COMPOUND TAIL RISK

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Risk Management

- A Risk is a bad event that may happen in the future.
  - How bad is it?
  - How likely is it?

- With this information the owner/investor will compare the risk with the return in a good outcome and decide whether to take the risk.
- This problem is widely applied and solutions are available for simple examples

COMPOUND RISK

- Maybe there are multiple risks that might occur. We call this compound risk.
- For example an investor might own stocks and bonds. There are two different risks – stocks can fall and bond values can fall. This is a very familiar problem but lets understand how we deal with it via stress tests.
  - We could analyze the risk of stock declines separately from the risk of bond declines. Lets consider a 40% drop in stocks and a 10% drop in bond prices, both of which are extreme but plausible. In fact each has occurred during roughly 1% of six monthly intervals since 2000. This involves doing two stress tests.
  - For a 60/40 portfolio, the loss from a stock collapse is -24% while the loss from the bond drop is only -4%.
  - How do we combine these two risks? Add them? Take the max? Shift into bonds?
- Risk managers are confronted with such problems every day.

MORE EXAMPLES

- Climate Risk and Market Risk
- Physical Risk and Transition Risk
- Market Risk and Liquidity Risk
- Market Risk and the Risk that Risk will rise
- Inflation Risk and Unemployment Risk
- Climate Risk and Pandemic Risk
- Macro Risk and Credit Risk and Interest Rate Risk
- Everything, Everywhere, All At Once

THE PROBABILITY OF RISK REALIZATION

- Clearly combining these risks into a compound risk requires knowing about the joint probability of their realization.

- In particular, what is the probability that one risk has an extreme realization when another risk has an extreme realization?

- In the next slide I will define the important properties of tail dependence for two factors.

THE PROBABILITY OF RISK REALIZATION

- Suppose the two risks are called x and y and they each have probability u quantiles defined by

$$P\left(x < q_x\right) = u = P\left(y < q_y\right)$$

- The conditional probability of tail events is defined by the function lambda $P\left(y < q_y \middle| x < q_x\right) = P\left(x < q_x \middle| y < q_y\right) = \lambda\left(u\right)$

- Tail dependence is defined as the limit of this conditional probability as u goes to zero. $\overline{\lambda} = \lim_{u \to 0} \lambda\left(u\right)$

PROPERTIES OF TAIL DEPENDENCE

- It is clear that these tail measures only require order statistics such as ranks. They therefore can be applied to any distribution. We say they depend only on the joint distribution of the ranks which is called the copula.
- Statistical theorems on extreme values can be applied to joint distributions to elucidate tail properties.
- Clearly, if x and y are disjoint so that they can never both occur, then .

$$\lambda\left(u\right)=0$$

- If x and y are independent or just "tail independent", then

$$\lambda\left(u\right)=u,\ \ \overline{\lambda}=0$$

- If x and y are perfectly dependent in the tails, then $\lambda\left(u\right)=1,\ \ \overline{\lambda}=1$

THE RISK MANAGER'S PROBLEM

- $FIND \quad (q_x, q_y) \ni P(x > q_x \cap y > q_y) = 1 - \alpha$
- Choosing (q_x, q_y) is choosing a scenario for both risks and $1-\alpha$ is the probability of not failing either stress test.

$$FIRM \quad VALUE: \quad d\log V = \beta_x x + \beta_y y + e, \quad E(e|y,x) = 0$$

$$and \ Let \ Stressed \ Loss \equiv \beta_x q_x + \beta_y q_y, \qquad then$$

$$For\,all \ \{x > q_x\}, \{y > q_y\} \ E(d\log V | x, y) \geq Stressed \ Loss$$

- Applying this to different firms and different times will give different exposures as the betas will be different. Risk can be compared across firms and over time because the confidence is held constant at $1-\alpha$.

SOLUTION

$$P\left(y < q_y\right) = P\left(x < q_x\right) = u^*$$

$$\alpha = 2u^* - P\left(y < q_y \bigcap x < q_x\right) \approx \left(2 - \bar{\lambda}\right)u^*$$

since $P\left(x < q_x \bigcap y < q_y\right) = u\lambda\left(u\right) \approx u\bar{\lambda}$

To get a size alpha scenario, choose u* based on the joint probability as above.

If you use a different u, then the second line will give the effective alpha of the scenario.

HOW TO SET U*

- Tail dependence can be estimated empirically or can be deduced theoretically from the economics of the risks. We will today assume that it is estimated.
- $$u^* = \alpha / \left(2 - \bar{\lambda}\right)$$
- Therefore if there is no tail dependence $\bar{\lambda} = 0, \quad u^* = \alpha / 2$
- If there is perfect tail dependence, $\bar{\lambda} = 1, \quad u^* = \alpha$

FOR THREE RISKS

- If there are three risks, then there are three measures of tail dependence and one measure of the probability that all three risks occur.
- Assuming that the probability that all three occur is negligible, then

$$u^* = \frac{\alpha}{\left(3 - \overline{\lambda}_{12} - \overline{\lambda}_{13} - \overline{\lambda}_{23}\right)}$$

BACK TO THE STOCK BOND EXAMPLE

- For stocks and bonds, it is unlikely that they both fall extremely over the same six month period. However, this is what happened this year so we might want to take that into account going forward.
- If we suppose that these events are tail independent, then we simply choose u* as half of alpha.
- From the same historical data, the compound stress scenario has bonds falling 11% and stocks 48% over the next six months.
- For the 60/40 portfolio, stressed loss=-33% (actual 1% quantile=-22%) . The probability is 99% that this portfolio will fall by less than 33% by this calculation.
- Fact: this portfolio did decline by 33% during the financial crisis but the .01 quantile is only -22% which satisfies the inequality. (I used SPY and IEF)
- Note: all returns are continuously compounded (i.e. log returns)

EXAMPLE: US VS INTERNATIONAL EQUITIES

- Using SPY and EFA over 20 years
- 1% daily quantile for SPY and EFA are -3.5% and -4%
- For daily data, tail dependence is close to 1 so u*=1%
- For an 80/20 portfolio Stressed Loss=-3.6% (actual 1% =-3.58)
- 1% semiannual quantile for SPY and EFA are -41.5% and -53%
- For semiannual tail dependence is roughly .6 so u*=.7%
- For this u*, SPY is -45% and EFA is -56%
- For 80/20 six month stressed loss= -47.2% (actual 1%=-44%)

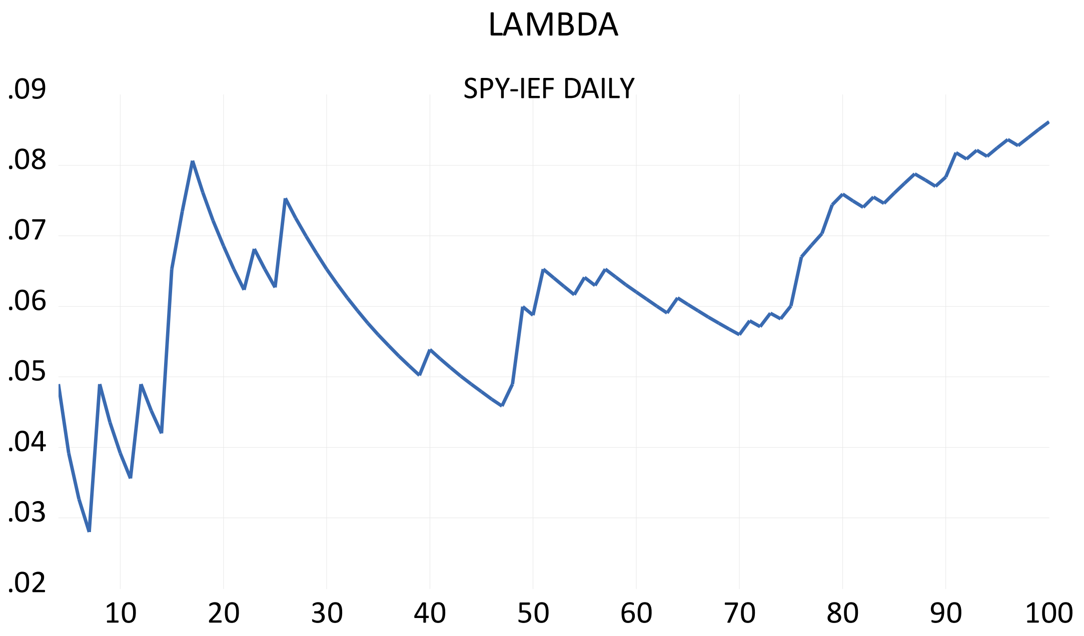
ESTIMATING TAIL DEPENDENCE

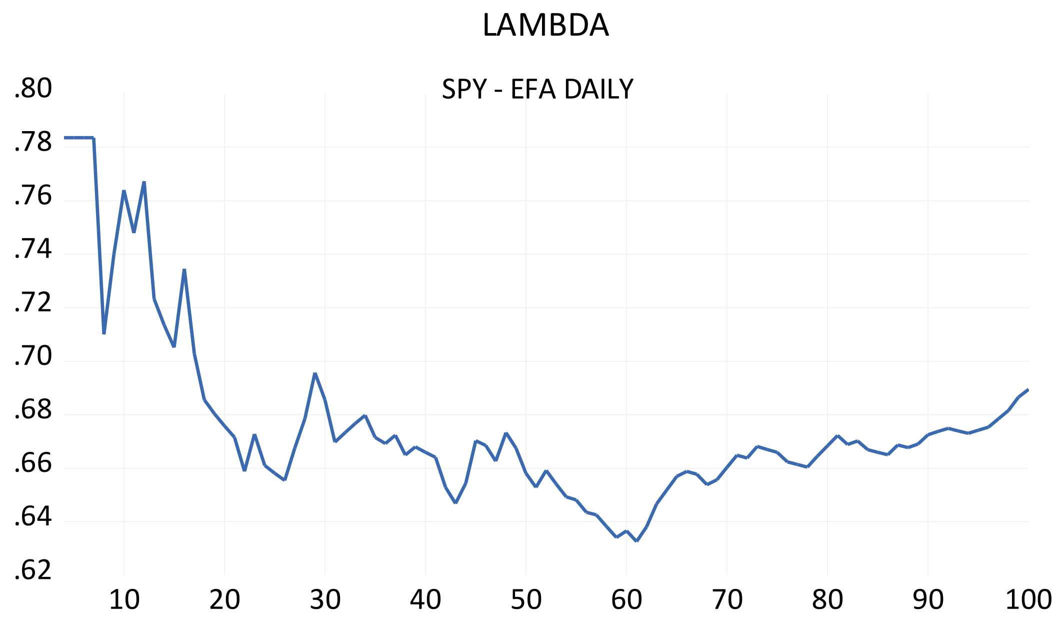
- This is a large literature in statistics.
- Frahm, Junker and Schmidt(2005) give a survey and monte carlo comparison of methods. Here are 4 classes of solutions.
- 1. Assume (x,y)~F(x,y) and F is known up to some parameters.
- 2. Assume only that the copula is known up to some parameters
- 3. Assume that observations beyond a threshold follow the tail distribution.
- 4. Non-parametric based on empirical copula (i.e. ranks)

A SIMPLE NON-PARAMETRIC ESTIMATOR

- From a sample of (y,x) observations, and a range of probabilities u, compute the conditional probabilities. Each point is 10 bp.

$$\lambda(u) = P\left(x < q_x \middle| y < q_y\right), \; u = P\left(x < q_x\right) = P\left(y < q_y\right)$$





MONTE CARLO EXAMINATION

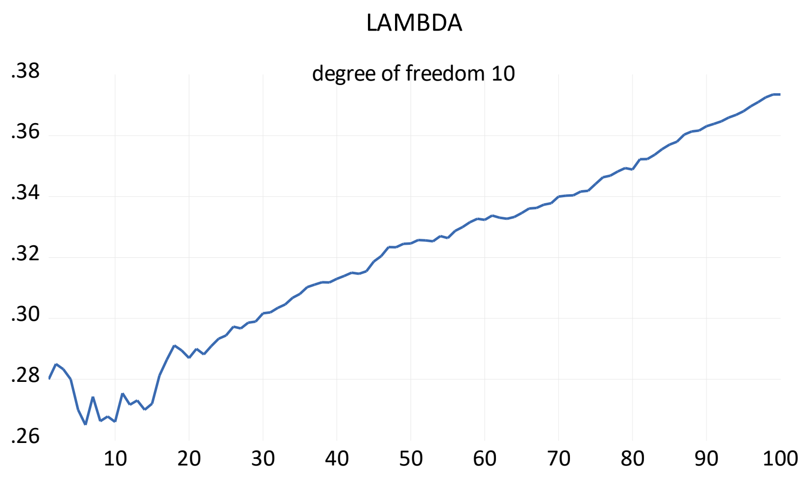
- DATA GENERATION, 100,000 observations

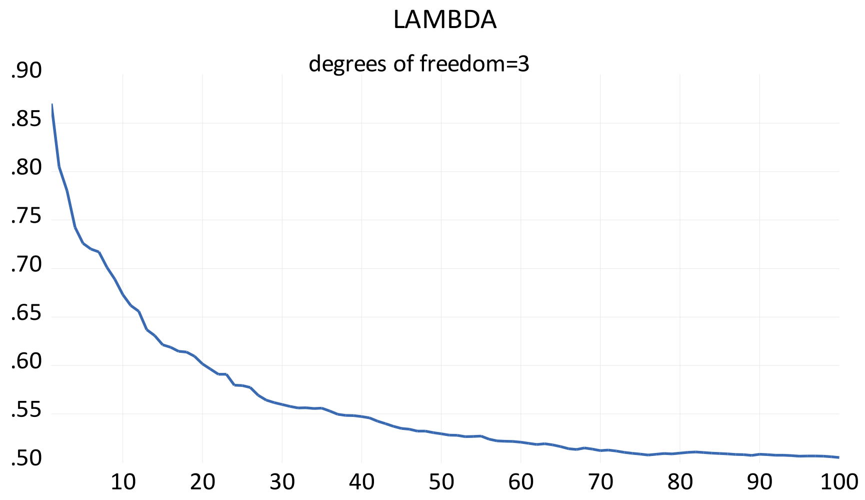
$$y = z + e_y$$

$$x = z + e_x$$

$$\left(e_y, e_x\right) \sim N(0,1)$$

$$z \sim student - t, df = 3,10$$





ANALYSIS OF MARKET AND CLIMATE RISK

- We now consider the compound risk of broad market decline and climate deterioration.
- Climate deterioration includes both physical risks and transition risks. In the US, the public policy driving transition risk is more in the future than the present but even future transition plans can have an important impact today.
- Why is this interesting?
  - If fossil energy is forced to reduce output, then the economy may collapse. These maybe should move same direction.
  - However market declines reduce emissions and may reduce the pressure on fossil energy to change. So they may move opposite ways.
  - From a big picture perspective, a healthy economy emits more GHG and therefore makes climate mitigation more likely. (opposite)
  - But climate change is a potentially big cost to the economy and should lead to market decline. (same)

REGULATORS ARE DEVELOPING APPROACHES TO CLIMATE STRESS TESTING FOR BANKS

- Is the banking sector vulnerable to the physical or transition risks of climate change?
- Central bankers around the world have joined the NGFS (Network of Central Banks and Supervisors for Greening the Financial System) and are developing tools to assess the stability of the financial system in the face of climate change.
- Should these scenarios include the risk of market decline as a compound risk?
- At VRI we have developed a market based climate stress test that includes this compound risk.

The Jung, Engle, Berner model implemented in VRI's VLAB

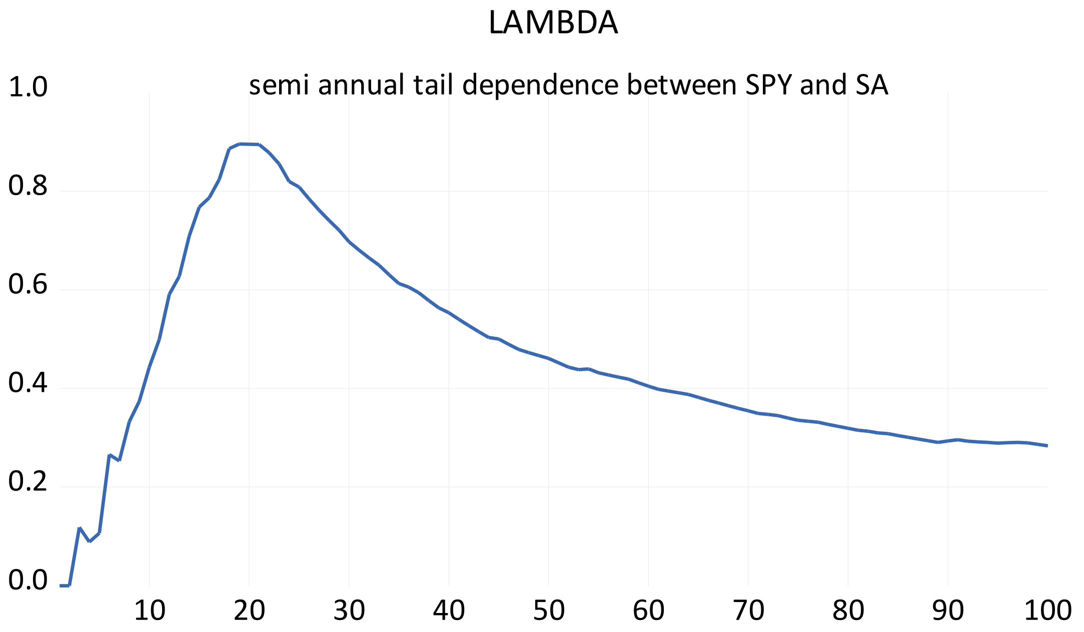
- The market cap of a financial institution is viewed as being sensitive to two factors, a market factor and a climate factor.
- The response to these factors varies over time as volatilities and correlations change. The dynamic conditional beta model forecasts beta from the forecasts of volatilities and correlations. At each point in time this gives both a climate beta and a market beta.
- The stressed loss depends upon these betas and upon the magnitude of the stress for each factor.

COMPOUND RISK ANALYSIS

- Market risk is the six month return on SPY etf.
- Climate risk is the Stranded Asset Risk Portfolio from VRI
  - SA=.7*KOL+.3*XLE-SPY where KOL is a coal etf, and XLE is an energy etf.
- From data since 2000 the 1% quantile of six month returns is -33% for SPY and -64% for SA
- In March 2020 as the pandemic was deepening, the Citibank beta for climate risk =.66 and for market risk = 1.53
- The damages from these two risks treated individually are therefore
- Climate Risk= -42%   and Market Risk= -50%

ADJUSTING FOR COMPOUND RISK

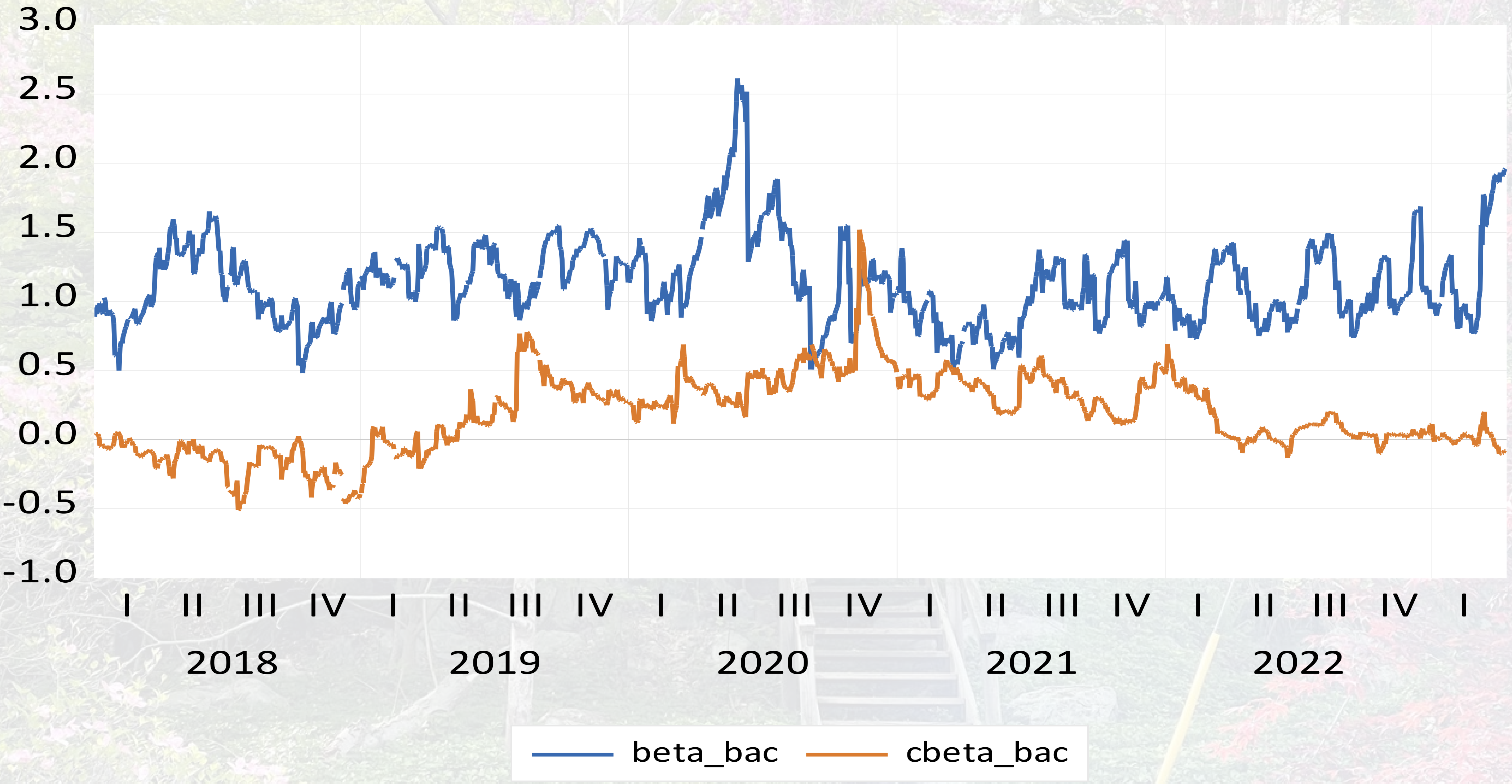
- To adjust these measures for compound risk, the confidence with which the losses are measured should be equalized.
- The dependence between climate risk and market risk is not simple to evaluate. The empirical tail dependence is shown here.



ADJUSTING FOR COMPOUND RISK

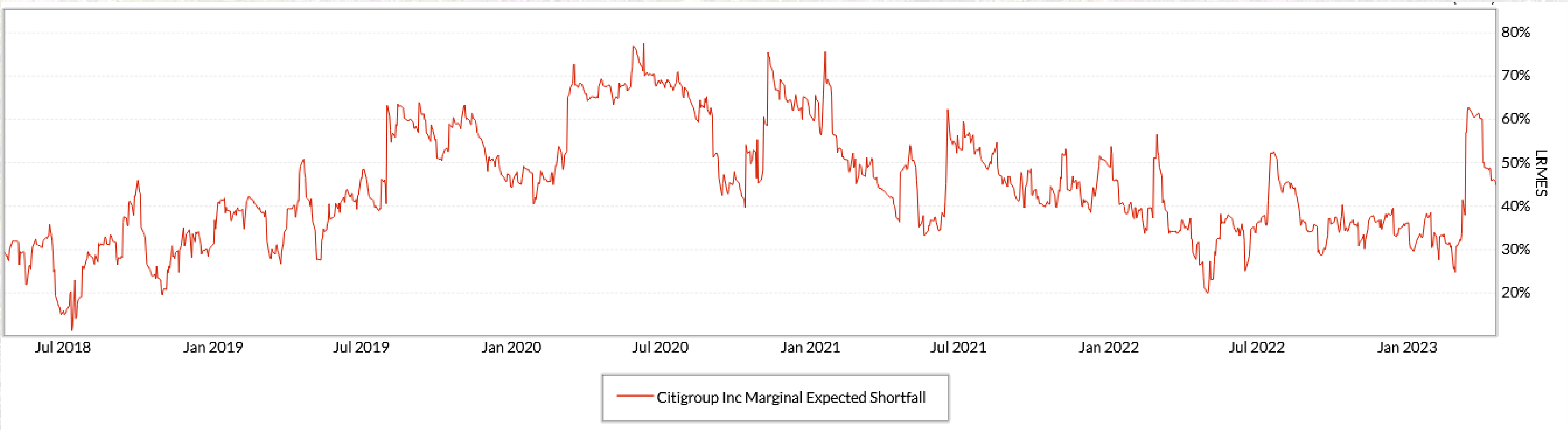
- The limit as u goes to zero appears to be zero but the hump at about 2% suggests that at higher probabilities it is headed for strong dependence. It turns out that for 2% and lower quantiles, the only joint events are during the great financial crisis.
- If we use $\bar{\lambda} = 0,\ u^* = \alpha / 2$
- Then the quantiles are -73% for SA and -38% for SPY leading to
- Stressed losses=-106% continuously compounded or -66% fractional.
- exp(-1.06)-1=-66%

CITI MARKET AND CLIMATE BETAS



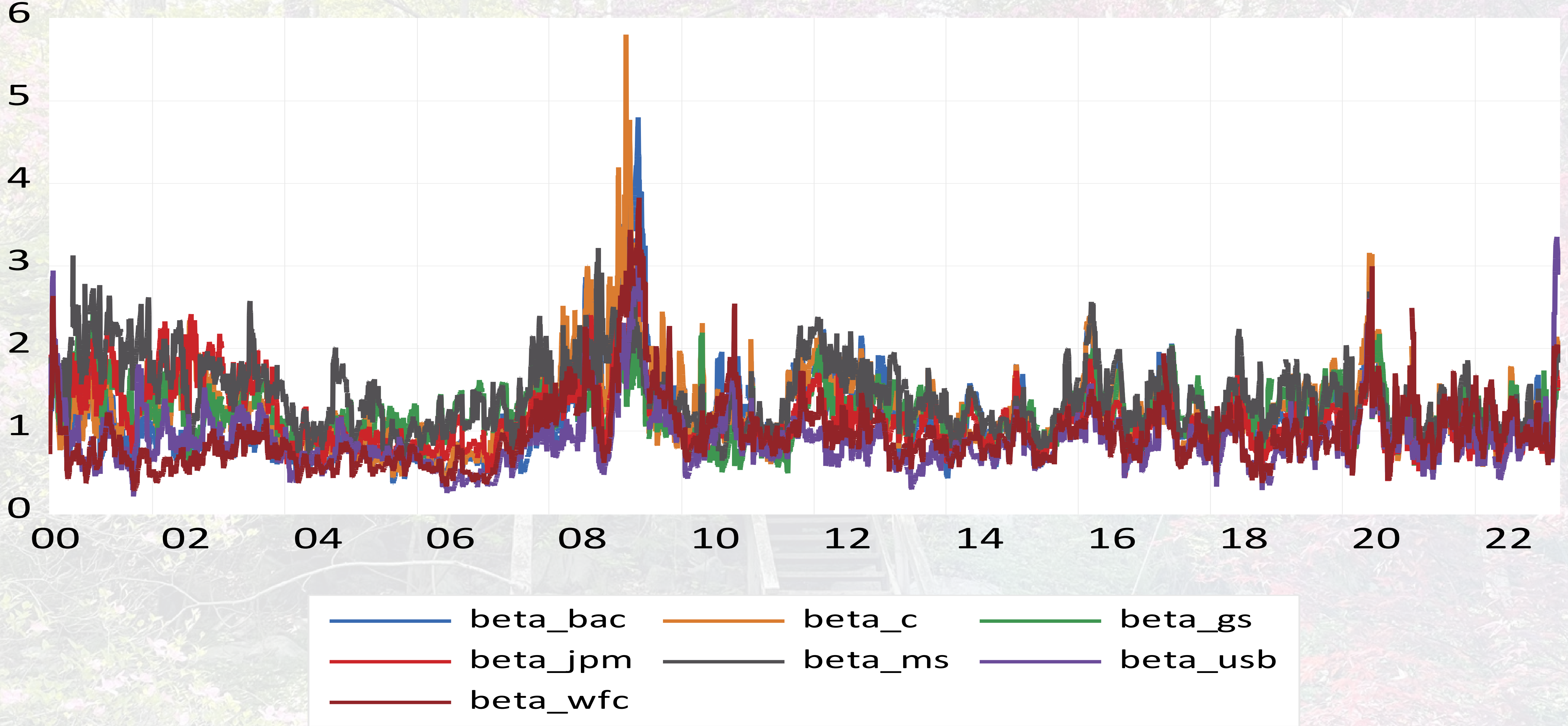
To compare Citi exposure over time and to other banks, we use the same scenario

Note: the quantile are converted to simple returns to insert in VLAB and the result is identical to LRMES measured in simple returns.

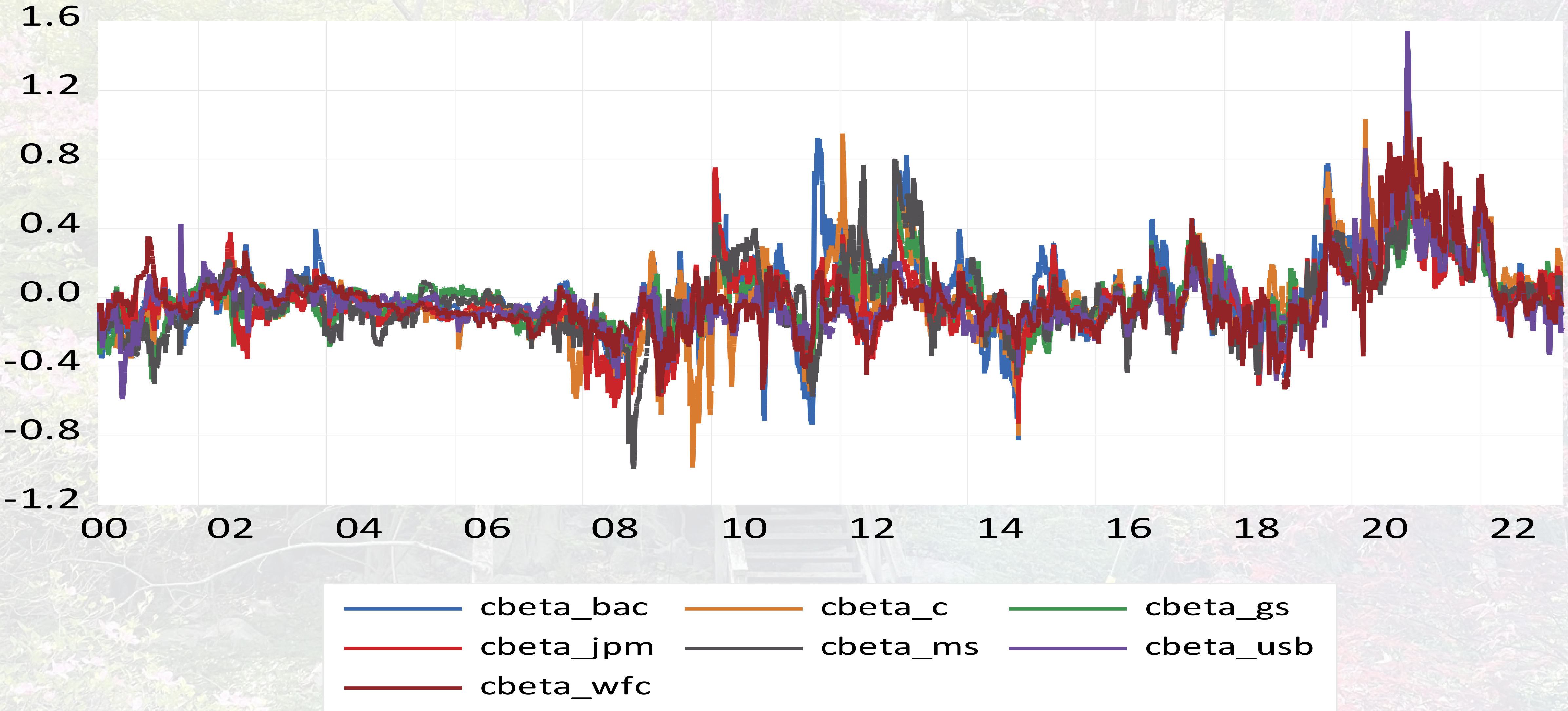


Stressed Losses for Citibank with Compound Risk from Market and Climate

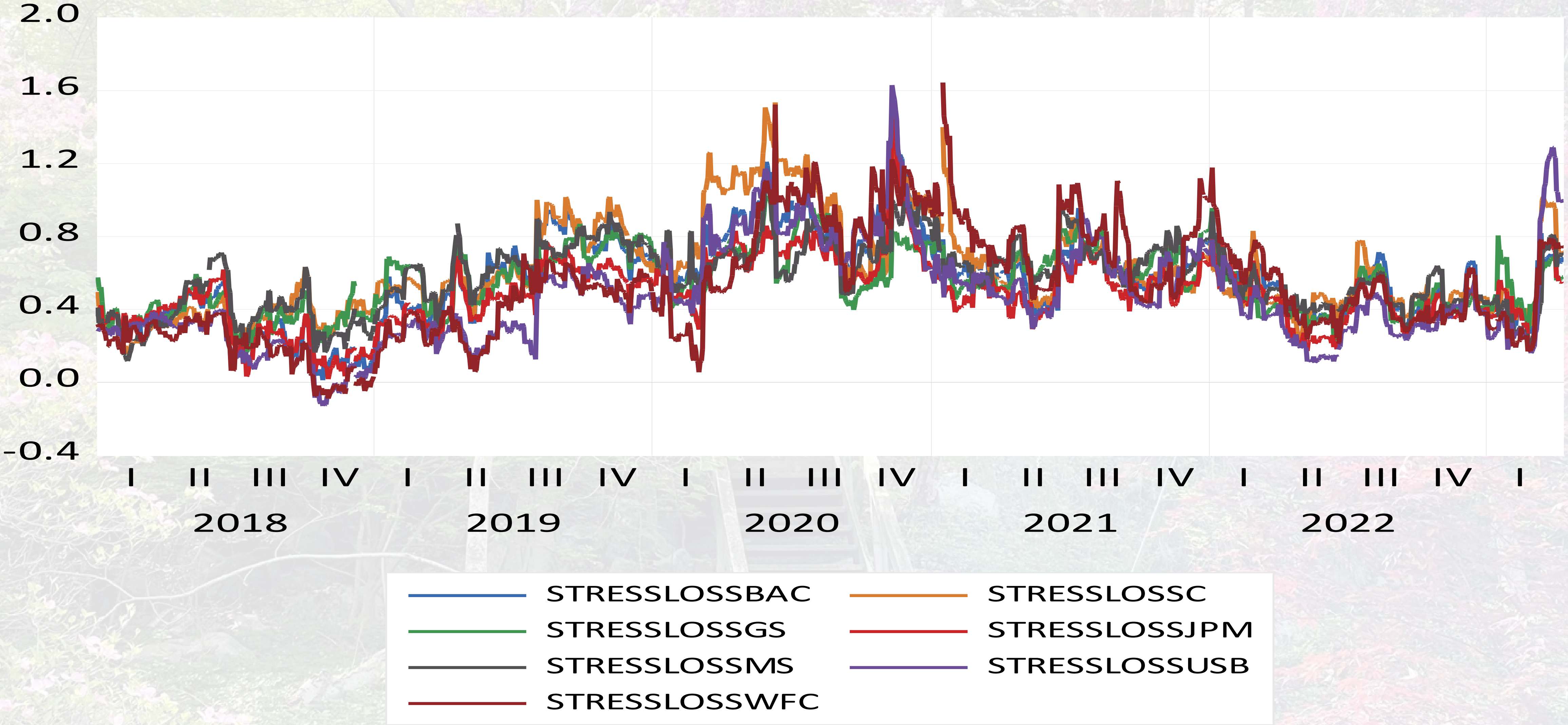
MARKET BETAS



CLIMATE BETAS



STRESSED LOSS



CORRELATIONS OF BETAS

	BETA_BAC	BETA_C	BETA_GS	BETA_JPM	BETA_MS	BETA_USB	BETA_WFC
BETA_BAC	1.000000	0.794153	0.510917	0.679454	0.564977	0.762116	0.832030
BETA_C	0.794153	1.000000	0.543176	0.726825	0.646339	0.664607	0.693750
BETA_GS	0.510917	0.543176	1.000000	0.633637	0.768188	0.413800	0.365330
BETA_JPM	0.679454	0.726825	0.633637	1.000000	0.732965	0.694270	0.576498
BETA_MS	0.564977	0.646339	0.768188	0.732965	1.000000	0.448274	0.366993
BETA_USB	0.762116	0.664607	0.413800	0.694270	0.448274	1.000000	0.817898
BETA_WFC	0.832030	0.693750	0.365330	0.576498	0.366993	0.817898	1.000000

	CBETA_BAC	CBETA_C	CBETA_GS	CBETA_JPM	CBETA_MS	CBETA_USB	CBETA_WFC
CBETA_BAC	1.000000	0.807466	0.739740	0.838389	0.703984	0.733598	0.739993
CBETA_C	0.807466	1.000000	0.766595	0.764367	0.726394	0.741551	0.690021
CBETA_GS	0.739740	0.766595	1.000000	0.779059	0.828940	0.657098	0.628200
CBETA_JPM	0.838389	0.764367	0.779059	1.000000	0.723263	0.769043	0.758306
CBETA_MS	0.703984	0.726394	0.828940	0.723263	1.000000	0.590984	0.565862
CBETA_USB	0.733598	0.741551	0.657098	0.769043	0.590984	1.000000	0.840554
CBETA_WFC	0.739993	0.690021	0.628200	0.758306	0.565862	0.840554	1.000000

VARIATION IN BETAS OVER TIME AND BANK

- Both climate and market betas are quite correlated:
  - Bank holdings move together or
  - Market impacts are sensitive to events outside the banks, or
  - Factor movements may impact the betas

AVENUES for future RESEARCH

- Better univariate quantiles based on EVT
- Better tail dependence measures using copulas and possibly threshold copulas
- Better measures of impact and whether the betas are actually non-linear
- Is there predictability of the betas based on other information?
- How to handle the natural complexity that comes with real examples.