Occupational Reallocation Within and Across Firms: Implications for Labor-Market Polarization*

Toshihiko Mukoyama† Georgetown University

Naoki Takayama‡ Hitotsubashi University

Satoshi Tanaka§ University of Queensland

April 2024

Abstract

This study analyzes how labor-market frictions interact with firms' decisions to reallocate workers across different occupations during labor-market polarization. We compare the patterns of occupational reallocation within and across firms in the United States and Germany in recent years. We find that within-firm reallocation contributes significantly to the decline in employment in routine occupations in Germany, but much less in the United States. We construct a general equilibrium model of firm dynamics and find that the model with different firing taxes can replicate the difference in firm-level adjustment patterns across these countries. We conduct two counterfactual experiments, highlighting the different roles played by the within-firm cost of reorganizing occupational mix and across-firm frictions created by firing taxes. The results suggest that the latter plays a more significant role in labor market polarization. Higher firing cost leads to greater and faster polarization.

Keywords: occupational reallocation, firing costs, labor-market polarization

JEL Classifications: E24, J24, J62

*For comments and discussions, we thank Gianluca Violante and Matthias Doepke as well as other seminar participants at the 2019 GRIPS-UT Macroeconomics and Policy Workshop in Tokyo, 2021 CEF, 2021 SED, VAMS, VEAMS, VII Santiago Macro Workshop, Canon Institute for Global Studies, Federal Reserve Bank of Cleveland, Federal Reserve Bank of New York, Federal Reserve Board, Hitotsubashi University, Kyoto University, National University of Singapore, Osaka University, Singapore Management University, UNC Chapel Hill, University of Virginia, and Waseda University. This study is conducted as a part of the Project "Macroeconomic and social security policies under demographic aging: dynamics of firms, individuals and inequality" undertaken at the Research Institute of Economy, Trade and Industry (RIETI). The draft of this paper was presented at the DP seminar of RIETI and we would also like to thank participants of the RIETI DP Seminar for their helpful comments. We acknowledge the financial support from the Joint Usage/Research Center, Institute of Economic Research at Hitotsubashi University (Grant ID IERPK2002) and Japan Society for the Promotion of Science (KAKENHI Grant Number JP21K13256).

†Contact information: tm1309@georgetown.edu

‡Contact information: ntakayama@ier.hit-u.ac.jp

§Contact information: s.tanaka@uq.edu.au

1 Introduction

In recent years, many advanced economies have experienced significant declines in employment in middle-skilled routine occupations. This phenomenon, often referred to as "labor-market polarization," has received considerable attention in the macroeconomic and labor economics literature.[1] The polarization is often attributed to technological change, which allows firms to automate routine tasks by substituting workers with machines.

From firms' perspective, automation requires occupational reallocation: reducing employment in occupations that are substituted by automation and increasing employment in occupations that complement automation. Given the heterogeneity in technology adoption and various (potentially time-varying) factors across firms, the transformation of the occupational mix likely accompanies reallocation of workers across firms.

How do firms reallocate workers across occupations under different labor-market environments? In particular, do firms change occupational mix by hiring and firing different workers, or by changing workers' tasks within the firm? We ask this question with a particular focus on differences in labor-market institutions. Several decades of research has shown that the US economy and continental European economies have very different labor-market institutions (see, for example, Mortensen et al., 1999; Ljungqvist and Sargent, 2008). One specific difference with extensive research attention is the ease of firing. In our context as well, across-firm occupational reallocation may be costlier in a labor market when firing is difficult.

Using micro-level panel datasets from the United States and Germany, we develop a novel decomposition method to compare the contributions of within-firm occupational reallocation to labor-market polarization in both countries. We show that within-firm reallocation contributes more to the decline of routine occupation employment in Germany than in the United States.

Motivated by the empirical observations, we build a dynamic general equilibrium model

[1]See, for example, Autor et al. (2006) for the United States, Goos and Manning (2007) for the UK, and Goos et al. (2009) for 16 European countries. Acemoglu and Autor (2011) survey the literature.

with heterogeneous workers, extending the standard firm-dynamics framework by Hopenhayn (1992) and Hopenhayn and Rogerson (1993). Our framework departs from the standard model by considering three different occupations and endogenous decisions of automation by firms. When the firm decides to automate, it optimally adjusts the occupational mix, given the costs of adjustments within and across firms.

Our theoretical framework includes two distinct variables that determine the firm-level productivity. The first variable affects productivity in a Hicks-neutral manner. This variable is formulated as an exogenous idiosyncratic shock. This type of shock is commonly employed in the Hopenhayn (1992)-type standard firm dynamics models. Essentially, this shock symmetrically affects the demand for all occupations. The second variable represents "automation productivity," which influences the marginal products of different tasks differently. The automation productivity is chosen by the firm, subject to a cost. We interpret the improvement in automation productivity as the costly adoption of new technology. Because the adoption induces changes in the occupational mix, the adoption decision is also affected by labor market frictions.

We calibrate the model to the US economy and find that the empirically relevant level of firing taxes in Germany can replicate the differences in the patterns of occupational reallocation between the United States and Germany. We then conduct two counterfactual experiments to assess how the frictions of occupational reallocation affect the degree and the speed of labor market polarization. In the first experiment, we reduce firms' reorganization cost, an adjustment cost for within-firm occupational reallocation. In the second experiment, we reduce firms' cost for firing.

We find that the within-firm reorganization cost has a small impact on the degree of polarization, whereas the firing cost has a significant impact on polarization. In particular, we find that the firing tax makes the labor market *more* polarized: the level of routine employment is higher without the firing taxes, whereas the level of cognitive employment is lower. Individual firms adjust the composition of occupational employment faster when the firing tax is larger. The reason for this seemingly counterintuitive result is that the firms are forward-looking. In the model, firms are constantly hit by idiosyncratic productivity shocks. Thus, when there is a firing tax, firms that are likely to adopt automation technology will reduce routine hires when they suffer a negative idiosyncratic shock, seeing it as an opportunity to prepare for future automation adoption. On the other hand, without a firing tax, the firm is more likely to keep the routine workers, because the firm can easily adjust the occupational composition in the future. This result has important implications for predicting how policies like the firing tax affect the polarizing labor market.

Our work is motivated by the recent empirical literature which documents sizable within-firm occupational reallocation in several European countries during labor-market polarization. Using French establishment data, Behaghel et al. (2012) is one of the earliest papers to find within-firm occupational reallocation following a firm's adoption of information and communication technologies (ICT). Battisti et al. (2017) and Dauth et al. (2018) report similar evidence using German establishment data after ICT or industrial robot-exposure shocks. Our empirical analysis of the United States and Germany builds on these empirical findings and finds patterns consistent with these studies. We further evaluate the role of labor-market institutions on the intra- and inter-firm reallocation of workers.

Several recent macroeconomic studies have built general equilibrium models and quantitatively analyzed the process of labor-market polarization. For example, Eden and Gaggl (2018), vom Lehn (2020), and Jaimovich et al. (2021) analyze the polarizing labor market using dynamic general equilibrium models. With a representative-firm assumption, the first two studies focus on accounting for the changes in occupational employment shares in the aggregate, whereas the last study analyzes the adverse effects of automation on workers and labor market policies. In contrast, we study worker reallocation across occupations *and* firms, while explicitly considering firm heterogeneity. We construct a novel theoretical framework, which is a natural extension of the standard heterogeneous-firm model a la Hopenhayn (1992) and Hopenhayn and Rogerson (1993). This framework enables us to analyze heterogeneous firm dynamics during the process of job polarization.

Recently, Humlum (2021) and Rodrigo (2021) analyze the firm-level adjustment after adopting robots. These authors quantitatively analyze heterogeneous-firm models using micro-level datasets (Humlum (2021) uses Danish data and Rodrigo (2021) uses Brazilian data), considering endogenous robot adoption and labor market responses. Meanwhile, we conduct a cross-country comparison of the United States and Germany and focus on the role of labor market adjustment costs. Neither of these studies considers across-firm labor adjustment costs (firing taxes), which is the main focus of our study.

Finally, our study also relates to the literature on the differences in labor-market institutions between the United States and European countries. Mortensen et al. (1999) is the closest to our study in spirit. They analyze how skill-biased shocks, interacting with different policy regimes, explain the rise in unemployment in Europe. While our focus is on labor-market polarization and occupational reallocation, the motivations are similar: different labor-market institutions can result in different responses to technology shocks.

The remainder of this article is organized as follows. Section 2 conducts the empirical analysis. Section 3 constructs a general equilibrium model of firm dynamics. Section 4 analyzes the model quantitatively and compares it with the data. Section 5 conducts counterfactual experiments using the calibrated model. Section 6 presents the conclusions of this study.

2 Empirical findings

Here, we document the patterns of occupational reallocation in the United States and Germany. Both countries have experienced significant changes in the occupational composition in their labor markets in the past decades.[2] The patterns of reallocation, however, are markedly different across these two countries as we show. We start by describing the data and then present the empirical results for the patterns of occupational reallocations.

[2] See Acemoglu and Autor (2011) for the United States and Böhm et al. (2024) for Germany.

2.1 Data

For the United States, we use the Survey of Income and Program Participation (SIPP), a household survey dataset that provides detailed information on individuals' labor market activities. We also use the Current Population Survey (CPS) for the United States for a supplementary purpose. For Germany, we use the Sample of Integrated Labor Market Biographies (SIAB), an administrative dataset that contains employment records for a 2% sample of the German labor market. The details of the datasets and data cleaning procedures are described in Appendix B.

2.1.1 United States: SIPP

SIPP is a dataset of a household-based panel survey administered by the US Census Bureau. We use the following seven panels of the SIPP for our analysis: 1990, 1991, 1992, 1993, 1996, 2001, and 2004. We do not use the panels before 1990 as no reliable job IDs are provided and thus cannot identify job switches. We don't use the 2008 panel or later, because the new data cleaning procedure that US Census introduced in 2004 removed the significant number of within-job occupational changes for the 2008 panel.[3] These panels have a sample of 14,000 to 52,000 individuals. We select observations where an individual is between ages 23 and 55. We drop observations where an individual works in the public sector or is self-employed.[4] Following Kambourov and Manovskii (2009), we also exclude managerial occupations from our analysis.[5]

Following the literature of the task-based approach (Acemoglu and Autor, 2011), we identify occupational switches when a worker changes their occupations across three broad occupational groups, *Cognitive*, *Routine*, and *Manual*, based on the nature of tasks per-

[3] This new data cleaning procedure pulls forward the occupation reported last time within the same job. This procedure allowed only a few of within-job occupation changes for the 2008 Panel.

[4] Further details of the dataset, data cleaning procedures, and the sample selection criteria are in Appendix B.1.

[5] This paper focuses on the effect of technological change on the horizontal reallocation of workers across occupations, not on career progression. Lee and Shin (2017) analyze the effect of technological changes on both workers (horizontal polarization) and managers (vertical polarization).

formed in an occupation. These occupation groups are listed in Appendix B.4. Among those occupational switches, we further identify *within-firm occupational switches*, those that involve employer changes, and *across-firm occupational switches*, those that do not involve employer changes, by examining changes in job IDs in SIPP. We identify these within-job and between-job occupational switches on an annual basis.

2.1.2 Germany: SIAB

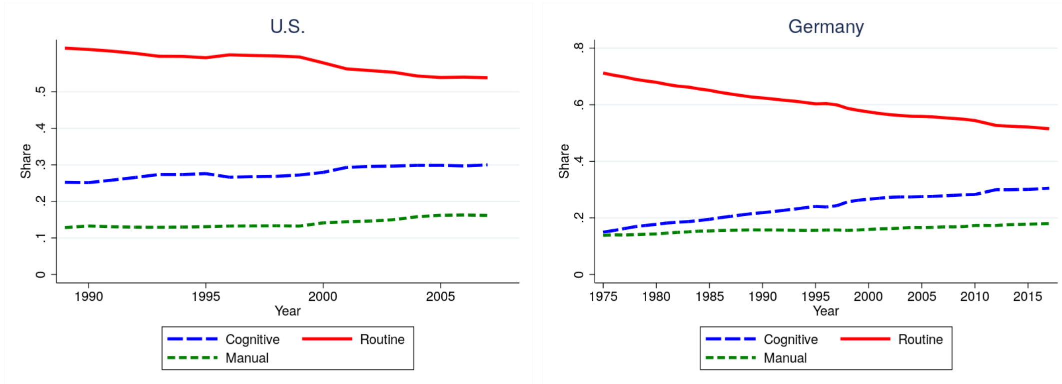
SIAB is a dataset based on the administrative employment records in Germany. The data are provided by the Institute for Employment Research (IAB) for research. The dataset has a 2% sample of employment histories from the entire German employment records for the period 1975–2017. It includes employees covered by social security, marginal part-time employment (since 1999), unemployment insurance benefit recipients, and individuals who are officially registered as job-seeking or who are participating in programs of active labor market policies. The dataset excludes the self-employed, civil servants, individuals performing military service, and those not in the labor force. It contains information on the starting and ending dates of each employment spell with an employer identification number and occupation classification code. Similar to SIPP, we select the sample of individuals between ages 23 and 55 and exclude managerial occupations.

With the occupation information, we first create three broad occupation classifications (Cognitive, Routine, and Manual) similar to those for the United States, following Böhm et al. (2024).[6] We then identify job and occupation switches at the annual frequency and document within-firm (within-establishment) and across-firm (across-establishment) patterns of occupational reallocation, similar to those in the United States.[7]

[6] Böhm et al. (2024) created task-based occupational groups for Germany that are comparable to those in Acemoglu and Autor (2011). The details of the occupational groups are in Appendix B.4.

[7] The SIAB dataset only provides establishment IDs, and thus, only the establishment switches can be identified. If a within-firm occupational switch is recognized as an across-firm one because the worker switches establishments but not an employer, our results for the within-firm occupational reallocation for Germany could be underestimated. Therefore, our result on the U.S.-German gap in within-firm occupational reallocation would be a lower bound. However, Gumpert et al. (2022) report that in 2012, 91% of German firms are single-establishment firms, and thus the effects of the underestimation could be limited.

Figure 1: Occupational Employment Share in the United States (Left) and Germany (Right)



Data Source: SIPP (United States); SIAB (Germany)

2.2 Time-series patterns of the stocks of occpational shares

Figure 1 plots the share of employment across occupations for the United States from 1989 to 2007 and for Germany from 1975 to 2017. As is commonly observed in the literature (see Acemoglu and Autor, 2011), the share of routine-occupation employment has declined both in the United States and Germany. In contrast, cognitive and manual occupations have gained employment shares. This phenomenon is often referred to as the labor-market polarization.[8]

2.3 Decomposition of the occupational employment share changes

Next, we investigate how firms reallocate workers behind the change in the stocks of occupational shares by analyzing the flows in and out of these stocks. To quantify the role of occupational switches within and across firms in the process of labor-market polarization, we decompose the change in each occupational employment share into contributions of the net flows of the internal and external occupational changes. *Internal* occupational changes occur when workers change occupations but remain with the same employer. *External* occupational changes occur when the workers switch both their employers and occupations.

[8]For the United States, we confirm the same pattern with the CPS in Figure 20 in Appendix A.

Previous studies, such as Moscarini and Thomsson (2007), have shown that there are substantial internal occupation changes in the United States. However, it has not been studied how the internal and external occupation changes separately contribute to the changes in occupation stocks.

Let ℓ_{it} be the stock of employment in occupation i at time t. The index i takes c, r, or m, where c represents cognitive, r represents routine, and m represents manual occupations. Further, let

$$E_t \equiv \sum_{i=c,r,m} \ell_{it}$$

be the total employment.

Now, we employ the following decomposition formula to quantify the contributions of different (net) flows to the change in the occupational stock. Let the employment share at time t for occupation i be ℓ_{it}/E_t. We decompose the change in the (log) employment share of occupation i from period t to period $t+T$:

$$\begin{aligned}
&\log\left(\frac{\ell_{i,t+T}}{E_{t+T}}\right) - \log\left(\frac{\ell_{it}}{E_t}\right) \\
&\quad \approx \Bigg[\underbrace{\sum_{\tau=0}^{T-1}\sum_{j\neq i}\frac{f^{ji,s}_{t+\tau,t+\tau+1} - f^{ij,s}_{t+\tau,t+\tau+1}}{\ell_{i,t+\tau}}}_{\text{internal net flow}} + \underbrace{\sum_{\tau=0}^{T-1}\sum_{j\neq i}\frac{f^{ji,d}_{t+\tau,t+\tau+1} - f^{ij,d}_{t+\tau,t+\tau+1}}{\ell_{i,t+\tau}}}_{\text{external } EE \text{ net flow}} \\
&\qquad + \underbrace{\sum_{\tau=0}^{T-1}\frac{f^{Ui}_{t+\tau,t+\tau+1} - f^{iU}_{t+\tau,t+\tau+1}}{\ell_{i,t+\tau}}}_{\text{external net flow from/to unemployment and OLF}} - \underbrace{\sum_{\tau=0}^{T-1}\Delta^{E}_{t+\tau,t+\tau+1}}_{\text{total employment effect}}\Bigg]. \qquad (1)
\end{aligned}$$

The derivation of equation (1) is in Appendix C. The equation shows that the cumulative change in employment share is decomposed into four components on the right-hand side. The first term (labeled as "internal net flow") is the contribution of within-firm occupational switches. The notation $f^{ji,s}_{t+\tau,t+\tau+1}$ is the gross worker flow from occupation j to occupation i between time $t+\tau$ and $t+\tau+1$, conditional on staying with the same employer (s for "the same employer"). The term $f^{ij,s}_{t+\tau,t+\tau+1}$ is the worker flow in the opposite direction. Therefore, $\sum_{j\neq i} f^{ji,s}_{t+\tau,t+\tau+1} - \sum_{j\neq i} f^{ij,s}_{t+\tau,t+\tau+1}$ is the sum of the total inflow minus the sum of the total outflow for occupation i. Thus, this term is the net inflow due to the internal occupational switches. Similarly, the second term (labeled as "external EE net flow") is the contribution of between-firm occupational switches. $f^{ji,d}_{t+\tau,t+\tau+1}$ represents the gross worker flow from occupation j to occupation i between time $t+\tau$ and $t+\tau+1$, conditional on workers switching to different employers (d represents "different employers"). The third term (labeled "external net from/to unemployment and OLF," where OLF means "out of labor force") represents the net inflow from unemployment and out of the labor force, where $f^{Ui}_{t+\tau,t+\tau+1}$ is the flow from U to occupation i employment and $f^{iU}_{t+\tau,t+\tau+1}$ is the opposite flow. Finally, the fourth term (labeled as "total employment effect") is the change in occupational employment share due to the change in total employment.

We call the first term on the right-hand side as *internal flow*, and the sum of the second to fourth terms as *external flow*. We do not distinguish between the second to fourth terms largely for the purpose of comparability. In particular, it is difficult to make comparable distinctions between the third and fourth terms across the United States (SIPP) and German (SIAB) datasets, because survey data, like SIPP, are often affected by sample attrition, which creates a spurious effect in the fourth term.[9] Given that our focus is on the internal flow, the most important task is to distinguish the internal flow and other occupational switches. The distinction across different external flows must be left to future research, as it would require different datasets.

Table 1 implements the decomposition equation (1) to SIPP for the United States and SIAB for Germany from the periods 1989–2007 and 1975–2019, respectively. The frequency is annual.

We observe striking differences between the United States and Germany. As Columns (4) and (5) show, internal switches play almost no role in explaining the rise of cognitive employment and decline in routine employment in the United States. This indicates that

[9] To check how the sample attrition affects our results, we create and analyze the balanced panels of SIPP in Appendix D.

Table 1: Decompositions of Occupational Employment Share Changes for the United States and Germany

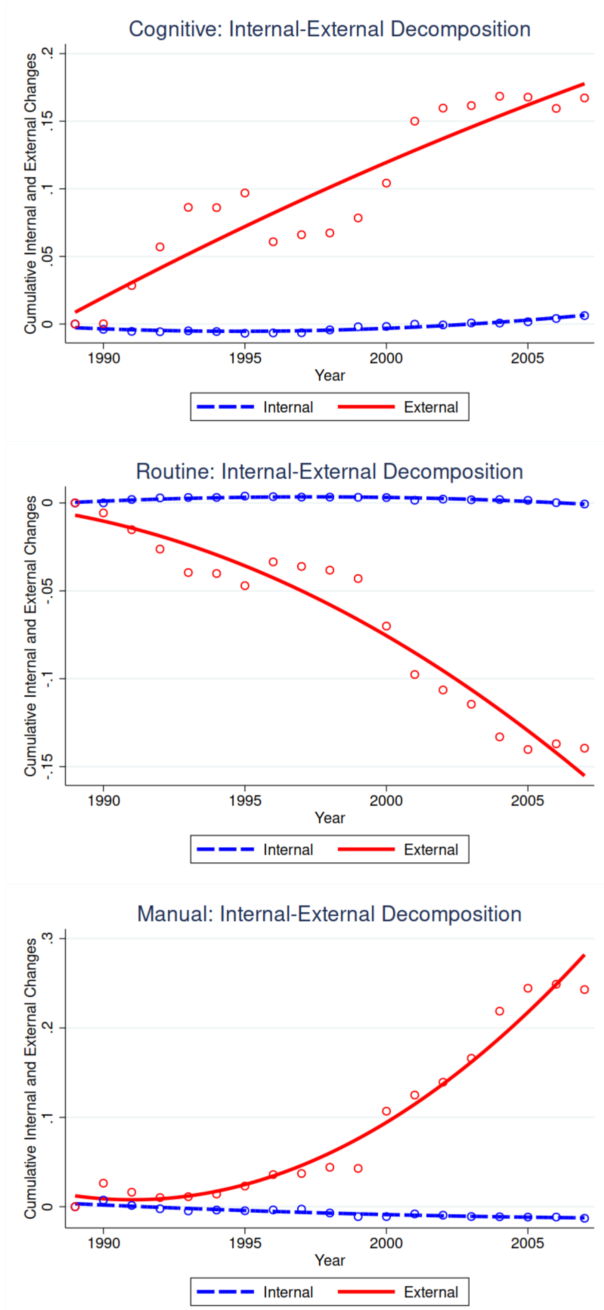
	Occupational employment share			Decomposed contributions	
	(1)	(2)	(3)	(4)	(5)
United States	1989	2007	log (Δshare)	Internal	External
Cognitive	0.25	0.30	0.17	0.01	0.17
Routine	0.62	0.54	−0.14	0.00	−0.14
Manual	0.13	0.16	0.23	−0.01	0.25
Germany	1975	2017	log (Δshare)	Internal	External
Cognitive	0.15	0.30	0.71	0.17	0.54
Routine	0.71	0.52	−0.32	−0.05	−0.27
Manual	0.14	0.18	0.26	−0.04	0.30

Data Source: SIPP (United States); SIAB (Germany)

most of these gross occupational movements of the workers offset each other. By contrast, internal switches make non-negligible contributions to the changes in occupational employment in Germany. We plot the cumulative contributions of net flows to occupational employment changes over time in Figures 2 and 3.[10] The figures show the dynamics that correspond to those in Columns (4) and (5) in Table 1.

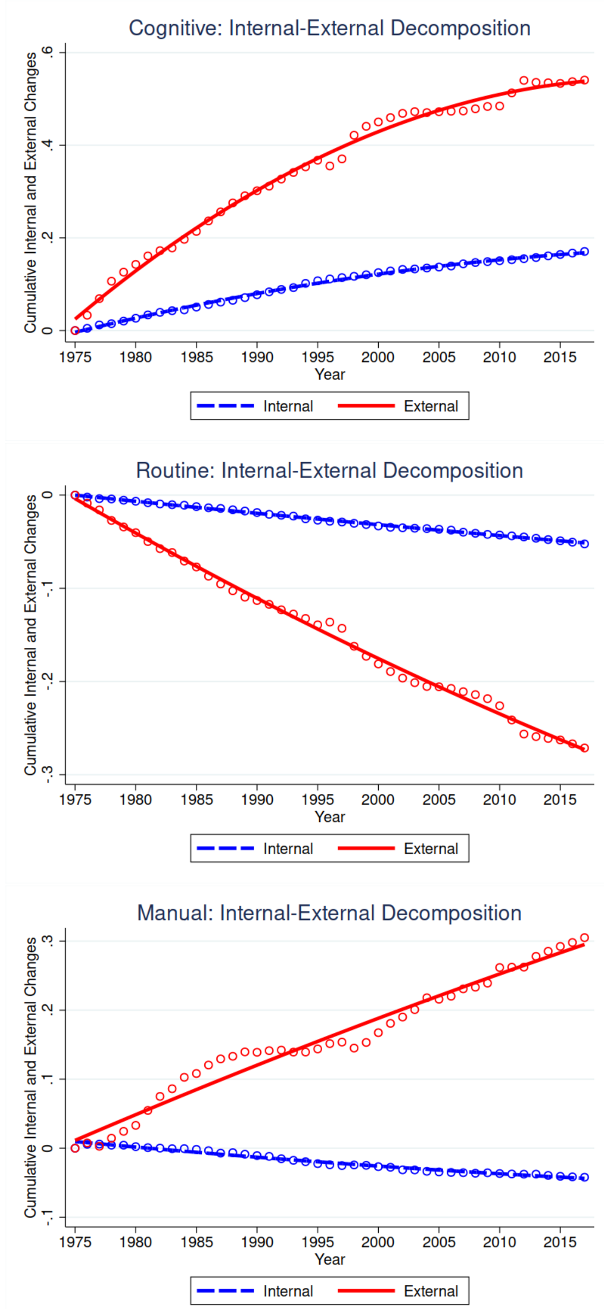
[10] As is usually the case with datasets based on surveys, the SIPP dataset is subject to sample attrition, which can potentially generate biases in the decomposition results. Therefore, we run a robustness check by conducting the decomposition with balanced panels of the SIPP in Appendix D. The results are essentially the same.

Figure 2: Cumulative Changes in Occupational Employment in the United States, SIPP, 1989–2007



Note: The data are from the SIPP 1990, 1991, 1992, 1993, 1996, 2001, and 2004 panels. The circular dots indicate data points, and the lines indicate quadratic fits to the data points.

Figure 3: Cumulative Changes in Occupational Employment in Germany, SIAB, 1975–2017



Note: The data are from the SIAB. The circular dots indicate data points, and the lines indicate quadratic fits to the data points.

What is the cause of the different patterns between the United States and Germany in Table 1? In the next two sections, we construct a model of heterogeneous firms to investigate the role of labor-market policies in the process of labor-market polarization.

3 Model

This section constructs a dynamic model with heterogeneous firms to examine the interaction between labor market policies and the process of polarization. Our model builds on Hopenhayn (1992) and Hopenhayn and Rogerson (1993), and features a CES production structure with three broad types of occupations (cognitive, routine, and manual) and two firm-level productivity variables. As in Hopenhayn (1992) and Hopenhayn and Rogerson (1993), the first variable is an exogenous Hicks-neutral productivity shock. The second variable, which represents the firm-level automation, is an endogenous choice variable for each firm. Automation affects different types of occupational labor demand differently. This differential labor demand drives the labor market polarization.

3.1 Setup

Time is discrete. We assume that an infinitely-lived representative consumer exists. The consumer supplies labor and receives wage income. They also own the firms and receive the profit. The consumer is a price-taker and maximizes utility

$$\sum_{t=0}^{\infty} \beta^t U(C_t, N_t)$$

subject to

$$C_t = w_t N_t + \Pi_t + R_t.$$

Here, $\beta \in (0,1)$, $U(\cdot,\cdot)$ is the period utility function, C_t is consumption at period t, and N_t is the labor supply. On the income side, w_t is the wage rate and Π_t is the profit from production in the firm. Firms pay firing taxes to the government, which is lump-sum rebated to the consumer as R_t. Below, we assume a quasi-linear period utility

$$U(C_t, N_t) = C_t - \xi \frac{N_t^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}},$$

where $\xi > 0$ and $\eta > 0$ are parameters. This specification implies that the equilibrium return to saving has to be equal to $1/\beta - 1$. To simplify the notation below, we adopt the recursive formulation, where the next-period variable is denoted by prime ($'$).

There is a unit mass of firms, and we abstract from entry and exit for simplicity. Firms produce the consumption goods using labor. They act competitively in both the product and labor markets. The production process involves three different tasks (which correspond to three different occupations): manual (m), cognitive (c), and routine (r). The production function also features two additional variables that affect worker productivity. The first is the standard Hicks-neutral total factor productivity (TFP) shock, denoted by s_h, which is exogenous and acts similarly to the standard firm dynamics model by Hopenhayn (1992) and Hopenhayn and Rogerson (1993). The second, denoted by s_a, is a choice variable for the firm's automation productivity. It represents the degree of introduction of new technology (machines). A firm can choose the next period s_a, denoted by s_a', subject to cost (denote it $\Gamma(s_a, s_a')$). The $\Gamma(s_a, s_a')$ function is assumed as follows. We assume that each firm's s_a can take two possible values, $\underline{s}_a$ and $\overline{s}_a$, where $\underline{s}_a < \overline{s}_a$. The interpretation of $\underline{s}_a$ is "before automation" and $\overline{s}_a$ is "after automation." The transition between these two values is one direction: from $\underline{s}_a$ to $\overline{s}_a$. Therefore, $\overline{s}_a$ is the absorbing state. The cost of transition is assumed to be $\bar{c}_a$. The cost is zero when the value of s_a does not change. Because $\overline{s}_a$ provides a higher productivity than $\underline{s}_a$, when the additional (present) value surpasses $\bar{c}_a$, firms prefer to pay the cost and transition from $\underline{s}_a$ to $\overline{s}_a$. A firm with $s_a = \underline{s}_a$ has an opportunity to automate with i.i.d. probability p. This restriction is introduced so that the automation process in the model economy can exhibit a realistic speed.

After observing the s_h shock, a firm makes hiring decisions (as well as the automation decision), where employment at task $i \in \{m, c, r\}$ is denoted as n_i. Note that, in the model, we build in the mechanism where the labor market polarization is driven by the firm's automation choice. Various existing studies have investigated the fundamental cause of labor market polarization and suggested several potential causes, including automation. Here, our goal is not to determine the fundamental cause of the polarization, and s_a should be interpreted more broadly than the strict definition of "automation." What is important here is that a change in s_a causes differential responses to the demands for different tasks (occupations).

The production function is specified as

$$f(\mathbf{n}, \mathbf{s}) = s_h \mathbf{F}^{\alpha},$$

where $\alpha \in (0, 1)$ is the returns-to-scale parameter,

$$\mathbf{F}(n_m, \mathbf{G}) = \left(\mu_m n_m^{\frac{\sigma_m - 1}{\sigma_m}} + (1 - \mu_m) \mathbf{G}^{\frac{\sigma_m - 1}{\sigma_m}} \right)^{\frac{\sigma_m}{\sigma_m - 1}},$$

where $\sigma_m \geq 0$ is the elasticity of substitution parameter and n_m is the manual labor,

$$\mathbf{G}(n_c, \mathbf{M}) = \left(\mu_c n_c^{\frac{\sigma_c - 1}{\sigma_c}} + (1 - \mu_c) \mathbf{M}^{\frac{\sigma_c - 1}{\sigma_c}} \right)^{\frac{\sigma_c}{\sigma_c - 1}},$$

where $\sigma_c \geq 0$ is the elasticity of substitution parameter and n_m is the cognitive labor,

$$\mathbf{M}(n_r, s_a) = \left(\mu_r n_r^{\frac{\sigma_r - 1}{\sigma_r}} + (1 - \mu_r) s_a^{\frac{\sigma_r - 1}{\sigma_r}} \right)^{\frac{\sigma_r}{\sigma_r - 1}},$$

where $\sigma_r \geq 0$ is the elasticity of substitution parameter and n_r is the routine labor. One can interpret s_a as the (automation) capital stock. This specification of the production function is in line with the existing literature on labor-market polarization. For example, $\sigma_r = \infty$ corresponds to Cortes et al. (2017) and $\sigma_c = 1$ case corresponds to Autor and Dorn (2013). Using the same specification as above, vom Lehn (2020) estimates the values of σ_i and μ_i. For simplicity, we assume that the workers are (ex-ante) homogeneous, and thus, each occupation pays the identical wage.

Changing occupational employment from one period to the next may require the firm to pay certain costs for adjustment. To describe these costs, we first introduce new notations. Let us denote the current period's employment in occupation $i \in \{m, c, r\}$ as n_i'.[11] The previous period's employment in occupation i is denoted as n_i. Firms decide on the vector $\mathbf{n}' \equiv \{n_m', n_c', n_r'\}$ for given $\mathbf{s} = \{s_h, s_a\}$ and $\mathbf{n} \equiv \{n_m', n_c', n_r'\}$. That is, $\mathbf{s}$ and $\mathbf{n}$ are the state variables for the firm's employment decision $\mathbf{n}'$.

While increasing the number of occupational hires ($n_i' > n_i$), the firm has to bring in new workers into that occupation from inside or outside the firm. Define $\tilde{n}_i' \in [0, n_i']$ as the internal workers (from any occupation but from the same firm) who now work in occupation i this period. Then, $\tilde{n}_i' - n_i$ is the number of internally-moved workers and $n_i' - \tilde{n}_i'$ is the number of workers who are brought from outside. Furthermore, define $\hat{n}_i' \in [0, \min\{n_i, \tilde{n}_i'\}]$ as internal workers who stayed in the same occupation i (that is, the same firm and occupation) from the previous period. Then, $\tilde{n}_i' - \hat{n}_i'$ is the number of workers who are internally brought into that occupation (from another occupation) within the firm.[12] Let $\tilde{\mathbf{n}}'$ be the vector of $\tilde{n}_i'$ and $\hat{\mathbf{n}}'$ be the vector of $\hat{n}_i'$.

In summary, the firm makes three layers of employment decisions: (i) how many people to hire this period $\mathbf{n}'$; (ii) within $\mathbf{n}'$, how many people come from the same firm ($\tilde{\mathbf{n}}'$), and (iii) how many people in $\tilde{\mathbf{n}}'$ are the ones from the same occupation ($\hat{\mathbf{n}}$). Clearly, $\hat{n}_i$ cannot exceed n_i and the sum of $\tilde{n}_i'$ must be less than the sum of n_i.

We assume two types of costs for employment adjustment. The first is the *firing taxes* imposed by the government. We denote it as $g(\mathbf{n}, \tilde{\mathbf{n}}')$ and assume that it takes the form

$$g(\mathbf{n}, \tilde{\mathbf{n}}') = \tau \left(\sum_{i=m,c,r} n_i - \sum_{i=m,c,r} \tilde{n}_i' \right),$$

where $\tau \geq 0$ is the tax rate. We also assume that a firm has to incur a *reorganization cost*,

[11] The convention of using $'$ for the current period employment follows Hopenhayn and Rogerson (1993).

[12] Note that we implicitly assume that the firm keeps the workers in the firm and in the same occupation whenever possible. This assumption can be justified by, for example, having an infinitesimally small amount of cost of moving workers across firms and occupations.

$h(\tilde{\mathbf{n}}', \hat{\mathbf{n}}')$, which takes the form

$$h(\tilde{\mathbf{n}}', \hat{\mathbf{n}}') = \sum_{i=m,c,r} H_i(\tilde{n}_i' - \hat{n}_i'),$$

where $H_i(\cdot)$ is an increasing function. In the quantitative analysis below, we consider a quadratic form of H_i function:

$$h(\tilde{\mathbf{n}}', \hat{\mathbf{n}}') = \sum_{i=0}^{k} \kappa_i (\max\{\tilde{n}_i' - \hat{n}_i', 0\})^2.$$

where $\kappa_i \geq 0$.

Formally, the firm's problem is

$$\begin{aligned} V(\mathbf{n}, \mathbf{s}; t) = \max_{\mathbf{n}', \tilde{\mathbf{n}}', \hat{\mathbf{n}}', s_a'} & f(\mathbf{n}', \mathbf{s}) - w\mathbf{1} \cdot \mathbf{n} - g(\mathbf{n}, \tilde{\mathbf{n}}') - h(\tilde{\mathbf{n}}', \hat{\mathbf{n}}') \\ & + \beta E_{s_h'} [p\{V(\mathbf{n}', s_h', s_a'; t+1) - \Gamma(s_a, s_a')\} + (1-p) V(\mathbf{n}', s_h', s_a; t+1)], \end{aligned}$$

subject to

$$\sum_{i=m,c,r} \tilde{n}_i' \leq \sum_{i=m,c,r} n_i,$$

where

$$g(\mathbf{n}, \tilde{\mathbf{n}}') = \tau \left(\sum_{i=m,c,r} n_i - \sum_{i=m,c,r} \tilde{n}_i' \right),$$

and

$$h(\tilde{\mathbf{n}}', \hat{\mathbf{n}}') = \sum_{i=m,c,r} H_i(\tilde{n}_i' - \hat{n}_i').$$

Here, we use the fact that the quasi-linear utility of consumers imply the firm's discount factor to be β.

In the competitive equilibrium of this economy, the wage w_t is determined by the labor market clearing condition. Next, we examine the transitional dynamics of the aggregate economy from one steady state to another.

4 Quantitative analysis

The empirical analysis in Section 2 highlights a significant difference in how firms react to the labor market polarization process. Motivated by this outcome, we quantitatively assess how labor market institutions affect the reallocation of workers across occupations and firms during the transition dynamics.

4.1 The transition economy

We consider the transition process of automation. The economy starts from the steady state where all firms have $s_a = \underline{s}_a$. In each period, some firms (who have an opportunity) choose to transition into $s_a = \overline{s}_a$ and the economy eventually converges to the new steady state where $s_a = \overline{s}_a$ for all firms. This process leads to gradual changes in the aggregate demand for each occupation. Therefore, the transition process entails labor-market polarization.

Below, we further impose a restriction that the *within-firm* occupation reallocation occurs only from the routine occupation to the cognitive occupation. This assumption simplifies the computation of the model dramatically, and is motivated by the analysis of the German data in Section 2, where the within-firm reallocation contributes to the polarization mainly through the transition from routine to cognitive occupation.

With this assumption, $\hat{n}'_m = \tilde{n}'_m = \min\{n_m, n'_m\}$ holds because there are no internal movements (both in and out) of workers for the manual occupation. For routine workers, suppose that $x' \geq 0$ number of workers move to the cognitive occupation. If $n'_r > n_r - x'$, $\hat{n}_r = \tilde{n}_r = n_r - x'$ and the remaining workers $(n'_r - (n_r - x'))$ must be brought in from outside the firm. If $n_r - x' \geq n'_r$, $\hat{n}_r = \tilde{n}_r = n'_r$ and the excess workers $((n_r - x') - n'_r = n_r - (n'_r + x'))$ must be fired. For cognitive workers, $n'_c - x'$ number of workers have to come from either previously cognitive workers or outside the firm. If $n'_c - x' > n_c$, $\hat{n}_c = n_c$, $\tilde{n}_c = n_c + x'$, and $(n'_c - x') - n_c$ workers must be brought in from outside. If $n_c \geq n'_c - x'$, $\hat{n}_c = n'_c - x'$, $\tilde{n}_c = n'_c$, and the excess workers $(n_c - (n'_c - x'))$ must be fired. In short, the firm chooses four numbers (n'_m, n'_r, n'_c, x'), the firing tax is $\tau(\max\{n_m - n'_m, 0\} + \max\{n_c - (n'_c - x'), 0\} +$

$\max\{n_r - (n'_r + x'), 0\})$, and the reorganization cost is $\kappa x'^2$.

4.2 Computation

Next, we outline how we compute the transition path. The details are described in Appendix E. We initially compute the steady state where $s_a = \underline{s}_a$ for all firms. In that steady state, no firm has a possibility of "automating" and moving to $s_a = \overline{s}_a$.

We assume that the economy is initially in a steady state where all firms have $s_a = \underline{s}_a$ and expect it to stay constant forever. Then, at a point in time (call time 0), the economy unexpectedly shifts to a new regime where a firm can endogenously switch to $s_a = \overline{s}_a$ when it is profitable. In particular, after time 0, with probability p, the firm (at any point in time) can decide whether it automates with adoption cost $\Gamma(\underline{s}_a, \overline{s}_a)$. The regime switch is permanent, and all economic agents understand the nature of the switch. At the firm level, the transition from $\underline{s}_a$ to $\overline{s}_a$ is one time and permanent: once they change s_a to $\overline{s}_a$, it stays at that value. The aggregate economy experiences a gradual transition from the steady state where all firms have $s_a = \underline{s}_a$ to another steady state where all firms have $s_a = \overline{s}_a$. We interpret this transition dynamics as the process of labor-market polarization, driven by automation at each firm.

To analyze the macroeconomic dynamics of this transition, we first compute the initial and final steady states. As in the previous section, let $\mathbf{n} = (n_m, n_c, n_r)$ be the previous period's occupational employment and $\mathbf{n}' = (n'_m, n'_c, n'_r)$ be the current period's employment decision. In the initial steady state where no firms automate, a firm's dynamic programming problem is

$$\begin{aligned}
\underline{V}(\mathbf{n}, s_h; \underline{s}_a) \quad = \quad & \max_{\mathbf{n}', x'} [-\tau(\max\{n_m - n'_m, 0\} + \max\{n_c - (n'_c - x'), 0\} + \max\{n_r - (n'_r + x'), 0\}) \\
& -\kappa x'^2 + f(\mathbf{n}', s_h; \underline{s}_a) - w\mathbf{1} \cdot \mathbf{n}' \\
& +\beta \mathbb{E}_{s'_h} [\underline{V}(\mathbf{n}', s'_h; \underline{s}_a) | s_h]],
\end{aligned}$$

subject to

$$
\begin{aligned}
n'_m &\geq 0, \\
n'_c &\geq x', \\
n'_r &\geq 0, \\
0 &\leq x' \leq n_r.
\end{aligned}
$$

Note that the time notation is not included because the only element of the model that is affected by calendar time is the automation decision (which is absent here). Here, we have already eliminated the notation of $\hat{n}'_i$ and $\tilde{n}'_i$ using the new notation of x'.

x' can be solved analytically once $\mathbf{n}$ and $\mathbf{n}'$ are given. Denote the solution as $x'(\mathbf{n}, \mathbf{n}')$. Then the problem can be rewritten as:

$$
\begin{aligned}
& \underline{V}(\mathbf{n}, s_h; \underline{s}_a) \\
= \; & \max_{\mathbf{n}' \geq \mathbf{0}} [-\tau(\max\{n_m - n'_m, 0\} + \max\{n_c - (n'_c - x'(\mathbf{n}, \mathbf{n}')), 0\} + \max\{n_r - (n'_r + x'(\mathbf{n}, \mathbf{n}')), 0\}) \\
& \qquad -\kappa x'(\mathbf{n}, \mathbf{n}')^2 + f(\mathbf{n}', s_h; \underline{s}_a) - \underline{w}\mathbf{1} \cdot \mathbf{n}' + \beta \mathbb{E}_{s'_h}[\underline{V}(\mathbf{n}', s'_h; \underline{s}_a) | s_h]].
\end{aligned}
$$

At the final state where all firms have completed the automation, the Bellman equation is

$$
\begin{aligned}
& \overline{W}(\mathbf{n}, s_h; \overline{s}_a) \\
= \; & \max_{\mathbf{n}' \geq \mathbf{0}} [-\tau(\max\{n_m - n'_m, 0\} + \max\{n_c - (n'_c - x'(\mathbf{n}, \mathbf{n}')), 0\} + \max\{n_r - (n'_r + x'(\mathbf{n}, \mathbf{n}')), 0\}) \\
& \qquad -\kappa x'(\mathbf{n}, \mathbf{n}')^2 + f(\mathbf{n}', s_h; \overline{s}_a) - \overline{w}\mathbf{1} \cdot \mathbf{n}' + \beta \mathbb{E}_{s'_h}[\overline{W}(\mathbf{n}', s'_h; \overline{s}_a) | s_h]].
\end{aligned}
$$

After computing the initial and final steady states, we compute the transition dynamics. Let $d = 1$ if firms plan to adopt and $d = 0$ otherwise. The value functions for the firms not yet automated are written as

$$
\begin{aligned}
& V_t(\mathbf{n}, s_h; \underline{s}_a) \\
= \; & \max_{\mathbf{n}' \geq \mathbf{0}, d \in \{0,1\}} [-\tau(\max\{n_m - n_m', 0\} + \max\{n_c - (n_c' - x'(\mathbf{n}, \mathbf{n}')), 0\} + \max\{n_r - (n_r' + x'(\mathbf{n}, \mathbf{n}')), 0\}) \\
& \qquad - \kappa x'(\mathbf{n}, \mathbf{n}')^2 + f(\mathbf{n}', s_h; \underline{s}_a) - w_t \mathbf{1} \cdot \mathbf{n}' \\
& \qquad + \beta \mathbb{E}_{s_h'} [p\{d(W_{t+1}(\mathbf{n}', s_h'; \overline{s}_a) - \Gamma(\underline{s}_a, \overline{s}_a)) + (1-d) V_{t+1}(\mathbf{n}', s_h'; \underline{s}_a)\} + (1-p) V_{t+1}(\mathbf{n}', s_h'; \underline{s}_a) | s_h],
\end{aligned}
$$

and the firms that are already automated solve the Bellman equation

$$
\begin{aligned}
& W_t(\mathbf{n}, s_h; \overline{s}_a) \\
= \; & \max_{\mathbf{n}' \geq \mathbf{0}} [-\tau(\max\{n_m - n_m', 0\} + \max\{n_c - (n_c' - x'(\mathbf{n}, \mathbf{n}')), 0\} + \max\{n_r - (n_r' + x'(\mathbf{n}, \mathbf{n}')), 0\}) \\
& \qquad - \kappa x'(\mathbf{n}, \mathbf{n}')^2 + f(\mathbf{n}', s_h; \overline{s}_a) - w_t \mathbf{1} \cdot \mathbf{n}' \\
& \qquad + \beta \mathbb{E}_{s_h'} [W_{t+1}(\mathbf{n}', s_h'; \overline{s}_a) | s_h].
\end{aligned}
$$

4.3 Calibration

The calibration of parameters is summarized in Table 2. First, the disutility for work ξ is set to clear the labor market at the initial steady state with $\tau = 0$ and $w = 1$. The Frisch elasticity η is in the range of standard values to calibrate macroeconomic models. The return-to-scale parameter α is based on Veracierto (2001). The initial level of s_a, $\underline{s}_a$, is set to unity. Meanwhile, the final level of s_a, $\overline{s}_a$, is calibrated jointly with the parameters of the production function except σ_r. Those parameters are determined so that the initial steady-state values of shares of tasks and the labor share without a firing tax for firms hit their counterparts of the United States in 1989 and the final steady-state values of those hit the level such that the changes between the initial and final steady-states are three times the changes between 1989 and 2007 of the United States. For instance, the initial level of the share of cognitive is set to 25 percent and the final level is set to 40 percent since the level of the United States in 2007 is 30 percent. Next, the remaining parameter of production function σ_r is assumed to make the automation capital stock a perfect substitute for the routine task. This assumption is also employed in Cortes et al. (2017). The discount factor β is set to be consistent with the annual safe interest rate of four percent. Regarding the idiosyncratic TFP shock s_h, we assume that $\log(s_h)$ follows an AR(1) process

$$\log(s'_h) = \rho_h \log(s_h) + \epsilon_h,$$

where

$$\epsilon_h \sim N(0, \sigma_h^2)$$

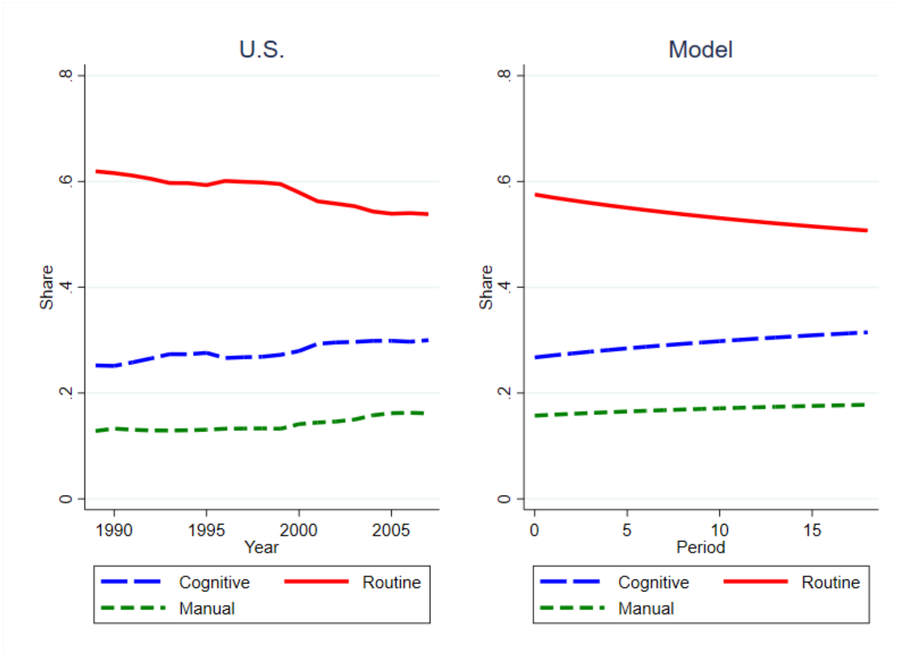
and the parameter values for ρ_h and σ_h are taken from Lee and Mukoyama (2015). The value for the cost of transition $\bar{c}_a$ is set to the highest value with which the transition immediately starts at $t = 0$ with $\tau = 0$. The probability p that a firm can make a transition decision is set so that the changes in shares of tasks in the first 19 periods of transition without firing tax match the changes in the counterparts of the United States between 1989 and 2007. Following Hopenhayn and Rogerson (1993), we interpret that the United States corresponds to the case with $\tau = 0$. The firing tax for Germany is set as $\tau = 0.3$, corresponding to the severance payment of 21.7 weeks equivalent of wages. This value corresponds to the mandatory severance payment for the workers with ten years of tenure, measured in the World Bank Doing Business 2020.[13] Finally, the reorganization cost κ is set to match the internal reallocation flow from the routine occupation in Germany.

[13] See `https://archive.doingbusiness.org/en/doingbusiness`.

Table 2: Calibrated Parameters

Parameter	Value	Description
ξ	5.272	To match $w = 1$ at the steady state with $s_a = \underline{s}_a$ and $\tau = 0$
η	2	Standard value
α	0.830	Veracierto (2001)
$\underline{s}_a$	1	Normalized to one
$\bar{s}_a$	2.908	Jointly determined to target the shares of tasks and labor share of United States
μ_m	0.169	
μ_c	0.002	
μ_r	0.988	
σ_m	1.281	
σ_c	0.195	
σ_r	∞	Cortes et al. (2017)
β	0.962	Annual safe interest rate of 4%
ρ_h	0.969	Lee and Mukoyama (2015)
σ_h	0.282	Lee and Mukoyama (2015)
$\bar{c}_a$	0.400	Highest value with which the transition immediately starts at $t = 0$
p	0.05	To set the United States 2007 to be one-third of the way through the transition
τ^{US}	0	Frictionless
τ^{DE}	0.3	To match the severance payment in World Bank Doing Business 2020
κ	4	To match the internal reallocation from the routine occupation in Germany

Figure 4: Occupation Share in Data versus Model: United States

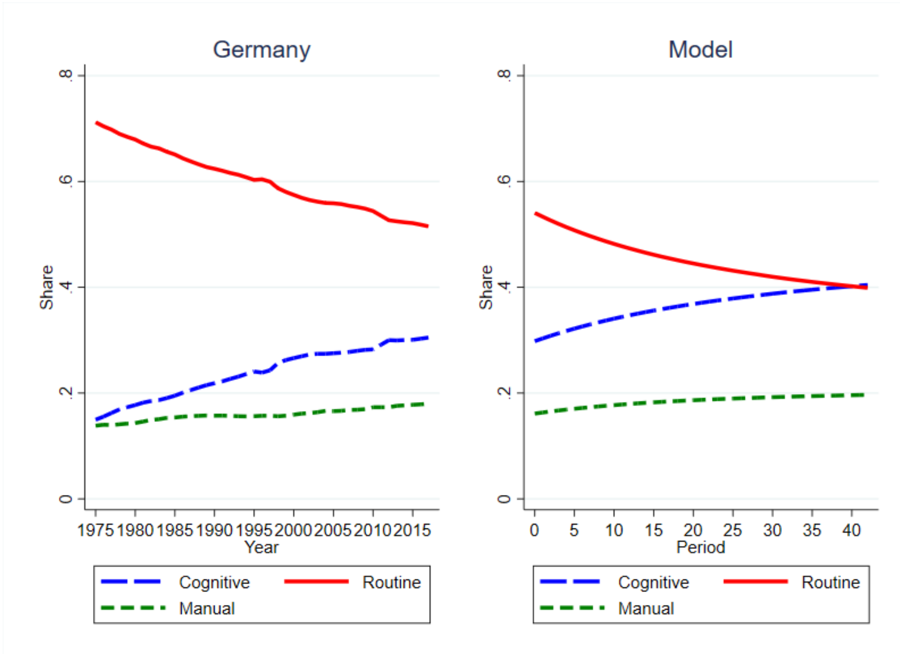


4.4 Model fit

We simulate the model's general equilibrium separately for the United States and Germany. Note that the computation of the US benchmark is substantially simpler than the German case because $\tau = 0$ implies that x' is always zero. This result follows because when $\tau = 0$, it is always cheaper to adjust the occupational composition through hiring and firing rather than internal reallocation.

Figure 4 plots the US data (SIPP), presented in Section 2.2, and the model simulation side-by-side. The model captures the main features of the data pattern very well: the routine share falls over time, whereas the manual and cognitive shares increase. This pattern of labor market polarization is driven by the endogenous automation (s_a moving from $\underline{s}_a$ to $\overline{s}_a$) of individual firms. Existing macroeconomic studies, such as Eden and Gaggl (2018) and vom Lehn (2020), generate similar pattern in the representative-firm framework. In our model, heterogeneous firms make the automation decision at different

Figure 5: Occupation Share in Data versus Model: Germany



timings from each other.

Figure 5 plots the corresponding figures for Germany. Note that we do not target any moments of the German data. Except for some level differences, the patterns of the labor market polarization in Germany are also captured well by the model.

Now, we move to the net flows. The model is targeted to match the cumulative change in the share of routine occupation via internal reallocation in Germany. Figures 6 through 11 compare the actual (presented in Section 2.3) and model-simulated data in terms of the cumulative change in the share of occupations. Again, the model does well in recovering the features of the data for other non-targeted components.

Figure 6: Cumulative Share Changes of Cognitive in Data versus Model: United States

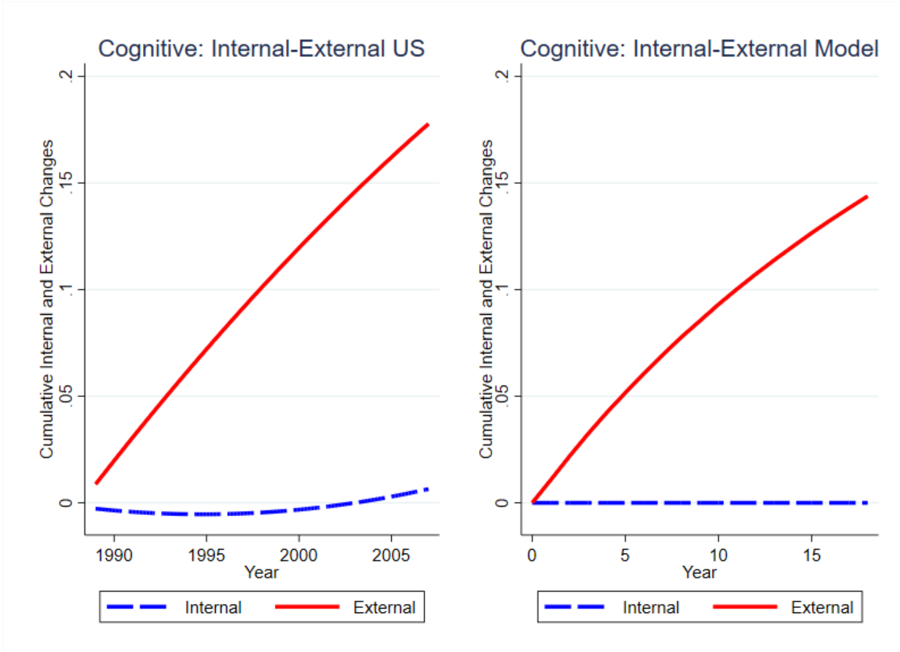


Figure 7: Cumulative Share Changes of Cognitive in Data versus Model: Germany

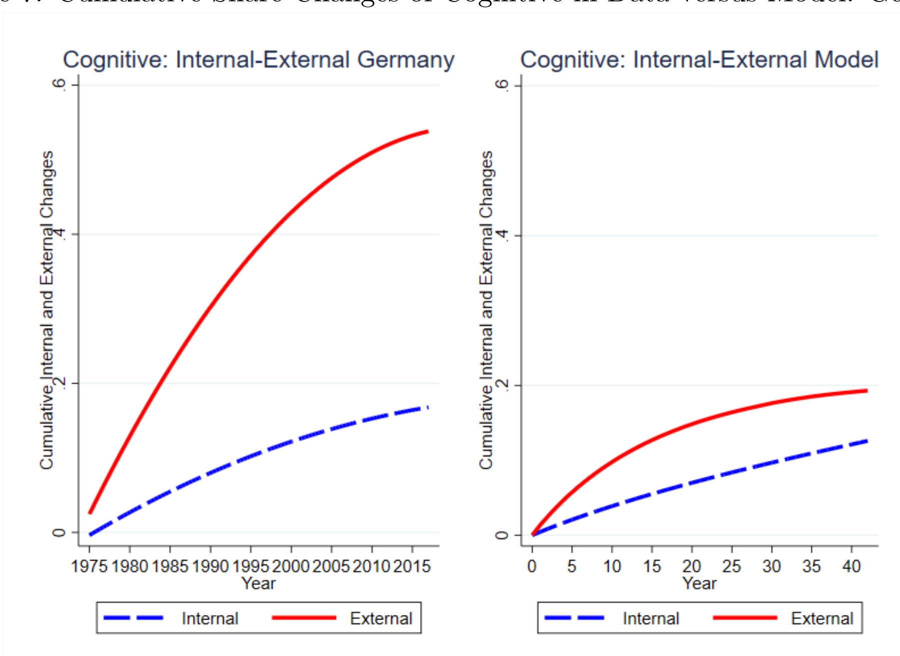


Figure 8: Cumulative Share Changes of Routine in Data versus Model: United States

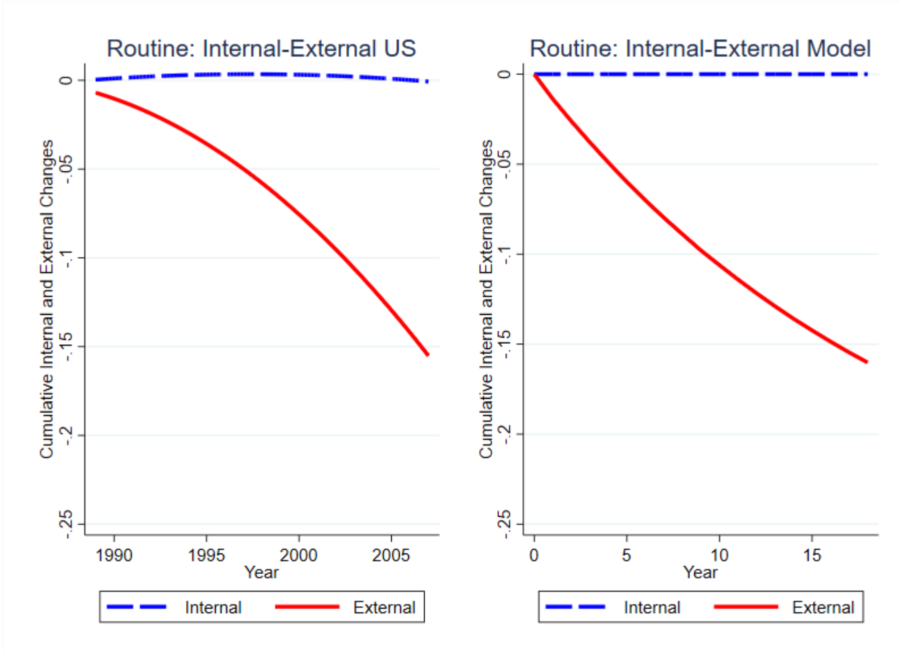


Figure 9: Cumulative Share Changes of Routine in Data versus Model: Germany

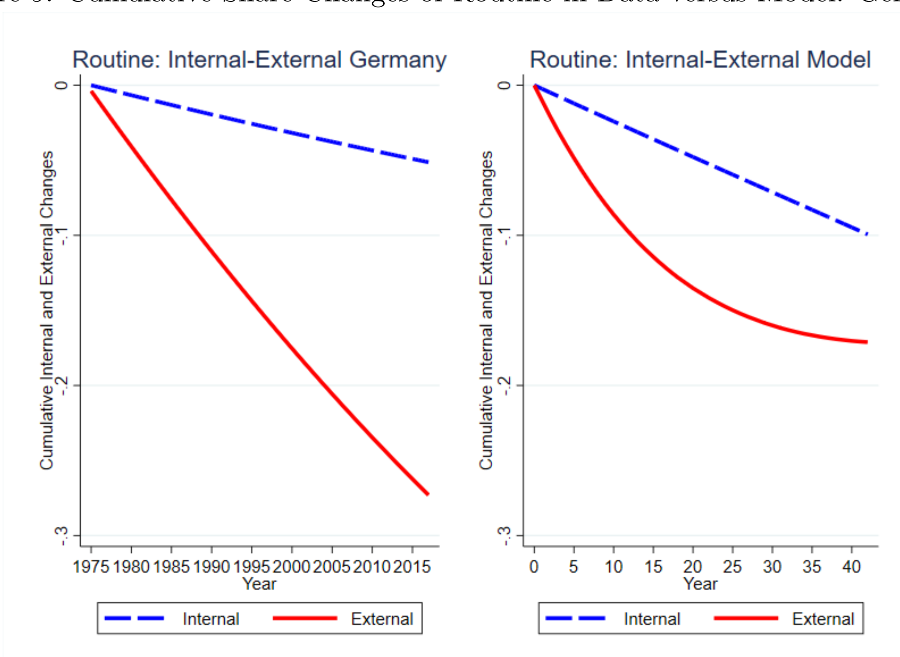


Figure 10: Cumulative Share Changes of Manual in Data versus Model: United States

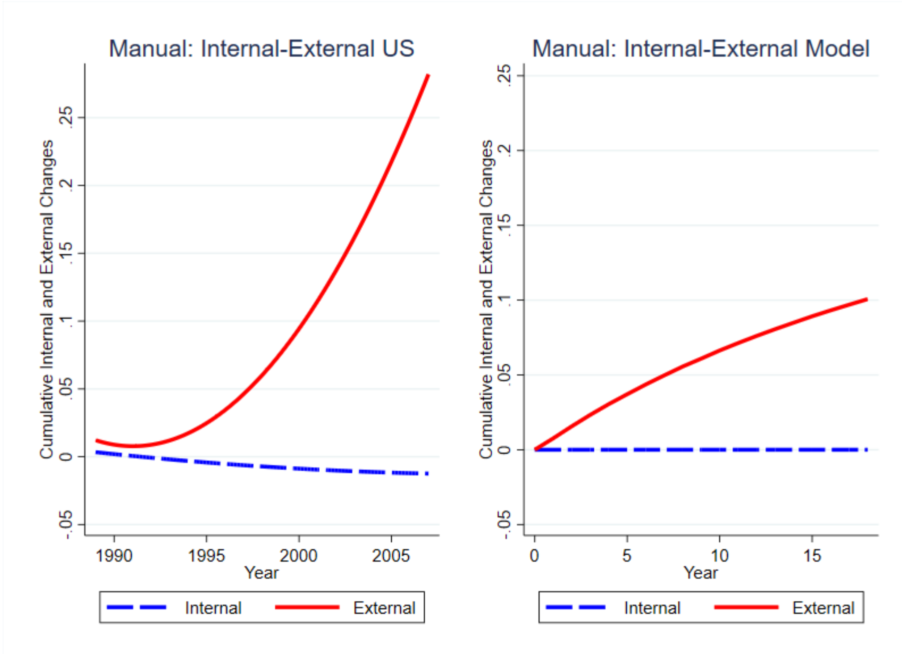


Figure 11: Cumulative Share Changes of Manual in Data versus Model: Germany

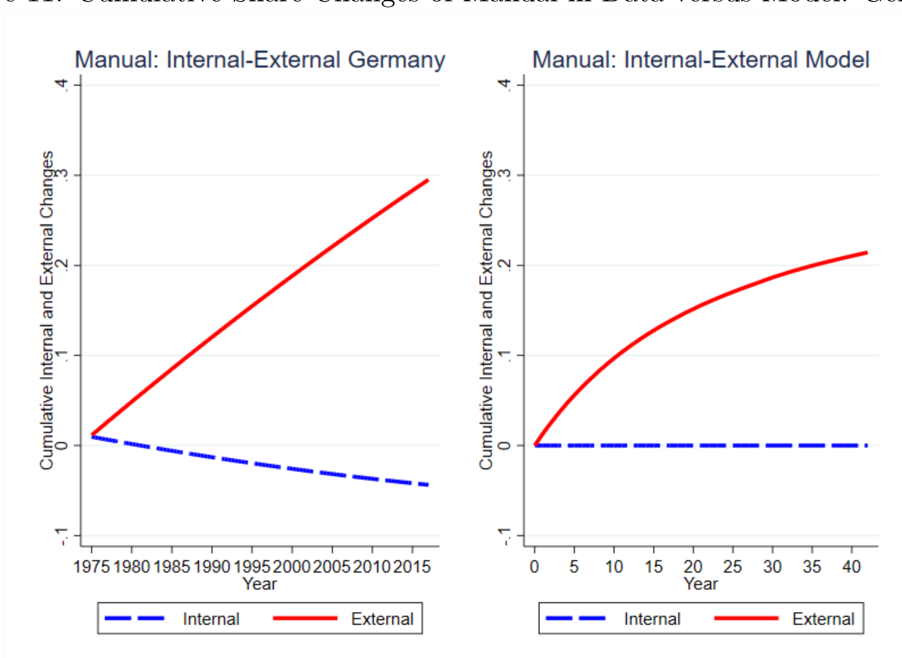
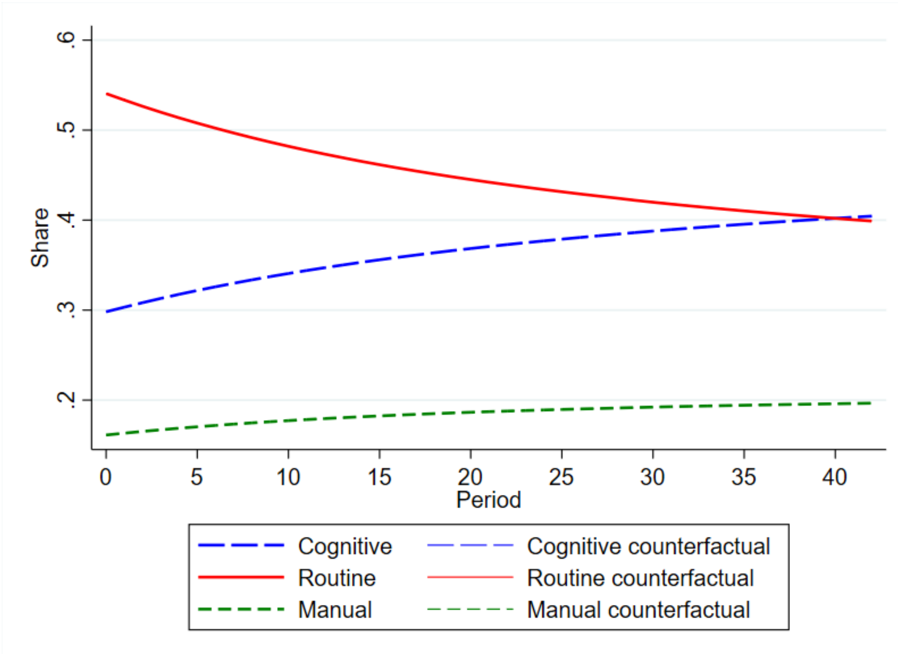


Figure 12: Counter-Factual on Occupation Share: Reduce κ by Half

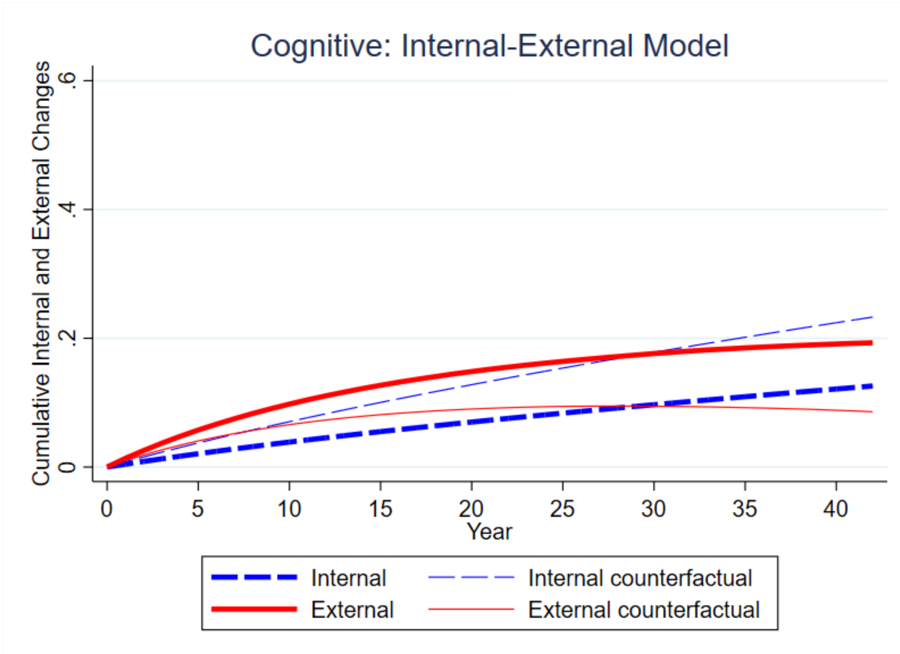


5 Counterfactual experiments

The previous section demonstrated that the model can match the observed patterns of the labor market adjustment during the polarization. In particular, the model calibrated to the US data can match the pattern of internal and external adjustments in Germany with firing tax $\tau = 0.3$ and an appropriate level of reorganization cost parameter κ.

Here, we further investigate the effects of these two parameters κ and τ by conducting counterfactual experiments, starting from the German outcome. In the first experiment, we reduce the value of κ to half. In the second experiment, we reduce the value of the firing tax τ to half. For both experiments, we highlight outcomes from two separate questions: How is the speed of labor market polarization affected by the change in these parameters? How is the margin of adjustments, internal or external, affected by these parameters?

Figure 13: Counter-Factual on Flow of Cognitive: Reduce κ by Half



5.1 Reorganization cost parameter κ

Our first experiment involves reducing the reorganization cost parameter κ by half ($\kappa = 2$). Figure 12 shows the time series of stocks. The reorganization cost has little impact on the path of stocks as the actual and counterfactual are nearly identical, as the thick (baseline) and thin lines (counterfactual) overlap and are not visible separately. This result implies that the timing of adjustment in individual firms is not significantly affected by the value of κ.

Figures 13, 14, and 15 show the effect on the margins of adjustment. Again, the thick lines are the baseline case and thin lines represent the counterfactual economy where $\kappa = 2$. The effect of κ on the internal versus external adjustment is quite large. When κ is small, the firm can shift a substantial part of the adjustment to internal worker movement. Therefore, this experiment reveals that the cost of internal adjustment plays an important role in *how* the labor market adjusts to the process of labor market polarization.

Figure 14: Counter-Factual on Flow of Routine: Reducing κ by Half

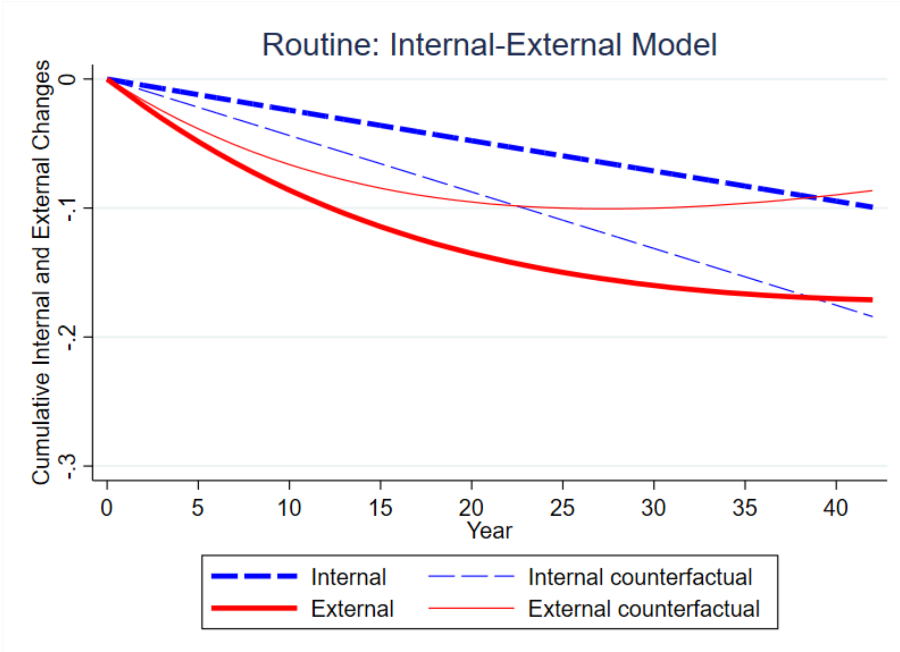


Figure 15: Counter-Factual on Flow of Manual: Reducing κ by Half

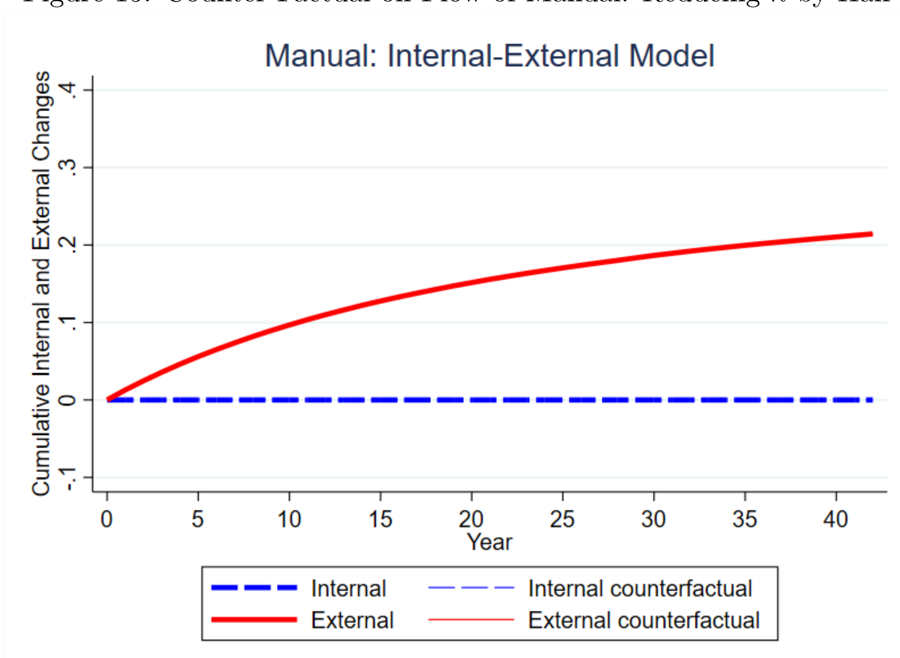
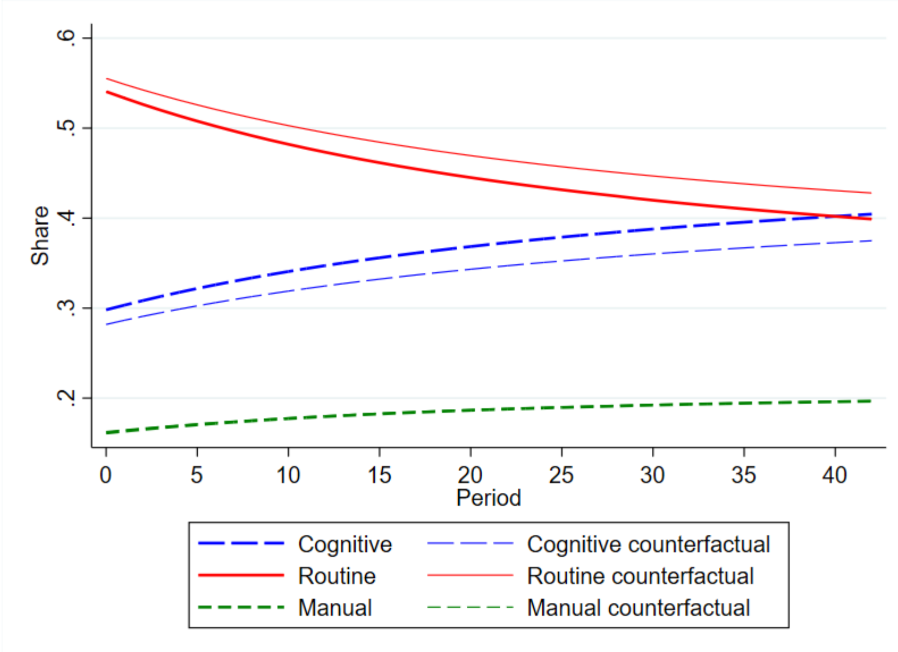


Figure 16: Counter-Factual on Occupation Share: Reduce τ by Half

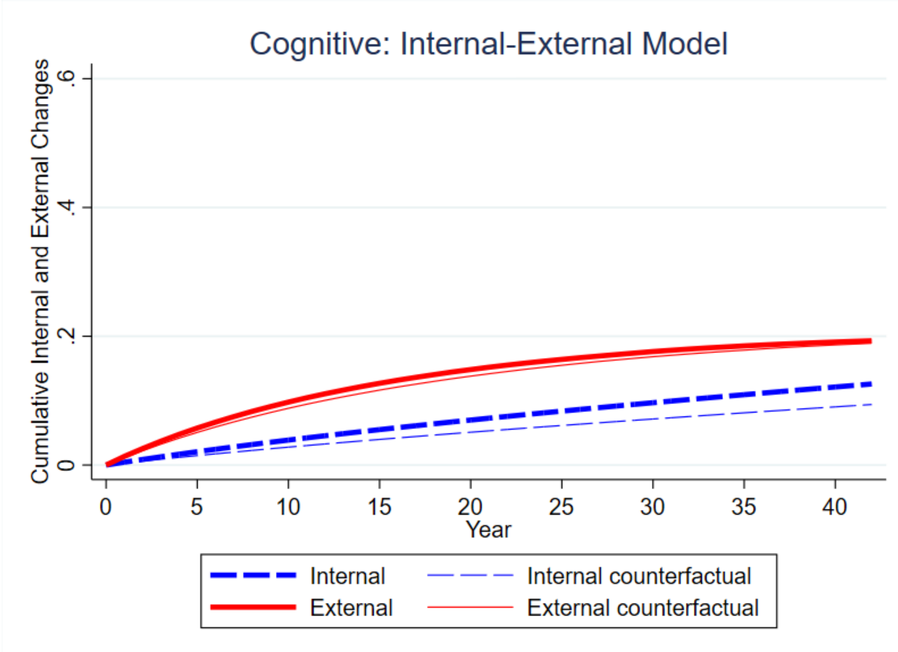


5.2 Firing tax parameter τ

The second experiment involves reducing the firing tax parameter τ by half ($\tau = 0.15$). Figure 16 shows the path of stocks. The thick lines are the baseline case and the thin lines are from the counterfactual economy with $\tau = 0.15$. The result indicates that the firing tax makes the labor market more polarized: the level of routine employment is higher without the firing taxes, whereas the level of cognitive employment is lower. Individual firms adjust the composition of labor *faster* when the firing tax is larger.

This result may sound counterintuitive, given that a larger τ implies greater labor market frictions. The intuition here is that the firms are forward-looking. In a more frictional economy, firms adjust their occupational composition *before* they change s_a. Firms are constantly hit by the s_h shocks and adjust employment in each occupation in response to these shocks. The timing when a positive shock to s_h hits the firm is an opportunity to expand cognitive employment. When a negative shock to s_h hits the

Figure 17: Counter-Factual on Flow of Cognitive: Reduce τ by Half



firm, the firm has to reduce its employment (by firing taxes). It uses this occasion as an opportunity to readjust the occupational composition. A firm that is likely to adopt the automation technology reduces routine employment at that timing, even though s_a is not yet upgraded. Therefore, the occupational composition under frictions is naturally "more biased."

It turns out that the speed of automation is almost identical between $\tau = 0.3$ and $\tau = 0.15$. On the one hand, a higher firing tax makes the reward of automation smaller and thus slows down the speed of automation. On the other hand, facing a higher firing cost, a large and unproductive firm has a large incentive to automate so that it can utilize a large employment.[14] These two forces are almost exactly offset with each other. Therefore, the difference in the speed of polarization in Figure 16 is almost entirely due to the firm's forward-looking ("precautionary") adjustment of labor given the path of automation.

Figures 17, 18, and 19 show the impact on the flow dimension. With lower τ, the total

[14] A similar intuition appear in Mukoyama and Osotimehin (2019) in a model of innovation and growth.

Figure 18: Counter-Factual on Flow of Routine: Reducing τ by Half

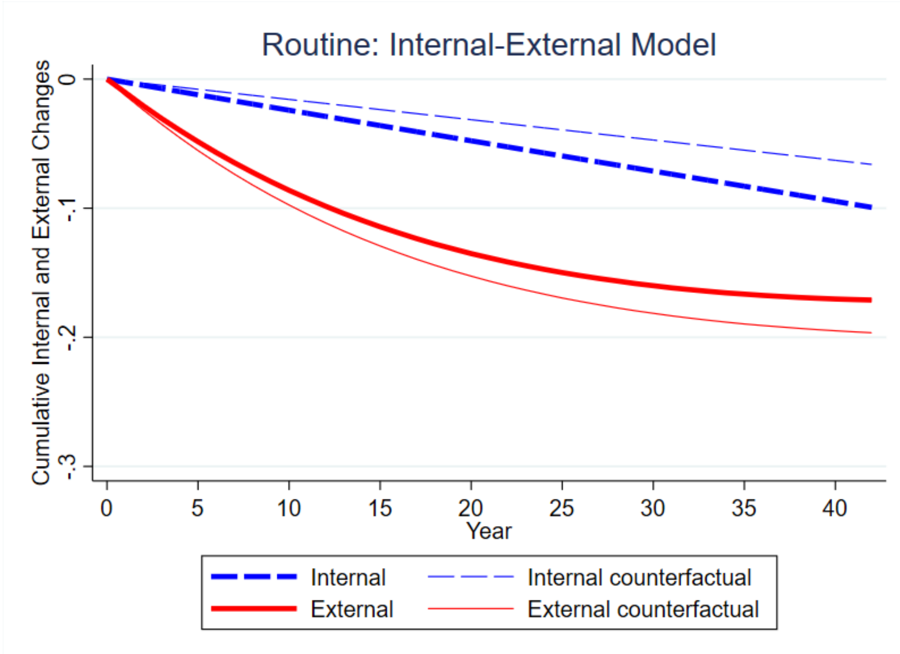
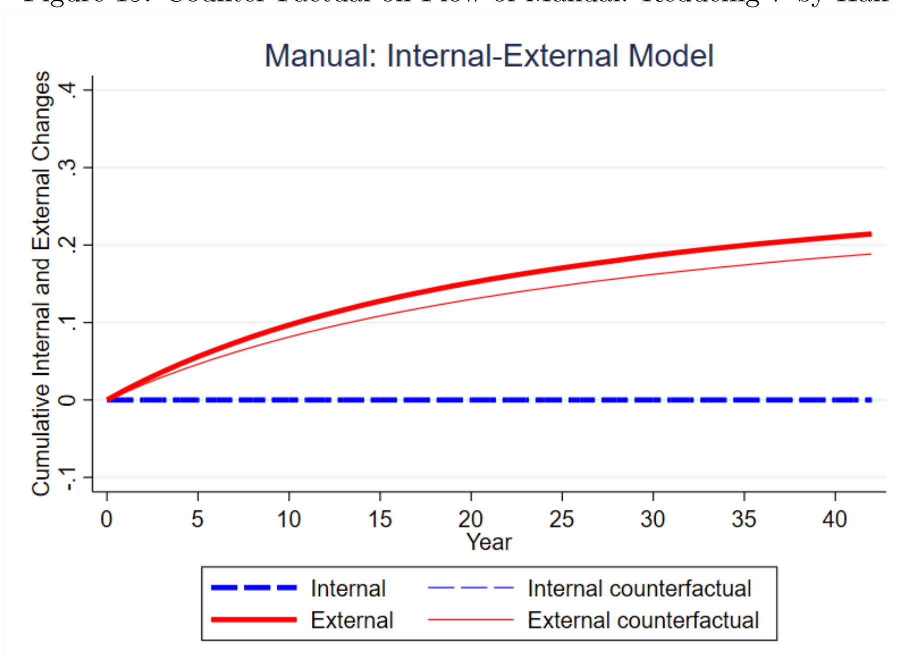


Figure 19: Counter-Factual on Flow of Manual: Reducing τ by Half



adjustment, especially the external adjustment, decreases. This outcome is consistent with the path of stocks.

6 Conclusion

We analyze how labor-market frictions interact with firms' decisions to reallocate workers across occupations when the economy faces labor-market polarization. Using datasets from the United States and Germany, we document that the pattern of occupational adjustments differs between these two countries. Although the aggregate pattern of labor-market polarization is very similar between the two countries, US firms adjust the occupational mix almost entirely through firing and hiring. In Germany, within-firm reallocation plays non-negligible roles in the decline in routine occupations and the increase in cognitive occupations.

We then build a model of firm dynamics with occupational mobility and labor-market frictions. Our model extends the standard firm-dynamics model in the tradition of Hopenhayn (1992) and Hopenhayn and Rogerson (1993) to multiple occupations and automation decisions. We calibrate the model to the US economy and find that the empirically relevant amount of firing taxes in Germany can replicate the different patterns of labor market adjustments during the labor market polarization across these two countries.

Using the calibrated model, we conduct two counterfactual experiments. First, we reduce a parameter that governs the cost of occupational adjustments within a firm. This parameter is interpreted as the level of reorganization costs when the occupational mix changes. We find that reducing this cost does not affect the speed of labor market polarization, whereas it significantly affects the combination of within- versus across-firm adjustments in the occupational mix. Second, we reduce the firing tax rate. Reducing the firing tax slows down the process of polarization because a forward-looking firm will start adjusting the occupational mix before automation when the tax is high. The net flows across occupations are also reduced by the reduction of tax. This change in flows mostly comes from the external adjustment margin.

This study highlights the distinction between within- and across-firm labor adjustments. This distinction also matters in the context of aggregate unemployment. One may easily imagine that one of the social costs of across-firm labor adjustments can be the unemployment of routine workers. Our model framework does not feature unemployment, although it is possible to extend the model by adding friction in hiring workers. We leave this topic to future research.

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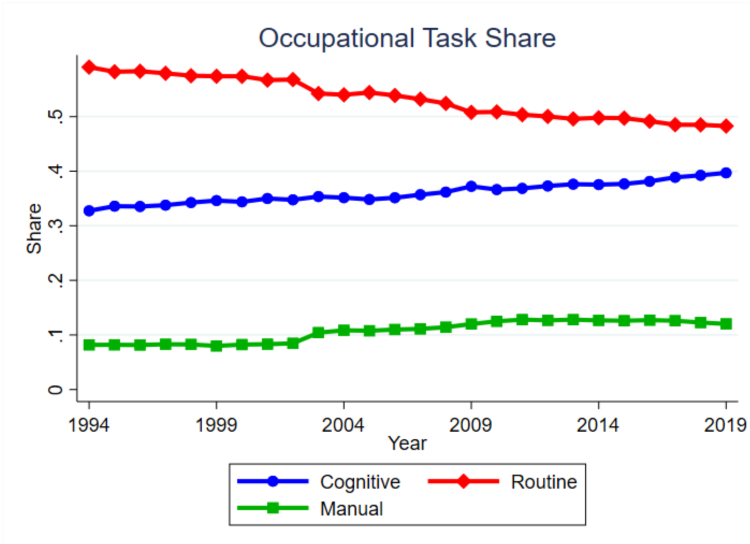
A Additional figure

Figure 20 corresponds to Figure 1 in the main text, but with a different dataset (Current Population Survey, CPS).

Figure 20: Occupational Employment Share in the United States, CPS, 1994–2019



Data Source: CPS

B Data details

B.1 Survey of Income and Program Participation (SIPP)

B.1.1 Data description

SIPP is a dataset of household-based panel survey, administrated by the US Census Bureau. We use the following seven panels from the SIPP for our analysis: 1990, 1991, 1992, 1993, 1996, 2001, and 2004. These panels have a sample of 14,000 to 52,000 individuals. Each panel is a nationally representative sample of households interviewed every 4 months. Individuals are asked to provide their employment information as detailed as in a weekly basis. With these SIPP panels, we identify the workers' job and occupation switches on an annual basis.

As noted in Stinson (2003), the 1990–1993 panels had substantial miscoding in their job IDs. Thus, we use the revised job IDs provided by the US Census Bureau. We do not use the panels before 1990 as no revised job IDs are provided. We are not able to use the 2008 panel because the US Census Bureau's data cleaning procedure has made occupational switches within firms unidentifiable for that panel.

B.1.2 Sample selection

We select observations where an individual is between ages 23 and 55. We drop observations where an individual works in the public sector or is self-employed. We also drop observations where no occupation information is available. We only focus on individuals who report valid employment status.

B.1.3 Data cleaning

In the SIPP, workers are asked to list up to two employers for each week. When a worker has two occupations at the same time, we select the occupation for the greater number of hours worked. We drop the observations with managerial occupations to eliminate the flows due to promotions. Those managerial occupations include:

- Legislators,
- Chief executives and general administrators, public administration
- Administrators and officials, public administration
- Administrators, protective services
- Financial managers
- Personnel and labor relations managers
- Purchasing managers
- Managers, marketing, advertising, and public relations
- Administrators, education and related fields
- Managers, medicine and health
- Postmasters and mail superintendents
- Managers, food serving and lodging establishments
- Managers, properties and real estate, funeral directors
- Managers, service organizations, n.e.c.
- Managers and administrators, n.e.c.

B.1.4 Attrition

One of the major problems in longitudinal survey data is that individuals can drop from the sample over time. The SIPP is not exempt from this problem as well, which creates biases in the decomposition results. Therefore, we run a robustness check by running the decomposition with the balanced panels of the SIPP in Appendix D.

B.1.5 Identifying job and occupational switches

We follow Xiong (2008) to identify occupaitonal switches of workers. We first define the three broad occupational groups from the as listed in Appendix B.4. When a report multiple occupations, we use the one for the job that report the largest number of hours worked in the month. Keeping the monthly frequency of the SIPP panel, we then identify the occupational switches by comparing the occupation of the worker in the current month and the 12 months ago. The identified switches are then aggregated to the annual frequency.

The identified occupational switches are classified into within-firm and across-firm switches by using the Job ID. The within-firm switches are the switches where the worker stays in the same firm. The across-firm switches are the switches where the worker moves to a different firm.

It is well-known in the literature that measurement errors in occupational codes might give rise to spurious transitions, as discussed for example in Kambourov and Manovskii (2009) and Moscarini and Thomsson (2007). Our approach here is similar to that in Carrillo-Tudela et al. (2022): We use a high degree of aggregation (i.e. three broad occupational groups) to minimize the coding errors. Furthermore, since 1986 the SIPP interviewing procedure has implied that if the worker declared he/she did not change type of job and employer in a given interview, the occupational code recorded in the previous interview was carried forward. This form of "dependent interviewing" reduces spurious occupational transitions among with-in-firm switchers.

B.2 Sample of Integrated Labor Market Biographies (SIAB)

B.2.1 Data description

We use the Sample of Integrated Labour Market Biographies (SIAB) for the period 1975–2017 for our analysis of German labor markets, provided by the Institute for Employment Research (IAB) in Germany. The dataset is a 2 % sample of the population of the Integrated Employment Biographies (IEB), which It includes employees covered by social security, marginal part-time employment (since 1999), unemployment insurance benefit recipients, and individuals who are officially registered as job-seeking or who are participating in programs of active labor market policies. It excludes the self-employed, civil servants, individuals performing military service, and those not in the labor force.

B.2.2 Sample selection

We select individuals who have German citizenship and have never worked in East Germany. We then select observations where the individual is between ages 23 and 55. We drop observations where no occupation information is available.

B.2.3 Data cleaning

When a worker has multiple job spells in a year, to identify the main occupation, we calculate the number of days worked for each job spell and select an occupation that is associated with the job spell with the most days. If the number of days worked are not available, we do the same for the earnings per day. We drop the observations of managerial occupations to eliminate the flows due to promotions. Those managerial occupations include:

- Foremen, master mechanics
- Entrepreneurs, managing directors
- Members of Parliament, ministers
- Senior government officials
- Association leaders, officials

B.2.4 Attrition

Workers may disappear from the social security records for various reasons (leave the labor force, migrate abroad, become a public servant or self-employed, or pass away). The IAB is adding new individuals to the sample every year to keep it as 2% of the entire population in Germany.

B.2.5 Identifying job and occupational switches

We follow Böhm et al. (2024) to identify occupational switches of workers. We first define the three broad occupational groups as listed in Appendix B.4. When a worker reports multiple occupations, we use the one for the job that reports the largest number of days worked in the year. Keeping the annual frequency of the SIAB panel, we then identify the occupational switches by comparing the occupation of the worker in the current year and the previous year. We classify within-firm and across-firm switches by using the establishment IDs.

B.3 Current Population Survey (CPS)

B.3.1 Data description

CPS, administered by the US Census Bureau, is conducted with a sample of around 60,000 households and consists of the basic monthly questions focusing on labor force participation and supplemental questions such as the annual March income supplement. Each individual shows up in the records at most eight times: respondents are contacted monthly for the first four consecutive months followed by eight months of gap and then the monthly interview resumes for the last four months. We use the Public Use Microdata File of the Basic Monthly CPS files from January 1994 to October 2019, which are obtained from the DataWeb FTP of the US Census Bureau. The respondents are matched based on Drew et al. (2014).

B.3.2 Sample selection

For comparability with the SIPP estimates, we restrict our focus to males between ages 23 and 55. We drop observations where an individual works in the public sector or is self-employed. We also drop observations where no occupation information is available.

B.4 Occupational groups

B.4.1 United States

We classify occupations into the three broad groups, as defined by Acemoglu and Autor (2011). For the SIPP and CPS, we aggregate the US Census' 1990/2000 Occupational Classification codes into these three broader categories:

1. Nonroutine cognitive: professional, technical, management, business and financial occupations.
2. Routine: clerical, administrative support, sales workers, craftsmen, foremen, operatives, installation, maintenance and repair occupations, production and transportation occupations, laborers.
3. Nonroutine manual: service workers.

B.4.2 Germany

For the SIAB, we follow Böhm et al. (2024) to group three-digit occupations (120 occupations according to the KLDB1988 classification) into nine categories, and define the three groups, which correspond to those in Acemoglu and Autor (2011), as follows:

1. Nonroutine cognitive: managers, professionals, and technicians.
2. Routine: craftspeople, sales personnel, office workers, production workers, operations and laborers.
3. Nonroutine manual: service personnel.

C Decomposition method

Let ℓ_{it} be the stock of employment of occupation i at time t. Further, let

$$E_t \equiv \sum_{i=c,r,m} \ell_{it}$$

be the employment. The employment share at time t for occupation i is

$$\frac{\ell_{it}}{E_t}.$$

We want to decompose

$$\log\left(\frac{\ell_{i,t+1}}{E_{t+1}}\right) - \log\left(\frac{\ell_{it}}{E_t}\right)$$

into net flows.

$$\log(\ell_{it}) = \log\left(\sum_{j=c,r,m,k=s,d} f^{ji,k}_{t-1,t} + f^{Ui}_{t-1,t}\right) = \log\left(\sum_{j=c,r,m,k=s,d} f^{ij,k}_{t,t+1} + f^{iU}_{t,t+1}\right).$$

Here, U includes unemployment, out-of-labor force, and dropped/added sample. s is for the same firm, and d is for the different firm. Thus,

$$\begin{aligned}
\log(\ell_{i,t+1}) - \log(\ell_{it}) &= \log\left(\frac{\sum_{j=c,r,m,k=s,d} f^{ji,k}_{t,t+1} + f^{Ui}_{t,t+1}}{\sum_{j=c,r,m,k=s,d} f^{ij,k}_{t,t+1} + f^{iU}_{t,t+1}}\right) \\
&= \log\left(1 + \frac{\sum_{j\neq i,k=s,d}(f^{ji,k}_{t,t+1} - f^{ij,k}_{t,t+1}) + (f^{Ui}_{t,t+1} - f^{iU}_{t,t+1})}{\sum_{j=c,r,m,k=s,d} f^{ij,k}_{t,t+1} + f^{iU}_{t,t+1}}\right) \\
&\approx \frac{\sum_{j\neq i,k=s,d}(f^{ji,k}_{t,t+1} - f^{ij,k}_{t,t+1}) + (f^{Ui}_{t,t+1} - f^{iU}_{t,t+1})}{\ell_{it}}
\end{aligned}$$

Note also that

$$\begin{aligned}
\log(E_{t+1}) - \log(E_t) &\approx \frac{E_{t+1} - E_t}{E_t} \\
&= \frac{1}{\ell_{it}} \ell_{it} \frac{E_{t+1} - E_t}{E_t} \\
&= \frac{1}{\ell_{it}} \left(\sum_{j=c,r,m,k=s,d} f_{t,t+1}^{ij,k} + f_{t,t+1}^{iU} \right) \frac{E_{t+1} - E_t}{E_t}
\end{aligned}$$

Let

$$\Delta_{t,t+1}^{E} \equiv \frac{E_{t+1} - E_t}{E_t}.$$

Combining the above, we have

$$\log\left(\frac{\ell_{i,t+1}}{E_{t+1}}\right) - \log\left(\frac{\ell_{it}}{E_t}\right) = \frac{1}{\ell_{it}} \left[\sum_{j \neq i} (f_{t,t+1}^{ji,s} - f_{t,t+1}^{ij,s}) + \sum_{j \neq i} (f_{t,t+1}^{ji,d} - f_{t,t+1}^{ij,d}) + (f_{t,t+1}^{Ui} - f_{t,t+1}^{iU}) - \ell_{it} \Delta_{t,t+1}^{E} \right].$$

To calculate the cumulative changes from period t to period $t+T$, note that

$$\log\left(\frac{\ell_{i,t+T}}{E_{t+T}}\right) - \log\left(\frac{\ell_{it}}{E_t}\right) = \sum_{\tau=0}^{T-1} \left[\log\left(\frac{\ell_{i,t+\tau+1}}{E_{t+\tau+1}}\right) - \log\left(\frac{\ell_{i,t+\tau}}{E_{t+\tau}}\right) \right].$$

Then, we can apply the decomposition formula to obtain

$$\begin{aligned}
&\log\left(\frac{\ell_{i,t+T}}{E_{t+T}}\right) - \log\left(\frac{\ell_{it}}{E_t}\right) \\
&\quad = \Bigg[\sum_{\tau=0}^{T-1} \sum_{j \neq i} \frac{f_{t+\tau,t+\tau+1}^{ji,s} - f_{t+\tau,t+\tau+1}^{ij,s}}{\ell_{i,t+\tau}} + \sum_{\tau=0}^{T-1} \sum_{j \neq i} \frac{f_{t+\tau,t+\tau+1}^{ji,d} - f_{t+\tau,t+\tau+1}^{ij,d}}{\ell_{i,t+\tau}} \\
&\qquad + \sum_{\tau=0}^{T-1} \frac{f_{t+\tau,t+\tau+1}^{Ui} - f_{t+\tau,t+\tau+1}^{iU}}{\ell_{i,t+\tau}} - \sum_{\tau=0}^{T-1} \Delta_{t+\tau,t+\tau+1}^{E} \Bigg].
\end{aligned}$$

D Balanced panel for SIPP

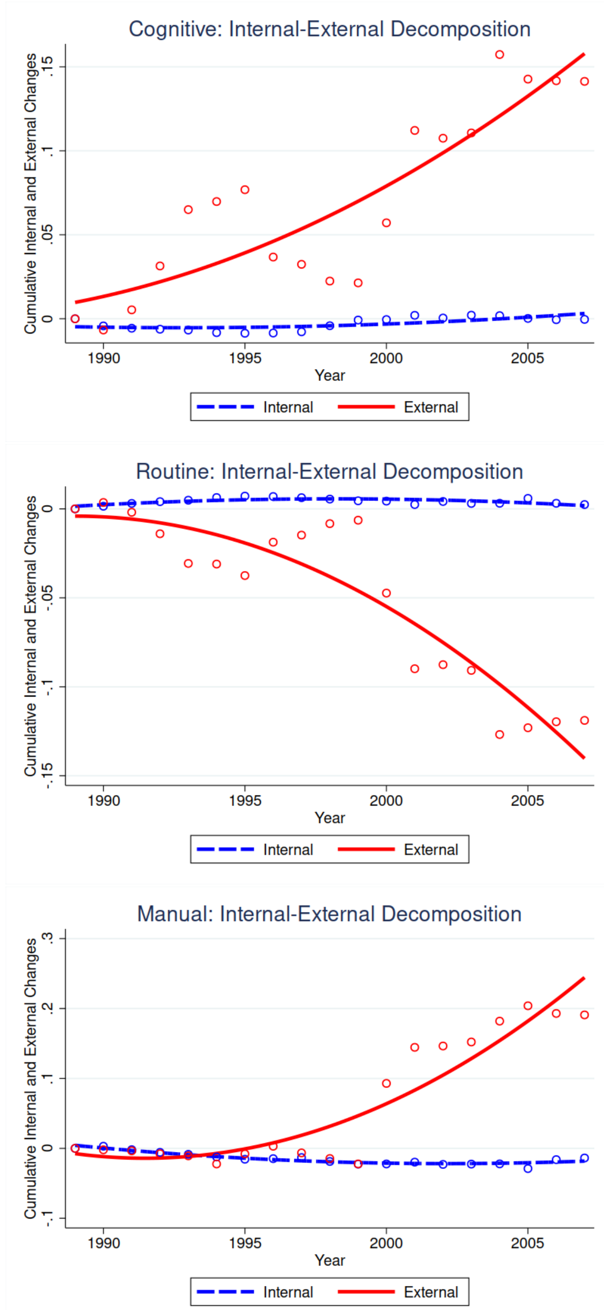
To check the robustness of our results in Table 1, and Figures 2 and 3 for the sample attrition issue of the SIPP sample, we create a balanced panel for the SIPP and run the decomposition again. That is, we select individuals who report their labor market status without any missing observations over the sample period of each SIPP panel and use the created balanced panel data for our analysis. Our internal-external decomposition results do not change the patterns even for the balanced panel case, as seen in Table 3 and Figure 21.

Table 3: Decompositions of Occupational Employment Share Changes for the United States, Balanced Panel

	Occupational employment share			Decomposed contributions	
	(1)	(2)	(3)	(4)	(5)
United States	1989	2007	log (Δshare)	Internal	External
Cognitive	0.29	0.33	0.14	0.00	0.14
Routine	0.60	0.53	−0.12	0.00	−0.12
Manual	0.11	0.13	0.18	−0.01	0.19

Data Source: SIPP; 1990, 1991, 1992, 1993, 1996, 2001, and 2004 panels.

Figure 21: Cumulative Changes in Occupational Employment in the United States, SIPP, 1989–2007, Balanced Panel



Data Source: SIPP; 1990, 1991, 1992, 1993, 1996, 2001, and 2004 panels.

E Computing the transition dynamics

We compute the transition path between the initial and final steady states. Let us overview the objects for computation. The notation is in the general form without simplifying.

First, the value functions are specified as follows. Assuming a linear utility with respect to consumption

$$U(C,N) \quad = \quad C - \xi \frac{N^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}},$$

we can set the discount factor for firms to be a constant β as $U_1(C',N')/U_1(C,N)$ is always unity. Thus, the value function at the initial state where all firms are non-automated is

$$\begin{aligned}
& \underline{V}(\mathbf{n}, s_h; \underline{s}_a) \\
= \; & \max_{\mathbf{n}' \geq \mathbf{0}} \Bigg[-\tau \max\{\sum_j (n_j - \tilde{n}'_j(\mathbf{n},\mathbf{n}')), 0\} - \sum_j \kappa_j (\max\{\tilde{n}'_j(\mathbf{n},\mathbf{n}') - n_j, 0\})^2 \\
& + f(\mathbf{n}', s_h; \underline{s}_a) - \underline{w}\mathbf{1}\cdot\mathbf{n}' + \beta \mathbb{E}_{s'_h}[\underline{V}(\mathbf{n}', s'_h; \underline{s}_a)|s_h] \Bigg],
\end{aligned}$$

and at the final state where all firms are automated is

$$\begin{aligned}
& \overline{W}(\mathbf{n}, s_h; \overline{s}_a) \\
= \; & \max_{\mathbf{n}' \geq \mathbf{0}} \Bigg[-\tau \max\{\sum_j (n_j - \tilde{n}'_j(\mathbf{n},\mathbf{n}')), 0\} - \sum_j \kappa_j (\max\{\tilde{n}'_j(\mathbf{n},\mathbf{n}') - n_j, 0\})^2 \\
& + f(\mathbf{n}', s_h; \overline{s}_a) - \overline{w}\mathbf{1}\cdot\mathbf{n}' + \beta \mathbb{E}_{s'_h}[\overline{W}(\mathbf{n}', s'_h; \overline{s}_a)|s_h] \Bigg].
\end{aligned}$$

Along the transition path, the value functions for non-automated and automated firms are

$$
\begin{aligned}
& V_t(\mathbf{n}, s_h; \underline{s}_a) \\
= \quad & \max_{\mathbf{n}' \geq \mathbf{0}, d \in \{0,1\}} \Bigg[-\tau \max\{\sum_j (n_j - \tilde{n}'_j(\mathbf{n}, \mathbf{n}')), 0\} - \sum_j \kappa_j (\max\{\tilde{n}'_j(\mathbf{n}, \mathbf{n}') - n_j, 0\})^2 \\
& + f(\mathbf{n}', s_h; \underline{s}_a) - w_t \mathbf{1} \cdot \mathbf{n}' \\
& + \beta \mathbb{E}_{s'_h} [p\{d(W_{t+1}(\mathbf{n}', s'_h; \overline{s}_a) - \Gamma(\underline{s}_a, \overline{s}_a)) + (1-d) V_{t+1}(\mathbf{n}', s'_h; \underline{s}_a)\} + (1-p) V_{t+1}(\mathbf{n}', s'_h; \underline{s}_a) | s_h] \Bigg],
\end{aligned}
$$

where d is the discrete choice to adopt, and

$$
\begin{aligned}
& W_t(\mathbf{n}, s_h; \overline{s}_a) \\
= \quad & \max_{\mathbf{n}' \geq \mathbf{0}} \Bigg[-\tau \max\{\sum_j (n_j - \tilde{n}'_j(\mathbf{n}, \mathbf{n}')), 0\} - \sum_j \kappa_j (\max\{\tilde{n}'_j(\mathbf{n}, \mathbf{n}') - n_j, 0\})^2 \\
& + f(\mathbf{n}', s_h; \overline{s}_a) - w_t \mathbf{1} \cdot \mathbf{n}' + \beta \mathbb{E}_{s'_h} [W_{t+1}(\mathbf{n}', s'_h; \overline{s}_a) | s_h] \Bigg],
\end{aligned}
$$

respectively.

Second, the distributions of firms are defined as below. Let $m_t^V(\mathbf{n}, s_h; \underline{s}_a)$ and $m_t^W(\mathbf{n}, s_h; \overline{s}_a)$ be the measures of non-automated and automated firms in the period t, and M_t^V and M_t^W be the total mass of the corresponding firms. The mass is defined as

$$
\begin{aligned}
M_t^V &= \sum_{\mathbf{g_n}} \sum_{g_h} m_t^V(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s}_a), \\
M_t^W &= \sum_{\mathbf{g_n}} \sum_{g_h} m_t^V(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \overline{s}_a).
\end{aligned}
$$

The counterparts at the initial steady state are denoted by $\underline{m}^V(\mathbf{n}, s_h; \underline{s}_a)$ and $\underline{M}^V$. At the final steady state, they are $\overline{m}^W(\mathbf{n}, s_h; \overline{s}_a)$ and $\overline{M}^W$. We assume $M_t^V = M_t^W = \underline{M}^V = \overline{M}^W = 1$, as we shut down entry-exit.

We compute these objects with the following steps.

E.1 Preparation

We discretize the labor and shock, and the grid points are denoted by $(n_m^{g_m}, n_c^{g_c}, n_r^{g_r}) = \mathbf{n}^{\mathbf{g_n}}$, respectively, and $s_h^{g_h}$ where integer $g_\cdot \in \{1, ..., g_\cdot^{max}\}$. Later, we redistribute the weight of an off-grid point $\mathbf{n}$ to the neighboring grid points, such as $\mathbf{n}^{\mathbf{g_n}}$, by the following discrete measure G such that

$$G(\mathbf{n}, \mathbf{n}^{\mathbf{g_n}}) = \begin{cases} \frac{\prod |n_j^{g_j'} - n_j|}{\prod_j |n_j^{g_j'} - n_j^{g_j}|} & \text{if } n_j \text{ is between } n_j^{g_j} \text{ and } n_j^{g_j'} \text{ including endpoint for all } j = m, c, r, \\ 0 & \text{otherwise,} \end{cases}$$

where g_j' is either $g_j - 1$ or $g_j + 1$. The transition probability from $s_h^{g_h}$ to $s_h^{g_h'}$ is denoted by $P(s_h^{g_h'} | s_h^{g_h})$.

While $(\beta, \eta, \phi, \tau, \kappa)$ are given from outside model, ξ is pinned down within the model. First, assuming $\tau = 0$ and $\underline{w}$=1, we solve for $\underline{V}$ and the corresponding decision rule $\underline{\mathbf{n}}'(\mathbf{n}, s_h; \underline{s}_a)$ by value function iteration. Next, simulating the above firms' decision rule repeatedly as

$$\underline{m}^{V,new}(\mathbf{n}^{\mathbf{g_n}'}, s_h^{g_h'}; \underline{s}_a) = \sum_{\mathbf{g_n}} \sum_{g_h} G(\underline{\mathbf{n}}'(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s}_a), \mathbf{n}^{\mathbf{g_n}'}) P(s_h^{g_h'} | s_h^{g_h}) \underline{m}^{V,old}(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s}_a),$$

we can obtain an invariant distribution of firms $\underline{m}^V(\mathbf{n}, s_h; \underline{s}_a)$. Then, the labor demand is computed as $\underline{N} = \sum_{\mathbf{g_n}} \sum_{g_h} \mathbf{1} \cdot \mathbf{n}'(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s}_a) \underline{m}^V(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s}_a)$. Then, by the intra-temporal optimality,

$$\xi = \frac{\underline{w}}{\underline{N}^{\frac{1}{\eta}}}.$$

E.2 Computing the initial and final steady states

Setting $\tau > 0$, we guess the GE wage $\underline{w}$, solve for $\underline{V}$ and the corresponding decision rule $\underline{\mathbf{n}}'(\mathbf{n}, s_h; \underline{s}_a)$ by value function iteration, and compute the invariant distribution $\underline{m}^V(\mathbf{n}, s_h; \underline{s}_a)$ by using the obtained decision rule similarly to the previous subsection. Then, we check if

$\underline{w}$ equates the demand and supply of labor as

$$\left(\frac{\underline{w}}{\xi}\right)^{\eta} = \sum_{\mathbf{g_n}}\sum_{g_h}\mathbf{1}\cdot\mathbf{n}'(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s}_a)\underline{m}^V(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s}_a).$$

If there is excess demand, we increase $\underline{w}$ and *vice versa.* Then, we repeat it until $\underline{w}$ equates the demand and supply of labor. We apply the same steps for $\overline{w}$ and $\overline{W}$.

E.3 Backward induction

First, we guess the path of w_t on the transition. Given w_t, we solve for V_t and W_t, and corresponding decision rules $\mathbf{n}'_t(\mathbf{n}, s_h; \underline{s}_a)$ and $\mathbf{n}'_t(\mathbf{n}, s_h; \overline{s}_a)$ by backward induction from T to 1, while we set $W_{T+1} = \overline{W}$ and $V_{T+1} = \overline{V}$. The latter is a hypothetical non-automated value function at the final steady state and obtained by solving

$$\begin{aligned}
& \overline{V}(\mathbf{n}, s_h; \underline{s}_a) \\
= \; & \max_{\mathbf{n}'\geq\mathbf{0}, d\in\{0,1\}} \Bigg[-\tau\max\{\sum_j (n_j - \tilde{n}'_j(\mathbf{n},\mathbf{n}')), 0\} - \sum_j \kappa_j(\max\{\tilde{n}'_j(\mathbf{n},\mathbf{n}') - n_j, 0\})^2 \\
& + f(\mathbf{n}', s_h; \underline{s}_a) - \overline{w}\mathbf{1}\cdot\mathbf{n}' \\
& + \beta\mathbb{E}_{s'_h}[p\{d(\overline{W}(\mathbf{n}', s'_h; \overline{s}_a) - \Gamma(\underline{s}_a, \overline{s}_a)) + (1-d)\overline{V}(\mathbf{n}', s'_h; \underline{s}_a)\} + (1-p)\overline{V}(\mathbf{n}', s'_h; \underline{s}_a)|s_h] \Bigg],
\end{aligned}$$

At each t, we solve for V_t and W_t and the decision rules, and proceed to $t-1$.

E.4 Simulating forward

Using the decision rules obtained above for $t = 1, ..., T$, we can compute $m_t^V(\mathbf{n}, s_h; \underline{s}_a)$, $m_t^W(\mathbf{n}, s_h; \overline{s}_a)$ as follows. Let $\phi_t(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s}_a)$ be the indicator of firms adopting at the grid point $(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h})$. First,

$$m_t^V(\mathbf{n}^{\mathbf{g_n}'}, s_h^{g'_h}; \underline{s}_a) = \sum_{\mathbf{g_n}}\sum_{g_h} G(\mathbf{n}'_t(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s}_a), \mathbf{n}^{\mathbf{g_n}'})P(s_h^{g'_h}|s_h^{g_h})(1-\phi_t(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s}_a))m_{t-1}^V(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s}_a).$$

Second,

$$
\begin{aligned}
m_t^W(\mathbf{n}^{\mathbf{g_n}\prime}, s_h^{g_h'}; \overline{s}_a) &= \sum_{\mathbf{g_n}} \sum_{g_h} G(\mathbf{n}_t'(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s}_a), \mathbf{n}^{\mathbf{g_n}\prime}) P(s_h^{g_h'} | s_h^{g_h}) \phi_t(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s}_a) m_{t-1}^V(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s}_a) \\
&+ \sum_{\mathbf{g_n}} \sum_{g_h} G(\mathbf{n}_t'(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \overline{s}_a), \mathbf{n}^{\mathbf{g_n}\prime}) P(s_h^{g_h'} | s_h^{g_h}) m_{t-1}^W(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \overline{s}_a)
\end{aligned}
$$

where the first term on the right-hand side represents the non-automated firms which become automated at the end of period t, and the second is the automated firms from the last period. As for the period 1 measure, we set $m_1^V(\mathbf{n}, s_h; \underline{s}_a) = \underline{m}^V(\mathbf{n}, s_h; \underline{s}_a)$ and $m_1^W(\mathbf{n}, s_h; \overline{s}_a) = 0$.

E.5 Updating the guess

We check if w_t for each t equates the demand and supply of labor as

$$
\left(\frac{w_t}{\xi}\right)^\eta = \sum_{\mathbf{g_n}} \sum_{g_h} \mathbf{1} \cdot \mathbf{n}_t'(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s}_a) m_t^V(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s}_a) + \sum_{\mathbf{g_n}} \sum_{g_h} \mathbf{1} \cdot \mathbf{n}_t'(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \overline{s}_a) m_t^W(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \overline{s}_a)
$$

where the first term on the right-hand side is the labor demand from non-automated firms and the second term is the demand from automated firms. If there is excess demand, increase w_t and *vice versa*. Then, we go back to the backward induction until w_t equates the demand and supply of labor for $t = 1, ..., T$.