Monetary Policy Risks in the Bond Markets and Macroeconomy

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Abstract

What is the role of monetary policy fluctuations for macroeconomic uncertainty and nominal bond risks? To answer this question we develop a structurally motivated asset-pricing framework which incorporates the flexible dynamics of a time-varying Taylor rule and macroeconomic factors, and risk pricing restrictions that arise from recursive preferences. In our model, movements in monetary regimes can affect short rate sensitivities to expected growth and expected inflation, and the conditional volatility of inflation expectations. We use a Bayesian MCMC particle filter approach to estimate the model using the macroeconomic, forecast, and term structure data. We find that aggressive policy regimes are associated with high volatility of inflation and high levels and volatilities of bond yields and bond risk premia. Movements in monetary policy significantly contribute to persistent fluctuations in bond risk premia. In aggressive regime, short rate sensitivity to inflation and the conditional uncertainty about future inflation both increase, which tends to raise bond risk premia. On the other hand, an increase in short rate sensitivity to expected real growth has an opposite, dampening effect.

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Introduction

There is a significant evidence that monetary policy fluctuates over time. In certain periods monetary authority has to react more strongly to fundamental concerns about real economic growth and inflation, and thus the sensitivity of short-term interest rates to macroeconomic risks can vary over time. What are the implications of these monetary policy fluctuations for the macroeconomy and bond markets? We develop and estimate an economically motivated asset-pricing model which allows us to quantify the effects of monetary policy fluctuations above and beyond standard macro-finance channels. We find that in aggressive policy regimes interest rates respond significantly more to expected real growth and inflation risks, and inflation uncertainty increases. Further, in aggressive regimes the levels and volatilities of yields and risk premia are on average higher relative to passive regimes. Movements in monetary policy significantly contribute to persistent fluctuations in bond risk premia. Interestingly, we find that the variation in inflation and real growth sensitivities of interest rates have an opposite effect on bond risk premia. In aggressive regime, short rate sensitivity to inflation increases, which tends to raise bond risk premia. On the other hand, an increase in short rate sensitivity to expected real growth has an opposite, dampening effect.

Our asset-pricing framework features a novel recursive-utility based representation of the stochastic discount factor (SDF), the exogenous dynamics for the macroeconomic factors, and the time-varying Taylor rule for the interest rates. Specifically, our specification for the stochastic discount factor incorporates pricing conditions of the recursive-utility investor, but does not force the inter-temporal restriction between the short rate and the fundamental macroeconomic processes. This representation is an alternative to the decompositions in Bansal, Kiku, Shaliastovich, and Yaron (2013) and Campbell, Giglio, Polk, and Turley (2012), and identifies long-run cash-flow, long-run interest rate news, and the uncertainty news as the key sources of risk for the investor. Our representation of the SDF is particularly convenient for our analysis. It allows us to incorporate the exogenous dynamics of the short rates, consumption and inflation risk factors into the stochastic discount factor, and derive prices for nominal bonds at longer maturities. Our approach is similar to the reduced-form, no-arbitrage models of the term structure which exogenously specify the dynamics of the short rates and the state variables.[1] An important difference is that in our specification, the stochastic discount factor is economically motivated, and the sources of risks and their

[1] See Singleton (2006) for the review of the no-arbitrage term structure models.

market prices of risks are disciplined to be consistent with the recursive utility models.

To model the short rate, we assume a forward looking, time-varying Taylor rule in which the sensitivities of the short rate to expected real growth and expected inflation can vary across the monetary policy regimes. As in Bansal and Shaliastovich (2013) and Piazzesi and Schneider (2005), the expected real growth and expected inflation follow persistent processes, and expected inflation can have a negative impact on future real growth. This inflation non-neutrality plays an important role to generate an average positive risk premium for long-term nominal bonds. Following Bansal and Shaliastovich (2013), we also incorporate exogenous fluctuations in inflation volatility which drive the quantity of macroeconomic risks and bond risk premia. Novel in our paper, we allow inflation volatility to directly depend on the monetary policy regime. In this sense, we introduce a link between the "risk exposure" channel coming from the time-variation in short-rate coefficients to economic risks, and the "quantity of risk" channel due to the fluctuations in macroeconomic uncertainty.

To estimate the model, we utilize quarterly data on realized consumption and inflation, the survey expectations on real growth and inflation, and the data on bond yields for short to long maturities. The model-implied observation equations are nonlinear in the states, and all economic factors are latent. Similar to Schorfheide, Song, and Yaron (2013) and Song (2014), we rely on Bayesian MCMC methods to draw model parameters, and use particle filter to estimate latent states and evaluate the likelihood function.

We find that the estimation produces plausible parameter values and delivers a good fit to the observed macroeconomic and yield data. The expected consumption, expected inflation, and inflation volatility are very persistent, and expected inflation has a strong and negative feedback to future expected consumption. We further find substantial fluctuations in the impact of monetary policy across the regimes. Indeed, the median short rate loadings are equal to 0.8 and 1.9 on the expected growth and the expected inflation, respectively, which are significantly larger than the estimated loadings of 0.3 and 0.8 in the passive regime. The inflation volatility is also significantly larger in the aggressive regime.

Monetary policy fluctuations affect the conditional dynamics of bond yields and bond risk premia. We document that the loadings of bond yields and bond risk premia are magnified in aggressive relative to passive regime. The aggressive regimes are associated with significantly higher means and volatilities of bond yields and bond risk premia. In terms of the marginal contributions of different channels, we find that introducing time-variation in short rate loadings to expected inflation and time-variation in inflation volatility across the regimes increases the volatility of bond risk premia by about 20% each. Adding further the time-variation in short rate loadings to expected growth actually decreases the volatility of risk premia. This happens because increases in Taylor rule coefficients on expected growth and expected inflation have opposite effect on the level of the bond risk premia, so that an increase in bond risk premia in aggressive regime due to an increased sensitivity of short rates to inflation risks is partially offset by a decrease in bond risk premia due to an increase in short rate sensitivity to expected real growth. On average, the inflation channel dominates, so the risk premia variations increase with monetary policy fluctuations. However, conditionally bond risk premia can turn negative when inflation volatility effect is small. This is indeed what happens in the later part of the sample when the estimated inflation uncertainty is relatively low.

Related Literature. There are several contributions of our approach to the existing literature. First, we rely on a novel representation of the stochastic discount factor which allows us to incorporate the flexible dynamics of a time-varying Taylor rule and macroeconomic factors, and yet impose economic pricing restrictions that arise from recursive preferences. Second, we consider movements in the stochastic volatility in addition to the monetary policy fluctuations, and allow for the interaction between the two. Finally, we estimate the model using the macroeconomic and asset price data, and perform a quantitative assessment of the importance of the model channels.

In a related literature, Song (2014) consider a general equilibrium, long-run risks type model to study the implications of monetary policy and the exogenous changes in consumption and inflation correlation for the dynamics of bond and equity prices, and specifically for the comovement between bond and equity returns. Our paper entertains an alternative and more flexible representation of the stochastic discount factor and macroeconomic factors, and further incorporates a novel link between the exogenous volatility and the monetary policy fluctuations, which we find to be important to explain bond risk premia fluctuations. Campbell, Pflueger and Viceira (2014) use New Keynesian habit formation model to consider the implications of the monetary policy fluctuations for the equity and bond correlation and the bond risk premia. They do not entertain movements in the macroeconomic uncertainties. Hasseltoft (2012) and Bansal and Shaliastovich (2013) document the importance of the fluctuations in macroeconomic uncertainty for the time-variation in bond risk premia. Gallmeyer et al. (2006) incorporate a constant coefficient Taylor rule into a habits model. These papers do not deal with the time-variation in monetary policy.

In terms of the reduced-form term structure literature, Bansal and Zhou (2002) implements regime shifting coefficients in the short rate of a one factor Cox-Ingersoll-Ross model. Beyond matching bond yield facts, such as the violation of the Expectations Hypothesis, they show that filtered state probabilities are correlated with business cycle movements. Dai, Singleton, and Yang (2007) show that incorporating regime shift factors helps increase time-variation in expected excess bond returns. Ang and Bekeart (2002) and Ang et al. (2008) further document the importance of regimes shifts for the term structure of interest rates. The latter uses regime shifts and time-varying prices of risks with a statistically-based stochastic discount factor to discuss implications of regime shifts for real economic rates.

In terms of the earlier literature, Hamilton (1989) was the first to perform a Markov-switching, regime shift model using purely macroeconomic data in a traditional VAR setting, finding that state parameters correspond to peaks and troughs in the business cycle. The seminal work of Sims and Zha (2006) extended the Markov-switching to a Bayesian framework with a structural-VAR setup. The large conclusion of this work was that monetary policy shifts have been brief if at all existent. In fact the model that fits the best is one where there is stochastic volatility in the disturbances of fundamental variables. Our study is fundamentally different than this line of work in that we principally utilize term structure data to infer monetary policy movements. Because financial data is inherently forward looking there is much to gain from utilizing bond yields. This insight is consistent with Chernov and Bikbov (2013), who show that a richer cross-section of bond yields matters for judging shifts in policy.

Our paper is organized as follows. The next section discusses the economic model. In the following two sections, we provide an overview of our estimation method and discuss our results. The last section concludes.

1 Economic Model

1.1 Stochastic Discount Factor

Similar to Bansal et al. (2013), we derive a convenient representation of the stochastic discount factor which incorporates pricing conditions of the recursive-utility investor, but which does not force the inter-temporal restriction between the short rate and the fundamental macroeconomic processes. This approach allows us to maintain economic restrictions on the fundamental risk sources and their market prices of risks, and at the same time flexibly model the dynamics of the short rates and macroeconomic risk factors. Effectively, our approach is similar to the reduced-form, no-arbitrage models of the term structure which exogenously specify the dynamics of the short rates and the state variables. An important difference is that in our specification, the stochastic discount factor is economically motivated, and the sources of risks and their market prices of risks are disciplined to be consistent with the recursive utility model.

Specifically, as shown in Epstein and Zin (1989), under the recursive utility the log real stochastic discount factor (SDF) is given by,

$$m^r_{t+1} = \theta \log \delta - \frac{\theta}{\psi} \triangle c_{t+1} + (\theta - 1) r_{c,t+1}, \tag{1}$$

where where Δc_{t+1} is the real consumption growth, and $r_{c,t+1}$ is the return to the aggregate wealth portfolio. Parameter γ is a measure of a local risk aversion of the agent, ψ is the intertemporal elasticity of substitution, and $\delta \in (0,1)$ is the subjective discount factor. For notational simplicity, parameter θ is defined as $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$. The nominal SDF is given by

$$\begin{aligned} m_{t+1} &= m^r_{t+1} - \pi_{t+1} \\ &= \theta \log \delta - \frac{\theta}{\psi} \triangle c_{t+1} + (\theta - 1) r_{c,t+1} - \pi_{t+1}, \end{aligned} \tag{2}$$

where π_{t+1} is the inflation process.

The standard first-order condition implies that for any nominal return r_{t+1}, the Euler equation should hold:

$$E_t \left[\exp(m_{t+1} + r_{t+1}) \right] = 1. \tag{3}$$

Using this condition for the one-period short-term nominal interest rate, i_t, we obtain that:

$$m_{t+1} = -i_t - V_t + N_{m,t+1}, \tag{4}$$

where $N_{m,t+1} = (E_{t+1} - E_t) m_{t+1}$ is the innovation in the SDF, and

$$V_t = \log E_t(\exp(N_{m,t+1})) \tag{5}$$

is the entropy of the SDF. Intuitively, in a conditionally Gaussian model, V_t just captures half of the conditional variance of the stochastic discount factor, that is why we refer to this component as capturing uncertainty or volatility risks.

Our goal is to rewrite a generic SDF innovation $N_{m,t+1}$ above in terms of the deeper economic shocks, consistent with the recursive utility specification in (2). To this end, consider the news into the current and future expected stochastic discount factor. Based on the recursive utility formulation in (2), it is equal to,

$$
\begin{aligned}
(E_{t+1}-E_t)\sum_{j=0}\kappa_1^j m_{t+j+1} = -\frac{\theta}{\psi}(E_{t+1}-E_t)\sum_{j=0}\kappa_1^j \Delta c_{t+j+1} \\
+(\theta-1)(E_{t+1}-E_t)\sum_{j=0}\kappa_1^j r_{c,t+j+1}-(E_{t+1}-E_t)\sum_{j=0}\kappa_1^j \pi_{t+j+1}.
\end{aligned}
\tag{6}
$$

Note that the return to the consumption claim $r_{c,t+1}$ satisfies the budget constraint:

$$
r_{c,t+1}=\log\frac{W_{t+1}}{W_t-C_t}\approx \kappa_0+wc_{t+1}-\frac{1}{\kappa_1}wc_t+\triangle c_{t+1}, \tag{7}
$$

where wc is the log wealth-consumption ratio and the parameter $\kappa_1\in(0,1)$ corresponds to the log-linearization coefficient in the investor's budget constraint. Iterating this equation forward, we obtain that the cash-flow news, defined as the current and future expected shocks to consumption, should be equal to the current and future expected shocks to consumption return:

$$
N_{CF,t+1}=(E_{t+1}-E_t)\sum_{j=0}\kappa_1^j\Delta c_{t+j+1}=(E_{t+1}-E_t)\sum_{j=0}\kappa_1^j\Delta r_{c,t+j+1}. \tag{8}
$$

With that, the right-hand side of (6) simplifies to,

$$
\begin{aligned}
(E_{t+1}-E_t)\sum_{j=0}\kappa_1^j m_{t+j+1} &= -\frac{\theta}{\psi}N_{CF,t+1}+(\theta-1)N_{CF,t+1}-N_{\pi,t+1} \\
&= -\gamma N_{CF,t+1}-N_{\pi,t+1},
\end{aligned}
\tag{9}
$$

where the long-run inflation news are defined as,

$$
N_{\pi,t+1}=(E_{t+1}-E_t)\sum_{j=0}\kappa_1^j\pi_{t+j+1}. \tag{10}
$$

Now consider the same news into the current and future expected stochastic discount factor, $(E_{t+1} - E_t) \sum_{j=0} \kappa_1^j m_{t+j+1}$ based on the reduced-form representation in (4):

$$\begin{aligned}(E_{t+1} - E_t) \sum_{j=0} \kappa_1^j m_{t+j+1} &= N_{m,t+1} - (E_{t+1} - E_t) \sum_{j=0} \kappa_1^j (i_{t+j} + V_{t+j}) \\ &= N_{m,t+1} - N_{i,t+1} - N_{V,t+1},\end{aligned} \tag{11}$$

for the interest rate and volatility news:

$$\begin{aligned}N_{i,t+1} &= (E_{t+1} - E_t) \sum_{j=0} \kappa_1^j (i_{t+j}), \\ N_{V,t+1} &= (E_{t+1} - E_t) \sum_{j=0} \kappa_1^j (V_{t+j}).\end{aligned} \tag{12}$$

Equating the recursive-utility and reduced-form representation for the long-run news to SDF, we obtain that the SDF shock can be written as,

$$N_{m,t+1} = -\gamma N_{CF,t+1} + (N_{i,t+1} - N_{\pi,t+1}) + N_{V,t+1}, \tag{13}$$

and the total SDF can thus be specified as follows:

$$m_{t+1} = -i_t - V_t - \gamma N_{CF,t+1} + (N_{i,t+1} - N_{\pi,t+1}) + N_{V,t+1}. \tag{14}$$

That is, under the recursive utility framework, the agent effectively is concerned about long-run real growth news $N_{CF,t+1}$, long-run risk free rate news (inflation-adjusted short rate news $N_{i,t+1} - N_{\pi,t+1}$), and long-run uncertainty news $N_{V,t+1}$. The market price of the cash-flow risks is equal to the risk-aversion coefficient γ, while the market prices of both the interest rate and volatility shocks are negative 1: the marginal utility increases one-to-one with a rise in uncertainty or interest rates.

It is important to emphasize that the SDF representation above is common to all the recursive-utility based models. Indeed, to derive it we only used the Euler condition and the budget constraint, and did not rely on any assumptions about the dynamics of the underlying economy. In general equilibrium environments, these macroeconomic model assumptions are going to determine the decomposition of these underlying cash-flow, interest rate, and volatility risks into primitive economic shocks. In particular, both the interest rates and the volatility shocks become endogenous and depend on the underlying model structure. In our approach, we rely on the SDF representation (14), instead of a more primitive specification in (2). This allows us to model short-term interest rates exogenously together with consumption and inflation processes, and yet maintain the pricing implications of the recursive utility SDF. Notably, the uncertainty term is still endogenous in our framework, as the volatility term V_t and the innovations $N_{V,t+1}$ should be consistent with the entropy of the SDF in (5).

1.2 Economic Dynamics

In this section we specify the exogenous dynamics of consumption, inflation, and the short rates. This, together with the specification of the SDF and the Euler condition, allows us to solve for the prices of long-term nominal bonds.

We first specify a Markov chain to represent the time-variation in monetary policy regimes s_t. We assume N states with a transition matrix, T, given by:

$$T = \begin{pmatrix} \pi_{11} & \pi_{12} & \dots & \pi_{1N} \\ \pi_{21} & \pi_{22} & \dots & \pi_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ \pi_{N1} & \pi_{N2} & \dots & \pi_{NN} \end{pmatrix} = \begin{pmatrix} \vdots & \vdots & \vdots & \vdots \\ T_1 & T_2 & \dots & T_N \\ \vdots & \vdots & \vdots & \vdots \end{pmatrix}$$

where each π_{ji} indicates the probability of moving from state i to state j. Each column, T_i, is the vector of probabilities of moving from state i to all other states next period.

Similar to Bansal and Shaliastovich (2013), we specify an exogenous dynamics for consumption and inflation, which incorporates persistent movements in the conditional expectations and volatilities. The realized consumption and inflation are given by,

$$\begin{aligned} \triangle c_{t+1} &= \mu_c + x_{c,t} + \sigma_c^* \epsilon_{c,t+1} \\ \pi_{t+1} &= \mu_\pi + x_{\pi,t} + \sigma_\pi^* \epsilon_{\pi,t+1}. \end{aligned} \tag{15}$$

Both processes include zero mean, persistent expected components, $x_{i,t}$, in the spirit of Bansal and Yaron (2004), and $\epsilon_{i,t+1}$ represent i.i.d. Gaussian short-run shocks, which for parsimony are assumed to be homoscedastic. The joint VAR process $X_t = [x_{ct}, x_{\pi t}]'$ is given by:

$$\begin{bmatrix} x_{c,t+1} \\ x_{\pi,t+1} \end{bmatrix} = \begin{bmatrix} \Pi_{cc} & \Pi_{c\pi} \\ \Pi_{\pi c} & \Pi_{\pi\pi} \end{bmatrix} \begin{bmatrix} x_{c,t} \\ x_{\pi,t} \end{bmatrix} + \Sigma_t \epsilon_{t+1}. \tag{16}$$

This representation allows to capture a persistence of expected consumption and inflation risks, as measured by the values of Π_{cc} and $\Pi_{\pi\pi}$. Further, this specification can also capture an "inflation non-neutrality," that is, a negative response of future expected consumption to high expected inflation. As in Bansal and Shaliastovich (2013), both of these channels are going to play an important role to account for the bond market data.

In general, the volatility of the expected growth and expected inflation are time-varying, and can depend on the monetary policy regime:[2]

$$\Sigma_t = \begin{pmatrix} \sigma_{c,0} & 0 \\ 0 & \sigma_{\pi,t} \end{pmatrix} = \begin{pmatrix} \sqrt{\delta^c(s_t) + \tilde{\sigma}_{c,t}^2} & 0 \\ 0 & \sqrt{\delta^\pi(s_t) + \tilde{\sigma}_{\pi,t}^2} \end{pmatrix}$$

Each conditional variance depends on the monetary policy regime and is also driven by an orthogonal component $\tilde{\sigma}_{i,t}$. This component captures movements in macroeconomic volatilities which are independent from the monetary policy. Their dynamics is specified as follows:

$$\begin{aligned} \tilde{\sigma}_{ct}^2 &= \tilde{\sigma}_{c,0}^2 + \varphi_c \tilde{\sigma}_{c,t-1}^2 + \omega_c \eta_{\sigma c}, \\ \tilde{\sigma}_{\pi t}^2 &= \tilde{\sigma}_{\pi,0}^2 + \varphi_\pi \tilde{\sigma}_{\pi,t-1}^2 + \omega_\pi \eta_{\sigma \pi}. \end{aligned} \tag{17}$$

For simplicity, the exogenous volatilities are driven by Gaussian shocks. The specification can be extended to square root processes, as in Tauchen (2005), or positive Gamma shocks.

Finally, we specify the dynamics of the short rate. It follows a modified Taylor rule, in which the monetary authority reacts to expected growth and expected inflation, and the stance of the monetary policy can vary across the regimes. Specifically,

$$\begin{aligned} i_t &= i_0 + \alpha_c(s_t)\left(x_{ct} + \mu_c\right) + \alpha_\pi(s_t)\left(x_{\pi,t} + \mu_\pi\right) \\ &= \underbrace{\left[i_0 + \alpha_c(s_t)\mu_c + \alpha_\pi(s_t)\mu_\pi\right]}_{\equiv \alpha_0(s_t)} + \alpha_c(s_t) x_{ct} + \alpha_\pi(s_t) x_{\pi,t}. \end{aligned} \tag{18}$$

The loadings $\{\alpha_c(s_t), \alpha_\pi(s_t)\}$ are the key regime-dependent parameters of monetary policy. The interpretation of this Taylor rule is that the short rate loads stochastically on both expected growth and inflation. The justification for a "forward-looking" Taylor rule has been empirically founded and shown in Clarida, Gali and Gertler (2003).

[2] In empirical implementation, we focus on the time-variation in inflation volatility, and set consumption volatility to be constant.

1.3 Model Solution

In Appendix we show that the long-run cash-flow, inflation, interest rate, and volatility news can be expressed in terms of the underlying macroeconomic, interest rate, and regime-shift shocks. Specifically, the cash flow, inflation news and interest rates news are given by,

$$
\begin{aligned}
N_{CF,t+1} &= F_{CF,0}(s_{t+1}, s_t) + F_{CF,\epsilon}(s_{t+1}, s_t)'\Sigma_t \epsilon_{t+1} + \sigma_c^* \epsilon_{c,t+1}, && (19)\\
N_{\pi,t+1} &= F_{\pi,0}(s_{t+1}, s_t) + F_{\pi,\epsilon}(s_{t+1}, s_t)'\Sigma_t \epsilon_{t+1} + \sigma_\pi^* \epsilon_{\pi,t+1}, && (20)\\
N_{I,t+1} &= F_{I,0}(s_{t+1}, s_t) + F_{I,X}(s_{t+1}, s_t)' X_t + F_{I,\epsilon}(\dots)'\Sigma_t \epsilon_{t+1}, && (21)
\end{aligned}
$$

where the functions F depend on the policy regimes and model parameters.

We can also show that the uncertainty term V_t is in general given by,

$$
V_t(s_t, X_t, \tilde{\sigma}^2_{c,t}, \tilde{\sigma}^2_{\pi,t}) = V_0(s_t) + V_1(s_t)'X_t + V_{2c}(s_t)\tilde{\sigma}^2_{c,t} + V_{2\pi}(s_t)\tilde{\sigma}^2_{\pi,t}, \tag{22}
$$

so that the volatility news are given by,

$$
\begin{aligned}
N_{V,t+1} = {} & F_{v,0}(s_{t+1}, s_t) + F_{v,X}(s_{t+1}, s_t)'X_t + F_{v,\sigma c}(s_{t+1}, s_t)'\tilde{\sigma}^2_{ct} + F_{v,\sigma\pi}(s_{t+1}, s_t)'\tilde{\sigma}^2_{\pi t} \\
& + F_{v,\epsilon}(\dots)\Sigma_t \epsilon_{t+1} + F_{v,\eta c}(\dots)\omega_c \eta_{\sigma c,t+1} + F_{v,\eta\pi}(\dots)\omega_\pi \eta_{\sigma\pi,t+1}.
\end{aligned} \tag{23}
$$

The coefficients are determined as part of the model solution, and are given in the Appendix.

Combining all the components together, we can represent the SDF in terms of the underlying macroeconomic, interest rate, and regime shift shocks:

$$
\begin{aligned}
m_{t+1} &= -i_t - V_t - \gamma N_{CF,t+1} + N_{R,t+1} + N_{V,t+1} \\
&= S_0 + S'_{1,X} X_t + S_{1,\sigma c}\tilde{\sigma}^2_{ct} + S_{1,\sigma\pi}\tilde{\sigma}^2_{\pi t} + S'_{2,\epsilon}\Sigma_t \epsilon_{t+1} + S_{2,\eta c}\omega_c \eta_{c,t+1} + S_{2,\eta\pi}\omega_\pi \eta_{\pi,t+1} \\
&\quad - \gamma \sigma_c^* \epsilon_{c,t+1} - \sigma_\pi^* \epsilon_{\pi,t+1}.
\end{aligned} \tag{24}
$$

Because short rate loadings are time-varying, the SDF coefficients generally depend on monetary policy regimes. Using the expression for the SDF, we can derive prices for long-term nominal bonds.

1.4 Nominal Term Structure

In our model, log bond prices, p_t^n, depend in a linear way on the underlying expected growth, expected inflation, and volatility states, and the loadings vary across the regimes:

$$p_t^n = \tilde{\mathcal{A}}^n(s_t) + \tilde{\mathcal{B}}_X^{n\prime}(s_t)X_t + \tilde{\mathcal{B}}_{\sigma c}^n(s_t)\tilde{\sigma}_{ct}^2 + \tilde{\mathcal{B}}_{\sigma\pi}^n(s_t)\tilde{\sigma}_{\pi t}^2. \tag{25}$$

For $n = 1$ we uncover the underlying Taylor rule parameters:

$$\begin{aligned}
\tilde{\mathcal{A}}^1(i) &= -\alpha_0(i), \\
\tilde{\mathcal{B}}_X^1(i)' &= -\alpha(i)', \\
\tilde{\mathcal{B}}_{\sigma c}^1(i) &= 0, \\
\tilde{\mathcal{B}}_{\sigma\pi}^1(i) &= 0.
\end{aligned}$$

The bond yields are given by:

$$y_t^n = -\frac{1}{n}p_t^n = \mathcal{A}^n(i) + \mathcal{B}_X^{n\prime}(i)X_t + \mathcal{B}_{\sigma c}^n(i)\tilde{\sigma}_{ct}^2 + \mathcal{B}_{\sigma\pi}^n(i)\sigma_{\pi t}^2. \tag{26}$$

Finally, we can define one-period excess returns on $n-$maturity bond,

$$rx_{t\to t+1,n} = ny_{t,n} - (n-1)y_{t+1,n-1} - y_{t,1}. \tag{27}$$

The risk premia on these bonds is approximately equal to,

$$\begin{aligned}
E_t(rx_{t\to t+1,n}) + \frac{1}{2}Var_t(rx_{t+1,n}) &\approx -Cov_t(m_{t+1}, rx_{t+1,n}) \\
&= Cons(s_t) + r_{\sigma c}(s_t)\tilde{\sigma}_{ct}^2 + r_{\sigma\pi}(s_t)\tilde{\sigma}_{\pi t}^2.
\end{aligned} \tag{28}$$

In particular, the risk premia in our economy is time varying because there are exogenous fluctuations in stochastic volatilities, and because bond exposures fluctuate across monetary policy regimes. The second, monetary policy channel is absent in standard macroeconomic models of the term structure which entertain constant bond exposures and rely on time-variation in macroeconomic volatilities to generate movements in the risk premia (see e.g., Bansal and Shaliastovich (2013)). In the next section we assess the importance of the monetary policy risks to explain the term structure dynamics, above and beyond traditional economic channels.

2 Model Estimation

2.1 Data Description

We use macroeconomic data on consumption and inflation, survey data on expected real growth and expected inflation, and asset-price data on bond yields to estimate the model. For our consumption measure we use log real growth rates of expenditures on non-durable goods and services from the Bureau of Economic Analysis (BEA). The inflation measure corresponds to the log growth in the GDP deflator. The empirical measures of the expectations are constructed from the cross-section of individual forecasts from the Survey of Professional Forecasts at the Philadelphia Fed. Specifically, the expected real growth corresponds to the cross-sectional average, after removing outliers, of four-quarters-ahead individual expectations of real GDP. Similarly, the expected inflation is given by the average of four-quarters-ahead expectations of inflation. The real growth and inflation expectation measures are adjusted to be mean zero, and are rescaled to predict next-quarter consumption and inflation, respectively, with a loading of one. The construction of these measures follows Bansal and Shaliastovich (2009). Finally, we use nominal zero-coupon bond yields of maturities one through five years, taken from the CRSP Fama-Bliss data files. We also utilize the nominal three-month rate from the Federal Reserve to proxy for the short rate. Based on the length of the survey data, our sample is quarterly, from 1969 through 2014.

Table 1 shows the summary statistics for our variables. In our sample, the average short rate is 5.2%. The term structure is upward sloping, so that the five-year rate reaches 6.4%. Bond volatilities decrease with maturity from 3.3% at short horizons to about 3% at five years. The yields are very persistent. As shown in the bottom panel of the Table, real growth and inflation expectations are very persistent as well. The AR(1) coefficients for the real growth and inflation forecasts are 0.87 and 0.98, respectively, and are much larger than the those for the realized consumption growth and inflation. Figure 1 shows the time series of the realized and expected consumption growth and inflation rate. As shown on the Figure, the expected states from the surveys capture quite well the low frequency movements in the realized macroeconomic variables.

2.2 Estimation Method

In our empirical implementation of the model, we focus on the stochastic volatility channel of the expected inflation, and set the volatility of the expected real growth to be constant.[3] To identify the volatility level parameters, we set the monetary policy component of the inflation volatility in state one to be zero; to identify the regimes, we impose that the short rate sensitivity to expected inflation is highest in regime 2. Finally, for parsimony, we set the mean consumption, inflation, and interest rate parameters to be equal to their sample counterparts, and set the log-linearization parameter κ_1 to a typical value of .99 in the literature.

To estimate the model and write down the likelihood of the data, we represent the evolution of the observable macroeconomic, survey, and bond yield variables in a convenient state-space form:

$$
\begin{aligned}
\text{(Measurement)} \qquad & y_{t+1}^{1:Ny} = \mathcal{A}^{1:Ny}(s_{t+1}) + \mathcal{B}_X^{1:Ny}(s_{t+1})X_{t+1} + \mathcal{B}_{\sigma\pi}^{1:Ny}(s_{t+1})\tilde{\sigma}^2_{\pi,t+1} + \Sigma_{u,y}u_{t+1,y}, \\
& \triangle c_{t+1} = \mu_c + e_1' X_t + \sigma_c^* \epsilon_{c,t+1}, \\
& \pi_{t+1} = \mu_\pi + e_2' X_t + \sigma_\pi^* \epsilon_{\pi,t+1}, \\
& X_{SPF,t+1} = X_{t+1} + \Sigma_{u,X} u_{t+1,X},
\end{aligned}
$$

$$
\begin{aligned}
\text{(Transition)} \qquad & X_{t+1} = \Pi X_t + \Sigma_t(\tilde{\sigma}_{\pi t}, s_t)\epsilon_{t+1}, \\
& \tilde{\sigma}^2_{\pi t} = \tilde{\sigma}^2_{\pi,0} + \varphi_\pi \tilde{\sigma}^2_{\pi,t-1} + \omega_\pi \eta_{\sigma\pi,t}, \\
& s_t \sim \text{ Markov Chain } (\mathbb{P}_s),
\end{aligned}
$$

where Ny is the number of bond yields in the data. Notably, in our estimation we allow for Gaussian measurement errors on the observed yields and survey expectations, captured by $u_{t+1,y}$ and $u_{t+1,X}$. For parsimony and to stabilize the chains, we fix the volatilities of the measurement errors to be equal to 20% of the unconditional volatilities of the factors. As we describe in the subsequent section, the ex-post measurement errors in the sample are much smaller than that.

The set of parameters, to be jointly estimated with the states, is denoted by Θ, is given

[3]Identification of real volatility is challenging in bond market data alone. In a related framework, Song (2013) estimates real and inflation volatility incorporating the data on equity prices, which are much more informative about movements in real uncertainty than nominal bond yields.

by:

$$\Theta = \{\Pi, \delta^{\alpha\pi}, \tilde{\sigma}^2_{c0}, \tilde{\sigma}^2_{\pi 0}, \varphi_\pi, \omega_\pi, \sigma^*_c, \sigma^*_\pi, i_0, \gamma, \mu_c, \mu_\pi, \alpha_c^{1:N}, \alpha_\pi^{1:N}, \mathbb{P}_s\}.$$

The estimation problem is quite challenging due to the fact that the observation equations are nonlinear in the state variables, and the underlying expectation, volatility, and regime state variables are latent. Because of these considerations, we cannot use the typical Carter and Kohn (1994) methodology which utilizes smoothed Kalman filter moments to draw states. Instead, to estimate parameters and latent state variables we rely a Bayesian MCMC procedure using particle filter methodology to evaluate the likelihood function. As in Andrieu et al. (2010) and Fernandez-Villaverde and Rubio-Ramirez (2007), we embed the particle filter based likelihood into a Random Walk Metropolis Hasting algorithm and sample parameters in this way. Schorfheide et al. (2013) and Song (2014) entertain similar approach to estimate the long-run risks models.

3 Estimation Results

3.1 Parameter and State Estimates

Table 2 shows the moments of the prior and posterior distributions of the parameters. We chose fairly loose priors which cover a wide range of economically plausible parameters to maximize learning from the data. For example, a two-standard deviation band for the persistence of expected inflation and expected consumption ranges from 0.5 to 1.0. The prior means for the scale parameters are set to typical values in the literature, and the prior standard deviations are quite large as well. Importantly, we are careful not to hardwire the fluctuations in monetary policy and their impact on inflation volatility through the prior selection. That is, in our prior we assume that the monetary policy coefficients are the same across the regimes, and are equal to 1 for expected inflation and 0.5 for expected growth. Likewise, our prior distribution for the role of monetary policy on inflation volatility is symmetric and is centered at zero, so we do not force any impact through the prior, and let the data determine the size and the direction of the effect.

The table further shows the posterior parameter estimates in the data. The expected consumption, expected inflation, and inflation volatility are very persistent: the median AR(1) coefficients are above 0.95. The expected inflation has a negative and non-neutral effect on future real growth: $\Pi_{c\pi}$ is negative, consistent with the findings in Bansal and Shaliastovich (2013) and Piazzesi and Schneider (2005). We further find that the monetary policy regimes are quite persistent as well, with the probability of remaining in a passive regime of 0.975, and in the aggressive regime of 0.93. There is a substantial fluctuation in the impact of monetary policy across the regimes. Indeed, the median short rate loadings are equal to 0.8 and 1.9 on the expected growth and expected inflation, respectively, which are significantly larger than 0.3 and 0.8 in the passive regime. Our estimates for these regime coefficients corroborate the prior macroeconomic and financial evidence for Taylor rule coefficients on inflation being above one. These results regarding Taylor rules are well documented in Cochrane (2011), Gallmeyer et al. (2006), and Backus, Chernov and Zin (2013). Interestingly, the novel finding in our paper is that inflation volatility is also larger in the aggressive regime: the value of δ^{π} is positive, and is equal to about one-fourth of the exogenous inflation volatility level. Most of the parameters are quite precisely estimated.

Our filtered series for the latent expected growth, expected inflation, inflation volatility, and monetary policy regimes are provided in Figures 2-5. Generally, the estimated expectations are quite close to the data counterparts, especially for the expectations of future real growth. Some of the noticeable deviations of model-implied inflation expectations from the data include post-2007 period, where model expectations are systematically below the data. Notably, this is a period of a zero lower bound and unconventional monetary policy, which are outside a simple Taylor rule specification considered in this model.

The exogenous component of inflation volatility is provided in Figure 4. It is apparent that non-policy related volatility spikes in the early to mid 1980's and gradually decreases over time. Notably, the inflation volatility is quite low in the recent period, which reflects low variability in inflation expectations in the data. This, we will show, have an important implication for the model-implied bond risk premia.

Finally, we provide model-implied estimates of the monetary regime in Figure 5. The figure suggests that a shift to an aggressive regime occurred in the late-70's / early-80's period, in accordance with the Volcker period. In the late 90's, there was a marked shift to a passive regime, perhaps consistent with the anecdotal evidence regarding the Greenspan loosening. In the crisis period however, our estimates suggest an aggressive regime.

3.2 Model Implications for Bond Prices

Figure 6 shows the time series of model-implied yields in the sample, while Table 3 reports the key unconditional moments of bond prices and bond risk premia calculated at the median parameter values and states of the model. The model matches quite well the yields in the sample: the average pricing errors range between 0.17% for 1 year yields to about 0.03% for 3 and 5 year yields, and a good fit is apparent from the Figure. The population statistics, computed analytically across the posterior distribution of parameters, are generally consistent with in-sample yield properties. The population levels of yields match quite well data counterparts. The unconditional volatilities of yields are quite close to, though, are somewhat bellow the data values. The ability of the model to match an upward-sloping term structure of bond risk premia is consistent with the positive bond risk premia levels implied by the model, as shown in the table.

We next consider the conditional dynamics of bond prices implied by the model. In Figure 7 we report standardized bond loadings on the expected growth, expected inflation and inflation volatility, and the overall level of yields across the two regimes. The Figure shows that bond yields increase at times of high expected real growth. This captures a standard inter-temporal trade-off between interest rates and the expected growth: at times of high expected real growth agents do not want to save, so bond prices fall and yields increase. Because we are looking at the nominal bonds which pay nominal dollars, their prices fall at times of higher anticipated inflation, so bond yields also increase with expected inflation. Finally, while by construction short rates do not respond to inflation volatility, long term yields increase at times of high volatility of inflation. This reflects a positive risk premium component which is embedded in long term yields, and which increases at time of high inflation volatility. The average level of yields increase over time, consistent with the evidence of a positive bond risk premium.

Interestingly, all the bond loadings are uniformly larger in aggressive relative to passive regime. In our estimation, we find that short rates directly load more on growth and inflation in the aggressive regimes. Our model shows that this difference persists at long maturities, and further, it leads to a difference in bond loadings on inflation volatility, and in the unconditional levels of bond yields.

3.3 Model Implications for Bond Premia

Table 4 shows the average bond risk premium in the model, and its decomposition to the underlying economic sources of risk. In our model bond prices are exposed to expected growth, expected inflation, and inflation volatility risks, as well as the changes in the monetary policy regimes. In the Table, we report the average risk premia, evaluated at the median parameter and state estimates. In the benchmark model, the market price of the expected growth risk is positive, while the market prices of risks are negative for expected inflation and volatility risks. Indeed, high marginal utility states are those associated with low expected real growth, high expected inflation, or high inflation volatility. As bond yield loadings are all positive to these risks, it implies that the bond exposure to expected real growth contributes negatively to bond risk premia, while bond exposures to inflation risks contribute positively to the bond risk premia. Quantitatively, bond sensitivity to expected inflation risks is quite large, so the average bond risk premia are positive.

One of the key parameters in the model which determines the magnitude of the inflation premium, and thus the level of the risk premia and slope of the nominal term structure, is the inflation non-neutrality coefficient $\Pi_{c\pi}$. When this parameter is negative, as in the current estimation, high expected inflation is bad news for future real growth. Because expected inflation is very persistent, the inflation non-neutrality implies that investors are significantly concerned about expected inflation news. Long-term bonds which are quite sensitive to expected inflation are thus quite risky, and require a positive inflation premium. In the bottom panel of Table 4 we show the risk premia implications when the inflation non-neutrality parameter is set to zero. In this case, expected inflation risk premium is virtually zero, the bond risk premia are negative, and the entire term structure is downward sloping.

In our model, the bond risk premium is a linear function of the exogenous inflation volatility, where the loadings depend on the underlying monetary policy regimes. Similar to the regime dependent structures in Bansal and Zhou (2002) and Dai et al. (2007), the time-variation in monetary policy coefficients creates nonlinearities in yields via regime dependent bond loadings that affect the fluctuations in the risk premia. We can think of this as a time-varying risk exposure channel, which is different from a time-varying quantity of risk generated through the conditional volatility present in the inflation expectations. This represents an alternative channel for generating risk premia variability. Figure 8 shows the loadings of the risk premia on the inflation volatility, and the average bond risk premia. Consistent with our earlier discussion, bond risk premia increase at times of high inflation volatility. Further, both the level and the sensitivity of bond risk premia increase in aggressive relative to passive regimes.

To examine the quantitative impact of monetary policy on risk premia fluctuations, in Figure 9 we present the levels and volatilities of the risk premia under different model specifications. First, we consider a case where all the regime-shifting parameters are set to the constant unconditional averages, and only the non-policy portion of inflation volatility is present. After this we add on time variation in short rate sensitivity to expected inflation α_π, followed by the sensitivity of inflation volatility to monetary policy regime δ^π. The final case is the baseline where we have time variation in all the mechanisms.

The figure shows that the fluctuations in monetary policy have fairly small contributions to the overall levels of the risk premia, but have quite significant impact on its volatility. Indeed, adding variation in α_π to the base case of only exogenous stochastic volatility increases the variation in risk premia by almost 20%, especially at the long end of the risk premia term structure. Adding variation in the policy portion of volatility (δ^π) increases risk premia volatility further, again by close to 20%. These two effects together suggest that the variation in short-rate exposures and quantities of risk on the inflation side play an important large role in increase bond risk premia. Interestingly, adding the time-variation in the short rate sensitivity to real growth decreases the risk premia variation back to its benchmark value. To understand that, recall that the risk premia loads both on the volatility of expected consumption, which assumed to be constant, and the volatility of expected inflation:

$$rp_t^n = Cons(s_t) + r_{\sigma c}(s_t)\tilde{\sigma}_{c0}^2 + r_{\sigma\pi}(s_t)\tilde{\sigma}_{\pi t}^2.$$

The last two components reflect the contributions of the expected consumption risk and expected inflation risk to the total bond risk premia. As we reported in Table 4, the expected consumption risk premium component is on average negative, while the expected inflation component is positive. In absolute value, the two risk premia components increase with the sensitivity of bond yields to expected consumption and expected inflation risks, however, because of the different signs, they are driving total risk premia in the opposite direction. Indeed, in aggressive regime, bond prices are more sensitive to expected growth risk, which tends to diminish risk premia, while an increase in their sensitivity to expected inflation risks tends to increase bond premia. Because Taylor coefficients move in the same direction across the regimes, the two risk premia components co-vary negatively, and thus the total variance of the risk premia goes down. On average, the inflation channel dominates, so the risk premia variations increase with monetary policy uncertainty. However, conditionally bond risk premia can turn negative when inflation volatility effect is small.

These conditional implications for the risk compensation can be seen in Figure 10 which shows the implied risk premia in the sample. The top panel shows that that there is a significant difference between the level and volatilities of the bond risk premia across the maturities. The bottom panel documents that the fluctuations in the risk premia increase when we consider adding regime-dependent fluctuations in inflation volatility and short rate exposures to inflation to the exogenous fluctuations in the volatility of inflation. Interestingly, the risk premia turn negative after the crisis. In our model, this occurs because the inflation volatility is quite low in this period, so that a positive risk premia due to inflation risks does not offset a decrease in bond risk premia due to a larger aggressiveness to real growth concerns in this period.

4 Conclusion

We investigate the effects of monetary policy fluctuations on inflation uncertainty and nominal bond risks. To do so we utilize a structurally motivated, nonlinear term structure model, arising from a time-varying Taylor rule and an Epstein-Zin based investor. In particular, movements in monetary regimes factor into the conditional volatility of fundamental inflation expectations. Through a Bayesian estimation procedure, we find that aggressive policy regimes lead to high volatility periods in fundamentals and financial markets. These periods are associated with higher yield levels, volatilities, and risk premia variation. Furthermore we find that variation in the Taylor rule's inflation sensitivity as well as the monetary policy portion of fundamental volatility increase risk premia volatility. Meanwhile, variation in the short term interest rate's sensitivity to growth has an opposite, dampening effect. Put together, we provide a new approach to understanding the effects of monetary policy aggressiveness on financial markets and the macroeconomy.

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Appendix

A Details on Analytical Model Solution

In this section we fill in details regarding the model solution. In particular, the derivation we provide below allows for a more general version of the model, where regime switches also affect the drift of consumption and inflation. In other words, $\mu_c(s_t)$ and $\mu_\pi(s_t)$ are allowed to be regime-dependent, in addition to the $\alpha(s_t)$ coefficients. When these coefficients are assumed to be constant, their effects wipe away in the stochastic discount factor, as provided in the main section of the paper.

A.1 Cash Flow and Inflation News

Recall that the cash flow news is solved through:

$$\begin{aligned}
N_{CF,t+1} &= (E_{t+1} - E_t) \sum_{j=0} \kappa_1^j \Delta c_{t+j+1} \\
&= (E_{t+1} - E_t) \sum_{j=0}^{\infty} \kappa_1^j \mu_c(s_{t+j}) + (E_{t+1} - E_t) \sum_{j=0}^{\infty} \kappa_1^j e_1' X_{t+j} + \sigma_c^* \epsilon_{c,t+1}
\end{aligned}$$

Let $N_{CF,t+1} = N_{CF}\left(s_t = i, s_{t+1} = k, X_t, \tilde{\sigma}_{ct}^2, \tilde{\sigma}_{\pi t}^2, \epsilon_{t+1}\right)$. That is we specifiy the cash flow news as one moving from state i to state k in terms of regimes with current continuous variables, $\{X_t, \tilde{\sigma}_{ct}^2, \tilde{\sigma}_{\pi t}^2\}$ and continuous realization ϵ_{t+1}. We start by calculating the first portion of the news (related to $\mu_c(\dots)$). Notice that:

$$\begin{aligned}
(E_{t+1} - E_t)\mu_c(s_{t+1}) &= \mu_c(k) - \sum_{j=1}^{N} \pi_{ji}\mu_c(j) = \mu_c(k) - T_i'\mu_c = \left(e_k' - T_i'\right)\mu_c \\
(E_{t+1} - E_t)\mu_c(s_{t+2}) &= T_k'\mu_c - \sum_{\tilde{j}} \pi_{\tilde{j}i} \sum_{j} \pi_{j\tilde{j}}\mu_c(j) = T_k'\mu_c - T_i'T'\mu_c \\
&\dots \\
(E_{t+1} - E_t)\mu_c(s_{t+j}) &= \left[T_k'(T')^{j-2} - T_i'(T')^{j-1}\right]\mu_c
\end{aligned}$$

Summing over j, we receive that:

$$(E_{t+1} - E_t) \sum_{j=1}^{\infty} \kappa_1^j \mu_c(s_{t+j}) = \left\{\kappa_1\left(e_k' - T_i'\right) + \kappa_1^2\left(T_k' - T_i'T'\right)\left(I - \kappa_1 T'\right)^{-1}\right\}\mu_c$$

It is fairly simple to show that:

$$(E_{t+1} - E_t) \sum_{j=0}^{\infty} \kappa_1^j e_1' X_{t+j} = \kappa_1 e_1' \left(I - \kappa_1 \Pi\right)^{-1} \Sigma_t \epsilon_{t+1}$$

Hence the final result for cash flow news is:

$$\begin{aligned} N_{CF,t+1} &= \left\{ \kappa_1 \left(e_k' - T_i'\right) + \kappa_1^2 \left(T_k' - T_i' T'\right) \left(I - \kappa_1 T'\right)^{-1} \right\} \mu_c \\ &+ \kappa_1 e_1' \left(I - \kappa_1 \Pi\right)^{-1} \Sigma_t \epsilon_{t+1} + \sigma_c^* \epsilon_{c,t+1} \\ \\ &= F_{CF,0}(s_{t+1}, s_t) + F_{CF,\epsilon}(\ldots)' \Sigma_t \epsilon_{t+1} + \sigma_c^* \epsilon_{c,t+1} \end{aligned}$$

In an analogous derivation way we can show that inflation news is:

$$\begin{aligned} N_{\pi,t+1} &= \left\{ \kappa_1 \left(e_k' - T_i'\right) + \kappa_1^2 \left(T_k' - T_i' T'\right) \left(I - \kappa_1 T'\right)^{-1} \right\} \mu_\pi \\ &+ \kappa_1 e_2' \left(I - \kappa_1 \Pi\right)^{-1} \Sigma_t \epsilon_{t+1} + \sigma_\pi^* \epsilon_{\pi,t+1} \\ \\ &= F_{\pi,0}(s_{t+1}, s_t) + F_{\pi,\epsilon}(\ldots)' \Sigma_t \epsilon_{t+1} + \sigma_\pi^* \epsilon_{\pi,t+1} \end{aligned}$$

A.2 (Nominal) Interest Rate News

The interest rate news is specified by:

$$N_{I,t+1} = (E_{t+1} - E_t) \sum_{j=1}^{\infty} \kappa_1^j \left[\alpha_0(s_{t+j}) + \alpha(s_{t+j})' x_{t+j}\right]$$

The first part will be similar to that of the cash flow news:

$$(E_{t+1} - E_t) \sum_{j=1}^{\infty} \kappa_1^j \alpha_0(s_{t+j}) = \left\{ \kappa_1 \left(e_k' - T_i'\right) + \kappa_1^2 \left(T_k' - T_i' T'\right) \left(I - \kappa_1 T'\right)^{-1} \right\} \alpha_0$$

For the second part:

$$\begin{aligned} (E_{t+1} - E_t) \alpha(s_{t+1})' x_{t+1} &= \alpha_k' x_{t+1} - \sum_{j=1}^{N} \pi_{ji} \alpha(j)' \left(\Pi X_t\right) \\ &= \alpha_k' \left(\Pi X_t + \Sigma_t \epsilon_{t+1}\right) - T_i' \alpha \left(\Pi X_t\right) = \left(\alpha_k' - T_i' \alpha\right) \Pi X_t + \alpha_k' \Sigma_t \epsilon_{t+1} \end{aligned}$$

where α indicates the stacked matrix of $\alpha(j)'$ of all states. To receive the first equaltiy I also use law of iterated expectations. The second guy will be given by:

$$\begin{aligned}
(E_{t+1} - E_t)\alpha(s_{t+2})'x_{t+2} &= T_k'\alpha\Pi X_{t+1} - \sum_j \pi_{ji} \sum_{\tilde{j}} \pi_{\tilde{j}j}\alpha(\tilde{j})' \left(\Pi^2 X_t\right) \\
&= T_k'\alpha\Pi X_{t+1} - T_i'T'\alpha\Pi^2 X_t \\
&= \left(T_k'\alpha\Pi - T_i'T'\alpha\Pi\right)\Pi X_t + T_k'\alpha\Pi\Sigma_t\epsilon_{t+1}
\end{aligned}$$

And more generally for $j \geq 2$:

$$(E_{t+1} - E_t)\alpha(s_{t+j})'x_{t+j} = \left[T_k' - T_i'T'\right]\left[(T')^{j-1}\alpha\Pi^j\right] X_t + T_k'(T')^{j-2}\alpha\Pi^{j-1}\Sigma_t\epsilon_{t+1}$$

Denote the infinite sums, $\{S^1_{0,I}, S^2_{0,I}\}$ which can be solved through Ricatti equations:

$$\begin{aligned}
S^1_{0,I} &= \sum_{j=2}^{\infty}(T')^{j-1}\alpha\Pi^j\kappa_1^j = \kappa_1^2 T'\alpha\Pi^2 + \kappa_1 T' S^1_{0,I}\Pi \\
S^2_{0,I} &= (T')^{-1}S^1_{0,I}(\Pi)^{-1}
\end{aligned}$$

After summing across j we recieve:

$$\begin{aligned}
N_{I,t+1} &= \left\{\kappa_1\left(e_k' - T_i'\right) + \kappa_1^2\left(T_k' - T_i'T'\right)\left(I - \kappa_1 T'\right)^{-1}\right\}\alpha_0 \\
&+ \left\{\kappa_1\left[\alpha_k'\Pi - T_i'\alpha\Pi\right] + \left[T_k' - T_i'T'\right]S^1_{0,I}\right\}X_t + \left\{\kappa_1\alpha_k' + T_k' S^2_{0,I}\right\}\Sigma_t\epsilon_{t+1} \\
&= F_{I,0}(s_{t+1}, s_t) + F_{I,X}(s_{t+1}, s_t)'X_t + F_{I,\epsilon}(\dots)'\Sigma_t\epsilon_{t+1}
\end{aligned}$$

A.3 Solving for Volatility Factor

To fully specify the stochastic discount factor we need to specify the value of V_t that will satisfy the Euler equation. We guess and verify V_t to be:

$$V_t(s_t, X_t, \tilde{\sigma}^2_{c,t}, \tilde{\sigma}^2_{\pi,t}) = V_0(s_t) + V_1(s_t)'X_t + V_{2c}(s_t)\tilde{\sigma}^2_{c,t} + V_{2\pi}(s_t)\tilde{\sigma}^2_{\pi,t}$$

for a $\{V_0, V_1, V_{2c}, V_{2\pi}\}$ that will be verified. First we calculate the volatility news. The first two terms we have done to some extent. Using results from before we have:

$$\begin{aligned}
N_{V,t+1} &= (E_{t+1} - E_t)\sum_{j=0}^{\infty}\kappa_1^j V_{t+j} \\
(E_{t+1} - E_t)\sum_{j=1}^{\infty}\kappa_1^j V_0(s_{t+j}) &= \left\{\kappa_1\left(e_k' - T_i'\right) + \kappa_1^2\left(T_k' - T_i'T'\right)\left(I - \kappa_1 T'\right)^{-1}\right\}V_0
\end{aligned}$$

where V_0 denotes the stacked matrix of $V_0(j)'$. The second portion of the volatility news is given by:

$$\begin{aligned}(E_{t+1} - E_t) \sum_{j=1}^{\infty} \kappa_1^j V_1(s_{t+j})' X_{t+j} &= \left\{ \kappa_1 \left[V_{1,k}' \Pi - T_i' V_1 \Pi \right] + \left[T_k' - T_i' T' \right] S_{0,V}^1 \right\} X_t \\ &\quad + \left\{ \kappa_1 V_{1,k}' + T_k' S_{0,V}^2 \right\} \Sigma_t \epsilon_{t+1}\end{aligned}$$

with:

$$\begin{aligned}S_{0,V}^1 &= \kappa_1^2 T' V_1 \Pi^2 + \kappa_1 T' S_{0,V}^1 \Pi \\ S_{0,V}^2 &= (T')^{-1} S_{0,V}^1 (\Pi)^{-1}\end{aligned}$$

The final two portions of the volatility news will be fairly similar to the above quantities, due to the analagous AR(1) structure of the volatilities.

$$\begin{aligned}(E_{t+1} - E_t) \sum_{j=1}^{\infty} \kappa_1^j V_{2c}(s_{t+j}) \tilde{\sigma}_{c,t+j}^2 &= \left\{ \kappa_1 \left[V_{2c}(k) \varphi_c - T_i' V_{2c} \varphi_c \right] + \left[T_k' - T_i' T' \right] S_{0,v2c}^1 \right\} \tilde{\sigma}_{c,t}^2 \\ &+ \left\{ \kappa_1 V_{2c}(k) + T_k' S_{0,v2c}^2 \right\} \omega_c \eta_{\sigma c,t+1} \\ (E_{t+1} - E_t) \sum_{j=1}^{\infty} \kappa_1^j V_{2\pi}(s_{t+j}) \tilde{\sigma}_{\pi,t+j}^2 &= \left\{ \kappa_1 \left[V_{2\pi}(k) \varphi_\pi - T_i' V_{2\pi} \varphi_\pi \right] + \left[T_k' - T_i' T' \right] S_{0,v2\pi}^1 \right\} \tilde{\sigma}_{\pi,t}^2 \\ &+ \left\{ \kappa_1 V_{2\pi}(k) + T_k' S_{0,v2\pi}^2 \right\} \omega_\pi \eta_{\sigma\pi,t+1}\end{aligned}$$

where for $i = c, \pi$ the vector terms again satisfy:

$$\begin{aligned}S_{0,v2i}^1 &= \kappa_1^2 T' V_1 \varphi_i^2 + \kappa_1 T' S_{0,v2i}^1 \varphi_i \\ S_{0,v2i}^2 &= (T')^{-1} S_{0,v2i}^1 (\varphi_i)^{-1}\end{aligned}$$

Aggregating these terms we receive the volatility news to be of the form:

$$\begin{aligned}N_{V,t+1} &= F_{v,0}(s_{t+1}, s_t) + F_{v,X}(s_{t+1}, s_t)' X_t + F_{v,\sigma c}(s_{t+1}, s_t)' \tilde{\sigma}_{ct}^2 + F_{v,\sigma\pi}(s_{t+1}, s_t)' \tilde{\sigma}_{\pi t}^2 \\ &+ F_{v,\epsilon}(\dots) \Sigma_t \epsilon_{t+1} + F_{v,\eta c}(\dots) \omega_c \eta_{\sigma c,t+1} + F_{v,\eta\pi}(\dots) \omega_\pi \eta_{\sigma\pi,t+1}\end{aligned}$$

where the functional forms are given by:

$$
\begin{aligned}
F_{v0} &= \left\{\kappa_1\left(e_k'-T_i'\right)+\kappa_1^2\left(T_k'-T_i'T'\right)\left(I-\kappa_1 T'\right)^{-1}\right\}V_0 \\
F_{v,X}' &= \left\{\kappa_1\left[V_{1,k}'\Pi-T_i'V_1\Pi\right]+\left[T_k'-T_i'T'\right]S_{0,V}^1\right\} \\
F_{v,\sigma c} &= \left\{\kappa_1\left[V_{2c}(k)\varphi_c-T_i'V_{2c}\varphi_c\right]+\left[T_k'-T_i'T'\right]S_{0,v2c}^1\right\} \\
F_{v,\sigma\pi} &= \left\{\kappa_1\left[V_{2\pi}(k)\varphi_\pi-T_i'V_{2\pi}\varphi_\pi\right]+\left[T_k'-T_i'T'\right]S_{0,v2\pi}^1\right\} \\
F_{v,\epsilon}' &= \left\{\kappa_1 V_{1k}'+T_k'S_{0v}^2\right\} \\
F_{v,\eta c} &= \left\{\kappa_1 V_{2c}(k)+T_k'S_{0,v2c}^2\right\} \\
F_{v,\eta\pi} &= \left\{\kappa_1 V_{2\pi}(k)+T_k'S_{0,v2\pi}^2\right\}
\end{aligned}
$$

Now we solve for coefficients on the volatility factor. From the Euler equation restriction evaluated and rearranged at the risk free rate we receive:

$$
\begin{aligned}
\exp\left(V_t\right) &= E_t\left[\exp\left(-\gamma N_{CF,t+1}+N_{I,t+1}-N_{\pi,t+1}+N_{V,t+1}\right)\right] \\
&= E_t\left[\exp\left(\underbrace{M_0+M_{1,X}'X_t+M_{1,\sigma c}\tilde{\sigma}_{ct}^2+M_{1,\sigma\pi}\tilde{\sigma}_{\pi t}^2+M_{2,\epsilon}'\Sigma_t\epsilon_{t+1}+M_{2,\eta c}\omega_c\eta_{c,t+1}+M_{2,\eta\pi}\omega_\pi\eta_{\pi,t+1}-\gamma\sigma_c^*\epsilon_{c,t+1}-\sigma_\pi^*\epsilon_{\pi,t+1}}_{*}\right)\right]
\end{aligned}
$$

where each of the coefficients are regime inter-dependent:

$$
\begin{aligned}
M_0(s_t,s_{t+1}) &= -\gamma F_{CF,0}+F_{I,0}-F_{\pi,0}+F_{v,0} \\
M_{1,X}(s_t,s_{t+1})' &= F_{I,X}'+F_{v,X}' \\
M_{1,\sigma c}(s_t,s_{t+1}) &= F_{v,\sigma c} \\
M_{1,\sigma\pi}(s_t,s_{t+1}) &= F_{v,\sigma\pi} \\
M_{2,\epsilon}(s_t,s_{t+1})' &= -\gamma F_{CF,\epsilon}'+F_{I,\epsilon}'-F_{\pi,\epsilon}'+F_{v,\epsilon}' \\
M_{2,\eta c}(s_t,s_{t+1}) &= F_{v,\eta c} \\
M_{2,\eta\pi}(s_t,s_{t+1}) &= F_{v,\eta\pi}
\end{aligned}
$$

Pre-conditioning on the $t+1$ regime will give us:

$$
\begin{aligned}
1=E_t\left[\exp\left(*\right)\right] &= E_t\left[E_t\left[\exp\left(*\right)|s_{t+1}\right]\right] \\
&= E_t\left[\exp\left(\underbrace{M_0+M_{1,X}'X_t+M_{1,\sigma c}\tilde{\sigma}_{ct}^2+M_{1,\sigma\pi}\tilde{\sigma}_{\pi t}^2+\frac{1}{2}\left(M_{2,\epsilon}'\Sigma_t\Sigma_t'M_{2,\epsilon}+M_{2,\eta c}^2\omega_c^2+M_{2,\eta\pi}^2\omega_\pi^2\right)+\frac{1}{2}\gamma^2\left(\sigma_c^*\right)^2+\frac{1}{2}\left(\sigma_\pi^*\right)^2}_{**}\right)\right]
\end{aligned}
$$

Recall the definition of $\Sigma_t\Sigma_t'$ from earlier. The inside term will become:

$$
M_{2,\epsilon}'\Sigma_t\Sigma_t'M_{2,\epsilon}=\left(M_{2,\epsilon}^1\right)^2\left(\delta^{\alpha c}\alpha_c(s_t)+\delta^{\sigma c}\tilde{\sigma}_{c,t}^2\right)+\left(M_{2,\epsilon}^2\right)^2\left(\delta^{\alpha\pi}\alpha_\pi(s_t)+\delta^{\sigma\pi}\tilde{\sigma}_{\pi,t}^2\right)
$$

We write the above expression as:

$$** \;=\; \tilde{M}_0 + \tilde{M}_{1,X}' X_t + \tilde{M}_{1,\sigma c}\tilde{\sigma}_{ct}^2 + \tilde{M}_{1,\sigma\pi}\tilde{\sigma}_{\pi t}^2$$

such that:

$$\begin{aligned}
\tilde{M}_0 &= M_0 + \frac{1}{2}\left[\left(M_{2,\epsilon}^1\right)^2 \delta^{\alpha c}\alpha_c(s_t) + \left(M_{2,\epsilon}^2\right)^2 \delta^{\alpha\pi}\alpha_\pi(s_t) + M_{2,\eta c}^2\omega_c^2 + M_{2,\eta\pi}^2\omega_\pi^2\right] + \frac{1}{2}\gamma^2\left(\sigma_c^*\right)^2 + \frac{1}{2}\left(\sigma_\pi^*\right)^2 \\
\tilde{M}_1' &= M_1' \\
\tilde{M}_{1,\sigma c} &= M_{1,\sigma c} + \frac{1}{2}\delta^{\sigma c}\left(M_{2,\epsilon}^1\right)^2 \\
\tilde{M}_{1,\sigma\pi} &= M_{1,\sigma\pi} + \frac{1}{2}\delta^{\sigma\pi}\left(M_{2,\epsilon}^2\right)^2
\end{aligned}$$

Let each coefficient be evaluated at $\{s_t = i, s_{t+1} = k\}$. We will now use the approximation, $\exp(y) \approx 1 + y$, which holds for small enough y.

$$\begin{aligned}
\exp\left(V_t\right) &= E_t\left[\exp\left(\tilde{M}_0(i,k) + \tilde{M}_{1,X}(i,k)' X_t + \tilde{M}_{1,\sigma c}(i,k)\tilde{\sigma}_{ct}^2 + \tilde{M}_{1,\sigma\pi}(i,k)\tilde{\sigma}_{\pi t}^2\right)\right] \\
&\approx E_t\left[1 + \tilde{M}_0(i,k) + \tilde{M}_{1,X}(i,k)' X_t + \tilde{M}_{1,\sigma c}(i,k)\tilde{\sigma}_{ct}^2 + \tilde{M}_{1,\sigma\pi}(i,k)\tilde{\sigma}_{\pi t}^2\right] \\
\Longrightarrow V_t &= T_i'\tilde{M}_0(i) + \left(T_i'\tilde{M}_{1,X}(i)\right) X_t + T_i'\tilde{M}_{1,\sigma c}(i)\tilde{\sigma}_{ct}^2 + T_i'\tilde{M}_{1,\sigma\pi}(i)\tilde{\sigma}_{\pi t}^2
\end{aligned}$$

where $\tilde{M}_0(i)$ is the stacked vector of $\tilde{M}_0(i,:)$. Similarly, $\tilde{M}_{1,X}(i)$ is the stacked matrix of $\tilde{M}_{1,X}(i,k)'$.

To guarantee this above expression is zero in all states of the world, (i, X_t), we need to find $\{V_0(i), V_1(i), V_{2c}(i), V_{2\pi}(i)\}_{i=1}^N$ such that:

$$\begin{aligned}
V_0(i) &= T_i'\tilde{M}_0(i) \\
V_{1X}(i) &= T_i'\tilde{M}_{1,X}(i) \\
V_{2c}(i) &= T_i'\tilde{M}_{1,\sigma c}(i) \\
V_{2\pi}(i) &= T_i'\tilde{M}_{1,\sigma\pi}(i)
\end{aligned}$$

for all i. This system is perfectly identified so we should be able to find a solution. More details to come on computation.

Once we have derived the coefficients on V_t we have fully derived the stochastic discount factor:

$$\begin{aligned}
m_{t+1} &= -i_t - V_t - \gamma N_{CF,t+1} + N_{R,t+1} + N_{V,t+1} \\
&= S_0 + S_{1,X}' X_t + S_{1,\sigma c}\tilde{\sigma}_{ct}^2 + S_{1,\sigma\pi}\tilde{\sigma}_{\pi t}^2 + S_{2,\epsilon}'\Sigma_t\epsilon_{t+1} + S_{2,\eta c}\omega_c\eta_{c,t+1} + S_{2,\eta\pi}\omega_\pi\eta_{\pi,t+1} \\
&\quad -\gamma\sigma_c^*\epsilon_{c,t+1} - \sigma_\pi^*\epsilon_{\pi,t+1}
\end{aligned}$$

where each of the coefficients are regime inter-dependent:

$$\begin{aligned}
S_0(s_t, s_{t+1}) &= M_0 - \alpha_0 - V_0 \\
S_{1,X}(s_t, s_{t+1})' &= M'_{1,X} - \alpha' - V'_1 \\
S_{1,\sigma c}(s_t, s_{t+1}) &= M_{1,\sigma c} - V_{2c} \\
S_{1,\sigma\pi}(s_t, s_{t+1}) &= M_{1,\sigma\pi} - V_{2\pi} \\
S_{2,\epsilon}(s_t, s_{t+1})' &= M'_{2,\epsilon} \\
S_{2,\eta c}(s_t, s_{t+1}) &= M_{2,\eta\pi} \\
S_{2,\eta\pi}(s_t, s_{t+1}) &= M_{2,\eta\pi}
\end{aligned}$$

A.4 Nominal Term Structure

We will show that log bond prices, p_t^n, take a quadratic structure in states:

$$p_t^1 = -i_t = \tilde{\mathcal{A}}^1(i) + \tilde{\mathcal{B}}_X^{1'}(i)X_t + \tilde{\mathcal{B}}_{\sigma c}^1(i)\tilde{\sigma}_{ct}^2 + \tilde{\mathcal{B}}_{\sigma\pi}^1(i)\tilde{\sigma}_{\pi t}^2$$

where the first set of coefficients is given by:

$$\begin{aligned}
\tilde{\mathcal{A}}^1(i) &= \alpha_0(i) \\
\tilde{\mathcal{B}}_X^1(i)' &= \alpha(i)' \\
\tilde{\mathcal{B}}_{\sigma c}^1(i) &= 0 \\
\tilde{\mathcal{B}}_{\sigma\pi}^1(i) &= 0
\end{aligned}$$

Conjecture that the statement holds for bonds of maturity $n-1$. Now prove for bonds of maturity n.

$$\begin{aligned}
p_t^n(i) &= \log E_t\left[\exp\left(\underbrace{m_{t+1} + p_{t+1}^{n-1}}_{***}\right)\right] \\
*** &= \left(S_0 + \tilde{\mathcal{A}}^{n-1} + \tilde{\mathcal{B}}_{\sigma c}^{n-1}\tilde{\sigma}_{c0}^2 + \tilde{\mathcal{B}}_{\sigma\pi}^{n-1}\tilde{\sigma}_{\pi 0}^2\right) + \left(S'_{1,X} + \tilde{\mathcal{B}}_X^{n-1'}\Pi\right)X_t + \left(S_{1,\sigma c} + \tilde{\mathcal{B}}_{\sigma c}^{n-1}\varphi_c\right)\tilde{\sigma}_{ct}^2 \\
&+ \left(S_{1,\sigma\pi} + \tilde{\mathcal{B}}_{\sigma\pi}^{n-1}\varphi_\pi\right)\tilde{\sigma}_{\pi t}^2 + \left(S'_{2,\epsilon} + \mathcal{B}_X^{n-1'}\right)\Sigma_t\epsilon_{t+1} + \left(S_{2,\eta c} + \tilde{\mathcal{B}}_{\sigma c}^{n-1}\right)\omega_c\eta_{c,t+1} + \left(S_{2,\eta\pi} + \tilde{\mathcal{B}}_{\sigma\pi}^{n-1}\right)\omega_\pi\eta_{\pi,t+1} \\
&- \gamma\sigma_c^*\epsilon_{c,t+1} - \sigma_\pi^*\epsilon_{\pi,t+1} \\
&= \tilde{S}_0 + \tilde{S}'_{1,X}X_t + \tilde{S}_{1,\sigma c}\tilde{\sigma}_{ct}^2 + \tilde{S}_{1,\sigma\pi}\tilde{\sigma}_{\pi t}^2 + \tilde{S}'_{2,\epsilon}\Sigma_t\epsilon_{t+1} + \tilde{S}_{2,\eta c}\omega_c\eta_{c,t+1} + \tilde{S}_{2,\eta\pi}\omega_\pi\eta_{\pi,t+1} - \gamma\sigma_c^*\epsilon_{c,t+1} - \sigma_\pi^*\epsilon_{\pi,t+1}
\end{aligned}$$

Similar to before:

$$
\begin{aligned}
p_t^n(i) &= \log E_t\left[E_t\left[\exp(***)|s_{t+1}\right]\right] \\
&= \log E_t\left[\exp\left(\underbrace{\tilde{S}_0+\tilde{S}_{1,X}'X_t+\tilde{S}_{1,\sigma c}\tilde{\sigma}_{ct}^2+\tilde{S}_{1,\sigma\pi}\tilde{\sigma}_{\pi t}^2+\frac{1}{2}\left(\tilde{S}_{2,\epsilon}'\Sigma_t\Sigma_t'\tilde{S}_{2,\epsilon}+\tilde{S}_{2,\eta c}^2\omega_c^2+\tilde{S}_{2,\eta\pi}^2\omega_\pi^2\right)+\frac{1}{2}\left(\gamma^2\left(\sigma_c^*\right)^2+\left(\sigma_\pi^*\right)^2\right)}_{****}\right)\right]
\end{aligned}
$$

We now use the earlier approximation twice – once to evaluate the inner expectation and a second time to simplify the outer log. We receive:

$$
\begin{aligned}
p_t^n(i) &\approx \sum_k \pi_{ki}\left(****\right) \\
&= \tilde{\mathcal{A}}^n(i)+\tilde{\mathcal{B}}_X^{n'}(i)X_t+\tilde{\mathcal{B}}_{\sigma c}^n(i)\tilde{\sigma}_{ct}^2+\tilde{\mathcal{B}}_{\sigma\pi}^n(i)\tilde{\sigma}_{\pi t}^2
\end{aligned}
$$

where:

$$
\begin{aligned}
\tilde{\mathcal{A}}^n(i) &= \frac{1}{2}\left(\gamma^2\left(\sigma_c^*\right)^2+\left(\sigma_\pi^*\right)^2\right) \\
&\quad+\sum_k \pi_{ki}\left\{\tilde{S}_0+\frac{1}{2}\left[\left(\tilde{S}_{2,\epsilon}^1\right)^2\delta^{\alpha c}\alpha_c(s_t)+\left(\tilde{S}_{2,\epsilon}^2\right)^2\delta^{\alpha\pi}\alpha_\pi(s_t)+\tilde{S}_{2,\eta c}^2\omega_c^2+\tilde{S}_{2,\eta\pi}^2\omega_\pi^2\right]\right\} \\
\tilde{\mathcal{B}}_X^{n'}(i) &= \sum_k \pi_{ki}\tilde{S}_{1,X}' \\
\tilde{\mathcal{B}}_{\sigma c}^{n'}(i) &= \sum_k \pi_{ki}\left\{\tilde{S}_{1,\sigma c}+\frac{1}{2}\delta^{\sigma c}\left(\tilde{S}_{2,\epsilon}^1\right)^2\right\} \\
\tilde{\mathcal{B}}_{\sigma\pi}^{n'}(i) &= \sum_k \pi_{ki}\left\{\tilde{S}_{1,\sigma c}+\frac{1}{2}\delta^{\sigma c}\left(\tilde{S}_{2,\epsilon}^1\right)^2\right\}
\end{aligned}
$$

Tables and Figures

Table 1: **Summary Statistics**

Bond Yields:

	3M	1Y	2Y	3Y	4Y	5Y
Mean (%)	5.19	5.66	5.90	6.09	6.26	6.39
Std. Dev. (%)	3.27	3.33	3.28	3.18	3.09	2.99
AR(1)	.942	.954	.961	.965	.967	.969

Macro and Survey Data:

	$\triangle c$	π	x_c	x_π
Mean (%)	1.81	3.64	0.00	0.00
Std. Dev. (%)	1.74	2.41	.957	2.13
AR(1)	.513	.892	.865	.983

The table shows summary statistics for bond yields, consumption growth, inflation, and expectations of real growth and inflation constructed from the Survey of Professional Forecasters. Quarterly observations from 1968Q3 to 2013Q4.

Table 2: **Prior and Posterior Distributions**

	Distr.	Prior Mean	Prior Std. Dev.	Posterior 5%	Posterior 50%	Posterior 95%
		Transition Parameters:				
Π_{cc}	N^T	.9	.2	.972	.991	.998
$\Pi_{\pi c}$	N^T	0	-	-	0	-
$\Pi_{c\pi}$	N^T	0	.2	-.032	-.011	-.004
$\Pi_{\pi\pi}$	N^T	.9	.2	.920	.955	.978
		Non-Policy Volatility Parameters:				
$\frac{\tilde{\sigma}^2_{c0}}{1-\varphi_c} \times 10^5$	IG	.01	.02	.013	.025	.068
$\frac{\tilde{\sigma}^2_{\pi 0}}{1-\varphi_\pi} \times 10^5$	IG	.02	.01	.009	.021	.043
φ_π	N^T	0.95	0.1	.962	.976	.992
$\omega_\pi \times 10^6$	IG	0.15	0.2	.186	.190	1.94
$\sigma^*_c \times 10$	IG	.05	.02	.029	.038	.050
$\sigma^*_\pi \times 10$	IG	.05	.02	.029	.039	.054
		Regime-Shifting Coefficients:				
$\delta^\pi(s_1) \times 10^5$	$-$	0.00	-	-	0.00	-
$\delta^\pi(s_2) \times 10^5$	N	0.00	0.1	.006	.008	.010
$\alpha_c(s_1)$	N	0.5	1	.023	.091	.274
$\alpha_c(s_2)$	N	0.5	1	.174	.315	.524
$\alpha_\pi(s_1)$	N	1	1	.622	.791	1.01
$\alpha_\pi(s_2)$	N	1	1	1.66	1.90	1.99
π_{11}	N^T	.9	.2	.945	.975	.994
π_{22}	N^T	.9	.2	.893	.929	.971
		Other Parameters:				
μ_c	$-$	.0045	-		.0045	-
μ_π	$-$	.0091	-	-	.0091	-
κ_1	$-$	.995	-	-	.995	-
γ	G	20	5	22.81	24.38	26.09
i_0	$-$	.013	-	-	.013	-

The table summarizes the prior and posterior distributions for the model parameter estimates. G refers to Gamma distribution, N to Normal distribution, N^T is truncated (at zero and/or one) Normal distribution, and IG is Inverse-Gamma. Dashed line indicates that the parameter value is fixed.

Table 3: **Model Implications for Yields**

	$n = 1$Y	2Y	3Y	4Y	5Y
$E(y^n)$ – Median	5.37	5.60	5.82	6.02	6.21
$E(y^n)$ – 5%	5.22	5.26	5.32	5.38	5.44
$E(y^n)$ – 95 %	5.70	6.29	6.81	7.24	7.63
$\sigma(y^n)$ – Median	2.45	2.27	2.16	2.08	2.02
$\sigma(y^n)$ – 5%	1.67	1.55	1.47	1.41	1.38
$\sigma(y^n)$ – 95 %	3.11	2.90	2.77	2.70	2.64
$E(rp^n)$ – Median	.346	.789	1.20	1.57	1.89
$E(rp^n)$ – 5%	.039	.145	.277	.420	.544
$E(rp^n)$ – 95 %	.989	2.07	2.92	3.64	4.22
$\sigma(rp^n)$ – Median	.531	1.08	1.50	1.83	2.10
$\sigma(rp^n)$ – 5%	.308	.653	.942	1.17	1.36
$\sigma(rp^n)$ – 95 %	.807	1.61	2.20	2.64	3.01
Abs(Mean Error)– Median	.165	.093	.028	.031	.104
Abs(Mean Error) – 5%	.123	.025	.002	.004	.026
Abs(Mean Error) – 95 %	.215	.168	.080	.077	.207

The table shows the median and (5%, 95%) credible sets for the posterior distributions of unconditional means and volatilities of bond yields and the risk premia, and of the absolute average mean squared errors across draws. All statistics reported are at the annual basis in percentage terms.

Table 4: **Decomposition of Average Risk Premia**

Baseline Parameters:

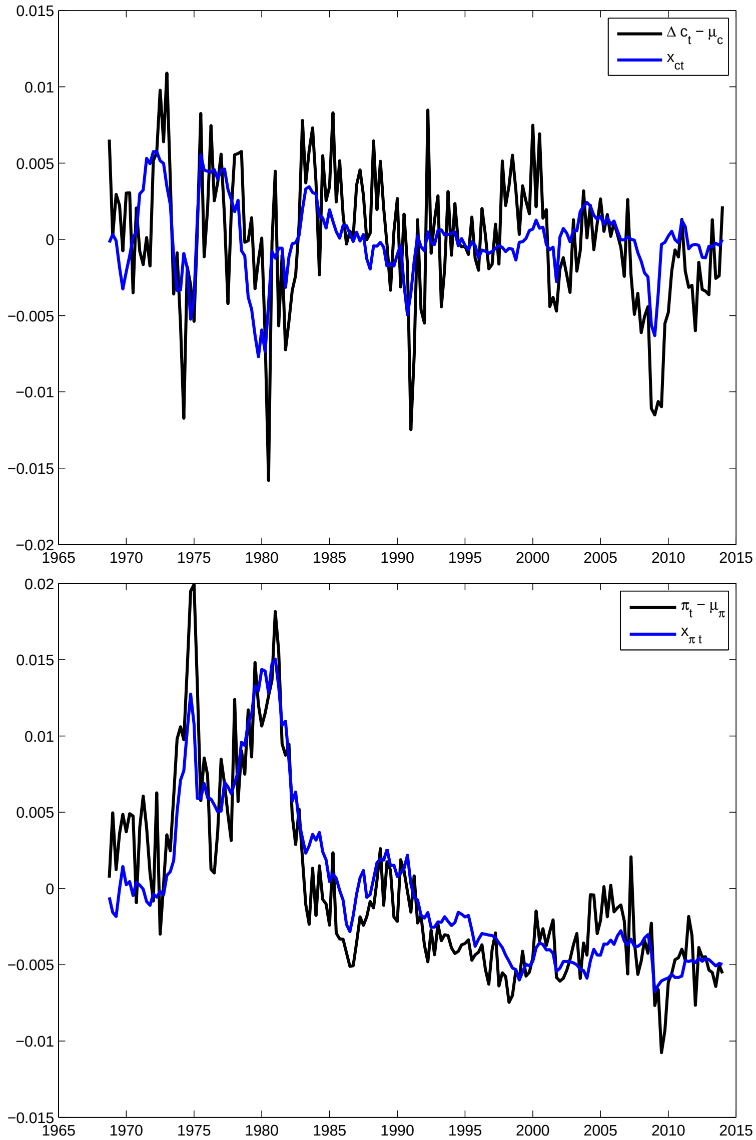
	$n = 6\text{M}$	1Y	3Y	5Y
Exp Growth Shocks (%)	-.254	-.756	-2.67	-4.45
Exp Inflation Shocks (%)	.406	1.16	3.49	5.04
Inflation Vol Shocks (%)	0.00	0.04	.607	1.56
Total (%)	.152	.440	1.42	2.15

Inflation Neutrality:

	$n = 6\text{M}$	1Y	3Y	5Y
Exp Growth Shocks (%)	-.254	-.756	-2.67	-4.45
Exp Inflation Shocks (%)	.004	.011	.030	.041
Inflation Vol Shocks (%)	0.00	0.00	0.00	0.00
Total (%)	-.250	-.745	-2.64	-4.41

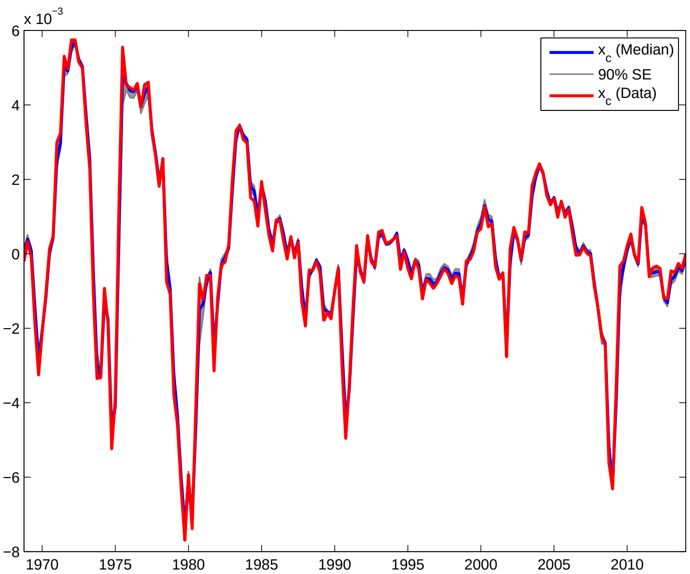
The table shows the decomposition of the average bond risk premia into the risk contributions of expected growth shocks, expected inflation shocks, and inflation volatility shocks. The Baseline case refers to the benchmark estimation of the model. For the Inflation Neutrality case the feedback between expected consumption and expected inflation is set to zero. All statistics are in annual terms and in percentages, and are computed at the median parameter draw.

Figure 1: **Realized and Expected Macroeconomic States**



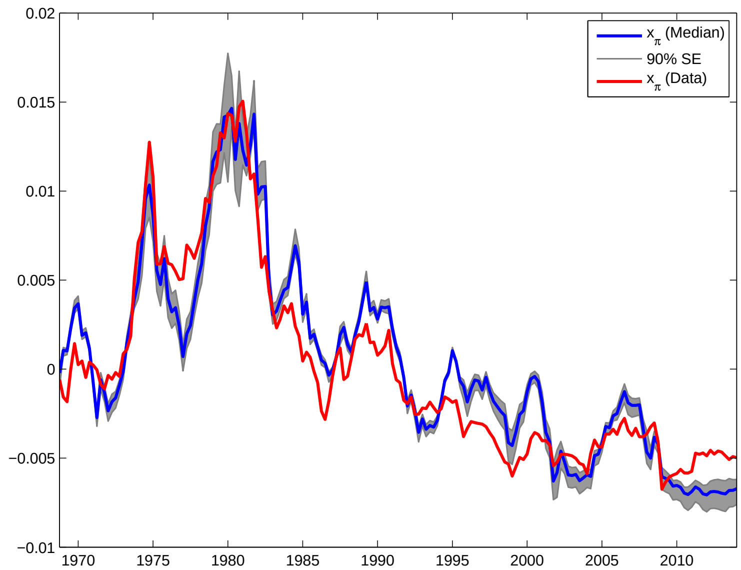
The figure shows the realized and expected consumption growth (top panel) and realized and expected inflation (bottom panel). Real growth and inflation expectations are constructed from the Survey of Professional Forecasters. Quarterly observations from 1968Q3 to 2013Q4.

Figure 2: **Estimated Expected Real Growth**



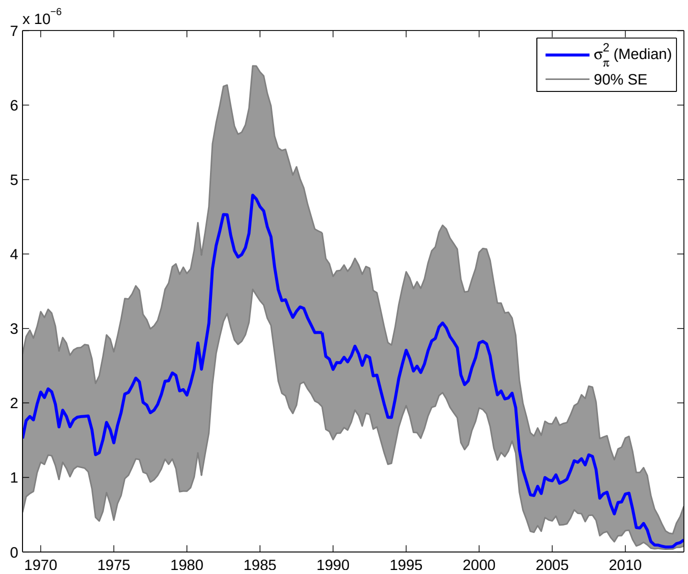
The figure shows the expected real growth in the data (red line), and the estimated posterior median from the model. Grey region represents posterior $(5\%, 95\%)$ credible sets. Data expectations are constructed from the Survey of Professional Forecasters. Quarterly observations from 1968Q3 to 2013Q4.

Figure 3: **Estimated Expected Inflation**



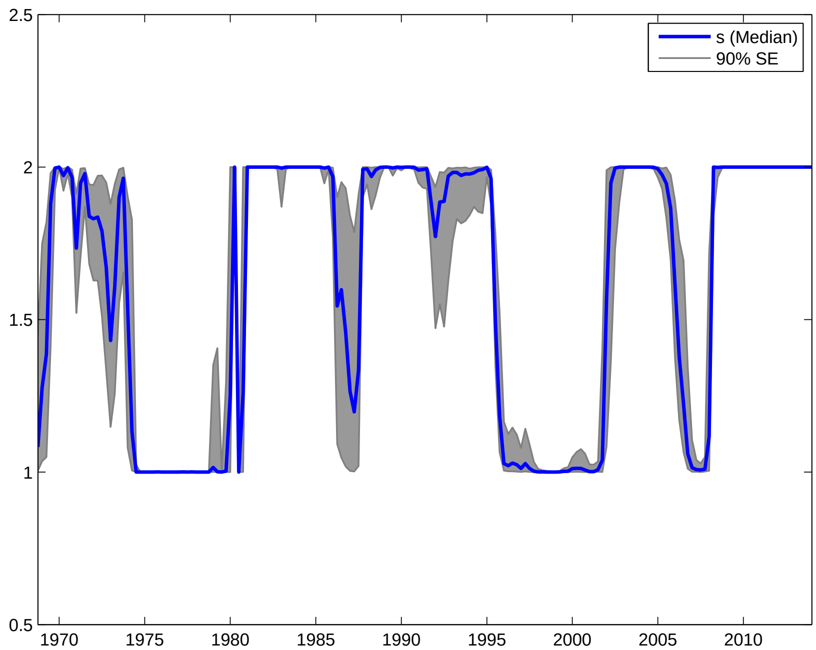
The figure shows the expected inflation in the data (red line), and the estimated posterior median from the model. Grey region represents posterior (5%, 95%) credible sets. Data expectations are constructed from the Survey of Professional Forecasters. Quarterly observations from 1968Q3 to 2013Q4.

Figure 4: **Estimated Inflation Volatility**



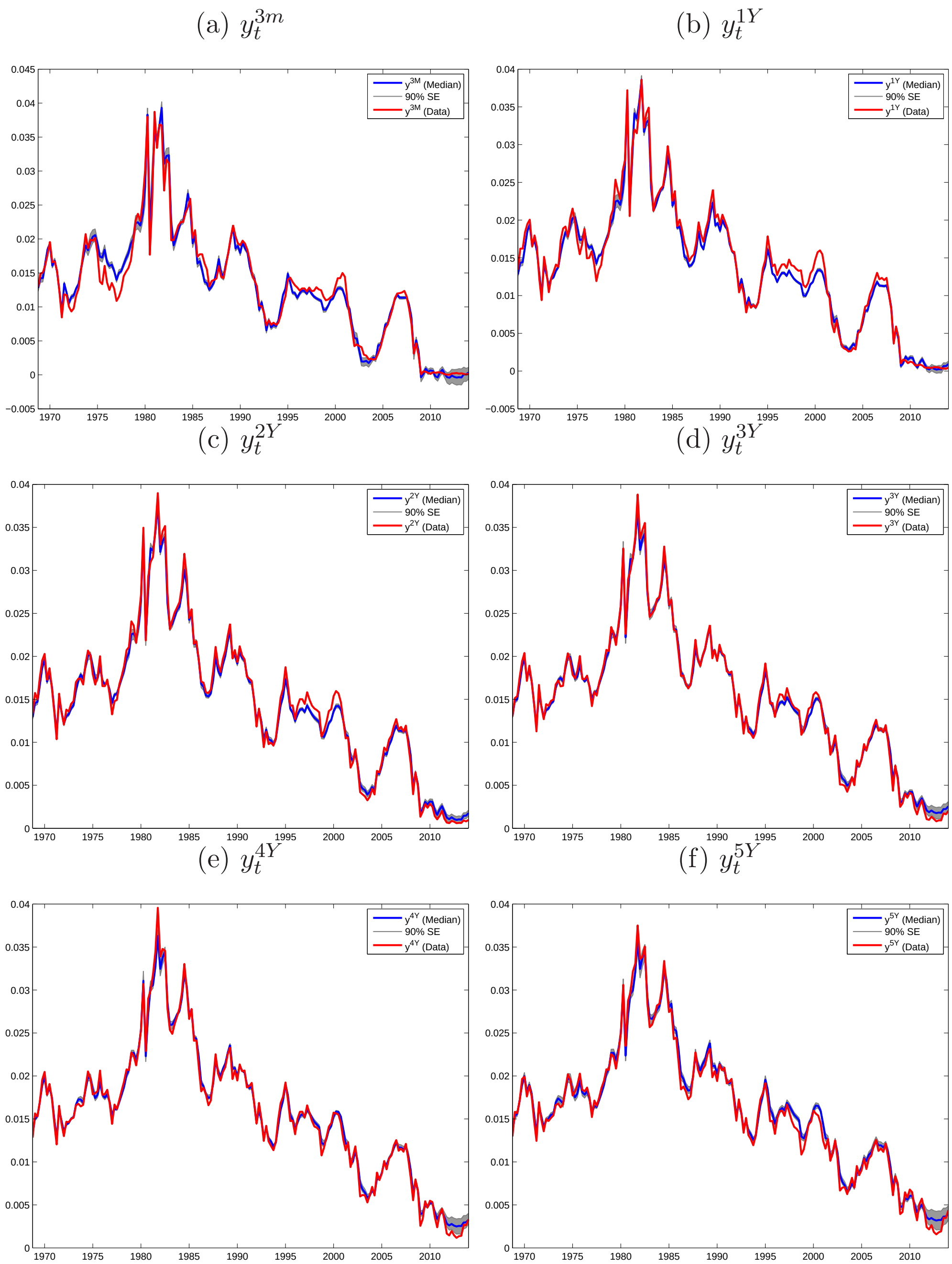
The figure shows the estimated posterior median of the exogenous component of inflation volatility. Grey region represents posterior (5%, 95%) credible sets. Quarterly observations from 1968Q3 to 2013Q4.

Figure 5: **Estimated Monetary Policy Regime**



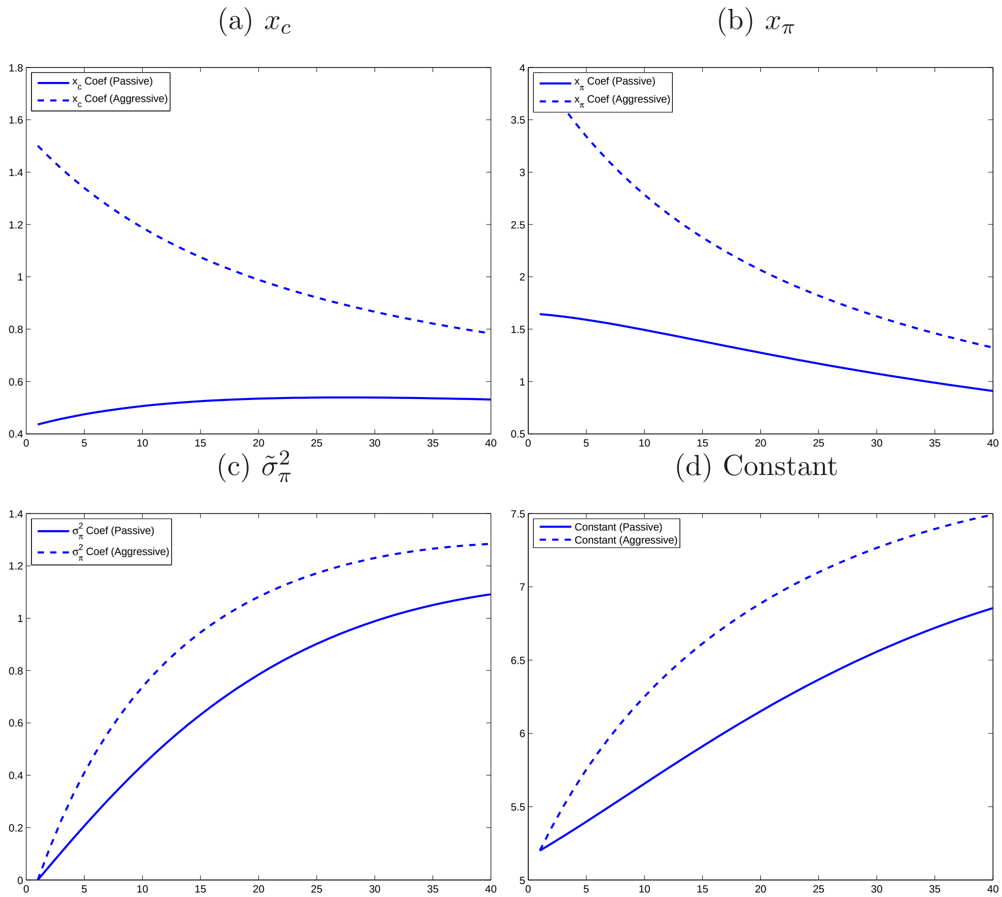
The figure shows the estimated posterior median of the monetary policy regime. Grey region represents posterior (5%, 95%) credible sets. Quarterly observations from 1968Q3 to 2013Q4.

Figure 6: **Estimated Yields**



The figure shows the nominal bond yields in the data (red line), and the estimated posterior median from the model. Grey region represents posterior (5%, 95%) credible sets. Quarterly observations from 1968Q3 to 2013Q4.

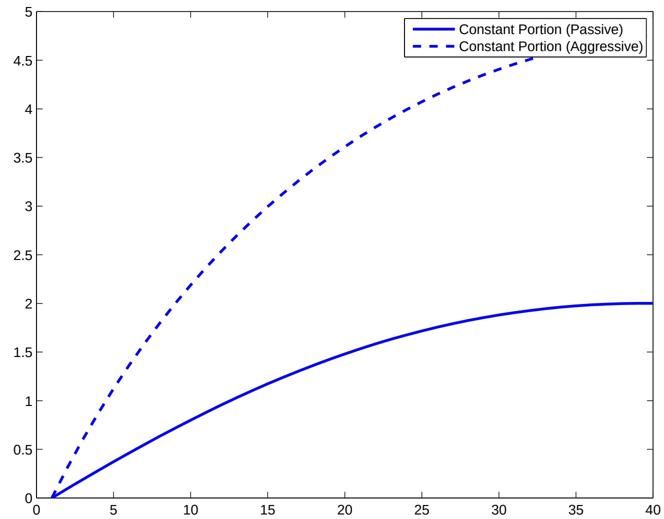
Figure 7: **Bond Loadings**



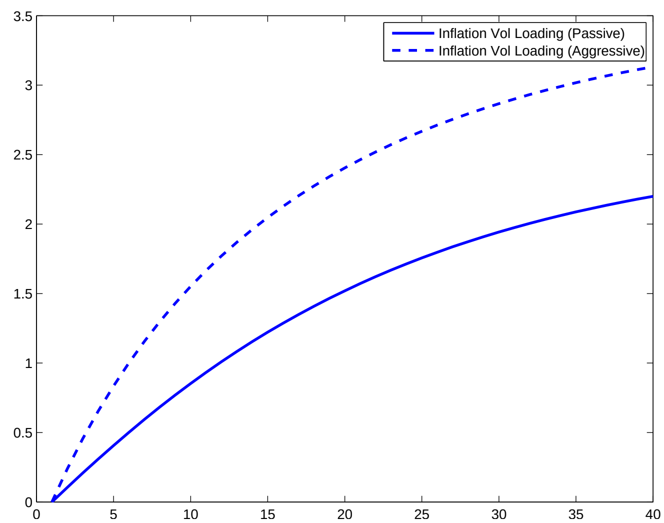
The figure shows the model-implied bond loadings on the expected real growth, expected inflation and inflation volatility, and the unconditional bond yields in aggressive and passive regimes. Bond loadings are standardized to capture a one standard deviation movement in each factor, and are computed at the median parameter draw.

Figure 8: **Risk Premia Loadings**

(a) Constant Portion of Risk Premia



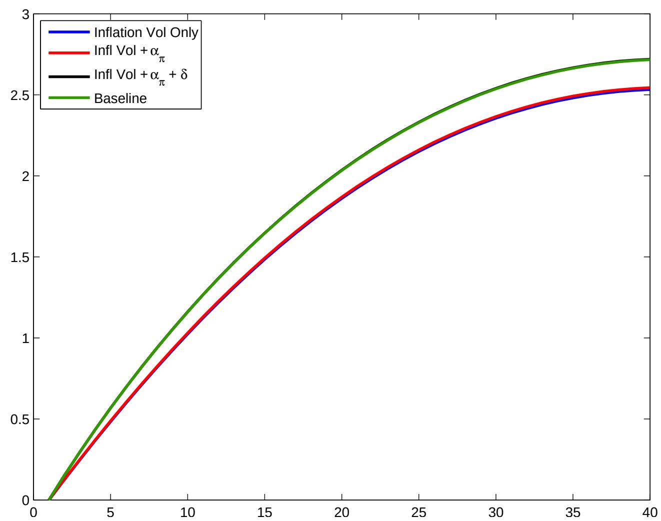
(b) Inflation Volatility Loading



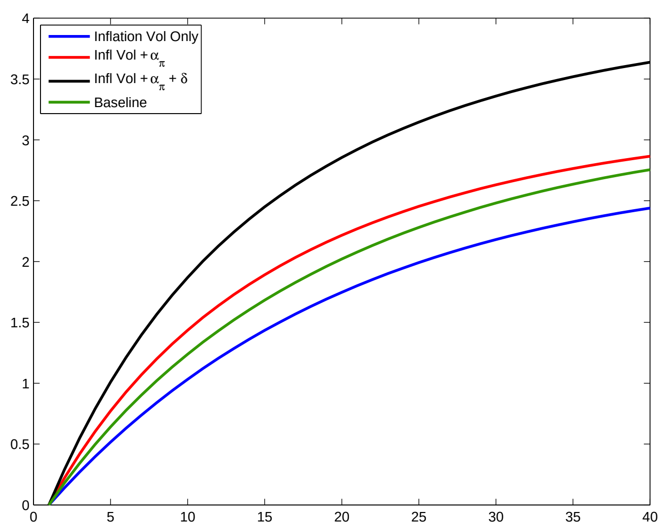
The figure shows the model-implied loadings of bond risk premia on inflation volatility and the unconditional bond risk premia levels in aggressive and passive regimes. Bond loadings are standardized to capture a one standard deviation movement in the factor, and are computed at the median parameter draw.

Figure 9: **Risk Premia Decompositions**

(a) Levels



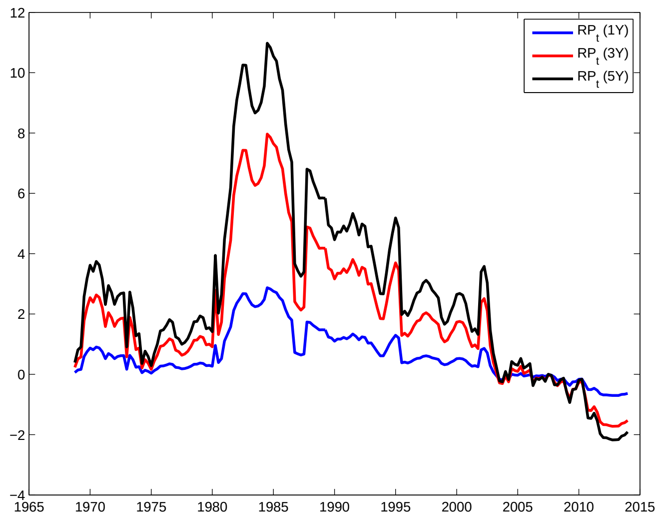
(b) Volatility



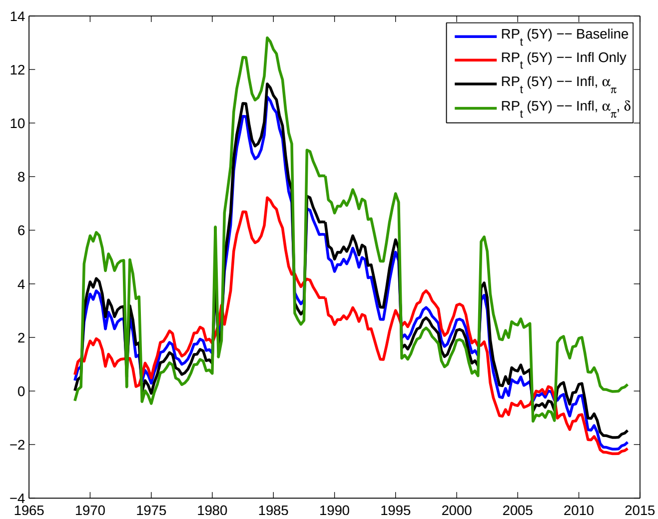
The figure shows the unconditional level and volatility of the bond risk premia in the benchmark case (Baseline) and across the restricted specifications. Infl vol Only refers to the case when only the exogenous inflation volatility component varies over time, and the monetary policy all regime-dependent coefficients are set to their unconditional values. Subsequent cases incorporate fluctuations in monetary policy loadings on the expected inflation, and the fluctuations in inflation volatility across the regimes.

Figure 10: **Model-Implied Risk Premia**

(a) Term Structure



(b) Counterfactual Paths



The figure shows the estimated bond risk premia in the sample. Top panel shows the one-quarter risk premia for one-, three-, and five-year to maturity bonds. The second panel shows the implied risk premia on a five-year bond in the benchmark case (Baseline) and across the restricted specifications. Infl vol Only refers to the case when only the exogenous inflation volatility component varies over time, and the monetary policy all regime-dependent coefficients are set to their unconditional values. Subsequent cases incorporate fluctuations in monetary policy loadings on the expected inflation, and the fluctuations in inflation volatility across the regimes.