understanding that the b function for external habit model depends on both π and s and differs from the b function for our baseline Epstein-Zin model.)

Importantly, unlike the welfare measure $(b(\pi))$ in our baseline planner's model of Section 3, $b(\pi, s)$ in our external habit model depends on not only belief π but also the (log) surplus consumption ratio s. In Online Appendix OB, we provide details summarizing how we obtain the PDE for $b(\pi, s)$ together with optimal policies and boundary conditions. Our external habit model is technically more challenging than our baseline model with Epstein-Zin utility, as the external habit becomes an additional state variable in addition to capital stock and belief.[8]

In (OA.12), $\delta(s_t)$ is the sensitivity function proportional to the conditional volatility of ds_t in response to $d\mathcal{W}_t^K$, which we assume is given by the following square-root function:

$$\delta(s) = \frac{1}{\overline{S}} \sqrt{1 - 2(s - \overline{s})} - 1 , \qquad s \leq s_{\max} \qquad \text{(OA.15)}$$

and $\delta(s) = 0$ for $s > s_{\max}$, where $s_{\max} = \overline{s} + \frac{1-\overline{S}^2}{2}$ and $\overline{S} = e^{\overline{s}}$.[9]

OB.2 Solution

Using the surplus consumption ratio process $\{s_t\}$ given in (OA.12) and the external habit utility function given in (OA.10), we obtain the HJB equation (OA.13) for the planner's

[8]Because of the homogeneity property of the Epstein-Zin utility, only capital stock and belief are state variables after simplifying the model solution.

[9]Additionally, we set $\overline{S} = \sigma_K \sqrt{\frac{\gamma}{1-\kappa_s}}$ as in Campbell and Cochrane (1999).

resource allocation problem. Substituting the value function given in (OA.14) into the HJB equation (OA.13), we obtain

$$
\begin{aligned}
0 \quad = \quad & \max_{\mathbf{c},\mathbf{i},\mathbf{x}^e\,\mathbf{x}^d} \frac{\rho}{1-\gamma}\left[\left(\frac{\mathbf{c}(\pi,s)e^s}{b(\pi,s)}\right)^{1-\gamma}-1\right]+(\mathbf{i}(\pi,s)-\delta_K)+\mu_\pi(\pi)\frac{b_\pi(\pi,s)}{b(\pi,s)}+(1-\kappa_s)(\overline{s}-s)\frac{b_s(\pi,s)}{b(\pi,s)} \\
& -\frac{\gamma\sigma_K^2}{2}+\frac{\sigma_K^2\delta(s)^2}{2}\left(\frac{b_{ss}(\pi,s)}{b(\pi,s)}-\gamma\frac{(b_s(\pi,s))^2}{b(\pi,s)^2}\right)+(1-\gamma)\sigma_K^2\delta(s)\frac{b_s(\pi,s)}{b(\pi,s)} \\
& +\frac{\lambda(\pi)}{1-\gamma}\left[\left(\frac{b\left(\pi^{\mathcal{J}},s\right)}{b(\pi,s)}\right)^{1-\gamma}\mathbb{E}^{\mathbf{x}^d}\left((1-N(\mathbf{x}^e(\pi,s))(1-Z))^{1-\gamma}\right)-1\right]. \qquad \text{(OA.16)}
\end{aligned}
$$

Using the resource constraint $\mathbf{c}=A-\mathbf{i}-\phi(\mathbf{i})-\mathbf{x}^d-\mathbf{x}^e$ to simplify the FOC for investment $\mathbf{i}$, we obtain the ODE system for $b(\pi,s)$, $\mathbf{i}(\pi,s)$, $\mathbf{x}^e(\pi,s)$ and $\mathbf{x}^d(\pi,s)$ in the region where $\pi\in(0,1)$ and $s\in(-\infty,s_{\max})$:

$$
\begin{aligned}
0 = & \frac{\rho}{1-\gamma}\left[\left(\frac{b(\pi,s)e^{-s}}{\rho(1+\phi'(\mathbf{i}(\pi,s)))}\right)^{1-\gamma^{-1}}-1\right]+(\mathbf{i}(\pi,s)-\delta_K)+(1-\kappa_s)(\overline{s}-s)\frac{b_s(\pi,s)}{b(\pi,s)} \\
& +\mu_\pi(\pi)\frac{b_\pi(\pi,s)}{b(\pi,s)}-\frac{\gamma\sigma_K^2}{2}+\frac{\sigma_K^2\delta(s)^2}{2}\left(\frac{b_{ss}(\pi,s)}{b(\pi,s)}-\gamma\frac{(b_s(\pi,s))^2}{b(\pi,s)^2}\right)+(1-\gamma)\sigma_K^2\delta(s)\frac{b_s(\pi,s)}{b(\pi,s)} \\
& +\frac{\lambda(\pi)}{1-\gamma}\left[\left(\frac{b\left(\pi^{\mathcal{J}},s\right)}{b(\pi,s)}\right)^{1-\gamma}\mathbb{E}^{\mathbf{x}^d(\pi,s)}\left((1-N(\mathbf{x}^e(\pi,s))(1-Z))^{1-\gamma}\right)-1\right],
\end{aligned}
$$

(OA.17)

$$b(\pi,s)=\left[A-\mathbf{i}(\pi,s)-\phi(\mathbf{i}(\pi,s))-\mathbf{x}^d(\pi,s)-\mathbf{x}^e(\pi,s)\right]^{\gamma/(\gamma-1)}\left[\rho\mathbf{q}(\pi,s)\right]^{1/(1-\gamma)}e^s, \qquad \text{(OA.18)}$$

$$\frac{1}{\mathbf{q}(\pi,s)}=\lambda(\pi)\left[\frac{b\left(\pi^{\mathcal{J}},s\right)}{b(\pi,s)}\right]^{1-\gamma}N'(\mathbf{x}^e(\pi,s))\mathbb{E}^{\mathbf{x}^d(\pi,s)}\left[(Z-1)(1-N(\mathbf{x}^e(\pi,s))(1-Z))^{-\gamma}\right],$$

(OA.19)

$$\frac{1}{\mathbf{q}(\pi,s)}=\frac{\lambda(\pi)}{1-\gamma}\left[\frac{b\left(\pi^{\mathcal{J}},s\right)}{b(\pi,s)}\right]^{1-\gamma}\int_0^1\left[\frac{\partial\xi(Z;\mathbf{x}^d(\pi,s))}{\partial\mathbf{x}^d}(1-N(\mathbf{x}^e(\pi,s))(1-Z))^{1-\gamma}\right]dZ,$$

(OA.20)

where $\mathbf{q}(\pi,s)$ is given by

$$\mathbf{q}(\pi,s)=1+\phi'(\mathbf{i}(\pi,s)). \qquad \text{(OA.21)}$$

Using the resource constraint $\mathbf{c}=A-\mathbf{i}-\phi(\mathbf{i})-\mathbf{x}^d-\mathbf{x}^e$ to simplify the FOCs for the two types of mitigation spending, $\mathbf{x}^e$ and $\mathbf{x}^d$, we obtain the optimal exposure mitigation and distribution mitigation spending rules, (OA.19) and (OA.20) for $\mathbf{x}^e$ and $\mathbf{x}^d$, respectively.

Since $\pi = 0$ is an absorbing state, we have the following boundary conditions at $\pi = 0$:

$$\begin{aligned}
0 &= \frac{\rho}{1-\gamma}\left[\left(\frac{b(0,s)e^{-s}}{\rho(1+\phi'(\mathbf{i}(0,s)))}\right)^{1-\gamma^{-1}} - 1\right] + (\mathbf{i}(0,s) - \delta_K) + (1-\kappa_s)(\bar{s}-s)\frac{b_s(0,s)}{b(0,s)} \\
&\quad -\frac{\gamma\sigma_K^2}{2} + \frac{\sigma_K^2\delta(s)^2}{2}\left(\frac{b_{ss}(0,s)}{b(0,s)} - \gamma\frac{(b_s(0,s))^2}{b(0,s)^2}\right) + (1-\gamma)\sigma_K^2\delta(s)\frac{b_s(0,s)}{b(0,s)} \\
&\quad +\frac{\lambda_G}{1-\gamma}\left[\mathbb{E}^{\mathbf{x}^d(\pi,s)}((1-N(\mathbf{x}^e(0,s))(1-Z))^{1-\gamma}) - 1\right], \qquad \text{(OA.22)} \\
b(0,s) &= [A - \mathbf{i}(0,s) - \phi(\mathbf{i}(0,s)) - \mathbf{x}^d(0,s) - \mathbf{x}^e(0,s)]^{\gamma/(\gamma-1)}\left[\rho\mathbf{q}(0,s)\right]^{1/(1-\gamma)} e^s, \qquad \text{(OA.23)} \\
\frac{1}{\mathbf{q}(0,s)} &= \lambda_G N'(\mathbf{x}^e(0,s))\mathbb{E}^{\mathbf{x}^d(\pi,s)}\left[(Z-1)(1-N(\mathbf{x}^e(0,s))(1-Z))^{-\gamma}\right], \qquad \text{(OA.24)} \\
\frac{1}{\mathbf{q}(0,s)} &= \frac{\lambda_G}{1-\gamma}\int_0^1 \left[\frac{\partial\xi(Z;\mathbf{x}^d(0,s))}{\partial\mathbf{x}^d}(1-N(\mathbf{x}^e(0,s))(1-Z))^{1-\gamma}\right] dZ, \qquad \text{(OA.25)}
\end{aligned}$$

where $\mathbf{q}(0,s) = 1 + \phi'(\mathbf{i}(0,s))$.

Similarly, at the $\pi = 1$ absorbing state, we have the following boundary conditions:

$$\begin{aligned}
0 &= \frac{\rho}{1-\gamma}\left[\left(\frac{b(1,s)e^{-s}}{\rho(1+\phi'(\mathbf{i}(1,s)))}\right)^{1-\gamma^{-1}} - 1\right] + (\mathbf{i}(1,s) - \delta_K) + (1-\kappa_s)(\bar{s}-s)\frac{b_s(1,s)}{b(1,s)} \\
&\quad -\frac{\gamma\sigma_K^2}{2} + \frac{\sigma_K^2\delta(s)^2}{2}\left(\frac{b_{ss}(1,s)}{b(1,s)} - \gamma\frac{(b_s(1,s))^2}{b(1,s)^2}\right) + (1-\gamma)\sigma_K^2\delta(s)\frac{b_s(1,s)}{b(1,s)} \\
&\quad +\frac{\lambda_B}{1-\gamma}\left[\mathbb{E}^{\mathbf{x}^d(1,s)}((1-N(\mathbf{x}^e(1,s))(1-Z))^{1-\gamma}) - 1\right], \qquad \text{(OA.26)} \\
b(1,s) &= [A - \mathbf{i}(1,s) - \phi(\mathbf{i}(1,s)) - \mathbf{x}^d(1,s) - \mathbf{x}^e(1,s)]^{\gamma/(\gamma-1)}\left[\rho\mathbf{q}(1,s)\right]^{1/(1-\gamma)} e^s, \qquad \text{(OA.27)} \\
\frac{1}{\mathbf{q}(1,s)} &= \lambda_B N'(\mathbf{x}^e(1,s))\mathbb{E}^{\mathbf{x}^d(1,s)}\left[(Z-1)(1-N(\mathbf{x}^e(1,s))(1-Z))^{-\gamma}\right], \qquad \text{(OA.28)} \\
\frac{1}{\mathbf{q}(1,s)} &= \frac{\lambda_B}{1-\gamma}\int_0^1 \left[\frac{\partial\xi(Z;\mathbf{x}^d(1,s))}{\partial\mathbf{x}^d}(1-N(\mathbf{x}^e(1,s))(1-Z))^{1-\gamma}\right] dZ, \qquad \text{(OA.29)}
\end{aligned}$$

where $\mathbf{q}(1,s) = 1 + \phi'(\mathbf{i}(1,s))$.

At $s = s_{\max}$, we have the following boundary condition:

$$\begin{aligned}
0 = &\frac{\rho}{1-\gamma}\left[\left(\frac{b(\pi,s_{\max})e^{-s_{\max}}}{\rho(1+\phi'(\mathbf{i}(\pi,s_{\max})))}\right)^{1-\gamma^{-1}} - 1\right] + (\mathbf{i}(\pi,s_{\max}) - \delta_K) + (1-\kappa_s)(\bar{s}-s_{\max})\frac{b_s(\pi,s_{\max})}{b(\pi,s_{\max})} \\
&- \frac{\gamma\sigma_K^2}{2} + \mu_\pi(\pi)\frac{b_\pi(\pi,s_{\max})}{b(\pi,s_{\max})} \\
&+ \frac{\lambda(\pi)}{1-\gamma}\left[\left(\frac{b\left(\pi^{\mathcal{J}},s_{\max}\right)}{b(\pi,s_{\max})}\right)^{1-\gamma}\mathbb{E}^{\mathbf{x}^d(\pi,s_{\max})}((1-N(\mathbf{x}^e(\pi,s_{\max}))(1-Z))^{1-\gamma}) - 1\right]. \qquad \text{(OA.30)}
\end{aligned}$$

Additionally, $\mathbf{i}(\pi,s_{\max})$, $\mathbf{x}^e(\pi,s_{\max})$ and $\mathbf{x}^d(\pi,s_{\max})$, satisfy (OA.18)- (OA.20) at $s = s_{\max}$.[10]

We summarize our model's solution in the following proposition.

[10]Note that as $s \to -\infty$ is not reachable in equilibrium, we can ignore the corresponding boundary conditions for our numerical analysis.

Proposition 7 *The first-best solution for our external habit model is given by the value function (OA.14) and the quartet policy rules, $b(\pi, s)$, $\mathbf{i}(\pi, s)$, $\mathbf{x}^d(\pi, s)$, and $\mathbf{x}^e(\pi, s)$, where $0 \leq \pi \leq 1$ and $-\infty < s \leq s_{\max}$, via the four-equation ODE system (OA.17), (OA.18), (OA.19) and (OA.20), together with the boundary conditions (OA.22)-(OA.25) for $\pi = 0$, (OA.26)-(OA.29) for $\pi = 1$, (OA.30) and (OA.18)-(OA.20) for $s = s_{\max}$.*

Next, we use the equilibrium resource allocation to derive our model's asset pricing implications.

OB.3 Asset Pricing Implications

Using the planner's solution, we can also infer the SDF process for the equilibrium outcome (under optimal taxation which supports the first-best equilibrium outcome) by applying the Ito's Lemma to $\mathbb{M}_t$ given below

$$\mathbb{M}_t = e^{-\rho t} \frac{U_C(C_t, \mathcal{H}_t)}{U_C(C_0, \mathcal{H}_0)} = e^{-\rho t} \left(\frac{C_t S_t}{C_0 S_0} \right)^{-\gamma} . \tag{OA.31}$$

We then use the no-arbitrage restriction for the SDF to obtain the equilibrium risk-free rate, the market price of risk, and the stock market risk premium.

Using (OA.31), we obtain the following expression for the logarithmic SDF, $\ln(\mathbb{M}_t)$:

$$\ln(\mathbb{M}_t) = -\rho t - \gamma \left(\ln(C_t) + \ln(S_t) - \ln(C_0) - \ln(S_0) \right) . \tag{OA.32}$$

Then using Ito's lemma, we obtain

$$\begin{aligned}
\frac{d\mathbb{M}_t}{\mathbb{M}_t} \quad = \quad & -\rho dt - \gamma \left(\mathbf{i}(\pi, s) - \delta_K - \frac{\sigma_K^2}{2} \right) dt + (1 - \kappa_s)(\overline{s} - s_t) \left((1-\gamma) \frac{b_s(\pi, s)}{b(\pi, s)} - \frac{\mathbf{q}_s(\pi, s)}{\mathbf{q}(\pi, s)} - 1 \right) dt \\
& + \mu_\pi(\pi) \left((1-\gamma) \frac{b_\pi(\pi, s)}{b(\pi, s)} - \frac{\mathbf{q}_\pi(\pi, s)}{\mathbf{q}(\pi, s)} \right) dt - \left[(1-\gamma) \frac{b_s(\pi, s)^2}{b(\pi, s)^2} - \frac{\mathbf{q}_s(\pi, s)^2}{\mathbf{q}(\pi, s)^2} \right] \frac{(\sigma_K \delta(s))^2}{2} dt \\
& + \left[(1-\gamma) \frac{b_{ss}(\pi, s)}{b(\pi, s)} - \frac{\mathbf{q}_{ss}(\pi, s)}{\mathbf{q}(\pi, s)} \right] \frac{(\sigma_K \delta(s))^2}{2} dt \\
& + \frac{\sigma_{\mathbb{M}}(\pi, s)^2}{2} dt - \sigma_{\mathbb{M}}(\pi, s) d\mathcal{W}_t^K + \left[\eta(\pi, s; Z, \mathbf{x}^e) - 1 \right] d\mathcal{J}_t ,
\end{aligned} \tag{OA.33}$$

where

$$\eta(\pi, s; Z, \mathbf{x}^e) = \frac{\mathbf{q}(\pi, s)}{\mathbf{q}(\pi^{\mathcal{J}}, s)} \left(\frac{b\left(\pi^{\mathcal{J}}, s\right)}{b(\pi, s)} \right)^{1-\gamma} \left(1 - N(\mathbf{x}^e(\pi, s))(1 - Z)\right)^{-\gamma} , \tag{OA.34}$$

and

$$\sigma_{\mathbb{M}}(\pi, s) = \left[\left(1 + \frac{\mathbf{q}_s(\pi, s)}{\mathbf{q}(\pi, s)} - (1-\gamma) \frac{b_s(\pi, s)}{b(\pi, s)} \right) \delta(s) + \gamma \right] \sigma_K . \tag{OA.35}$$

Using the equilibrium restriction that the drift of $\frac{d\mathbb{M}_t}{\mathbb{M}_t}$ equals $-r_{t-}dt$, we obtain the following expression for the equilibrium risk-free rate:

$$
\begin{aligned}
r(\pi,s) = \rho + \gamma\left(\mathbf{i}(\pi,s) - \delta_K - \frac{\sigma_K^2}{2}\right) - (1-\kappa_s)(\bar{s}-s_t)\left((1-\gamma)\frac{b_s(\pi,s)}{b(\pi,s)} - \frac{\mathbf{q}_s(\pi,s)}{\mathbf{q}(\pi,s)} - 1\right) \\
- \mu_\pi(\pi)\left((1-\gamma)\frac{b_\pi(\pi,s)}{b(\pi,s)} - \frac{\mathbf{q}_\pi(\pi,s)}{\mathbf{q}(\pi,s)}\right) + \left[(1-\gamma)\frac{b_s(\pi,s)^2}{b(\pi,s)^2} - \frac{\mathbf{q}_s(\pi,s)^2}{\mathbf{q}(\pi,s)^2}\right]\frac{(\sigma_K\delta(s))^2}{2} \\
- \left[(1-\gamma)\frac{b_{ss}(\pi,s)}{b(\pi,s)} - \frac{\mathbf{q}_{ss}(\pi,s)}{\mathbf{q}(\pi,s)}\right]\frac{(\sigma_K\delta(s))^2}{2} \\
- \frac{\sigma_{\mathbb{M}}(\pi,s)^2}{2} - \lambda(\pi)\left[\mathbb{E}^{\mathbf{x}^d}(\eta(\pi,s;Z,\mathbf{x}^e)) - 1\right]. \qquad \text{(OA.36)}
\end{aligned}
$$

Using the equilibrium SDF, we may calculate firm value, $Q(K,\pi,s)$ by using

$$
Q(K_t,\pi_t,s_t) = \int_t^\infty \frac{\mathbb{M}_v}{\mathbb{M}_t}(AK_v - I_v - \Phi(I_v,K_v) - X_v^e)dv\,. \qquad \text{(OA.37)}
$$

Applying the Ito's Lemma to firm value $Q(K,\pi,s) = q(\pi,s)K$ and using (OA.33), we obtain the following PDE for $q(\pi,s)$:

$$
\begin{aligned}
r(\pi,s)q(\pi,s) = \max_{i,\,x^e}\; A - i - \phi(i) - x^e + (i - \sigma_{\mathbb{M}}(\pi,s)\sigma_K)q(\pi,s) + \mu_\pi(\pi)q_\pi(\pi,s) \\
+ \left[(1-\kappa_s)(\bar{s}-s) + \delta(s)\sigma_K^2 - \sigma_{\mathbb{M}}(\pi,s)\delta(s)\sigma_K\right]q_s(\pi,s) + \frac{\sigma_K^2\delta(s)^2}{2}q_{ss}(\pi,s) \\
+ \lambda(\pi)\mathbb{E}^{\mathbf{x}^d}\left[\eta(\pi,s;Z,\mathbf{x}^e)\left(q(\pi^{\mathcal{J}},s)(1-N(x^e)(1-Z)) - q(\pi,s)\right)\right]. \qquad \text{(OA.38)}
\end{aligned}
$$

The cum-dividend return dR_t over the period dt is given by

$$
\begin{aligned}
dR_t &= \frac{(AK_{t-} - I_t - \Phi(I_{t-},K_{t-}) - X_{t-}^e)dt}{Q_{t-}} + \frac{dQ_t}{Q_{t-}} \\
&= \frac{A - i_{t-} - \phi(i_{t-}) - x_{t-}^e}{q(\pi_{t-},s_{t-})}dt + \frac{dq(\pi_t,s_t)}{q(\pi_{t-},s_{t-})} + \frac{dK_t}{K_{t-}} + \frac{< dq(\pi_t,s_t), dK_t >}{q(\pi_{t-},s_{t-})K_{t-}} \\
&= \frac{A - i_{t-} - \phi(i_{t-}) - x_{t-}^e + (1-\kappa_s)(\bar{s}-s_{t-})q_s(\pi_{t-},s_{t-}) + \frac{\sigma_K^2\delta(s_{t-})^2}{2}q_{ss}(\pi_{t-},s_{t-})}{q(\pi_{t-},s_{t-})}dt \\
&\quad + \frac{\mu_\pi(\pi_{t-})q_\pi(\pi_{t-},s_{t-})}{q(\pi_{t-},s_{t-})}dt + (i_{t-} - \delta_K)dt + \frac{q_s(\pi_{t-},s_{t-})\delta(s_{t-})}{q(\pi_{t-},s_{t-})}\sigma_K^2 dt + \left[\frac{q_s(\pi_{t-},s_{t-})\delta(s_{t-})}{q(\pi_{t-},s_{t-})} + 1\right]\sigma_K d\mathcal{W}_t^K \\
&\quad + \left[\frac{(1-N(x_{t-}^e)(1-Z))q\left(\pi_t^{\mathcal{J}},s_{t-}\right)}{q(\pi_{t-},s_{t-})} - 1\right]d\mathcal{J}_t \\
&= \left[r(\pi_{t-},s_{t-}) + \sigma_{\mathbb{M}}(\pi_{t-},s_{t-})\left(\sigma_K + \delta(s_{t-})\sigma_K\frac{q_s(\pi_{t-},s_{t-})}{q(\pi_{t-},s_{t-})}\right)\right]dt \\
&\quad -\lambda(\pi)\mathbb{E}^{\mathbf{x}_{t-}^d}\left[\eta(\pi_{t-},s_{t-};Z,\mathbf{x}_{t-}^e)\left(\frac{(1-N(x_{t-}^e)(1-Z))q\left(\pi_t^{\mathcal{J}},s_{t-}\right)}{q(\pi_{t-},s_{t-})} - 1\right)\right]dt \\
&\quad + \left[\frac{q_s(\pi_{t-},s_{t-})\delta(s_{t-})}{q(\pi_{t-},s_{t-})} + 1\right]\sigma_K d\mathcal{W}_t^K + \left[\frac{(1-N(x_{t-}^e)(1-Z))q\left(\pi_t^{\mathcal{J}},s_{t-}\right)}{q(\pi_{t-},s_{t-})} - 1\right]d\mathcal{J}_t\,. \qquad \text{(OA.39)}
\end{aligned}
$$

Finally, using the equilibrium conditions $q(\pi,s) = \mathbf{q}(\pi,s)$ and $x^e(\pi,s) = \mathbf{x}^e(\pi,s)$, we write

$$
\begin{aligned}
\frac{d\mathbf{Q}_t + \mathbf{D}_{t-}dt}{\mathbf{Q}_{t-}} &= \left(\mu_{\mathbf{Q}}(\pi_{t-},s_{t-}) + \lambda(\pi_{t-})\left(\frac{\mathbf{Q}_t^{\mathcal{J}}}{\mathbf{Q}_{t-}} - 1\right)\right)dt + \left[\frac{\mathbf{q}_s(\pi_{t-},s_{t-})\delta(s_{t-})}{\mathbf{q}(\pi_{t-},s_{t-})} + 1\right]\sigma_K d\mathcal{W}_t^K \\
&\quad + \left(\frac{\mathbf{Q}_t^{\mathcal{J}}}{\mathbf{Q}_{t-}} - 1\right)(d\mathcal{J}_t - \lambda(\pi_{t-})dt)\,, \qquad \text{(OA.40)}
\end{aligned}
$$

where

$$\frac{\mathbf{Q}_t^{\mathcal{J}}}{\mathbf{Q}_{t-}} = \frac{(1 - N(\mathbf{x}_{t-}^e)(1-Z))\mathbf{q}\left(\pi_t^{\mathcal{J}}, s_{t-}\right)}{\mathbf{q}(\pi_{t-}, s_{t-})}\,, \tag{OA.41}$$

and

$$\begin{aligned} \mu_{\mathbf{Q}}(\pi_{t-}, s_{t-}) &= r(\pi_{t-}, s_{t-}) + \sigma_{\mathbb{M}}(\pi_{t-}, s_{t-}) \left(1 + \delta(s_{t-})\frac{\mathbf{q}_s(\pi_{t-}, s_{t-})}{\mathbf{q}(\pi_{t-}, s_{t-})}\right)\sigma_K \\ &\quad + \lambda(\pi_{t-})\mathbb{E}^{\mathbf{x}_{t-}^d}\left[\eta(\pi_{t-}, s_{t-}; Z, \mathbf{x}_{t-}^e)\left(1 - \frac{\mathbf{Q}_t^{\mathcal{J}}}{\mathbf{Q}_{t-}}\right)\right]. \end{aligned} \tag{OA.42}$$

The market risk premium is

$$\begin{aligned} rp(\pi_{t-}, s_{t-}) &= \mu_{\mathbf{Q}}(\pi_{t-}, s_{t-}) + \lambda(\pi_{t-})\left(\frac{\mathbf{Q}_t^{\mathcal{J}}}{\mathbf{Q}_{t-}} - 1\right) - r(\pi_{t-}, s_{t-}) \\ &= \sigma_{\mathbb{M}}(\pi_{t-}, s_{t-})\left(1 + \delta(s_{t-})\frac{\mathbf{q}_s(\pi_{t-}, s_{t-})}{\mathbf{q}(\pi_{t-}, s_{t-})}\right)\sigma_K \\ &\quad - \lambda(\pi_{t-})\mathbb{E}^{\mathbf{x}_{t-}^d}\left[\left(\eta(\pi_{t-}, s_{t-}; Z, \mathbf{x}_{t-}^e) - 1\right)\left(\frac{\mathbf{Q}_t^{\mathcal{J}}}{\mathbf{Q}_{t-}} - 1\right)\right]. \end{aligned} \tag{OA.43}$$

Next, we calibrate the model and provide a quantitative analysis.

Table 6: Parameter Values for External Habit Model

Parameters	Symbol	Value
jump arrival rate in State G	λ_G	0.1
power law exponent absent mitigation	β_0	39
mitigation technology parameter	β_1	1800
mitigation technology parameter	ζ	0.4
jump arrival rate in State B	λ_B	0.8
prior of being in State B	π_0	0.08
surplus consumption parameter	κ_s	0.87
time rate of preference	ρ	5%
productivity	A	27%
quadratic adjustment cost parameter	θ	17
coefficient of relative risk aversion	γ	8
capital diffusion volatility	σ_K	8%

All parameter values, whenever applicable, are continuously compounded and annualized.

OB.4 Quantitative Analysis

Calibration. We first calibrate our model with the Campbell-Cochrane external habit model to match the key global warming and macro moments.

The key new parameter for the external habit model is the (log) surplus consumption parameter κ_s. We set the persistence parameter for external habit at $\kappa_s = 0.87$ per annum as in Campbell and Cochrane (1999).

We calibrate β_0, β_1 and ζ in a world with no or low global warming risk, i.e., under the assumption that countries are optimally mitigating cyclone arrivals with belief $\pi_0 = 0.08$. We use the following three moments at the steady state level of the surplus consumption ratio $\overline{S}$:[11] 1.) the optimal public mitigation of 0.1% of the capital stock, $\mathbf{x}^d(0.08) = 0.1\%$; 2.) the optimal private mitigation of 0.04% of the capital stock, $\mathbf{x}^e(0.08) = 0.04\%$; and 3.) a reduction of the expected annual GDP growth rate by 1.3% per annum caused by the arrival of a major cyclone, $N(\mathbf{x}^e)\mathbb{E}^{\mathbf{x}^d}(1 - Z) = 1.3\%$. As in our baseline model with Epstein-Zin utility, we calibrate the adjustment cost parameter θ along with the time rate of preference ρ, risk aversion γ, diffusion volatility σ_K, and productivity A by targeting five key moments for state G. These include the annual (real) risk-free rate of 2.5%, the expected annual stock market risk premium of 7%, the annual stock market return volatility of $\sqrt{0.0206} = 14\%$, the expected growth rate of 4.4%, and Tobin's q of 2.5 (e.g., in line with Eberly, Rebelo and Vincent, 2012), when the prior is $\pi_0 = 0.104$. The resulting parameter values are $\sigma_K = 14\%$, $\theta = 11$, $\gamma = 3$, $A = 15\%$, and $\rho = 4\%$. These parameter values are in line with those used in the literature. Moreover, these calibrated parameter values are close to those in our baseline calibration with Epstein-Zin utility, even though the building blocks of the two models differ significantly.

We report the values for all the twelve parameters in Table 6.

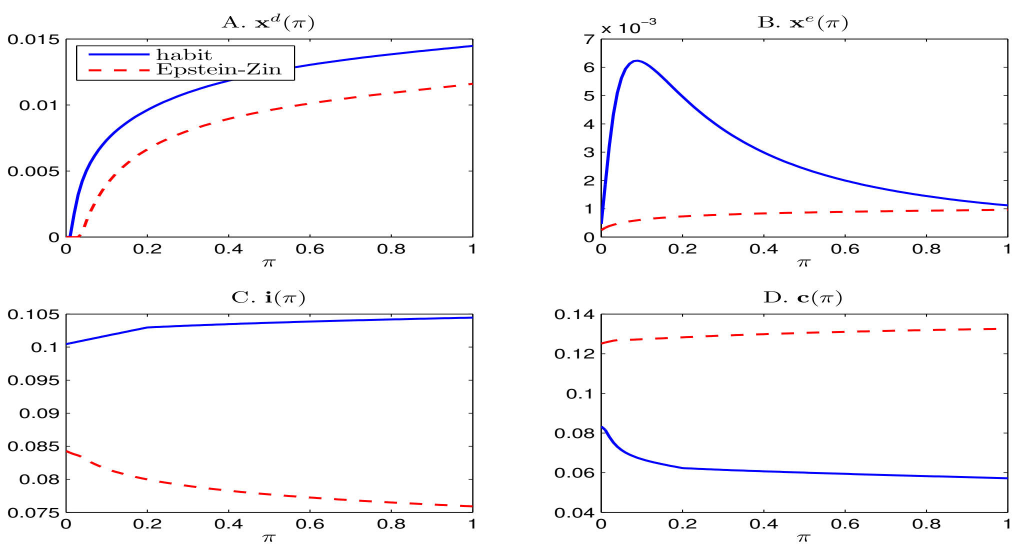

Figure O-4: This figure compares the first-best planner's model solutions for the external habit model (solid blue lines) and the baseline model with Epstein-Zin recursive utility. The parameter values for our baseline (Epstein-Zin) model are summarized in Table 4 and those for the external habit (Campbell-Cochrane) model are summarized in Table 6.

[11] The steady-state value of S is $\overline{S} = 0.63$ and $S_{\max} = 0.85$.

OB.5 Quantitative Results

In Figures O-4 and O-5, we compare the external habit model at the steady state where $S = \overline{S} = 0.63$ with the Epstein-Zin recursive utility model. Recall that both models are recalibrated to match climate change and macro finance moments at the belief level of $\pi = 8\%$. Panel A of Figure O-4 shows that the distribution mitigation $\mathbf{x}^d(\pi)$ policies for the two (different utility) models are quite close to each other. Similarly, Panel B of Figure O-4 shows that the exposure mitigation $\mathbf{x}^e(\pi)$ policies for the two models are also quite close. These two findings suggest that our main results on how changes of belief impact disaster distribution and exposure mitigation spendings are reasonably robust to preference specifications. This is encouraging as our key results are not sensitive to the choices of our preferences.

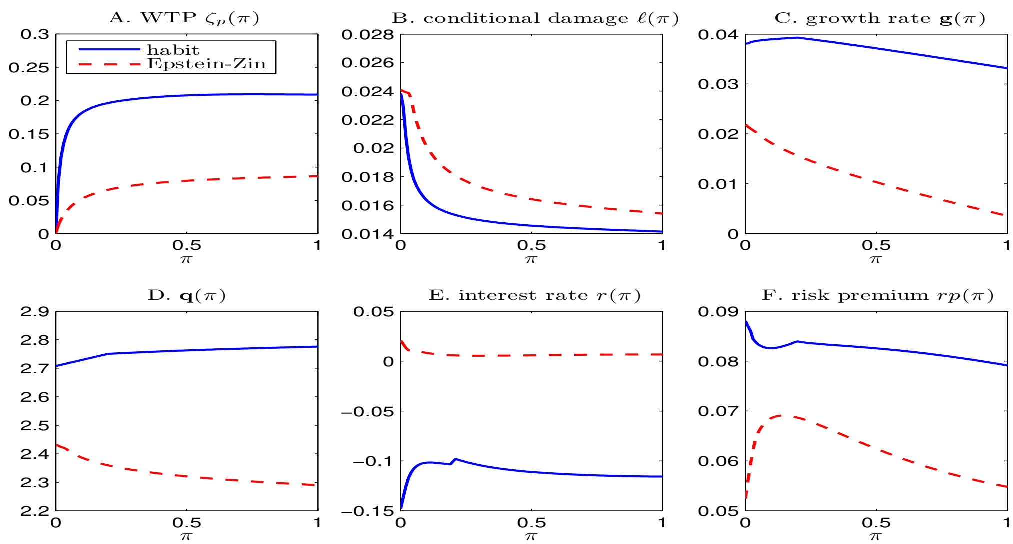

Figure O-5: This figure compares the first-best planner's model solutions for the external habit model (solid blue lines) and the baseline model with Epstein-Zin recursive utility. The parameter values for our baseline (Epstein-Zin) model are summarized in Table 4 and those for the external habit (Campbell-Cochrane) model are summarized in Table 6.

Panel C of Figure O-4 shows that the investment-capital ratio is a bit higher with Epstein-Zin preferences than with external habit at the steady state where $S = \overline{S} = 0.63$. Panel D of Figure O-4 shows that the consumption-capital ratio is a bit lower with Epstein-Zin preferences than with external habit, which is expected as the sum of total mitigation spending, investment, and consumption is the same and equals the productivity A in the two models. Nonetheless, the quantitative differences between the two models in terms of consumption and investment are of the second order. Again, this is good news as our results seem robust to changing preferences assumptions.

It is interesting to note that while $\mathbf{i}(\pi)$ decreases with π for the Epstein-Zin utility model, $\mathbf{i}(\pi)$ increases with π in the external habit model. This difference is caused by the long-run risk force in the Epstein-Zin utility specification, where the EIS $\psi > 1$. To generate the prediction that worsening belief (increasing π) lowers Tobin's q and equivalently investment (as investment increases with Tobin's q), we require $\psi > 1$.

The external habit model differs from the baseline Epstein-Zin utility model in two ways. First, risk aversion is significantly enhanced by and also varies with external habit. Second, the EIS implied by our external habit model also generates a time-varying elasticity of intertemporal substitution (EIS). As risk aversion increases with habit stock, the EIS decreases. This is why our model predicts investment (and hence Tobin's q) increases with belief. Figure O-5 reports the WTP, conditional damage $\ell(\pi)$, the expected growth rate $\mathbf{g}(\pi)$, Tobin's average $q(\pi)$, the risk-free rate $r(\pi)$, and the market risk premium $rp(\pi)$. While there are some differences, we see that these two models, calibrated to match key moments, generate quantitatively similar results.

In sum, these findings are encouraging when it comes to interpreting our key results.

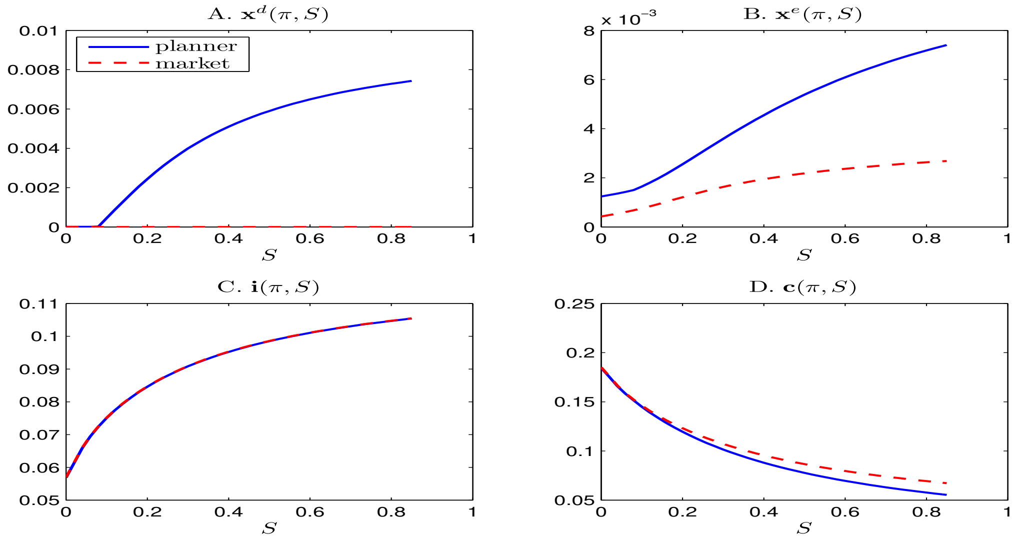

Figure O-6: This figure plots the optimal policies for the planner's first-best economy (solid blue lines) and the market economy (dashed red lines) as functions of surplus consumption ratio S, for the external habit (Campbell-Cochrane) model, where $\pi = 0.104$.

In Figure O-6, we focus on the external habit utility model by comparing two formulations: the planner's first-best economy (solid blue lines) with the market economy solution (dashed red lines). We plot the two mitigation spending, investment, and consumption policies for varying levels of S, for a given belief $\pi = 0.08$.

Panel A of Figure O-6 shows that there is no public mitigation in a competitive market economy for the same externality argument as in our baseline model with Epstein-Zin utility. This Panel also shows that $\mathbf{x}^d$ increases as the surplus consumption ratio increases. Similarly, both the exposure mitigation spending and investment increase with S (Panels B and C). The intuition for these results is as follows. As we increase S, the marginal utility of consumption (and SDF $\mathbb{M}_t$) decrease, which causes $\mathbf{c}$ to decrease with S (see Panel D). Additionally, the marginal value of investment and that of mitigation (for both types) increase, which causes $\mathbf{x}^d$, $\mathbf{x}^e$, and $\mathbf{i}$ to increase with S as shown in Panels A, B, and C).

Finally, we note that the private mitigation spending $\mathbf{x}^e$ is larger for the market economy than for the planner's economy. This is because the marginal benefit of private mitigation is higher in the market economy as there is no public mitigation. In contrast, as the public mitigation spending $\mathbf{x}^d$ is positive and significant under the planner's economy, the additional value of private mitigation spending in the planner's economy is much smaller and hence $\mathbf{x}^e$ is much smaller under the planner's economy than under the market economy (a substitution effect.)

In sum, we show that time-varying risk aversion induced by external habit influences optimal mitigation policies, but the general results that we obtain from our baseline model with Epstein-Zin utility remains valid in our external habit model.

OC Details on Numerical Analysis

In this appendix, we offer a detailed discussion of numerical analysis used in our paper. We proceed as follows. First, we compare the technical differences between our baseline model of Section 2, where the unobservable disaster arrival rate λ is constant, and Pindyck and Wang (2013). Second, we discuss the additional technical complication in our generalized learning model of Section OA, where the disaster arrival rate λ is stochastic and unobservable. Third, we discuss how our model with external habit of Section 7.7 further brings technical complications to our analysis.

OC.1 Comparing Baseline Model with Pindyck and Wang (2013)

Recall that Pindyck and Wang (2013), henceforth PW (2013), is a jump diffusion model with stochastic capital recovery, Epstein-Zin recursive utility, and capital adjustment costs, but features no learning and no mitigation. As a result, the state variable in PW (2013) is capital stock K. Then using the homogeneity property, PW (2013) show that their model solution can be further simplified. To be precise, to solve the PW (2013) model, one first solves a simple nonlinear equation for the optimal constant investment-capital ratio i^* (given by equation (12) in their paper), then calculates a welfare measure (proportional to certainty equivalent wealth) b by substituting i^* into equation (11) in their paper, and finally obtain equilibrium asset pricing implications and conduct willingness-to-pay (WTP) calculations. That is, there is no differential equation or even coupled nonlinear equations involved. Therefore, in terms of numerical solution, PW (2013) is very simple. The PW (2013) model is purposefully designed with parsimony and transparency to highlight the key features of disasters in mind.

The economics and technical details for our baseline model (with constant unobservable disaster arrival rate λ) are inevitably more involved than PW (2013), as we need to incorporate learning and two types of mitigation into PW (2013). As a result, there are two state variables in our baseline model: belief π and capital stock K. After using the homogeneity property, we still need to deal with a numerical problem that has one more dimension than PW (2013). Specifically, this one-dimensional problem involves a system of ordinary differential equations (ODEs). To obtain solutions for four unknown functions, $b(\pi)$, $\mathbf{i}(\pi)$, $\mathbf{x}^e(\pi)$, and $\mathbf{x}^d(\pi)$, we need to solve the ODE system of four inter-connected nonlinear differential equations subject to various boundary conditions. It is worth noting that this ODE system is more difficult to work with than some ODEs that we see in various economics and finance applications, e.g., the ODEs appearing in dynamic contracting, e.g., DeMarzo and Sannikov (2006) and Sannikov (2008), are easier to work with.

It is also worth emphasizing that our model has both jumps and diffusion shocks. Jumps further complicate our numerical analysis. For diffusion models, finite difference methods only require local information, as discretizing a second-order ODE (of diffusion models) calls for analyzing tridiagonal matrix. In contrast, as belief may jump in our model, to solve the model at a given level of π, we also need to take into account the nonlocal effect of jump on value function and policy rules.

As a reference to the technical difficulty of our ODE system, our baseline model's technical difficulty is at least at par with the technical difficulty level of Brunnermeier and Sannikov (2014), which is a diffusion model and hence analyzing tridiagonal matrices is sufficient when solving the coupled ODEs in that paper. In terms of numerical analysis, jumps effectively increase the difficulty of our numerical analysis by increasing the dimension of our problem by "0.5 dimension."

To solve the interconnected ODE system, we also need a set of four interconnected non-linear equations for the boundary $\pi = 0$ and similarly another set of four interconnected nonlinear equations for the boundary $\pi = 1$. Since both boundaries in our baseline model are absorbing, they are relatively easy to work with but are technically still more involved than the full PW (2013) model. This is because for our boundary conditions at $\pi = 0$ and $\pi = 1$, we solve for four unknowns simultaneously while in PW (2013), we only need to sequentially solve one unknown using one nonlinear equation.

OC.2 Additional Difficulties in Stochastic λ Model of Section OA

In our generalized learning model where the arrival rate is stochastic and unobservable, while we still characterize the solution with four interconnected ODEs in the interior belief region where $\pi \in (0, 1)$, the boundary conditions are more complicated posing additional technical and numerical challenges. To be precise, with stochastic transitions between the G and B states, i.e., $\varphi_G > 0$ and/or $\varphi_B > 0$, the two belief boundaries, $\pi = 0$ and $\pi = 1$, are no longer absorbing. Therefore, we can no longer first solve the four nonlinear equation system to pin down the values of welfare b and policy functions ($\mathbf{i}$, $\mathbf{x}^e$, and $\mathbf{x}^d$) at each boundary. To be precise, consider the boundary $\pi = 0$, the term $\frac{\varphi_G b'(0)}{b(0)}$ in the ODE (OA.8) is no longer zero. Indeed, to solve for $b(0)$, we need information about $b'(0)$, which depends on the solution in the interior region $\pi \in (0, 1)$.

In sum, the interconnected ODE system in the $\pi \in (0, 1)$ region and the nonlinear equation systems at the boundaries, $\pi = 0$ and $\pi = 1$, are interdependent, as summarized in Proposition 6. This interdependence between the interior region and the boundary conditions further complicate our numerical analysis. We can no longer solve the ODE by first solving the boundary values and then focus on the ODE system for the interior region as we do for our baseline model with constant unobservable λ.

Despite these challenges, we are able to obtain very high precision for our numerical solution.

OC.3 Additional Difficulties of External Habit Model of Section 7.7

Replacing Epstein-Zin recursive utility with Campbell-Cochrane external habit model invites a new state variable and inevitably we face an optimization problem with three state variables: (log) surplus consumption ratio s being the new state variable in addition to belief π and capital stock K. As we have shown in Section OB, using the homogeneity property, we can simplify our model to a two-dimensional problem, which yields an interconnected partial differential equation (PDE) system.

The interconnected PDE system in the $\pi \in (0,1)$ region and the nonlinear equation systems at the two belief boundaries, $\pi = 0$ and $\pi = 1$, as well as the boundary conditions, (OA.30) and (OA.18)-(OA.20) for $s = s_{max}$ have to be solved jointly. Proposition 7 summarizes the entire PDE system with 4 interconnected PDEs in the interior region with 12 nonlinear (differential) equations for the boundaries. This system is numerically quite challenging.

Moreover, we note that as the boundary $s = s_{\max}$ is not absorbing, the value function $b(\pi, s)$ at $s = s_{\max}$ depends on $b_s(\pi, s_{\max})$ and other equilibrium objects, which have to be solved jointly with the PDEs in the interior region where $s \in (-\infty, s_{\max})$. This further complicates our numerical analysis.

In sum, compared with our Epstein-Zin-utility-based models which require us to solve interconnected ODE system, Campbell-Cochrane external-habit-based model is technically much more challenging, as we have to solve an involved interconnected PDE problem described above.

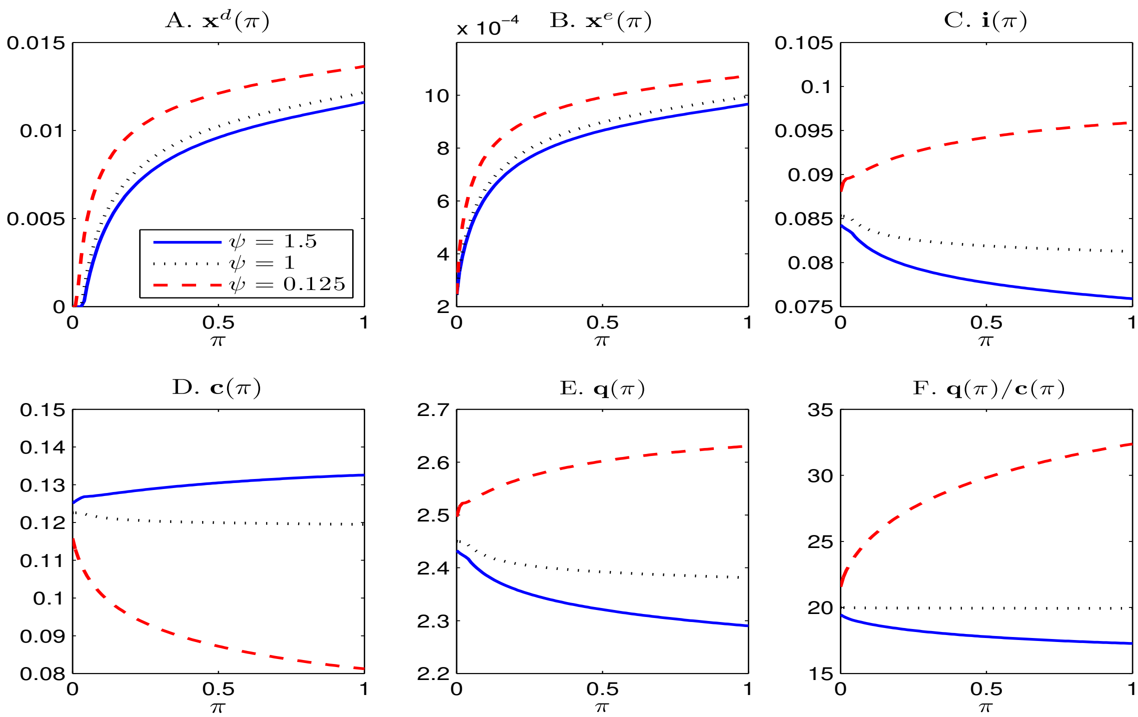

Figure O-7: This figure plots the planner's first-best solution for three values of the EIS ψ: $1/\gamma = 0.125, 1, 1.5$ for our baseline learning model (with Epstein-Zin utility). The other parameter values are given in Table 4.

Summary. In this online appendix, we have summarized the numerical challenges for the various models developed in this paper. The technical difficulties for our numerical solution are substantial and our numerical solution is significantly different from the ones in the literature.

OD Comparative Statics

OD.1 Elasticity of Intertemporal Substitution ψ

In Figure O-7, we plot the first-best solutions for three levels of the EIS ψ: $\gamma = 0.125, 1, 1.5$. Panels A and B show that the larger the EIS ψ the higher both public mitigation $\mathbf{x}^d$ and private mitigation $\mathbf{x}^e$ spendings. Quantitatively, these differences are not very large. Panel C shows that the larger the EIS ψ the higher the investment-capital ratio $\mathbf{i}(\pi)$. Panel D shows that the higher the EIS ψ the lower the consumption-capital ratio $\mathbf{c}(\pi)$, as $\mathbf{c} = A - (\mathbf{i} + \mathbf{x}^d + \mathbf{x}^e)$. Panel E shows that the larger the EIS ψ the higher Tobin's average $\mathbf{q}(\pi)$. This follows directly from the comparative static result of changing ψ on $\mathbf{i}$ (Panel C), as Tobin's q is increasing with $\mathbf{i}$: $\mathbf{q}(\pi) = 1 + \phi'(\mathbf{i}(\pi))$. Panel F shows that the larger the EIS ψ the higher the price-dividend ratio $\mathbf{q}(\pi)/\mathbf{c}(\pi)$, which follows from the comparative effects shown in Panels D and E.

The intuition for these results is as follows. The higher the EIS ψ, the more marginal propensity to consume as in partial equilibrium model consistent with Ramsey/Friedman consumption rule. As a result, the agent spends less on mitigation and also invests less for the future.

Additionally, we show that whether the price-dividend ratio $\mathbf{q}(\pi)/\mathbf{c}(\pi)$ increases or decreases when disaster arrives (which increases (worsens) belief π) crucially depends on whether the EIS ψ is larger or smaller than one. In our baseline case where $\psi = 1.5 > 1$, the equilibrium price-dividend ratio $\mathbf{q}(\pi)/\mathbf{c}(\pi)$ decreases when a disaster arrives (i.e., when π increases). This result is consistent with Bansal and Yaron (2004) and the subsequent long-run risk literature, who show that the price-dividend ratio decreases in response to a negative growth shock when the EIS parameter ψ is set to be larger than one. Unlike Bansal and Yaron's pure exchange economy, our model features production and hence we need to compute the endogenous dividend $\mathbf{c}$ together with value of capital, Tobin's $\mathbf{q}$, in order to obtain the price-dividend ratio. However, we obtain the same results for the effect of EIS on the price-dividend ratio.

For the unity EIS ($\psi = 1$) Epstein-Zin utility case, which is a generalized version of expected logarithmic utility (with a flexible choice of risk aversion parameter γ), the wealth and the substitution effects exactly offset each other. As a result, the equilibrium price-dividend ratio remains constant, i.e., $\mathbf{q}(\pi)/\mathbf{c}(\pi) = 1/\rho = 20$ at all levels of π (See the dotted line in Panel F.) Finally, with $\psi = 1/\gamma = 0.125 < 1$, the wealth effect is stronger than the substitution effect. For this case, as belief worsens (increases), the price-dividend ratio $\mathbf{q}(\pi)/\mathbf{c}(\pi)$ increases, which is empirically counterfactual. This is one reason (among others) why Epstein-Zin utility with an EIS larger than one ($\psi > 1$) is a more appealing utility specification than commonly used expected utility for asset pricing.

In Figure O-8, we show that the the quantitative effects of EIS ψ on the WTP is large (Panel A). In Panel B, the higher the EIS ψ, the higher the conditional damages $\ell(\pi)$. This is because the agent with a higher EIS mitigates less as we show in Panels A and B of Figure O-7. As a result, the higher EIS the higher the conditional damages $\ell(\pi)$.

Figure O-9 of Panel A shows that the higher the EIS ψ, the lower the expected growth rate $\mathbf{g}(\pi)$. This result follows from 1.) the higher the EIS the lower investment result (as shown in Panel C in Figure O-7) and 2.) the higher the EIS the larger damage $\ell(\pi)$ (as shown in Panel B of Figure O-8.)

Note that the effects of the EIS on the interest rate is ambiguous which depends on the agent's belief (Panel B). Panel C of Figure O-9 shows the higher the EIS the lower mitigation in equilibrium the higher risk premium.

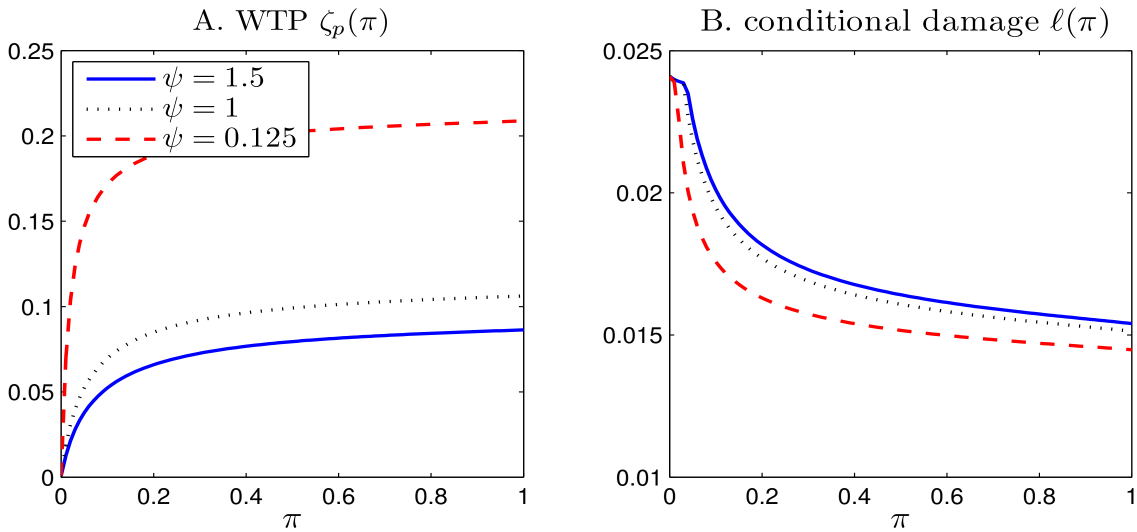

Figure O-8: This figure plots the planner's first-best solution for three values of the EIS ψ: $1/\gamma = 0.125, 1, 1.5$ for our baseline learning model (with Epstein-Zin utility). The other parameter values are given in Table 4.

OD.2 Disaster Arrival Rate λ_B in State B

In Figure O-10, we plot the first-best solutions for three levels of the disaster arrival rate in state B: $\lambda_B = 0.4, 0.8, 1$. Panel A shows that the higher the disaster arrival rate λ_B in state B, the higher the public mitigation spending $\mathbf{x}^d$. Moreover, the more pessimistic the agent's belief the stronger this effect. Note that the wedge between the lines for two different levels of λ widens as π increases.

Panel B shows that increasing the arrival rate λ_B has a highly nonlinear effect on the private mitigation spending $\mathbf{x}^e$. Increasing λ_B from 0.4 to 0.8 significantly increases the mitigation spending (for sufficiently large values of π.) However, further increasing λ_B from 0.8 to 1 has limited effects on the mitigation spending.

Panel C shows that as λ_B increases, investment falls. The higher the belief level π (the more pessimistic the agent) the larger the impact of λ_B on $\mathbf{i}$. Panel D shows that the impact of λ_B on consumption $\mathbf{c}(\pi)$ is ambiguous due to the general equilibrium effect.

In Figure O-11, we show that λ_B has a large effect on the WTP ζ_p (Panel A). For example, when the belief changes from $\pi = 0$ to $\pi = 1$, the WTP increases from about 0 to 13% when $\lambda_B = 1$. In contrast, when $\lambda_B = 0.4$, the WTP barely changes from 0 to 2% in response to the same change of the belief. Panel B shows that the higher the arrival rate λ_B the smaller the conditional damage $\ell(\pi)$. This is intuitive as mitigation spending is higher when λ_B is larger. However, as investment is lower when λ_B is larger, the impact of λ_B on the growth rate $\mathbf{g}(\pi)$ is minimal as the two channels (investment and conditional damage) offset each other (Panel C). Panel D shows that the higher the arrival rate λ_B the lower Tobin's $\mathbf{q}$, tracking the impact of λ_B on $\mathbf{i}(\pi)$ as $\mathbf{q}(\pi) = 1 + \theta\mathbf{i}(\pi)$. Panel E and Panel F show that the quantitative effects of λ_B on the risk-free rate r and the market risk premium rp are moderate at best.

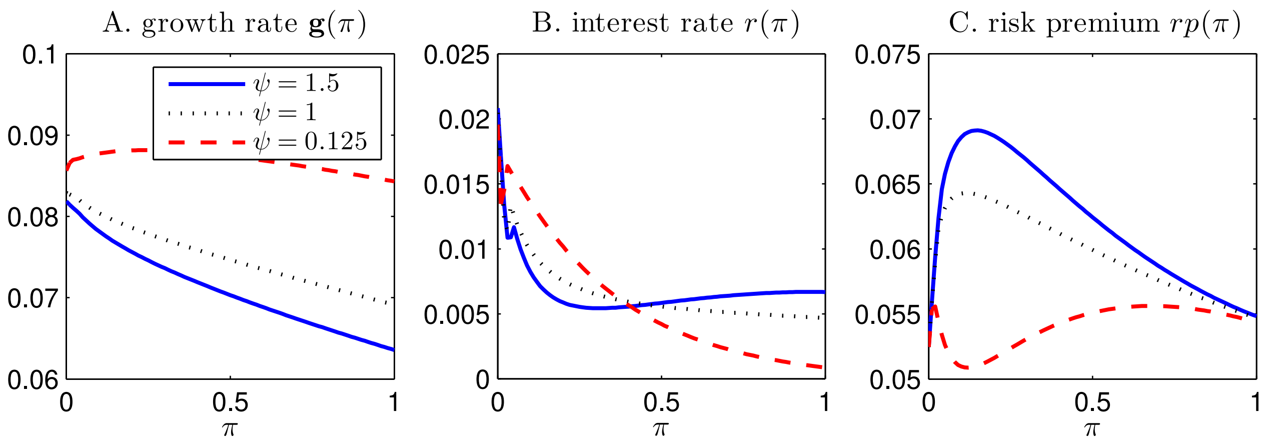

Figure O-9: This figure plots the planner's first-best solution for three values of the EIS ψ: $1/\gamma = 0.125, 1, 1.5$ for our baseline learning model (with Epstein-Zin utility). The other parameter values are given in Table 4.

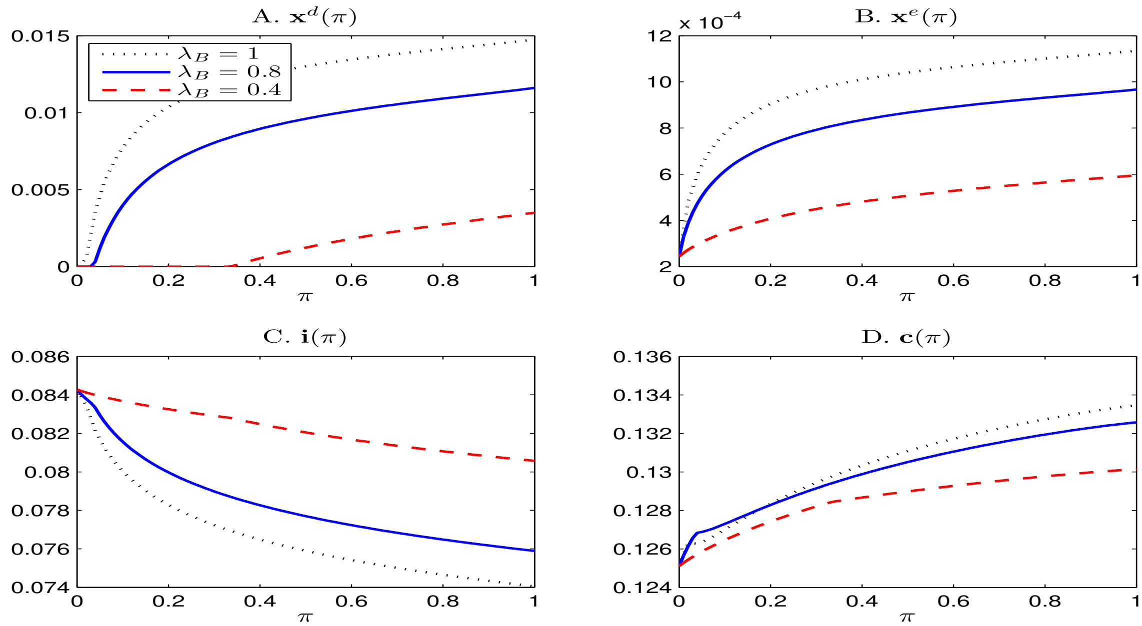

Figure O-10: This figure plots the planner's first-best solution for three values of the annual disaster arrival rate λ_B: $0.4, 0.8, 1$ for our baseline learning model (with Epstein-Zin utility). The other parameter values are given in Table 4.

OD.3 Time Rate of Preference ρ

In our baseline calculation, we set the time rate of preference ρ at 5% per annum, a commonly used value. Next, we compare our baseline model results with two other economies with lower discount rates: $\rho = 4.5\%$ and $\rho = 6\%$.

Panels A and B of Figure O-12 show that the higher the time rate of preference ρ, the less the planner spends on both types of mitigation spendings, $\mathbf{x}^d$ and $\mathbf{x}^e$. Similarly, Panel C of

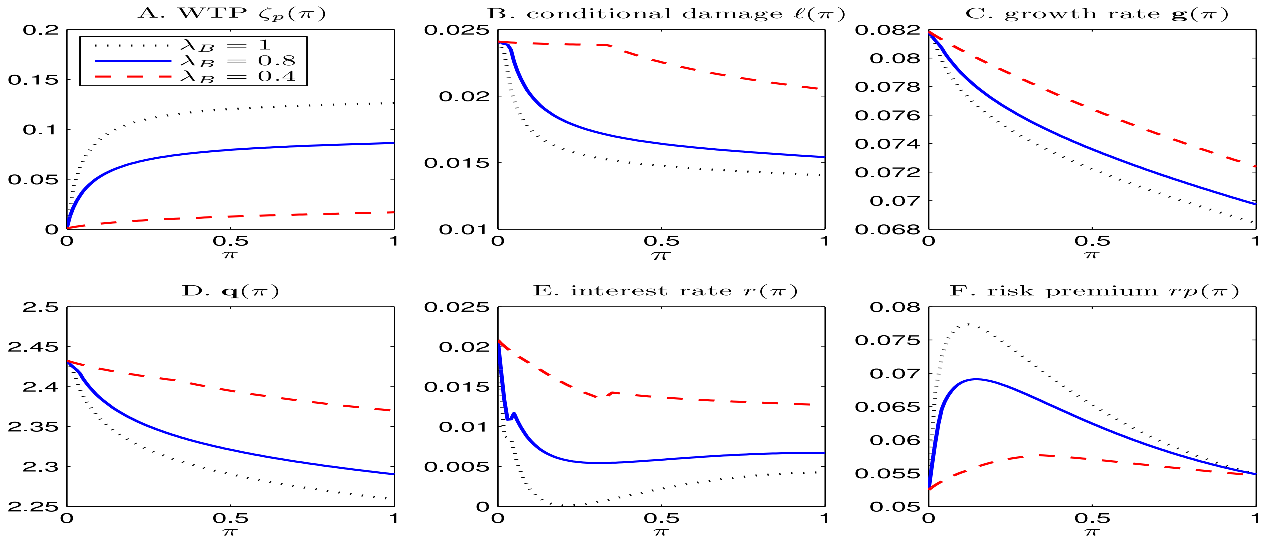

Figure O-11: This figure plots the planner's first-best solution for three values of the annual disaster arrival rate λ_B: $0.4, 0.8, 1$ for our baseline learning model (with Epstein-Zin utility). The other parameter values are given in Table 4.

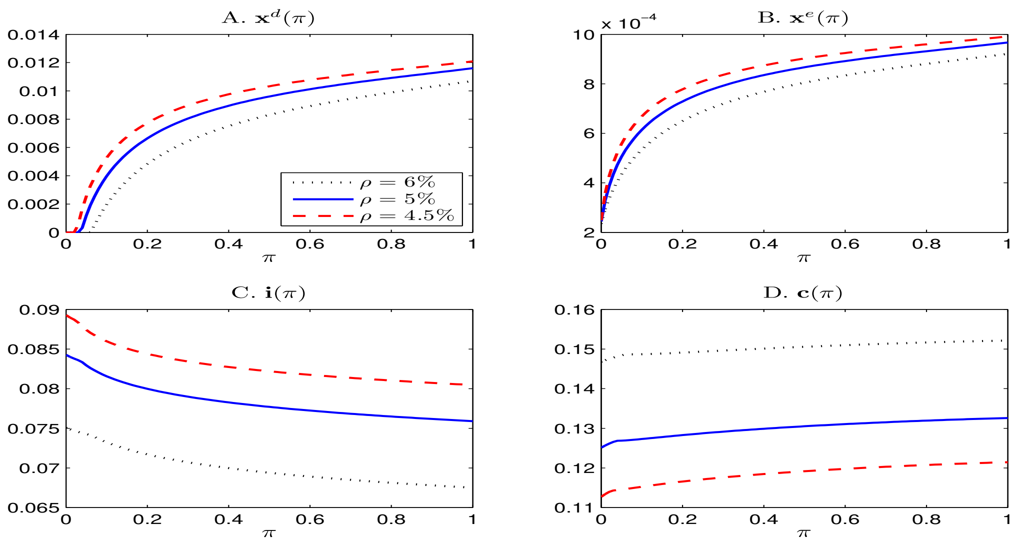

Figure O-12: This figure plots the planner's first-best solution for three values of the annual time rate of preference ρ: $4.5\%, 5\%, 6\%$ for our baseline learning model (with Epstein-Zin utility). The other parameter values are given in Table 4.

Figure O-12 shows that the higher the time rate of preference ρ, the less the planner invests and Panel D shows that the higher the time rate of preference ρ the more the agent consumes. The quantitative effects on consumption are large. For example increasing ρ from 4.5% to 6% roughly increases consumption $\mathbf{c}$ from 12% to 15% per annum.

In Figure O-13, we show that the quantitative effects of the time rate of preference ρ on the WTP is significant (Panel A). For example, when the belief changes from $\pi = 0$ to $\pi = 1$, the WTP increases from about 0 to 6.7% when $\rho = 6\%$, and increases from 0 to 10% when

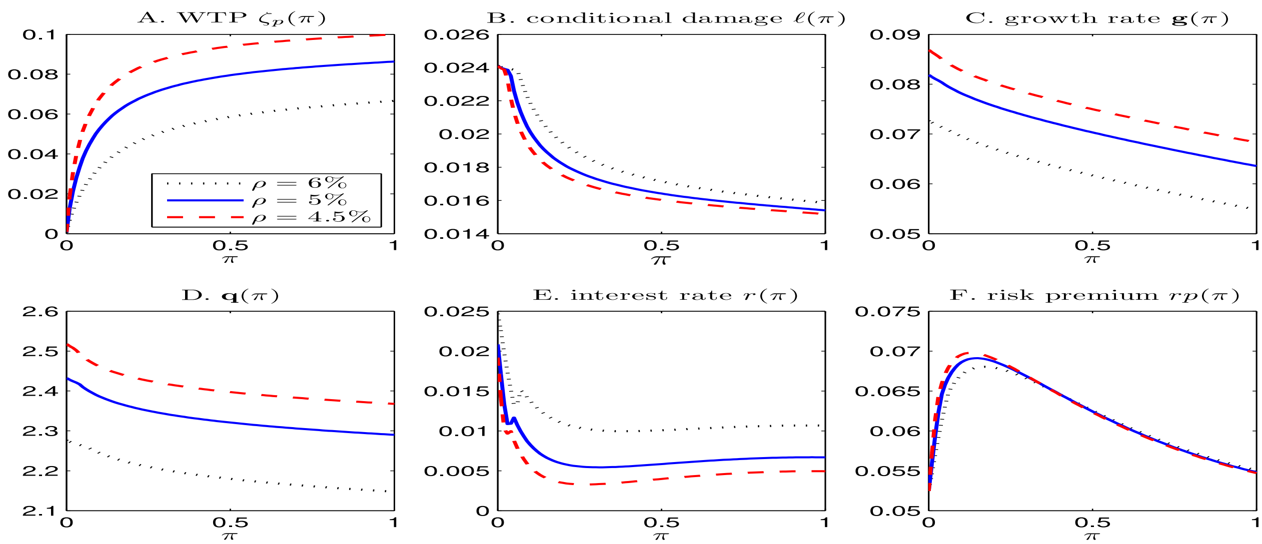

Figure O-13: This figure plots the planner's first-best solution for three values of the annual time rate of preference ρ: $4.5\%, 5\%, 6\%$ for our baseline learning model (with Epstein-Zin utility). The other parameter values are given in Table 4.

$\rho = 4.5\%$.

The higher the time rate of preference ρ the higher the conditional damage $\ell(\pi)$ (Panel B) and the lower the Tobin's $\mathbf{q}$ (Panel D) as the agent is less patient and puts a smaller weight on the future. Since these two forces push towards the same direction, the higher the discount rate ρ the lower growth rate $\mathbf{g}$ (Panel C).

Finally, Panel E shows that the quantitative effect of ρ on the risk-free rate r is moderate at best and Panel F shows that the effect of ρ on the market risk premium rp is very small.

OD.4 Coefficient of Relative Risk Aversion γ

In our baseline calculation, we set the coefficient of relative risk aversion γ at 8, which is within the range of widely used values (e.g., 2 to 10). Next, we compare our baseline model results to two other economies with $\gamma = 4$ and $\gamma = 10$.

Panel A of Figure O-14 shows that the higher the coefficient of relative risk aversion γ, the more the planner spends on distribution mitigation $\mathbf{x}^d$ and the less the planner spends on exposure mitigation $\mathbf{x}^e$. The higher the coefficient of relative risk aversion γ the less the planner invests (Panel C), the more the agent consumes (Panel D).

In Figure O-15, we show that the quantitative effects of increasing risk aversion from $\gamma = 4$ to $\gamma = 10$ on the WTP is large (Panel A). For example, as we increase γ from 4 to 10, the WTP ζ_p increases from 6.6% to 9.8% when the agent's belief is $\pi = 1$.

The higher the coefficient of relative risk aversion γ the lower the conditional damage $\ell(\pi)$ (Panel B of Figure O-15) and the lower the growth rate $\mathbf{g}(\pi)$ (Panel C of Figure O-15). This is because a more risk-averse agent mitigates more but invests less. Quantitatively, the negative effect of increasing γ via investment on growth dominates the positive effect of increasing γ via mitigation. As a result, the net effect of increasing γ on growth is negative.

Finally, Panels E and F of Figure O-15 show that the quantitative effects of γ on the risk-free rate r and the market risk premium rp are very large, as we expect (in line with

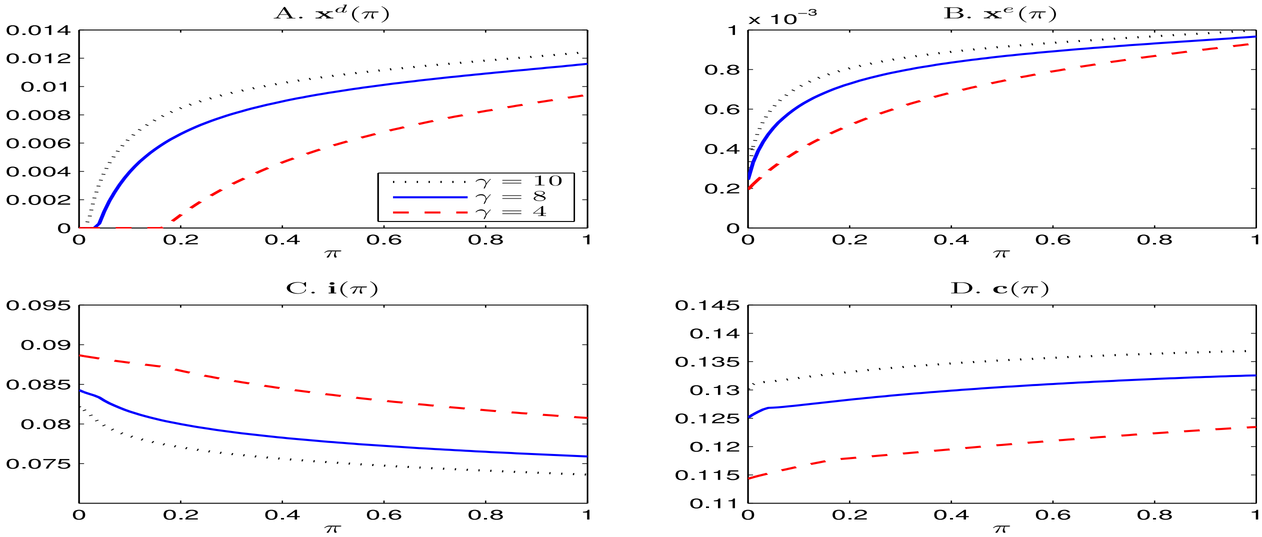

Figure O-14: This figure plots the planner's first-best solution for three values of coefficient of relative risk aversion γ: $4, 8, 10$ for our baseline learning model (with Epstein-Zin utility). The other parameter values are given in Table 4.

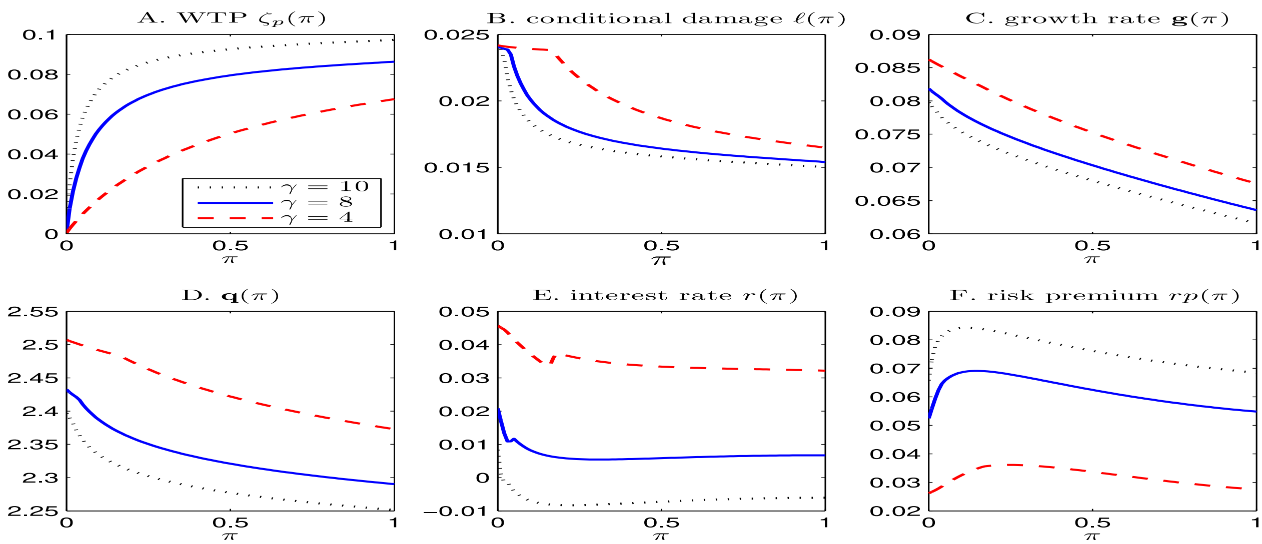

Figure O-15: This figure plots the planner's first-best solution for three values of the coefficient of relative risk aversion γ: $4, 8, 10$ for our baseline learning model (with Epstein-Zin utility). The other parameter values are given in Table 4.

standard asset pricing results.)

Panel A of Figure O-14 shows that the higher the coefficient of relative risk aversion γ, the more the planner spends on distribution mitigation $\mathbf{x}^d$ and the less the planner spends on exposure mitigation $\mathbf{x}^e$. The higher the coefficient of relative risk aversion γ the less the planner invests (Panel C), the more the agent consumes (Panel D).

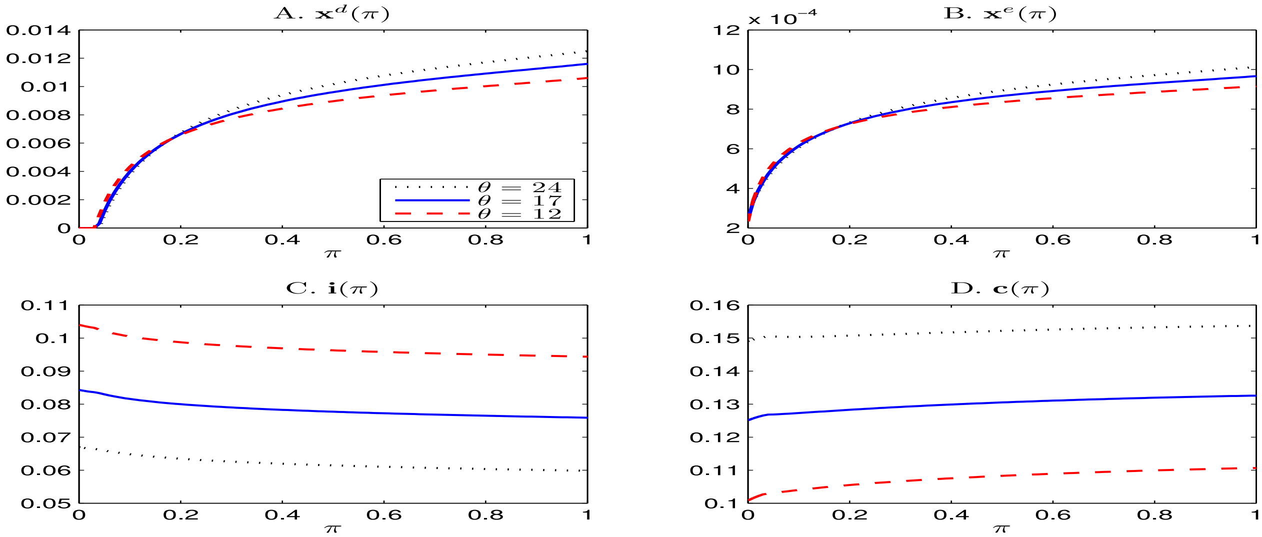

Figure O-16: This figure plots the planner's first-best solution for three values of adjustment cost θ: $12, 17, 24$ for our baseline learning model (with Epstein-Zin utility). The other parameter values are given in Table 4.

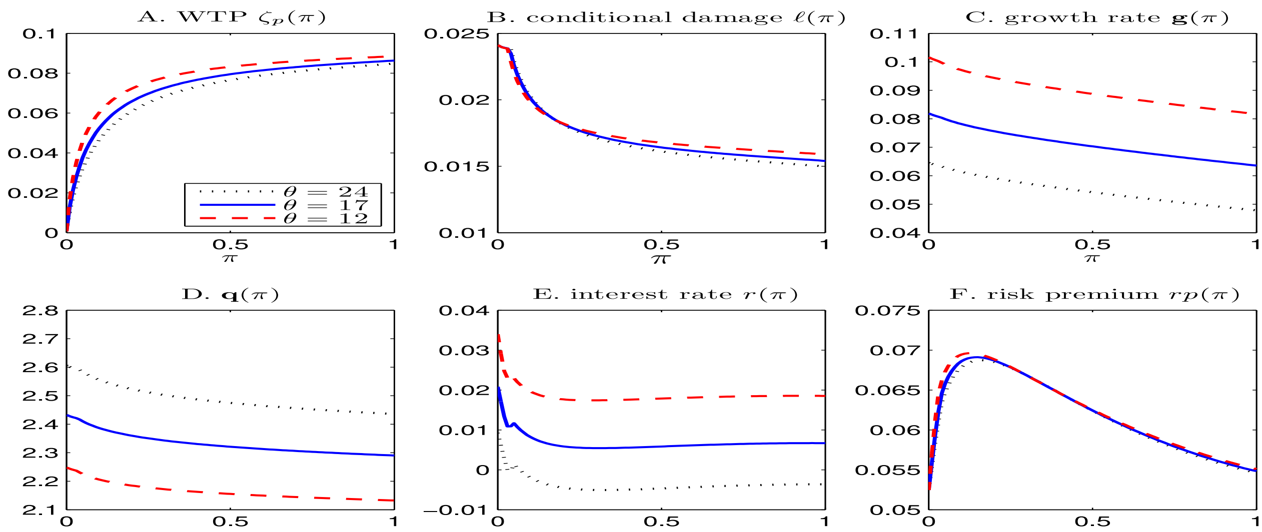

Figure O-17: This figure plots the planner's first-best solution for three values of adjustment cost θ: $12, 17, 24$ for our baseline learning model (with Epstein-Zin utility). The other parameter values are given in Table 4.

OE Carbon Stock Extension

OE.1 Asset Prices Under Planner's Solution

Dynamics of s. Ito's lemma gives

$$\begin{aligned} d\mathbf{s}_t &= d\left(\frac{\mathbf{S}_t}{K_t}\right) = \frac{d\mathbf{S}_t}{K_{t-}} - \frac{\mathbf{S}_{t-}dK_t}{K_{t-}^2} + \frac{\mathbf{S}_{t-}dK_t^2}{K_{t-}^3} - \frac{< d\mathbf{S}_t, dK_t >}{K_{t-}^2} \\ &= \mathbf{s}_{t-}\left[\left(\frac{\mathbf{h}_{t-}}{\mathbf{s}_{t-}} - \delta_S - \mathbf{i}_{t-} + \delta_K + \sigma_K^2 - \vartheta\sigma_K\sigma_S\right)dt + \sigma_S d\mathcal{W}_t^S - \sigma_K d\mathcal{W}_t^K + N_{t-}(1-Z)d\mathcal{J}_t\right] \\ &= \mu_s(\pi_{t-}, \mathbf{s}_{t-})dt + \mathbf{s}_{t-}\left[\sigma_S d\mathcal{W}_t^S - \sigma_K d\mathcal{W}_t^K + N_{t-}(1-Z)d\mathcal{J}_t\right], \end{aligned} \quad \text{(OA.44)}$$

where

$$\mu_s(\pi_{t-}, \mathbf{s}_{t-}) = \mathbf{h}_{t-} - \left(\mathbf{i}_{t-} - \delta_K + \delta_S - \sigma_K^2 + \vartheta \sigma_K \sigma_S\right) \mathbf{s}_{t-} \,. \tag{OA.45}$$

Duffie and Epstein (1992) show that the SDF $\{\mathbb{M}_t : t \geq 0\}$ implied by the planner's solution is given by:

$$\mathbb{M}_t = \exp\left[\int_0^t f_V(\mathbf{C}_s, V_s)\, ds\right] f_{\mathbf{C}}(\mathbf{C}_t, V_t) \,. \tag{OA.46}$$

Using the FOC for investment, the value function, and the resource constraint, we obtain:

$$f_{\mathbf{C}}(\mathbf{C}, V) = \frac{1}{1+\phi'(\mathbf{i}(\pi, \mathbf{s}))} b(\pi, \mathbf{s})^{1-\gamma} \mathbf{K}^{-\gamma} = \frac{1}{\mathbf{q}(\pi, \mathbf{s})} b(\pi, \mathbf{s})^{1-\gamma} \mathbf{K}^{-\gamma} \tag{OA.47}$$

and

$$f_V(\mathbf{C}, V) = \frac{\rho}{1-\psi^{-1}} \left[\frac{(1-\omega)\mathbf{C}^{1-\psi^{-1}}}{((1-\gamma))^{\omega-1}} V^{-\omega} - (1-\gamma)\right] = -\epsilon(\pi, \mathbf{s}) \,, \tag{OA.48}$$

where

$$\epsilon(\pi, \mathbf{s}) = -\frac{\rho(1-\gamma)}{1-\psi^{-1}} \left[\left(\frac{\mathbf{c}(\pi, \mathbf{s})}{b(\pi, \mathbf{s})}\right)^{1-\psi^{-1}} \left(\frac{\psi^{-1}-\gamma}{1-\gamma}\right) - 1\right]. \tag{OA.49}$$

Using the equilibrium relation between $b(\pi, \mathbf{s})$ and $\mathbf{c}(\pi, \mathbf{s})$, we simplify (OA.49) as:

$$\begin{aligned}
\epsilon(\pi, \mathbf{s}) &= \rho + \left(\psi^{-1} - \gamma\right) \left[\mathbf{i}(\pi, \mathbf{s}) - \delta_K - \frac{\gamma \sigma_K^2}{2} + \mu_\pi(\pi) \frac{b_\pi(\pi, \mathbf{s})}{b(\pi, \mathbf{s})}\right] \\
&+ \left(\psi^{-1} - \gamma\right) \left[(\mathbf{h}(\pi, \mathbf{s}) - \delta_S \mathbf{s}) \frac{b_{\mathbf{s}}(\pi, \mathbf{s})}{b(\pi, \mathbf{s})} + \frac{\sigma_S^2 \mathbf{s}^2}{2} \left(\frac{b_{\mathbf{ss}}(\pi, \mathbf{s})}{b(\pi, \mathbf{s})} - \gamma \frac{(b_{\mathbf{s}}(\pi, \mathbf{s}))^2}{b(\pi, \mathbf{s})^2}\right) + (1-\gamma)\vartheta \sigma_K \sigma_S \mathbf{s} \frac{b_{\mathbf{s}}(\pi, \mathbf{s})}{b(\pi, \mathbf{s})}\right] \\
&+ \left(\psi^{-1} - \gamma\right) \left[\frac{\lambda(\pi)}{1-\gamma} \left[\mathbb{E}^{\mathbf{x}^d(\pi, \mathbf{s})} \left(\frac{(1 - N(\mathbf{x}^e(\pi, \mathbf{s}))(1-Z)) b\left(\pi^{\mathcal{J}}, \mathbf{s}^{\mathcal{J}}\right)}{b(\pi, \mathbf{s})}\right)^{1-\gamma} - 1\right]\right],
\end{aligned} \tag{OA.50}$$

Using Ito's Lemma and the optimal allocation, we obtain

$$\begin{aligned}
\frac{d\mathbb{M}_t}{\mathbb{M}_{t-}} &= -\epsilon(\pi, \mathbf{s}) dt - \gamma \left[(\mathbf{i}(\pi, \mathbf{s}) - \delta_K)\, dt + \sigma_K d\mathcal{W}_t\right] + \left[(1-\gamma) \frac{b_\pi(\pi, \mathbf{s})}{b(\pi, \mathbf{s})} - \frac{\mathbf{q}_\pi(\pi, \mathbf{s})}{\mathbf{q}(\pi, \mathbf{s})}\right] \mu_\pi(\pi) dt \\
&+ \left[(1-\gamma) \frac{b_{\mathbf{s}}(\pi, \mathbf{s})}{b(\pi, \mathbf{s})} - \frac{\mathbf{q}_{\mathbf{s}}(\pi, \mathbf{s})}{\mathbf{q}(\pi, \mathbf{s})}\right] \left[(\mu_s(\pi, \mathbf{s}) + \mathbf{s}\gamma(\sigma_K^2 - \vartheta \sigma_S \sigma_K)) dt + \sigma_S d\mathcal{W}_t^S - \sigma_K d\mathcal{W}_t^K\right] \\
&+ \frac{\gamma(\gamma+1)}{2} \sigma_K^2 dt + \frac{(\sigma_S^2 - 2\vartheta \sigma_S \sigma_K + \sigma_K^2) \mathbf{s}^2}{2} \left[(1-\gamma) \left(\frac{b_{\mathbf{ss}}}{b} - \frac{\gamma b_{\mathbf{s}}^2}{b^2} - \frac{b_{\mathbf{s}}}{b} \frac{\mathbf{q}_{\mathbf{s}}}{\mathbf{q}}\right) - \frac{\mathbf{q}_{\mathbf{ss}}}{\mathbf{q}} + \frac{2\mathbf{q}_{\mathbf{s}}^2}{\mathbf{q}^2}\right] dt \\
&+ \left[\eta(\pi, \mathbf{s}; Z, \mathbf{x}^e, \mathbf{x}^d) - 1\right] d\mathcal{J}_t \,,
\end{aligned} \tag{OA.51}$$

where

$$\eta(\pi, \mathbf{s}; Z, \mathbf{x}^e, \mathbf{x}^d) = \frac{\mathbf{q}(\pi, \mathbf{s})}{\mathbf{q}(\pi^{\mathcal{J}}, \mathbf{s}^{\mathcal{J}})} \left(\frac{b\left(\pi^{\mathcal{J}}, \mathbf{s}^{\mathcal{J}}\right)}{b(\pi, \mathbf{s})}\right)^{1-\gamma} (1 - N(\mathbf{x}^e(\pi, \mathbf{s}))(1-Z))^{-\gamma} \,. \tag{OA.52}$$

As the expected percentage change of $\mathbb{M}_t$ equals $-r_t$ per unit of time (Duffie, 2001), we obtain the following expression for the interest rate:

$$
\begin{aligned}
r(\pi,\mathbf{s}) &= \rho+\psi^{-1}\left(\mathbf{i}-\delta_K\right)-\frac{\gamma(\psi^{-1}+1)\sigma_K^2}{2}-\left[(1-\psi^{-1})\frac{b_\pi}{b}-\frac{\mathbf{q}_\pi}{\mathbf{q}}\right]\mu_\pi(\pi)-\left[(1-\gamma)\frac{b_{\mathbf{s}}}{b}-\frac{\mathbf{q}_{\mathbf{s}}}{\mathbf{q}}\right](\mu_s(\pi,\mathbf{s})+\mathbf{s}\gamma(\sigma_K^2-\vartheta\sigma_S\sigma_K)) \\
&+(\psi^{-1}-\gamma)\left[(\mathbf{h}-\delta_S\mathbf{s})\frac{b_{\mathbf{s}}}{b}+\frac{\sigma_S^2\mathbf{s}^2}{2}\left(\frac{b_{\mathbf{ss}}}{b}-\frac{\gamma b_{\mathbf{s}}^2}{b^2}\right)+(1-\gamma)\vartheta\sigma_K\sigma_S\mathbf{s}\frac{b_{\mathbf{s}}}{b}\right] \\
&-\frac{(\sigma_S^2-2\vartheta\sigma_S\sigma_K+\sigma_K^2)\mathbf{s}^2}{2}\left[(1-\gamma)\left(\frac{b_{\mathbf{ss}}}{b}-\frac{\gamma b_{\mathbf{s}}^2}{b^2}-\frac{b_{\mathbf{s}}}{b}\frac{\mathbf{q}_{\mathbf{s}}}{\mathbf{q}}\right)-\frac{\mathbf{q}_{\mathbf{ss}}}{\mathbf{q}}+\frac{2\mathbf{q}_{\mathbf{s}}^2}{\mathbf{q}^2}\right] \\
&-\lambda(\pi)\left[\mathbb{E}^{\mathbf{x}^d}\left(\eta(\pi,\mathbf{s};Z,\mathbf{x}^e,\mathbf{x}^d)\right)-1\right]-\lambda(\pi)\left[\frac{\psi^{-1}-\gamma}{1-\gamma}\left(1-\mathbb{E}^{\mathbf{x}^d}\left(\left(\frac{(1-N(\mathbf{x}^e)(1-Z))b\left(\pi^{\mathcal{J}},\mathbf{s}^{\mathcal{J}}\right)}{b(\pi,\mathbf{s})}\right)^{1-\gamma}\right)\right)\right].
\end{aligned}
\tag{OA.53}
$$

Using the equilibrium SDF, we may calculate firm value, $Q(K,\pi,\mathbf{s})$ by using

$$
Q(K_t,\pi_t,\mathbf{s}_t)=\int_t^\infty\frac{\mathbb{M}_v}{\mathbb{M}_t}(AK_v^\alpha\mathbf{H}_v^{1-\alpha}-p_H\mathbf{H}_v-I_v-\Phi(I_v,K_v)-X_v^e-\mathbf{X}_v^d)dv\,. \tag{OA.54}
$$

Applying the Ito's Lemma to firm value $Q(K,\pi,\mathbf{s})=q(\pi,\mathbf{s})K$, we obtain the following PDE for $q(\pi,\mathbf{s})$:

$$
\begin{aligned}
r(\pi,\mathbf{s})q(\pi,\mathbf{s}) &= A\mathbf{h}^{1-\alpha}-p_H\mathbf{h}-i-\phi(i)-x^e-\mathbf{x}^d+\left(i-\delta_K-\eta_{\mathbb{M}}^k(\pi,\mathbf{s})\sigma_K\right)q(\pi,\mathbf{s})+\mu_\pi(\pi)q_\pi \\
&+\left[\mu_s(\pi,\mathbf{s})+\vartheta\sigma_S\sigma_K-\sigma_K^2-(\eta_{\mathbb{M}}^s(\pi,\mathbf{s})\vartheta\sigma_S-\eta_{\mathbb{M}}^k(\pi,\mathbf{s})\sigma_K)\right]\mathbf{s}q_{\mathbf{s}}+\frac{(\sigma_S^2-2\vartheta\sigma_S\sigma_K+\sigma_K^2)\mathbf{s}^2}{2}q_{\mathbf{ss}} \\
&+\lambda(\pi)\mathbb{E}^{\mathbf{x}^d}\left[\eta(\pi,\mathbf{s};Z,\mathbf{x}^e,\mathbf{x}^d)\left(q(\pi^{\mathcal{J}},\mathbf{s}^{\mathcal{J}})(1-N(\mathbf{x}^e)(1-Z))-q(\pi,\mathbf{s})\right)\right].
\end{aligned}
\tag{OA.55}
$$

where

$$
\eta_{\mathbb{M}}^k(\pi,\mathbf{s})=\gamma\sigma_K+\left[(1-\gamma)\frac{\mathbf{s}b_{\mathbf{s}}(\pi,\mathbf{s})}{b(\pi,\mathbf{s})}-\frac{\mathbf{s}\mathbf{q}_{\mathbf{s}}(\pi,\mathbf{s})}{\mathbf{q}(\pi,\mathbf{s})}\right](\sigma_K-\vartheta\sigma_S)\,, \tag{OA.56}
$$

and

$$
\eta_{\mathbb{M}}^s(\pi,\mathbf{s})=\gamma\sigma_K+\left[(1-\gamma)\frac{\mathbf{s}b_{\mathbf{s}}(\pi,\mathbf{s})}{b(\pi,\mathbf{s})}-\frac{\mathbf{s}\mathbf{q}_{\mathbf{s}}(\pi,\mathbf{s})}{\mathbf{q}(\pi,\mathbf{s})}\right]\left(\sigma_K-\frac{\sigma_S}{\vartheta}\right). \tag{OA.57}
$$

The cum-dividend return dR_t over the period dt is given by

$$
\begin{aligned}
dR_t &= \frac{(AK_{t-}^\alpha\mathbf{H}_{t-}^{1-\alpha}-p_H\mathbf{H}_{t-}-I_t-\Phi(I_{t-},K_{t-})-X_{t-}^e-\mathbf{X}_{t-}^d)dt}{Q_{t-}}+\frac{dQ_t}{Q_{t-}} \\
&= \frac{A\mathbf{h}_{t-}^{1-\alpha}-p_H\mathbf{h}_{t-}-i_{t-}-\phi(i_{t-})-x_{t-}^e-\mathbf{x}_{t-}^d}{q(\pi_{t-},\mathbf{s}_{t-})}dt+\frac{dq(\pi_t,\mathbf{s}_t)}{q(\pi_{t-},\mathbf{s}_{t-})}+\frac{dK_t}{K_{t-}}+\frac{<dq(\pi_t,\mathbf{s}_t),dK_t>}{q(\pi_{t-},\mathbf{s}_{t-})K_{t-}} \\
&= \frac{A\mathbf{h}_{t-}^{1-\alpha}-p_H\mathbf{h}_{t-}-i_{t-}-\phi(i_{t-})-x_{t-}^e-\mathbf{x}_{t-}^d+\mu_s(\pi_{t-},\mathbf{s}_{t-})q_{\mathbf{s}}(\pi_{t-},\mathbf{s}_{t-})+\frac{(\sigma_S^2-2\vartheta\sigma_S\sigma_K+\sigma_K^2)\mathbf{s}_{t-}^2}{2}q_{\mathbf{ss}}(\pi_{t-},\mathbf{s}_{t-})}{q(\pi_{t-},\mathbf{s}_{t-})}dt \\
&+\frac{\mu_\pi(\pi_{t-})q_\pi(\pi_{t-},\mathbf{s}_{t-})}{q(\pi_{t-},\mathbf{s}_{t-})}dt+(i_{t-}-\delta_K)dt+\frac{\mathbf{s}_{t-}q_{\mathbf{s}}(\pi_{t-},\mathbf{s}_{t-})}{q(\pi_{t-},\mathbf{s}_{t-})}(\vartheta\sigma_S\sigma_K-\sigma_K^2)dt+\sigma_Kd\mathcal{W}_t^K \\
&+\frac{\mathbf{s}_{t-}q_{\mathbf{s}}(\pi_{t-},\mathbf{s}_{t-})}{q(\pi_{t-},\mathbf{s}_{t-})}\left[\sigma_Sd\mathcal{W}_t^S-\sigma_Kd\mathcal{W}_t^K\right]+\left[\frac{(1-N(x_{t-}^e)(1-Z))q\left(\pi_t^{\mathcal{J}},\mathbf{s}_{t-}^{\mathcal{J}}\right)}{q(\pi_{t-},\mathbf{s}_{t-})}-1\right]d\mathcal{J}_t \\
&= \left[r(\pi_{t-},\mathbf{s}_{t-})+\eta_{\mathbb{M}}^k(\pi_{t-},\mathbf{s}_{t-})\sigma_K+(\eta_{\mathbb{M}}^s(\pi_{t-},\mathbf{s}_{t-})\vartheta\sigma_S-\eta_{\mathbb{M}}^k(\pi_{t-},\mathbf{s}_{t-})\sigma_K)\frac{\mathbf{s}_{t-}q_{\mathbf{s}}(\pi_{t-},\mathbf{s}_{t-})}{q(\pi_{t-},\mathbf{s}_{t-})}\right]dt \\
&-\lambda(\pi)\mathbb{E}^{\mathbf{x}_{t-}^d}\left[\eta(\pi_{t-},\mathbf{s}_{t-};Z,\mathbf{x}_{t-}^e,\mathbf{x}_{t-}^d)\left(\frac{(1-N(x_{t-}^e)(1-Z))q\left(\pi_t^{\mathcal{J}},\mathbf{s}_{t-}^{\mathcal{J}}\right)}{q(\pi_{t-},\mathbf{s}_{t-})}-1\right)\right]dt \\
&+\sigma_Kd\mathcal{W}_t^K+\frac{\mathbf{s}_{t-}q_{\mathbf{s}}(\pi_{t-},\mathbf{s}_{t-})}{q(\pi_{t-},\mathbf{s}_{t-})}\left[\sigma_Sd\mathcal{W}_t^S-\sigma_Kd\mathcal{W}_t^K\right]+\left[\frac{(1-N(x_{t-}^e)(1-Z))q\left(\pi_t^{\mathcal{J}},\mathbf{s}_{t-}^{\mathcal{J}}\right)}{q(\pi_{t-},\mathbf{s}_{t-})}-1\right]d\mathcal{J}_t\,.
\end{aligned}
\tag{OA.58}
$$

Finally, using the equilibrium conditions $q(\pi,\mathbf{s})=\mathbf{q}(\pi,\mathbf{s})$ and $x^e(\pi,\mathbf{s})=\mathbf{x}^e(\pi,\mathbf{s})$, we write

$$\begin{aligned}\frac{d\mathbf{Q}_t+\mathbf{D}_{t-}dt}{\mathbf{Q}_{t-}} &= \left(\mu_{\mathbf{Q}}(\pi_{t-},\mathbf{s}_{t-})+\lambda(\pi_{t-})\left(\frac{\mathbf{Q}_t^{\mathcal{J}}}{\mathbf{Q}_{t-}}-1\right)\right)dt+\sigma_K d\mathcal{W}_t^K+\frac{\mathbf{s}_{t-}\mathbf{q}_{\mathbf{s}}(\pi_{t-},\mathbf{s}_{t-})}{\mathbf{q}(\pi_{t-},\mathbf{s}_{t-})}\left[\sigma_S d\mathcal{W}_t^S-\sigma_K d\mathcal{W}_t^K\right]\\ &\quad+\left(\frac{\mathbf{Q}_t^{\mathcal{J}}}{\mathbf{Q}_{t-}}-1\right)(d\mathcal{J}_t-\lambda(\pi_{t-})dt)\,, \end{aligned} \tag{OA.59}$$

where

$$\frac{\mathbf{Q}_t^{\mathcal{J}}}{\mathbf{Q}_{t-}}=\frac{(1-N(\mathbf{x}_{t-}^e)(1-Z))\mathbf{q}\left(\pi_t^{\mathcal{J}},\mathbf{s}_{t-}^{\mathcal{J}}\right)}{\mathbf{q}(\pi_{t-},\mathbf{s}_{t-})}\,, \tag{OA.60}$$

and

$$\begin{aligned}\mu_{\mathbf{Q}}(\pi_{t-},\mathbf{s}_{t-}) &= r(\pi_{t-},\mathbf{s}_{t-})+\eta_{\mathbb{M}}^k(\pi_{t-},\mathbf{s}_{t-})\sigma_K+(\eta_{\mathbb{M}}^s(\pi_{t-},\mathbf{s}_{t-})\vartheta\sigma_S-\eta_{\mathbb{M}}^k(\pi_{t-},\mathbf{s}_{t-})\sigma_K)\frac{\mathbf{s}_{t-}\mathbf{q}_{\mathbf{s}}(\pi_{t-},\mathbf{s}_{t-})}{\mathbf{q}(\pi_{t-},\mathbf{s}_{t-})}\\ &\quad+\lambda(\pi_{t-})\mathbb{E}^{\mathbf{x}_{t-}^d}\left[\eta(\pi_{t-},\mathbf{s}_{t-};Z,\mathbf{x}_{t-}^e,\mathbf{x}_{t-}^d)\left(1-\frac{\mathbf{Q}_t^{\mathcal{J}}}{\mathbf{Q}_{t-}}\right)\right]. \end{aligned} \tag{OA.61}$$

The market risk premium is

$$\begin{aligned}rp(\pi_{t-},\mathbf{s}_{t-}) &= \mu_{\mathbf{Q}}(\pi_{t-},\mathbf{s}_{t-})+\lambda(\pi_{t-})\left(\frac{\mathbf{Q}_t^{\mathcal{J}}}{\mathbf{Q}_{t-}}-1\right)-r(\pi_{t-},\mathbf{s}_{t-})\\ &= \eta_{\mathbb{M}}^k(\pi_{t-},\mathbf{s}_{t-})\sigma_K+(\eta_{\mathbb{M}}^s(\pi_{t-},\mathbf{s}_{t-})\vartheta\sigma_S-\eta_{\mathbb{M}}^k(\pi_{t-},\mathbf{s}_{t-})\sigma_K)\frac{\mathbf{s}_{t-}\mathbf{q}_{\mathbf{s}}(\pi_{t-},\mathbf{s}_{t-})}{\mathbf{q}(\pi_{t-},\mathbf{s}_{t-})}\\ &\quad-\lambda(\pi_{t-})\mathbb{E}^{\mathbf{x}_{t-}^d}\left[\left(\eta(\pi_{t-},\mathbf{s}_{t-};Z,\mathbf{x}_{t-}^e,\mathbf{x}_{t-}^d)-1\right)\left(\frac{\mathbf{Q}_t^{\mathcal{J}}}{\mathbf{Q}_{t-}}-1\right)\right]. \end{aligned} \tag{OA.62}$$

OE.2 Some technical details for numerical solutions

Variable transformation. Denote $s=\frac{f}{1-f}$, and $b(\pi,s)=b(\pi,f)$ we could rewrite (74) as

$$\begin{aligned}0 &= \frac{\rho}{1-\psi^{-1}}\left[\left[\frac{b(\pi,f)}{\rho(1+\phi'(\mathbf{i}(\pi,f)))}\right]^{1-\psi}-1\right]+\mathbf{i}(\pi,f)-\delta_K-\frac{\gamma\sigma_K^2}{2}+\mu_\pi(\pi)\frac{b_\pi(\pi,f)}{b(\pi,f)}\\ &\quad+\left[\frac{(1-f)\mathbf{h}}{f}-\delta_S+(1-\gamma)\vartheta\sigma_K\sigma_S\right]\frac{f(1-f)b_f}{b}+\frac{\sigma_S^2f^2(1-f)^2}{2}\left(\frac{b_{ff}-\frac{2b_f}{1-f}}{b}-\frac{\gamma b_f^2}{b^2}\right)\\ &\quad+\frac{\lambda(\pi)}{1-\gamma}\left[\mathbb{E}^{\mathbf{x}^d(\pi,f)}\left(\frac{(1-N(\mathbf{x}^e(\pi,f))(1-Z))b\left(\pi^{\mathcal{J}},f^{\mathcal{J}}\right)}{b(\pi,f)}\right)^{1-\gamma}-1\right], \end{aligned} \tag{OA.63}$$

or equivalently

$$\begin{aligned}0 &= \frac{\rho}{1-\psi^{-1}}\left[\left[\frac{b(\pi,f)}{\rho(1+\phi'(\mathbf{i}(\pi,f)))}\right]^{1-\psi}-1\right]+\mathbf{i}(\pi,f)-\delta_K-\frac{\gamma\sigma_K^2}{2}+\mu_\pi(\pi)\frac{b_\pi(\pi,f)}{b(\pi,f)}\\ &\quad+\left[\frac{(1-f)\mathbf{h}}{f}-\delta_S+(1-\gamma)\vartheta\sigma_K\sigma_S-\sigma_S^2 f\right]\frac{f(1-f)b_f}{b}+\frac{\sigma_S^2f^2(1-f)^2}{2}\left(\frac{b_{ff}}{b}-\frac{\gamma b_f^2}{b^2}\right)\\ &\quad+\frac{\lambda(\pi)}{1-\gamma}\left[\mathbb{E}^{\mathbf{x}^d(\pi,f)}\left(\frac{(1-N(\mathbf{x}^e(\pi,f))(1-Z))b\left(\pi^{\mathcal{J}},f^{\mathcal{J}}\right)}{b(\pi,f)}\right)^{1-\gamma}-1\right], \end{aligned} \tag{OA.64}$$

where $f^{\mathcal{J}} = \frac{f}{f+(1-N(\mathbf{x}^e(\pi,f))(1-Z))(1-f)}$.

And we could rewrite (76) as

$$\frac{(1-f)^2 b_f(\pi,f)}{b(\pi,f)-f(1-f)b_f(\pi,f)} = \frac{p_H-(1-\alpha)A\mathbf{h}(\pi,f)^{-\alpha}}{1+\phi'(\mathbf{i}(\pi,f))} \,. \tag{OA.65}$$

OE.3 Competitive Equilibrium Solution

Firm's Optimization Problem. When making its decisions, the firm takes the equilibrium risk-free rate r_t given by (OA.53) and the market price of (diffusion and jump) risks as given by (OA.52), we maximizes its market value, $Q(K,\pi,s)$ as folows

$$Q(K_t,\pi_t,\mathbf{s}_t) = \max_{H,I,X^e,X^d} \int_t^\infty \frac{\mathbb{M}_v}{\mathbb{M}_t}(AK_v^\alpha H_v^{1-\alpha} - p_H H_v - I_v - \Phi(I_v,K_v) - X_v^e - X_v^d)dv \,. \tag{OA.66}$$

Applying the Ito's Lemma to firm value $Q(K,\pi,s) = q(\pi,s)K$, we obtain the following PDE for $q(\pi,s)$:

$$\begin{aligned} r(\pi,\mathbf{s})q(\pi,\mathbf{s}) = &\max_{i,\,x^e,\,x^d,h} \; Ah^{1-\alpha} - p_H h - i - \phi(i) - x^e - x^d + \left(i - \delta_K - \eta^k_{\mathbb{M}}(\pi,\mathbf{s})\sigma_K\right) q(\pi,\mathbf{s}) + \mu_\pi(\pi) q_\pi \\ &+ \left[\mu_s(\pi,\mathbf{s}) + \vartheta\sigma_S\sigma_K - \sigma_K^2 - (\eta^s_{\mathbb{M}}(\pi,\mathbf{s})\vartheta\sigma_S - \eta^k_{\mathbb{M}}(\pi,s)\sigma_K)\right]\mathbf{s}q_{\mathbf{s}} + \frac{(\sigma_S^2 - 2\vartheta\sigma_S\sigma_K + \sigma_K^2)\mathbf{s}^2}{2} q_{\mathbf{ss}} \\ &+ \lambda(\pi)\mathbb{E}^{\mathbf{x}^d}\left[\eta(\pi,\mathbf{s};Z,\mathbf{x}^e,\mathbf{x}^d)\left(q(\pi^{\mathcal{J}},\mathbf{s}^{\mathcal{J}})(1-N(x^e)(1-Z)) - q(\pi,\mathbf{s})\right)\right] \,. \end{aligned} \tag{OA.67}$$

By using the FOC for i, x^e, x^d, we obtain

$$q(\pi,\mathbf{s}) = 1 + \phi'(i(\pi,\mathbf{s}))\,, \tag{OA.68}$$

$$1 = -\lambda(\pi)\mathbb{E}^{\mathbf{x}^d}\left[(1-Z)\eta(\pi,\mathbf{s};Z,\mathbf{x}^e,\mathbf{x}^d)q(\pi^{\mathcal{J}},\mathbf{s}^{\mathcal{J}})N'(x^e)\right] \,, \tag{OA.69}$$

$$x^d = 0\,, \tag{OA.70}$$

which are similarly with solutions of the baseline model under competitive equilibrium. And then by using the FOC for h, we obtain

$$(1-\alpha)Ah(\pi,\mathbf{s})^{-\alpha} = p_H \,, \tag{OA.71}$$

which implies

$$h(\pi,\mathbf{s}) = \overline{\mathbf{h}} = \left(\frac{(1-\alpha)A}{p_H}\right)^{\frac{1}{\alpha}} \,. \tag{OA.72}$$

Recall that the optimal fossil fuel under planner problem is given by (72) as

$$\frac{(1-\alpha)Y}{H} = (1-\alpha)Ah(\pi,\mathbf{s})^{-\alpha} = p_H + m \,. \tag{OA.73}$$

We have the wedge of the optimal fossil fuel between social planner and competitive equilibrium is the SCC, m, as

$$m(\pi,\mathbf{s}) = -\frac{V_{\mathbf{S}}(\mathbf{K},\pi,\mathbf{S})}{f_{\mathbf{C}}(\mathbf{C},V)} = -\frac{b_s(\pi,\mathbf{s})}{\rho}\left(\frac{\mathbf{c}(\pi,\mathbf{s})}{b(\pi,\mathbf{s})}\right)^{\psi^{-1}} \,. \tag{OA.74}$$

Household's Optimization Problem. When making its decisions, the household takes the equilibrium risk-free rate r_t given by (OA.53) and the market price of (diffusion and jump) risks as given by (OA.52), we maximizes its value function J_t, and we show that household's value function $J_t = J(W_t, \pi_t, s_t)$ as follows

$$J(W, \pi, \mathbf{s}) = \frac{1}{1-\gamma} (u(\pi, \mathbf{s})W)^{1-\gamma} \,, \tag{OA.75}$$

where $u(\pi, \mathbf{s})$ is a welfare measure that will be endogenously determined. The HJB equation for the household in our decentralized market setting is given by

$$\begin{aligned} 0 = \max_{C, \widehat{H}, X^e, X^d} \quad & f(C, J) + \mu_\pi(\pi) J_\pi + \lambda(\pi) \int_0^1 \left[J\left(W^{\mathcal{J}}, \pi^{\mathcal{J}}, s^{\mathcal{J}}\right) - J(W, \pi, \mathbf{s}) \right] \xi(Z; \mathbf{x}^d) dZ \\ & + \left[r(\pi, \mathbf{s})W + (\mu_{\mathbf{Q}}(\pi, \mathbf{s}) - r(\pi, \mathbf{s}))\widehat{H} - C \right] J_W + \mu_s(\pi, \mathbf{s}) J_{\mathbf{s}} + \frac{(\sigma_S^2 - 2\vartheta\sigma_S\sigma_K + \sigma_K^2)\mathbf{s}^2 J_{\mathbf{ss}}}{2} \\ & + \frac{\left(\left(\frac{\mathbf{q}(\pi,\mathbf{s}) - \mathbf{s}\mathbf{q}_{\mathbf{s}}(\pi,\mathbf{s})}{\mathbf{q}(\pi,\mathbf{s})} \sigma_K \right)^2 + 2\vartheta \frac{\mathbf{q}(\pi,\mathbf{s}) - \mathbf{s}\mathbf{q}_{\mathbf{s}}(\pi,\mathbf{s})}{\mathbf{q}(\pi,\mathbf{s})} \frac{\mathbf{s}\mathbf{q}_{\mathbf{s}}(\pi,\mathbf{s})}{\mathbf{q}(\pi,\mathbf{s})} \sigma_K \sigma_S + \left(\frac{\mathbf{s}\mathbf{q}_{\mathbf{s}}(\pi,\mathbf{s})}{\mathbf{q}(\pi,\mathbf{s})} \sigma_S \right)^2 \right) \widehat{H}^2 J_{WW}}{2} \\ & + \left(\frac{s\mathbf{q}_{\mathbf{s}}(\pi, \mathbf{s})}{\mathbf{q}(\pi, \mathbf{s})} (\sigma_S^2 - \vartheta\sigma_K\sigma_S) + \frac{\mathbf{q}(\pi, \mathbf{s}) - s\mathbf{q}_{\mathbf{s}}(\pi, \mathbf{s})}{\mathbf{q}(\pi, \mathbf{s})} (\vartheta\sigma_S\sigma_K - \sigma_K^2) \right) \widehat{H} \mathbf{s} J_{W\mathbf{s}} \,, \end{aligned} \tag{OA.76}$$

where $\mu_{\mathbf{Q}}(\pi, \mathbf{s})$ is given by (OA.61).

OE.4 Firm and Household Optimization under Capital Taxation and Carbon Taxation

The government taxes the firm's capital stock K_t at a rate of $\tau_t^x = \mathbf{x}_{fb,t}^d$, where $\mathbf{x}_{fb,t}^d$ is the first-best mitigation spending to change the distribution of Z, obtained in Subsection 8.1. Then, the government spends $\mathbf{X}_t^d = \tau_t^x \mathbf{K}_t$ to reduce the tail risk of the disaster distribution. Similarly, the government taxes the using of fossil fuel H_t at a rate of $\tau_t^h = \mathbf{h}_{fb,t}^d$, where $\mathbf{h}_{fb,t}^d$ is the first-best using of fossil fuel, obtained in Subsection 8.1. We make the dependence of the tax rate τ_t^x and τ_t^h on π_t explicit by writing $\tau_t^x = \tau^x(\pi_t, \mathbf{s}_t) = \mathbf{x}_{fb,t}^d = \mathbf{x}_{fb}^d(\pi_t, \mathbf{s}_t)$ and $\tau_t^h = \tau^h(\pi_t, \mathbf{s}_t) = m(\pi_t, \mathbf{s}_t)$ where $m(\pi, \mathbf{s}) \equiv -\frac{V_S(\mathbf{K}, \pi, \mathbf{S})}{f_{\mathbf{C}}(\mathbf{C}, V)}$ is the SCC given by (72).

Facing a capital tax rate of $\tau^x(\pi_t, \mathbf{s}_t)$ and a carbon tax rate of $\tau^h(\pi_t, \mathbf{s}_t)$, each firm solves the following problem:

$$\max_{I, X^e, X^d, H} \mathbb{E}^{\mathbf{x}_d} \left[\int_0^\infty \left(\frac{\mathbb{M}_t}{\mathbb{M}_0} \left[AK_t^\alpha H_t^{1-\alpha} - \tau^x(\pi_t, \mathbf{s}_t)K_t - \tau^h(\pi_t, \mathbf{s}_t)(H_t - \mathbf{H}_t) - I_t - \Phi_t - X_t^e - X_t^d - p_H H_t \right] \right) dt \right] \,, \tag{OA.77}$$

taking the equilibrium SDF $\mathbb{M}_t$ as given. First, the firm has no incentive to spend on disaster distribution mitigation, again as doing so is costly but yields no benefit for the firm. Thus, $X^d = 0$. The tax makes the firm behave as if its productivity is lowered from $AK_t^\alpha H_t^{1-\alpha}$ to $AK_t^\alpha H_t^{1-\alpha} - \tau^x(\pi_t, \mathbf{s}_t)K_t - \tau^h(\pi_t, \mathbf{s}_t)H_t$. Applying the Ito's Lemma to firm value $Q(K_t, \pi_t, \mathbf{s}_t) = q(\pi_t, \mathbf{s}_t)K_t$ given in (6) and using (32), we obtain the following HJB equation for $q(\pi_t, \mathbf{s}_t)$:

$$\begin{aligned} r(\pi, \mathbf{s})q(\pi, \mathbf{s}) = \max_{i, x^e, x^d, h} \; & Ah^{1-\alpha} - \tau^x(\pi, \mathbf{s}) - \tau^h(\pi, \mathbf{s})h - p_H h - i - \phi(i) - x^e - x^d + \left(i - \delta_K - \eta_{\mathbb{M}}^k(\pi, \mathbf{s})\sigma_K \right) q(\pi, \mathbf{s}) + \mu_\pi(\pi) q_\pi \\ & + \tau^h(\pi, \mathbf{s})\mathbf{h} + \mu_\pi(\pi) q_\pi(\pi, \mathbf{s}) + \left[\mu_s(\pi, \mathbf{s}) + \vartheta\sigma_S\sigma_K - \sigma_K^2 - (\eta_{\mathbb{M}}^s(\pi, \mathbf{s})\vartheta\sigma_S - \eta_{\mathbb{M}}^k(\pi, \mathbf{s})\sigma_K) \right] \mathbf{s} q_{\mathbf{s}} \\ & + \frac{(\sigma_S^2 - 2\vartheta\sigma_S\sigma_K + \sigma_K^2)\mathbf{s}^2}{2} q_{\mathbf{ss}} + \lambda(\pi) \mathbb{E}^{\mathbf{x}^d} \left[\eta(\pi, \mathbf{s}; Z, \mathbf{x}^e, \mathbf{x}^d) \left(q(\pi^{\mathcal{J}}, \mathbf{s}^{\mathcal{J}})(1 - N(x^e)(1 - Z)) - q(\pi, \mathbf{s}) \right) \right] . \end{aligned} \tag{OA.78}$$

Note that the tax rate $\tau^x(\pi, \mathbf{s})$ and $\tau^h(\pi, \mathbf{s})$ appears in (OA.78). The FOCs for i and x^e are given by (OA.68) and (OA.69), respectively, the same as in the no-tax competitive-market economy model of subsection OE.3. Importantly, the FOCs for h is given by

$$(1-\alpha)Ah(\pi, \mathbf{s})^{-\alpha} = p_H + \tau^h(\pi, \mathbf{s})\,. \tag{OA.79}$$

OE.5 Optimal Taxation in Markets Restores First-Best

In this section, we show that the household's value function in the competitive economy with optimal taxes is the same as the value function under the first-best. As the household's value function in a market economy depends on wealth W while the planner's value function depends on $\mathbf{K}$, we use the equilibrium result $W_t = \mathbf{q}(\pi_t, \mathbf{s}_t)\mathbf{K}_t$ to write the household's value function as $J(W_t, \pi_t) = J(\mathbf{q}(\pi_t, \mathbf{s}_t)\mathbf{K}_t, \pi_t, \mathbf{s}_t)$ in the market economy with taxation. Therefore, the value functions in the two economies are equal, $V(\mathbf{K}_t, \pi_t, \mathbf{s}_t) = J(W_t, \pi_t, \mathbf{s}_t)$, if and only if $b(\pi, \mathbf{s})$ in the first-best economy equals the product $u(\pi, \mathbf{s})\mathbf{q}(\pi, \mathbf{s})$ in the competitive economy with taxes.

Specifically, we show the following results: (1.) the first-order conditions for $\mathbf{i}(\pi, \mathbf{s})$ and $\mathbf{x}^e(\pi, \mathbf{s})$ in the competitive economy with an optimal tax rate (set at the planner's first best distribution mitigation $\mathbf{x}^d_{fb,t}$) are the same as the corresponding first-order conditions in the planner's economy and carbon tax is the same as the SCC in the planner's economy; (2.) the implied ODE for $u(\pi, \mathbf{s})\mathbf{q}(\pi, \mathbf{s})$ in the competitive market economy is the same as the ODE (74) for $b(\pi, \mathbf{s})$ in the planner's economy; (3.) all the boundary conditions at $\pi = 0$ and $\pi = 1$ in the two economies are the same. Below is a step-by-step proof.

First, combining the equilibrium aggregate investment FOC, $\mathbf{q}(\pi, \mathbf{s}) = 1 + \phi'(\mathbf{i}(\pi, \mathbf{s}))$, implied by (36) with the optimal scaled consumption rule $\mathbf{c}(\pi, \mathbf{s}) = \rho^{\psi} u(\pi, \mathbf{s})^{1-\psi}\mathbf{q}(\pi, \mathbf{s}) = (\rho \mathbf{q}(\pi, \mathbf{s}))^{\psi} [u(\pi, \mathbf{s})\mathbf{q}(\pi, \mathbf{s})]^{1-\psi}$, implied by (41) and $W = \mathbf{q}(\pi, \mathbf{s})\mathbf{K}$, we obtain the following expression for consumption:

$$\mathbf{c}(\pi, \mathbf{s}) = [\rho(1 + \phi'(\mathbf{i}(\pi, \mathbf{s})))]^{\psi} [u(\pi, \mathbf{s})\mathbf{q}(\pi, \mathbf{s})]^{1-\psi}\,. \tag{OA.80}$$

Using the goods market clear condition $\mathbf{c}(\pi, \mathbf{s}) = Ah(\pi, \mathbf{s})^{1-\alpha} - \tau^x(\pi, \mathbf{s}) - \tau^h(\pi, \mathbf{s})(h(\pi, \mathbf{s}) - \mathbf{h}(\pi, \mathbf{s})) - p_H h(\pi, \mathbf{s}) - \mathbf{i}(\pi, \mathbf{s}) - \phi(\mathbf{i}(\pi, \mathbf{s})) - \mathbf{x}^e(\pi, \mathbf{s})$ and the conjecture $b(\pi, \mathbf{s}) = u(\pi, \mathbf{s})\mathbf{q}(\pi, \mathbf{s})$, we obtain the following expression:

$$\begin{aligned} b(\pi, \mathbf{s}) \;=\; & [Ah(\pi, \mathbf{s})^{1-\alpha} - \tau^x(\pi, \mathbf{s}) - \tau^h(\pi, \mathbf{s})(h(\pi, \mathbf{s}) - \mathbf{h}(\pi, \mathbf{s})) - p_H h(\pi, \mathbf{s}) - \mathbf{i}(\pi, \mathbf{s}) \\ & -\phi(\mathbf{i}(\pi, \mathbf{s})) - \mathbf{x}^e(\pi, \mathbf{s})]^{1/(1-\psi)} [\rho(1 + \phi'(\mathbf{i}(\pi, \mathbf{s})))]^{-\psi/(1-\psi)}\,, \end{aligned} \tag{OA.81}$$

which is the same as the investment FOC, given in (75), for the planner's problem, provided that the capital tax rate equals $\mathbf{x}^d_{fb}(\pi, \mathbf{s})$: $\tau^x(\pi, \mathbf{s}) = \mathbf{x}^d_{fb}(\pi, \mathbf{s})$, and $h(\pi, \mathbf{s}) = \mathbf{h}(\pi, \mathbf{s}) = \mathbf{h}_{fb}(\pi, \mathbf{s})$ for $\tau^h(\pi, \mathbf{s}) = m(\pi, \mathbf{s})$. Note that (OA.81) summarizes both the consumer's and the firm's optimization FOCs in the market economy with optimal taxes.

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