A Probability-Based Stress Test of Federal Reserve Assets and Income[☆]

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Abstract

To support the economic recovery, the Federal Reserve amassed a large portfolio of long-term bonds. We assess the Fed's associated interest rate risk—including potential losses to its Treasury and mortgage-backed securities holdings and declines in the Fed's remittances to the Treasury. In assessing this interest rate risk, we use probabilities of alternative interest rate scenarios that are obtained from a dynamic term structure model that respects the zero lower bound on yields. The resulting probability-based stress tests indicate that large portfolio losses or a cessation of remittances to the Treasury are unlikely to occur over the next few years.

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1. Introduction

The fundamental business model of many private financial institutions uses shorter-term liabilities to finance longer-term assets. The resulting balance sheet maturity mismatch implies that the financial institutions are bearing interest rate risk, namely, a financial exposure to adverse movements in interest rates. Accepting some interest rate risk is a natural part of financial intermediation and can be an important source of profitability; however, excessive interest rate risk can pose a significant threat to the earnings and capital base of a financial institution. There are two separate, but complementary, perspectives commonly used for assessing interest rate risk exposure (e.g., Basel Committee on Banking Supervision, 2004). From the earnings perspective, the focus of analysis is on the effect of interest rate fluctuations on future firm cash flows. From the economic value (or capital) perspective, the focus of analysis is on the sensitivity of firm assets to fluctuations in interest rates, especially relative to the firm's capital base. Financial regulators have long cautioned financial institutions to avoid excessive interest rate risk from both perspectives. For example, recent supervisory guidance (Board of Governors of the Federal Reserve System, 2010) encourages financial institutions to forecast the effect of various interest rate scenarios on their future income statements and balance sheets. It is ironic then that many commentators have recently accused the Federal Reserve itself of taking on excessive interest rate risk along both dimensions. We consider such criticisms by examining forecast probability distributions of the Fed's own income statement and balance sheet.

In late 2008, in response to the severe financial crisis and recession, the Federal Reserve reduced its target for a key monetary policy rate—the overnight federal funds rate—to a range between 0 and 25 basis points. To provide additional monetary stimulus to spur economic growth and avoid deflation, the Fed then conducted three rounds of large-scale asset purchases—commonly referred to as quantitative easing (QE). These purchases left the Fed's portfolio of longer-term securities several times larger than its pre-crisis level and greatly increased the Fed's liabilities of bank reserves, which are commercial bank deposits with the Fed. Although the Fed's expanded securities portfolio carries essentially no credit risk, its market value will fluctuate over time, and its greater size and longer duration exposes the Fed to greater interest rate risk. The criticisms of the Fed's greater exposure to interest rate risk have generally taken either the standard earnings or economic value perspectives used for private financial institutions.[1] We label these two forms of interest rate risk as income risk and balance sheet risk, respectively, and they are the centerpieces of our analysis.

What we call income risk (also known in the literature as "carry" risk) is the risk that increases in *short-term* interest rates, notably the short-term interest rate that the Fed pays on bank reserves, will significantly increase the funding cost of the Fed's securities portfolio. Because the Fed's interest income is generated from fixed coupon payments on longer-maturity securities, rising short-term interest rates and increased payments on reserves would reduce the Fed's net interest income, which in turn would lower the Fed's remittances to the U.S. Treasury. For example, such worries were noted in the minutes of the March 20, 2013 Federal Open Market Committee meeting, which stated that "[s]ome participants were concerned that a substantial decline in remittances might lead to an adverse public reaction or potentially undermine Federal Reserve credibility or effectiveness."[2] Balance sheet risk (also known as "duration" risk) is the risk that increases in *longer-term* interest rates will erode the market value of the Fed's portfolio. For example, former Fed Governor Frederic Mishkin (2010) argued that "major holdings of long-term securities expose the Fed's balance sheet to potentially large losses if interest rates rise. Such losses would result in severe criticism of the Fed and a weakening of its independence." Similarly, former Fed Vice Chairman Donald Kohn (2014) worried: "As long-term rates rise, the Federal Reserve will have mark-to-market losses on its balance sheet. These losses are not a threat to the Federal Reserve's ability to tighten nor do they have any economic significance, but losses could be used as a political weapon by those who seek to curtail the Federal Reserve's independence or limit its powers."

To understand and assess the Fed's income and balance sheet risks, it is crucial to quantify

[1]Such balance sheet concerns have affected many central banks. For example, the Bank of Japan previously limited bond purchases from a fear that capital losses could tarnish its credibility; the Bank of England obtained an explicit indemnity from the British Treasury in advance of losses stemming from their QE (McLaren and Smith, 2013); and the Swiss National Bank faced a public referendum on the composition of its balance sheet. In response to these concerns, the literature on central bank financial accounting has recently grown in tandem with the latest expansion of central bank balance sheets and notably includes Bindseil et al. (2009), Archer and Moser-Boehm (2013), Hall and Reis (2013), and Del Negro and Sims (2014).

[2]See also Rudebusch (2011), Dudley (2013), and Goodfriend (2014).

them. Two recent papers—Carpenter et al. (2013) and Greenlaw et al. (2013), henceforth GHHM—have made great progress in doing so. Both studies generated detailed projections of the market value and cash flow of the Fed's assets and liabilities under a few specific interest rate scenarios. In essence, their projections are akin to the "stress tests" that large financial institutions undergo to gauge whether they have enough capital to survive adverse economic scenarios.[3] As is common, these stress tests do not place probabilities on the alternative interest rate scenarios but simply consider a few arbitrary scenarios including, say, shifting the level of the entire yield curve up or down from its baseline projection by 100 basis points. Clearly, it is also of great interest to know what probabilities should be attached to the range of considered outcomes.[4] Attaching likelihoods to the alternative scenarios—or more generally, looking at the entire distributional forecast—results in what we term probabilistic or "probability-based" stress tests. In this paper, we illustrate such a probability-based methodology by examining potential mark-to-market losses on the Fed's Treasury and mortgage-backed securities (MBS) holdings as well as the potential cessation of its remittances to the Treasury. Importantly, having information from the probability distribution of future interest rate scenarios enables us to assess the likelihood of certain events, such as the possibility that losses on the Fed's securities holdings will exceed a certain threshold or that net interest income will be negative for more than one year.

A key component of our probability-based stress test methodology is a dynamic term structure model that generates yield curve projections consistent with historical interest rate variation. Since nominal yields on Treasury debt are near their zero lower bound (ZLB), we use the shadow-rate, arbitrage-free Nelson-Siegel (AFNS) model class developed by Christensen and Rudebusch (2014) to generate the requisite, potentially asymmetric, distributional interest rate forecasts. Shadow-rate models are latent-factor models in which the state variables have standard Gaussian dynamics, but the standard short rate is replaced

[3] Stress testing financial institutions, and the financial system more broadly, has taken on great importance in the wake of the financial crisis; see Schuermann (2013) and Borio et al. (2014). Our analysis is directly related to interest rate risk stress testing, which is discussed by Drehmann et al. (2010) and Abdymomunov and Gerlach (2014).

[4] Berkowitz (2000) and Pritsker (2011) make a similar point regarding bank stress tests. In contrast, Borio et al. (2014) express the common view that stress tests should focus only on a few scenarios.

by a shadow short rate that may be negative, as in Black (1995). Since the short rate equals the shadow short rate truncated at zero, the model-generated observed short rate and yield forecasts respect the ZLB. Despite its inherent nonlinearity, shadow-rate AFNS models remain as flexible and empirically tractable as standard term structure models. Critically for our purposes, these models are able to accurately price the Fed's portfolio of Treasury securities.

For our empirical assessment of the Fed's balance sheet risk, we generate Treasury yield curve projections using the shadow-rate AFNS model favored by Christensen and Rudebusch (2013, henceforth CR) in their analysis of U.S. Treasury yields near the ZLB. We examine distributional forecasts of the value of the Fed's Treasury and MBS securities that are based on 10,000 yield curve simulations, which indicate that potential mark-to-market losses on the Fed's securities holdings are unlikely to be large. In particular, based on the Fed's Treasury holdings as of the second quarter of 2014, the projected median value of the portfolio does not fall below face value over the three-year horizon of our exercise; indeed, such a projected securities valuation shortfall only occurs at about the tenth percentile of the simulated distribution. With respect to the joint holdings of Treasury and MBS securities, the added exposure from the MBS holdings does raise the portfolio's interest rate sensitivity and thus risk. However, even then, only the projected portfolio value at the 25th percentile falls below its face value.

To assess the Fed's income risk, we use model-based yield curve projections and the GHHM modeling framework to generate distributional projections of the Fed's remittances to the Treasury up to seven years ahead. In more than 90 percent of the simulations as of year-end 2013, remittances are projected to remain positive over the seven-year horizon. Even at the lower fifth percentile of the distribution of outcomes, the cumulative remittance shortfall peaks at less than $5 billion in 2018. Accordingly, our probability-based stress test suggests that the risk of a "negative carry" resulting in a significant halt of remittances to the Treasury is fairly remote. A probability-based approach also allows us to assess the distribution of cumulative remittances, which shows that the Treasury likely will receive more remittances in total with the Fed's QE purchases than it would have otherwise.

Although we have motivated our analysis by analogy to interest rate risk management at private financial institutions, it is important to recognize that a central bank is fundamentally different with respect to its objectives, stakeholders, and financial accounting. With regard to objectives, we are not conducting a comprehensive assessment of the macroeconomic costs and benefits of the Fed's QE program, as discussed by Rudebusch (2011). Indeed, our probability-based stress test captures only part of the financial consequences of the Fed's securities purchases and, notably, excludes two key fiscal benefits accruing to the Treasury as longer-term interest rates were pushed lower by the Fed's securities purchases.[5] First, the lower interest rates likely resulted in higher output and household income, which boosted federal tax revenue and reduced federal outlays. Second, the lower interest rates associated with QE helped lower the Treasury's borrowing costs for issuing new debt. However, even if we included these two effects in our analysis, any such financial or fiscal accounting is ancillary to the Fed's statutory mission. The Fed's statutory goal for setting monetary policy is to promote maximum employment and price stability, and these macroeconomic goals are the fundamental metrics for judging monetary policy. Financial considerations—even potentially large capital losses—are at best secondary, and the Fed could be criticized for profit maximization at the expense of its macroeconomic goals.

Similarly, the Fed's overseers are not private shareholders but the public and their political representatives. Relative to the private sector, this governance undergirds the concerns of the commentators quoted above. Unlike a private institution that depends on earnings and solvency to function, strictly operational concerns are less important for the Fed largely because of the unique status of a central bank as the monopoly holder of a currency franchise. The central bank can always "print money" to satisfy financial obligations and continue to operate, even with negative equity (as several central banks have done in the past). Furthermore, regardless of its portfolio losses or income expenses, the Fed still has operational control of short-term interest rates because its ability to pay interest on bank reserves allows it to conduct monetary policy independently of the size of its balance sheet.

With respect to accounting, the Fed's balance sheet is not marked to market, so such declines in market value would constitute unrealized capital losses, which would only become

[5] Regarding the effect of QE on yields, see Gagnon et al. (2011), Christensen and Rudebusch (2012), and Bauer and Rudebusch (2014), among many others.

realized if the securities were sold. While realized portfolio losses are recorded, any resulting negative net income would not diminish the Fed's capital; instead, the Fed would maintain its capital by reducing projected future remittances to the Treasury via creation of a deferred asset.[6] In essence, under current institutional arrangements, the Fed would simply recapitalize itself with future seigniorage income generated from its currency franchise. However, as noted above, severe political fallout could be triggered by large declines in the Fed's income and remittances to the Treasury or by large capital losses (even mark-to-market unrealized losses, which the Fed does report as an adjunct to its official balance sheet). The repercussions of such political problems are difficult to gauge but could be quite severe (Cukierman, 2011). Goodfriend (2014) argues that the Fed's "operational credibility" depends on the public's confidence that the independent pursuit of monetary policy involves little fiscal cost and so does not invade the province of the fiscal authority. Specifically, he notes (p. 37) that a "period in which the central bank is seen as having to create reserves (to pay interest on its liabilities) to stabilize the purchasing power of money will rightly unnerve and very possibly unhinge inflation expectations, especially if the period is at all protracted." Furthermore, loss of confidence in the Fed's financial actions could cause institutional changes—say, new arrangements diverting seigniorage income for other purposes—that would potentially constrain monetary policy.[7] Such risks are challenging to model but need to be kept in mind when conducting central bank stress tests.

The rest of the paper is structured as follows. Section 2 describes the evolution of the Fed's securities portfolio since the onset of the financial crisis and our data sample. Section 3 describes the shadow-rate AFNS model. The next two sections examine the Fed's interest rate risk from the standard economic value and earnings perspectives, respectively. Section 4 presents our probability-based stress tests of the Fed's Treasury and MBS holdings—that

[6]The Fed remits to the U.S. Treasury all income and realized net capital gains less the interest paid on reserves, operating expenses, dividends paid on member banks' capital holdings, and contributions to capital. If the Fed's net income falls below zero—an operating loss—the realizations of negative net income are capitalized as an asset. That is, no remittances are made until accumulated earnings are sufficient to cover that loss. The value of the earnings that would need to be retained to cover this loss is called a "deferred asset" and is booked as a negative liability and counterbalanced by the creation of additional reserves.

[7]Indeed, Goodfriend (2014) strongly argues that the Fed should build up its capital in advance of any losses rather than counting on future seigniorage revenue.

is, an assessment of the Fed's balance sheet risk. Section 5 details our probability-based stress test of the Fed's remittances to the U.S. Treasury—that is, an assessment of the Fed's income risk. Section 6 concludes.

2. The Fed's Securities Portfolio

[Figure 1 about here]

We start with a brief description of the Fed's securities holdings and the associated yield data. Figure 1 shows the evolution of the assets of the Federal Reserve System at a weekly frequency since the start of 2008. In the early stages of the financial crisis, the Fed's balance sheet expanded through various emergency lending facilities, most notably the Term Auction Facility.[8] In the figure, this lending appears in the "Other Assets" category, which as of June 2014 represented less than 5 percent of the Fed's assets. The "Non-Treasury Securities" category is composed almost exclusively of agency MBS, much of which was purchased during the Fed's first and third large-scale asset purchase programs (QE1 and QE3). As of June 25, 2014, the Fed held 68,557 individual MBS with a total face value of $1.66 trillion—almost 41 percent of the Fed's total securities portfolio. This MBS portfolio is made up of many small, heterogeneous, and difficult-to-value securities. For example, about 8 percent of the portfolio was spread across 53,193 securities, each with a face value of less than $10 million.

The "Treasury Securities" category experienced a large expansion during the second and third purchase programs (QE2 and QE3).[9] As of June 25, 2014, the Fed held 237 different nominal Treasury securities with a face value of $2.28 trillion—almost 56 percent of the Fed's total securities portfolio. The long duration of these securities is also relevant for assessing balance sheet risk. From September 2011 through the end of 2012, the Fed conducted a Maturity Extension Program that sold essentially all of its Treasury securities with remaining maturities of three years or less and purchased a similar amount of Treasury securities with remaining maturities of six to thirty years. As a result of this policy, by the

[8] See Christensen et al. (2014) for details on this facility.

[9] We ignore the small amount of inflation-indexed, Treasury inflation protected securities (TIPS) discussed in Christensen and Gillan (2014). As of June 25, 2014, TIPS totaled $97 billion in principal and another $16 billion in accrued inflation compensation.

end of 2012, the Fed had essentially no short-term Treasuries. By June 25, 2014, due to maturity reduction of the remaining Treasury portfolio, the short-term Treasury share had risen to 13 percent, but it still remains far below its historical average.

[Table 1 about here]

As is usual, this description of the Fed's portfolio is based on the face value of the securities held, as shown in the first column of Table 1 as of June 2014.[10] However, there are two other accounting methods shown in Table 1 that can be used to measure the size of the Fed's securities holdings: amortized historical cost and fair (or market) value. Historical cost values securities according to their purchase prices; thus, it reflects any premiums or discounts paid relative to the securities' face values. However, the Fed does not report the actual historical cost (or value) that adjusts the acquisition cost of the securities for amortization of premiums or discounts over the maturity of the bonds.[11] The Fed has long argued that such amortized historical-cost accounting more accurately reflects the quantity of reserves in the banking system and is especially appropriate given the Fed's macroeconomic policy objectives and the buy-and-hold securities strategy the Fed has traditionally followed. Thus, the Fed only registers capital gains and losses when securities are sold. In contrast, the fair value approach records the market value of the securities at a given point in time. The Fed reports the fair value of its holdings on a quarterly basis (though not for the nominal Treasuries category that we project here), which allows calculation of unrealized capital gains and losses on its securities portfolio.[12]

In our analysis, we project forward the fair value of the Fed's portfolio and use its face value as a benchmark for comparison. Amortized historical cost could be an alternative benchmark, but it is not possible to project that cost into the future as it requires security-by-security accounting of premiums and discounts. This complexity, especially as applied to the MBS, and the fact that only aggregate information of this type is reported led us to choose the face value of the Fed's holdings as our benchmark for comparison to their

[10] Face values for aggregate holdings are reported in the H.4.1 release and for individual securities at http://www.newyorkfed.org/markets/soma/sysopen_accholdings.html.

[11] To be specific, U.S. Treasury and federal agency debt securities are amortized on a straight-line basis, while mortgage-backed securities are amortized on an effective-interest basis.

[12] See http://www.federalreserve.gov/monetarypolicy/bst_fedfinancials.htm#quarterly.

projected market values. In addition, if an important component of the Fed's balance sheet risk is political in nature, the valuation shortfall relative to face value is an obvious calculation that the public could monitor and understand. Finally, the relatively small difference between the face value and amortized cost approaches—on the order of 5% for all securities as shown in Table 1—suggests that a comparison based on the face value of the securities should not be much affected by the accounting treatment.

As noted in the introduction, the enlarged portfolio of longer-term securities greatly increases the Fed's interest rate risk. To model the market value of the Fed's Treasury and MBS holdings, we use the data set of zero-coupon Treasury yields described in Gürkaynak et al. (2007).[13] We use daily yields from January 2, 1986, to June 25, 2014, for the following 11 maturities: 3-month, 6-month, 1-year, 2-year, 3-year, 5-year, 7-year, 10-year, 15-year, 20-year, and 30-year.[14] Treasury yields were at the lower end of their historical range towards the end of our sample, and short-term yields remained at the effective ZLB on nominal yields.

3. A Shadow-Rate Model of U.S. Treasury Yields

A key ingredient for our probability-based stress test is a data-generating process for the Treasury yield curve, and this section describes the term structure model we use for this purpose. Because short-term interest rates have been near zero since 2009, the proximity of the ZLB affects the pricing of Treasuries and induces a notable asymmetry into distributional forecasts of future yields. To respect the ZLB, we employ a shadow-rate term structure model.

3.1. The Option-Based Approach to the Shadow-Rate Model

The concept of a shadow interest rate as a modeling tool to account for the ZLB can be attributed to Black (1995). He noted that the observed nominal short rate will be nonnegative because currency is a readily available asset to investors that carries a nominal interest

[13]For each business day, a zero-coupon yield curve is fitted to price a large pool of underlying off-the-run Treasury bonds. For up-to-date data, see the related website http://www.federalreserve.gov/pubs/feds/2006/index.html.

[14]The longest maturity Treasury yields are not available prior to November 25, 1985. Also, between October 2001 and February 2006, the U.S. Treasury temporarily halted its issuance of 30-year bonds, but this has only a minuscule effect on our estimation results, which are primarily determined by the yields with 10 years or less to maturity.

rate of zero. Therefore, the existence of currency sets a zero lower bound on yields. To account for this, a shadow short rate s_t, which is unconstrained by the ZLB, was proposed as a modeling tool. The usual observed instantaneous risk-free rate r_t, which is used for discounting cash flows when valuing securities, is then given by the greater of the shadow rate or zero:

$$r_t = \max\{0, s_t\}.$$

Accordingly, as s_t falls below zero, the observed r_t simply remains at the zero bound.

While Black (1995) described circumstances under which the zero bound on nominal yields might be relevant, he did not provide specifics for implementation. The small set of empirical research on shadow-rate models has relied on numerical methods for pricing.[15] To overcome the computational burden of numerical-based estimation that limits the use of shadow-rate models, Krippner (2013) suggested an alternative option-based approach that makes shadow-rate models almost as easy to estimate as the standard model.[16] To illustrate this approach, consider two bond-pricing situations: one without currency as an alternative asset, and the other that has a currency in circulation with a constant nominal value and no transaction costs. In the world without currency, the price of a shadow-rate zero-coupon bond, $P_t(\tau)$, may trade above par; that is, its risk-neutral expected instantaneous return equals the risk-free shadow short rate, which may be negative. In contrast, in the world with currency, the price at time t for a zero-coupon bond that pays \$1 when it matures in τ years is given by $\underline{P}_t(\tau)$. This price will never rise above par, so nonnegative yields will never be observed.

Now consider the relationship between the two bond prices at time t for the shortest (say, overnight) maturity available, δ. In the presence of currency, investors can either buy the zero-coupon bond at price $P_t(\delta)$ and receive one unit of currency the following day or just

[15] For example, Kim and Singleton (2012) and Bomfim (2003) use finite-difference methods to calculate bond prices, while Ichiue and Ueno (2007) employ interest rate lattices.

[16] Wu and Xia (2014) derive a discrete-time version of the Krippner framework and implement a three-factor specification using U.S. Treasury data. In related research, Priebsch (2013) derives a second-order approximation to the Black (1995) shadow-rate model and estimates a three-factor version thereof, but it requires the calculation of a double integral in contrast to the single integral needed to fit the yield curve in the Krippner framework.

hold the currency. As a consequence, this bond price, which would equal the shadow bond price, must be capped at 1:

$$\begin{aligned}\underline{P}_t(\delta) &= \min\{1, P_t(\delta)\}\\ &= P_t(\delta) - \max\{P_t(\delta) - 1, 0\}.\end{aligned}$$

That is, the availability of currency implies that the overnight claim has a value equal to the zero-coupon shadow bond price minus the value of a call option on the zero-coupon shadow bond with a strike price of 1. More generally, we can express the price of a bond in the presence of currency as the price of a shadow bond minus the call option on values of the bond above par:

$$\underline{P}_t(\tau) = P_t(\tau) - C_t^A(\tau, \tau; 1),$$

where $C_t^A(\tau, \tau; 1)$ is the value of an American call option at time t with maturity in τ years and strike price 1 written on the shadow bond maturing in τ years. In essence, in a world with currency, the bond investor has had to forgo any possible gain from the bond rising above par at any time prior to maturity.

Unfortunately, analytically valuing this American option is complicated by the difficulty in determining the early exercise premium. However, Krippner (2013) argues that there is an analytically close approximation based on tractable European options. Specifically, Krippner (2013) shows that the ZLB instantaneous forward rate, $\underline{f}_t(\tau)$, is

$$\underline{f}_t(\tau) = f_t(\tau) + z_t(\tau),$$

where $f_t(\tau)$ is the instantaneous forward rate on the shadow bond, which may go negative, while $z_t(\tau)$ is an add-on term given by

$$z_t(\tau) = \lim_{\delta \to 0} \Big[\frac{\partial}{\partial \delta} \frac{C_t^E(\tau, \tau + \delta; 1)}{P_t(\tau + \delta)}\Big],$$

where $C_t^E(\tau, \tau + \delta; 1)$ is the value of a European call option at time t with maturity $t + \tau$ and strike price 1 written on the shadow discount bond maturing at $t + \tau + \delta$. Thus, the observed yield-to-maturity is

$$\begin{aligned}
\underline{y}_t(\tau) &= \frac{1}{\tau}\int_t^{t+\tau} \underline{f}_t(s)ds \\
&= \frac{1}{\tau}\int_t^{t+\tau} f_t(s)ds + \frac{1}{\tau}\int_t^{t+\tau} \lim_{\delta\to 0}\Big[\frac{\partial}{\partial\delta}\frac{C_t^E(s,s+\delta;1)}{P_t(s+\delta)}\Big]ds \\
&= y_t(\tau) + \frac{1}{\tau}\int_t^{t+\tau} \lim_{\delta\to 0}\Big[\frac{\partial}{\partial\delta}\frac{C_t^E(s,s+\delta;1)}{P_t(s+\delta)}\Big]ds.
\end{aligned}$$

Hence, bond yields constrained at the ZLB can be viewed as the sum of the yield on the unconstrained shadow bond, denoted $y_t(\tau)$, which is modeled using standard tools, and an add-on correction term derived from the price formula for the option written on the shadow bond that provides an upward push to deliver the higher nonnegative yields actually observed.

As highlighted by Christensen and Rudebusch (2014), the Krippner (2013) framework should be viewed as an approximation to an arbitrage-free model. The size of the approximation error near the ZLB has been determined in Christensen and Rudebusch (2013, 2014) to be quite modest.[17] Of course, away from the ZLB, with a negligible call option, the model will match the standard arbitrage-free term structure representation.

3.2. The Shadow-Rate AFNS Model

In theory, the option-based shadow-rate result is quite general and applies to any assumptions about the dynamics of the shadow-rate process. However, as implementation requires the calculation of the limit term under the integral, option-based shadow-rate models are limited practically to the Gaussian model class where option prices are available in analytical form. The arbitrage-free Nelson-Siegel (AFNS) representation developed by Christensen et al. (2011, henceforth CDR) is well suited for this extension.[18] Its three factors correspond

[17]Christensen and Rudebusch (2013, 2014) analyze how closely the option-based bond pricing from their shadow-rate AFNS models matches an arbitrage-free bond pricing obtained from the Black (1995) approach based on Monte Carlo simulations of maturities out to 10 years. We extended these simulation results to consider bond maturities up to 30 years, as needed for pricing the longest bonds in the Fed's portfolio. At the 30-year maturity, the approximation errors are understandably larger but still do not exceed 6 basis points, which are notably smaller than the model's fitted errors.

[18]For details of this derivation, see Christensen and Rudebusch (2014). For general discussion of the AFNS model, see Diebold and Rudebusch (2013).

to the level, slope, and curvature factors commonly observed for Treasury yields and are denoted L_t, S_t, and C_t, respectively. The state vector is thus defined as $X_t = (L_t, S_t, C_t)$.[19]

In the shadow-rate AFNS model, the instantaneous risk-free rate is the nonnegative constrained process of the shadow risk-free rate, which is defined as the sum of level and slope as in the original AFNS model class:

$$s_t = L_t + S_t, \quad r_t = \max\{0, s_t\}. \tag{1}$$

Also, the dynamics of the state variables used for pricing under the Q-measure remain as in the regular AFNS model:

$$\begin{pmatrix} dL_t \\ dS_t \\ dC_t \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -\lambda & \lambda \\ 0 & 0 & -\lambda \end{pmatrix} \begin{pmatrix} L_t \\ S_t \\ C_t \end{pmatrix} dt + \Sigma \begin{pmatrix} dW_t^{L,Q} \\ dW_t^{S,Q} \\ dW_t^{C,Q} \end{pmatrix}, \tag{2}$$

where Σ is the constant covariance (or volatility) matrix.[20]

Based on this specification of the Q-dynamics, the yield on the shadow discount bond maintains the popular Nelson and Siegel (1987) factor loading structure

$$y_t(\tau) = L_t + \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau}\right) S_t + \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right) C_t + \frac{A(\tau)}{\tau},$$

where $A(\tau)/\tau$ is a maturity-dependent yield-adjustment term. The corresponding instantaneous shadow forward rate is given by

$$f_t(\tau) = L_t + e^{-\lambda\tau} S_t + \lambda\tau e^{-\lambda\tau} C_t + A^f(\tau),$$

where the final term is another maturity-dependent yield-adjustment term.

Christensen and Rudebusch (2014) show that, within the shadow-rate AFNS model,

[19] Note that this factor structure fits U.S. supervisory guidance on stress testing depository institution interest rate risk quite well. As summarized in Supervision SR Letter 10-1 (Board of Governors of the Federal Reserve System, 2010), firms are instructed to examine large changes in the level, slope, and shape of the yield curve.

[20] As per CDR, Σ is a diagonal matrix, and θ^Q is set to zero without loss of generality.

the zero-coupon bond yields that observe the zero lower bound, denoted $\underline{y}_t(\tau)$, are readily calculated as

$$\underline{y}_t(\tau) = \frac{1}{\tau}\int_t^{t+\tau}\Bigg[f_t(s)\Phi\Big(\frac{f_t(s)}{\omega(s)}\Big) + \omega(s)\frac{1}{\sqrt{2\pi}}\exp\Big(-\frac{1}{2}\Big[\frac{f_t(s)}{\omega(s)}\Big]^2\Big)\Bigg]ds, \tag{3}$$

where $\Phi(\cdot)$ is the cumulative probability function for the standard normal distribution, $f_t(\tau)$ is the shadow forward rate, and $\omega(\tau)$ takes the following simple form

$$\omega(\tau)^2 = \sigma_{11}^2\tau + \sigma_{22}^2\Big(\frac{1-e^{-2\lambda\tau}}{2\lambda}\Big) + \sigma_{33}^2\Big(\frac{1-e^{-2\lambda\tau}}{4\lambda} - \frac{1}{2}\tau e^{-2\lambda\tau} - \frac{1}{2}\lambda\tau^2 e^{-2\lambda\tau}\Big),$$

when the volatility matrix Σ is assumed diagonal.

As in the affine AFNS model, the shadow-rate AFNS model is completed by specifying the price of risk using the essentially affine risk premium specification introduced by Duffee (2002), so the risk premium Γ_t is defined by the measure change

$$dW_t^Q = dW_t^P + \Gamma_t dt,$$

with $\Gamma_t = \gamma^0 + \gamma^1 X_t$, $\gamma^0 \in \mathbf{R}^3$, and $\gamma^1 \in \mathbf{R}^{3\times3}$. Therefore, the real-world dynamics of the state variables can be expressed as

$$dX_t = K^P(\theta^P - X_t)dt + \Sigma dW_t^P. \tag{4}$$

In the unrestricted case, both K^P and θ^P are allowed to vary freely relative to their counterparts under the Q-measure just as in the original AFNS model.

In state-space form, the model is characterized by a standard Gaussian affine transition equation (4) and a measurement equation (3), where measurement errors assumed *i.i.d.* with standard deviations unique to each yield maturity are added in the model estimation. Finally, we note that, due to the nonlinear measurement equation for the yields in the shadow-rate AFNS models, their estimation is based on the extended Kalman filter as described in Christensen and Rudebusch (2014).

3.3. The B-CR Model Estimates and Fit

In this subsection, we briefly describe the specific model underlying our analysis, which is the shadow-rate equivalent of the AFNS model preferred by Christensen and Rudebusch (2012). Using both in-sample and out-of-sample performance measures, the authors determined that the zero-value restrictions on the K^P matrix in the following dynamic system for the P-dynamics were empirically warranted; i.e.,

$$\begin{pmatrix} dL_t \\ dS_t \\ dC_t \end{pmatrix} = \begin{pmatrix} 10^{-7} & 0 & 0 \\ \kappa_{21}^P & \kappa_{22}^P & \kappa_{23}^P \\ 0 & 0 & \kappa_{33}^P \end{pmatrix} \left(\begin{pmatrix} 0 \\ \theta_2^P \\ \theta_3^P \end{pmatrix} - \begin{pmatrix} L_t \\ S_t \\ C_t \end{pmatrix} \right) dt + \Sigma \begin{pmatrix} dW_t^{L,P} \\ dW_t^{S,P} \\ dW_t^{C,P} \end{pmatrix}, \quad (5)$$

where the covariance matrix Σ is assumed diagonal and constant. Throughout, we refer to the shadow-rate AFNS model given by equations (1), (2), and (5) as the B-CR model.

Note that the level factor is restricted to be an independent unit-root process under both probability measures. As discussed in Christensen and Rudebusch (2012), this restriction helps improve forecast performance independent of the specification of the remaining elements of K^P.[21] Second, we test the significance of the four parameter restrictions imposed on K^P in the model relative to the corresponding model with an unrestricted K^P matrix.[22] The test results from rolling weekly re-estimations show that the four parameter restrictions have been either statistically insignificant or at most borderline significant since the early 2000s.[23] Thus, the B-CR model is flexible enough to capture the relevant information in the data. Third and more importantly, as shown in CR, the B-CR model outperforms its standard AFNS model equivalent in the most recent period with yields near the ZLB in terms of forecasting future policy rates in real time and matching the compression in yield volatility. To summarize, the B-CR model has desirable dynamic properties in the current yield environment in addition to enforcing the ZLB.

[21]As described in Bauer et al. (2012), bias-corrected K^P estimates are typically very close to a unit-root process, so we view the imposition of the unit-root restriction as a simple shortcut to overcome small-sample estimation bias. Due to the unit-root property of the first factor, we can arbitrarily fix its mean at $\theta_1^P = 0$. In model estimation, factor nonstationarity is handled as described in Christensen and Rudebusch (2012).

[22]That is, we test the hypotheses $\kappa_{12}^P = \kappa_{13}^P = \kappa_{31}^P = \kappa_{32}^P = 0$ jointly using a standard likelihood ratio test.

[23]Figure 1 in Christensen and Rudebusch (2012) provide similar evidence for the corresponding AFNS model.

In our analysis below, we employ estimates of the B-CR model over three different samples to provide real-time yield curve forecasts. All three samples start in January 1986, but end on January 2, 2013, December 31, 2013, or June 25, 2014. We use the estimates corresponding to the first and third samples for analysis of the Fed's balance sheet in Section 4. The estimates from the second sample are used for income projections in Section 5. The estimated model parameters and summary statistics on the fit of the model to the yield curve are available in an online appendix. In particular, while the estimated B-CR model captures the important dynamics of the term structure in a parsimonious, theoretically consistent framework, to properly stress test the Fed's portfolio, the model must also accurately price the wide variety of securities in the Fed's portfolio. In the appendix, we show that the B-CR model can match raw bond price data that were not directly used in the model estimation, which is a strong test of the model fit.

3.4. Model-Based Short Rate Projections

Treasury yield curve projections based on the estimated B-CR model allow us to assign probabilities to specific yield curve outcomes. As the yield function in equation (3) is nonlinear in the state variables, we use Monte Carlo simulations to generate the yield curve distributional projections.[24] Specifically, we simulate 10,000 sample paths of the state variables, denoted $\widehat{X}_t = (\widehat{L}_t, \widehat{S}_t, \widehat{C}_t)$, up to three years ahead, each starting from the filtered values at the end of each of our samples. The simulated state variables are converted into a full yield curve at each quarter during the three-year simulation horizon.

[Figure 2 about here]

To provide some insight into the variation in the resulting yield curve forecasts, Figure 2(a) compares forecasts of the short rate as of early 2013 from our model, from Blue Chip professional forecasters, and from eurodollar options data. For the model-based forecasts, the figure shows the median, 5th, and 95th percentile values for the simulated short rate for a 10-year forecast horizon.[25] For the short rate, the median simulated yield remains at the

[24] We approximate the continuous-time process in equation (5) using the Euler approximation with a uniform Δt increment of 0.0001 measured in years, see Thompson (2008) for an example.

[25] Note that the lines do not represent yield curves from a single simulation run over the forecast horizon; instead, they delineate the distribution of all simulation outcomes at a given point in time.

ZLB for the first two years of the forecast horizon and gradually rises to 3 percent at the 10-year horizon. The upper 95th percentile rises more rapidly and reaches 7 percent at 10 years out, while the lower 5th percentile remains at the ZLB throughout. Long-term projections of the federal funds rate from the Blue Chip survey are also shown. The median (or consensus) forecast fits well within the range of our simulated projections. Finally, the distribution of future three-month LIBOR implied by options on eurodollar futures with maturities up to three years ahead provides another benchmark for comparison.[26] The median, 5th and 95th percentile values from these distributions are shown in Figure 2(a). The model-implied distributions easily encompass the option-based ones, which suggests that the model is able to account adequately for future likely outcomes of short rates over the projection horizon.

Figure 2(b) presents a similar short-rate forecast comparison based on data as of June 25, 2014. The model's median short-rate projections remain at roughly 3% for the longer horizon. As before, the Blue Chip projections are within the model's range of outcomes, and the consensus projections are now closer to the model projections over the entire period. Similarly, the option-implied distribution for three-month LIBOR is in line with the other two projections, although its 95th percentile bound is a bit higher than the B-CR model after mid-2016.

4. Stress Testing the Fed's Securities Holdings

In this section, we conduct probability-based stress tests of the Fed's securities portfolio using model-based distributional forecasts of the yield curve over three-year forecast horizons at two different dates: the start of 2013 and mid-2014. The earlier date provides both interesting historical perspective (i.e., what were the potential interest rate risks at the beginning of 2013) and an opportunity for model validation in that we can compare subsequent realizations of the portfolio values to the model's real-time projections. The latter date gives a more up-to-date reading on the interest rate risks within the Fed's balance sheet. We

[26] See Bauer (2014) for a description of these data. Note that these are risk-neutral distributions with no correction for risk premiums, while the model-implied distributions reflect objective probabilities. Also, three-month LIBOR represents unsecured lending at term and has historically been above the short end of the Treasury curve, which accounts for the basis difference between these series.

conduct separate stress tests at these two dates on the Fed's holdings of Treasuries alone and on the Fed's combined portfolio of Treasury and MBS securities.

4.1. Stress Testing the Fed's Treasury Portfolio

To stress test the Fed's Treasury portfolio, we use 10,000 simulated yield curves (as discussed above) to price the Fed's individual securities, which numbered 241 in January 2013 and 237 in June 2014. That is, for each quarter, we use each fitted yield curve from the B-CR model associated with one of the 10,000 values of the state variables to discount the cash flows associated with each individual security in the Fed's portfolio. This computation provides the net present value of each security as of that quarter, which is then multiplied by the notional amount of the Fed's holdings of that security. We then sum the net present values of all Treasury securities to calculate the overall portfolio value at the end of that quarter. Taken together, these values provide a distributional forecast of the market value of the Fed's Treasury portfolio.

[Figure 3 about here]

Figure 3(a) presents the median and lower percentiles of the projected Treasury portfolio value over a three-year horizon as of January 2, 2013—our first real-time projection start date with this model. The median portfolio value declines as the forecast horizon increases as the general upward trend in short rate projections shown in Figure 2(a) leads to a decline in bond values. Still, the results show that the projected value of the Fed's Treasury holdings was unlikely to fall below face value over the forecast horizon, and at most, losses are expected to occur with only a 5% probability by the end of 2015. Also shown in Figure 3(a) as crosses are the subsequent monthly realizations of the value of the Treasury portfolio that the Fed was holding as of January 2, 2013.[27] According to the model, the sell-off in global financial markets during the summer of 2013 (the so-called "taper tantrum") represented about a 5% event from the perspective of what could reasonably have been expected as of January 2, 2013. However, yields fell thereafter through the first half of 2014. As a consequence, by

[27]The realizations plotted in the figure are derived by valuing the Treasury securities that remain outstanding at the end of each month using the fitted yield curve from an updated estimation of the B-CR model at the end of each month. The 18 crosses represent each month from January 2013 through June 2014.

mid-2014, the net yield changes since early 2013 represented an outcome that the model expected to occur with about 25% chance. Although this is just one out-of-sample path realization, it suggests the model is not unrealistic in its interest rate projections.

Figure 3(b) presents an updated stress test of the Fed's Treasury portfolio by providing a distributional forecast of the Fed's Treasury portfolio as of June 25, 2014, based on an updated set of simulated yield curves. The decline in the portfolio face value over the scenario horizon is due to the maturing of the bonds with the shortest remaining terms. As before, the projected median portfolio value remains above the face value, but relative to the 2013 analysis, the much larger Treasury portfolio that the Fed was holding—due to continued QE3 purchases throughout 2013 and the first half of 2014—increased the risk of large dollar losses. For this stress test, the probability that the portfolio value falls below face value rises from about 10% at the end of 2015 to 25% at the end of the scenario horizon in mid-2017.

[Figure 4 about here]

To provide a sense of the projected yields associated with these lower tail outcomes for the portfolio values, Figure 4 shows the projected yield curves that produce the first, fifth, and median percentiles of portfolio values by mid-2017 in Figure 3(b). The figure shows that the projected yield curve that pushes the value of the Fed's Treasury portfolio below its face value at the first percentile is associated with a federal funds rate above 5% percent and a corresponding dramatic increase in the entire yield curve relative to the median projection. This simulated yield curve represents a very different state of monetary policy actions and corresponding economic conditions than currently expected. On the other hand, the simulated yield curve generating the median outcome reflects only a slight change from the yield curve as of June 25, 2014 (not shown); i.e., the simulated curve matches the compression in yield volatilities near the ZLB, which reduces the magnitude of the yield curve changes and the variation in the model's projected market valuations. Finally, to put the three shown yield curves in the context of the entire distribution of projected yield curves, the figure also shows the band between the 5th and 95th percentile values for the B-CR model's yield curve simulations by mid-2017 across all maturities. Clearly, the tail outcomes for the portfolio values by mid-2017 are associated with short- and medium-term yields outside of the 90 percent band of simulated outcomes, even though these curves move with the band at longer maturities.

4.2. Inclusion of MBS Holdings in the Stress Testing Analysis

As noted in Section 2, agency MBS constitute a large fraction of the Fed's portfolio holdings; thus, stress testing the Fed's Treasury securities alone is a limited exercise. However, using our methodology to value the large number of different MBS held by the Fed (almost 70,000 individual securities as of mid-2014)—and especially taking into account their prepayment option—is a Herculean task. Instead, in this section, we use a simplified approach that converts the expected duration of an MBS portfolio into ten-year Treasury equivalents.[28] This dollar-weighted duration measure provides an estimate of the amount of ten-year Treasury bonds that an investor would need to hold in order to be exposed to the same degree of interest rate risk. This conversion allows us to simply augment the Fed's Treasury portfolio with a large amount of one additional ten-year Treasury bond and use our same probability-based stress test methodology.[29] As of January 2, 2013, the Fed's MBS holdings represented \$271.2 billion in ten-year Treasury equivalents.[30] According to the B-CR model, the ten-year par-coupon yield was 190.6 basis points as of that date. Thus, to account for these MBS holdings, we add \$271.2 billion of a ten-year Treasury with a coupon of 1.906 percent to the Fed's Treasury portfolio on that date. Doing so, we raise the total face value of the augmented portfolio to \$1.85 trillion, while its market value is \$2.12 trillion.

[Figure 5 about here]

Figure 5(a) shows the stress test results as of January 2, 2013, using the B-CR model to simulate the market value of the augmented portfolio of Treasury securities. As compared to the Treasuries-only stress test in Figure 3(a), the augmented portfolio has a higher probability of falling below its face value by the end of 2015. However, the potential loss remains relatively small with the projected portfolio values dipping below face value with less than a

[28]See Greenwood et al. (2014) and Li and Wei (2013) for other applications involving ten-year Treasury equivalents.

[29]The MBS prepayment options are not well priced using this approach. In a rising interest rate environment, MBS prepayment speeds fall and duration increases (i.e., negative convexity arises), which is a further hit to MBS valuations. Further research is required to measure this interest rate risk component.

[30]These data are available from chart 9 of Federal Reserve Bank of New York (2014).

10% probability by late 2015. Again, the subsequent monthly realizations of the aggregate Treasury and MBS portfolio valuations, denoted as crosses, fall very close to the central tendency of the projection into 2014.

The interest rate risk of the joint Treasury and MBS portfolio does increase in an updated stress test as of June 25, 2014. By that date, the Fed had increased its notional holdings of Treasuries and MBS by $704 billion and $737 billion, respectively. In the absence of an officially reported number, we determine the ten-year Treasury bond equivalent of the MBS holdings at that date using a simplified approach. Since yields did not change much, on net, since the end of 2013, we make the simplifying assumption that the average duration of the MBS portfolio also did not change much since then, when it was reported to be 5.8 years.[31] According to the B-CR model, as of June 25, 2014, the duration of the ten-year par-coupon Treasury bond was 8.83 years, while its coupon was 2.585 percent. Hence, as the Fed's MBS holdings totalled $1,663.9 billion in notional value, the interest rate sensitivity of the MBS portfolio can be approximated by replacing it with $\$1{,}663.9 \times 5.8/8.83 = \$1{,}092.5$ billion of ten-year Treasury bonds with a coupon of 2.585 percent.[32] The results of stress-testing this augmented Treasury portfolio are shown in Figure 5(b). The continued purchase of MBS during 2013 and the first half of 2014 clearly increased the portfolio's interest rate risk. While the median projected value remains above the portfolio face value up through the second quarter of 2017, about 30% of the simulated yield curve paths now cause the projected portfolio value to fall below face value by early 2016.

5. Stress Testing the Fed's Net Income

Our probability-based stress testing approach can also be applied to questions regarding the Fed's income risk; that is, the sensitivity of its net income to alternative interest rate scenarios. The Fed's interest income is relatively fixed by the set coupon payments from its securities holdings, although MBS payments are sensitive to prepayment risk. However, the Fed's interest expenses will vary directly with the level of short-term interest rates through

[31] See Chart 7 of Federal Reserve Bank of New York (2014).

[32] The calculation of the dollar value of ten-year Treasury equivalent amounts is the dollar value of the portfolio multiplied by the ratio of the portfolio duration to that of the ten-year Treasury bond.

payment of interest on bank reserves. The primary concern is that certain interest rate outcomes could lead the Fed's net income to decline sufficiently that it would halt remittances (i.e., payments of excess net interest income beyond operational expenses) to the Treasury Department. For example, Carpenter et al. (2013) and GHHM consider whether the Fed's remittances would remain positive under several specific interest rate scenarios. In this section, we address this policy question using our model-based approach to generating yield curve distributions in conjunction with the accounting framework of GHHM, which encompasses the projection horizon from year-end 2013 through 2020 at an annual frequency.[33]

For this analysis, we maintain the GHHM assumptions regarding future MBS prepayment, Fed liability growth, capital accretion, and operating expenses. However, we do update and alter four other assumptions to generate our own "CLR baseline scenario." First, we update the expected path of Federal Reserve asset purchases using publicly announced results of the New York Fed Primary Dealer Survey as of June 2014, which has purchases ending in October 2014.[34] Second, we assume that the Fed does not sell any securities through 2020, which is consistent with the announced plans in the FOMC's "Policy Normalization Principles and Plans" released on September 17, 2014. Third, we extend the assumed reinvestment path for bond principal payments into Treasuries of various maturities up through the end of 2015. A final modification is that we set the path for the interest on excess reserves (IOER) rate—which is the rate the Fed pays on the reserves that banks hold and thus determines its main interest expense—equal to the overnight rate as implied by our yield curve simulations.[35] Given the variation in our simulated short rates, we generate notable variation in the Fed's interest expenses going forward. Indeed, a 90 percent confidence interval for the Fed's annual interest expenses ranges from \$5 billion to almost \$90 billion at its widest point in 2017.

[Figure 6 about here]

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[33] We greatly appreciate the GHHM authors sharing their model with us for this analysis. Our stress testing results rely on projected values over the period from 2014 through 2020 as generated from yield curve simulations using data as of year-end 2013.

[34] The survey is available at: http://www.newyorkfed.org/markets/primarydealer_survey_questions.html.

[35] The overnight rate is approximated by an instantaneous short rate given by $r_t = \max\{0.25\%, s_t\}$; i.e., we impose a minimum of 25 basis points for the IOER rate consistent with current policy.

[Figure 7 about here]

Figures 6 and 7 present the key results of our simulation-based approach using this set of modified GHHM assumptions. Figure 6 presents the projected range of the Fed's remittances to the Treasury over the period from 2014 to 2020. The median value (represented as the black, dashed line) declines over the period, both as interest income declines based on maturing securities and as interest rate expenses rise due to interest rates rising from their current values near the ZLB. However, by the end of the forecast horizon, these declines are back in line with linearly projected remittances (the gray dashed line) based on data from 1990 to 2007.

Remittances are only projected to stop with a 5% probability in 2018 before resuming again. Figure 7 shows the corresponding projected range of cumulative negative remittances or, in accounting terms, the deferred asset. As before, our results imply zero remittances and a need for the deferred asset for the lower fifth percentile of outcomes. In these cases, the low point is only in 2018, and the deferred asset does not exceed $5 billion. The figure also presents the lower first percentile of simulated outcomes, which consists of a much longer period of paused remittances and a much greater magnitude of deferred assets. However, as of year-end 2013, our probabilistic results suggest that the Federal Reserve is unlikely to stop earning net interest income and making Treasury remittances over the next seven years under reasonable assumptions.

[Figure 8 about here]

To provide further insight on the use of deferred assets, Figure 8 shows our simulated probability distribution of the maximum deferred asset amount over the forecast horizon up through 2020. The results are heavily left-skewed with more than 92 percent of the probability mass at zero—that is, no cessation of remittances. For the rest of the distribution, the probability of observing a maximum deferred asset of less than $10 billion is 2.4 percent, and the probability of a maximum value greater than $10 billion is just shy of 5 percent.

[Figure 9 about here]

As a final exercise, we try to assess the cumulative remittances to the Treasury from the Fed's expansion of its balance sheet starting in 2008. Figure 9 shows our simulated probability distribution for the cumulative remittances from 2008 to 2020 net of the projected linear trend based on remittances from 1990 to 2007. The trend totals nearly $400 billion in cumulative remittances during the 2008-2020 period—about $30 billion per year. This amount, which is a benchmark for the absence of any QE programs or balance sheet expansion by the Fed, is then deducted from the sum of projected remittances (including the known 2008-2013 remittances of $402 billion) in each of the 10,000 simulation runs to produce the distribution.

There are two things to note in the figure. First, with near certainty, the expansion of the Fed's balance sheet is likely to generate hundreds of billions of dollars in excess remittances to the U.S. Treasury over the entire 2008-2020 period. Thus, the extraordinary monetary policy initiatives most likely will have provided a direct financial benefit to the Treasury, in addition to any indirect benefits from improved economic outcomes noted in the introduction. Second, the chance that these policies will ultimately produce below-trend net remittances is near zero according to this exercise, as the smallest outcome across the 10,000 simulations is $51 billion in remittances above trend.

6. Conclusion

Financial stress tests, including those that have examined the Fed's own financial position, usually only consider a small number of hand-picked scenarios. The selection of a few ad hoc scenarios introduces a seemingly large degree of arbitrariness into the stress test results and makes it difficult to judge the plausibility of the projected outcomes. Our methodological contribution is to introduce a probabilistic structure into a stress test of the Fed's balance sheet and income risks. We argue that attaching likelihoods to adverse outcomes based on interest rate fluctuations is a crucially important addition to the policy debate.

In terms of substantive results, we use a model-based approach to generate Treasury yield curve projections to analyze the Fed's interest rate risk exposures. Our stress test results suggest that in all likelihood the potential losses to the Fed's Treasury and agency MBS holdings over the next several years should be quite moderate. We also generate more comprehensive projections of the Fed's future income and find only a small chance of a temporary halt in the remittances to the Treasury; furthermore, the magnitude of the deferred asset created during this period likely would be modest. In addition, cumulative remittances to the Treasury over the period from 2008 to 2020 are almost surely to be greater than in a counterfactual scenario in which remittances were a linear projection of what they were from 1990 through 2007. In summary, our probability-based scenario analysis provides generally reassuring results regarding questions related to the financial costs of the Fed's balance sheet policy.

Our analysis should not be taken as a blanket portrayal or endorsement of benign fiscal consequences of QE programs. We are merely stress testing the Fed at two points in time. The Fed may have been "lucky" in this episode (in a financial if not a macroeconomic sense) with a slow recovery, unusually low and stable inflation, perceptions of a declining real equilibrium interest rate, and delayed liftoff from the ZLB—all of which contributed to the likely favorable fiscal outcome. In future episodes, financial considerations may be more prominent and adverse. Our analysis does not show—in an unconditional sense—that a central bank conducting QE cannot have severe financial losses.

Our analysis leaves much room for further research. For example, the analysis relies on historical data to estimate forecast distributions, and these may not be completely appropriate for assessing all future circumstances. Consideration of a distribution with fatter tails may be warranted. Also, we do not consider distributional projections of all possible relevant conditioning factors—such as the inflation path or possible asset sales by the Fed. Finally, as noted earlier, unlike for a stress test of a commercial enterprise, it is the political consequences of the financial costs that are of key concern to the Fed. It may be that in those states of the world in which the Fed bears large losses owing to higher long rates, economic growth is also likely to be strong, which may mitigate the political risks. Alternatively, in scenarios with very small Fed remittances to the Treasury because of a high IOER rate, substantial payments of interest on reserves would be paid to large commercial banks, likely boosting political risk. Further research could expand and refine our probabilistic structure in these directions.

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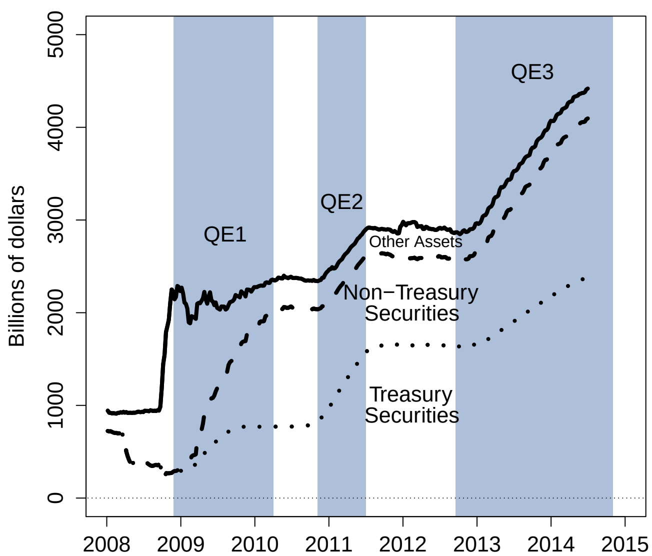
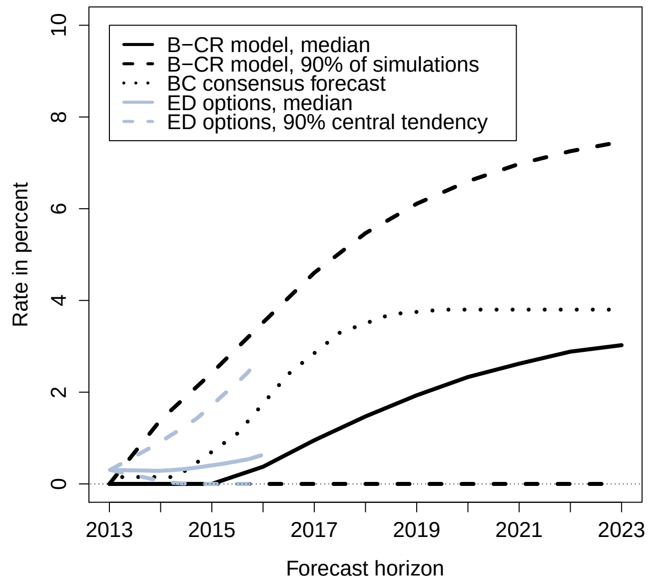


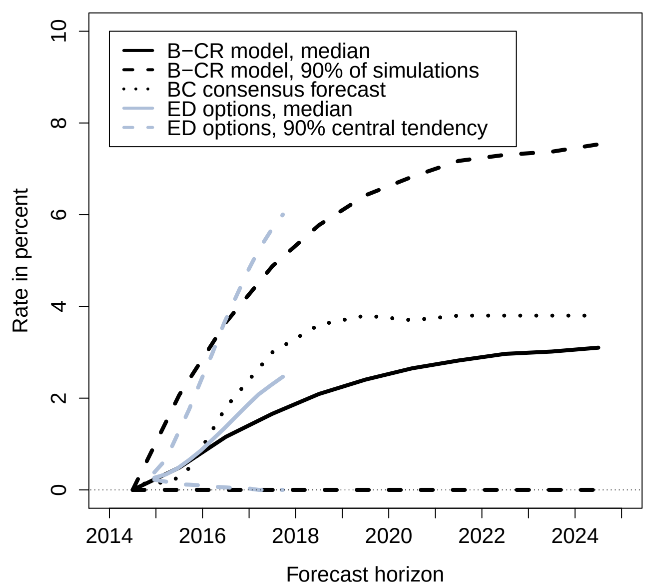
Figure 1: **Assets of the Federal Reserve System.**
Illustration of the total assets of the Federal Reserve System divided into Treasury securities, non-Treasury securities, and other assets. The data are weekly covering the period from January 2, 2008, to June 25, 2014. Source: H.4.1 statistical release.

	Face value	Amortized value	Fair value
All securities	4,105	4,299	4,390
All Treasuries	2,397	2,541	2,614
Nominal Treasuries	2,284	n.a.	n.a.
MBS	1,664	1,713	1,727

Table 1: **Fed's Portfolio Value Based on Different Accounting Measures.**
The table reports the value of the Fed's securities holdings in billions of dollars as of the second quarter of 2014 according to three different accounting measures as explained in the text and for four categories: all securities, all Treasury securities, nominal Treasury securities, and MBS holdings. The face value measure is from the H.4.1 statistical release and dated June 25, 2014, while the amortized value and fair value measures are from the unaudited Federal Reserve Banks Combined Quarterly Financial Report for the second quarter of 2014 and dated June 30, 2014.

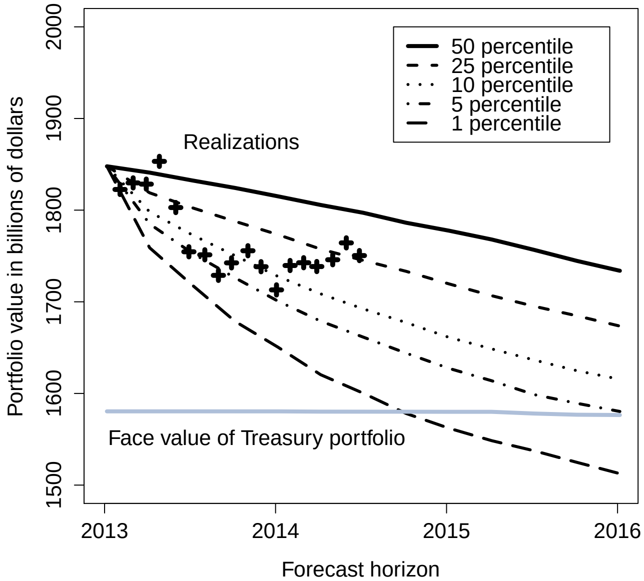


(a) January 2, 2013.

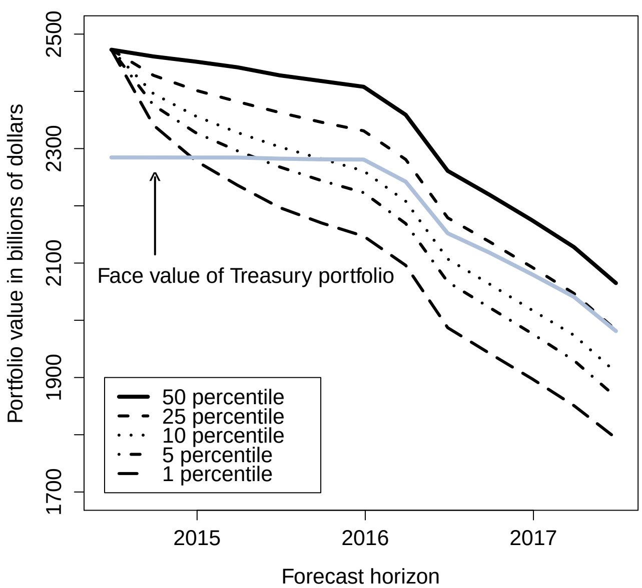


(b) June 25, 2014.

Figure 2: **Short Rate Projections.**
Panel (a) presents the median and $[5\%, 95\%]$ range of the fed funds rate from the B-CR model's simulated interest rate scenarios as of January 2, 2013. Note that the lines do not represent yield curves from a single simulation run over the forecast horizon; instead, they delineate the distribution of all simulation outcomes at a given point in time. The graph also shows the consensus federal funds rate forecast from the Blue Chip Financial Forecasts (BC) survey released on December 1, 2012. Finally, the graph shows the median and 90% central tendency of the risk-neutral distribution of three-month LIBOR implied by options on eurodollar futures (ED) with maturities up to three years ahead as of January 2, 2013. Panel (b) shows the corresponding results as of June 25, 2014, with a comparison to the BC survey released on June 1, 2014.



(a) January 2, 2013.



(b) June 25, 2014.

Figure 3: **Projected Market Value of Fed's Treasury Securities.**
Panel (a) presents the percentiles ranging from 1% to 50% in the distribution of the market value of the Fed's Treasury securities portfolio projected between 3 and 36 months ahead based on 10,000 Monte Carlo simulations of the B-CR model estimated on data through January 2, 2013. Crosses show the monthly realizations of the remaining Treasury values for the 18 months from January 2013 through June 2014. The gray solid line shows the face value of the MBS-augmented portfolio. Panel (b) shows the corresponding simulation results as of June 25, 2014.

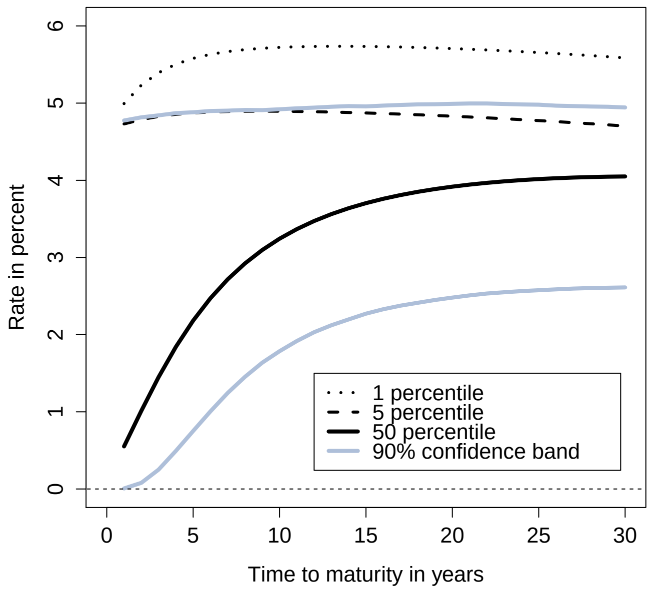
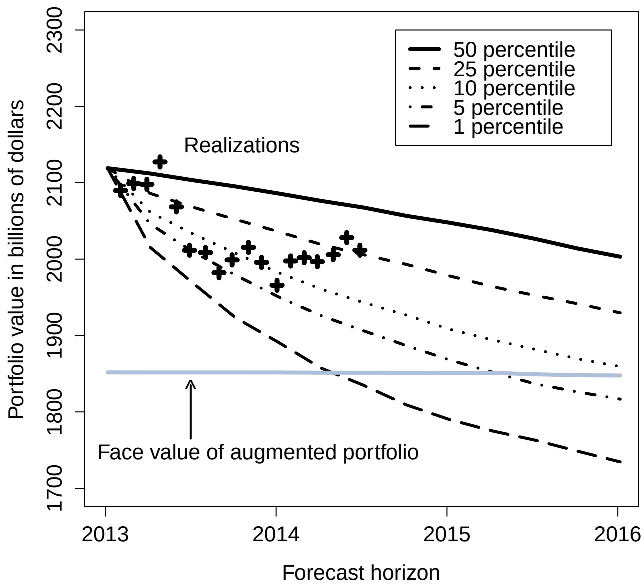
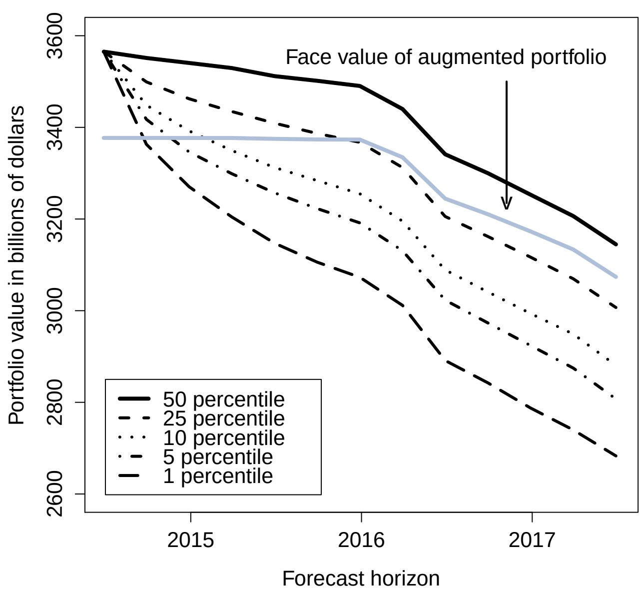


Figure 4: **Projected Yield Curves for Mid-2017.**
Illustration of the projected yield curves that produce the first percentile, fifth percentile, and fiftieth percentile (median) Treasury portfolio values by mid-2017 in Figure 3(b). Also shown is the $[5\%, 95\%]$ range of yields for each maturity from the B-CR model's simulated interest rate scenarios by mid-2017.



(a) January 2, 2013.



(b) June 25, 2014.

Figure 5: **Projected Market Value of Fed's MBS-Augmented Portfolio.**
Panel (a) shows the percentiles ranging from 1% to 50% in the distribution of the market value of the Fed's MBS-augmented Treasury securities portfolio projected between 3 and 36 months ahead based on 10,000 Monte Carlo simulations of the B-CR model estimated on data through January 2, 2013. Crosses show the monthly realizations of the remaining Treasury securities for the 18 months from January 2013 through June 2014. The gray solid line shows the face value of the MBS-augmented portfolio. Panel (b) shows the corresponding updated projections as of June 25, 2014, as described in the text.

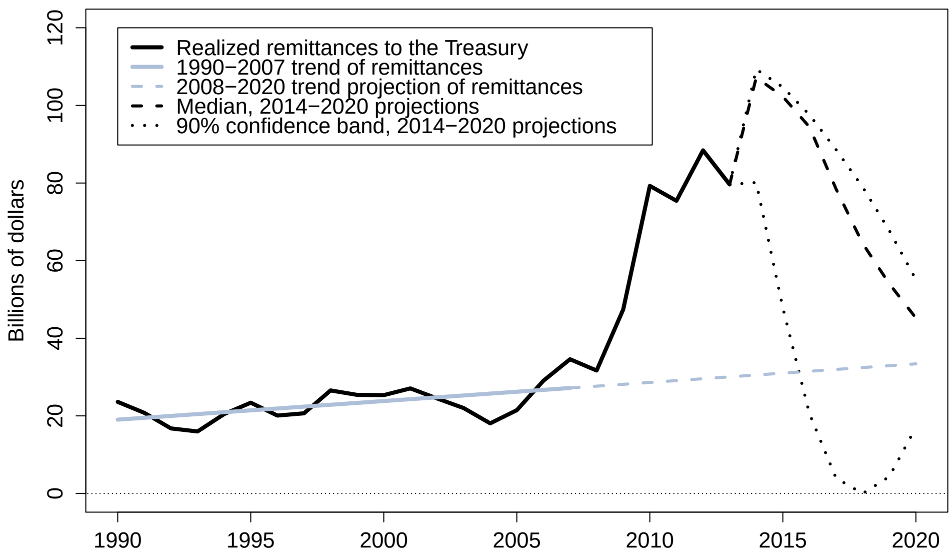


Figure 6: **Projected Fed Remittances to U.S. Treasury.**
The solid black line shows the realized remittances to the U.S. Treasury over the period from 1990 to 2013. The solid gray line indicates the simple linear trend of the remittances from 1990 to 2007, while the dashed gray line shows the extrapolation of that trend to the 2008-2020 period. Also shown are the median (dashed black line) and the 5th and 95th percentiles (dotted black lines) of the projected remittances to the U.S. Treasury over the 2014-2020 period based on the CLR baseline scenario combined with 10,000 Monte Carlo simulations of the B-CR model as of year-end 2013.

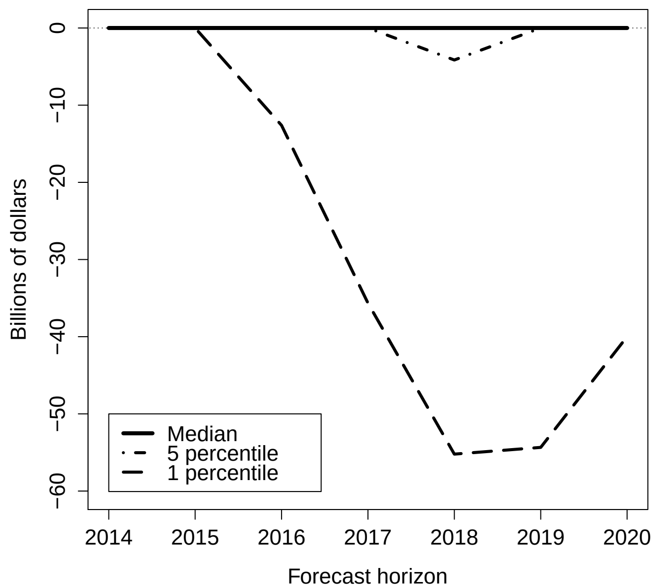


Figure 7: **Projected Cumulative Negative Remittances (i.e., the Deferred Asset).** Illustration of the median, the 5th percentile, and the 1st percentile of projections of the Fed's deferred asset based on the CLR baseline scenario combined with 10,000 Monte Carlo simulations of the B-CR model.

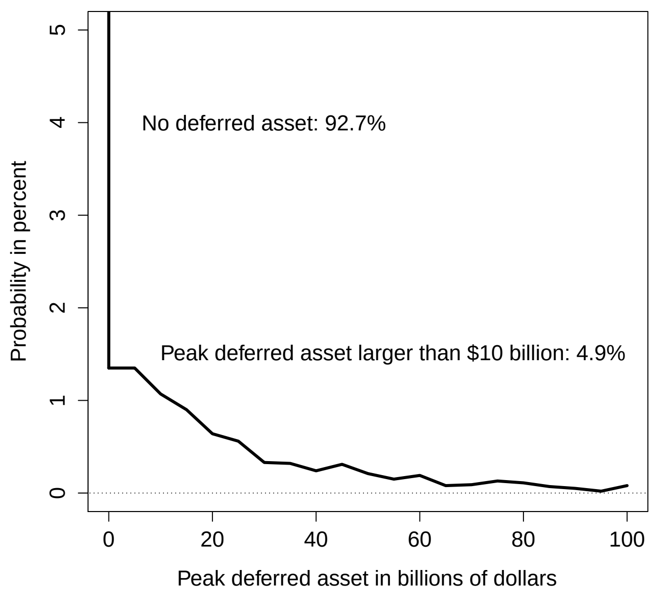


Figure 8: **Simulated Distribution of Maximum Deferred Asset.**
The graph depicts the simulated probability distribution function for 10,000 Monte Carlo simulations of the maximum deferred asset over the seven-year forecast horizon. More than 92 percent of the values are zero.

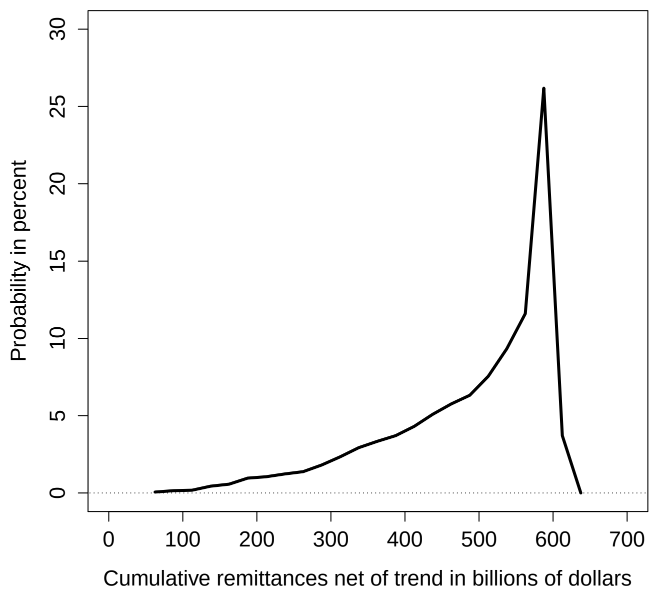


Figure 9: **Simulated Distribution of Cumulative Remittances Net of Trend.**
The graph depicts the simulated probability distribution function for 10,000 Monte Carlo simulations of the cumulative remittances by the Fed to the U.S. Treasury from 2008 to 2020, net of the projected trend of remittances from the 1990-2007 period.