NYU Stern School of Business
Department of Information, Operations & Management Sciences
STATISTICS RESEARCH SEMINAR

TOPIC: Asymptotically Efficient Estimation of Gaussian Graphical Model and A Fast Algorithm
SPEAKER: Zhao Ren (University of Pittsburgh)
DATE: Friday, May 15, 2015
TIME: 12:30 PM - 1:30 PM (Lunch served at 12:15pm)
PLACE: KMC 4-80

Abstract

The Gaussian graphical model, a popular paradigm for studying relationship among variables in a wide range of applications, has attracted great attention in recent years. This talk considers a fundamental question: When is it possible to estimate low-dimensional parameters at parametric square-root rate in a large Gaussian graphical model? A novel tuning-free regression approach is proposed to obtain asymptotically efficient estimation of each entry of a precision matrix under a sparseness condition relative to the sample size. When the precision matrix is not sufficiently sparse, a lower bound is established to show that it is no longer possible to achieve the parametric rate in the estimation of each entry through a novel construction of a subset of sparse precision matrices in an application of Le Cam's Lemma. Moreover, the proposed estimator is proven to have optimal convergence rate when the parametric rate cannot be achieved, under a minimal sample requirement.

Starting from this fundamental inference result, we can obtain almost all results in estimating Gaussian graphical model in literature. The asymptotic efficiency estimator is applied to test the presence of each edge and provide its confidence interval, do adaptive support recovery, to obtain adaptive rate-optimal estimation of the en- tire precision matrix under various matrix ℓq operator norms, to make inference in latent variable graphical model and to make inference in covariate-adjusted graphical model. All these are achieved under a minimal sparsity condition on the precision matrix and a side condition on the range of its spectrum. This significantly relaxes the commonly imposed uniform signal strength condition, irrepresentable condition or the $\ell 1$ constraint on the precision matrix.

Computational issue is investigated. A fast algorithm which computes the exact solution is proposed. Surprisingly, this new algorithm makes computation possible even if the dimension $p \geq 10000$ and yields confidence intervals of $p(p-1)/2$ edges simultaneously. Numerical results confirm our theoretical findings. Our method significantly outperforms the popular GLasso algorithm for support recovery.

Bio

Dr. Zhao Ren is an Assistant Professor in the Department of Statistics at the University of Pittsburgh. Dr. Ren obtained his Ph.D. in Statistics at Yale University under the supervision of Prof. Harrison

Zhou in 2014. His research interests are in high-dimensional statistical inference, covariance/precision matrix estimation, graphical models, statistical machine learning, nonparametric function estimation, robust covariance estimation and applications in statistical genomics.