A Theory of Labor Markets with Allocative Wages*

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Abstract

We develop a model of a frictional labor market with idiosyncratic and aggregate shocks, sticky wages, and two-sided lack of commitment between workers and firms. In this environment, wages are allocative in the sense that inefficient job separations arise when the wage-to-productivity ratio falls outside of a supported range. We analytically characterize equilibrium job separation policies and show how to use microdata on wage changes and worker transitions between jobs in order to recover the unobserved distribution of wage-to-productivity ratios. The latter is a sufficient statistic for the prevalence of inefficient job separations as well as the aggregate response of employment and real wages to monetary shocks.

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JEL Classification: Efficiency of job separations; sticky wages; lack of commitment; nonzero-sum stochastic differential game with stopping times

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"Not everything that can be counted counts, and not everything that counts can be counted."

William Bruce Cameron (1963)

1 Introduction

What is the allocative role of wages in the labor market? A common view is based on the assumption that wages flexibly adjust to shocks to ensure that all gains from trade between workers and firms are exploited. Under this assumption, wages are not allocative in the sense that the viability of a worker-firm match depends only on the sign of the match surplus and all job separations are efficient.[1] An alternative view holds that wages are rigid and that, consequently, the way the surplus is split between workers and firms matters for employment outcomes. According to this alternative view, wages are allocative in the sense that some job separations may be inefficient from a welfare perspective.

While abstracting from any allocative role of wages is technically convenient in that it allows for significant model tractability, it is also at odds with mounting empirical evidence of wages being less than fully flexible (Bewley, 2007; Hazell and Taska, 2020; Grigsby *et al.*, 2021; Blanco *et al.*, 2022a). This raises the possibility that some job separations may be inefficient. Yet quantifying the prevalence of inefficient job separations is a difficult task, chiefly because the surplus of a job cannot be directly measured and thus inefficient job separations cannot be *counted*. Doing so requires a theory of labor markets with allocative wages, which proves challenging due to the dynamic strategic considerations this introduces. Although technically difficult to model, the prevalence of inefficient job separations has direct implications for macroeconomic and labor market policies. For example, the efficacy of monetary policy depends on the response of employment to aggregate money supply, and the optimal design of unemployment insurance depends on which workers lose their jobs in response to productivity shocks. Therefore, measuring the allocativeness of wages *counts* in the eyes of both economists and policymakers.

In this paper, we develop a theory of labor markets with allocative wages. To this end, we depart from the canonical DMP model of search and matching by incorporating two additional frictions into worker-firm relationships. First, wages within job spells are sticky and unresponsive to productivity shocks. Second, neither workers nor firms can commit to their future decisions to dissolve a match. The interaction between productivity shocks, wage rigidity, and two-sided lack of commitment gives rise to inefficient job separations. That is, one party's value of continuing in the match may fall below a critical

[1] For example, under Nash bargaining in the canonical DMP model (Diamond, 1982; Pissarides, 1985; Mortensen and Pissarides, 1994), wages are determined in a way that ensures that both the worker's and the firm's surplus are positive whenever the match surplus is positive, which results in all matches with positive match surplus being viable.

value that triggers job separation in spite of the match surplus being strictly positive. Our contribution is to provide a theoretical characterization of inefficient job separations stemming from productivity shocks, wage rigidity, and two-sided lack of commitment, and to link the unobserved prevalence of inefficient job separations to empirically measurable objects.

To this end, we proceed in three steps. In the first step, we *characterize equilibrium behavior under allocative wages*. Both a worker's decision to quit and a firm's decision to fire the worker depend not only on the current wage and productivity but also on dynamic strategic considerations, which we formulate in a nonzero-sum stochastic differential game with stopping times. We show that an agent's choice to remain in a presently unprofitable match resembles a costly investment decision, which becomes relatively less attractive under lack of commitment. In the second step, we show how to *identify the microeconomic implications of allocative wages*. We infer the unobserved steady-state distribution of workers' wage-to-productivity ratios and the prevalence of inefficient job separations based on microdata on wage changes and worker flows between jobs. This yields several testable microeconomic implications of our theory. Specifically, if the between-job wage differentials are larger in absolute magnitude, this intuitively indicates greater deviations of wages from fundamentals, which in turn are associated with higher prevalence of inefficient job separations. In the third step, we *analyze the macroeconomic consequences of allocative wages*. A key insight emerging from this analysis is that labor markets and the macroeconomy are tightly linked to one another. Specifically, we find that the transmission of monetary policy to employment depends on the prevailing inflation regime. We show that in stable economies with low trend inflation monetary shocks do not affect aggregate employment, despite wages being allocative at the micro level. Deviations from low and stable inflation lead to an incrased responsiveness of the labor market to aggregate shocks. We theoretically explore the mechanisms behind these effects and and provide sufficient statistics for their magnitude. Next, we describe each of the three steps in more detail.

Step 1: Characterizing Equilibrium Policies under Allocative Wages. Our study remains analytically tractabe by leveraging the powerful tools of optimal control in continuous time. The model labor market is populated by continua of workers and firms. A worker's income depends on their employment state and their idiosyncratic productivity, which follows a Brownian motion with drift. Employed workers receive a wage, while unemployed workers derive consumption from home production. Output in both employment states depends on a worker's productivity. Job search is directed, as in Moen (1997), and segmented across submarkets according to the wage rate and productivity. In each submarket, a set of homogeneous firms post vacancies to recruit workers. Existing matches become obsolete at an exogenous Poisson rate. In addition, worker-firm relationships are characterized by two key frictions.

First, contracted wages are fixed within a match. Second, neither workers nor firms can commit to future actions. Together, the two key frictions give rise to the distinguishing feature of our theory: endogenous job separations that can be unilaterally initiated by either the worker or the firm.

The strategic interaction between workers and firms has three features. First, agents in a match play a dynamic nonzero-sum game since one party's payoffs from continuing in the match depend on the other party's future actions, and the joint value of a match is greater than the sum of the two agents' outside options. Second, agents' payoffs are stochastic due to fluctuations in worker productivity. Third, agents' strategies consist of stopping times, which define the stochastic arrival of unilateral job dissolution. In summary, the strategic interaction between workers and firms can be formulated as a nonzero-sum stochastic differential game with stopping times (Bensoussan and Friedman, 1977).

Using the theory of optimal control in continuous time, we prove the existence of a unique block recursive equilibrium. We analytically characterize workers' and firms' decisions to dissolve a match, which we show are functions of only a single state variable, namely the wage-to-productivity ratio. Agents' optimal policy functions reflects both static and dynamic considerations. In terms of static considerations, workers' and firms' respective value functions depend on their flow payoffs and flow opportunity costs from being matched. In the special case when the discount rate tends to infinity, agents behave myopically. In this case, job separations occur either if the wage-to-productivity ratio falls below a threshold that depends on the efficiency of home production, in which case the worker quits, or if the wage-to-productivity ratio rises above unity, in which case the firm fires the worker. More generally, both sides of a match solve a dynamic optimization problem that leads them to optimally delay job separation beyond the stopping time dictated by static calculus. For example, firms continue in a match beyond the time when the wage-to-productivity ratio falls below unity, either because future productivity may increase due to its stochastic component (i.e., the *option value effect*) or because productivity may have a positive drift (i.e., the *anticipatory effect*). Analogously, workers continue in a match either because of the option value of productivity falling stochastically or in anticipation of productivity's negative drift. Surprisingly and unlike in other model contexts, these option value effects are bounded and finite due to the match relationship being characterized by two-sided lack of commitment.

Step 2: Identifying the Microeconomic Implications of Allocative Wages. In theory, knowing the distribution of wage-to-productivity ratios is key for measuring the prevalence of inefficient job separations through the lens of our model. In practice, although wages are commonly available in appropriate microdata, individual workers' productivity levels are not directly observed. To get around this challenge, we use our model to derive a mapping between the unobserved prevalence of inefficient job separations on one hand and observed labor market outcomes on the other hand.

To this end, we proceed in four steps. First, we recast agents' state variable—the wage-to-productivity ratio—in terms of the negative of the cumulative productivity shocks since the beginning of the employment spell. We show that this alternative choice of state variable delivers an equivalent representation of both workers' and firms' problems. Second, we identify the parameters governing the stochastic process of idiosyncratic productivity from data on wage changes between employment spells. To achieve this, we exploit properties of continuous-time stochastic processes as summarized in Doob's Optional Stopping Theorem. Third, due to the model's homogeneity in worker productivity, we recover the distribution of cumulative productivity shocks from observed wage changes between employment spells, given the already-identified stochastic process of idiosyncratic productivity. Fourth and finally, we derive a Kolmogorov forward equation guiding the evolution of the distribution of cumulative productivity shocks, which we show incorporates all the relevant information needed to quantify the prevalence of inefficient job separations in the economy.

Step 3: Analyzing the Macroeconomic Consequences of Allocative Wages. Our model highlights two distinct ways in which aggregate shocks can impact the distribution of employment in the labor market. The first way is by changing the size of the match surplus (i.e., the *surplus channel*). The second way is by changing the way the match surplus is split between workers and firms (i.e., the *redistribution channel*). Because the first channel is the standard one considered in myriad previous studies of labor markets, and also because the allocativeness of wages is our prime focus here, we restrict attention to the effects of the redistribution channel on employment across workers. To this end, we extend our model to a monetary economy, in which wages are nominally sticky while the aggregate price level fluctuates. Since changes in money supply translate one-for-one into inflation and nominal wages are rigid, monetary shocks redistribute the match surplus between workers and firms in existing jobs by moving real wages.

In such a monetary economy, we characterize analytically the transition dynamics of aggregate employment and the average real wage following a one-off monetary shock. On impact, a monetary shock causes real wages of incumbent workers to fall, while the real wages of new hires from unemployment remain constant as their nominal wages adjust one-for-one with inflation. Consequently, the labor market undergoes an adjustment in employment driven entirely by changes in the job separation rate due to increased quits by workers and decreased layoffs by firms. Following such an adjustment, the economy converges back to the previous steady state, with nominal wage growth compensating for the one-off increase in the aggregate price level.

We quantify the effect of a monetary shock on employment and average real wages, relative to their steady-state values, by computing the *cumulative impulse response (CIR)* as the area under the respective impulse response function.[2] With flexible wages among new hires, the CIR of wages is linked to the variance of cumulative productivity shocks during employment and the covariance of cumulative productivity shocks with the tenure of employed workers. These two moments reflect both the response of employed workers to idiosyncratic productivity shocks in the steady state and also the response of average wages to an aggregate shock. The CIR of employment depends on the steady-state unemployment rate, the average of cumulative productivity shocks in employment, and the average drift of productivity in employment—three objects that with the help of our theory can be inferred from the data. We provide economic intuition behind these results and explore special cases of the model that shed light on the different mechanisms at play. Finally, we highlight the importance of combining theory and microdata to tease out the prevalence of inefficient job separations under allocative wages, which could go undetected in aggregate time series data.

Related Literature. We highlight three contributions. Our first contribution is to provide a methodology to infer the prevalence of inefficient job separations based on microdata on wage changes and worker flows between jobs. Our model-based approach complements recent reduced-form evidence by Jäger *et al.* (2019), who find evidence consistent with wage rigidity driving inefficient job separations in the context of changes to unemployment benefit extensions in Austria. Our work complements the analysis in Hall (2005) and Shimer (2005a) by allowing for inefficient job separations as an endogenous equilibrium outcome. In the models of staggered Nash wage bargaining by Gertler and Trigari (2009) and Gertler *et al.* (2020), inefficient job separations may occur in theory whenever large enough aggregate shocks take a worker-firm match's wage out of the bargaining set, although this possibility is ignored in practice. In our setting, the interaction between idiosyncratic worker productivity shocks, wage rigidity, and two-sided lack of commitment gives rise to inefficient job separations. We show that such a framework has very different implications for the employment and output response to aggregate shocks, compared to alternative frameworks in which all job separations are efficient.

Our second contribution is to prove the existence and uniqueness of and then characterize a block recursive equilibrium in a directed search environment with rigid wages and two-sided lack of commitment. A technical challenge posed by this environment concerns the discontinuities in agents' value functions, and thus the inapplicability of standard dynamic programming results such as the contraction mapping theorem, due to dynamic strategic considerations in discrete time. To overcome this challenge, we leverage the powerful tools of optimal control in continuous time by casting the problem as a

[2]See Álvarez *et al.* (2016) for a similar approach in the context of the product pricing literature.

nonzero-sum stochastic differential game with stopping times (Bensoussan and Friedman, 1977).[3] Similar continuous-time methods have recently been employed by Bilal *et al.* (2021a,b) to tractably study firm dynamics with random matching and on-the-job search. A distinguishing feature of our analysis is that it allows for the possibility of inefficient job separations, which have been absent in previous work assuming full commitment (Moen, 1997; Acemoglu and Shimer, 1999a,b) or one-sided lack of commitment (Shi, 2009; Menzio and Shi, 2010a,b, 2011; Schaal, 2017; Herkenhoff, 2019; Balke and Lamadon, 2020; Fukui, 2020). While Sigouin (2004), Rudanko (2009, 2021), and Bilal *et al.* (2021a,b) also study environments with two-sided lack of commitment, their analysis remains tractable precisely because their assumptions lead to a privately efficient solution (i.e., all agents' decisions maximize the joint value of a match) of the game between workers and firms. In contrast, our focus explicitly lies on the privately inefficient solution (i.e., some of agents' decisions lower the joint value of a match) under sticky wages.

Our third contribution is to extend the method of sufficient statistics from heterogeneous-agent models of inaction to a labor market setting. In the context of the product pricing literature, Álvarez *et al.* (2016) link the CIR of output to monetary shocks to the ratio of kurtosis to the frequency of price changes. In related work, Baley and Blanco (2021a) characterize the CIR of output in terms of unobserved steady-state objects, which can be mapped to observed data on price changes by use of their model. Álvarez *et al.* (2020) and Baley and Blanco (2022) extend this theoretical result to general hazard models and multiple reset points, respectively. Before the current paper, these tools have not been imported to the labor literature. To make this possible, a notable technical challenge we overcome lies in tractably incorporating endogenous transitions from and to unemployment, which are central to our quest to quantify the prevalence of inefficient job separations. While our theory is developed in the context of labor markets, our core insights are generalizable to alternative settings with endogenous transitions between a discrete set of states, including firms' entry and exit decisions, traders sorting across segmented asset markets, and individual mobility decisions across geographic locations.

Outline. The rest of the paper is organized as follows. Section 2 lays out the model environment, defines an equilibrium, and characterizes equilibrium policies. Section 3 establishes the one-to-one mapping between the model's unobservable state variable and data. Section 4 describes the dynamic response of aggregate employment and real wages to a monetary shock. Finally, Section 5 concludes.

[3] Related models of inaction are studied by Dixit (1991) and Sheshinski and Weiss (1977) in the context of price setting and by Bloom (2009) in the context of investment. See also the overviews contained in Dixit (2001) and Stokey (2008).

2 A Model of Allocative Wages

We develop a labor market model with search and matching in the spirit of DMP with the additional features that workers are subject to idiosyncratic productivity shocks, wages are sticky within a job spell, and neither workers nor firms can commit to future actions. We show that the equilibrium interaction between idiosyncratic worker productivity shocks, wage rigidity, and two-sided lack of commitment gives rise to allocative wages, which allows us to study the prevalence of inefficient job separations.

2.1 Environment

Time is continuous and indexed by t. The economy is populated by a unit mass of workers and an endogenously determined mass of firms who meet in a frictional labor market. Workers maximize the expected discounted utility from consumption, while firms maximize the net present value of profits. All agents discount the future at a common rate $\rho > 0$.

Preferences and Technology. Workers value an expected discounted consumption stream $\{c_t\}_{t=0}^{\infty}$ with risk-neutral preferences:

$$\mathbb{E}\left[\int_0^{\infty} e^{-\rho t} c_t \, \mathrm{d}t\right],$$

Without loss of generality, we assume that workers consume their flow income y_t. A worker's flow income depends on her employment state E_t, which can be either employed (h) or unemployed (u), and the worker's productivity level Z_t. While employed, a worker produces output of a homogeneous good equal to the worker's productivity and receives flow income equal to a wage level W_t, which we assume is constant within a job spell. While unemployed, a worker receives flow income $B(Z_t)$ from home production.

Henceforth, we use lower-case letters to denote the natural logarithm of variables in upper-case letters. For example, z_t denotes the log of the worker's productivity and w_t denotes the log wage.

Stochastic Process for Worker Productivity. A worker's idiosyncratic productivity follows a Brownian motion in logs and can be written as

$$dz_t = \gamma \, \mathrm{d}t + \sigma \, \mathrm{d}\mathcal{W}_t^z,$$

where γ is the drift, σ is the volatility, and $\mathcal{W}_t^z$ is a Wiener process. For the time being, we focus on a stationary environment in which the only shocks are to idiosyncratic worker productivity, but we introduce aggregate shocks in Section 4.

Search Frictions. Unemployed workers search for jobs in a frictional labor market. Search is directed, as in Moen (1997), and segmented across submarkets according to the log wage w and the worker's log productivity z. In each submarket (w, z), firms post vacancies $\mathcal{V}$ at cost $K(Z_t)$. Given $\mathcal{U}$ unemployed workers and $\mathcal{V}$ vacancies in a submarket, a Cobb-Douglas matching function with constant returns produces $m(\mathcal{U}, \mathcal{V}) = \mathcal{U}^\alpha \mathcal{V}^{1-\alpha}$ matches, where α is the elasticity of matches to the unemployment rate. Thus, a worker's job finding rate is $f(w,z) = m(w,z)/\mathcal{U}(w,z) = \theta(w,z)^{1-\alpha}$ and a firm's job filling rate is $q(w,z) = m(w,z)/\mathcal{V}(w,z) = \theta(w,z)^{-\alpha}$, where $\theta(w,z) = \mathcal{V}(w,z)/\mathcal{U}(w,z)$ denotes the market tightness in submarket (w,z). Existing matches can get exogenously dissolved according to a Poisson process with arrival rate δ, or they can be endogenously and unilaterally dissolved by either the worker or the firm.

Wage Determination. We assume that entry wages are competitively set and constant throughout a match.

Agents' Choices. An unemployed worker's choice of a submarket (w,z) induces a stopping time τ^u, which is distributed according to a Poisson process with arrival rate $p(w,z)$. Once matched, the worker chooses the duration of the match before quitting, summarized by the stopping time τ^h, while the firm chooses the duration of the match before firing the worker, summarized by the stopping time τ^j. Given these two choices and the exogenous stopping time τ^δ, the actual duration of a match is determined by the minimum stopping time in the vector $\vec{\tau}^m = (\tau^h, \tau^j, \tau^\delta)$, which we denote by $\tau^m = \min\{\tau^h, \tau^j, \tau^\delta\}$.

Value Functions. In what follows, we describe agents' value functions, which depend on the worker's productivity z and, if matched, the match-specific wage rate w. In theory, value functions may also depend on the aggregate state, which consists of the joint distribution of workers' productivities, wages, and employment states. However, we show below that our model features a unique block recursive equilibrium, as in Shi (2009) and Menzio and Shi (2010a,b, 2011)—i.e., equilibrium objects do not depend on the distribution of workers' idiosyncratic states. Thus, we omit the aggregate state in all notation.

The value of an unemployed worker with productivity z is

$$U(z) = \max_{\{w_t\}_{t=0}^{\tau^u}} \mathbb{E}_0 \left[\int_0^{\tau^u} e^{-\rho t} B(e^{z_t})\, \mathrm{d}t + e^{-\rho \tau^u} H(w_{\tau^u}, z_{\tau_u}, \vec{\tau}^m(w_{\tau^u}, z_{\tau^u})) \right]. \tag{1}$$

That is, an unemployed worker searches for a job in submarket (w_t, z_t) at time $t \leq \tau^u$, after which she becomes employed at wage w_{τ^u} and receives the value of employment $H(w_{\tau^u}, z_{\tau_u}, \vec{\tau}^m(w_{\tau^u}, z_{\tau^u}))$.

Given a vector of stopping times $\vec{\tau}^m$, the value of a worker employed at wage w with productivity z is

$$H(w, z, \vec{\tau}^m) = \mathbb{E}_0 \left[\int_0^{\tau^m} e^{-\rho t} e^w \, \mathrm{d}t + e^{-\rho \tau^m} U(z_{\tau^m}) \right].$$

That is, an employed worker consumes a constant wage w until time τ^m, when she either endogenously or exogenously transitions to unemployment. Similarly, given a vector of stopping times $\vec{\tau}^m$, the value of a firm matched with a worker with wage wage w and productivity z is

$$J(w, z, \vec{\tau}^m) = \mathbb{E}_0 \left[\int_0^{\tau^m} e^{-\rho t} \left[e^{z_t} - e^w \right] \mathrm{d}t \right]. \tag{2}$$

That is, the match produces e^{z_t}, of which e^w is paid to the worker, until it gets dissolved at time τ^m.

Free Entry. Firms, in choosing the number of vacancies to post in each submarket, trade off the benefit of posting a vacancy—i.e., the product of the vacancy filling rate $q(w, z)$ and the value of a filled vacancy $J(w, z, \vec{\tau}^m(w, z))$—with the vacancy posting cost. In each submarket, firms post vacancies up to the point at which the marginal vacancy posting cost exceeds its expected benefits. Thus, free entry requires that

$$K(e^{z_t}) - q(w, z) J(w, z, \vec{\tau}^m(w, z)) \geq 0 \quad \forall (w, z) \tag{3}$$

and $\theta(w, z) \geq 0$, with complementary slackness, for all (w, z).

Equilibrium Definition. Having described agents' problem, we are now ready to define an equilibrium.

Definition 1. *An equilibrium consists of a set of value functions* $\{H(w, z, \vec{\tau}^m), J(w, z, \vec{\tau}^m), U(z)\}$, *market tightness* $\theta(w, z)$, *and policy functions* $\left\{\tau^{h*}(w, z), \tau^{j*}(w, z), w^*(z_t)\right\}$, *such that:*

1. *Given* $U(z)$, $(\tau^{h*}(w, z), \tau^{j*}(w, z))$ *is a Nash equilibrium. That is, all stopping times* (τ^h, τ^j) *satisfy*

$$H(w, z, \tau^{h*}(w, z), \tau^{j*}(w, z), \tau^\delta) \geq H(w, z, \tau^h, \tau^{j*}(w, z), \tau^\delta), \quad \forall (w, z) \tag{4}$$

$$J(w, z, \tau^{h*}(w, z), \tau^{j*}(w, z), \tau^\delta) \geq J(w, z, \tau^{h*}(w, z), \tau^j, \tau^\delta), \quad \forall (w, z) \tag{5}$$

   with $Pr(\tau^{m*}(w^*(z), z) = 0) = 0$.

2. *Given* $H(w, z, \vec{\tau}^{m*}(w, z))$, $U(z)$, *and* $\theta(w, z)$, $\{w^*(z_t)\}_{t=0}^{\tau^{u*}}$ *solves equation* (1).

3. *Given* $J(w, z, \vec{\tau}^{m*}(w, z))$, $\theta(w, z)$ *solves the free entry condition* (3).

Part 1 of Definition 1 requires that agents' strategies form a Nash equilibrium. That is, the worker's optimal quitting strategy τ^{h*} is a best response to the firm's firing strategy τ^{j*}, and vice versa—see equations (4)–(5). Our equilibrium definition rules out the trivial trivial Nash equilibrium, in which both the worker and the firm choose to dissolve the match immediately. Part 2 requires that unemployed workers' search strategies are optimal. Finally, Part 3 requires that free entry holds.

Commitment. The equilibrium definition reflects the two-sided lack of commitment. We can see this property by noting that the stopping times depend on the history of shocks and (τ^j, τ^h) are optimal for all (w, z). Lack of commitment affects the unemployed worker's policies through $H(w, z, \vec{\tau}^{m*}(w, z))$ as well as the market tightness through $J(w, z, \vec{\tau}^{m*}(w, z))$. Therefore, lack of commitment affects both job finding and job separation rates in the economy.

Homotheticity. Shocks to worker productivity affect agents' choices because they change the relative values of three margins: wages while employed, w, home production while unemployed, $B(Z_t)$, and vacancy posting costs, $K(Z_t)$, all relative to a worker's productivity level Z_t. In order to focus on the margin pertaining to the relative value of wages, thereby abstracting from the other two margins, we assume search costs and unemployment income are homothetic in workers' productivity. That is, $B(Z_t) = \tilde{B} Z_t$ for $\tilde{B} \in (0, 1)$ and $K(Z_t) = \tilde{K} Z_t$ for $\tilde{K} > 0$.

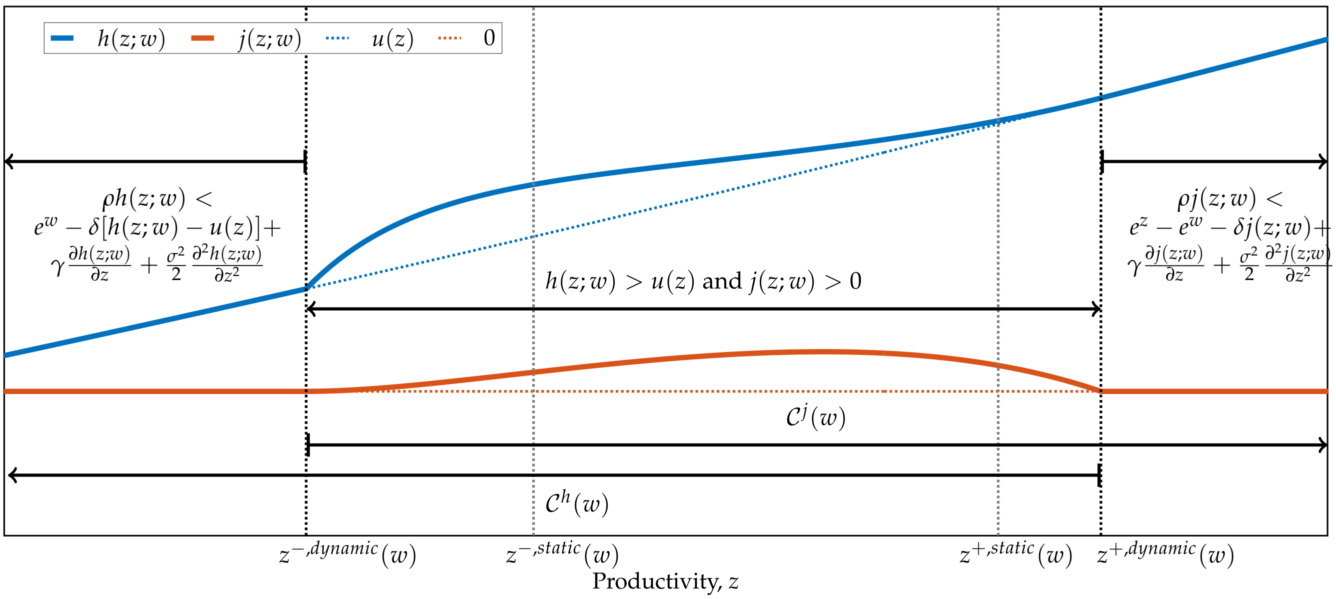
Strategic Interaction Between Workers and Firms. The strategic interaction between workers and firms has three features. First, agents play a nonzero-sum game since the value of a match e^z exceeds the value of unemployment $\tilde{B} e^z$ for $\tilde{B} < 1$. Second, agents' payoffs are stochastic and follow an Itô process. Third, agents' strategies consist of stopping times. Thus, the strategic interaction between workers and firms can be formulated as a nonzero-sum stochastic differential game with stopping times (Bensoussan and Friedman, 1977), the equilibrium of which we characterize in the next section.

Allocative Wages and Inefficient Job Separations. We refer to wages as *allocative* whenever they affect the duration of the match, that is, whenever there exist $w, w' \in \mathbb{R}$ such that $\vec{\tau}^m(w, z) \neq \vec{\tau}^m(w', z)$. Relatedly, we refer to a job separation as *inefficient* whenever a match is dissolved in spite of a positive joint match surplus $S(w, z, \vec{\tau}^m) := H(w, z, \vec{\tau}^m) + J(w, z, \vec{\tau}^m)$. In our setting, inefficient job separations will be a consequence of allocative wages, which arise due to the interaction between idiosyncratic worker productivity shocks, wage rigidity, and two-sided lack of commitment.

2.2 Equilibrium Characterization

Illustration. We describe the equilibrium conditions for a worker-firm match with the aid of Figure 1, which illustrates the equilibrium values, outside options, and policies for both agents. To this end, let $u(z)$, $h(z;w)$, and $j(z;w)$ denote the values of an unemployed worker, an employed worker, and a filled vacancy evaluated at equilibrium policies, where the index w references the constant log wage.

FIGURE 1. EQUILIBRIUM VALUES AND OPTIMAL POLICIES



Notes: The figure plots the value functions of workers and firms for a given log wage w as a function of log productivity z. The blue and red solid lines show the value functions for the worker and the firm, respectively. The blue and red dashed lines show the opportunity costs for the worker and the firm, respectively. The firm's optimal static job separation trigger based on flow profits is $z^{-,static}(w) := w$. The worker's optimal static job separation trigger under the optimal choice of submarket w^* is $z^{+,static}(w^*)$ and satisfies $e^{w^*} = \tilde{B}e^{z^{+,static}(w^*)} + p(w^*, z^{+,static}(w^*))[h(z^{+,static}; w^* - u(z^{+,static})]$. The optimal dynamic job separation triggers for the worker and the firm are $z^{+,dynamic}(w) := \sup_z\{z : h(z;w) > u(z)\}$ and $z^{-,dynamic}(w) := \inf_z\{z : j(z;w) > 0\}$, respectively. The parameter values used in the example are $(\gamma, \sigma, \rho, \alpha, \tilde{K}, \delta, \tilde{B}) = (0.00, 0.005, 0.04, 0.5, 1, 0.021, 0.55)$.

The possibility that both the worker and the firm can unilaterally walk away from a match at any point in time imposes lower bounds on the agents' values $h(z;w)$ and $j(z;w)$. Formally, *individual rationality* of the worker and the firm requires that

$$h(z;w) \geq u(z) \quad \forall z, \tag{6}$$

$$j(z;w) \geq 0 \quad \forall z. \tag{7}$$

Let $\mathcal{C}^h(w)$ denote the interior of the set of productivities for which the worker prefers to continue a match with wage w. Importantly, this set is made up of two productivity ranges: one over which both the firm and the worker opt to continue the match and one over which the firm opts to dissolve the match in spite of the worker's preference for it to continue. Let $\mathcal{C}^j(w)$ denote the analogous object for the firm. The boundaries of $\mathcal{C}^h(w)$ and $\mathcal{C}^j(w)$ are defined by both static and dynamic considerations.[4]

Under static considerations alone, the worker chooses to continue the match as long as the flow value of employment exceeds that of unemployment. This happens for productivity levels $z \leq z^{+,static}$, where $z^{+,static}$ satisfies $e^w = \tilde{B}e^{z^{+,static}} + p(w^*, z^{+,static}) \left[h(z^{+,static}; w^*) - u(z^{+,static})\right]$, given the optimal choice w^*. The range $z \in (-\infty, z^{+,static})$ can be partitioned according to the firm's choice. For $z \in (z^{-,static}, z^{+,static})$, with $z^{-,static}$ characterized below, the firm chooses to continue the match and the worker's value satisfies $h(z; w) > u(z)$. For $z \in (-\infty, z^{-,static})$, the firm chooses to dissolve the match in spite of the worker's preference for it to continue and the worker's value is $h(z; w) = u(z)$.

Analogously, under static considerations alone, the firm chooses to retain the worker as long as it makes positive flow profits, which happens for productivity levels $z \geq z^{-,static} := w$. The range $z \in (z^{-,static}, \infty)$ can be partitioned according to the worker's choice. For $z \in (z^{-,static}, z^{+,static})$, with $z^{+,static}$ characterized above, the worker chooses to remain in the match and the firm's value satisfies $j(z; w) > 0$. For $z \in (z^{+,static}, \infty)$, the worker chooses to dissolve the match in spite of the firm's preference for it to continue and the firm's value is $j(z; w) = 0$.

The decision to continue or dissolve a match also involves dynamic considerations. From the worker's perspective, quitting a job may be suboptimal in spite of the current flow value of unemployment exceeding the wage, given that future productivity could decrease. From the firm's perspective, firing the worker may be suboptimal in spite of current flow profits being negative, given that future productivity could increase. Such dynamic considerations widen agents' inaction regions to $(-\infty, z^{+,dynamic})$ for the worker and to $z^{-,dynamic}, \infty$ for the firm. Whenever the firm fires the worker, the worker prefers to continue the match and the worker's equilibrium value is increasing in expectectation:

$$\rho h(z; w) \leq e^w - \delta[h(z; w) - u(z)] + \gamma \frac{\partial h(z; w)}{\partial z} + \frac{\sigma^2}{2} \frac{\partial^2 h(z; w)}{\partial z^2}. \tag{8}$$

Analogously, whenever the worker quits, the firm prefers to continue the match and the firm's equilibrium value is increasing in expectectation:

$$\rho j(z; w) \leq e^z - e^w - \delta j(z; w) + \gamma \frac{\partial j(z; w)}{\partial z} + \frac{\sigma^2}{2} \frac{\partial^2 j(z; w)}{\partial z^2}. \tag{9}$$

[4] By static considerations, we mean agents' flow payoffs while holding fixed the productivity level. By dynamic considerations, we mean the full problem minus static considerations.

Continuation Sets. To summarize the above discussion, every match is in one of three states. First, if both agents prefer to continue the match, then conditions (6)–(7) hold. Second, if only the worker but not the firm prefer to continue, then condition (8) applies. Third, if only the firm but not the worker prefer to continue, then condition (9) applies. Thus, the continuation set for the worker is

$$\mathcal{C}^h(w) := int\left\{z \in \mathbb{R} : h(z;w) > u(z) \text{ or } \rho h(z;w) \leq e^w + \gamma\frac{\partial h(z;w)}{\partial z} + \frac{\sigma^2}{2}\frac{\partial^2 h(z;w)}{\partial z^2} - \delta\left[h(z;w) - u(z)\right]\right\}, \tag{10}$$

while the continuation set for the firm is

$$\mathcal{C}^j(w) := int\left\{z \in \mathbb{R} : j(z;w) > 0 \text{ or } \rho j(z;w) \leq e^z - e^w + \gamma\frac{\partial j(z;w)}{\partial z} + \frac{\sigma^2}{2}\frac{\partial^2 j(z;w)}{\partial z^2} - \delta j(z;w)\right\}. \tag{11}$$

In theory, agents' continuation strategies would continue to form a Nash equilibrium if one agent decided to match the other agent's decision to dissolve a match, which would result in two-sided separation thresholds for each agent. That agents' continuation sets (10)–(11), as stated, feature a separation threshold on one side but not the other follows from a trembling-hand refinement of the equilibrium.

Equilibrium Conditions. We now derive sufficient conditions for a block recursive equilibrium. First, the Hamilton-Jacobi-Bellman (HJB) equation of an unemployed worker is

$$\rho u(z) = \tilde{B}e^z + \gamma\frac{\partial u(z)}{\partial z} + \frac{\sigma^2}{2}\frac{\partial^2 u(z)}{\partial z^2} + \max_w p(w,z)[h(z;w) - u(z)]. \tag{12}$$

Next, we state the variational inequalities that characterize the worker's and the firm's values for productivity levels inside and outside the other agent's continuation set. The HJB equation of a worker employed at log wage w with log productivity $z \in \mathcal{C}^j(w)$, for which the firm prefers to continue, is

$$\rho h(z;w) = \max\left\{e^w + \gamma\frac{\partial h(z;w)}{\partial z} + \frac{\sigma^2}{2}\frac{\partial^2 h(z;w)}{\partial z^2} + \delta\left[u(z) - h(z;w)\right], \rho u(z)\right\}. \tag{13}$$

Similarly, the HJB equation of a firm employing a worker at log wage w with log productivity $z \in \mathcal{C}^h(w)$, for which the worker prefers to continue, is

$$\rho j(z;w) = \max\left\{e^z - e^w + \gamma\frac{\partial j(z;w)}{\partial z} + \frac{\sigma^2}{2}\frac{\partial^2 j(z;w)}{\partial z^2} - \delta j(z;w), 0\right\}. \tag{14}$$

On the other hand, if any one agent dissolves the match, then the other agent receives the value of their outside option. Therefore, the worker's and the firm's values of a match with log productivity z and log wage w satisfy the following conditions:

$$h(z;w) = u(z) \quad \forall z \in (\mathcal{C}^j(w))^c, \tag{15}$$

$$j(z;w) = 0 \quad \forall z \in (\mathcal{C}^h(w))^c, \tag{16}$$

where $X^c := \mathbb{R}\backslash X$. Equations (15)–(16) define the game's *value matching conditions*, which imply continuity of one agent's value function at the boundary of the other agent's continuation set. Similarly, equations (13)–(14) imply value matching at the boundary of the agent's own continuation set. A new match with productivity z continues if the intersection of both agents' continuation sets at the optimal entry wage $w^*(z)$ is nonempty:

$$\mathcal{C}^h(w^*(z)) \cap \mathcal{C}^j(w^*(z)) \neq \emptyset.$$

The worker quits and the firm fires the worker at times τ^h and τ^j, respectively. These stopping times denote the stochastic time at which productivity falls outside of the worker's and the firm's respective continuation sets. Agents' optimal stopping times are given by

$$\begin{aligned} \tau^{j*}(w,z) &= \inf\left\{t \geq 0 : z_t \in (\mathcal{C}^j(w))^c, z_0 = z\right\}, \\ \tau^{h*}(w,z) &= \inf\left\{t \geq 0 : z_t \in (\mathcal{C}^h(w))^c, z_0 = z\right\}. \end{aligned} \tag{17}$$

The optimality of workers' search decisions implies that the competitive entry wage satisfies

$$w^*(z) = \arg\max_w \theta(w,z)^{1-\alpha}\left[h(z;w) - u(z)\right]. \tag{18}$$

Finally, free entry requires that the equilibrium market tightness satisfies

$$K(e^z) - q(w,z)j(z;w) \geq 0 \quad \forall (w,z) \tag{19}$$

and $\theta(w,z) \geq 0$, with complementary slackness, for all (w,z).

The following lemma summarizes the sufficient conditions for a block recursive equilibrium.

Lemma 1. *Suppose that the value functions $\{u(z), h(z;w), j(z;w)\}$ and market tightness $\theta(w,z)$ satisfy the HJB equations (12)–(14), the game's value matching conditions (15)–(16) given the continuation sets (10)–(11), and the free entry condition (19). Then the policy functions $\{\tau^{h*}, \tau^{j*}, w^*(z)\}$ given by (17)–(18) together with the value functions $\{U(z), H(w,z,\vec{\tau}^m), J(w,z,\vec{\tau}^m)\}$ given by (1)–(2) and market tightness $\theta(w,z)$ form a block recursive* *equilibrium with*

$$u(z) = U(z),$$
$$h(z; w) = H(w, z, \tau^{h*}(w, z), \tau^{j*}(w, z), \tau^{\delta}),$$
$$j(z; w) = J(w, z, \tau^{h*}(w, z), \tau^{j*}(w, z), \tau^{\delta}).$$

Proof. All proofs are contained in the Online Appendix. □

It is worth noting that we do not expect to find a *classical solution* to each of the HJB equations (12)–(14).[5] In our setting, two-sided lack of commitment gives rise to nondifferentiable kinks in agents' value functions. Thus, we are interested in a weaker notion of solution to each of the HJB equations (12)–(14), namely the *viscosity solution* (Crandall and Lions, 1983).

Equilibrium Existence and Uniqueness. Equipped with the equilibrium conditions summarized in Lemma 1, we now demonstrate the existence and uniqueness of a block recursive equilibrium.

Proposition 1. *There exists a unique block recursive equilibrium.*

While the result stated in Proposition 1 is essential for any model of directed search with private inefficiency (i.e., some of agents' choices lower the joint value of a match), it does not directly follow from existing results. Theorems for the existence of a block recursive equilibrium with exogenous job separations rely on Schauder's fixed point theorem—see, for instance, Menzio and Shi (2010a,b) or Schaal (2017). Two conditions are critical for Schauder's fixed point theorem to apply: continuity in the value functions and continuity in the mapping between value functions characterizing the block recursive equilibrium. In our setting, idiosyncratic worker productivity shocks, wage rigidity, and two-sided lack of commitment jointly generate endogenous job separations, which break the regularity conditions on which traditional arguments in discrete time (e.g., Menzio and Shi, 2010a,b; Schaal, 2017) rely.

To see how private inefficiency violates the conditions needed to prove the existence and uniqueness of an equilibrium in discrete time, we focus on continuity of the value functions. Starting from a productivity level such that the firm is indifferent between firing or retaining the worker at a given wage, a marginal reduction in productivity leads the firm to fire the worker. In doing so, the firm's value remains unchanged but the worker's value discontinuously decreases. To see this, note that the negotiated wage must have been strictly higher than the flow value of unemployment, and the continuation value of employment is weakly higher than the value of unemployment. In contrast, our continuous-time setup gets around this

[5] A classical solution to an ordinary differential equation is differentiable as many times as needed to satisfy the relevant equation—i.e., twice for the HJB equations (12)–(14).

complication due to the value matching conditions guaranteeing continuity of agents' value functions, which is necessary to prove the existence and uniqueness of a block recursive equilibrium.

2.3 Understanding the Mechanisms

Having described the equilibrium conditions, we proceed to characterize the mechanisms that determine workers' and firms' equilibrium behavior.

Finding the state. To understand the dependence of equilibrium outcomes on the state variables, we first note that we can recast the equilibrium conditions in terms of a smaller state space. Since the flow income of unemployed workers and firms' vacancy costs are both proportional to productivity, Z, it turns out that the relevant state variable for both workers and firms is the log wage-to-productivity ratio, $\hat{w} := w - z$. This result allows us to express agents' values and policies as functions of the scalar $\hat{w}$ instead of the duplet (w, z). To simplify notation, we define the transformed drift $\hat{\gamma} := \gamma + \sigma^2$ and the transformed discount factor $\hat{\rho} := \rho - \gamma - \sigma^2/2$.

Lemma 2. *Suppose that the functions $(u(z), h(z; w), j(z; w), \theta(w, z))$ satisfy the equilibrium conditions in (12)–(14) and (19), given the continuation sets $\mathcal{C}^h(w)$ and $\mathcal{C}^j(w)$ defined in (10)–(11). Then the transformed value functions and market tightness given by*

$$(\hat{U}, \hat{J}(w - z), \hat{W}(w - z), \hat{\theta}(w - z)) = \left(\frac{u(z)}{e^z}, \frac{j(z; w)}{e^z}, \frac{h(z; w) - u(z)}{e^z}, \theta(w, z) \right)$$

equivalently characterize the equilibrium if the following conditions are satisfied:

1. *The transformed value function of an unemployed worker, $\hat{U}$, satisfies*

$$\hat{\rho}\hat{U} = \tilde{B} + \hat{\theta}(\hat{w}^*)^{1-\alpha}\hat{W}(\hat{w}^*), \tag{20}$$

where the optimal choice of submarket for an unemployed worker to search in is $\hat{w}^* = w^*(z) - z$.

2. *The lower bounds of the game's values for workers and firms are:*

$$\hat{W}(\hat{w}) \geq 0, \tag{21}$$

$$\hat{J}(\hat{w}) \geq 0. \tag{22}$$

3. *The variational inequalities for workers and firms are satisfied: Given*

$$\hat{\mathcal{C}}^h := int\left\{\hat{w} \in \mathbb{R} : \hat{W}(\hat{w}) > 0 \text{ or } (\hat{\rho}+\delta)\hat{W}(\hat{w}) \leq e^{\hat{w}} - \hat{\rho}\hat{U} - \hat{\gamma}\hat{W}'(\hat{w}) + \frac{\sigma^2}{2}\hat{W}''(\hat{w})\right\},$$
$$\hat{\mathcal{C}}^j := int\left\{\hat{w} \in \mathbb{R} : \hat{J}(\hat{w}) > 0 \text{ or } (\hat{\rho}+\delta)\hat{J}(\hat{w}) \leq 1 - e^{\hat{w}} - \hat{\gamma}\hat{J}'(\hat{w}) + \frac{\sigma^2}{2}\hat{J}''(\hat{w})\right\},$$

the transformed value function of an employed worker, $\hat{W}(\hat{w})$, and that of a filled vacancy, $\hat{J}(\hat{w})$, satisfy

$$(\hat{\rho}+\delta)\hat{W}(\hat{w}) = \max\{e^{\hat{w}} - \hat{\rho}\hat{U} - \hat{\gamma}\hat{W}'(\hat{w}) + \frac{\sigma^2}{2}\hat{W}''(\hat{w})\,,0\},\ \forall \hat{w} \in \hat{\mathcal{C}}^j, \tag{23}$$
$$(\hat{\rho}+\delta)\hat{J}(\hat{w}) = \max\{1 - e^{\hat{w}} - \hat{\gamma}\hat{J}'(\hat{w}) + \frac{\sigma^2}{2}\hat{J}''(\hat{w})\,,0\},\ \forall \hat{w} \in \hat{\mathcal{C}}^h, \tag{24}$$

with $\tau^{j*} = \inf\{t \geq 0 : \hat{w}_t \notin \hat{\mathcal{C}}^j, w_0 = \hat{w}^*\}$ and $\tau^{h*} = \inf\{t \geq 0 : \hat{w}_t \notin \hat{\mathcal{C}}^h, w_0 = \hat{w}^*\}$.

4. *The value matching conditions are satisfied:*

$$\hat{W}(\hat{w}) = 0 \quad \forall \hat{w} \in (\hat{\mathcal{C}}^j)^c, \tag{25}$$
$$\hat{J}(\hat{w}) = 0 \quad \forall \hat{w} \in (\hat{\mathcal{C}}^h)^c. \tag{26}$$

5. *The free entry condition for $\hat{\theta}(\hat{w})$ is satisfied:*

$$\tilde{K} - \hat{\theta}(\hat{w})^{-\alpha}\hat{J}(\hat{w}) \geq 0 \tag{27}$$

and $\hat{\theta}(\hat{w}) \geq 0$, with complementary slackness.

6. *Trivial Nash equilibria are ruled out:*

$$\hat{\mathcal{C}}^h \cap \hat{\mathcal{C}}^j \neq \emptyset. \tag{28}$$

The equilibrium conditions in Lemma 2 are transformed versions of those stated above and follow similar intuitions. Equation (20) of Part 1 of the lemma encodes the payoff under the optimal log wage-to-productivity ratio of newly employed workers. For an unemployed worker, the optimal wage w^* trades off the job finding rate $\hat{\theta}(\hat{w}^*)^{1-\alpha}$ with the value of employment $\hat{W}(\hat{w}^*)$. Equations (21)–(22) of Part 2 describe the lower bounds on agents' transformed values. From equations (23)–(24) of Part 3, we can infer the thresholds that render the worker's and the firm's respective transformed flow payoffs negative. If $e^{\hat{w}} < \hat{\rho}\hat{U}$ then the worker's wage is below the flow value of unemployment. Similarly, if $e^{\hat{w}} > 1$, then the firm's flow profits are negative. Equations (26)–(25) of Part 4 state the transformed value matching conditions. Equation (27) of Part 5 states the transformed free entry condition. Finally, equation (28) of part 6 rules out trivial Nash equilibria of the game in its transformed notation.

Equilibrium Policies. Based on the transformed state variable $\hat{w}$ and equilibrium conditions (20)–(28), we can characterize agents' equilibrium policies. Recalling the definition of the transformed state variable $\hat{w} := w - z$, we postulate that there are exist optimal policies $\hat{w}^- < \hat{w}^* < \hat{w}^+$, where $\hat{w}^-$ is the worker's optimal job separation threshold, $\hat{w}^*$ is the optimal value of $\hat{w}$ at match formation, and $\hat{w}^+$ is the firm's optimal job separation threshold. We define the transformed surplus of the match as $\hat{S}(\hat{w}) := \hat{J}(\hat{w}) + \hat{W}(\hat{w})$ and the worker's share of the transformed surplus as $\eta(\hat{w}) := \hat{W}(\hat{w})/\hat{S}(\hat{w})$. The following proposition characterizes properties of the block recursive equilibrium in its transformed notation.

Proposition 2. *The block recursive equilibrium has the following properties:*

1. *The joint match surplus satisfies*

$$\hat{S}(\hat{w}) = (1 - \hat{\rho}\hat{U})\mathcal{T}(\hat{w}, \hat{\rho}) > 0 \quad \forall \hat{w} \in (\hat{w}^+, \hat{w}^-), \tag{29}$$

where

$$\mathcal{T}(\hat{w}, \hat{\rho}) := \mathbb{E}\left[\int_0^{\tau^{m*}} e^{-\hat{\rho}t}\, \mathrm{d}t | \hat{w}_0 = \hat{w}\right] \tag{30}$$

is the expected discounted match duration and $1 > \hat{\rho}\hat{U} > \tilde{B}$.

2. *The competitive entry wage* $\hat{w}^*$ *coincides with the Nash bargaining solution with worker's weight* α*:*

$$\hat{w}^* = \arg\max_{\hat{w}} \left\{ \hat{W}(\hat{w})^\alpha \hat{J}(\hat{w})^{1-\alpha} \right\} = \arg\max_{\hat{w}} \left\{ \eta(\hat{w})^\alpha (1 - \eta(\hat{w}))^{1-\alpha} \mathcal{T}(\hat{w}, \hat{\rho}) \right\}, \tag{31}$$

with optimality condition

$$\underbrace{\eta'(\hat{w}^*)\left(\frac{\alpha}{\eta(\hat{w}^*)} - \frac{1-\alpha}{1-\eta(\hat{w}^*)}\right)}_{\textit{Share channel}} = -\underbrace{\frac{\mathcal{T}_{\hat{w}}(\hat{w}^*, \hat{\rho})}{\mathcal{T}(\hat{w}^*, \hat{\rho})}}_{\textit{Surplus channel}}. \tag{32}$$

3. *Given* $\eta(\hat{w}^*)$ *and* $\mathcal{T}(\hat{w}^*, \hat{\rho})$*, the equilibrium job finding rate* $p(\hat{w}^*)$ *and the flow opportunity cost of employment* $\hat{\rho}\hat{U}$ *are given by*

$$p(\hat{w}^*) = \left[(1 - \eta(\hat{w}^*))(1 - \hat{\rho}\hat{U})\mathcal{T}(\hat{w}^*, \hat{\rho})/\tilde{K}\right]^{\frac{1-\alpha}{\alpha}}, \tag{33}$$

$$\hat{\rho}\hat{U} = \tilde{B} + \left(\tilde{K}^{\alpha-1}\left(1-\eta(\hat{w})\right)^{1-\alpha}\eta(\hat{w})^{\alpha}\left(1-\hat{\rho}\hat{U}\right)\mathcal{T}(\hat{w}^*,\hat{\rho})\right)^{\frac{1}{\alpha}}. \tag{34}$$

4. *Assume $\gamma \neq 0$ or $\sigma \neq 0$. Given $\hat{U}$, the worker's and the firm's continuation sets are connected and bounded and satisfy:*

$$\{\hat{w} : \hat{w} > \log(\hat{\rho}\hat{U})\} \subset \{\hat{w} : \hat{w} > \hat{w}^-\} = \hat{\mathcal{C}}^h, \tag{35}$$

$$\{\hat{w} : \hat{w} < 0\} \subset \{\hat{w} : \hat{w} < \hat{w}^+\} = \hat{\mathcal{C}}^j, \tag{36}$$

where the worker's and firm's value functions satisfy smooth pasting conditions *at $\hat{w}^-$ and $\hat{w}^+$, respectively:*

$$\hat{W}'(\hat{w}^-) = \hat{J}'(\hat{w}^+) = 0. \tag{37}$$

Starting with Part 1 of Proposition 2, equation (29) states that the surplus of the match is equal to the product between the transformed flow surplus $1 - \hat{\rho}\hat{U}$ and the expected discounted match duration $\mathcal{T}(\hat{w},\hat{\rho})$ defined in equation (30), which depends on the widths of the agents' continuation sets, $\hat{w}^* - \hat{w}^-$ and $\hat{w}^+ - \hat{w}^*$. Additionally, the flow opportunity cost of employment $\hat{\rho}\hat{U}$ is bounded between one (i.e., the value of output in the match) and $\tilde{B}$ (i.e., the value of home production). As $1 > \hat{\rho}\hat{U}$, the joint match surplus is always strictly positive—thus, all endogenous job separations are inefficient.

Equations (31)–(32) of Part 2 shows that the competitive entry wage $\hat{w}^*$ balances a *share channel* and a *surplus channel*. Unemployed workers search for wages that are competitively set in a way that coincides with the Nash bargaining solution with worker's weight α, thereby satisfying the well-known efficiency condition due to Hosios (1990). This result obtains due to the free entry condition, which implies that a worker's job finding rate is proportional to the value of a firm. A larger initial wage increases the worker's share by $\eta'(\hat{w}^*)\alpha/\eta(\hat{w}^*)$ but at the same time reduces the job finding probability by $\eta'(\hat{w}^*)(1-\alpha)/(1-\eta(\hat{w}^*))$. This trade-off is reflected in the share channel and is standard in models with directed search (e.g., Moen, 1997; Menzio and Shi, 2010a).

With allocative wages, a novel surplus channel arises. Intuitively, the surplus channel captures the fact that the wage set at match formation affects the expected match duration and therefore the expected surplus. The higher the entry wage, the sooner the firm will dissolve the match in expectation. Conversely, the lower the entry wage, the sooner the worker will dissolve the match in expectation. Only if $\mathcal{T}_{\hat{w}}(\hat{w}^*,\hat{\rho}) = 0$ will the worker's share of the surplus equal $\eta(\hat{w}^*) = \alpha$, as in efficient models with nonallocative wages. These considerations are unique to our environment with allocative wages.

Part 3 characterizes the unemployed worker's job finding rate (33) and the flow opportunity cost of employment (34) as functions of the worker's surplus share and the expected discounted match duration.

Part 4 shows that the continuation set of the worker (35) and that of the firm (36) follow a threshold rule in the log wage-to-productivity ratio $\hat{w}$. Workers refrain from quitting as long as $\hat{w} > \hat{w}^-$, while firms refrain from firing the worker as long as $\hat{w} < \hat{w}^+$. Thus, the continuation set for the match is given by $\hat{\mathcal{C}}^h \cap \hat{\mathcal{C}}^j = (\hat{w}^-, \hat{w}^+)$. These thresholds satisfy $\hat{w}^- < \log(\hat{\rho}\hat{U})$ and $\hat{w}^+ > 0$, reflecting the fact that both parties are willing to accept flow payoffs below that from their respective outside option. Finally, the smooth pasting conditions in (37) apply at the worker's quitting trigger $\hat{w}^-$ and at the firm's firing trigger $\hat{w}^+$, reflecting the optimality of agents' continuation policies.

Static Considerations. Before further characterizing the dynamic problem, it is instructive to consider equilibrium policies when productivity is fixed—i.e., $\gamma = \sigma = 0$.[6] The following proposition characterizes the static considerations in this case.

Proposition 3. *Assume $\gamma = \sigma = 0$. Then optimal policies are given by*

$$(\hat{w}^-, \hat{w}^*, \hat{w}^+) = \log(\hat{\rho}\hat{U}, \alpha + (1-\alpha)\hat{\rho}\hat{U}, 1),$$

with $\eta(\hat{w}^*) = \alpha$ and $\mathcal{T}(\hat{w}^*, \hat{\rho}) = 1/(\hat{\rho} + \delta)$.

Note that $\hat{w}^- < \hat{w}^* < \hat{w}^+$ and $\hat{w} = \hat{w}^*$ for the duration of the match, absent productivity fluctuations, so there are no endogenous job separations. From this, we see that lack of commitment and wage rigidity by themselves do not generate any inefficient job separations. Absent productivity fluctuations, agents' behavior is privately efficient in that it maximizes the joint match surplus.

In addition to the forces outlined in this static example, two important dynamic incentives guide workers' and firms' choices, namely the *option value effect* and the *anticipatory effect*.

Dynamic Considerations I: The Option Value Effect. To understand the role of productivity fluctuations in creating the option value effect, we assume away, for now, the drift of worker productivity—i.e., $\hat{\gamma} = 0$. The following proposition characterizes the option value effect in this case.

Proposition 4. *Assume $\hat{\gamma} = 0$ and $\alpha = 1/2$. Then, to a first-order approximation, the optimal entry wage is given by $\hat{w}^* = \log\left((1 + \tilde{\rho}\tilde{U})/2\right)$ and the job separation triggers satisfy $\hat{w}^{\pm} = \hat{w}^* \pm h(\varphi, \Phi)$ for some function $h(\varphi, \Phi)$ with $\varphi := \sqrt{2(\rho + \delta)}/\sigma$ and $\Phi := (1 - \tilde{\rho}\tilde{U})/(1 + \tilde{\rho}\tilde{U})$. The following properties apply:*

1. *$h(\varphi, \Phi)$ is decreasing in φ.*

2. $\lim_{\varphi \to 0} h(\varphi, \Phi) = 3\Phi$.

[6] Observe that if $\gamma = \sigma = 0$, then the smooth pasting conditions do not apply.

3. $\lim_{\varphi\to\infty} h(\varphi,\Phi) = \Phi$*.*

4. $h(\varphi,\Phi)$ *is increasing in* Φ*.*

5. $\varphi h(\varphi,\Phi)$ *is increasing in* φ*.*

Furthermore, the equilibrium surplus share is $\eta(\hat{w}) = \alpha = 1/2$ *and the expected discounted match duration*

$$\mathcal{T}(\hat{w}^*,\hat{\rho}) = \frac{1 - 2\left(e^{\varphi h(\varphi,\Phi)} + e^{-\varphi h(\varphi,\Phi)}\right)^{-1}}{\tilde{\rho}+\delta}, \tag{38}$$

is increasing in φ *and* Φ *and satisfies* $\mathcal{T}_{\hat{w}}(\hat{w}^*,\hat{\rho}) = 0$*.*

Proposition 4 demonstrates that idiosyncratic volatility, by itself, does not affect the split of the match surplus between the worker and the firm. Such an econonmy is symmetric in the sense that $\mathcal{T}_{\hat{w}}(\hat{w}^*,\rho) = 0$ and $\eta(\hat{w}) = \alpha$. Thus, a larger $\hat{w}^*$ reduces the match duration by increasing the likelihood of a layoff but increases the match duration by reducing the likelihood of a quit. Weighing both forces, $\mathcal{T}(\cdot,\rho)$ is maximized at $\hat{w}^* = (1+\tilde{\rho}\tilde{U})/2$ and $\eta(\hat{w}^*) = 1/2$.

This result provides a tight characterization of the worker's and the firm's optimal policy functions, which result in the continuation region of the match $(\hat{w}^-,\ \hat{w}^+)$ being symmetrically centered around the optimal entry wage $\hat{w}^*$. Second, the width of the continuation region is increasing in the volatility σ (Part 1) and decreasing in $\hat{\rho}\hat{U}$ (Part 4). The width of the inaction region increases with σ due to the option value effect: Although the worker's productivity might be low today, the firm is willing to wait before firing the worker in case productivity improves in the future. The width of the inaction region decreases with $\hat{\rho}\hat{U}$ because a higher opportunity cost of employment makes it less attractive to delay job separations.

However, our model features a departure from canonical models of inaction (e.g., Dixit, 1991). In those models, the width of the continuation region typically grows unboundedly with the level of volatility σ. Instead, in our model, the width of the continuation region has an upper bound (Part 2). To see the intuition behind this result, consider the problem of a firm that finds itself in a match with negative flow profits. The marginal benefit from remaining in a currently unprofitable match is that, with some probability in the future, productivity increases enough to make the match profitable by rendering the wage-to-productivity ratio less than unity. At the same time, inaction on part of the firm is risky for two reasons. First, productivity may fall, in which case the match value declines further, eventually leading the firm to fire the worker. Second, productivity may increase by a large enough amount for the worker to choose to quit. Given the two job separation triggers, as the volatility goes to infinity, the probability of remaining in the profitable part of the inaction region approaches zero. Thus, two-sided lack of commitment imposes an upper bound on the option value associated with remaining in a match with negative flow profits.

The inefficiency generated by the lack of commitment also manifests itself in the expected duration of the match given by equation (38). It is easy to see that a small and bounded option value effect, as indexed by $h(\varphi, \Phi)$, implies a lower expected duration as the volatility increases.

Dynamic Considerations II: The Anticipatory Effect. To understand the role of a nonzero drift of productivity in generating the anticipatory effect, we assume away, for now, the volatility of worker productivity—i.e., $\sigma = 0$—and focus on the case with weakly positive drift—i.e., $\hat{\gamma} \geq 0$. The following proposition characterizes the anticipatory effect in this case.

Proposition 5. *Assume $\sigma = 0$ and $\hat{\gamma} \geq 0$. Then, $\hat{w}^- = \log\left(\hat{\rho}\hat{U}\right)$ and*

$$w^* = \hat{w}^- + \tilde{T}\left(\frac{\alpha + (1-\alpha)\hat{\rho}\hat{U}}{\hat{\rho}\hat{U}}, \frac{\hat{\rho}+\delta}{\gamma}, \frac{(1-\alpha)(1-\hat{\rho}\hat{U})}{\hat{\rho}\hat{U}}\right),$$

where $\tilde{T}(\cdot)$ is increasing in the first argument and decreasing in the second argument—see equation (B.28) in the Online Appendix for its implicit definition. Moreover,

1. *If $\hat{\gamma} = 0$, then*

$$\left(\tilde{T}(\cdot), \mathcal{T}(\hat{w}^*, \hat{\rho}), \eta(\hat{w}^*)\right) \to \left(\log\left(\frac{\alpha + (1-\alpha)\hat{\rho}\hat{U}}{\hat{\rho}\hat{U}}\right), \frac{1}{\hat{\rho}+\delta}, \alpha\right).$$

2. *If $\hat{\gamma} \to \infty$, then $\tilde{T}(\cdot) \to \tilde{T}^{limit}$, $\mathcal{T}(\hat{w}^*, \hat{\rho}) \to 0$, and $\eta(\hat{w}^*) \to \eta^{limit}$, where $\tilde{T}^{limit}$ and η^{limit} are implicitly defined as*

$$\frac{\alpha + (1-\alpha)\hat{\rho}\hat{U}}{\hat{\rho}\hat{U}} = \frac{e^{\tilde{T}^{limit}} - 1 - \frac{(1-\alpha)(1-\hat{\rho}\hat{U})}{\hat{\rho}\hat{U}}\left(1 - \frac{\tilde{T}^{limit}}{e^{\tilde{T}^{limit}}-1}\right)}{\tilde{T}^{limit}},$$
$$\eta^{limit} = \alpha + \frac{1-\alpha}{\tilde{T}^{limit}} \frac{(1-\hat{\rho}\hat{U})\eta^{limit}}{\eta^{limit} + \hat{\rho}\hat{U}(1-\eta^{limit})}. \tag{39}$$

When productivity grows at a constant rate, the job separation trigger $\hat{w}^-$ equals the static opportunity cost of employment. The novel mechanism is embedded in the entry wage $\hat{w}^*$ and, therefore, in the function $\tilde{T}(\cdot)$. From Proposition 5, we can see that $\hat{w}^*$ is increasing in the Nash bargaining target and also in the drift. We refer to the latter as the anticipatory effect: Workers anticipate a higher future productivity and modify their search strategy accordingly. Two limiting cases illustrate this point.

As $\gamma \to 0$ (Part 1), the equilibrium entry wage $\hat{w}^*$ is the same as in the case without drift; thus, $\eta(\hat{w}^*) = \alpha$. As the drift increases, the workers partially compensate for it by searching for a job with a higher entry wage. Therefore, the average wage in the economy increases above the Nash bargaining target—recall that $\hat{w}^-$ remains fixed. This results from the worker internalizing the trade-off that a higher wage implies a sub-optimal job finding rate but lowers the frequency of inefficient job separations. As the drift increases unboundedly (Part 2), the entry wage w^* becomes unresponsive to the drift because the job finding rate becomes so small that it starts to dominate. Finally, as we can see in (39), the anticipatory effect makes the worker's share of surplus increase in the drift.

Relative to the case with no drift, the worker's lack of commitment decreases the value of searching for a job, which is captured by the null response of $\hat{w}^-$ to changes in the drift. To understand this, assume that the worker commits to any given $\hat{w}^-$ and $\delta \to 0$. Under these assumptions, the job separation rate is given by $s = \hat{\gamma}/(w^* - \hat{w}^-)$. Thus, the worker minimizes the frequency of inefficient job separations by increasing the width of $w^* - \hat{w}^-$, which is captured by the surplus channel. At the same time, workers choose an entry wage that takes into account the trade-off captured by the share channel. Let $T^* := (\hat{w}^* - \hat{w}^-)/\hat{\gamma}$. If $\mathcal{T}_{\hat{w}}(\hat{w}^*, \hat{\rho}) \approx 0$, then the optimality condition for the entry wage implies

$$\frac{\int_0^{T^*} e^{\hat{\rho}t + \hat{w}^* - \hat{\gamma}t}\, \mathrm{d}t - \hat{\rho}\hat{U} \int_0^{T^*} e^{\hat{\rho}t}\, \mathrm{d}t}{\left(1 - \hat{\rho}\hat{U}\right) \int_0^{T^*} e^{\hat{\rho}t}\, \mathrm{d}t} = \eta(\hat{w}^*) = \alpha,$$

$$\iff \qquad \alpha + (1-\alpha)\hat{\rho}\hat{U} = \frac{\int_0^{T^*} e^{\hat{\rho}t + \hat{w}^* - \hat{\gamma}t}\, \mathrm{d}t}{\int_0^{T^*} e^{\hat{\rho}t}\, \mathrm{d}t} \to_{\hat{\rho}\to 0} \frac{e^{\hat{w}^*} - e^{\hat{w}^-}}{\hat{w}^* - \hat{w}^-} \approx 1 + \frac{\hat{w}^* + \hat{w}^-}{2},$$

which follows from the definition of $\eta(\hat{w}^*)$ and the second-order approximation $e^x \approx 1 + x + x^2/2$.

The share channel leads to an entry wage that replicates the solution to a Nash bargaining problem with worker weight α. As discounting vanishes, this wage is approximately linear in $\hat{w}^* + \hat{w}^-$. For a given $\hat{w}^-$, the worker has only the choice of $\hat{w}^*$ to achieve two opposing objectives: increase $\hat{w}^*$ to avoid inefficient job separations (i.e., the surplus channel) or keep $\hat{w}^*$ close to the Nash bargaining target (i.e., the share channel). Thus, lack of commitment distorts both the expected duration of the match and the equilibrium job finding rates.

3 Identifying the Microeconomic Implications of Allocative Wages

This section proceeds in two steps. First, we show that the prevalence of inefficient job separations in our model critically depends on moments of the distribution of wage-to-productivity ratios $\hat{w}$. Second, we demonstrate how to use microdata on wage changes and worker transitions between jobs to recover the unobserved distribution of wage-to-productivity ratios.

Notation. Our model has a set of testable implications. First, agents' policies imply transitions from employment to unemployment at rate s, from unemployment to employment at rate $p(\hat{w}^*)$, and a level of aggregate employment $\mathcal{E}$. Second, the model predicts a joint distribution over the duration of completed employment spells τ^m, the duration of completed unemployment spells τ^u, and the log wage change between consecutive job spells Δw. We denote the joint distribution of $(\tau^m, \tau^u, \Delta w)$ with $l(\tau^m, \tau^u, \Delta w)$ and the marginal distribution of each variable with $l^m(\tau^m)$, $l^u(\tau^u)$, and $l^w(\Delta w)$. Let $\mathcal{D} := \{\mathcal{E},\ s,\ p(\hat{w}^*),\ l(\tau^m,\ \tau^u,\ \Delta w)\}$ summarize the model's observable implications in the data. Finally, we define $\tau := \tau^m + \tau^u$ as the time elapsed between the starting dates of two consecutive jobs, and we use $\mathbb{E}_{\mathcal{D}}[\cdot]$ to denote the expectation operator under the distribution $l(\tau^m,\ \tau^u,\ \Delta w) \in \mathcal{D}$.

Before proceeding, it will be useful to find the minimum model ingredients needed to characterize $\mathcal{D}$. In principle, we could characterize $\mathcal{D}$ as a function of the joint distribution of workers' employment states, wages, and productivities. In practice, given the parameters guiding the stochastic process of a worker's productivity, all that is needed to characterize $\mathcal{D}$ is the distribution of the negative sum of worker productivity shocks since the beginning of a spell of employment or unemployment. We denote this variable by Δz and refer to it as *cumulative productivity shocks*. [7] Using cumulative productivity shocks as the state variable has three advantages: (i) it is unidimensional; (ii) it is well-defined during spells of employment and unemployment; (iii) it follows a stationary distribution. By definition of Δz, its law of motion is given by $\mathrm{d}\Delta z = -\gamma\, \mathrm{d}t + \sigma\, \mathrm{d}\mathcal{W}^z_t$.

Let $g^h(\Delta z)$ and $g^u(\Delta z)$ be the distributions of Δz across employed and unemployed workers, respectively. The support of $g^h(\Delta z)$ is given by $[-\Delta^-, \Delta^+]$, where $\Delta^- := \hat{w}^* - \hat{w}^-$ and $\Delta^+ := \hat{w}^+ - \hat{w}^*$. We denote by $\mathbb{E}_h[\cdot]$ and $\mathbb{E}_u[\cdot]$ the expectation operators under the distributions $g^h(\Delta z)$ and $g^u(\Delta z)$, respectively. Let $\mathcal{M} = \{g^h(\Delta z), g^u(\Delta z), \gamma, \sigma\}$ denote the set of model objects sufficient to characterize $\mathcal{D}$. Online Appendix C provides the analytical mapping from model objects $\mathcal{M}$ to $\mathcal{D}$ in the data. Here, our goal is to link $\mathcal{M}$ to the prevalence of inefficient job separations and to deduce the elements in $\mathcal{M}$ from objects $\mathcal{D}$ that are measurable in labor market microdata.

Characterizing the Equilibrium Distributions of Cumulative Productivity Shocks $g^h(\Delta z)$ and $g^u(\Delta z)$. The equilibrium policies $(\hat{w}^-, \hat{w}^*, \hat{w}^+)$ together with the stochastic process guiding Δz and the exogenous job separation rate, determine the equilibrium distributions of cumulative productivity shocks $g^h(\Delta z)$

[7] Formally, the negative of cumulative productivity shocks of worker i at time t are $\Delta z_{it} := z_{it_0} - z_{it}$, where t_0 denotes the beginning of the current spell of employment or unemployment. Note that this reflects the negative sum of productivity changes since t_0.

and $g^u(\Delta z)$. Due to the law of motion for Δz being independent of the worker's employment state, the Kolmogorov forward equations (KFEs) for employed and unemployed workers are

$$\delta g^h(\Delta z) = \gamma (g^h)'(\Delta z) + \frac{\sigma^2}{2}(g^h)''(\Delta z) \quad \forall \Delta z \in (-\Delta^-, \Delta^+) \backslash \{0\}, \tag{40}$$

$$p(\hat{w}^*) g^u(\Delta z) = \gamma (g^u)'(\Delta z) + \frac{\sigma^2}{2}(g^u)''(\Delta z) \quad \forall \Delta z \in \mathbb{R} \backslash \{0\}. \tag{41}$$

Here, δ is the exogenous exit rate of employed workers and $p(\hat{w}^*)$ the job finding rate of unemployed workers. Since the entry state for a newly employed or unemployed worker is $\Delta z = 0$, the KFEs (40)–(41) do not hold at this point, but $g^h(\cdot)$ and $g^u(\cdot)$ must be continuous there.

The boundary conditions impose a zero measure of workers at the borders of the support,

$$g^h(-\Delta^-) = g^h(\Delta^+) = 0,$$
$$\lim_{\Delta z \to -\infty} g^u(\Delta z) = \lim_{\Delta z \to \infty} g^u(\Delta z) = 0.$$

These distributions must also be consistent with (i) a unit measure of workers, and (ii) a flow balance equation implying constant steady-state employment:

$$1 = \int_{-\infty}^{\infty} g^u(\Delta z)\, \mathrm{d}\Delta z + \int_{-\Delta^-}^{\Delta^+} g^h(\Delta z)\, \mathrm{d}\Delta z, \tag{42}$$

$$\underbrace{p(\hat{w}^*)(1-\mathcal{E})}_{u\text{-to-}h \text{ flows}} = \underbrace{\delta \mathcal{E} + \frac{\sigma^2}{2}\left[\lim_{\Delta z \downarrow -\Delta^-} (g^h)'(\Delta z) - \lim_{\Delta z \uparrow \Delta^+} (g^h)'(\Delta z)\right]}_{h\text{-to-}u \text{ flows}}. \tag{43}$$

In equation (42), the unit measure of workers is composed of $\int_{-\infty}^{\infty} g^u(\Delta z)\, \mathrm{d}\Delta z = 1 - \mathcal{E}$ unemployed and $\int_{-\Delta^-}^{\Delta^+} g^h(\Delta z)\, \mathrm{d}\Delta z = \mathcal{E}$ employed workers. In equation (43), the mass of u-to$h$ flows is $p(\hat{w}^*)(1-\mathcal{E})$, while the mass of h-to-u flows is $\delta\mathcal{E} + \frac{\sigma^2}{2}\left[\lim_{\Delta z \downarrow -\Delta^-} (g^h)'(\Delta z) - \lim_{\Delta z \uparrow \Delta^+} (g^h)'(\Delta z)\right]$—i.e., the sum of exogenous and endogenous job separations.

To summarize, equations (40)–(43), together with continuity of $g^u(\Delta z)$ and $g^h(\Delta z)$ at $\Delta z = 0$, constitute the equilibrium conditions for the steady-state distributions of cumulative productivity shocks. Next, we show that $g^h(\Delta z)$ incorporates all the relevant information needed to quantify the prevalence of inefficient job separations in the economy.

The Distribution of Cumulative Productivity Shocks in Employment $g^h(\Delta z)$ is a Sufficient Statistic for the Prevalence of Inefficient Job Separations. In our model, the ratio of the measure of endogenous job separations s^{end} to the measure of all job separations s is given by

$$\frac{s^{end}}{s} = \frac{\frac{\sigma^2}{2\mathcal{E}}\left[\lim_{\Delta z \downarrow -\Delta^-}(g^h)'(\Delta z) - \lim_{\Delta z \uparrow \Delta^+}(g^h)'(\Delta z)\right]}{s}. \tag{44}$$

The numerator on the right-hand side of equation (44) is the share of employment resulting in endogenous job separations, which are triggered by cumulative productivity shocks hitting the boundary $-\Delta^-$ from above or the boundary Δ^+ from below. Recall that, by Proposition 2, the match surplus is always strictly positive in equilibrium, which implies that all endogenous job separations are inefficient. Therefore, this ratio summarizes the prevalence of inefficient job separations in the economy. A challenge in operationalizing equation (44) is that the distribution of cumulative productivity shocks in employment $g^h(\Delta z)$ is unobserved. Next, we show how to recover this distribution from labor market microdata.

Inferring the Distribution of Cumulative Productivity Shocks in Employment $g^h(\Delta z)$. A key insight is that, given the parameters of the stochastic process guiding worker productivity, the distribution of wage changes between jobs contains sufficient information to recover $g^h(\Delta z)$ and therefore the prevalence of endogenous job separations. We guide the discussion with the aid of Figure 2, which shows the marginal distribution of wage changes between jobs $l^w(\Delta w)$ (left panel) and the marginal distribution of cumulative productivity changes in employment $g^h(\Delta z)$ (right panel). Each panel plots the respective distribution for two extreme calibrations, one that renders almost all job separations endogenous (blue solid line) and one that renders almost all job separations exogenous (red dashed line).

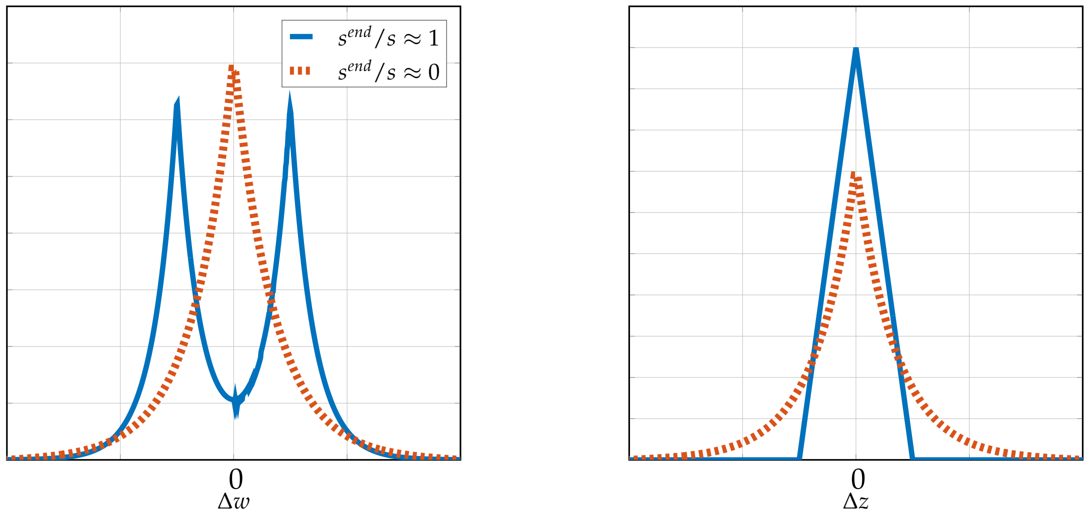
If, on the one hand, most job separations are endogenous, then most separated workers experienced cumulative productivity shocks during employment of either $-\Delta^-$ or Δ^+. As a result, the probability mass associated with positive wage changes between jobs is concentrated around $-\Delta^-$, and the probability mass associated with negative wage changes is concentrated around Δ^+. This results in a bimodal distribution of wage changes between jobs, with additional dispersion around the two modes caused by cumulative productivity shocks in unemployment.

If, on the other hand, most job separations are exogenous, then most separated workers experienced cumulative productivity shocks in employment close to zero. Because the probability of finding a job is independent of the shocks experienced during unemployment, the shape of the distribution of wage changes between jobs mimics the distribution of cumulative productivity shocks in employment, being symmetric and single peaked at zero.

With this intuition in mind, we formalize the argument for the identification of $g^h(\Delta z)$ in three steps. First, we infer the drift γ and volatility σ of worker productivity from microdata on wage changes and

FIGURE 2. DISTRIBUTIONS OF WAGE CHANGES BETWEEN JOBS AND CUMULATIVE PRODUCTIVITY SHOCKS IN EMPLOYMENT

A. Distribution of wage changes between jobs, Δw B. Distribution of cumulative productivity shocks in employment, Δz



Notes: The figure plots the distribution of wage changes between jobs $l^w(\Delta w)$ and the distribution of cumulative worker shocks in employment $g^h(\Delta z)$ for two calibrations. In the first calibration, we set $(\Delta^-, \Delta^+, \gamma, \sigma, \delta, p(\hat{w}^*)) = (0.05, 0.05, 0, 0.02, 0, 0.5)$ so that $s^{end}/s \approx 1$ (blue solid line). In the second calibration, we set $(\Delta^-, \Delta^+, \gamma, \sigma, \delta, p(\hat{w}^*)) = (0.2, 0.2, 0, 0.1, 0.04, 0.05)$ so that $s^{end}/s \approx 0$ (red dashed line).

worker transitions between jobs. Second, we measure the job finding rate $p(\hat{w}^*)$ and marginal distribution of wage changes between jobs $l^w(\Delta w)$ in order to deduce the CDF of cumulative productivity shocks conditional on a job separation event $\bar{G}^h(\Delta z)$. Third, we recover $g^h(\Delta z)$ along with $g^u(\Delta z)$.

Step 1: Identifying the Parameters of the Stochastic Process Guiding Worker Productivity. A challenge in recovering the drift γ and volatility σ of the stochastic process guiding worker productivity lies in the endogenous job separation of workers into unemployment. The following lemma shows how to recover γ and σ from observables $\mathcal{D}$ by use of Doob's Optional Stopping Theorem.

Lemma 3. *The drift γ and volatility σ of the stochastic process guiding cumulative productivity shocks can be recovered from $\mathcal{D}$ with*

$$\gamma = \frac{\mathbb{E}_{\mathcal{D}}[\Delta w]}{\mathbb{E}_{\mathcal{D}}[\tau]}, \tag{45}$$

$$\sigma^2 = \frac{\mathbb{E}_{\mathcal{D}}[(\Delta w - \gamma\tau)^2]}{\mathbb{E}_{\mathcal{D}}[\tau]}. \tag{46}$$

Lemma 3 provides a mapping between the drift γ and volatility σ of worker productivity and measurable labor market objects. Equation (45) states that the drift of productivity γ simply equals the mean wage change between jobs $\mathbb{E}_{\mathcal{D}}[\Delta w]$ divided by the mean time elapsed between the starting dates of two consecutive jobs $\mathbb{E}_{\mathcal{D}}[\tau]$. Equation (46) shows that the volatility of productivity σ equals the dispersion of wage changes around the expected wage change between jobs $\mathbb{E}_{\mathcal{D}}[(\Delta w - \gamma\tau)^2]$ divided by the mean time elapsed between the starting dates of two consecutive jobs $\mathbb{E}_{\mathcal{D}}[\tau]$.

Step 2: Identifying the Distribution of Cumulative Productivity Shocks Conditional on Job Transitions. Having identified (γ, σ), we next characterize the distribution of cumulative productivity shocks conditional on a job separation event $\bar{G}^h(\Delta z)$. To understand how to identify this distribution, we first turn to the dynamics of h-to-u and u-to-h worker flows. Consider a worker who at time t_0 starts a job with wage w_{t_0}, at time $t_0 + \tau^m$ separates, and at time $t_0 + \tau^m + \tau^u$ finds a new job with wage $w_{t_0+\tau^m+\tau^u}$. This worker's wage change between jobs is given by

$$
\begin{aligned}
\Delta w &= w_{t_0+\tau^m+\tau^u} - w_{t_0}, && (47)\\
&= \underbrace{w_{t_0+\tau^m+\tau^u} - z_{t_0+\tau^m+\tau^u}}_{=\,\hat{w}^*} - \underbrace{(w_{t_0} - z_{t_0})}_{=\,\hat{w}^*} + \underbrace{z_{t_0+\tau^m+\tau^u} - z_{t_0},}_{=\,\Delta z \text{ after } h\text{-}u\text{-}h \text{ transition}} && (48)\\
&= \underbrace{\hat{w}^* - \hat{w}^*}_{=\,0} + \underbrace{z_{t_0+\tau^m} - z_{t_0}}_{\Delta z | h\text{-}u \text{ transition starting from } z_{t_0}} + \underbrace{z_{t_0+\tau^m+\tau^u} - z_{t_0+\tau^m}.}_{\Delta z | u\text{-}h \text{ transition starting from } z_{t_0+\tau^m}} && (49)
\end{aligned}
$$

Equation (47) applies the definition of Δw. Equation (48) adds and subtracts $z_{t_0+\tau^h+\tau^u} - z_{t_0}$ before grouping terms into the wage-to-productivity ratio in the old job, the wage-to-productivity ratio in the new job, and the cumulative productivity shocks between the starting dates of the old and new jobs. Finally, equation (49) adds and subtracts $z_{t_0+\tau^m}$ before applying the definition of $\hat{w}^*$ and that of Δz. In summary, equations (47)–(49) show that the wage change across jobs is equal to the sum of three random variables: (i) the difference of entry wage-to-productivity ratios across jobs, which equals zero, (ii) Δz conditional on a job separation starting from productivity z_{t_0}, and (iii) Δz conditional on finding a new job, which is independent of the productivity z_t for $t\ in (t_0 + \tau^m, t_0 + \tau^m + \tau^u)$. Based on these arguments, we derive the following proposition.

Proposition 6. *The distribution of Δz conditional on a job separation is given by*

$$\bar{G}^h(\Delta z) = \frac{\sigma^2}{2p(\hat{w}^*)} \frac{\mathrm{d}l^w(-\Delta z)}{\mathrm{d}z} - \frac{\gamma}{p(\hat{w}^*)} l^w(-\Delta z) - [1 - L^w(-\Delta z)], \tag{50}$$

where $L^w(\Delta w)$ denotes the cumulative distribution function (CDF) corresponding to the marginal distribution $l^w(\Delta w)$.

Step 3: Identifying the Distribution of Cumulative Productivity Shocks in Employment. Given the distribution of cumulative productivity shocks conditional on a job separation event $\bar{G}^h(\Delta z)$ and $\bar{g}^h(\Delta z)$, we can recover the steady-state cross-sectional distribution of wage-to-productivity ratios in employment.

Proposition 7. *Assume $\gamma \neq 0$. The distribution of cumulative productivity shocks $g^h(\Delta z)$ is given by*

$$g^h(\Delta z) = \frac{s\mathcal{E}}{\gamma}\left[\int_{-\Delta^-}^{\Delta z}\left(1 - e^{\frac{2\gamma}{\sigma^2}(y-\Delta z)}\right)\bar{g}^h(y)\,\mathrm{d}y + \bar{G}^h(-\Delta^-)\left[1 - e^{-\frac{2\gamma}{\sigma^2}(\Delta z + \Delta^-)}\right]\right]. \tag{51}$$

Proposition 7 provides the functional equation (51) that, when combined with equation (50), maps $l^w(\Delta w)$ into $g^h(\Delta z)$. Depending on the application, one needs to compute specific moments of the distribution $g^h(\Delta z)$. For example, the next section shows that the response of aggregate job separations after a monetary shocks depends only on average tenure—which is directly measurable in the data—and $\mathbb{E}_h[\Delta z]$. Online Appendix D.5 shows how to recover the required moments of the distribution $g^h(\Delta z)$ using moments of the observed distribution of Δw.

We conclude this section with a brief discussion of the assumptions underlying the method described in Propositions 6 and 7. The first assumption is the threshold nature of job separation policies, according to which the job separation rate is equal to δ for $\Delta z \in [-\Delta^-, \Delta^+]$ and infinite for $\Delta z \in \{-\Delta^-, \Delta^+\}$. This assumption is not crucial, and it can be replaced with a general job separation hazard as in Álvarez *et al.* (2020). The second assumption is the lack of other types of wage adjustments, such as those arising from job-to-job transitions or wage adjustments within a job spell. This assumption could be relaxed following the methodology in Baley and Blanco (2021b). Finally, while we assume a particular stochastic process for $\mathrm{d}\Delta z_t$, this assumption can be empirically tested and adjusted if deemed necessary, as in Baley and Blanco (2021a). For example, it would be straightforward to make the parameters of the productivity process depend on the worker's employment state. The critical assumption behind Propositions 6 and 7 is that we have sufficient information about $\bar{g}^u(\Delta z)$, the distribution of productivity changes during unemployment. Given our model assumptions, this is indeed the case, as the lack of selection in job finding and the pre-identified stochastic process for Δz together yield the strong identification result.

4 Analyzing the Macroeconomic Consequences of Allocative Wages

How does the interaction between productivity shocks, wage rigidity, and two-sided lack of commitment—which gives rise to inefficient job separations—matter for the transmission of monetary shocks? To answer this question, we add money as a numeraire to the economic environment.

4.1 A Monetary Economy

We modify the baseline model in four dimensions. First, we introduce preferences over real money holdings:

$$\mathbb{E}_0 \left[\int_{t=0}^{\infty} e^{-\rho t} \left(c_{it} + \mu \log \left(\frac{\hat{M}_{it}}{P_t} \right) \right) \mathrm{d}t \right], \tag{52}$$

where $\hat{M}_{it}$ denotes a worker's money holdings, P_t is the relative price of the good in terms of money, and μ is a preference weight on real money holdings.

Second, workers face a budget constraint that reflects access to complete financial markets and ownership of firms' profits. Given a history of labor market decisions regarding job search, acceptance, and dissolution, $lm_i^t := \{lm_{it'}\}_{t'=0}^{t}$, a worker's private income is $y(lm_i^t)$, which equals the nominal value of the wage while employed and the nominal value of home production while unemployed. In addition, each worker receives transfers of T_{it} from the government and profits of a fully diversified portfolio claims on the individual firms. On the spending side, a worker pays for consumption expenditures $P_t c_{it}$ and the opportunity cost of holding money $i_t \hat{M}_{it}$ at a given interest rate $i_t \geq 0$. Letting Q_t denote the time-0 Arrow-Debreu price under complete markets, the worker's budget constraint is

$$\mathbb{E}_0 \left[\int_{t=0}^{\infty} Q_t \left(P_t c_{it} + i_t \hat{M}_{it} - y(lm_i^t) - T_{it} \right) \mathrm{d}t \right] \leq \hat{M}_{i0}. \tag{53}$$

The worker's problem is to choose a consumption stream $\{c_t\}_{t=0}^{\infty}$, labor market decisions $\{lm_t\}_{t=0}^{\infty}$, and money holdings $\{\hat{M}_{it}\}_{t=0}^{\infty}$ to maximize utility (52) subject to the budget constraint (53) at time 0.

Third, the economy is subject to shocks to the aggregate money supply M_t. We assume that the log of the aggregate money supply m_t follows a Brownian motion with drift π and volatility ζ:

$$\mathrm{d}m_t = \pi \, \mathrm{d}t + \zeta \, \mathrm{d}\mathcal{W}_t^m,$$

where $\mathcal{W}_t^m$ is a Wiener process. Because the aggregate money supply moves stochastically over time, fluctuations in m_t constitute aggregate shocks to the economy.

Fourth and finally, we assume that the vacancy posting cost $K(Z_t)$ and the value of home production $B(Z_T)$ are both denominated in real terms.

Given these modifications, the market-clearing conditions for goods and money, respectively, are

$$\int_{i=0}^{1} c_{it} + \theta_{it} \mathbb{1}[E_{it} = u] K(Z_{it}) \, \mathrm{d}i = \int_{i=0}^{1} Z_{it} \mathbb{1}[E_{it} = h] + B(Z_{it}) \mathbb{1}[E_{it} = u] \, \mathrm{d}i, \tag{54}$$

$$\int_{i=0}^{1} \hat{M}_{it} \, \mathrm{d}i = M_t, \tag{55}$$

where $\mathbb{1}[\cdot]$ is an indicator function that takes a logical expression as its argument. Equation (54) states that the sum of real consumption and recruiting expenses must equal the total market and home production of the good. Equation (55) states that the total demand of nominal money holdings across workers equals the aggregate money supply.

The following proposition characterizes the worker's problem in this monetary economy.

Proposition 8. *Let $Q_0 = 1$ be the numeraire and assume $\mu = \rho + \pi + \zeta^2/2$. Then, $P_t = M_t$ and the value of a worker at time 0 is*

$$V_0 = \max_{\{lm_{it}\}_{t=0}^{\infty}} \mathbb{E}_0 \left[\int_0^\infty e^{-\rho t} \frac{y(lm_i^t)}{P_t} \, \mathrm{d}t \right] + k,$$

where k is a constant independent of the worker's choices and the present discounted value of financial wealth.

Proposition 8 shows that the price level is equal to the aggregate money supply and that maximizing (52) subject to (53) is equivalent to maximizing expected discounted real income. The result relies on the following assumptions: (i) markets are complete, (ii) workers have quasi-linear preferences in consumption, and (iii) the log of aggregate money supply follows a random walk with drift. The first two assumptions imply a constant marginal value of nominal wealth, which combined with the last assumption leads to a constant real interest rate and a one-for-one pass-through of money shocks to inflation.

The introduction of a monetary economy requires minor adjustments to our previous solution approach. Given fluctuations in the log price level p, the relevant state variable becomes the *real wage-to-productivity ratio* $\hat{w} := w - z - p$. Similarly, we keep track of the negative of a worker's cumulative shocks to *revenue productivity* $z + p$ since the beginning of the current employment or unemployment spell, which we denote by Δz.[8] By definition, $\hat{w} = \hat{w}^* + \Delta z$ and the law of motion for Δz is

$$\Delta z = -(\gamma + \pi) \, \mathrm{d}t + \sigma \, \mathrm{d}\mathcal{W}_t^z + \xi \, \mathrm{d}\mathcal{W}_t^m.$$

All policies $(\hat{w}^+, \hat{w}^*, \hat{w}^-)$ are expressed in real terms. Since productivity growth γ and trend inflation π symmetrically affect revenue productivity, without loss of generality, we set $\pi = 0$. Finally, let $G_h(z, a)$ denote the steady-state joint distribution of cumulative revenue productivity shocks z and tenure a of a job spell. For any integers $k, l \in \mathbb{N}$, we define the moments of this distribution as

$$\mathbb{E}_h(\Delta z^k a^l) \equiv \int_{\Delta z} \int_a \Delta z^k a^l \, \mathrm{d}G^h(\Delta z, a).$$

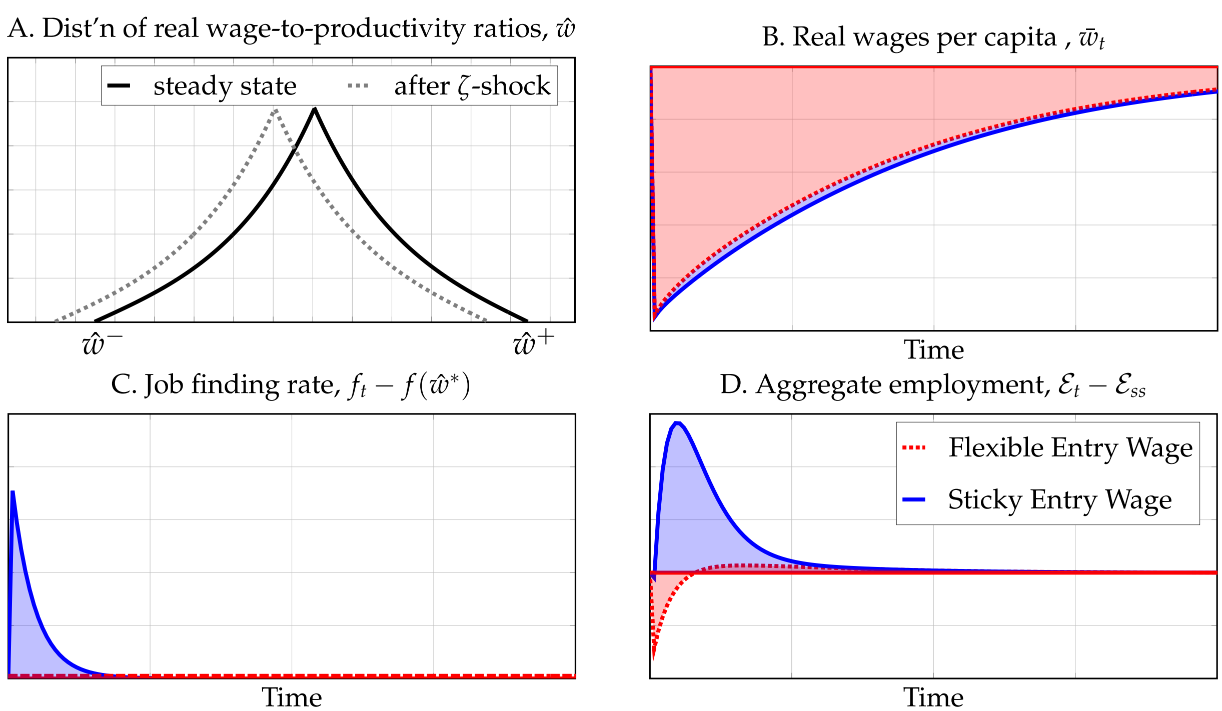
[8] We choose this notation to avoid defining new objects. Below, we set $\pi = 0$; thus, steady-state moments of cumulative productivity shocks are equal to the corresponding moments of revenue productivity.

4.2 Monetary Multipliers for Aggregate Employment and Real Wages

Starting from the steady state without aggregate shocks, so that $\zeta = 0$, we consider a small, unanticipated, one-standard-deviation shock $\zeta > 0$ to aggregate money supply at time $t = 0$—i.e., $\log(M_0) = \lim_{t\uparrow 0}\log(M_t) + \zeta$. The shock leads to a one-for-one increase in the price level. We are interested in the economy's *impulse response function (IRF)* and *cumulative impulse response (CIR)* of aggregate employment and real wages to such a monetary shock.[9]

An Illustration. Figure 3 shows the distribution of real wage-to-productivity ratios $\hat{w}$ before and after the monetary shock together with the IRFs of average real per capita wages $\bar{w}_t := \int_0^1 (\mathbb{1}[E_{it} = h]w_{it} - \mathbb{E}_h[\hat{w}^* + \Delta z])\, di$, the job-finding rate $f_t - f_{ss}$, and aggregate employment $\mathcal{E}_t - \mathcal{E}_{ss}$.

FIGURE 3. IMPULSE RESPONSE FUNCTIONS OF LABOR MARKET VARIABLES



Notes: Panel A shows the distribution of real wage-to-productivity ratios $\hat{w} := \log(W_{it}/(Z_{it}P_t))$ in the steady state and after a monetary shock of size ζ. Panels B, C, and D show the impulse response functions of the average log real per capita wage $\bar{w}_t$, the job-finding rate $f_t - f_{ss}$, and aggregate employment $\mathcal{E}_t - \mathcal{E}_{ss}$, respectively. The parameter values are $(\gamma, \pi, \sigma, \rho, \alpha, \tilde{K}, \delta, \tilde{B}) = (0, 0.001, 0.007, 0.03, 0.5, 1, 0.005, 0.4)$.

After the initial increase in the price level, the distribution of real wage-to-productivity ratios shifts to

[9] By the certainty equivalence principle, the impulse response function following a money shock departing from the steady-state with steady-state policies is equivalent to the solution based on a first-order perturbation of the model with business cycles fluctuations.

the left (Panel A), leading to a sudden decrease in the aggregate log real wage (Panel B). Consequently, the monetary shock affects both the endogenous job separation rate and aggregate employment (Panel D). Given that the wages of new matches are critical for the job-finding rate (Pissarides, 2009), we study two separate cases: flexible entry wages and sticky entry wages. With flexible entry wages, we assume that unemployed workers fully adjust their search behavior to incorporate the higher price level, so that $\hat{w}^*$ remains at its steady-state level. Thus, the real entry wage and the job-finding probability remain constant (Panel C). The aggregate log real wage is affected by the shock only because the nominal wages of workers already employed at $t = 0$ are rigid. Since entry wages adjust one-for-one with the price level, the firm's real value of a filled vacancy is unaffected, so that both vacancy-filling and job-finding rates remain at their steady-state levels. Therefore, the employment effects are only driven by the effects of the aggregate shock on endogenous job separations.

In the sticky entry wage case, we assume that unemployed workers do not adjust their search behavior to incorporate the higher price level. In this case, the real entry wage reverts to its steady-state level following the worker's first job separation after the shock. Thus, after the shock, the real entry wage also decreases, which induces firms to post more vacancies and the job-finding rate to increase (Panel C). As a consequence, employment dynamics are driven by both the job-separation and job-finding rates. The assumption of sticky entry wages is motivated by the empirical evidence in Grigsby *et al.* (2021), which documents that new hire wages evolve similarly to incumbent workers within a firm at business cycle frequencies, and Hazell and Taska (2020), which shows that wages for new hires rarely change between successive vacancies at the same job. Micro-founding this assumption is outside the scope of this paper. Nevertheless, observe that since the steady-state entry wage is optimal, any perturbation around that level has a second-order welfare effect on the worker. Thus, any first-order cost of wage adjustment would replace this assumption as a result. There is abundant literature that provides a plethora of alternative models to think about imperfect knowledge about aggregate shocks—we chose a simple one to focus on our contribution.[10] For alternative models of rigid entry wages, see Fukui (2020) and Menzio (2022).

Defining IRFs and CIRs. Our goal is to characterize the effects of a monetary shock on aggregate employment $\mathcal{E}$ and aggregate real wages $\bar{w}$. To this end, we denote by $IRF_x(\zeta, t)$ the IRF for variable $x \in \{\mathcal{E}, \bar{w}\}$ at time t relative to its steady-state value, following a monetary shock ζ at time 0. The IRF for aggregate employment is

$$IRF_{\mathcal{E}}(\zeta, t) = \mathcal{E}_t - \mathcal{E}_{ss},$$

[10] A few examples are sticky information in Mankiw and Reis (2002), rational inattention in Woodford (2009) and Maćkowiak and Wiederholt (2009), dispersed knowledge in Hellwig *et al.* (2014), level-k reasoning in Farhi and Werning (2017), among many others.

where $\mathcal{E}_{ss}$ is the steady-state employment rate. Analogously, the IRF for aggregate real wages is

$$IRF_{\bar{w}}(\zeta, t) = \int_{\hat{w}^-}^{\hat{w}^+} \hat{w}[\mathrm{d}G_t(\hat{w}) - \mathrm{d}G_{ss}(\hat{w})], \tag{56}$$

where $G_t(\hat{w})$ is the CDF of real log wage-to-productivity ratios at time t and $G_{ss}(\hat{w})$ is its steady-state counterpart. It is worth noting that equation (56) implicitly makes use of the fact that the IRF of the mean log real wage $\bar{w}$ is identical to that of the mean log real wage-to-productivity ratio $\hat{w}$, given that the process governing a worker's productivity is unresponsive to the monetary shock.

Following Álvarez *et al.* (2016), we define the CIR of a variable $x \in \{\mathcal{E}, \bar{w}\}$ to a monetary shock ζ as

$$CIR_x(\zeta) = \int_0^\infty IRF_x(\zeta, t)\,\mathrm{d}t,$$

which measures the area under the $IRF_x(\zeta, t)$ curve for $t \in [0, \infty)$. The CIR summarizes the response on impact and the persistence of the response of the labor market to the monetary shock in a single scalar. Therefore, the CIR can be interpreted as a *monetary multiplier*. To illustrate the logic behind the CIR, suppose that there are no nominal rigidities so that nominal wages of both newly hired and incumbent workers respond one-for-one to the price level. In this case, $IRF_x(\zeta, t) = 0$ for all t and thus $CIR_x(\zeta) = 0$ for $x \in \{\mathcal{E}, \bar{w}\}$, reflecting the fact that there are no real consequences of inflation. With nominal rigidities, an inflationary shock affects both employment and wages, the magnitude of which is reflected in the CIR.

Characterizing the CIR of employment. Now, we characterize the CIR of aggregate employment. The first proposition relates the CIR to a perturbation of two Bellman equations describing future employment fluctuations for initially employed and unemployed workers. The idea behind the proof is to exchange the order of integration; we first integrate over time for a given worker and then integrate across workers.

Proposition 9. *Given steady-state policies $(\hat{w}^-, \hat{w}^*, \hat{w}^+)$ and distributions $(g^h(\Delta z), g^u(\Delta z))$, the CIR is given by*

$$CIR_{\mathcal{E}}(\zeta) = \underbrace{\int_{-\infty}^{\infty} m_{\mathcal{E},h}(\Delta z) g^h(\Delta z + \zeta)\,\mathrm{d}\Delta z}_{CIR_{\mathcal{E}} \text{ of initially employed workers}} + \underbrace{\int_{-\infty}^{\infty} m_{\mathcal{E},u}(\Delta z, \zeta) g^u(\Delta z + \zeta)\,\mathrm{d}\Delta z}_{CIR_{\mathcal{E}} \text{ ofinitially unemployed workers}},$$

where the value functions $m_{\mathcal{E},h}(\Delta z)$ and $m_{\mathcal{E},u}(\Delta z, \zeta)$ are defined as:

$$m_{\mathcal{E},h}(\Delta z) = \mathbb{E}\left[\int_0^{\tau^m} (1 - \mathcal{E}_{ss})\,\mathrm{d}t + m_{\mathcal{E},u}(\Delta z, 0) \middle| \Delta z_0 = \Delta z\right], \tag{57}$$

$$m_{\mathcal{E},u}(\Delta z, \zeta) = \mathbb{E}\left[\int_0^{\tau^u(\zeta)} (-\mathcal{E}_{ss})\,\mathrm{d}t + m_{\mathcal{E},h}(-\zeta) \middle| \Delta z_0 = \Delta z\right]. \tag{58}$$

$$0 = \int_{-\infty}^{\infty} m_{\mathcal{E},h}(\Delta z) g^h(\Delta z)\,\mathrm{d}\Delta z + \int_{-\infty}^{\infty} m_{\mathcal{E},u}(\Delta z, 0) g^u(\Delta z)\,\mathrm{d}\Delta z. \tag{59}$$

with $\tau^u(\zeta)$ being distributed according to a Poisson process with arrival rate $f(\hat{w}^* - \zeta)$.

The proposition shows that to characterize the CIR, we need to keep track of future employment dynamics of initially employed and unemployed workers. When the shock arrives, the real wages of initially employed workers decrease (since $\Delta z_0 = \lim_{t \uparrow 0} \Delta z_t - \zeta$, we have that the starting point is given by the distribution $g^h(\Delta z + \zeta)$) affecting their future employment spells, which is captured by $m_{\mathcal{E},h}(\Delta z)$. During employment, the employed worker's value function accumulates positive deviations from the steady-state level $(1 - \mathcal{E}_{ss})$, and the unemployed worker's value function accumulates negative deviations from the steady-state level $(-\mathcal{E}_{ss})$. Thus, $m_{\mathcal{E},h}(\Delta z)$ measures the cumulative deviations of employment from its steady-state level conditional on being initially employed at revenue productivity Δz. Similarly, $m_{\mathcal{E},u}(\Delta z, \zeta)$ measures the cumulative employment deviations from the steady-state level conditional on being initially unemployed at revenue productivity Δz and search for a job in submarket $\hat{w}^* - \zeta$. Searching in submarket $\hat{w}^* - \zeta$ increases the job-finding probability $f(\hat{w}^* - \zeta)$ and also changes the wages of new hires. Finally, since the Bellman equations (57) and (58) lack discounting (i.e., they simply count non-discounted deviations), it is easy to show that they have infinitely many solutions. The unique relevant solution is pinned down by equation (59), which requires that an economy that departs from the steady-state and experiences no shock must have a null $\text{CIR}_{\mathcal{E}}$—i.e., $\text{CIR}_{\mathcal{E}}(0) = 0$.

Next, we characterize up to first order the CIR of aggregate employment as a set of measurable objects in labor market microdata. Specifically, we argue that certain moments of the joint distribution of tenure and wages in steady-state are informative of the CIR. The key insight below is that the CIR of aggregate employment, which is a summary statistic of its dynamic response, can be characterized only in terms of steady-state cross-sectional moments. The intuition behind this result is that changes in a worker's idiosyncratic productivity and changes in the aggregate price level affect the real wage-to-productivity ratio $W_{it}/(Z_{it}P_t)$ of a match in symmetric ways. Therefore, the response of a match to productivity changes in the steady-state is informative of the aggregate effects of changes in the price level.

CIR of employment with flexible entry wages. To facilitate the exposition of the analysis, we first present the case with flexible entry wages. Proposition 10 characterizes the CIR up to first order.[11]

Proposition 10. *Assume flexible entry wages. Up to first order, the CIR of employment is given by:*

$$\frac{CIR_{\mathcal{E}}(\zeta)}{\zeta} = -(1 - \mathcal{E}_{ss}) \frac{\gamma \mathbb{E}_h[a] + \mathbb{E}_h[\Delta z]}{\sigma^2} + o(\zeta). \tag{60}$$

[11] That is, $CIR_x(\zeta) = CIR_x(0) + (CIR_x)'(0)\zeta + o(\zeta^2)$, where $CIR_x(0) = 0$.

The conventional wisdom in macroeconomics is that fluctuations in the job separation rate are not the main driver of aggregate employment dynamics (e.g., Shimer, 2005b). In the context of a monetary shock, equation (60) points to conditions under which aggregate employment fluctuations due to endogenous job separations can be small. More importantly, it also highlights the conditions for these effects to be large. This new result, combined with Corollary 1 below, provides a guide to verify those conditions in the data.

In light of this conventional wisdom, one might also be tempted to conclude that sticky wages cannot lead to inefficiencies at the micro-level. However, equation (60) allows for a small CIR of aggregate employment to a monetary shock *despite* the presence of allocative wages and inefficient job separations. Thus, aggregate time-series data on job flows cannot be used as model discrimination devices between theories of the allocativeness of wages or in assessing the prevalence of inefficient job separations at the micro-level.

To build the intuition behind this result, we first consider the implications of equation (60) under zero productivity drift, $\gamma = 0$. In two cases do the job separation rate and aggregate employment not respond to aggregate shocks. In the first case, all job separations are exogenous and, therefore, the IRF of the job separation rate is zero. In the second case, all job separations are endogenous but the mass of additional worker quits due to lower real wages is exactly compensated by the mass of workers who would have been fired by firms in the absence of the monetary shock. In both cases, the key sufficient statistic referenced by equation (60) is $\mathbb{E}_h[\Delta z] = 0$.

Importantly, the sufficient statistic in equation (60) captures more than the on-impact response of endogenous quits and layoffs. Rather, it measures the response at all times along the IRF. Therefore, the relative mass of workers near the two job separation triggers is not a sufficient statistic for characterizing the CIR of aggregate employment. To illustrate this, consider the following example in which all endogenous separations are quits, but nevertheless, the CIR is zero. Suppose $\gamma > 0$ and $\sigma \downarrow 0$ in this environment. The equilibrium approaches a situation in which workers quit because wages lag behind productivity growth, and firms do not have incentives to fire any worker. Then, $\Delta z_{it} + \gamma a_{it} = \sigma \mathcal{W}_t^z \to 0$ and, therefore, $\gamma \mathbb{E}_h[a] + \mathbb{E}_h[\Delta z] = 0$. Intuitively, the increase in worker quits on impact is exactly offset by a reduction in future worker quits, resulting in a null net effect as captured by the CIR.

The sufficient statistic in equation (60) also points to scenarios in which inefficient job separations matter for aggregate employment dynamics. For example, if trend inflation π is large in magnitude, then—all else equal—the rate of inefficient job separations will be more responsive to an inflationary shock. Alternatively, following an unexpected sequence of negative productivity shocks, an inflationary shock reduces the incidence of inefficient job separations due to firings.[12] Furthermore, if it is easy for workers to quit but costly for firms to fire workers or vice versa—for example, due to the presence of mandatory severance pay or unemployment insurance programs—then inflationary shocks can interact with such asymmetries in job separation policies leading to inefficient job separations.

Finally, notice that the CIR is scaled by the steady-state unemployment rate, $1 - \mathcal{E}_{ss}$. This is because the steady-state unemployment rate is informative of workers' steady-state job finding rate $f(\hat{w}^*)$. When this rate is high, relative to the separation rate, then a monetary shock causes temporary unemployment fluctuations but those workers quickly become matched again with new firms. Consequently, aggregate employment remains relatively stable, resulting in a relatively small CIR of aggregate employment to an inflationary shock.

Next, we leverage the mapping from the data to the model provided in Section 3 to express the response of aggregate employment to an inflation shock in terms of observable moments of the distribution of wage changes and tenure. Here, we focus and explain the case with no drift, i.e., $\gamma = 0$. See Online Appendix Section E.4 for the case with non-zero drift, i.e., $\gamma \neq 0$. While the required moments to measure the CIR are different in the case with non-zero drift, they reflect similar economic mechanisms.

Corollary 1. *Assume $\gamma = 0$. Up to first order, the $CIR_{\mathcal{E}}(\zeta)$ can be expressed in terms of data moments as follows:*

$$\frac{CIR_{\mathcal{E}}(\zeta)}{\zeta} = \underbrace{\frac{1}{f(\hat{w}^*)}}_{avg.\ u\ dur.} \underbrace{\frac{1}{\mathbb{V}ar_{\mathcal{D}}[\Delta w]}}_{dispersion} \left[\underbrace{\frac{1}{3}\mathbb{E}_{\mathcal{D}}\left[\Delta w \frac{\Delta w^2}{\mathbb{E}_{\mathcal{D}}[\Delta w^2]}\right]}_{asymmetries} \right] + o(\zeta). \tag{61}$$

Equation (61) shows that, for zero drift, the effect of an inflationary shock on aggregate employment is determined by three statistics: (i) the average duration of unemployment spells, (ii) the inverse of the dispersion of wage changes, and (iii) a measure of the asymmetry of the wage change distribution. Each statistic in turn determines the persistence, initial absolute size, and sign of the effect.

The steady-state average duration of unemployment spells naturally amplifies the CIR as it captures how quickly a separated worker recovers from unemployment. Larger unemployment duration is indicative of larger search frictions, which makes the on-impact effect on employment more persistent. A similar result has been found in price-setting models (Álvarez *et al.*, 2016) and investment models with inaction (Baley and Blanco, 2021a).

In an environment with zero drift, a larger dispersion of wage changes is indicative of a wider inaction region and the presence of more resilient matches to idiosyncratic shocks. A large dispersion arises when

[12] See also Blanco *et al.* (2022b) for empirical evidence consistent with this theoretical result.

the pool of workers experiencing wage changes not only includes previously endogenously separated workers (with large but similar absolute wage changes) but also many exogenously separated ones (with smaller but more dispersed absolute wage changes). Thus, the larger this dispersion, the smaller the share of endogenous separations and the smaller the propagation of shocks.

Finally, the CIR is also affected by the degree of asymmetry of the distribution of wage changes, as captured by the last term in brackets in (61), which is a weighted average of wage changes that puts more weight on larger changes. When the drift is zero, this term captures how asymmetric the policies $\hat{w}^-$ and $\hat{w}^+$ are around the entry wage $\hat{w}^*$. While the previous two statistics capture the degree of amplification of the monetary shock, the asymmetry of the distribution will determine the direction of the effect. A negatively skewed distribution has a longer left tail and the mass concentrated on the right, which reflects a larger fraction of workers quitting to obtain a wage increase relative to the number of workers experiencing a wage cut due to layoffs. Thus, the increase in the price level and the fall in real wages make a large mass of workers quit and the CIR is negative. The opposite holds when the distribution of wage changes is positively skewed.[13] Instead, a symmetric distribution of wages chances is indicative of symmetric policies when the drift is zero, and monetary shocks do not affect employment.

CIR of employment with sticky entry wages. With the understanding of employment dynamics when entry wages are flexible, we now characterize the case with sticky entry wages.

Proposition 11. *Assume sticky entry wages. Up to first order, the CIR of employment is given by:*

$$\frac{CIR'_{\mathcal{E}}(\zeta)}{\zeta} = (1-\mathcal{E}_{ss})\left[-\frac{[\gamma\mathbb{E}_h[a]+\mathbb{E}_h[\Delta z]]}{\sigma^2} + \frac{1}{f(\hat{w}^*)+s}\left[\underbrace{\frac{\eta'(\hat{w}^*)}{\eta(\hat{w}^*)} + \frac{\mathcal{T}'_{\hat{w}}(\hat{w}^*,\hat{\rho})}{\mathcal{T}(\hat{w}^*,\hat{\rho})}}_{\text{job finding effect on } \mathcal{E}} + \underbrace{\frac{\mathcal{T}'_{\hat{w}}(\hat{w}^*,0)}{\mathcal{T}(\hat{w}^*,0)}}_{\text{separation effect on } \mathcal{E}}\right]\right] + o(\zeta).$$

Proposition 11 characterizes the new mechanisms affecting employment dynamics when entry wages are sticky. The elasticity of the worker's share of the surplus with respect to the entry wage (i.e., $\eta'(\hat{w}^*)/\eta(\hat{w}^*)$) together with the elasticity of the expected discounted duration of the match (i.e., $\mathcal{T}'_{\hat{w}}(\hat{w}^*,\hat{\rho})/\mathcal{T}(\hat{w}^*,\hat{\rho})$) captures the effect of the increase in the job-finding probability following the decrease in the real entry wage. A drop in the real entry wage increases the firm's share and their incentive to post vacancies. On top of this standard mechanism in search and matching models, a drop in the real

[13] When the distribution is positively skewed (i.e., with a longer right tail and the mass concentrated on the left), there is a large mass of workers experiencing wage cuts, which signals a relatively high layoff risk. Thus, higher inflation reduces real wages and increases the firms' incentives to keep their workers; as a result, aggregate employment increases.

entry wage could also change the expected duration and, therefore, the total surplus of the match. This new effect also shapes firms' incentives to post vacancies for a given share. These first two mechanisms affect aggregate employment through changes in the job-finding rate. The last term, which is also new, captures the effect of a lower real entry wage on aggregate employment that arises from fluctuations in the job separation rate of initially unemployed workers.

We now characterize the elasticity of the discounted duration of the match to an increase in the entry wage by focusing on two dimensions: (i) how the equilibrium policies determine this elasticity and (ii) how we can discipline this elasticity with data on $g^h(\Delta z)$.

Proposition 12. *The following properties hold for $\mathcal{T}'_{\hat{w}}(\hat{w}^*,\hat{\rho})/\mathcal{T}(\hat{w}^*,\hat{\rho})$.*

1. *Assume that $\Delta^- = \Delta^+$ and $\gamma = 0$. Then, $\mathcal{T}'_w(\hat{w}^*,\hat{\rho}) = 0$ and, up to a 4th order approximation around $\hat{w}^*$,*

$$\mathcal{T}(\hat{w}^*,\hat{\rho}) = \frac{1}{\hat{\rho}+\delta+(\sigma/\Delta^+)^2}.$$

2. *Up to a 3rd order approximation around $\hat{w}^*$,*

$$\frac{\mathcal{T}'_{\hat{w}}(\hat{w}^*,\hat{\rho})}{\mathcal{T}(\hat{w}^*,\hat{\rho})} = \frac{\Delta^+ - \Delta^-}{\Delta^+(2\Delta^+ - \Delta^-)}.$$

3. *If $\hat{\rho} = 0$, then*

$$\frac{\mathcal{T}'_{\hat{w}}(\hat{w}^*,0)}{\mathcal{T}(\hat{w}^*,0)} = \frac{1}{\sigma^2 g^h(0)}\left[s^{end}\left(\mathcal{E}_{ss} - 2G^h(0)\right) + \frac{\sigma^2}{2}\left(\lim_{\Delta z\uparrow\Delta^+}(g^h)'(\Delta z) + \lim_{\Delta z\downarrow-\Delta^-}(g^h)'(\Delta z)\right)\right]. \qquad (62)$$

4. *If $\hat{\rho} > 0$, then*

$$\frac{\mathcal{T}'_{\hat{w}}(\hat{w}^*,\hat{\rho})}{\mathcal{T}(\hat{w}^*,\hat{\rho})} = \frac{\mathcal{T}(\hat{w}^*,0)}{\mathcal{T}(\hat{w}^*,\hat{\rho})\mathcal{E}_{ss}}\left[\hat{\rho}\frac{\gamma\mathbb{E}_h[a]+\mathbb{E}_h[\Delta z]}{\sigma^2} + \sigma^2\left[\lim_{\Delta z\downarrow\Delta^-} - \lim_{\Delta z\uparrow\Delta^+}\right]\frac{\mathrm{d}^2\left[\mathcal{T}(\hat{w}^*+\Delta z,\hat{\rho})g^h(\Delta z)\right]}{\mathrm{d}\Delta z^2}\right] + o(\hat{\rho}^2),$$

with

$$\lim_{\Delta z\downarrow\Delta^-}\frac{\mathrm{d}^2\left[\mathcal{T}(\hat{w}^*+\Delta z,\hat{\rho})g^h(\Delta z)\right]}{\mathrm{d}\Delta z^2} = \lim_{\Delta z\downarrow-\Delta^-}\frac{(g^h)'(\Delta z)^2}{g^h(0)}H^-(g^h,\hat{\rho},\mathcal{T}(\hat{w}^*,\hat{\rho}),\mathcal{T}(\hat{w}^*,0))$$

$$\lim_{\Delta z\uparrow\Delta^+}\frac{\mathrm{d}^2\left[\mathcal{T}(\hat{w}^*+\Delta z,\hat{\rho})g^h(\Delta z)\right]}{\mathrm{d}\Delta z^2} = \lim_{\Delta z\uparrow\Delta^+}\frac{(g^h)'(\Delta z)^2}{g^h(0)}H^+(g^h,\hat{\rho},\mathcal{T}(\hat{w}^*,\hat{\rho}),\mathcal{T}(\hat{w}^*,0))$$

where $H^{-}(\cdot)$ and $H^{+}(\cdot)$ are described equation (E.43) of the Online Appendix.

Items 1 and 2 of Proposition 12 characterize an approximation of the elasticity of the discounted duration with respect to the entry wage as a function of the separation triggers $(-\Delta^{-}, \Delta^{+})$ and model parameters. The proof is based on a Taylor approximation of $\mathcal{T}(\hat{w}, \rho)$ around $\hat{w}^*$ and the HJB equation and border conditions that characterize $\mathcal{T}(\hat{w}, \rho)$. In symmetric economies—i.e., zero drift and symmetric separation triggers—the elasticity of the expected duration with respect to the entry wage is zero. Intuitively, an increase in the entry wage lowers the probability of a quit but increases the probability of a layoff in a similar proportion. Surprisingly, in asymmetric economies, the only mechanism affecting the elasticity of expected duration is the asymmetry of the separation triggers; thus, it is independent of $(\rho, \delta, \gamma, \sigma)$ conditional on Δ^{-} and Δ^{+}. For example, a higher discount factor $\hat{\rho}$ decreases the expected discounted duration and, at the same time, it decreases the effect of the initial wage $\hat{w}^*$ on the expected duration. Thus, the ratio $\mathcal{T}'_{\hat{w}}(\hat{w}^*, \rho)/\mathcal{T}(\hat{w}^*, \rho)$ is independent of $\hat{\rho}$.

While items 1 and 2 show the elasticity of the expected duration with respect to the entry wage and the mechanisms that shape it, items 3 and 4 show how to discipline these mechanisms with information about the distribution of Δz for the cases with $\hat{\rho} = 0$ and $\hat{\rho} > 0$. Mechanically, the steady-state distribution of Δz is proportional to the time workers spend at productivity Δz; thus, up to normalization, $\mathcal{T}(\hat{w}^*, 0)$ could be obtained from $g^h(\Delta z)$. Equation (62) shows the two conditions to generate positive effect on duration. If the quit rate ($\frac{\sigma^2}{2} \lim_{\Delta z \downarrow -\Delta^{-}} (g^h)'(\Delta z)$) is larger than the firing rate ($-\frac{\sigma^2}{2} \lim_{\Delta z \downarrow \Delta^{+}} (g^h)'(\Delta z)$), then a higher entry wage increases expected match duration; if these rates are equal, then this term is zero. Similarly, the product between endogenous separations and the CDF of Δz evaluated at 0 also determines the duration elasticity since it measures the asymmetries of the distribution within the separation triggers. While these two statistics measure the effect of asymmetries in Δz among employed workers, its effect on expected duration at the entry wage is obtained by re-scaling the statistics by $1/(\sigma^2 g^h(0))$.

Finally, when $\hat{\rho} > 0$, the elasticity of expected duration with respect to the entry wage depends on: (i) the product between $\hat{\rho}$ and the negative of the CIR with flexible entry wages and, (ii) the product between the curvature of the expected discounted duration and the mass of workers at the separation triggers. The main reason the CIR shows up in the elasticity of discounted duration is that a marginal increase in the entry wage has a similar effect on quits and layoffs, and therefore on marginal duration, as a lower price level (once the effect of discounting is properly accounted for).

To finish understanding the role of allocative wages for aggregate fluctuations, we characterize their effect on the elasticity of a worker's share of the surplus. The worker's share is given by the ratio of two Bellman equations—one for the worker's value and the other for the surplus of the match. The first step is to show that their ratio also satisfies a Bellman equation, once properly adjusted due to the endogenous duration of the match. Proposition 13 shows this result.

Proposition 13. *Define*

$$\tau^{end} = \inf\{t \geq 0 : \Gamma_t \notin (\hat{w}^-, \hat{w}^+)\}$$

where $(\hat{w}^-, \hat{w}^+)$ is a Nash equilibrium. Then, the worker's share $\eta(\hat{w})$ satisfies the following Bellman equation

$$\eta(\hat{w}) = \mathbb{E}\left[\int_0^{\tau^{end}} e^{-(\hat{\rho}+\delta)t}(\hat{\rho}+\delta)\frac{e^{\Gamma_t} - \hat{\rho}\hat{U}}{1-\hat{\rho}\hat{U}}\,\mathrm{d}t + e^{-(\hat{\rho}+\delta)\tau^{end}}\mathbb{1}[\Delta z_{\tau^m} = \Delta^+]|\Gamma_0 = \hat{w}\right] \tag{63}$$

with

$$\mathrm{d}\Gamma_t = (\hat{\rho}+\delta)(-\gamma\mathcal{T}(\Gamma_t,\hat{\rho}) + \sigma^2\mathcal{T}'_{\hat{w}}(\Gamma_t,\hat{\rho}))\,\mathrm{d}t + \sigma\sqrt{\mathcal{T}(\Gamma_t,\hat{\rho})(\hat{\rho}+\delta)}\,\mathrm{d}\mathcal{W}^z{}_t$$

Before discussing this proposition, some properties are important to mention. First, for the Bellman equation (63) to describe the worker's share, $(\hat{w}^-, \hat{w}^+)$ must be a Nash equilibrium of the game between the firm and the worker. This guarantees that (63) properly characterizes a share $\eta(\hat{w}) \in [0,1]$. Second, observe that $\lim_{\hat{w}\downarrow\hat{w}^-}\eta(\hat{w}) = 0$ and $\lim_{\hat{w}\uparrow\hat{w}^+}\eta(\hat{w}) = 1$. In other words, if the wage is close to the quitting trigger, then the worker's share of the surplus is zero; if the wage is close to the firing trigger, then the worker's share is one. Third, the flow payoff of $\eta(\cdot)$ is equal to the flow share of the worker relative to the surplus of the match, adjusted for discounting and the rate of exogenous separations.

The law of motion of the relevant state Γ_t is a transformation of the law of motion of $\hat{w}_t$ that reflects the share of inefficient separations. To understand the state Γ_t, suppose first that the width of the inaction region is large enough such that there exists a Γ_t such that, if the match starts from this point, all future separations are exogenous—which implies $\mathcal{T}(\Gamma_t,\hat{\rho}) = 1/(\hat{\rho}+\delta)$ and $\mathcal{T}'_{\hat{w}}(\Gamma_t,\hat{\rho}) = 0$. In this case, the local law of motion of Γ_t is given by $\mathrm{d}\Gamma_t = \mathrm{d}\hat{w}_t = -\gamma\,\mathrm{d}t + \sigma\,\mathrm{d}W_t$. In the opposite case, if Γ_t is sufficiently close to a separation trigger—e.g., the upper Ss band $\hat{w}^+$—then $\mathcal{T}'_{\hat{w}}(\Gamma_t,\hat{\rho}) < 0$ and $\mathcal{T}(\Gamma_t,\hat{\rho})(\hat{\rho}+\delta) \approx 0$, therefore $\mathrm{d}\Gamma_t = \sigma^2(\hat{\rho}+\delta)\mathcal{T}'_{\hat{w}}(\Gamma_t,\hat{\rho})\,\mathrm{d}t$. The argument here is the following: A high enough wage increases the probability of a layoff, which has a first order effect that is equivalent to reducing wages in the same amount of the drop in the expected duration.

From Proposition 13, we can characterize the elasticity of the share in symmetric economies.

Proposition 14. *Assume that $\gamma = 0$.*

1. *If $(\hat{w}^-, \hat{w}) \to (-\infty, \infty)$, then*

$$\left.\frac{\mathrm{d}\log(\eta(\hat{w}))}{\mathrm{d}\hat{w}}\right|_{\hat{w}=\hat{w}^*} = \frac{\left[\alpha + (1-\alpha)\hat{\rho}\hat{U}\right]}{\alpha(1-\hat{\rho}\hat{U})} \tag{64}$$

2. *Assume that $\Delta^+ = \Delta^-$, and Δ^+ is small enough, then*

$$\left.\frac{\mathrm{dlog}(\eta(\hat{w}))}{\mathrm{d}\hat{w}}\right|_{\hat{w}=\hat{w}^*} = \frac{1}{\alpha(\Delta^+ + \Delta^-)} = \frac{\sqrt{s^{end}}}{\alpha\sigma} \tag{65}$$

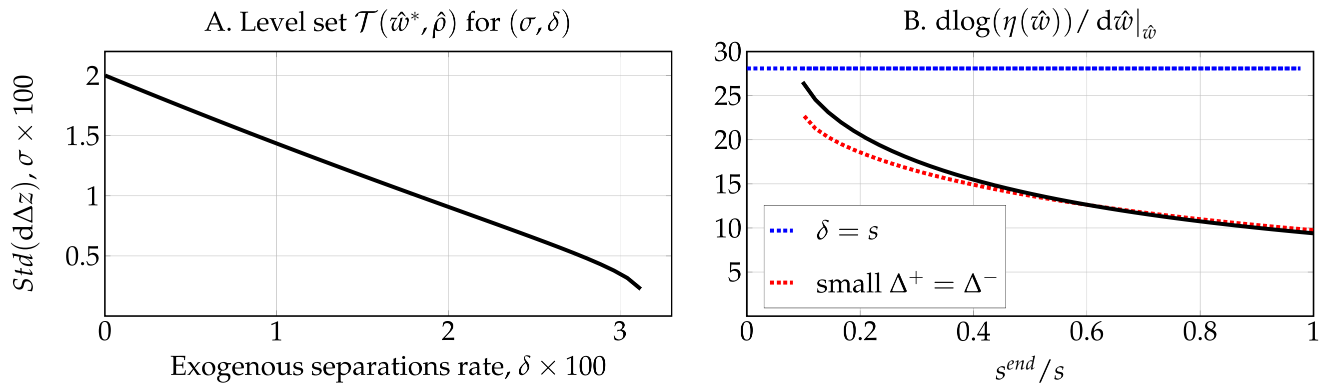
We explain Proposition 14 with the help of Figure 4, which is computed in two steps. First, we set $\delta = 0$ and calibrate the model to match the average job-finding and separation rates in the US economy, together with a replacement ratio for new employed workers of 0.46. We choose α such that $\mathcal{T}'_{\hat{w}}(\hat{w}^*, \hat{\rho}) = 0$. Second, we compute the function $\sigma(\delta)$ that keeps $\mathcal{T}(\hat{w}^*, \hat{\rho})$, and therefore the aggregate separation rate, constant. What the function $\sigma(\delta)$ does is to keep, by construction, the opportunity cost $\hat{\rho}\hat{U}$ and the duration of the match constant, but change the share of endogenous separations from 0 to 1. Figure 4-Panels A and B show $\sigma(\delta)$ and $\mathrm{dlog}(\eta(\hat{w}))/\left.\mathrm{d}\hat{w}\right|_{\hat{w}=\hat{w}^*}$, respectively.

As a starting point, assume the case with $\delta = s^{data}$ and $s^{end}/s = 0$. Equation (64) characterize this limit. In this case, all separations are exogenous, and a marginal increase in the entry wage increases the worker's share since wages during the match are higher. Equation (64) also shows the well-known result that, in this limiting case, the elasticity of the share is proportional to the inverse of flow surplus $1 - \hat{\rho}\tilde{U}$ (Shimer, 2005a).

When the share of inefficient separations increases, the elasticity of the worker's share to the entry wage decreases. In this case, a new mechanism that reduces the elasticity arises. With a higher entry wage, the probability that the worker gets fired increases, and the probability that the worker chooses to quit decreases. By construction, the expected duration of the match does not change; thus, the match's joint surplus—i.e., the denominator in the worker's share—does not change. In addition, by the envelope condition, the change in the probability of quitting does not affect the worker's value. Nevertheless, up to a first-order approximation, the increase in the probability of being laid off reduces the worker's value since she did not choose this separation trigger. This mechanism reduces the elasticity of the worker's share to the entry wage whenever the ratio of endogenous to total separation increases. In the limit, equation (65) disciplines this elasticity as a function of observables.

Discussion. We relegate several additional results to the Online Appendix F. There, we present the CIR for large shocks by characterizing the CIR of employment up to a second-order approximation. We also characterize the CIR for average real per capita wages when entry wages are flexible.

FIGURE 4. $\mathrm{dlog}(\eta(\hat{w}))/\,\mathrm{d}\hat{w}|_{\hat{w}=\hat{w}^*}$ FOR DIFFERENT (δ,σ) AND CONSTANT $\mathcal{T}(\hat{w}^*,\hat{\rho})$



Notes: Panel A shows the level set of $\mathcal{T}(\hat{w}^*,\hat{\rho})$ for different values of (δ,σ). Panels B shows the elasticity of the worker's share with the entry wage and two theoretical limits when $\delta = s$ and $\delta = 0$, respectively. The parameter values for $\delta = 0$ are $(\gamma,\pi,\sigma,\rho,\alpha,\tilde{K},\delta,\tilde{B}) = (0,0,0.02,0.0033,0.45,2.2,0,0.45)$. The steady-state targets for this calibration are: $(f(\hat{w}^*),s) = (0.45,0.032)$ with a replacement ratio of 0.46 and $\mathcal{T}'_{\hat{w}}(\hat{w}^*,\hat{\rho}) = 0$.

5 Conclusion

There is mounting empirical evidence of wages being less than fully flexible. To understand the consequences of wage rigidity at the micro and macro levels, we developed a theory of labor markets with allocative wages. The realistic ingredients of this theory included fluctuations in individual output (i.e., productivity shocks), fixed pay within jobs (i.e., wage rigidity), and the possibility that workers can quit and firms can dissolve jobs at any point in time (i.e., two-sided lack of commitment). Our theory embedded these ingredients in an environment with search frictions, which are central to many macroeconomic analyses of labor markets.

We demonstrated that this theory is useful because it enables us to study the prevalence of inefficient job separations by first identifying the microeconomic implications and then analyzing the macroeconomic consequences of allocative wages. Our study remains analytically tractable by leveraging the powerful tools of optimal control in continuous time. We establish that both a worker's decision to quit and a firm's decision to dissolve a job can be formulated as a nonzero-sum stochastic differential game with stopping times. This formulation allows us to prove the existence of a unique block recursive equilibrium and provide a sharp characterization of agents' equilibrium policies. We show that our theory also has empirical content, as it can be inverted to identify the unobserved distribution of an appropriately defined state variable from microdata on wage changes and worker flows between jobs. The identified model allows us to study the monetary multipliers for aggregate employment and real wages through the use of sufficient statistics, which we show are closely linked to the prevalence of inefficient job separations.

Our work points to several interesting avenues for future research. Possible extensions to our framework include the introduction of on-the-job search, wage renegotiations subject to adjustment frictions à la Rotemberg (1982) or Calvo (1983), a notion of a firm, and alternative types of idiosyncratic and aggregate shocks. While we have abstracted from these features, integrating them into a unified model would make possible a rich, quantitative analysis of labor markets and the macroeconomy.

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