Arrested Development: Theory and Evidence of Supply-Side Speculation in the Housing Market*

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Abstract

This paper studies the role of speculation in amplifying housing cycles. Speculation is easier in the land market than in the housing market due to rental frictions. Therefore, speculation amplifies house price booms the most in cities with ample undeveloped land. This observation reverses the standard intuition that cities where construction is easier experience smaller house price booms. It also explains why the largest house price booms in the United States between 2000 and 2006 occurred in areas with elastic housing supply. These episodes are most likely to occur in elastic cities approaching a long-run development constraint.

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1 Introduction

How do prices aggregate information? We take up this question in a setting of particular macroeconomic importance: housing markets. Housing is a key driver of the business cycle (Leamer, 2007), and the causes of the financial crisis of 2008 and the Great Recession originated in housing markets (Mian and Sufi, 2009, 2011). An enduring feature of these markets is booms and busts in prices that coincide with widespread disagreement about fundamentals (Shiller, 2005). This paper argues that these cycles are caused by how housing markets aggregate beliefs.

Studying belief aggregation allows us to address some of the most puzzling aspects of the U.S. housing boom that occurred between 2000 and 2006. According to the standard model of housing markets, elastic housing supply prevents house price booms by allowing new construction to absorb rising demand.[1] But the episode from 2000 to 2006 witnessed several major anomalies, in which historically elastic cities experienced house price booms despite continuing to build housing rapidly. And house prices rose more in many of these cities—located in Arizona, Nevada, inland California, and Florida—than in cities where it was difficult to build new housing. Further complicating the puzzle, house prices remained flat in other elastic cities that were also rapidly building housing. Why was rapid construction able to hold down house prices in some cities and not others?

We solve this puzzle by adding two ingredients to the standard model. The first is a friction that makes owner-occupancy more efficient than renting. The second is disagreement about long-run growth paths. In this framework the way housing markets aggregate beliefs depends on a city's land availability. Prices appear more optimistic when land is plentiful and building houses is easy, reversing the standard model's intuition for how land supply influences prices. Crucially, optimism amplifies prices most when a city nears but has not yet reached a long-run development constraint. This mechanism matches the data. The anomalous cities are those that, as the boom began, found themselves in just this state of "arrested development."

We model a city of developers and residents with a fixed amount of land available for development. Developers decide how many houses to build and how much land to buy. Residents decide how much housing to consume and whether to buy or rent. They prefer owning their houses over renting because of frictions in the rental market.[2] Residents can invest in the equity of developers, which provides exposure to land prices. Short-selling land and housing is impossible, but residents can short-sell developer equity. Over time,

[1] See, for example, Glaeser, Gyourko and Saiz (2008), Gyourko (2009), and Saiz (2010).

[2] Such frictions include the effort spent monitoring tenants to prevent property damage (Henderson and Ioannides, 1983), tax disadvantages (Poterba, 1984), and difficulty renting properties like single-family homes that are designed for owners (Glaeser and Gyourko, 2009).

new residents arrive in the city, leading developers to build houses using their holdings of undeveloped land. Because of this growth, the city gradually exhausts its land supply. What today's investors believe about future inflows determines the price of undeveloped land.

House construction is instantaneous and developers bear a constant unit cost per house. As a result, all variation in house prices is caused by movements in land prices and not construction costs. Data from the U.S. boom support this feature of the model. Rapidly rising land prices account for most of the house price increases across cities. In contrast, construction costs remained relatively stable throughout the boom, and cost changes hardly varied across cities. These aspects of the data distinguish our theory from those that stress "time-to-build" factors such as input shortages or delivery lags (Mayer and Somerville, 2000; Gao, 2014).

We study a demand shock that raises the current inflow of new residents and also creates uncertainty about future inflows. Disagreement about long-run demand leads to disagreement about future house prices. The most optimistic residents seek to speculate through buying housing and through buying the equity of optimistic developers who are buying land.

Our first result is that speculation is crowded out of the housing market and into the land market. Consider an optimistic resident who wishes to speculate on future house prices. Buying a house and renting it out is difficult because of the widespread preference for owner-occupancy. And buying more housing for personal consumption is unappealing because of diminishing marginal utility. Land however offers a pure, frictionless bet on real estate. The optimistic resident chooses to invest in land through buying developer equity.

With data from the U.S. housing boom, we confirm several of the model's predictions about land speculation. In the model, developers run by optimistic CEOs use resident financing to amass large land portfolios, buying land from less optimistic developers. Consistent with this prediction, we find that supply-side speculation figures prominently in the data. Between 2000 and 2006, the eight largest U.S. public homebuilders tripled their land investments, an increase far exceeding their additional construction needs. Their market equity then fell 74%, with most of the losses coming from write-downs on their land portfolios. The model also predicts that short-selling of developer equity increases during a boom because pessimistic residents disagree with the high valuations of the developer land portfolios. Matching this prediction, the short interest in homebuilder stocks rose from 2% in 2001 to 12% in 2006. Rising short interest provides evidence of disagreement over the value of homebuilder land portfolios and thus over future house prices.

Our second result concerns how house prices aggregate beliefs. Speculators are crowded into the land market, while homeownership remains dispersed among residents of all beliefs. Therefore, house prices reflect a weighted average of the optimistic belief of speculators and the average owner-occupant belief. The weight on the optimistic belief equals the share of the housing market on the margin that consists of the land market. Prices look most optimistic where land is plentiful and building easy—that is, in cities where the short-run elasticity of housing supply is large.

This optimism bias affects prices most when the city's housing supply will become inelastic soon. This observation, which constitutes our third result, explains why house price booms occur in some elastic cities and not others. Consider a city in which the land available for development is large relative to the city's current size. Here, new construction fully absorbs the demand shock now and in the foreseeable future, and so beliefs about future house prices remain unchanged. The shock raises future price expectations only in cities where construction will be difficult in the near future.

Speculation amplifies house price booms most in cities that exist in a state of arrested development: they have ample land for construction today, but also face land barriers that will restrict growth in the near future. This theoretical supply condition characterizes the anomalous elastic cities during the U.S. housing boom. For instance, Las Vegas faces a development boundary put in place by Congress in 1998 and depicted in Figure 1. During the 2000-2006 housing boom, many investors believed the city would soon run out of land.[3] Likewise, Phoenix's long-run development is constrained by Indian reservations and National Forests that surround the metropolitan area (Land Advisors, 2010). In inland California, much of the farmland around cities is protected by a state law that penalizes real estate development on these parcels (Onsted, 2009).

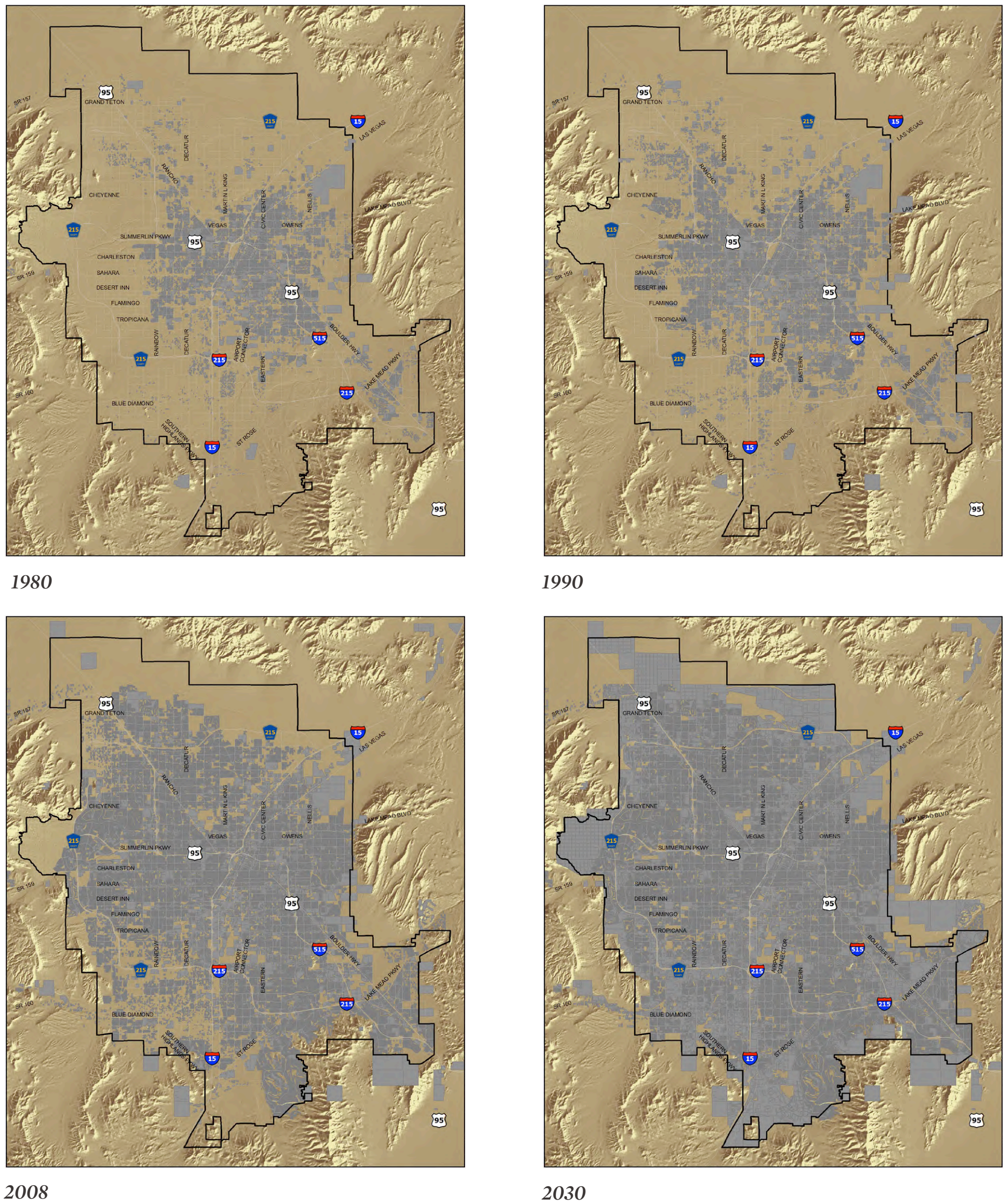
When disagreement is strong enough, house prices increase more in these nearly developed cities than in a fully developed city. In the nearly developed cities, the extreme optimistic beliefs of land speculators determine house prices, amplifying the house price boom. Prices remain more stable in the fully developed city because they reflect the average belief. This result explains the puzzling house price booms in elastic areas that motivate this paper. Supply conditions in these places—elastic current supply, inelastic long-run supply—lead disagreement to have the largest possible amplification effect on a house price boom.

Our theory differs from several other explanations for the strong house price booms that

[3]Las Vegas provides a particularly clear illustration of our model. The ample raw land available in the short-run allowed Las Vegas to build more houses per capita than any other large city in the U.S during the boom. At the same time, speculation in the land markets caused land prices to quadruple between 2000 and 2006, rising from $150,000 per acre to $650,000 per acre, and then lose those gains. This in turn led to a boom and bust in house prices. The high price of $150,000 for desert land before the boom and after the bust demonstrates the binding nature of the city's long-run development constraint. A *New York Times* article published in 2007 cites investors who believed the remaining land would be fully developed by 2017 (McKinley and Palmer, 2007). The dramatic rise in land prices during the boom resulted from optimistic developers taking large positions in the land market. In a striking example of supply-side speculation, a single land development fund, Focus Property Group, outbid all other firms in every large parcel land auction between 2001 and 2005 conducted by the federal government in Las Vegas, obtaining a 5% stake in the undeveloped land within the barrier. Focus Property Group declared bankruptcy in 2009.

FIGURE 1

Long-Run Development Constraints in Las Vegas



Notes: This figure comes from Page 51 of the Regional Transportation Commission of Southern Nevada's *Regional Transportation Plan 2009-2035* (RTCSNV, 2012). The first three pictures display the Las Vegas metropolitan area in 1980, 1990, and 2008. The final picture represents the Regional Transportation Commission's forecast for 2030. The boundary is the development barrier stipulated by the Southern Nevada Public Land Management Act. The shaded gray region denotes developed land.

occurred in elastic areas between 2000 and 2006. One possibility is that these cities experienced much larger demand shocks than the rest of the United States.[4] Our analysis assumes a constant demand shock across cities; the heterogeneity in city house prices booms results entirely from differences in supply conditions. An additional possibility is that uncertainty increased land values due to the embedded option to develop land with different types of housing (Titman, 1983; Grenadier, 1996), and that this option value increase was largest in cities with an intermediate amount of land. In our model, all housing is identical, so this option does not exist. A final explanation is that developers hoarded land to gain monopoly power, and the incentive to do so was strongest in cities about to run out of land. This effect does not appear in our model because homebuilding is perfectly competitive, as is the case empirically at the metro-area level.[5] Unlike these stories, our approach explores the cross-sectional implications of disagreement, an understudied aspect of housing cycles for which we provide direct evidence.

In addition to explaining the city-level cross-section, our model offers new predictions on the cross-section of neighborhoods within a city. We allow some residents to prefer renting over owner-occupancy, so that both rental and owner-occupied housing exist in equilibrium. During periods of disagreement, optimistic speculators hold the rental housing, just as they hold land. Prices appear more optimistic, and hence house price booms are larger, in neighborhoods where a greater share of housing is rented. This prediction matches the data: house prices increased more from 2000 to 2006 in neighborhoods where the share of rental housing in 2000 was higher.

A long literature in macroeconomics and finance has studied how prices aggregate information. When markets are complete and investors share a common prior, prices usually are efficient and reflect the information of all market participants (Fama, 1970; Grossman, 1976; Hellwig, 1980). Our paper sits among a body of work showing that prices reflect only a limited and potentially biased subset of information when investors persistently disagree with each other, and markets are incomplete. Many of these papers focus on strategic considerations that arise in this setting, and the implications for asset prices (Harrison and Kreps, 1978; Scheinkman and Xiong, 2003). A related literature, starting with Miller (1977), demonstrates that prices can be biased even in the absence of strategic considerations because optimists end up holding the asset.[6] We show that this optimism bias is strongest

[4]For instance, the expansion of credit described by Mian and Sufi (2009) may have been largest in these cities. Alternatively, historical increases in house prices in nearby areas may have spread to these cities, either through behavioral contagion (DeFusco et al., 2013) or long-distance gentrification (Guerrieri, Hartley and Hurst, 2013).

[5]Somerville (1999) demonstrates the high level of homebuilder competition at the metro-area level, although he points out that construction is less competitive at the neighborhood level. Hoberg and Phillips (2010) argue that price booms often occur in competitive industries because firms mistakenly believe they will obtain future monopoly power.

[6]In these papers, all market participants are fundamental investors who ignore other investors' beliefs

in housing markets when land is plentiful or when much of the housing stock is rented. In contrast, prices aggregate beliefs well in cities where the housing stock is fixed and owner-occupied. In these areas, house prices reflect the average of all resident beliefs, even though they are agreeing to disagree and short-selling housing is impossible.

The paper proceeds as follows. In Section 2, we document the puzzling aspects of the cross-section of the U.S. housing boom, as well as the importance of supply-side speculation in land markets. Section 3 models the housing market environment. Section 4 contains our analysis of how house prices aggregate beliefs. In Section 5, we derive implications of the model to explain the empirical cross-section of housing markets during the U.S. boom. Section 6 contains new predictions on the cross-section of neighborhoods within a city, and Section 7 concludes.

2 Stylized Facts of the U.S. Housing Boom and Bust

2.1 The Cross-Section of Cities

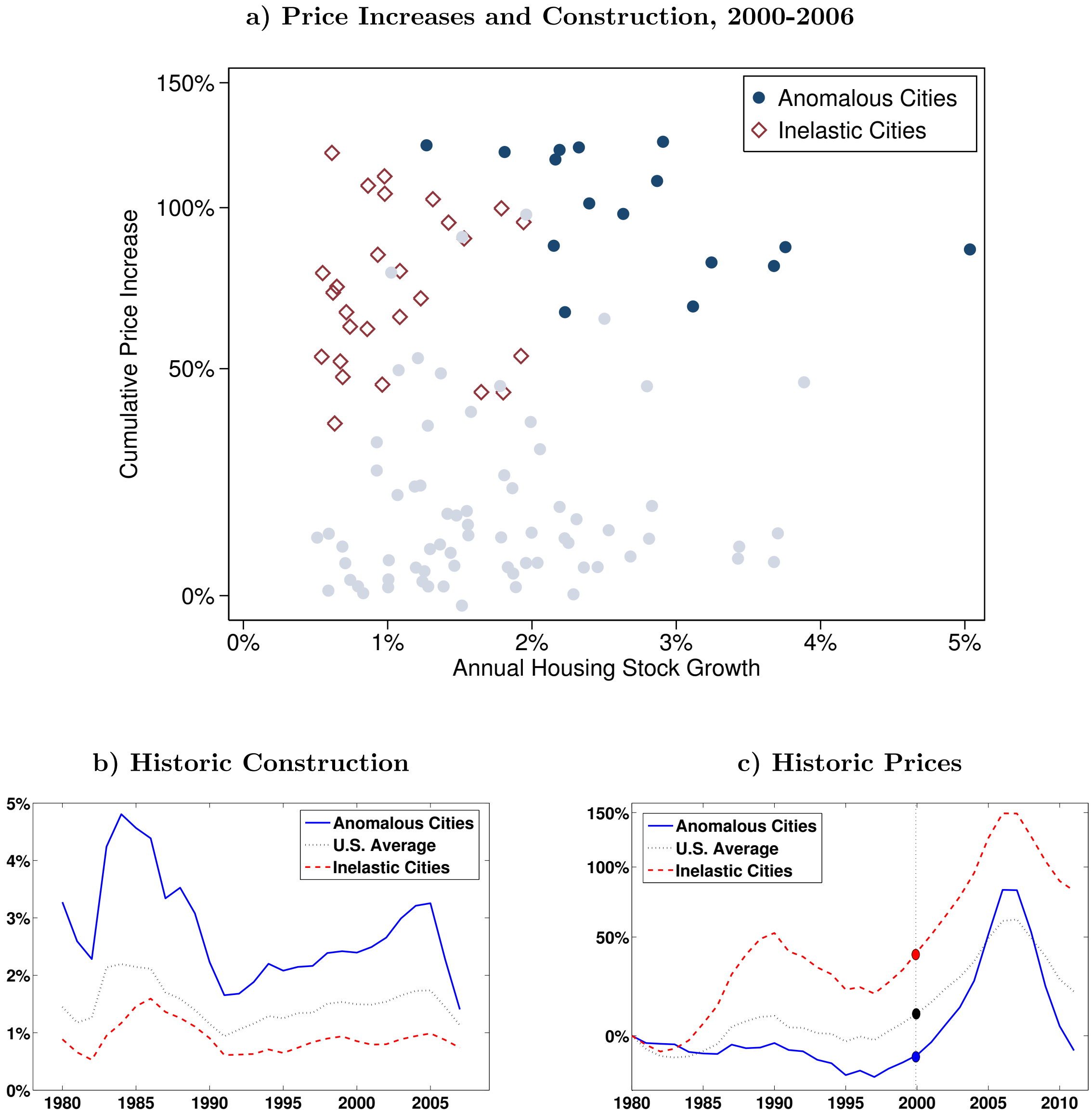
The Introduction mentions three puzzles about the cross-section of city experiences during the boom. First, large house price booms occurred in elastic cities where new construction historically had kept prices low. Second, the price booms in these elastic areas were as large as, if not larger than, those happening in inelastic cities at the same time. Finally, house prices remained flat in other elastic cities that were also rapidly building housing.

We document these puzzles using city-level house price and construction data. House price data come from the Federal Housing Finance Agency's metropolitan statistical area quarterly house price indices. We measure the housing stock in each city at an annual frequency by interpolating the U.S. Census's decadal housing stock estimates with its annual housing permit figures. Throughout, we focus on the 115 metropolitan areas for which the population in 2000 exceeds 500,000. The boom consists of the period between 2000 and 2006, matching the convention in the literature to use 2006 as the end point (Mian, Rao and Sufi, 2013).

Figure 2(a) plots construction and house price increases across cities during the boom. The house price increases vary enormously across cities, ranging from 0% to 125% over this brief six-year period. The largest price increases occurred in two groups of cities. The first group, which we call the Anomalous Cities, consists of Arizona, Nevada, Florida, and inland

(Chen, Hong and Stein, 2002; Geanakoplos, 2009; Hong and Sraer, 2012; Simsek, 2013*a*,*b*). Pástor and Veronesi (2003, 2009) also study environments in which investors care only about long-run fundamentals during booms and busts, but their focus is on learning, and all investors agree as they are all identical. Piazzesi and Schneider (2009) and Burnside, Eichenbaum and Rebelo (2013) also apply models of disagreement to the housing market. Papers in which strategic behavior matters include Abreu and Brunnermeier (2003), Allen, Morris and Shin (2006), and Hong, Scheinkman and Xiong (2006).

FIGURE 2
The U.S. Housing Boom and Bust Across Cities



Notes: Anomalous Cities include those in Arizona, Nevada, Florida, and inland California. Inelastic Cities are Boston, Providence, New York, Philadelphia, and all cities on the west coast of the United States. We measure the housing stock in each city at an annual frequency by interpolating the U.S. Census's decadal housing stock estimates with its annual housing permit figures. House price data come from the second quarter FHFA house price index deflated by the CPI-U. The figure includes all metropolitan areas with populations over 500,000 in 2000 for which we have data. **(a)** The cumulative price increase is the ratio of the house price in 2006 to the house price in 2000. The annual housing stock growth is the log difference in the housing stock in 2006 and 2000 divided by six. **(b), (c)** Each series is an average over cities in a group weighted by the city's housing stock in 2000. Construction is annual permitting as a fraction of the housing stock. Prices represent the cumulative returns from 1980 on the housing in each group.

California. The other large price booms happened in the Inelastic Cities, which comprise Boston, Providence, New York, Philadelphia, and the west coast of the United States.

The history of construction and house prices in the Anomalous Cities before 2000 constitute the first puzzle. As shown in Figures 2(b) and 2(c), from 1980 to 2000 these cities provided clear examples of elastic housing markets in which prices stay low through rapid construction activity. Construction far outpaced the U.S. average while house prices remained constant. The standard model of housing cycles would have predicted the surge in U.S. housing demand between 2000 and 2006 to increase construction in these cities but not to raise prices. Empirically, the shock did increase construction, as shown in Panel (b). The puzzle is that house prices rapidly increased as well.

The second puzzle is that the price increases in the Anomalous Cities were as large as those in the Inelastic Cities. The Inelastic Cities consist of markets where house prices rise because regulation prohibits construction from absorbing higher demand. We document this relationship in Panels (b) and (c) of Figure 2, which show that construction in these cities was lower than the U.S. average before 2000 while house price growth greatly exceeded the U.S. average. The standard housing cycle model would have predicted the Inelastic Cities to lead the nation in house price growth in the boom after 2000. Although house prices did sharply rise, the price increases in the Inelastic Cities were no larger than those in the Anomalous Cities where the boom led to rapid construction.

The final puzzle is that some elastic cities built housing quickly during the boom but, unlike the Anomalous Cities, experienced stable house prices. These cities appear in the bottom-right corner of Figure 2(a), and are located in the southeastern United States (e.g. Texas and North Carolina). Their construction during the boom quantitatively matches that in the Anomalous Cities, but the price changes are significantly smaller. Why was rapid construction able to hold down house prices in some cities and not others?

One response to these three puzzles is that the Anomalous Cities simply experienced much larger demand shocks than the rest of the nation during the boom. Although differential demand shocks surely explain part of the cross-section, they cannot account for all aspects of the Anomalous Cities just documented. These cities had been experiencing abnormally large demand shocks for years before 2000. Figure 2(b) shows that they were some of the fastest growing cities in the United States. Yet the surging demand to live in these areas did not increase prices. The departure from this pattern after 2000 requires a more nuanced theory than the hypothesis that housing demand increased particularly strongly in the Anomalous cities during the boom.

2.2 The Central Importance of Land Prices

This paper argues that speculation in land markets explains the variation in the house price boom across cities just documented. Our model demonstrates that land market speculation amplifies house price increases by making prices look more optimistic, and that this amplification is strongest in areas at the same level of development as the Anomalous Cities. In our framework, all movements in house prices arise from changes in land prices that reflect optimistic beliefs. Matching this premise, land price increases empirically account for nearly all of the increase in house prices during the boom, as we now show.

Tracing house price increases to land prices distinguishes our argument from "time-to-build" theories. According to the time-to-build hypothesis, house prices rise during a boom because of a temporary failure of homebuilders to expand construction. This delivery lag derives from obstacles erected by local regulators or from temporary shortages of inputs such as drywall and skilled labor. Under this theory, the price of undeveloped land should remain constant during the boom. Because land prices reflect the long-run, temporary housing shortages have no effect on the price of undeveloped land. These shortages instead raise construction costs and the shadow price of regulatory building permission.

To assess the importance of land prices, we gather data on land prices and construction costs at the city level. Data on land prices come from the indices developed by Nichols, Oliner and Mulhall (2013) using land parcel transaction data. They run hedonic regressions to control for parcel characteristics and then derive city-level indices from the coefficients on city-specific time dummies. We measure construction costs using the R.S. Means construction cost survey. This survey asks homebuilders in each city to report the marginal cost of building a square foot of housing, including all labor and materials costs. Survey responses reflect real differences across cities in construction costs. In 2000, the lowest cost is \$54 per square foot and the highest is \$95; the mean is \$67 per square foot and the standard deviation is \$9.

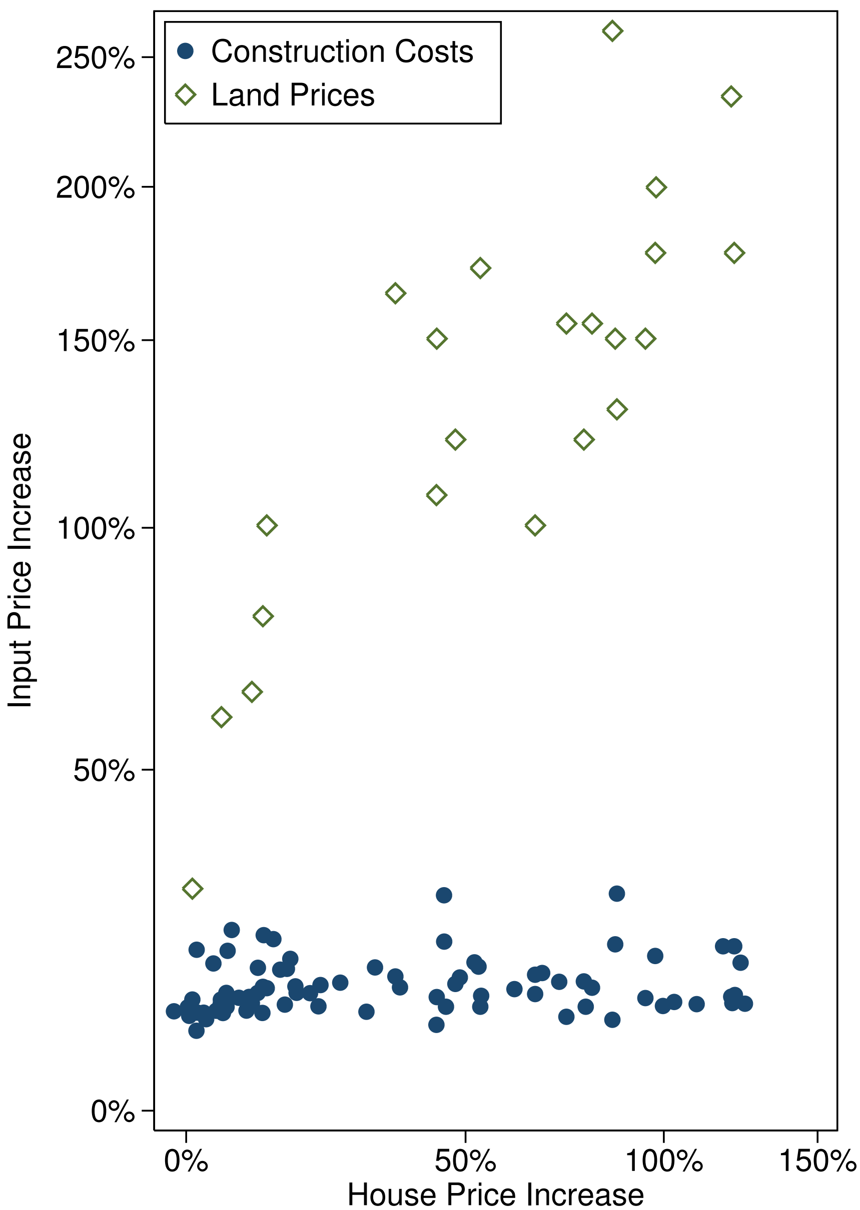
Competition among homebuilders implies that, when construction is positive, house prices must equal land prices plus construction costs: $p_t^h = p_t^l + K_t$. Log-differencing this equation between 2000 and 2006 yields

$$\Delta \log p^h = \alpha \Delta \log p^l + (1-\alpha)\Delta \log K,$$

where Δ denotes the difference between 2000 and 2006 and α is land's share of house prices in 2000. The factor that matters more should vary more closely with house prices across cities. Because α and $1-\alpha$ are less than 1, the critical factor should also rise more than house prices do.

Figure 3 plots for each city the real growth in construction costs and land prices between

FIGURE 3
Input Price and House Price Increases Across Cities, 2000-2006



Notes: We measure construction costs for each city using the R.S. Means survey figures for the marginal cost of a square foot of an average quality home, deflated by the CPI-U. Gyourko and Saiz (2006) contains further information on the survey. Land price changes come from the hedonic indices calculated in Nichols, Oliner and Mulhall (2013) using land parcel transactions, and house prices come from the second quarter FHFA housing price index deflated by the CPI-U. The figure includes all metropolitan areas with populations over 500,000 in 2000 for which we have data.

2000 and 2006 against the corresponding growth in house prices. Construction costs rose relatively little during this period, and growth in these costs does not vary in relation to the size of house price increases. Land prices display the opposite pattern, rising substantially during the boom and exhibiting a high correlation with house prices. Each city's land price increase also exceeds its house price increase. This evidence underscores the central importance of land prices for understanding the cross-section of house price booms.

2.3 Land Market Speculation by Homebuilders

The land price booms just documented were driven by speculation in land markets. The term "speculation" refers to the process in which optimists buy up an asset that cannot be shorted, biasing its price. Our model describes two implications of this behavior. First, the owners of the land during the boom increase their positions as they crowd out less optimistic landowners. Second, when their beliefs are revealed to be more optimistic than reality, optimists suffer capital losses. We document both of these features among a class of landowners for whom rich data are publicly available: public homebuilders. We focus on the eight largest firms and hand-collect landholding data from their annual financial statements between 2001 and 2010.

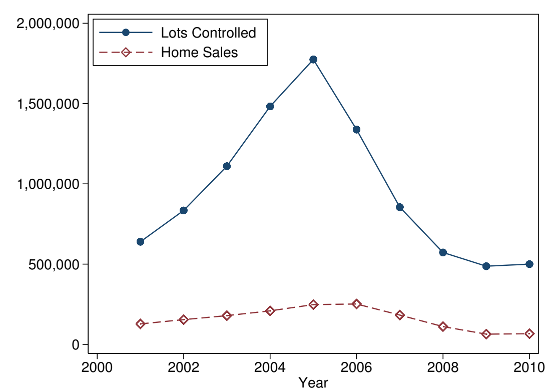
Consistent with speculative behavior, these firms nearly tripled their landholdings between 2001 and 2005, as shown in Figure 4(a). These land acquisitions far exceed additional land needed for new construction. Annual home sales increased by 120,000 between 2001 and 2005, while landholdings increased by 1,100,000 lots. One lot can produce one house, so landholdings rose more than *nine times* relative to home sales. In 2005, Pulte changed the description of its business in its 10-K to say, "We consider land acquisition one of our core competencies." This language appeared until 2008, when it was replaced by, "Homebuilding operations represent our core business."

Having amassed large land portfolios, these firms subsequently suffered large capital losses. Figure 4(b) documents the dramatic rise and fall in the total market equity of these homebuilders between 2001 and 2010. Homebuilder stocks rose 430% and then fell 74% over this period. The majority of the losses borne by homebuilders arose from losses on the land portfolios they accumulated from 2001 to 2005. In 2006, these firms began reporting write-downs to their land portfolios. At \$29 billion, the value of the land losses between 2006 and 2010 accounts for 73% of the market equity losses over this time period. The homebuilders bore the entirety of their land portfolio losses. The absence of a hedge against downside risk supports the theory that homebuilder land acquisitions represented their optimistic beliefs.

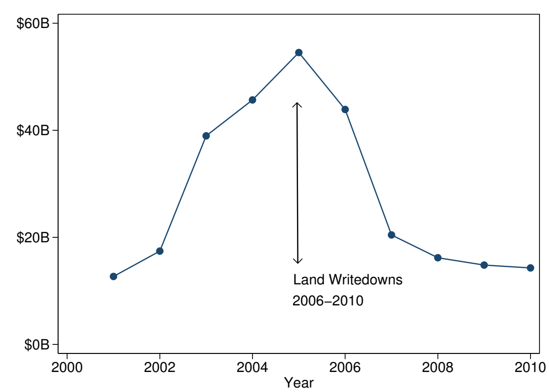
Further evidence of homebuilder optimism comes from short-selling of their market equity. If the homebuilders buying land are more optimistic than most investors, then other investors should bet against them by shorting their stock. Figure 4(c) plots monthly short interest

FIGURE 4
Supply-Side Speculation Among U.S. Public Homebuilders, 2001-2010

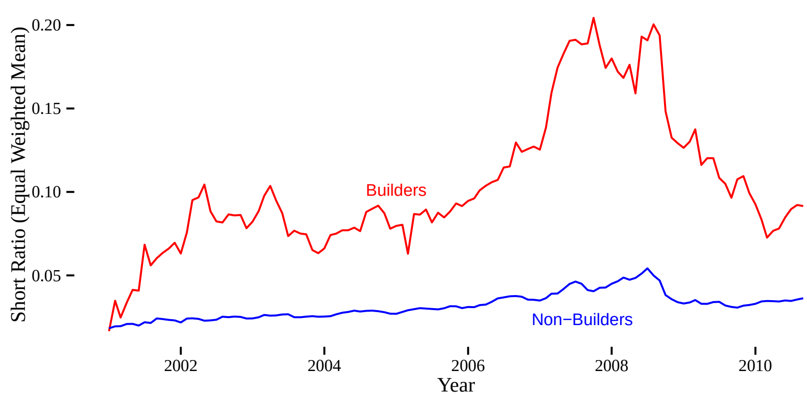
a) Land Holdings and Home Sales



b) Market Equity



c) Short Interest



Notes: **(a), (b)** Data come from the 10-K filings of Centex, Pulte, Lennar, D.R. Horton, K.B. Homes, Toll Brothers, Hovnanian, and Southern Pacific, the eight largest public U.S. homebuilders in 2001. "Lots Controlled" equals the sum of lots directly owned and those controlled by option contracts. The cumulative writedowns to land holdings between 2006 and 2010 among these homebuilders totals $29 billion. **(c)** Short interest is computed as the ratio of shares currently sold short to total shares outstanding. Monthly data series for shares short come from COMPUSTAT and for shares outstanding come from CRSP. Builder stocks are classified as those with NAICS code 236117.

ratios, defined as the ratio of shares currently sold short to total shares outstanding, for homebuilder stocks and non-homebuilder stocks between 2001 and 2010. Throughout the boom, short interest of homebuilder stock sharply increased, rising from 2% in 2001 to 12% in 2006. It further increased as homebuilders began to announce their land losses in 2006. Rising short interest provides direct evidence of disagreement over the value of homebuilder land portfolios and thus over future house prices.

3 A Housing Market with Homeowners and Developers

Housing Supply. The city we study has a fixed amount of space S. This space can either be used for housing, or it remains as undeveloped land. The total housing stock in the city at time t is H_t and the remaining undeveloped land is L_t, so $S = H_t + L_t$ for all t.

A continuum of real estate developers invest in land and construct housing from the land at a cost of K per unit of housing. The aggregate supply of new housing is ΔH_t. Construction is instantaneous, and housing does not depreciate: $H_t = \Delta H_t + H_{t-1}$. Construction is also irreversible: $\Delta H_t \geq 0$. Both housing and land are continuous variables, and one unit of housing requires one unit of land.

The developers rent out land on spot markets at a price of r_t^l. Rental demand for undeveloped land comes from firms, such as farms, that use the city's land as an input. These firms buy their inputs and sell their products on the global market. Therefore, their aggregate demand for land depends only on r_t^l and not on any other local market conditions. This aggregate rental demand curve is $D^l(r_t^l)$, where $D^l(\cdot)$ is decreasing positive function such that $D^l(0) \geq S$.

The profit flow of a developer j at time t is

$$\pi_{j,t} = \underbrace{r_t^l L_{j,t} + p_t^l (L_{j,t-1} - L_{j,t})}_{\text{development profit}} + \underbrace{(p_t^h - p_t^l - K)\Delta H_{j,t}}_{\text{homebuilding profit}}, \tag{1}$$

where p_t^h is the price of housing and p_t^l is the price of land. The real estate development industry faces no entry costs, so the industry is perfectly competitive. Because homebuilding is instantaneous and does not depend on prior land investments, profits from this line of business must be zero due to perfect competition. We denote the aggregate homebuilding profit by $\pi_t^{hb} = (p_t^h - p_t^l - K)\Delta H_t$.

Each developer begins with a land endowment and issues equity to finance its land investments. It maximizes its expected net present value of profits $\mathbf{E}_j \sum_{t=0}^{\infty} \beta^t \pi_{j,t}$. The operator $\mathbf{E}_j$ reflects firm j's expectation of future land prices. Firm-specific beliefs represent the beliefs of the firm's CEO, who owns equity, cannot be fired, and decides the firm's land investments. The number of each developer's equity shares equals the amount of land it holds, and each developer pays out its land rents as dividends. The market price of developer equity therefore equals the market price p^l_t of land.

Individual Housing Demand. A population of residents live in the city and hold its housing. These residents receive direct utility from consuming housing. Lower-case h denotes the flow consumption of housing, whereas upper-case H denotes the asset holding. Flow utility from housing depends on whether housing is consumed through owner-occupancy or under a rental contract. Residents also derive utility from non-local consumption c. Each resident i maximizes the expected present value of utility, given by

$$\mathbf{E}_i \sum_{t=0}^{\infty} \beta^t u_i(c_t, h^{own}_t, h^{rent}_t),$$

where β is the common discount factor.

Flow utility $u_i(\cdot,\cdot,\cdot)$ has three properties. First, it is separable and linear in non-real estate consumption c. This quasi-linearity eliminates risk aversion and hedging motives. Second, owner-occupied and rented housing are substitutes, and residents vary in which type of contract they prefer and to what degree. Substitutability of owner-occupied and rented housing fully sorts residents between the two types of contracts; no resident consumes both types of housing simultaneously. Finally, residents face diminishing marginal utility of owner-occupied housing. This property leads homeownership to be dispersed among residents in equilibrium.

The utility specification we adopt that features these three properties is

$$u_i(c, h^{own}, h^{rent}) = c + v(a_i h^{own} + h^{rent}) \tag{2}$$

where $a_i > 0$ is resident i's preference for owner-occupancy, and $v(\cdot)$ is an increasing, concave function for which $\lim_{h\to 0} v'(h) = \infty$. The distribution of the owner-occupancy preference parameter a_i across residents is given by a continuously differentiable cumulative distribution function F_a, which is stable over time. Owner-occupancy utility is unbounded: dF_a has full support on $\mathbb{R}^+$. The functional form of the owner-occupancy preference in (2) results from a moral hazard problem we describe in the Appendix.

Resident Optimization. Residents hold three assets classes: bonds B, housing H, and developer equity Q. Global capital markets external to the city determine the gross interest rate on bonds, which is $R_t = 1/\beta$, where β is the common discount factor. Residents may borrow or lend at this rate by buying or selling these bonds in unlimited quantities.

In contrast, housing and developer equity are traded within the city, and equilibrium conditions determine their prices p_t^l and p_t^h. Homeowners earn income by renting out the housing they own in excess of what they consume. The spot rental price for housing is r_t^h; landlord revenue is therefore $r_t^h(H_{i,t} - h_{i,t}^{own} - h_{i,t}^{rent})$. Shorting housing is impossible, but residents can short developer equity. Doing so is costly. Residents incur a convex cost $k_s(Q)$ to short Q units of developer stock, where $k_s(0) = 0$ and $k_s', k_s'' > 0$. These costs reflect fees paid to borrow stock, as well as time spent locating available stock (D'Avolio, 2002).

Short-sale constraints in the housing market result from a lack of asset interchangeability. Although housing is homogeneous in the model, empirical housing markets involve large variation in characteristics across houses. This variation in characteristics makes it essentially impossible to cover a short. Unlike in the housing market, asset interchangeability holds in the equity market, where all of a firm's shares are equivalent.

The Bellman equation representing the resident optimization problem is

$$V(B_{i,t-1}, H_{i,t-1}, Q_{i,t-1}) = \max_{\substack{B_{i,t}, H_{i,t}, Q_{i,t} \\ c_{i,t}, h_{i,t}^{own}, h_{i,t}^{rent}}} \underbrace{c_{i,t} + v(a_i h_{i,t}^{own} + h_{i,t}^{rent})}_{\text{flow utility}} + \underbrace{\beta \mathbf{E}_{i,t} V(B_{i,t}, H_{i,t}, Q_{i,t})}_{\text{continuation value}}, \tag{3}$$

where the maximization is subject to the short-sale constraint

$$0 \leq H_{i,t},$$

the ownership constraint

$$h_{i,t}^{own} \leq H_{i,t},$$

and the budget constraint

$$\begin{aligned} \underbrace{R_t B_{i,t-1} - B_{i,t}}_{\text{borrowing costs}} + \underbrace{c_{i,t}}_{\text{consumption}} \leq \; & \underbrace{p_t^h(H_{i,t-1} - H_{i,t})}_{\text{housing returns}} + \underbrace{r_t^h(H_{i,t} - h_{i,t}^{own} - h_{i,t}^{rent})}_{\text{housing rental income}} \\ & + \underbrace{p_t^l(Q_{i,t-1} - Q_{i,t})}_{\text{equity returns}} + \underbrace{r_t^l Q_{i,t}}_{\text{dividends}} - \underbrace{\max(0, k_s(-Q_{i,t}))}_{\text{shorting costs}}. \end{aligned}$$

Aggregate Demand and Beliefs. Aggregate demand to live in the city equals the number of residents N_t. This aggregate demand consists of a shock and a trend:

$$\underbrace{\log N_t}_{\text{demand}} = \underbrace{z_t}_{\text{shock}} + \underbrace{\log \overline{N}_t}_{\text{trend}}.$$

The trend component grows at a constant positive rate g: for all $t > 0$,

$$\log \overline{N}_t = g + \log \overline{N}_{t-1}.$$

The shocks z_t have a common factor x. The dependence of the time-t shock on the common factor x is μ_t, so that

$$z_t = \mu_t x.$$

Without loss of generality, $\mu_0 = 1$: the time 0 shock z_0 equals the common factor x. We denote $\boldsymbol{\mu} = \{\mu_t\}_{t\geq 0}$.

At time 0, residents observe the following information: the current and future values of trend demand $\overline{N}_t$, the trend growth rate g, the current demand N_0, the current shock z_0, and the common factor x of the future shocks. They do *not* observe $\boldsymbol{\mu}$, the data needed to extrapolate the factor x to future shocks. Residents learn the true value of the entire vector $\boldsymbol{\mu}$ at time $t = 1$. The resolution of uncertainty at time $t = 1$ is common knowledge at $t = 0$.

Residents agree to disagree about the true value of $\boldsymbol{\mu}$. At time 0, resident i's subjective prior of $\boldsymbol{\mu}$ is given by F_i, an integrable probability measure on the compact space M of all possible values of $\boldsymbol{\mu}$. These priors vary across residents. The resulting subjective expected value of each μ_t is $\mu_{i,t} = \int_M \mu_t dF_i$, and the vector of resident i's subjective expected values of each μ_t is $\boldsymbol{\mu}_i = \{\mu_{i,t}\}_{t\geq 0}$. The subjective expected value $\boldsymbol{\mu}_i$ uniquely determines the prior F_i. The distribution of $\boldsymbol{\mu}_i$ itself across residents admits an integrable probability distribution F_μ on M, which is independent from the distribution F_a of owner-occupancy preferences. The CEOs of the development firms are city residents, so their beliefs are drawn from the same distribution F_μ.

Resident disagreement reflects the unprecedented nature of the demand shock $\mathbf{z}$. As argued by Morris (1996), this heterogeneous prior assumption is most appropriate when investors face an unprecedented situation in which they have not yet had a chance to collect information and engage in rational updating. The events surrounding housing booms are precisely these types of situations. Glaeser (2013) meticulously shows that in each of the historical booms he analyzes, reasonable investors could agree to disagree about future real estate prices. In the case of the U.S. housing boom between 2000 and 2006, we follow Mian and Sufi (2009) in thinking of the shock as the arrival of new securitization technologies that expanded credit to low-income borrowers. The initial shock to housing demand is x, and $\boldsymbol{\mu}$ represents the degree to which this expansion of credit in 2000-2006 persists after 2006.

Equilibrium. Equilibrium consists of time-series vectors of prices $\mathbf{p}^L(\boldsymbol{\mu})$, $\mathbf{p}^H(\boldsymbol{\mu})$, $\mathbf{r}^l(\boldsymbol{\mu})$, $\mathbf{r}^h(\boldsymbol{\mu})$ and quantities $\mathbf{L}(\boldsymbol{\mu})$, $\mathbf{H}(\boldsymbol{\mu})$ that depend on the realized value of $\boldsymbol{\mu}$. These pricing and quantity functions constitute an equilibrium when housing, land, and equity markets clear while residents and developers maximize utility and profits:

Consider pricing functions $\mathbf{p}^h(\boldsymbol{\mu}), \mathbf{p}^l(\boldsymbol{\mu}), \mathbf{r}^h(\boldsymbol{\mu}), \mathbf{r}^l(\boldsymbol{\mu})$ *and quantity functions* $\mathbf{H}(\boldsymbol{\mu})$, $\mathbf{L}(\boldsymbol{\mu})$. *Let* $H^*_{i,t}$, $Q^*_{i,t}$, $(h^{own}_{i,t})^*$, *and* $(h^{rent}_{i,t})^*$ *be resident* i*'s solutions to the Bellman equation* (3) *given his owner-occupancy preference* a_i*, his beliefs* $\boldsymbol{\mu}_i$*, and these pricing functions. Let* $L^*_{j,t}$ *be de-*

veloper j's land holdings that maximize expected net present value of profits in equation (1)*, given the pricing functions;* L_t^* *is the sum of these land holdings across developers. The pricing and quantity functions constitute a* recursive competitive equilibrium *if at each time* t*:*

1. *The sum of undeveloped land and housing equals the city's endowment of open space:*

$$S = L_t(\boldsymbol{\mu}) + H_t(\boldsymbol{\mu}).$$

2. *Flow demand for land equals investment demand from developers, which equals the resident demand for their equity:*

$$L_t(\boldsymbol{\mu}) = L_t^* = D^l(r_t^l(\boldsymbol{\mu})) = \int_0^\infty \int_M Q_{i,t}^* dF_\mu dF_a.$$

3. *Resident stock and flow demand for housing clear:*

$$H_t(\boldsymbol{\mu}) = N_t(\boldsymbol{\mu}) \int_0^\infty \int_M H_{i,t}^* dF_\mu dF_a = N_t(\boldsymbol{\mu}) \int_0^\infty \int_M ((h_{i,t}^{own})^* + (h_{i,t}^{rent})^*) dF_\mu dF_a.$$

4. *Construction maximizes developer profits:*

$$H_t(\boldsymbol{\mu}) - H_{t-1}(\boldsymbol{\mu}) \in \arg\max_{\Delta H_t} \pi_t^{hb}.$$

5. *Developer profit from homebuilding is zero:*

$$\max_{\Delta H_t} \pi_t^{hb} = 0.$$

Elasticity of Housing Supply. The housing supply curve is the city's open space S less the rental demand for land $D^l(r_t^l)$. We denote the elasticity of this supply curve with respect to housing rents r_t^h by ϵ_t^S. The supply elasticity determines the construction response to the shocks $\{z_t\}$. It will also serve as a sufficient statistic for the extent to which land speculation affects house prices. This section describes the supply elasticity ϵ_t^S along the city's trend growth path, which obtains when $x = 0$.

The relationship between land rents r_t^l and house rents r_t^h allows us to calculate this elasticity. Because trend growth $g > 0$, new residents perpetually arrive to the city, and developers build new houses each period. Perpetual construction ties together land and house prices. In particular, as developers must be indifferent between building today or tomorrow, house rents equal land rents plus flow construction costs:

$$r_t^h = r_t^l + (1-\beta)K.$$

The supply of housing is open space net of flow land demand: $S - D^l(r_t^h - (1-\beta)K)$. The elasticity of housing supply is thus $\epsilon_t^S \equiv -r_t^h(D^l)'/(S-D^l)$. When the flow land demand D^l features a constant elasticity ϵ^l, the elasticity of housing supply takes on the simple form

$$\epsilon_t^S = \frac{r_t^h}{r_t^h - (1-\beta)K}\left(\frac{S}{H_t} - 1\right)\epsilon^l, \tag{4}$$

where H_t is the housing stock at time t. The arrival of new residents increases both rents r_t^h and the level of development H_t/S. The supply elasticity given in (4) unambiguously falls (see Appendix for proof):

Lemma 1. *Define* housing supply *to be the residual of the city's open space S minus the flow demand for land: $S - D^l$. The elasticity ϵ_t^S of housing supply with respect to housing rents r_t^h decreases with the level of city development H_t/S along the city's trend growth path.*

4 Supply-Side Speculation

At time 0, residents disagree about the future path of housing demand. Speculative trading behavior results from this disagreement. This section describes how owner-occupancy frictions crowd speculators out of owner-occupied housing and into rental housing and land. Demand and supply elasticities determine how prices aggregate the beliefs of owner-occupants and of optimistic speculators holding rental housing and land.

4.1 Land Speculation and Dispersed Homeownership

We first consider the developer decision to hold land at time 0. Developer j's first-order condition on its land-holding $L_{j,0}$ is

$$\underbrace{1/\beta}_{\text{risk-free rate}} \quad \geq \quad \underbrace{\mathbf{E}_j p_1^l/(p_0^l - r_0^l)}_{\text{expected land return}} \quad ,$$

with equality if and only if $L_{j,0} > 0$. A developer invests in land if and only if it expects land to return the risk-free rate. At time 0, developers disagree about this expected return on land because they disagree about the future path of housing demand. The developers that expect the highest returns invest in land, while all other developers sell to these optimistic firms and exit the market. We denote the optimistic belief of the developers who invest in land by $\widetilde{\mathbf{E}}p_1^l \equiv \max_{\boldsymbol{\mu}_j} \mathbf{E}(p_1^l \mid \boldsymbol{\mu}_j)$.

Optimistic residents finance developer investments in land through purchasing their equity. Less optimistic residents choose to short-sell developer stock. Developer stock allows residents to hold land indirectly: its price is p_0^l and it pays a dividend of r_0^l. Resident i holds this equity only if he agrees with the land valuation of the optimistic developers, in which case $\mathbf{E}_i p_1^l = \widetilde{\mathbf{E}}p_1^l$. Otherwise, he shorts the equity, and his first-order condition is

$$k_s'(-Q_{i,0}^*) = \beta(\widetilde{\mathbf{E}}p_1^l - \mathbf{E}_i p_1^l).$$

Disagreement increases the short interest in the equity of the developers holding the land. Without disagreement, $\widetilde{\mathbf{E}}p_1^l = \mathbf{E}_i p_1^l$ for all residents, so no one shorts.

Only the most optimistic residents hold housing as landlords. A resident is a *landlord* if he owns more housing than he consumes through owner-occupancy: $H_i > h_i^{own}$. The first-order condition of the Bellman equation (3) with respect to $H_{i,0}$ when it is in excess of $h_{i,0}^{own}$ is

$$\underbrace{1/\beta}_{\text{risk-free rate}} \quad \geq \quad \underbrace{\mathbf{E}_i p_1^h/(p_0^h - r_0^h)}_{\text{expected housing return}} \quad , \tag{5}$$

with equality if and only if $H_{i,0} > h_{i,0}^{own}$. Only the most optimistic residents invest in rental housing, just as only the most optimistic developers invest in land. Land and rental housing share this fundamental property. During periods of uncertainty, the most optimistic investors are the sole holders of these asset classes.

Owner-occupancy utility crowds these optimistic investors out of owner-occupied housing, which remains dispersed among residents of all beliefs. The decision to own or rent emerges from the first-order conditions of the Bellman equation (3) with respect to $h_{i,0}^{own}$ and $h_{i,0}^{rent}$. We express these equations jointly as

$$\underbrace{v'(a_i(h_{i,0}^{own})^* + (h_{i,0}^{rent})^*)}_{\text{marginal utility of housing}} \quad = \min\left(\underbrace{a_i^{-1}(p_0^h - \beta\mathbf{E}_i p_1^h)}_{\text{owning}}, \underbrace{r_0^h}_{\text{renting}}\right). \tag{6}$$

The left term in the parentheses denotes the expected flow price of marginal utility v' from owning a house; the right term denotes the flow price of renting. A resident owns when the owner-occupancy price is less than the rental price. As long as the owner-occupancy preference a_i is large enough, resident i decides to own even if his belief $\mathbf{E}_i p_1^h$ is quite pessimistic. Homeownership remains dispersed among residents of all beliefs.

We gain additional intuition about the own-rent margin by substituting (5) into (6). We denote the most optimistic belief about future house prices, the one held by landlords investing in rental housing, by $\widetilde{\mathbf{E}}p_1^h \equiv \max_{\boldsymbol{\mu}_i} \mathbf{E}(p_1^h \mid \boldsymbol{\mu}_i)$. The decision to own rather than rent reduces to

$$a_i \geq 1 + \frac{\beta(\widetilde{\mathbf{E}}p_1^h - \mathbf{E}_i p_1^h)}{r_0^h}. \tag{7}$$

Without disagreement, a resident owns exactly when he intrinsically prefers owning to renting, so that $a_i \geq 1$. Disagreement sets the bar higher. Some pessimists for whom $a_i \geq 1$ choose to rent because they expect capital losses on owning a home. Other pessimists continue to own because their owner-occupancy utility is high enough to offset the fear of capital losses. Proposition 1 summarizes these results.

Proposition 1. *Owner-occupancy utility crowds speculators out of the owner-occupied housing market and into the land and rental markets. The most optimistic residents—those holding the highest value of $\mathbf{E}_i p_1^h$—buy up all rental housing and finance optimistic developers who purchase all the land. In contrast, owner-occupied housing remains dispersed among residents of all beliefs.*

Proposition 1 yields two corollaries that match stylized facts presented in Section 2. The most optimistic developers buy up all the land. Unless they start out owning all the land, these optimistic developers increase their land positions following the demand shock. They hold this land as an investment rather than for immediate construction.

Implication 1. *The developers who hold land at time 0 increase their aggregate land holdings at time 0. They buy land in excess of their immediate construction needs.*

This implication explains the land-buying activities of large public U.S. homebuilders documented in Figure 4(a).

The second corollary concerns short-selling. Residents who disagree with the optimistic valuations of developers short their equity.

Implication 2. *Disagreement increases the short interest of developer equity at time 0.*

Figure 4(c) documents the rising short interest in the stocks of U.S. public homebuilders who were taking on large land positions during the boom. This short interest provides direct evidence of disagreement during the boom.

4.2 Belief Aggregation

Prices aggregate the heterogeneous beliefs of residents and developers holding housing and land. The real estate market consists of three components: land, rental housing, and owner-occupied housing. The most optimistic residents hold the first two, while the third remains dispersed among owner-occupants. House prices reflect a weighted average of the optimistic belief and the average belief of all owner-occupants. The weight on the optimistic belief is the share of the real estate market consisting of land and rental housing; the weight on the average owner-occupant belief is owner-occupied housing's share of the market.

To derive these results, we take a comparative static of the form $\partial p_0^h/\partial x$. The shock $\mathbf{z} = \boldsymbol{\mu} x$ scales with the common factor x. We differentiate with respect to x at $x = 0$ to explore how prices change as the shocks, and hence the ensuing disagreement, increase. Our partial derivative holds current demand N_0 constant to isolate the aggregation of future beliefs.

We first use (5) to write $p_0^h = r_0^h + \beta\widetilde{\mathbf{E}}p_1^h$. The shock increases the optimistic belief $\beta\widetilde{\mathbf{E}}p_1^h$, directly increasing prices. It also changes the market rent r_0^h. This rent is determined by the intersection of housing supply and housing demand:

$$\underbrace{S - D^l\left(r_0^h - (1-\beta)K\right)}_{\text{housing supply}} = \underbrace{D_0^h(r_0^h)}_{\text{housing demand}}, \tag{8}$$

where

$$D_0^h(r_0^h) = \underbrace{N_0 \int_M \int_0^{1+\beta(\widetilde{\mathbf{E}}p_1^h - \mathbf{E}_i p_1^h)/r_0^h} (v')^{-1}(r_0^h) dF_a dF_\mu}_{\text{rental housing}} \tag{9}$$
$$+ \underbrace{N_0 \int_M \int_{1+\beta(\widetilde{\mathbf{E}}p_1^h - \mathbf{E}_i p_1^h)/r_0^h}^{\infty} a_i^{-1}(v')^{-1}\left(a_i^{-1}(r_0^h + \beta(\widetilde{\mathbf{E}}p_1^h - \mathbf{E}_i p_1^h))\right) dF_a dF_\mu}_{\text{owner-occupied housing}}.$$

The housing demand equation follows from (6) and (7). We determine the shock's effect on rents by totally differentiating (8) with respect to x at $x = 0$, keeping current demand N_0 constant. When the elasticity of housing demand ϵ^D is constant, the resulting comparative static $\partial p_0^h/\partial x$ adopts the simple form given in the following proposition, which we prove in the Appendix.

Proposition 2. *Consider the partial effect of the shock in which current demand N_0 stays constant but future house price expectations $\mathbf{E}_i p_1^h$ change. The change in house prices averages the changes in the optimistic resident belief and the average belief:*

$$\frac{\partial p_0^h}{\partial x} = \frac{\epsilon_0^S + (1-\chi)\epsilon^D}{\epsilon_0^S + \epsilon^D}\frac{\partial \beta\widetilde{\mathbf{E}}p_1^h}{\partial x} + \frac{\chi\epsilon^D}{\epsilon_0^S + \epsilon^D}\frac{\partial \beta\overline{\mathbf{E}}p_1^h}{\partial x}, \tag{10}$$

where $\widetilde{\mathbf{E}}p_1^h = \max_i \mathbf{E}_i p_1^h$ is the most optimistic belief, $\overline{\mathbf{E}}p_1^h = \int_M \mathbf{E}_i p_1^h dF_\mu$ is the average belief, ϵ_0^S is the elasticity of housing supply at time 0, ϵ^D is the elasticity of housing demand, and

$\chi = \int_0^\infty (h_{i,0}^{own})^* dF_a / H_0$ *is the share of housing that is owner-occupied when* $x = 0$.

The weight on the optimistic belief in Proposition 2 represents the share, on the margin, of the real estate market owned by speculators. The supply elasticity ϵ_0^S represents land, and $(1-\chi)\epsilon^D$ represents rental housing. The remaining $\chi\epsilon^D$ represents owner-occupied housing and is the weight on the average owner-occupant belief. The average owner-occupant belief coincides with the unconditional average belief because at $x = 0$, beliefs and tenure choice are independent.

Proposition 2 yields four corollaries on the difference in belief aggregation across cities and neighborhoods. Prices look more optimistic when the weight $(\epsilon_0^S + (1-\chi)\epsilon^D)/(\epsilon_0^S + \epsilon^D)$ is higher. This ratio is greater when the supply elasticity ϵ_0^S is higher:

Implication 3. *Prices look more optimistic when the housing supply elasticity is higher, i.e. in less developed cities.*

Disagreement reverses the common intuition relating housing supply elasticity and movements in house prices. Elastic supply keeps prices low by allowing construction to respond to demand shocks. But land constitutes a larger share of the real estate market when supply is elastic. Speculators are drawn to the land markets, so elastic supply amplifies the role of speculators in determining prices during periods of disagreement. When supply is perfectly elastic, $\epsilon_0^S = \infty$ and prices reflect only the beliefs of these optimistic speculators:

Implication 4. *When housing supply is perfectly elastic, house prices incorporate only the most optimistic beliefs; they reflect the beliefs of developers and not of owner-occupants.*

Recent research has measured owner-occupant beliefs about the future evolution of house prices.[7] In cities with elastic housing supply, such as the cities motivating this paper, developer rather than owner-occupant beliefs determine prices. Data on the expectations of homebuilders would supplement the research on owner-occupant beliefs to explain prices in these elastic areas.

Prices aggregate beliefs much better when housing supply is perfectly inelastic ($\epsilon_0^S = 0$) and all housing is owner-occupied ($\chi = 1$). In this case, the price change depends only on the average belief $\overline{\mathbf{E}} p_1^h$:

Implication 5. *When the housing stock is fixed and all housing is owner-occupied, prices reflect the average belief about long-run growth.*

In many settings, such as when investor information equals a signal plus mean zero noise, prices reflect all information when they incorporate the average private belief of all investors.

[7] See Landvoigt (2011), Case, Shiller and Thompson (2012), Burnside, Eichenbaum and Rebelo (2013), Soo (2013), Suher (2013), and Cheng, Raina and Xiong (2014).

Owner-occupied housing markets with a fixed housing stock display this property, even though short-selling is impossible and residents persistently disagree. These frictions fail to bias prices because homeownership remains dispersed among residents of all beliefs, due to the utility flows that residents derive from housing.

The weight $(\epsilon_0^S + (1-\chi)\epsilon^D)/(\epsilon_0^S + \epsilon^D)$ on optimistic beliefs is also higher when χ is lower:

Implication 6. *Prices look more optimistic when a greater share of housing is rented.*

Speculators own a greater share of the real estate market when the rental share $1 - \chi$ is higher. Prices bias towards optimistic beliefs in market segments where more of the housing stock is rented.

5 The Cross-Section of City Experiences During the Boom

This section explains three puzzling aspects of the U.S. housing boom that occurred between 2000 and 2006. First, large house price booms occurred in elastic cities where new construction historically had kept prices low. Second, the price booms in these elastic areas were as large as, if not larger than, those happening in inelastic cities at the same time. Finally, house prices remained flat in other elastic cities that were also rapidly building housing.

To explain these cross-sectional facts, we derive a formula for the total effect of the shock $\mathbf{z}$ on house prices. This formula expresses the house price boom as a function of the city's level of development when the shock occurs. Our analysis up to this point has explored the partial effect of how prices aggregate beliefs $\mathbf{E}_i p_1^h$, without specifying how these beliefs are formed. To derive the total effect of the shock, we express the changes in these beliefs in terms of city characteristics and the exogenous demand process. Specifically, we calculate the partial derivative $\partial \log p_0^h / \partial x$ holding all beliefs fixed at $\boldsymbol{\mu}_i = \boldsymbol{\mu}$, and then use Proposition 2 to derive the total effect of the shock x on house prices. As before, we evaluate derivatives at $x = 0$.[8]

At time 0, each resident expects the shock z_t to raise log-demand at time t by $\mu_t x$. The resulting expected change in rents r_t^h depends on the elasticities of supply and demand at time t:

$$\frac{\partial \log \mathbf{E}_0 r_t^h}{\partial x} = \frac{\mu_t}{\epsilon_t^S + \epsilon^D}.$$

This equation follows from price theory. When a demand curve shifts up, a good's price increases by the inverse of the total elasticity of supply and demand. The total effects of

[8] Evaluating derivatives at $x = 0$ describes the model when construction occurs in each period. When x is large enough and the shock $\mathbf{z}$ might mean-revert, a construction stop at $t = 1$ is possible and anticipated by residents at $t = 0$. This feature of housing cycles distracts from our focus on housing booms and how they vary across cities.

the shocks $\{z_t\}$ on the current house price p_0^h follows from aggregating the above equation across all time periods, using the relation $p_0 = \mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t r_t^h$:

$$\frac{\partial \log p_0^h}{\partial x} = \frac{\mu}{\widetilde{\epsilon}^S + \epsilon^D}. \tag{11}$$

The mean persistence of the shock is $\mu = \sum_{t=0}^{\infty} \mu_t \beta^t r_t^h (\epsilon_t^S + \epsilon^D)^{-1} / \sum_{t=0}^{\infty} \beta^t r_t^h (\epsilon_t^S + \epsilon^D)^{-1}$, and $\widetilde{\epsilon}^S$ is the *long-run supply elasticity* given by the weighted harmonic mean of future supply elasticities in the city:

$$\widetilde{\epsilon}^S \equiv -\epsilon^D + \frac{\sum_{t=0}^{\infty} \beta_t r_t^h}{\sum_{t=0}^{\infty} \beta^t r_t^h (\epsilon_t^S + \epsilon^D)^{-1}}.$$

The higher this long-run supply elasticity, the smaller the shock's impact on current house prices, holding $\boldsymbol{\mu}$ fixed.

We now put together the two channels through which the shock changes prices. Equation (11) expresses the price change that results when $\boldsymbol{\mu}$ is known, and (10) describes how prices aggregate residents' heterogeneous beliefs about $\boldsymbol{\mu}$. Proposition 3 states the total effect $d \log p_0^h / dx$, which we formally calculate in the Appendix.

Proposition 3. *The total effect of the shock x on current house prices is*

$$\frac{d \log p_0^h}{dx} = \underbrace{\left(\frac{\epsilon_0^S + (1-\chi)\epsilon^D}{\epsilon_0^S + \epsilon^D} \widetilde{\mu} + \frac{\chi \epsilon^D}{\epsilon_0^S + \epsilon^D} \overline{\mu} \right)}_{\textit{aggregate belief}} \underbrace{\frac{1}{\widetilde{\epsilon}^S + \epsilon^D}}_{\textit{pass-through}}, \tag{12}$$

where ϵ_0^S is the current elasticity of housing supply, $\widetilde{\epsilon}^S$ is the long-run supply elasticity, ϵ^D is the elasticity of housing demand, χ is the share of housing that is owner-occupied, $\widetilde{\mu}$ is the mean persistence of the most optimistic belief about $\boldsymbol{\mu}$, and $\overline{\mu}$ is the mean persistence of the average belief.

The first puzzle (12) explains is how a city with perfectly elastic housing supply can experience a house price boom. Housing supply is perfectly elastic when $\epsilon_0^S = \infty$. In this case, the house price boom is $\widetilde{\mu} x / (\widetilde{\epsilon}^S + \epsilon^D)$. This price increase is positive as long as the *long-run* supply elasticity $\widetilde{\epsilon}^S$ is not also infinite.

Implication 7. *A house price boom occurs in a city where current housing supply is completely elastic, construction costs are constant, and construction is instantaneous. Supply must be inelastic in the future for such a price boom to occur.*

In the Appendix, we prove that a limiting case exists in which $\epsilon_0^S = \infty$ while $\widetilde{\epsilon}^S < \infty$.

A house price boom results from a shock to current demand accompanied by news of future shocks. When supply is inelastic in the long-run, these future shocks raise future rents, and prices rise today to reflect this fact. This price change occurs even if supply is perfectly elastic today, because residents anticipate the near future in which supply will not be able to adjust as easily.

This supply condition—elastic short-run supply, inelastic long-run supply—occurs in cities at an intermediate level of development. Figure 5(a) demonstrates the possible combinations of short-run and long-run supply elasticities in a city. We plot the pass-through $1/(\epsilon^S + \epsilon^D)$; a higher pass-through corresponds to a lower elasticity. Lightly developed cities have highly elastic short-run and long-run supply, and heavily developed cities have inelastic short-run and long-run supply. In the intermediate case, current supply is elastic while long-run supply is inelastic, reflecting the near future of constrained supply.

As we discussed in the Introduction, this theoretical supply condition describes the elastic markets that experienced large house price booms between 2000 and 2006. These cities found themselves in a state of arrested development as the boom began in 2000. Although ample land existed for current construction, long-run barriers constrain their future growth.

The second puzzle (12) explains is why the price booms in these elastic cities were as large as those happening in inelastic cities at the same time. Disagreement amplifies the house price boom the most in exactly these nearly developed elastic cities. The amplification effect of disagreement equals the extent to which optimists bias the price increase given in (12). When owner-occupancy frictions are present ($\chi = 1$), the difference between the price boom under disagreement and under the counterfactual in which all residents hold the average belief $\overline{\mu}$ is

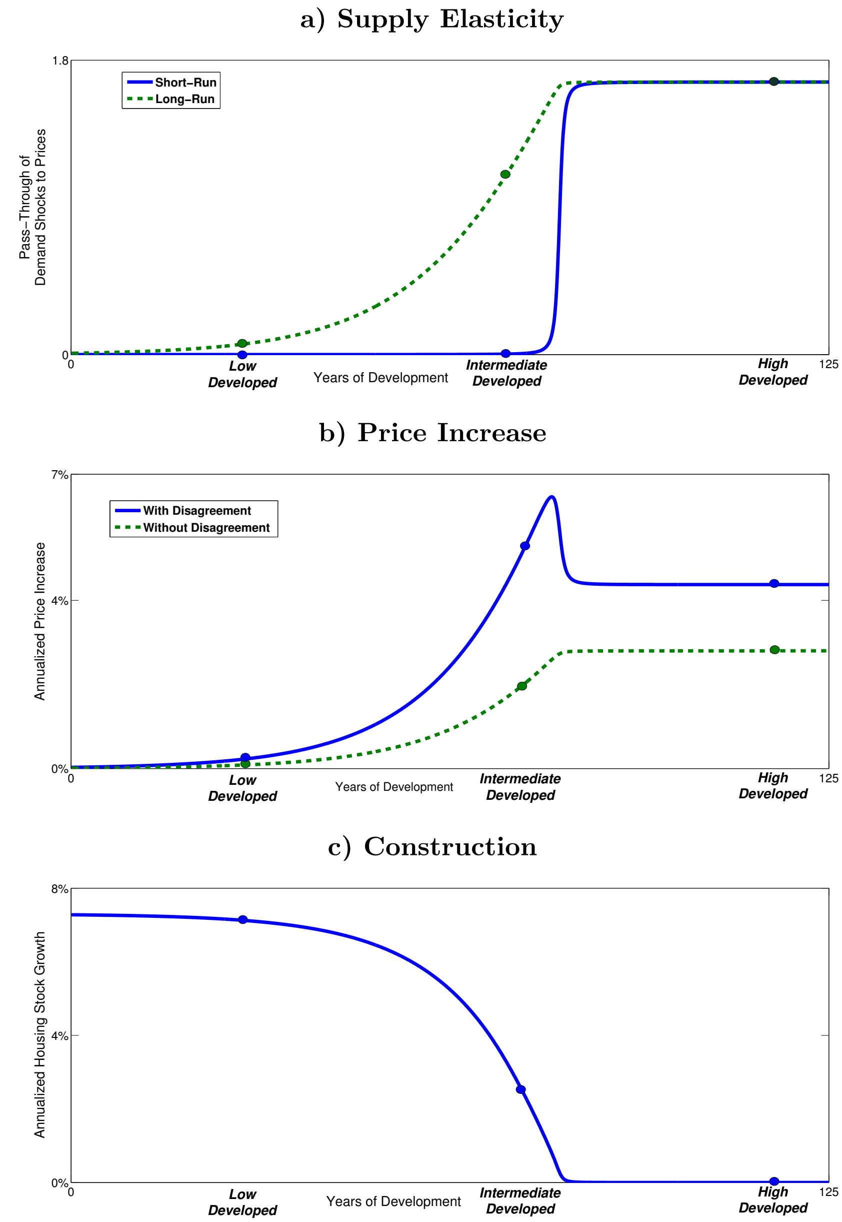
$$\frac{\epsilon_0^S}{\epsilon_0^S + \epsilon^D} \frac{\widetilde{\mu} - \overline{\mu}}{\widetilde{\epsilon}^S + \epsilon^D}.$$

This amplification is largest in nearly developed elastic cities, where ϵ_0^S is large and $\widetilde{\epsilon}^S$ is small. Because this amplification increases in ϵ_0^S and decreases in $\widetilde{\epsilon}^S$, nearly developed elastic cities provide the *ideal* condition for disagreement to amplify a house price boom. Implication 8, which we prove in the Appendix, states this result formally.

Implication 8. *Disagreement amplifies house price booms most in cities at an intermediate level of development, as long as owner-occupancy frictions are large enough. Define Δ to be the difference between the price boom given in* (12) *and the counterfactual in which all residents hold the average belief $\overline{\mu}$. Then there exists $\chi^* < 1$ such that for $\chi^* \leq \chi \leq 1$, Δ is strictly largest at an intermediate level of initial development $\overline{N}_0^* < \infty$.*

Figure 5(b) plots the house price boom given by (12) across different levels of city development, for both the case of disagreement and the case in which all residents hold the average belief. The amplification effect of disagreement is the difference between the two curves. Optimistic speculators amplify the price boom the most in the intermediate city. Highly elastic short-run supply facilitates speculation in land markets, biasing prices towards

FIGURE 5
Model Simulations For Different Cities



Notes: The parameters we use are $\widetilde{\mu} = 1$, $\overline{\mu} = 1$, $x = 0.06$, $g = 0.013$, $\epsilon^D = 1$, $\beta = 0.93^6$, and $\epsilon^l = 1$. We hold the amount of space S fixed and vary the initial trend demand $\overline{N}_0$. The x-axis reports annualized trend demand given by $\log \overline{N}_0/g$. **(a)** Short-run pass-through is $1/(\epsilon_0^S + \epsilon^D)$; long-run pass-through is $1/(\widetilde{\epsilon}^S + \epsilon^D)$. We calculate the rent and housing stock at each level of development using (A1) in the Appendix, and then calculate the supply elasticities using (4). **(b)** Each curve reports the derivative in (12) times x, which we calculate using the elasticities shown in panel (a). The "without disagreement" counterfactual uses $\widetilde{\mu} = \overline{\mu} = 0.2$ instead of $\widetilde{\mu} = 1 > \overline{\mu} = 0.2$. **(c)** We plot the construction equation (A2) using the elasticities shown in panel (a), as well as rents at each stage of development from (A1) and prices at each development stage from $p_0 = \sum_{t=0}^{\infty} \beta^t r_t^h$, which we calculate at $x = 0$.

their optimistic belief. This bias significantly increases house prices because housing supply is constrained in the near future. The optimism bias is smaller in the highly developed city. As a result, price increases in intermediate cities are as large as the price boom in the highly developed areas.

In fact, the price boom in some intermediate cities can *exceed* that in the highly developed cities. The parameters we use in Figure 5(b) generate an example of this phenomenon. This surprising result reverses the conclusion of standard models of housing cycles, in which the most constrained areas always experience the largest price increases. This reversal occurs as long as owner-occupancy frictions are high and the extent of disagreement is sufficiently large:

Implication 9. *If disagreement and owner-occupancy frictions are large enough, then the largest house price boom occurs in a city at an intermediate level of development. There exists $\chi^* < 1$ and $\delta > 0$ such that for $\chi^* \leq \chi \leq 1$ and $\widetilde{\mu} - \overline{\mu} \geq \delta$, the price boom $d \log p_0^h / dx$ is strictly largest at an intermediate level of development $\overline{N}_0^* < \infty$.*

Our model has succeeded in explaining the large house price booms in the elastic cities without arguing that these cities experienced abnormally large housing demand shocks. These markets experienced some of the largest house price booms in the country because of their supply conditions, not in spite of them.

The final puzzle explained by (12) is why large house price booms occurred in some elastic cities but not in others. Elastic cities are those for which $\epsilon_0^S \approx \infty$. As shown in Figure 5(a), these cities differ in their long-run supply elasticity $\widetilde{\epsilon}^S$. When $\widetilde{\epsilon}^S = \infty$, the house price boom $d \log p_0^h / dx = 0$. Prices remain flat because construction can freely respond to demand shocks now and for the foreseeable future. House prices increase in elastic cities only when the development constraint will make construction difficult in the near future.

The elastic American cities which experienced stable house prices between 2000 and 2006 possess characteristics that lead long-run supply to be elastic. These cities, located in Texas and other central American areas, are characterized by flat geography, a lack of future regulation, and homogeneous sprawl (Glaeser and Kahn, 2004; Glaeser and Kohlhase, 2004; Burchfield et al., 2006; Glaeser, Gyourko and Saiz, 2008; Saiz, 2010). These conditions allow the cities to expand indefinitely, leading S to be infinite or very high. Unlimited land leads the elasticity of supply to remain infinite forever, according to (4).

The level of house prices before the shock identifies the difference between the elastic cities that can expand indefinitely and elastic cities that face constraints in the near future. House prices increase with development. Therefore, the elastic cities nearing their development constraints should have higher house prices before the shock than the other elastic cities. The following implication summarizes these results.

Implication 10. *Consider two cities that experience the same demand shock and in which current housing supply is perfectly elastic ($\epsilon_0^S = \infty$). House prices rise more in the city in which the long-run supply elasticity $\widetilde{\epsilon}^S$ is lower. Before the shock occurs, a greater share of the land in this city is already developed, and the level of house prices is higher.*

In practice, calculating a metro-area house price level is difficult because characteristics such as construction costs vary widely within and across metro areas, although valiant attempts have been made (Glaeser and Gyourko, 2005; Davis and Heathcote, 2007; Nichols, Oliner and Mulhall, 2013). With the appropriate data, we would be able to distinguish the low-developed from the medium-developed cities.

We have used (12) to explain the large house price increases in certain elastic housing markets in the U.S. between 2000 and 2006. An additional salient feature of these booms is that they coincided with rapid construction. As we document in Section 2, these cities experienced some of the most intense permitting activity in the nation during this period. Our model captures this phenomenon. Figure 5(c) plots the construction response to the shock in different cities. In cities where current housing supply is elastic , new construction accommodates the shock. The elastic cities include both the lightly developed and intermediate developed areas.

6 Variation in House Price Booms Within Cities

The model also makes predictions on the variation in house price increases within a given city. Optimistic speculators hold rental housing, just as they hold land. Prices appear more optimistic, and hence house price booms are larger, in market segments where a greater share of housing is rented.

This result emerges from (12). Recall that χ is the share of the housing stock that is owner-occupied rather than rented when $x = 0$. It is a sufficient statistic for the distribution F_a of owner-occupancy utility. When χ is larger, the price increase $d \log p_0^h / dx$ is smaller:

$$\frac{\partial}{\partial \chi} \frac{d \log p_0^h}{dx} = -\frac{\epsilon^D}{\epsilon_0^S + \epsilon^D} \frac{\widetilde{\mu} - \overline{\mu}}{\widetilde{\epsilon}^S + \epsilon^D} < 0.$$

This derivative is negative because the optimistic belief $\widetilde{\mu}$ exceeds the average belief $\overline{\mu}$.

A city's housing market consists of a number of market segments, which are subsets of the housing market that attract distinct populations of residents. Because they attract distinct populations, we can analyze them using (12), which was formulated at the city-level. All else equal, housing submarkets in which χ is higher experience smaller house price booms:

Implication 11. *Suppose market segments within a city differ only in χ, the relative share of renters versus owner-occupants they attract: the shock x and the short-run and long-run* *supply elasticities ϵ_0^S and $\widetilde{\epsilon}^S$ are constant within a city. Then house price booms are smaller in market segments where χ is larger.*

6.1 Location

We first consider variation in χ across neighborhoods. Neighborhoods provide an example of market segments because they differ in the amenities they offer. For instance, some areas offer proximity to restaurants and nightlife; others are characterized by access to good public schools. These amenities appeal differentially to different populations of residents. Variation in amenities hence leads χ to vary across space. Neighborhoods whose amenities appeal relatively more to owner-occupants (high a residents) than to renters (low a residents) are characterized by a higher value of χ.

Consistent with Implication 11, house prices increased more between 2000 and 2006 in neighborhoods where χ was higher in 2000. We obtain ZIP-level data on χ from the U.S. Census, which reports the share of occupied housing that is owner-occupied, as opposed to rented, in each ZIP code in 2000. The fraction χ varies considerably within cities. Its national mean is 0.71 and standard deviation is 0.17, while the R^2 of regressing χ on city fixed-effects is only 0.12. We calculate the real increase in house prices from 2000 to 2006 using Zillow.com's ZIP-level house price indices. We regress this price increase on χ and city fixed-effects, and find a negative and highly significant coefficient of -0.10 (0.026), where the standard error is clustered at the city level.

However, this negative relationship between χ and price increases may not be causal. Housing demand shocks in this boom were larger in neighborhoods with a lower value of χ. The housing boom resulted from an expansion of credit to low-income households (Mian and Sufi, 2009; Landvoigt, Piazzesi and Schneider, 2013), and ZIP-level income strongly covaries with χ.[9]

The appeal of χ is that it predicts price increases in any housing boom in which there is disagreement about future fundamentals. In general, χ predicts price increases because it is negatively correlated with speculation, not because it is correlated with demand shocks. Empirical work can test Implication 11 by examining housing booms in which the shocks are independent from χ.

6.2 Structure Type

The second approach to measuring χ is to exploit variation across different types of housing structures. According to the U.S. Census, 87% of occupied detached single-family houses in 2000 were owner-occupied rather than rented. In contrast, only 14% of occupied multifamily

[9]The IRS reports the median adjusted gross income at the ZIP level. We take out city-level means, and the resulting correlation with χ is 0.40.

housing was owner-occupied. According to Implication 11, the enormous difference in χ between these two types of housing causes a larger price boom in multifamily housing, all else equal.

This result squares with accounts of heightened investment activity in multifamily housing during the boom.[10] For instance, a consortium of investors—including the Church of England and California's pension fund CalPERS—purchased Stuyvesant Town & Peter Cooper Village, Manhattan's largest apartment complex, for a record price of $5.4 billion in 2006. Their investment went into foreclosure in 2010 as the price of this complex sharply fell (Segel et al., 2011). Multifamily housing attracts speculators because it is easier to rent out than single-family housing. Optimistic speculators bid up multifamily house prices and cause large price booms in this submarket during periods of uncertainty.

7 Conclusion

In this paper, we argue that speculation explains an important part of housing cycles. Speculation amplifies house price booms by biasing prices toward optimistic valuations. We document the central importance of land price increases for explaining the U.S. house price boom between 2000 and 2006. These land price increases resulted from speculation directly in the land market. Consistent with this theory, homebuilders significantly increased their land investments during the boom and then suffered large capital losses during the bust. Many investors disagreed with this optimistic behavior and short-sold homebuilder equity as the homebuilders were purchasing land.

Our emphasis on speculation allows us to explain aspects of the boom that are at odds with existing theories of house prices. Many of the largest price increases occurred in cities that were able to build new houses quickly. This fact poses a problem for theories that stress inelastic housing supply as the source of house price booms. But it sits well with our theory, which instead emphasizes speculation. Undeveloped land facilitates speculation due to rental frictions in the housing market. In our model, large price booms occur in elastic cities facing a development barrier in the near future—cities in arrested development.

Our approach also makes some new predictions. Price booms are larger in submarkets within a city where a greater share of housing is rented. Although we presented some evidence for this prediction, further empirical work is needed to test it more carefully.

In all, we have presented a different but complementary story of the sources of housing cycles than the literature has offered. Our theory explains several puzzles and suggests new directions for empirical research.

[10] Bayer, Geissler and Roberts (2013) develop a method to identify speculators in the data. A relevant extension of their work would be to look at the types of housing speculators invest in.

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Appendix

Micro-foundation of owner-occupancy utility. We present a moral hazard framework in which ownership utility matches the specification of (2). Our framework follows the spirit of Henderson and Ioannides (1983)'s treatment of tenure choice, in which maintenance frictions lead some residents to own instead of rent.

Residents derive utility from the particular way their house is "customized": e.g. the color of the walls, the way the lawn is maintained, et cetera. The set of possible customizations is $\mathcal{K}$. Resident i's utility from housing is $v(\sum_{k\in\mathcal{K}} a_{i,k}h_k)$, where $a_{i,k} > 0$ is his preference for k and h_k is the quantity of housing customized that way. Individual customization choices are not contractible, but the right to customize one's house is. If a landlord retains the customization rights, then the tenant cannot customize the house, and the null customization $k = 0$ occurs for which $a_{i,0} = 1$ for all residents i. If the tenant holds these rights, he may choose any $k \in \mathcal{K} \setminus \{0\}$.

Moral hazard arises due to a doomsday customization $k = d$. This customization incurs a cost $\eta(h_d)$ to the owner of the house. All residents prefer this customization to all others: $d = \arg\max_{k\in\mathcal{K}} a_{i,k}$. However, the costs of d outweigh the benefits: for all i and h,

$$v(a_{i,d}h) < \eta(h).$$

The doomsday customization represents the proclivity of residents to damage a house when they do not bear the costs of doing so.

This inequality prevents landlords from ever selling customization rights to tenants. Suppose the landlord sells the rights. Then the tenant chooses his preferred customization, without taking into account the resultant costs, which the landlord bears. The tenant therefore chooses $k = d$. Knowing this, the landlord demands at least $\eta(h)$ for the customization rights. But the most the tenant is willing to pay is $v(a_{i,d}h) - v(h)$, which is less than $\eta(h)$. Therefore they agree not to trade. The landlord keeps the rights, and $k = 0$. The utility from renting is $v(h)$ because $a_{i,0} = 1$.

An owner-occupant chooses the customization, but also bears the costs if he chooses $k = d$. Let $k(i)$ denote the solution to his optimization problem $\max_{k\in\mathcal{K}\setminus\{0\}} v(a_{i,k}h) - \eta(h)\mathbf{1}_{k=d}$. Due to the costliness of the doomsday customization, the resident never chooses it: $k(i) \neq d$. Indeed, if k' is any other customization, then $v(a_{i,k'}h) > v(a_{i,d}h) - \eta(h)$ due to the above inequality. We define $a_i \equiv a_{i,k(i)}$. The utility from owning is $v(a_ih)$. This form corresponds exactly to (2).

Proof of Lemma 1. First we prove that construction occurs in each period. Construction occurs at time 0 because the housing stock starts at 0, and the housing demand equation (9) is positive. For a contradiction, let $t_1 > 0$ denote the first period in which construction does not occur. Let $t_2 > t_1$ denote the next time construction occurs (t_2 may be infinite).

We now claim that $r_t^h > r_{t_1-1}^h$ for $t_1 \le t < t_2$. Along the trend growth path, $x = 0$, so no uncertainty exists and by (7), a resident rents if and only if $a_i < 1$. Because F_a has full support on $\mathbb{R}^+$, some residents must rent. Landlords hence exist in equilibrium, and their arbitrage equation $p_t^h = r_t^h + \beta p_{t+1}^h$ holds. Aggregate housing demand resulting from the first-order condition (6) is

$$D_t^h(r_t^h) = N_t\left(\int_0^1 (v')^{-1}(r_t^h)dF_a + \int_1^\infty (v')^{-1}(r_t^h/a_i)/a_i dF_a\right). \tag{A1}$$

By assumption, the housing stock and hence housing demand is the same for $t_1 - 1 \le t < t_2$. Equation (A1) decreases in r_t^h because $v'' < 0$. Because $x = 0$ and $g > 0$, N_t increases with t. The left side of (A1) stays constant for $t_1 - 1 \le t < t_2$ while N_t increases. Therefore, r_t increases for $t_1 - 1 \le t < t_2$.

Because construction occurs at $t_1 - 1$, we have $p^h_{t_1-1} = p^l_{t_1-1} + K$, which results from zero homebuilder profits. Zero construction at t_1 can only occur when $p^h_{t_1} \leq p^l_{t_1} + K$, from homebuilder profit maximization. The landlord and landowner arbitrage equations at $t_1 - 1$ deliver $r^h_{t_1-1} \geq r^l_{t_1-1} + (1-\beta)K$. The quantity of undeveloped land stays constant for $t_1 - 1 \leq t < t_2$ and hence r^l_t does as well, because firm land demand D^l does not change over time. Therefore $r^h_t > r^l_t + (1-\beta)K$ for $t_1 \leq t < t_2$. Then

$$\begin{aligned} p^h_{t_1} &= \sum_{t_1 \leq t < t_2} \beta^{t-t_1} r^h_t + \beta^{t_2-t_1} p^h_{t_2} \\ &> \sum_{t_1 \leq t < t_2} \beta^{t-t_1} (r^l_t + (1-\beta)K) + \beta^{t_2-t_1}(p^l_{t_2} + K) \\ &= p^l_t + K, \end{aligned}$$

which contradicts the zero construction inequality $p^h_{t_1} \leq p^l_t + K$. This contradiction proves that construction occurs at all times t.

We now show that rents r^h_t increase over time. Because construction occurs at all t, $p^h_t = p^l_t + K$ for all t. Undeveloped land must always exist because perpetual construction occurs. Therefore, landowners are indifferent between holding land until tomorrow or selling it, so $p^l_t = r^l_t + \beta p^l_{t+1}$. Together with the landlord arbitrage equation, this equation gives $r^h_t = p^h_t - \beta p^h_{t+1} = p^l_t + K - \beta(p^l_{t+1} + K) = r^l_t + (1-\beta)K$. Equilibrium rents are determined by $S - D^l(r^h_t - (1-\beta)K) = D^h_t(r^h_t)$, where housing demand comes from (A1). The left side increases in r^h_t, whereas the right side deceases. N_t increases over time, which shifts up D^h_t. Therefore r^h_t increases as well.

Finally, we can show directly that the supply elasticity decreases over time. The elasticity by definition is

$$\epsilon^S_t = \frac{r^h_t (D^l)'(r^h_t - (1-\beta)K)}{S - D^l(r^h_t - (1-\beta)K)} = \frac{r^h_t}{r^h_t - (1-\beta)K} \frac{D^l(r^h_t)}{D^h_t(r^h_t)} \frac{r^l_t (D^l)'(r^l_t)}{D^l(r^l_t)} = \frac{r^h_t}{r^h_t - (1-\beta)K} \left(\frac{S}{H_t} - 1 \right) \epsilon^l,$$

which coincides with (4). We have shown directly that H_t and r^h_t increase over time. Therefore, when ϵ^l is constant, ϵ^S_t decreases over time.

Proof of Proposition 2. We use (5) to write $p^h_0 = r^h_0 + \beta \widetilde{\mathbf{E}} p^h_1$. Let $\partial/\partial x$ denote the partial derivative in which N_0 stays constant. Then $\partial p^h_0/\partial x = \partial r^h_0/\partial x + \partial \beta \widetilde{\mathbf{E}} p^h_1/\partial x$. We calculate $\partial r^h_0/\partial x$ by differentiating (8) at $x = 0$. Let $d(\cdot) = (v')^{-1}(\cdot)$, and let $b_i = 1 + \beta(\widetilde{\mathbf{E}} p^h_1 - \mathbf{E}_i p^h_1)/r^h_0$. Note that when $x = 0$, $b_i = 1$ for all i. Then

$$\begin{aligned} -(D^l)' \frac{\partial r^h_0}{\partial x} &= N_0 \int_M \int_0^1 d'(r^h_0) \frac{\partial r^h_0}{\partial x} dF_a dF_\mu + N_0 \int_M d(r^h_0) \frac{\partial b_i}{\partial x} dF_\mu \\ &+ N_0 \int_M \int_1^\infty a_i^{-2} d'(r^h_0/a_i) \left(\frac{\partial r^h_0}{\partial x} + \frac{\partial \beta \widetilde{\mathbf{E}} p^h_1}{\partial x} - \frac{\partial \beta \mathbf{E}_i p^h_1}{\partial x} \right) dF_a dF_\mu - N_0 \int_M d(r^h_0) \frac{\partial b_i}{\partial x} dF_\mu. \end{aligned}$$

The extensive margins terms for the rental and owner-occupied populations cancel. We simplify this equation to

$$\frac{\partial r^h_0}{\partial x} = -\frac{N_0 \int_M \int_1^\infty a_i^{-2} d'(r_0/a_i) \left(\partial \beta \widetilde{\mathbf{E}} p^h_1/\partial x - \partial \beta \mathbf{E}_i p^h_1/\partial x \right) dF_a dF_\mu}{(D^l)' + N_0 \int_M \int_0^1 d'(r^h_0) dF_a dF_\mu + N_0 \int_M \int_1^\infty a_i^{-2} d'(r^h_0/a_i) dF_a dF_\mu}.$$

The proposition assumes a constant elasticity of housing demand ϵ^D. This property occurs when individual demand $d(\cdot)$ displays the same constant elasticity. Indeed, from (A1), the elasticity of housing demand when $x = 0$ is

$$\epsilon^D = -\frac{\int_0^1 rd'(r)dF_a + \int_1^\infty ra_i^{-2}d'(r/a_i)dF_a}{\int_0^1 d(r)dF_a + \int_1^\infty a_i^{-1}d(r/a_i)dF_a},$$

which holds when $rd'(r)/d(r) = -\epsilon^D$ for all r. We can therefore rewrite $\partial r_0^h/\partial x$ as

$$\frac{\partial r_0^h}{\partial x} = -\frac{\epsilon^D N_0 \int_M \int_1^\infty a_i^{-1}d(r_0^h/a_i)\left(\partial\beta\widetilde{\mathbf{E}}p_1^h/\partial x - \partial\beta\mathbf{E}_i p_1^h/\partial x\right)dF_a dF_\mu}{r_0^h(D^l)' + \epsilon^D N_0 \int_M \int_0^1 d(r_0^h)dF_a dF_\mu + \epsilon^D N_0 \int_M \int_1^\infty a_i^{-1}d(r_0^h/a_i)dF_a dF_\mu}.$$

Because F_a and F_μ are independent, we can write

$$\begin{aligned}\int_M \int_1^\infty a_i^{-1}d(r_0^h/a_i)\mathbf{E}_i p_1^h dF_a dF_\mu &= \int_1^\infty a_i^{-1}d(r_0^h/a_i)dF_a \int_M \mathbf{E}_i p_1^h dF_\mu \\ &= \int_1^\infty a_i^{-1}d(r_0^h/a_i)dF_a \overline{\mathbf{E}}p_1^h,\end{aligned}$$

where $\overline{\mathbf{E}}p_1^h \equiv \int_M \mathbf{E}_i p_1^h dF_\mu$ is the average belief about p_1^h. Recall from (A1) that $(h_{i,0}^{rent})^* = d(r_0^h)$ if $a_i < 1$ (and 0 otherwise) and $(h_{i,0}^{own})^* = d(r_0^h/a_i)/a_i$ if $a_i \geq 1$ (and 0 otherwise). The share of housing that is owner-occupied is $\chi = \int_1^\infty a_i^{-1}d(r_0^h/a_i)dF_a/(\int_0^1 d(r_0^h)dF_a + \int_1^\infty a_i^{-1}d(r_0^h/a_i)dF_a)$. We can therefore divide through the equation for $\partial r_0^h/\partial x$ by the total housing stock to get

$$\frac{\partial r_0^h}{\partial x} = -\frac{\epsilon^D\chi\left(\partial\beta\widetilde{\mathbf{E}}p_1^h/\partial x - \partial\beta\overline{\mathbf{E}}p_1^h/\partial x\right)}{\epsilon_0^S + \epsilon^D}.$$

Substituting into $\partial p_0^h/\partial x = \partial r_0^h/\partial x + \partial\beta\widetilde{\mathbf{E}}p_1^h/\partial x$ yields (10) of the proposition.

Proof of Proposition 3. We will calculate the effect of the shock z_t on r_t^h by differentiating the equation $S - D^l(r_t^h - (1-\beta)K) = D_t^h(r_t^h)$ with respect to x at $x = 0$, where $D_t^h(r_t^h)$ is given by (A1). This derivative is valid if and only if this equilibrium condition holds for x around 0. The condition holds as long as construction occurs at t. Our first task is thus proving the existence of an open set $I \in \mathbb{R}$ such that $0 \in I$ and for $x \in I$, construction occurs for all t.

As in the proof of Lemma 1, we can prove that construction must occur at t_1 if, conditional on the absence of construction at t_1, $r_t > r_{t_1-1}^h$ for $t_1 \leq t < t_2$ where t_2 is the next time construction occurs. The key step in that proof was that N_t increases with t. We define an open set I_1 containing 0 such that N_t still increases in t for $x \in I_1$. Because M is uniformly bounded, there exist μ^{min} and μ^{max} such that $\mu^{min} \leq \mu' \leq \mu^{max}$ for all μ' that are coordinates of vectors in M. Recall that $N_{t+1}/N_t = e^{g+(\mu_{t+1}-\mu_t)x}$. Because $g > 0$, the set $I_1 = \left(-g/(\mu^{max} - \mu^{min}), g/(\mu^{max} - \mu^{min})\right)$ is open. For any $x \in I_1$, $N_{t+1}/N_t > 1$. With this result, the proof of this increasing rent condition matches verbatim the proof given in the proof of Lemma 1 when $t_1 > 1$. When $t_1 = 1$, $D_{t_1-1}^h$ is no longer given by (A1) but instead by (9).

The only new fact we must show therefore is that if construction fails to occur at $t = 1$, then $r_0^h < r_1^h$. To do this, we first show that $\widetilde{\mathbf{E}}p_1^h - \mathbf{E}_i p_1^h = O(x)$ as $x \to 0$ for all i. We have $p_1^h = \sum_{t=1}^\infty \beta^{t-1}r_t^h$. All residents agree on H_0 and N_0 because they are observable at $t = 0$. Let t_2 be the next time construction occurs given H_0. Once it occurs it will occur afterward forever due to the arguments in the proof of Lemma 1. In principle residents could disagree about t_2, but we will now show that for x small enough they do not. While construction does not occur, rents are determined by $H_0 = N_t D_t^h(r_t^h)$ and $S - H_0 = D^l(r_t^l)$. Because N_t increases over time, r_t^h must as well. When construction occurs next period but not today at t, $p_t^h < p_t^l + K$ while $p_{t+1}^h = p_{t+1}^l + K$, so using the landlord and landowner arbitrage equations defined in the proof of Lemma 1, we find that $(D_t^h)^{-1}(H_0/N_t) < (D^l)^{-1}(S - H_0) + (1-\beta)K$ while construction fails to occur. The first time construction does occur, $t = t_2$, is defined as the lowest value of t for which this inequality fails to hold. Because we are in discrete time, and because the relationships $N_t = N_0 e^{gt+(\mu_t-1)x}$ and $\mu^{min} \leq \mu_t \leq \mu^{max}$ hold, there exists an open $I_2 \ni 0$ such that when $x \in I_2$, t_2 is the same for all realizations of $\boldsymbol{\mu} \in M$. For $1 \leq t < t_2$, r_t^h is the solution to $H_0 = N_t D_t^h(r_t^h)$, and for $t \geq t_2$, r_t^h solves $S - D^l(r_t^h - (1-\beta)K) = N_t D_t^h(r_t^h)$. In each case, because $N_t = N_0 e^{gt+(\mu_t-1)x}$, the resulting r_t^h is a differentiable function of x for any value of μ_t and is the same at $x = 0$ for any value of μ_t. Therefore, $\widetilde{\mathbf{E}} r_t^h - \mathbf{E}_i r_t^h = O(x)$ as $x \to 0$ for all i, and the same then holds for p_1^h.

We now return to showing that if construction fails to occur at $t = 1$, then $r_0^h < r_1^h$. Using (9), we write $D_0^h(r_0^h) = N_0 f_0(r_0^h)$, and using (A1), we write $D_1^h(r_1^h) = N_1 f_1(r_1^h)$. Without construction at $t = 1$, we have $N_0 f_0(r_0^h) = N_1 f_1(r_1^h)$. Note from (9) and (A1) that $f_0 = f_1 + O(x)$ as $x \to 0$; this fact follows because $\widetilde{\mathbf{E}} p_1^h - \mathbf{E}_i p_1^h = O(x)$ as $x \to 0$ for all i. Using $N_1 = N_0 e^{g+(\mu_1-1)x}$, we can conclude that $e^{g+(\mu_1-1)x} f_1(r_1^h) = f_1(r_0^h) + O(x)$ as $x \to 0$. Because $e^{g+(\mu_1-1)x} > 1$ as $x \to 0$ and f_1 is decreasing, there exists an open $I_3 \ni 0$ such that for $x \in I_3$, $r_1^h > r_1^0$. This inequality is what we needed to show to prove that construction occurs at time 1, which is all that remained to prove that construction always occurs. We set $I = I_1 \cap I_2 \cap I_3$.

All of that proved that for $t > 0$, the effect of the shock z_t on r_t^h results from differentiating the equation $S - D^l(r_t^h - (1-\beta)K) = D_t^h(r_t^h)$ with respect to x at $x = 0$. Doing so yields $-(D^l)' dr_t^h/dx = \mu_t D_t^h + (D_t^h)' dr_t^h/dx$, from which it follows that $dr_t^h/dx = -\mu_t D_t^h/((D^l)' + (D_t^h)') = \mu_t r_t^h/(\epsilon_t^S + \epsilon^D)$. Similarly, the partial effect of the shock on current rents r_0^h, holding beliefs constant and letting N_0 change, is $\partial r_0^h/\partial x = r_0^h/(\epsilon_0^S + \epsilon^D)$. Putting together this partial effect with the one in Proposition 2 yields

$$\frac{dp_0^h}{dx} = \frac{r_0^h}{\epsilon_0^S + \epsilon^D} + \sum_{t=1}^{\infty} \left(\frac{\epsilon_0^S + (1-\chi)\epsilon^D}{\epsilon_0^S + \epsilon^D} \widetilde{\mu}_t + \frac{\chi \epsilon^S}{\epsilon_0^S + \epsilon^S} \overline{\mu}_t \right) \frac{\beta^t r_t^h}{\epsilon_t^S + \epsilon^S},$$

where $\widetilde{\mu}_t$ is the most optimistic belief of μ_t and $\overline{\mu}_t$ is the average belief of μ_t. Because all residents agree that $\mu_0 = 1$, we may rewrite this expression as

$$\frac{dp_0^h}{dx} = \sum_{t=0}^{\infty} \left(\frac{\epsilon_0^S + (1-\chi)\epsilon^D}{\epsilon_0^S + \epsilon^D} \widetilde{\mu}_t + \frac{\chi \epsilon^S}{\epsilon_0^S + \epsilon^S} \overline{\mu}_t \right) \frac{\beta^t r_t^h}{\epsilon_t^S + \epsilon^S}.$$

The text defines the mean persistence of the shock $\boldsymbol{\mu}$ to be $\mu = \sum_{t=0}^{\infty} \mu_t \beta^t r_t^h (\epsilon_t^S + \epsilon^D)^{-1} / \sum_{t=0}^{\infty} \beta^t r_t^h (\epsilon_t^S + \epsilon^D)^{-1}$. We use this definition, and divide through by $p_0 = \sum_{i=0}^{\infty} \beta^t r_t^h$, which holds at $x = 0$, to derive

$$\begin{aligned} \frac{d \log p_0^h}{dx} &= \left(\sum_{t=0}^{\infty} \beta^t r_t^h \right)^{-1} \sum_{t=0}^{\infty} \left(\frac{\epsilon_0^S + (1-\chi)\epsilon^D}{\epsilon_0^S + \epsilon^D} \widetilde{\mu} + \frac{\chi \epsilon^S}{\epsilon_0^S + \epsilon^S} \overline{\mu} \right) \frac{\beta^t r_t^h}{\epsilon_t^S + \epsilon^S} \\ &= \left(\frac{\epsilon_0^S + (1-\chi)\epsilon^D}{\epsilon_0^S + \epsilon^D} \widetilde{\mu} + \frac{\chi \epsilon^S}{\epsilon_0^S + \epsilon^S} \overline{\mu} \right) \frac{1}{\widetilde{\epsilon}^S + \epsilon^D}, \end{aligned}$$

where we have used the definition of the long-run supply elasticity $\widetilde{\epsilon}^S$ given in the text. This equation for $d \log p_0^h/dx$ matches (12) in Proposition 3.

Proof of Implication 7. We demonstrate a limiting case in which $\epsilon_0^S = \infty$ while $\widetilde{\epsilon}^S < \infty$. Let $D^l(r) = br^{-\epsilon^l}$ for some constant $b > 0$. Consider the limit as $b \to 0$. We know that $r_t^h \geq (1-\beta)K$

because $r_t^h = r_t^l + (1-\beta)K$ and $r_t^l \geq 0$. Define N^* to be the value of N_t that solves the equation $S = D_t^h((1-\beta)K)$, where D_t^h is given by (A1). For $N_t < N^*$, housing demand fails to exceed available land at the minimum rent, and there is no demand for land in the limit, so the market clearing rent must be $r_t^h = (1-\beta)K$ while $H_t < S$. By (4), $\epsilon_t^S = \infty$ in this case. But for $N_t > N^*$, demand exceeds supply at the minimum rent, so $r_t^h > (1-\beta)K$ and $H_t > 0$, leading to a finite elasticity. Since N_t grows at a constant rate g, for any $N_t < N^*$ we have $\epsilon_0^S = \infty$ but $\widetilde{\epsilon}^S < \infty$.

Proof of Implication 8. Disagreement amplification Δ equals

$$\Delta = \frac{\epsilon_0^S + (1-\chi)\epsilon^D}{\epsilon_0^S + \epsilon^D} \frac{\widetilde{\mu} - \overline{\mu}}{\widetilde{\epsilon}^S + \epsilon^D}.$$

We calculate this difference from subtracting from (12) the counterfactual in which we substitute $\overline{\mu}$ for $\widetilde{\mu}$. Define $\overline{N}_0^*(\chi)$ to be the value of development (which determines the supply elasticities; see above) that maximizes Δ. When $\chi = 1$, Δ is 0 in the limits as $\overline{N}_0 \to 0$ and $\overline{N}_0 \to \infty$, because $\widetilde{\epsilon}^S = 0$ in the first case and $\epsilon_0^S = 0$ in the second. But $\Delta > 0$ for $\chi = 1$, so $0 < \overline{N}_0^*(1) < \infty$ by continuity. But $\overline{N}_0^*(\chi)$ is continuous in χ as long as it exists and is finite, so there must exist $\chi^* < 1$ such that for $\chi^* \leq \chi \leq 1$, $\overline{N}_0^*(\chi)$ exists and is finite.

Proof of Implication 9. When $\chi = 1$, the limit as $\overline{N}_0 \to \infty$ of $d \log p_0^h/dx$ is $\overline{\mu}/\epsilon^D$. For *any* $0 < \overline{N}_0 < \infty$, we can choose $\widetilde{\mu}$ to be large enough so that the price change given by (12) is larger than $\overline{\mu}/\epsilon^D$, because this price change becomes arbitrarily large with $\widetilde{\mu}$. By continuity, we can do the same for some $\chi < 1$.

Construction equation. By the definition of supply elasticity, the change in the log housing stock is $\epsilon_0^S d\log r_0^h/dx$. The total effect of the shock on rents combines the effect in the end of the proof of Proposition 2 and the direct effect of the shock on N_0 derived in the proof of Proposition 3. It is $dr_0^h/dx = r_0^h/(\epsilon_0^S + \epsilon^D) - \chi\epsilon^D(\partial\beta\widetilde{\mathbf{E}}p_1^h\partial x - \partial\beta\overline{\mathbf{E}}p_1^h/\partial x)/(\epsilon_0^S + \epsilon^D)$. We substitute in for the beliefs from the Proof of Proposition 3 and divide through by r_0^h, and then multiply by ϵ_0^S to get

$$\frac{d\log H_0}{dx} = \frac{\epsilon_0^S}{\epsilon_0^S + \epsilon^D}\left(1 - \frac{\chi\epsilon^D}{\widetilde{\epsilon}^S + \epsilon^D}\rho(\widetilde{\mu} - \overline{\mu})\right), \tag{A2}$$

where $\rho \equiv p_0^h/r_0^h$ is the price-rent ratio of the city before the shock at $x = 0$.