Optimal Trade Policy with Trade Imbalances*

Mostafa Beshkar
Indiana University

Ali Shourideh
Carnegie Mellon University

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Abstract

Trade imbalances are a salient feature of international trade, yet we know little about their implications for optimal trade policy. For example, does a government have more incentives to restrict imports when the country runs a larger trade deficit? Are optimal import tariffs counter-cyclical? Does capital control obviate or mitigate the need to manipulate trade policies in response to trade shocks? To answer these questions, we characterize optimal trade policy under a dynamic trade model in which trade imbalances are generated endogenously. Our key finding is that optimal import taxes are counter-cyclical and optimal export taxes are pro-cyclical, and the size or direction of trade imbalances have no bearing on optimal trade policy. Nevertheless, under certain growth paths, optimal trade policy and equilibrium trade deficit are correlated. Finally, we find that the optimal policy will discourage (encourage) the accumulation of foreign debt when the country is growing faster (slower) than the rest of the world. This aspect of optimal policy may be implemented using capital control.

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1 Introduction

A salient feature of international trade is the presence of trade imbalances. The level and direction of these imbalances may be affected by various policies including trade policy and capital controls. How should governments conduct their trade policy under trade imbalances? The economic literature on trade policy has largely avoided this question by focusing on static models in which trade is balanced by assumption and, thus, various dynamic aspects of trade policies are overlooked. Our goal in this paper is to explore the relationship between inter-temporal trade and optimal trade policy.

The presence of inter-temporal dynamics poses several important questions regarding the optimal conduct of trade policy. First, in the presence of international capital flows, which make trade imbalances possible, governments could supplement trade policy with capital controls to manipulate the flow of goods and services across borders. The potential interdependencies between capital controls and trade policies may have important implications about the design and benefits of trade agreements. For example, following negotiated trade liberalizations, governments may have an incentive to use capital controls more actively to manipulate their terms of trade, thereby frustrating the intent of trade agreements to some degree.[1] To what extent are such arguments valid? As a step toward addressing these questions, in this paper we characterize the interdependence of capital controls and trade policy.[2]

A second question is related to the pattern of optimal tariff over business cycles. Are optimal tariffs related to the size of the economy or to its growth rate? A country that is expecting a high growth rate in the future may find it optimal to run a deficit at the current period to smooth its consumption over time. Does the growth rate of the economy have any bearing on optimal conduct of trade policy?

Our point of departure is two previous papers on optimal trade policy (Beshkar and Lashkaripour 2019, henceforth BL) and optimal capital control (Costinot, Lorenzoni, and Werning 2014, henceforth CLW). BL find the structure of optimal im-

[1]It is notable that shortly after its accession to the World Trade Organization, China was frequently accused of manipulating its exchange rate to affect the flow of goods and services.

[2]A key difference between tariffs and capital controls is that tariffs could be imposed on trade flows at the sectoral level, while capital flow taxes or exchange rate manipulations cannot replicate a sector-specific tax on trade flows. In other words, compared to sector-specific tariffs, these policies are more blunt instruments that affect aggregate trade flows across all sectors.

port and export taxes under a static general-equilibrium Ricardian model in which trade is balanced by assumption and, hence, inter-temporal considerations are absent. To focus on capital control, CLW assume free trade and adopt a dynamic endowment model in which the endowments are subject to exogenous changes over time. We go beyond these studies to analyze trade policy and capital control simultaneously under a dynamic trade model. We can, thus, address issues regarding cyclicality of trade policy, the interdependence of trade policy and capital control, and the potential effects of trade imbalances on trade policy.

We work within a two-country Ricardian model with time-varying labor productivity. The variation in productivity over time creates a role for international lending and borrowing for consumption-smoothing purposes. We assume that the policy instruments at the disposal of the government include import and export tax/subsidy as well as a tax on foreign borrowing and lending of domestic households.

First, we show that under general intra-period preferences, the optimal structure of trade policy is profoundly different under dynamic and static versions of a Ricardian model. In particular, as shown by BL and Costinot et al. 2015, optimal import tariffs are uniform across products under a static Ricardian model. In contrast, we show that under a dynamic model, optimal tariffs vary across products. The cross-product variation in optimal tariffs generally depends on the elasticity of demand and the expenditure share of each product.

To obtain an intuition about this result, note that the optimality of uniform tariffs in a static Ricardian model follows because, due to the assumption of constant unit-labor requirement, import tariffs do not affect the relative prices of home imports or the relative prices of home exports. However, allowing for inter-temporal trade introduces a new price, i.e., the asset prices, the relative magnitude of which is not pinned down by Ricardian technologies.

Next, in order to focus on inter-temporal trade and the pattern of optimal policy over time, we restrict our attention to CES preferences, which yields a uniform optimal import and export tax in each period. Therefore, the problem of optimal trade policy under a multi-product model with CES preferences reduces to the problem of optimal trade policy for a two-good economy. Under this environment, we find that, in the absence of capital control, both import and export taxes/subsidies are necessary for the implementation of optimal policy. This is in contrast to a static two-good model in which one of the trade tax instruments are redundant.

We find that the productivity of the home country relative to the rest of the world is the key parameter that explains the variation in optimal trade taxes across periods. In particular, we find that import taxes are counter-cyclical and export taxes are pro-cyclical. Intuitively, this result is obtained because the government is interested in manipulating its inter-temporal terms of trade by reducing exports in booms and limiting imports in recessions. In other words, from the government's point of view, the households save too much in booms and, thus, they consume too much of the foreign good in recessions. The government could, therefore, achieve its desired inter-temporal allocation by a higher import tax in low-productivity periods and a higher export tax in high-productivity periods.

We then ask to what degree can capital control could substitute trade policy. We show that capital control in the form of a tax on international asset trade, could substitute one (and only one) trade tax instrument. Our main finding regarding capital control policy is that for a given relative productivity, the optimal tax on foreign asset positions in period t is decreasing function of the home country's relative growth rate. More specifically, during growth periods, it is optimal to discourage the accumulation of foreign debt by subsidizing net foreign asset holdings, or equivalently, by taxing the export of financial capital.

Literature (To be completed) Staiger and Sykes (2010) take on the issue of interdependence between trade policy and currency manipulation and ask if governments could frustrate the intent of trade agreements by manipulating the value of their currency. They conclude that the trade effects of such policies could not be identified well-enough to make a judgement about whether these policies frustrate the intent of trade agreements. Bagwell and Staiger (1990) consider trade wars and self-enforcing trade agreement in a dynamic environment in which the countries' endowments are subject to shock, but no inter-temporal trade takes place. The dynamics in the model come from the fact that governments could exchange trade policy concessions over time. Keeping the assumption of balanced trade in each period, Bagwell and Staiger (2003) extend their previous work to study trade policy over persistent business-cycle shocks. Other items: Benigno et al. (2016), Fernández et al. (2015), Lake and Linask (2016), Schmitt-Grohé and Uribe (2017) Syropoulos (2002).

In Section 2, we present the basic model and the planner's problem and establish our first result about the pattern of optimal trade policy under a dynamic trade model. In Section 3, we derive optimal trade policy and characterize its variation over the business cycle. In Section 4, we consider capital control taxes as an additional policy instrument at the government's disposal and find its interdependence with trade taxes. In Section 5, we consider a simple growth path and compute the optimal trade policy and the equilibrium levels of deficit and surplus. Finally, in Section 6, we conclude by discussing some of the potential implications of our analysis as well as further questions for future research.

2 The Basic Model and the Planner's Problem

We use an infinite-horizon , two-country, K-industry, model in which consumption and production takes place in each period of time, t. Each country, $i, j \in \{h, f\}$, produces a distinct variety in each industry. We let $x^{j}_{t,i,k}$ denote the period-t quantity of country i's variety of good k that is consumed in country j. With appropriate interpretation of subscripts and superscripts, bold-faced variables denote vectors and capitalized variables denote aggregate values.

Technologies Each variety is produced using labor as the only input to production. Labor productivity in industry k of country i at time t, denoted by $a_{t,i,k}$, is independent of the quantity of production. Labor is perfectly mobile across industries within the same country. The population of labor in each country is assumed to be constant over time, and we normalize the population in each country to 1.

Preferences The consumer's utility in country j is given by

$$\sum_{t} \beta^{t} u\left(g\left(\boldsymbol{x}^{j}_{t}\right)\right), \tag{1}$$

where $\boldsymbol{x}^{j}_{t}$ is country j's vector of consumption in period t, and $g\left(\boldsymbol{x}^{j}_{t}\right)$ is the aggregate consumption in period j. We assume that $g\left(\boldsymbol{x}^{j}_{t}\right)$ takes a nested CES form such that

$$g\left(\boldsymbol{x}_t^j\right) \equiv \left[\sum_k \alpha_k \left(\left[\left(x_{t,h,k}^j\right)^{\frac{\sigma_k-1}{\sigma_k}} + \left(x_{t,f,k}^j\right)^{\frac{\sigma_k-1}{\sigma_k}}\right]^{\frac{\sigma_k}{\sigma_k-1}}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{2}$$

After establishing some results with these preferences, we further simplify the analysis by assuming $\sigma_k = \sigma$ for all k.

Policy Instruments We assume that the home government is policy active, while the foreign government takes a laissez faire approach. We assume that, generally, the policy instruments at the disposal of the home government include import and export tax/subsidy as well as a tax on international borrowing and lending of domestic households. We also consider scenarios in which the government's policy space is constrained.

The Planner's Problem

To solve the home government's problem, we first take a *primal* approach, by solving a planning problem, as in CLW, in which the equilibrium quantities are directly chosen by the government. We then find the set of policies that implement the optimal allocation.

The planner's problem is to choose an allocation to maximize the welfare of the representative consumer in the home country:

$$\max_{\left\{\boldsymbol{x}_t^h\right\}} \sum_{t=0}^{\infty} \beta^t u\left(g\left(\boldsymbol{x}_t^h\right)\right),$$

subject to

1. Per-period labor-market clearing conditions:

$$\left(\boldsymbol{x}_{t,h}^h + \boldsymbol{x}_{t,h}^f\right) . \frac{1}{\boldsymbol{a}_{t,h}} = 1, \tag{3}$$
$$\left(\boldsymbol{x}_{t,f}^h + \boldsymbol{x}_{t,f}^f\right) . \frac{1}{\boldsymbol{a}_{t,f}} = 1,$$

2. Budget constraint of foreign country's representative consumer:

$$\sum_{t=0}^{\infty} \beta^t \nabla u \left(g \left(\boldsymbol{x}_t^f \right) \right) \cdot \boldsymbol{x}_t^f = \sum_{t=0}^{\infty} \beta^t \left[\nabla u \left(g \left(\boldsymbol{x}_t^f \right) \right) \right]_{\boldsymbol{x}_{t,f}^f} . \mathbf{y}_t^f. \tag{4}$$

3. No domestic distortion in either country (given that the available tax instruments are levied only on international exchanges), which implies that marginal utilities from consumption of the domestically-produced goods in each country are proportional to their input requirement:

$$\begin{aligned} \frac{\partial g \left(\boldsymbol{x}_t^f \right)}{\partial x_{t,f,k}^f} &= \frac{\lambda_t^f}{a_{t,f,k}}, \\ \frac{\partial g \left(\boldsymbol{x}_t^h \right)}{\partial x_{t,h,k}^h} &= \frac{\lambda_t^h}{a_{t,h,k}}. \end{aligned} \tag{5}$$

The optimal policy problem above highlights a few features. First, since the government of the domestic home country distributes the revenue from its taxes on imports and exports in a lump-sum fashion, the budget constraint of the domestic consumer imposes no extra constraint on the planning problem. Second, since taxes are only imposed on international flows, production is domestically efficient. Equation (5) specifies that for any two goods produced in each country, marginal rate of substitution must be equal to marginal rate of transformation which is the relative productivities.

Our first result establishes the effect of trade imbalances on the structure of optimal trade taxes by comparing the optimum under dynamic and static environments:

Proposition 1. *Under a dynamic Ricardian model, the optimal import and export taxes within each period are generally differential across products. Under the assumption of balanced trade in each period, optimal tariffs are uniform.*

This proposition shows a profound difference in the pattern of optimal trade taxes under dynamic and static models of international trade. As shown by BL, under a static Ricardian model with general assumptions on preferences, optimal import tariffs are uniform. This proposition, however, shows that this result does not generally hold under a dynamic Ricardian model in which households can engage in inter-temporal trade to smooth their consumption over time.

The assumption of fixed unit-labor requirement under Ricardian models precludes import tariffs from affecting the relative prices of home imports or the relative prices of home exports *within each period*. Therefore, in a one-period (i.e., static) model, the terms-of-trade effects of differential tariffs could be replicated by a uniform tariff on all products. Additionally, uniform tariffs create less distortions for domestic consumption, hence, the optimality of uniform tariffs under static Ricardian models. In contrast, under a dynamic setting, import tariffs could affect the relative prices of home imports across periods and, thus, the above argument for the uniformity of import tariffs is no longer valid.

In other words, our result shows that the fact that intra-temporal relative prices are determined by productivities does not necessarily imply that taxes must be uniform. This feature is mainly due to the presence of cross elasticities coming from individual preferences together with the fact that trade is not balanced within a period. More specifically, since relative prices across periods are not fixed, a change in the relative price of two imports within a period can lead to a trade deficit in the current period which in turn can change the dynamic relative prices in the direction that favors the home country.

In general, the above result implies that providing a full characterization of optimal trade taxes depends heavily on the structure of preferences and cross-elasticities. In what follows, we restrict attention to CES preferences to make some progress in providing analytical results on the behavior of optimal trade policies.

3 Optimal Trade Policies

In comparison to static trade policy analyses, the problem of optimal policy under a dynamic setting has at least two novel features. First, trade policy could fluctuate over time, which creates the possibility for tariffs to affect household savings in expectation of future changes in trade policy.[3] For instance, if governments announce a commitment to gradually reduce import tariffs over time, households

[3]For example, a surge in the United States' trade deficit in 2018 was attributed to a looming trade war with China and other countries. Other examples include increased investment in the export sector in expectation of free trade agreements.

are induced to decrease their current consumption in order to save for consumption at more favorable prices in the future. A second feature of trade policy in dynamic settings is the emergence of an additional tax instrument, namely, an inter-temporal trade or capital control tax, which may complement or substitute trade policy.

Our objective in this Section is to shed light on the *inter*-temporal features of trade policy and capital control. To this end, we first establish the following result, which helps us adopt a framework that abstracts away from *intra*-temporal variations in trade policy:

Lemma 1. *If the within-period preferences, g, takes a CES form, the optimal import and export taxes are uniform within each period.*

In order to focus on inter-temporal trade policy, we henceforth restrict our attention to CES preferences within each period, which, according to the above lemma yields a uniform optimal import and export tax in each period. Assuming within-period CES preferences reduces the problem of optimal policy to a two-good model in which each country naturally imports one and exports the other.

Under within-period CES preferences, quantities, prices, and productivities could be written in aggregate form. Formally, assuming that $g\left(\boldsymbol{x}_t^j\right) \equiv \left[\sum_k \alpha_k \sum_i \left(x_{t,i,k}^j\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$ and $u(X_t^j) = \frac{\gamma}{\gamma-1}\left(X_t^j\right)^{\frac{\gamma-1}{\gamma}}$, the planner's problem may be written as:

$$\max_{\left\{X_{t,h}^h, X_{t,f}^h\right\}_{t=1,\dots,\infty}} \sum_{t=1}^{\infty} \beta^t u(X_t^h)$$

subject to

$$\begin{aligned} X_{t,h}^h + X_{t,h}^f &= A_{t,h}, \\ X_{t,f}^h + X_{t,f}^f &= A_{t,f}, \end{aligned} \tag{6}$$

$$\sum_{t=0}^{\infty} \beta^t u'\left(X_t^f\right) X_t^f = \sum_{t=0}^{\infty} \beta^t u'\left(X_t^f\right) \frac{dX_t^f}{dX_{t,f}^f} A_{t,f} \tag{7}$$

In the above planning problem, we have used the result in Lemma 1 that under CES preferences, optimal import and export taxes are uniform. This together with the fact that households in both countries have identical preferences imply that we can define a consumption bundle for home and foreign goods given by $X^i_{t,i'}$ where $i, i' = h, f$. These bundles are defined as

$$X^j_{t,h} = \left[\sum_k \alpha_k \left(x^j_{t,h,k}\right)^{1-\frac{1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}},$$
$$X^j_{t,f} = \left[\sum_k \alpha_k \left(x^j_{t,f,k}\right)^{1-\frac{1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}.$$

These aggregate bundles coming from each country then form the aggregate consumption bundle

$$X^j_t = \left[\left(X^j_{t,f}\right)^{1-\frac{1}{\sigma}} + \left(X^j_{t,h}\right)^{1-\frac{1}{\sigma}}\right].$$

Moreover, it also implies that production of each bundle, denominated in units of the bundle is given by $A_{t,i}$ where $A_{t,i}$ is defined as

$$A_{t,i} = 2\left[\sum_k (\alpha_k)^{\sigma} \left(a_{t,i,k}\right)^{\sigma-1}\right]^{\frac{1}{\sigma-1}}.$$

In essence, uniform taxes implies that our economy is equivalent to an endowment economy with endowments of countries being imperfect substitutes.

Using the Lagrangian form

$$\mathcal{L} = \sum_{t=1}^{\infty} \beta^t u(X^h_t) + \mu \sum_{t=1}^{\infty} \beta^t \left(u'\left(X^f_t\right) X^f_t - u'\left(X^f_t\right) \frac{dX^f_t}{dX^f_{t,f}} A_{t,f}\right),$$

we can write the first-order conditions for optimality of $X^h_{t,h}$ as follows:

$$\begin{aligned} \beta^t u'(X^h_t) \frac{dX^h_t}{dX^h_{t,h}} + \mu\beta^t &\left[-u''\left(X^f_t\right) X^f_t \frac{dX^f_t}{dX^f_{t,h}} - u'\left(X^f_t\right) \frac{dX^f_t}{dX^f_{t,h}}\right. \\ &+ \left.\left(u''\left(X^f_t\right) \frac{dX^f_t}{dX^f_{t,f}} \frac{dX^f_t}{dX^f_{t,h}} + u'\left(X^f_t\right) \frac{d^2 X^f_t}{dX^f_{t,f} dX^f_{t,h}}\right) A_{t,f}\right] = 0 \end{aligned} \tag{8}$$

Similarly, the FOC with respect to $X^h_{t,f}$ will be:

$$\beta^t u'(X^h_t)\frac{dX^h_t}{dX^h_{t,f}} + \mu\beta^t \left[-u''\left(X^f_t\right)X^f_t\frac{dX^f_t}{dX^f_{t,f}} - u'\left(X^f_t\right)\frac{dX^f_t}{dX^f_{t,f}}\right. \tag{9}$$
$$\left. + \left(u''\left(X^f_t\right)\left(\frac{dX^f_t}{dX^f_{t,f}}\right)^2 + u'\left(X^f_t\right)\frac{d^2X^f_t}{d\left(X^f_{t,f}\right)^2}\right)A_{t,f}\right] = 0.$$

The necessary conditions for optimality would be also sufficient if our programming problem is convex, which requires the concavity of the implementability constraint (i.e., the foreign budget constraint 7). To ensure the convexity of the programing problem, we assume:

Assumption 1. $\gamma \geq 1$ *and* $\sigma > \gamma$.

This assumption guarantees that the following conditions hold:

1. $\sum_{t=0}^{\infty}\beta^t u'\left(X^f_t\right)X^f_t \equiv \sum_{t=0}^{\infty}\beta^t\left(X^f_t\right)^{1-\frac{1}{\gamma}}$ is concave in $\left(X^f_1, X^f_2, ...\right)$.
2. $\sum_{t=0}^{\infty}\beta^t u'\left(X^f_t\right)\frac{dX^f_t}{dX^f_{t,f}}A_{t,f} \equiv \sum_{t=0}^{\infty}\beta^t\left(X^f_t\right)^{\frac{1}{\sigma}-\frac{1}{\gamma}}\left(X^f_{t,f}\right)^{-\frac{1}{\sigma}}A_{t,f}$ is convex in $\left(X^f_1, X^f_2, ...\right)$ and $\left(X^f_{1,f}, X^f_{2,f}, ...\right)$.

These conditions enable us to replace the equation (7) with an inequality

$$\sum_{t=0}^{\infty}\beta^t\left(X^f_t\right)^{1-\frac{1}{\gamma}} \geq \sum_{t=0}^{\infty}\beta^t\left(X^f_t\right)^{\frac{1}{\sigma}-\frac{1}{\gamma}}\left(X^f_{t,f}\right)^{-\frac{1}{\sigma}}A_{t,f}$$

Given Assumption 1, this inequality will be binding in optimum. Then standard arguments – see Luenberger (1997) – imply that we have convex optimization problem and that first order conditions are necessary and sufficient. Under this assumption, therefore, the FOCs are also sufficient for optimality.

3.1 Implementation of the Optimal Allocation

So far we have considered the problem of a planner, who could directly choose the quantities of production and consumption subject to implementability of the allocation. We now find the corresponding tax rates that would recreate the planner's optimal allocation under a competitive market. To this end, consider the optimal choice of consumers in h, which maximizes the following Lagrangian:

$$\sum_t \beta^t u(X_t^h) + \lambda^h \sum_t \left[P_{t,h}^h X_{t,h}^h + P_{t,f}^h X_{t,f}^h - I_t^h \right]$$

where $P_{t,i}^h$ denote period-t price of the (aggregate) good produced in i for consumers in h, and I_t^h is the income of country h in period t. The home consumer choice will satisfy

$$\beta^t u'(X_t^h) \frac{dX_t^h}{dX_{t,h}^h} + \lambda^h P_{t,h}^h = 0.$$

Similarly, the foreign consumer choice will satisfy

$$\beta^t u'(X_t^f) \frac{dX_t^f}{dX_{t,h}^f} + \lambda^f P_{t,h}^f = 0.$$

Defining export tax as $1 + \tau_{t,h} \equiv \frac{P_{t,h}^f}{P_{t,h}^h}$, we must have

$$1 + \tau_{t,h} \equiv \frac{\lambda^h}{\lambda^f} \frac{u'(X_t^f) \frac{dX_t^f}{dX_{t,h}^f}}{u'(X_t^h) \frac{dX_t^h}{dX_{t,h}^h}},$$

Similarly, the import tax, $1 + \tau_{t,f} \equiv \frac{P_{t,f}^h}{P_{t,f}^f}$, is given by

$$1 + \tau_{t,f} \equiv \frac{\lambda^f}{\lambda^h} \frac{u'(X_t^h) \frac{dX_t^h}{dX_{t,f}^h}}{u'(X_t^f) \frac{dX_t^f}{dX_{t,f}^f}}.$$

To reflect the corresponding tax rates, we can rewrite the FOC with respect to

$X^h_{t,h}$ as follows:

$$\frac{u'\left(X^h_t\right)\left(X^h_t\right)^{\frac{1}{\sigma}}\left(X^h_{t,h}\right)^{-\frac{1}{\sigma}}}{\mu u'\left(X^f_t\right)\left(X^f_t\right)^{\frac{1}{\sigma}}\left(X^f_{t,h}\right)^{-\frac{1}{\sigma}}} = 1-\frac{1}{\gamma}+\left(\frac{1}{\gamma}-\frac{1}{\sigma}\right)(1-\alpha)\left(\frac{X^f_{t,f}}{X^f_t}\right)^{-\frac{1}{\sigma}}\frac{A_{t,f}}{X^f_t}. \tag{10}$$

Similarly, the FOC with respect to $X^h_{t,f}$ may be written as

$$\frac{u'\left(X^h_t\right)\left(X^h_t\right)^{\frac{1}{\sigma}}\left(X^h_{t,f}\right)^{-\frac{1}{\sigma}}}{\mu u'\left(X^f_t\right)\left(X^f_t\right)^{\frac{1}{\sigma}}\left(X^f_{t,f}\right)^{-\frac{1}{\sigma}}} = 1-\frac{1}{\gamma}+\left(\frac{1}{\gamma}-\frac{1}{\sigma}\right)(1-\alpha)\left(\frac{X^f_{t,f}}{X^f_t}\right)^{-\frac{1}{\sigma}}\frac{A_{t,f}}{X^f_t}+\frac{1}{\sigma}\frac{A_{t,f}}{X^f_{t,f}} \tag{11}$$

The left-hand side of equation (10) is the relative marginal utilities of Home and Foreign from consumption of the home good in period t under the CES assumption. Therefore the left-hand side of (10) may be replaced with $\frac{\lambda^h}{\mu\lambda^f}\frac{1}{1+\tau_{t,h}}$. Similarly, the left-hand side of (11) may be written as $\frac{\lambda^h}{\mu\lambda^f}(1+\tau_{t,f})$. Substituting these values for the left-hand side of the FOCs and dividing the FOCs of each period yields

$$\left(1+\tau_{t,f}\right)\left(1+\tau_{t,h}\right) = 1+\frac{\frac{1}{\sigma}\frac{A_{t,f}}{X^f_{t,f}}}{1-\frac{1}{\gamma}+\left(\frac{1}{\gamma}-\frac{1}{\sigma}\right)(1-\alpha)\left(\frac{X^f_{t,f}}{X^f_t}\right)^{-\frac{1}{\sigma}}\frac{A_{t,f}}{X^f_t}}. \tag{12}$$

Moreover, combining the FOC (10) for periods t and zero yields

$$\frac{1+\tau_{t,h}}{1+\tau_{0,h}} = \frac{1-\frac{1}{\gamma}+\left(\frac{1}{\gamma}-\frac{1}{\sigma}\right)(1-\alpha)\left(\frac{X^f_{0,f}}{X^f_0}\right)^{-\frac{1}{\sigma}}\frac{A_{0,f}}{X^f_0}}{1-\frac{1}{\gamma}+\left(\frac{1}{\gamma}-\frac{1}{\sigma}\right)(1-\alpha)\left(\frac{X^f_{t,f}}{X^f_t}\right)^{-\frac{1}{\sigma}}\frac{A_{t,f}}{X^f_t}}. \tag{13}$$

Equations (12-13) characterize import and export taxes in period t relative to the export tax in period 0, which we henceforth normalize to zero.[4]

Note that in contrast to the static version of the model in which the import and export taxes are perfectly substitutable (and, hence, one of the policy instruments is redundant), equations (12) and (13) show that both import and export taxes are generally necessary for the implementation of the optimal policy. The result that both instruments are necessary arises due to the assumption that the elasticity of substitution for goods within and across periods are different. That is, if we assume $\sigma = \gamma$ the optimal export tax will be acyclic and, thus, could be set to zero. Nevertheless, under the dynamic model, even in the case were $\sigma = \gamma$, the import and export taxes are not substitutable.

3.2 Variation of optimal taxes over time

How do optimal trade taxes vary over time? To answer this question, it is useful to rewrite the optimality conditions using fraction of outputs consumed in each country. Letting $\pi_t^f = \frac{X_{t,f}^f}{A_{t,f}}$ and $\pi_t^h = \frac{X_{t,h}^h}{A_{t,h}}$ and $z_t = \frac{A_{t,h}}{A_{t,f}}$, the FOC (10) may be written as :

$$\frac{\left(\pi_{t,h}^h\right)^{-\frac{1}{\sigma}}\left(\left[\alpha\left(\pi_{t,h}^h A_{t,h}\right)^{1-\frac{1}{\sigma}}+(1-\alpha)\left(\pi_{t,f}^h A_{t,f}\right)^{1-\frac{1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}\right)^{\frac{1}{\sigma}-\frac{1}{\gamma}}}{\mu\left(\pi_{t,h}^f\right)^{-\frac{1}{\sigma}}\left(\left[\alpha\left(\pi_{t,h}^f A_{t,h}\right)^{1-\frac{1}{\sigma}}+(1-\alpha)\left(\pi_{t,f}^f A_{t,f}\right)^{1-\frac{1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}\right)^{\frac{1}{\sigma}-\frac{1}{\gamma}}}$$

$$=1-\frac{1}{\gamma}+\left(\frac{1}{\gamma}-\frac{1}{\sigma}\right)(1-\alpha)\frac{\left(\pi_{t,f}^f\right)^{-\frac{1}{\sigma}}}{\alpha\left(\pi_{t,h}^f z_t\right)^{1-\frac{1}{\sigma}}+(1-\alpha)\left(\pi_{t,f}^f\right)^{1-\frac{1}{\sigma}}}.$$

The FOC 11 may be also written as a function of only two endogenous variables, π_t^f and π_t^h. Therefore, in each period, the fraction of home and foreign production that is consumed at home is pinned down by the relative productivity, z_t, in that period. The effect of future and past productivities on the current allocation operates through the time-invariant Lagrange multiplier, μ. This observation also

[4]Note that as shown by Bond (1990), the solution to optimal trade policy determines all tariff levels relative to a numeriarie.

implies that

Proposition 2. *The optimal import and export taxes/subsidies in period t, relative to export tax in period 0, is uniquely determined by the relative productivities in period t.*

This proposition implies that the size and direction of current trade balance has no bearing on the current optimal trade policy. In particular, while, according to Proposition 2, the optimal trade policy is identical in periods with equal relative productivities, trade imbalances could be widely different across such periods. Therefore, in general, there is no relationship between optimal trade policy and trade balance in a given period.

We can further establish a monotonic relationship between the level of taxes and the relative productivity of the home country. In particular:

Proposition 3. *Optimal export tax (import tariff) is increasing in the relative productivity of the home country, z_t. Moreover, the optimal intra-temporal wedge between the home and foreign relative price of the foreign good, $\left(1+\tau_{t,f}\right)\left(1+\tau_{t,h}\right)-1$, is always positive and increasing in z_t.*

This Proposition implies that import taxes are counter-cyclical and export taxes are pro-cyclical. To obtain intuition about this result, note that the government is interested in reducing the national saving rate in booms, in order to reduce the country's demand for imports during recessions, thereby achieving a better *inter-temporal* term-of-trade.[5] This goal may be achieved by a higher import tariff during recessions or a higher export tax in booms.

4 Interdependence of Capital Control and Trade Policy

In this section, we characterize the interdependence of capital control and trade policies. Policy interdependence concerns the effect of policy choices in one area

[5] In other words, from the perspective of the government, under free trade, consumers consume too much of the foreign good in recessions and too little of the domestic good in booms. Relatedly, the saving rate of the consumers in booms (i.e., periods with high relative productivities) are too high from the government's point of view.

on the tradeoffs that policymakers face in other areas. For example, due to differences in policymaking institutions, in some countries (like the United States) capital control policies are harder to implement than other countries (like China). Do these institutional differences in the use of capital control induce a systematic difference in the use of *trade* policies across countries?

Evaluating incomplete agreements also requires an understanding of policy interdependence. Incomplete agreements are contracts that constrain—but do not eliminate—the government's policy space. For example, most trade agreements limit the use of export subsidy and impose caps on import tariffs but leave other trade-related instruments—such as capital control and exchange rate policies—to the discretion of the governments.

Our analysis so far shows that trade policy instruments alone are sufficient to implement the optimal allocation and, thus, capital control taxes are redundant when the government has unconstrained access to trade policy instruments. We now ask to what extent capital control taxes can substitute for import and export taxes.

To study capital control, suppose that in each period households can trade a one-period bond that is denominated in the home good. The rate of return on this bond for the home and foreign consumers is given by $\frac{p^h_{t,h}}{p^h_{t-1,h}}$ and $\frac{p^f_{t,h}}{p^f_{t-1,h}}$, respectively. Assuming zero export taxes/subsidies, a tax, ϕ_t, on period-t bond creates a wedge between these two rates of return such that

$$\frac{p^f_{t,h}}{p^f_{t-1,h}} = (1+\phi_t)\,\frac{p^h_{t,h}}{p^h_{t-1,h}}.$$

This wedge between the inter-temporal price of the home good in the two countries could be generated levying export taxes in each period such that $1+\phi_t = \frac{1+\tau_{t,h}}{1+\tau_{t-1,h}}$.

First, suppose that export taxes/subsidies are exogenously set to zero for all periods, but the home government has access to import tariffs and capital control taxes. The above discussion makes it clear that the government could achieve the optimal allocation with these two policy instruments.[6] In particular, after eliminating export tax/subsidy, the government will respond with an adjustment in

[6]Optimal policy implies a set of relative prices within and across periods. To set intra-period

import tariffs that preserves the terms of trade within each period. Moreover, the government uses capital control taxes/subsidies to preserve the inter-period terms of trade. Moreover, using the definition of capital control tax, $1+\phi_t = \frac{1+\tau_{t,h}}{1+\tau_{t-1,h}}$, and Proposition 3, we can establish the following result:

Proposition 4. *Suppose that the home government can choose import and capital control taxes. Then, for a given relative productivity, the optimal tax on foreign asset positions in period t is a decreasing function of the home country's relative growth rate. Moreover, with zero relative growth rate, optimal capital control tax is zero.*[7]

This proposition implies that when the economy is growing (shrinking), the optimal capital control policy is to subsidize (tax) net foreign asset positions. In other words,

Corollary 1. *It is optimal to discourage (encourage) the accumulation of foreign debt when the country is growing faster (slower) than the rest of the world.*

5 Economic Growth, Trade Imbalances and Optimal Trade Policy

In this section, we discuss the implications of economic growth on trade imbalances and optimal trade policy. To do so, we use first use our two-good model with exogenous growth and compute optimal trade taxes and equilibrium trade imbalances over time. We then extend the model to allow for endogenous growth a la Rebelo (1991) and study its implications.

5.1 Exogenous Growth

In light of proposition 2, under the unilaterally optimal allocation, the households in the home country consume a greater fraction of domestic production in periods in which home output is relatively larger. Moreover, during these periods, the fraction of foreign production that is consumed at home decreases. As we illustrate

relative prices, one trade policy instrument (i.e., import or export tax) is sufficient. To set inter-period relative prices, we can either use a capital control tax which creates a wedge between current and future consumptions, or a combination of import and export taxes.

[7] This qualitative result on capital control is also valid under free trade.

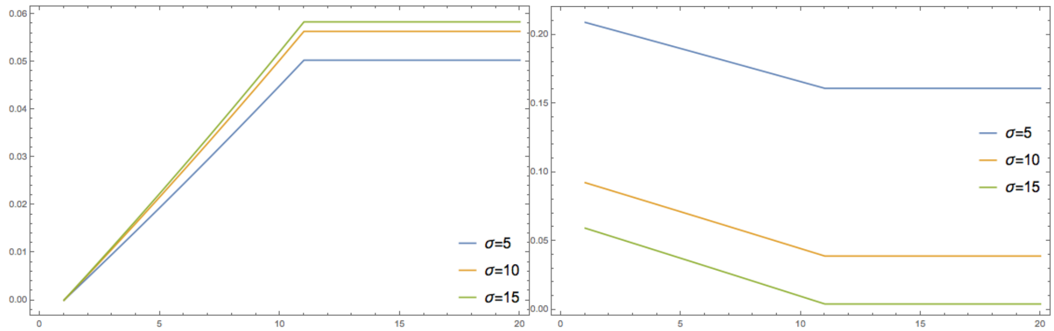

Figure 1: Export taxes (top) and import tariffs (bottom) over time

in Section 3, this trade-off leads to an import tariff that decreases with relative endowment and an export tax that increases with relative endowment.

This implies that over a period of high growth, one in which the productivity of the home country increases relative to the foreign country, we must have that export taxes increase while import tariffs decrease. This is illustrated in Figure 1. In this figure, we consider two countries that start at the same level of endowment while the home country grows at an annual constant rate of 4% while the foreign country grows annually at rate 2%. We assume that this lasts for 10 years and afterwards, the two countries have constant endowments. We normalize import tariffs to 0 at time 0. As we see, over this period the home country increases its export subsidy while at the same time it reduces its import tariffs. Moreover, the increase in export subsidies is more pronounced that the decline in import tariffs.

As we have shown, the difference between inter-temporal and intra-temporal elasticity of substitution is the main determinant of the variation in tax policies. In our calculations, we have assumed that $\gamma = 1.1$ while we allow σ to vary. The values of σ we consider are 5–15. Note that σ is equivalent to the trade elasticity in an Armington model – see Caliendo and Parro (2014) – and its estimated values are in this range. As σ increases, we see that the level of export subsidies declines while its variation increases. This illustrates a trade-off between intra- and inter-temporal terms of trade manipulation. As σ increases, imports become very elastic relative to import and thus within period terms of trade manipulation is not very beneficial to the home government. Furthermore, the benefits of inter-temporal terms of trade manipulation increases and thus both export subsidies and import taxes change more.

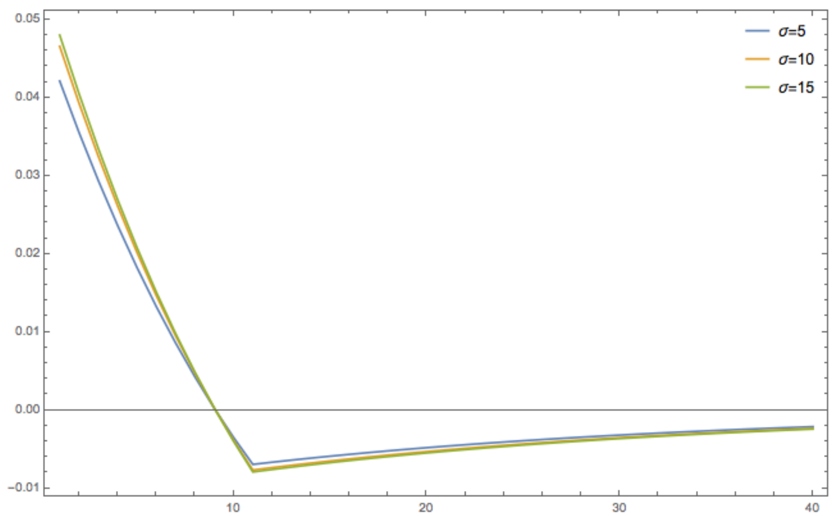

Figure 2: Trade deficit in the home country over time

In Figure 2, we plot trade deficit in the home country over time and as it varies with σ. While in our model, there is no particular relationship between deficit and trade policies, since home country finds it optimal to borrow – due to the fact that its income is growing relative to its trading partner – as deficit decreases export subsidies increase and import taxes decrease. Finally, Figure 3 depicts the intra-temporal relative price of imports to exports in the home country. The fact that this relative price is higher than 1 reflects the bias towards export at home, i.e., the fact that intra-temporal wedge is positive. Moreover, this relative price increases over time. In other words, as the home country becomes richer, its incentive for intra-temporal terms of trade manipulation of its imports becomes stronger.

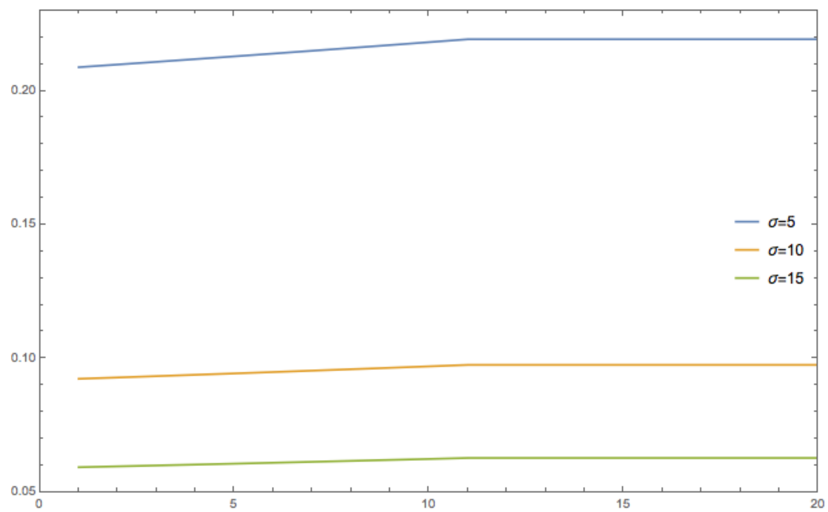

Figure 3: Intra-temporal wedge in the relative price of imports between home and foreign

5.2 Endogenous Growth [In Progress]

While the example above shed light on the behavior of optimal taxes, it is somewhat unsatisfactory since growth is exogenously imposed on endowments. In this section, we provide a a model of endogenous growth a la Rebelo (1991) to study the behavior of optimal trade policy.

TBC

6 Conclusion

In this paper, we analyzed unilaterally-optimal trade policy under a dynamic model with one factor of production. We find that the relative productivity of the home country to the rest of the world is the key time-varying parameter that determines the fluctuations in the optimal policy.

We characterized the interdependence of capital control and trade policy for a simple two-good model. In particular, we find that after entry in a trade agreement that constrains trade taxes, the government could use capital control to restore a fraction of its lost policy space. An interesting question that could be addressed in subsequent research is whether capital control could serve a useful purpose as a *flexibility* mechanism in trade agreements. Flexibility may be a desired feature for trade agreements for at least two reasons. First, if political economy preferences are subject to shocks in the future (as in Beshkar 2010, Maggi and Staiger 2011, and Beshkar and Bond 2017, among others) governments will negotiate an agreement that includes a mechanism for policy flexibility such as the WTO Agreement on Safeguards. Second, similar to Bagwell and Staiger (1990), if trade agreements must be self-enforcing, flexibility in capital control policies could reduce the governments' incentive to renege on the agreement at times when a surge in imports or a widening trade deficit increases temptations to leave an international agreement.

The possibility of resorting to the use of capital control after negotiating a trade agreement could complicate the negotiation process especially if the governments' ability to use capital controls is asymmetric. For example, due to differences in policymaking institutions, in some countries (e.g., the United States) capital control policies are harder to implement than other countries (e.g., China). Moreover, the potency of capital controls as an instrument to manipulate terms of trade depends on the magnitude of trade imbalances, which could vary substantially across countries. It may be, therefore, argued that giving up trade policy space is more costly for the former type of countries. As a result, the calculus of 'balanced concessions' in trade deals becomes a more complicated issue when countries differ in their ability to use capital control to manipulate their terms of trade.

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A Equal intra- and inter-period elasticities

To further simplify the analysis, we now assume that the intra- and inter-period elasticity of substitutions are equal, i.e., $\gamma = \sigma$. In this case, our first order conditions become

$$\frac{\left(\overline{x}^1_{t,d}\right)^{-\frac{1}{\sigma}}}{\left(\overline{x}^2_{t,f}\right)^{-\frac{1}{\sigma}}} = \mu\left(1-\frac{1}{\gamma}\right)$$
$$\frac{\left(\overline{x}^1_{t,f}\right)^{-\frac{1}{\sigma}}}{\left(\overline{x}^2_{t,d}\right)^{-\frac{1}{\sigma}}} = \mu\left(1-\frac{1}{\gamma}\right) + \mu\frac{1}{\sigma}\left(\frac{\overline{x}^2_{t,d}}{\overline{x}^2_t}\right)^{-\frac{1}{\sigma}}\frac{\overline{A}^2_t}{\overline{x}^2_{t,d}}$$

Therefore, under the optimal allocation, the fraction of country 1's output that is consumed domestically and abroad is constant and independent of time. That is

$$\begin{aligned}\overline{x}^1_{t,d} &= \frac{1}{1+\left(\mu\left(1-\frac{1}{\gamma}\right)\right)^{\sigma}}\frac{\overline{A}^1_t}{\alpha}, \overline{x}^2_{t,f} \\ &= \frac{\left(\mu\left(1-\frac{1}{\gamma}\right)\right)^{\sigma}}{1+\left(\mu\left(1-\frac{1}{\gamma}\right)\right)^{\sigma}}\frac{\overline{A}^1_t}{\alpha} = (1-\kappa)\frac{\overline{A}^1_t}{\alpha}\end{aligned}$$

Let π^2 be the fraction of output in country 2 that is consumed domestically. Then, we have

$$\overline{x}^2_{t,d} = \pi^2\frac{\overline{A}^2_t}{1-\alpha}, \overline{x}^1_{t,f} = \left(1-\pi^2\right)\frac{\overline{A}^2_t}{1-\alpha}$$

We can then write the

$$\frac{\left(\pi^2\right)^{1/\sigma}}{\left(1-\pi^2\right)^{1/\sigma}} = \mu\left(1-\frac{1}{\gamma}\right)$$
$$+\mu\frac{1}{\sigma}\left(\frac{\pi^2\overline{A}_t^2/\left(1-\alpha\right)}{\left[\alpha\left(\frac{(1-\kappa)\overline{A}_t^1}{\alpha}\right)^{1-\frac{1}{\sigma}}+\left(1-\alpha\right)\left(\frac{\pi^2\overline{A}_t^2}{1-\alpha}\right)^{1-\frac{1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}}\right)^{-\frac{1}{\sigma}}\frac{1-\alpha}{\pi^2}$$

$$\frac{\left(\pi^2\right)^{1/\sigma}}{\left(1-\pi^2\right)^{1/\sigma}} = \mu\left(1-\frac{1}{\gamma}\right) \tag{14}$$
$$+\mu\frac{1}{\sigma}\left[\alpha^{\frac{1}{\sigma}}\left(\frac{\left(1-\kappa\right)\overline{A}_t^1}{\pi^2\overline{A}_t^2}\right)^{1-\frac{1}{\sigma}}+\left(1-\alpha\right)^{\frac{1}{\sigma}}\right]^{\frac{1}{\sigma-1}}\frac{\left(1-\alpha\right)^{1+\frac{1}{\sigma}}}{\pi^2} \tag{15}$$

The above equation has a unique solution in π^2 since the LHS is increasing in π^2 while the right hand side is decreasing in π. As we see, π^2 is pinned down by the ratio of aggregate production $\frac{\overline{A}_t^1}{\overline{A}_t^2}$. Moreover, as $\overline{A}_t^1/\overline{A}_t^2$ increases π^2 increases. In other words, as one country grows relative to other one, import tariffs must increase over time. Intuitively, an increase in $\overline{x}_{t,d}^2$ reduces the income in country 2 while it increases expenditure. Therefore, this increase in $\overline{x}_{t,d}$ is beneficial in periods where $\frac{\overline{x}_{t,d}^2}{\overline{x}_t^2}$ is low. Given the formulas, optimal tariffs are also higher when $\overline{A}_t^2/\overline{x}_{t,d}^2$ is higher. Holding the ratio, $\overline{A}_t^2/\overline{x}_{t,d}^2$ constant. An increase in output of country 1 leads to a lower $\overline{x}_{t,d}^2/\overline{x}_{t,d}$ which leads to higher tariffs and a higher share of good 2's output shifting to country 1.

What can we say about trade deficit? Note that allocations are given by

$$\overline{x}_{t,d}^1 = \kappa\frac{\overline{A}_t^1}{\alpha}, \overline{x}_{t,f}^1 = \left(1-\pi^2\right)\frac{\overline{A}_t^2}{1-\alpha}$$

the relative price of exports to imports for country 2, p_t, is given by

$$p_t = \left(\frac{\overline{x}^2_{t,f}}{\overline{x}^2_{t,d}}\right)^{\frac{1}{\sigma}} = \left(\frac{(1-\kappa)\,\overline{A}^1_t/\alpha}{\pi^2\overline{A}^2_t/\,(1-\alpha)}\right)^{\frac{1}{\sigma}}$$

Trade deficit in country 1 is therefore equal to – trade surplus in country 2

$$p_t\pi^2\frac{\overline{A}^2_t}{1-\alpha} - (1-\kappa)\,\frac{\overline{A}^1_t}{\alpha}$$

with units being in terms of output of country 1. We can write this as

$$\begin{aligned}
d^1_t &= p_t\pi^2\frac{\overline{A}^2_t}{1-\alpha} - (1-\kappa)\,\frac{\overline{A}^1_t}{\alpha} \\
&= \frac{\overline{A}^1_t}{\alpha}\,(1-\kappa)\left[p_t\frac{\pi^2\frac{\overline{A}^2_t}{1-\alpha}}{(1-\kappa)\,\overline{A}^1_t/\alpha} - 1\right] \\
&= \frac{\overline{A}^1_t}{\alpha}\,(1-\kappa)\left[\left(\frac{(1-\kappa)\,\overline{A}^1_t/\alpha}{\pi^2\overline{A}^2_t/\,(1-\alpha)}\right)^{\frac{1}{\sigma}-1} - 1\right]
\end{aligned}$$

Note that as $\overline{A}^1_t/\overline{A}^2_t$ rises, so does π^2. As a result, the LHS of (14) rises. In order for the RHS to rise, we must have that $\left[\alpha^{\frac{1}{\sigma}}\left(\frac{(1-\kappa)\overline{A}^1_t}{\pi^2\overline{A}^2_t}\right)^{1-\frac{1}{\sigma}} + (1-\alpha)^{\frac{1}{\sigma}}\right]^{\frac{1}{\sigma-1}}$ increases since $\frac{(1-\alpha)^{1+\frac{1}{\sigma}}}{\pi^2}$ decreases. Now, as the above shows whether deficit rises or declines in response to a change in productivities depends on its sign as well as whether $\sigma > 1$ or not. In the empirical relevant case of $\sigma > 1 - \sigma$ is related to export elasticity in this model which is often bigger than 1, an increase in $\overline{A}^1_t/\overline{A}^2_t$ decreases the term in the bracket. It can , however, increase the term outside the bracket. Thus how tariffs are correlated with deficit really depends on the correlation between level of output in country 1 and the term inside the bracket.

What about capital control or inter-temporal distortions? We can do this in multiple ways. One idea is to assume away any export tax/subsidies. In that case, the budget constraint of households in country 1 is given by

$$\overline{x}^1_{t,d} + p_t\left(1+\tau_t\right)\overline{x}^1_{t,f} + a_{t+1}\left(1-\tau_{a,t+1}\right) \leq R_t a_t + \frac{\overline{A}^1_t}{\alpha}$$

This would imply the following optimality conditions

$$\begin{aligned}\beta^t\left(\overline{x}^1_t\right)^{-\frac{1}{\gamma}}\alpha\left(\overline{x}^1_{t,d}\right)^{-\frac{1}{\sigma}}\left(\overline{x}^1_t\right)^{\frac{1}{\sigma}} &= \lambda_t \\ \lambda_t\left(1-\tau_{a,t+1}\right) &= \lambda_{t+1}R_{t+1}\end{aligned}$$

Using the optimality conditions of the households in country 2, we can write

$$\beta\left(\frac{\overline{x}^1_{t+1,d}}{\overline{x}^1_{t,d}}\right)^{-\frac{1}{\sigma}}\left(1-\tau_{a,t+1}\right) = \beta\left(\frac{\overline{x}^2_{t+1,f}}{\overline{x}^2_{t,f}}\right)^{-\frac{1}{\sigma}}$$

The above together with our main first order condition above that states that all exports are uniformly taxed implies that $\tau_{a,t+1} = 0$. In other words, there is no need for capital control and a procyclial tariff is sufficient to achieve optimality. Intuitively, as we have shown above, the main reason to distort the margins in this model is to reduce the income of country 2. In this special case where the elasticity of inter-temporal and intra-temporal are identical, relative prices of exports for country 2 are independent of the allocation of imports. As a result, country 1's exports should not be distorted.

B Proof of Proposition 1

Separable and homothetic utility

Let us start with some examples to understand better what is going on. Suppose that utility function is separable across periods. That is suppose that

$$V\left(\mathbf{q}; I\right) = \max\sum_j V_j\left(\left[\mathbf{q}\right]_{\Omega_j}; I_j\right)$$

subject to

$$\sum_j I_j = I$$

where $\Omega_j = \Omega_j^1 \cup \Omega_j^2$. Moreover, suppose that preferences within the period are homothetic. That is there exists a function $\nu_j \left([\mathbf{q}]_{\Omega_j} \right)$ – reciprocal price index – which is increasing in all of its elements and homogeneous of degree -1 as well as concave functions $U_j (\cdot)$ such that

$$V_j \left([\mathbf{q}]_{\Omega_j} ; I \right) = U_j \left(I \nu_j \left([\mathbf{q}]_{\Omega_j} \right) \right)$$

Note that in this case, demand function for each period/sector j is given by

$$\mathbf{x}^j = -I \nabla_{[\mathbf{q}]_{\Omega_j}} \nu_j$$

This implies that we can think of $I_j \nu_j$ as a separate composite commodity with its price given by $\frac{1}{\nu_j}$. Let the Hicksian demand associated with utility function be given by $\hat{\mathbf{h}} (\boldsymbol{\nu}; u) \in \mathbb{R}^J$ where $\boldsymbol{\nu}$ is the vector of price indexes. More specifically $\hat{\mathbf{h}}$ is the solution to the following optimization

$$e (\boldsymbol{\nu}; u) = \min_{\hat{\mathbf{h}}} \sum_j \frac{\hat{h}_j}{\nu_j}$$

subject to

$$U \left(\hat{h}_1, \cdots, \hat{h}_J \right) \geq u$$

Then we have that

$$[\mathbf{h}]_{\Omega_j} = -\frac{\hat{h}_j (\boldsymbol{\nu}; u)}{\nu_j} \nabla_{[\mathbf{q}]_{\Omega_j}} \nu_j$$

Note that when this is the case, instead of compensating the consumer with distributions, we can always compensate the consumer with changes in other prices. In particular, consider $i, k \in \Omega_j^2$ and a perturbation of their prices δq_i and δq_k so that period j's price index remains unchanged. In other words,

$$\frac{\partial \nu_j}{\partial q_i} \delta q_i + \frac{\partial \nu_j}{\partial q_k} \delta q_k = 0$$

The above thus can be written as

$$\frac{\delta q_i}{\delta q_k} = -\frac{\frac{\partial \nu_j}{\partial q_k}}{\frac{\partial \nu_j}{\partial q_i}}$$

Note that since utility only depends on the price index, this perturbation of prices leaves the utility unchanged. Moreover, due to the separability shown above, the demand for other goods in other periods/sectors does not change. This implies that the only effect of this perturbation is on the imports and exports in period j. Since foreign prices are constant, exports do not change. Therefore, we have the following optimality condition

$$\begin{aligned}
&\frac{1}{\frac{\partial \nu_j}{\partial q_i}}\gamma_j^1 \nabla_{q_i}\left[\mathbf{h}\right]_{\Omega_j^1}\cdot\left[\mathbf{q}\right]_{\Omega_j^1} - \frac{1}{\frac{\partial \nu_j}{\partial q_k}}\gamma_j^1 \nabla_{q_k}\left[\mathbf{h}\right]_{\Omega_j^1}\cdot\left[\mathbf{q}\right]_{\Omega_j^1}\\
+&\frac{1}{\frac{\partial \nu_j}{\partial q_i}}\gamma_j^2 \nabla_{q_i}\left[\mathbf{h}\right]_{\Omega_j^2}\cdot\left[\mathbf{p}\right]_{\Omega_j^2} - \frac{1}{\frac{\partial \nu_j}{\partial q_k}}\gamma_j^2 \nabla_{q_k}\left[\mathbf{h}\right]_{\Omega_j^2}\cdot\left[\mathbf{p}\right]_{\Omega_j^2}\\
&+\left[\mathbf{q}-\mathbf{p}\right]_{\Omega_j^2}\cdot\left\{\frac{1}{\frac{\partial \nu_j}{\partial q_i}}\nabla_{q_i}\left[\mathbf{h}\right]_{\Omega_j^2} - \frac{1}{\frac{\partial \nu_j}{\partial q_k}}\nabla_{q_k}\left[\mathbf{h}\right]_{\Omega_j^2}\right\} = 0
\end{aligned}$$

Note that since this perturbation leaves ν_j unchanged, the only effect of a change in q_i or q_k is through the term $\nabla_{[\mathbf{q}]_{\Omega_j}}\nu_j$. Note further that ν_j is inverse of the price index and thus it is homogeneous of degree -1. Therefore, we can write

$$\nu_j\left(\lambda\left[\mathbf{q}\right]_{\Omega_j}\right) = \frac{1}{\lambda}\nu_j\left(\left[\mathbf{q}\right]_{\Omega_j}\right)$$

Taking a derivative with respect to λ and setting λ equal to 1, we have

$$\nabla_{[\mathbf{q}]_{\Omega_j}}\nu_j\left(\left[\mathbf{q}\right]_{\Omega_j}\right)\cdot\left[\mathbf{q}\right]_{\Omega_j} = -1$$

Hence,

$$\begin{aligned}
\frac{\partial}{\partial q_i}\left(\nabla_{[\mathbf{q}]_{\Omega_j}}\nu_j\left(\left[\mathbf{q}\right]_{\Omega_j}\right)\cdot\left[\mathbf{q}\right]_{\Omega_j}\right) &= 0\\
\nabla_{q_i}\nabla_{[\mathbf{q}]_{\Omega_j}}\nu_j\cdot\left[\mathbf{q}\right]_{\Omega_j} + \frac{\partial}{\partial q_i}\nu_j &= 0
\end{aligned}$$

Therefore, we can write

$$\frac{\nabla_{q_i}\left[\mathbf{h}\right]_{\Omega_j^1}\cdot\left[\mathbf{q}\right]_{\Omega_j^1}}{\frac{\partial v_j}{\partial q_i}}-\frac{\nabla_{q_k}\left[\mathbf{h}\right]_{\Omega_j^1}\cdot\left[\mathbf{q}\right]_{\Omega_j^1}}{\frac{\partial v_j}{\partial q_k}}=\frac{\nabla_{q_k}\left[\mathbf{h}\right]_{\Omega_j^2}\cdot\left[\mathbf{q}\right]_{\Omega_j^2}}{\frac{\partial v_j}{\partial q_k}}-\frac{\nabla_{q_i}\left[\mathbf{h}\right]_{\Omega_j^2}\cdot\left[\mathbf{q}\right]_{\Omega_j^2}}{\frac{\partial v_j}{\partial q_i}}$$

Hence, the above optimality condition becomes

$$\begin{aligned}
&\gamma_j^1\left\{\frac{1}{\frac{\partial v_j}{\partial q_k}}\nabla_{q_k}\left[\mathbf{h}\right]_{\Omega_j^2}\cdot\left[\mathbf{q}\right]_{\Omega_j^2}-\frac{1}{\frac{\partial v_j}{\partial q_i}}\nabla_{q_i}\left[\mathbf{h}\right]_{\Omega_j^2}\cdot\left[\mathbf{q}\right]_{\Omega_j^2}\right\}\\
&+\frac{1}{\frac{\partial v_j}{\partial q_i}}\gamma_j^2\nabla_{q_i}\left[\mathbf{h}\right]_{\Omega_j^2}\cdot\left[\mathbf{p}\right]_{\Omega_j^2}-\frac{1}{\frac{\partial v_j}{\partial q_k}}\gamma_j^2\nabla_{q_k}\left[\mathbf{h}\right]_{\Omega_j^2}\cdot\left[\mathbf{p}\right]_{\Omega_j^2}\\
&\quad+\left[\mathbf{q}-\mathbf{p}\right]_{\Omega_j^2}\cdot\left\{\frac{1}{\frac{\partial v_j}{\partial q_i}}\nabla_{q_i}\left[\mathbf{h}\right]_{\Omega_j^2}-\frac{1}{\frac{\partial v_j}{\partial q_k}}\nabla_{q_k}\left[\mathbf{h}\right]_{\Omega_j^2}\right\}=0
\end{aligned}$$

A solution to the above is a uniform tariff that satisfies

$$\tau_j^m=\frac{\gamma_j^2-\gamma_j^1}{\gamma_j^1+1}$$

The question is is this the only optimal tariff that satisfies the above.

To see this from another perspective, let us do the usual perturbation: increase q_i by dq_i and T by $x_i dq_i$ in order to compensate the representative consumer in country 1. As we have shown in the above,

$$\begin{aligned}
\frac{\partial}{\partial q_i}\left[\mathbf{h}\right]_{\Omega_j}&=-\frac{\partial\hat{h}_j}{\partial\nu_k}\frac{\partial\nu_k}{\partial q_i}\nabla_{\left[\mathbf{q}\right]_{\Omega_j}}\left[\log\nu_j\right],\forall k\neq j,i\in\Omega_k\\
\frac{\partial}{\partial q_i}\left[\mathbf{h}\right]_{\Omega_k}&=-\frac{\partial\hat{h}_k}{\partial\nu_k}\frac{\partial\nu_k}{\partial q_i}\nabla_{\left[\mathbf{q}\right]_{\Omega_k}}\left[\log\nu_k\right]+\hat{h}_k\nabla_{q_i}\nabla_{\left[\mathbf{q}\right]_{\Omega_k}}\left[\log\nu_k\right]
\end{aligned}$$

Note that again from above, we know that ν_k is homogeneous of degree -1 and therefore

$$\nabla_{\left[\mathbf{q}\right]_{\Omega_k}}\log\nu_k\cdot\left[\mathbf{q}\right]_{\Omega_k}=-1$$

Note further that by differentiating the above with respect to a q_i with $i\in\Omega_k$ we arrive at

$$\nabla_{q_i} \nabla_{[\mathbf{q}]_{\Omega_k}} \log \nu_k \cdot [\mathbf{q}]_{\Omega_k} + \frac{\partial \nu_k}{\partial q_i} = 0$$

Now, the optimality equation is given by

$$\begin{aligned} &\sum_j \gamma_j^1 \nabla_{q_i} [\mathbf{h}]_{\Omega_j^1} \cdot [\mathbf{q}]_{\Omega_j^1} \\ &+ \sum_j \gamma_j^2 \nabla_{q_i} [\mathbf{h}]_{\Omega_j^2} \cdot [\mathbf{p}]_{\Omega_j^2} \\ &+ \sum_j \nabla_{q_i} [\mathbf{h}]_{\Omega_j^2} \cdot [\mathbf{q} - \mathbf{p}]_{\Omega_j^2} = 0 \end{aligned}$$

We have that

$$\begin{aligned} &\sum_j \gamma_j^1 \frac{\partial \hat{h}_j}{\partial \nu_k} \frac{\partial \nu_k}{\partial q_i} \nabla_{[\mathbf{q}]_{\Omega_j^1}} [\log \nu_j] \cdot [\mathbf{q}]_{\Omega_j^1} + \gamma_k^1 \hat{h}_k \nabla_{q_i} \nabla_{[\mathbf{q}]_{\Omega_k^1}} [\log \nu_k] \cdot [\mathbf{q}]_{\Omega_k^1} \\ &\sum_j \gamma_j^2 \frac{\partial \hat{h}_j}{\partial \nu_k} \frac{\partial \nu_k}{\partial q_i} \nabla_{[\mathbf{q}]_{\Omega_j^2}} [\log \nu_j] \cdot [\mathbf{p}]_{\Omega_j^2} + \gamma_k^2 \hat{h}_k \nabla_{q_i} \nabla_{[\mathbf{q}]_{\Omega_k^2}} [\log \nu_k] \cdot [\mathbf{p}]_{\Omega_k^2} \\ &\qquad + \sum_j \frac{\partial \hat{h}_j}{\partial \nu_k} \frac{\partial \nu_k}{\partial q_i} \nabla_{[\mathbf{q}]_{\Omega_j^2}} [\log \nu_j] \cdot [\mathbf{q} - \mathbf{p}]_{\Omega_j^2} = 0 \end{aligned}$$

So we can write the above as

$$\begin{aligned} &\frac{\partial \nu_k}{\partial q_i} \left[\sum_j \frac{\partial \hat{h}_j}{\partial \nu_k} \left(\gamma_j^1 \nabla_{[\mathbf{q}]_{\Omega_j^1}} [\log \nu_j] \cdot [\mathbf{q}]_{\Omega_j^1} + \nabla_{[\mathbf{q}]_{\Omega_j^2}} [\log \nu_j] \cdot [\mathbf{p}]_{\Omega_j^2} + \nabla_{[\mathbf{q}]_{\Omega_j^2}} [\log \nu_j] \cdot [\mathbf{q} - \mathbf{p}]_{\Omega_j^2} \right) \right] \\ &\qquad + \gamma_k^1 \hat{h}_k \left(-\frac{\partial \nu_k}{\partial q_i} - \nabla_{q_i} \nabla_{[\mathbf{q}]_{\Omega_k^2}} [\log \nu_k] \cdot [\mathbf{q}]_{\Omega_k^2} \right) \\ &\qquad + \gamma_k^2 \hat{h}_k \nabla_{q_i} \nabla_{[\mathbf{q}]_{\Omega_k^2}} [\log \nu_k] \cdot [\mathbf{p}]_{\Omega_k^2} = 0 \end{aligned}$$

$$\frac{\partial \nu_k}{\partial q_i} \alpha_k = \hat{h}_k \nabla_{q_i} \nabla_{[\mathbf{q}]_{\Omega_k^2}} [\log \nu_k] \left(\gamma_k^1 [\mathbf{p}]_{\Omega_k^2} - \gamma_k^2 [\mathbf{q}]_{\Omega_k^2} \right)$$

Thus, we can write the above equation as

$$\frac{\alpha_k}{\hat{h}_k} \mathbf{e} = \operatorname{diag} \left(\nabla_{[\mathbf{q}]_{\Omega_k^2}} \nu_k \right)^{-1} \nabla_{[\mathbf{q}]_{\Omega_k^2}} \nabla_{[\mathbf{q}]_{\Omega_k^2}} [\log \nu_k] \left(\gamma_k^1 [\mathbf{p}]_{\Omega_k^2} - \gamma_k^2 [\mathbf{q}]_{\Omega_k^2} \right)$$

Suppose preferences are CES. Then

$$
\begin{aligned}
\nu_k &= \left[\sum_{i\in\Omega_k} \alpha_i q_i^{1-\sigma}\right]^{-\frac{1}{1-\sigma}} \\
\nu_k^{1-\sigma} &= \left[\sum_{i\in\Omega_k} \alpha_i q_i^{1-\sigma}\right]^{-1} \\
\nabla_{[\mathbf{q}]_{\Omega_k^2}} \nu_k^{1-\sigma} &= -\left(1-\sigma\right) \left[\boldsymbol{\alpha} \odot \mathbf{q}^{-\sigma}\right]_{\Omega_k^2} \nu_k^{2-2\sigma} \\
\nabla_{[\mathbf{q}]_{\Omega_k^2}} \nu_k &= -\left[\boldsymbol{\alpha} \odot \mathbf{q}^{-\sigma}\right]_{\Omega_k^2} \nu_k^{2-\sigma} \\
\log \nu_k &= -\frac{1}{1-\sigma} \log \left[\sum_{i\in\Omega_k} \alpha_i q_i^{1-\sigma}\right] \\
\nabla_{[\mathbf{q}]_{\Omega_k^2}} \log \nu_k &= -\left[\boldsymbol{\alpha} \odot \mathbf{q}^{-\sigma}\right]_{\Omega_k^2} \nu_k^{1-\sigma} \\
\nabla_{[\mathbf{q}]_{\Omega_k^2}} \nabla_{[\mathbf{q}]_{\Omega_k^2}} \log \nu_k &= \sigma \mathrm{diag}\left(\left[\boldsymbol{\alpha} \odot \mathbf{q}^{-1-\sigma}\right]_{\Omega_k^2}\right) \nu_k^{1-\sigma} - \left(1-\sigma\right) \left[\boldsymbol{\alpha} \odot \mathbf{q}^{-\sigma}\right]_{\Omega_k^2} \left[\boldsymbol{\alpha} \odot \mathbf{q}^{-\sigma}\right]_{\Omega_k^2}^T \nu_k^{2-2\sigma}
\end{aligned}
$$

Therefore

$$
\begin{aligned}
\mathrm{diag}\left(\nabla_{[\mathbf{q}]_{\Omega_k^2}} \nu_k\right)^{-1} \nabla_{[\mathbf{q}]_{\Omega_k^2}} \nabla_{[\mathbf{q}]_{\Omega_k^2}} \left[\log \nu_k\right] \cdot [\mathbf{q}]_{\Omega_k^2} &= \left(\sigma \nu_k^{-1} \mathrm{diag}\left[\mathbf{q}^{-1}\right]_{\Omega_k^2} - \left(1-\sigma\right) [\mathbf{e}]_{\Omega_k^2} \left[\boldsymbol{\alpha} \odot \mathbf{q}^{-\sigma}\right]_{\Omega_k^2}^T \nu_k \right. \\
&= \sigma \nu_k^{-1} [\mathbf{e}]_{\Omega_k^2} - \left(1-\sigma\right) [\mathbf{e}]_{\Omega_k^2} \nu_k^{-\sigma} \sum_{j\in\Omega_k^2} \alpha_j q_j^{1-\sigma} \\
&= \left\{\sigma \nu_k^{-1} - \left(1-\sigma\right) \nu_k^{-\sigma} \sum_{j\in\Omega_k^2} \alpha_j q_j^{1-\sigma}\right\} [\mathbf{e}]_{\Omega_k^2}
\end{aligned}
$$

This implies that in this case, import tariffs are uniform.

However, as the above equation shows, there is no reason that generally speaking tariffs would be uniform for the imports in the same period. What is the intuitition?

Another example of utility function is nested CES

$$
\begin{aligned}
\nu_k &= \left[\sum_l \left(\sum_{i\in S_l} \alpha_i q_i^{1-\sigma}\right)^{\frac{1-\rho}{1-\sigma}}\right]^{-\frac{1}{1-\rho}} \\
\log \nu_k &= -\frac{1}{1-\rho} \log \sum_l \left(\sum_{i\in S_l} \alpha_i q_i^{1-\sigma}\right)^{\frac{1-\rho}{1-\sigma}} \\
\frac{\partial}{\partial q_i} \log \nu_k &= -\frac{\alpha_i q_i^{-\sigma} \left(\sum_{r\in S_l} \alpha_r q_r^{1-\sigma}\right)^{\frac{\sigma-\rho}{1-\sigma}}}{\sum_l \left(\sum_{r\in S_l} \alpha_r q_r^{1-\sigma}\right)^{\frac{1-\rho}{1-\sigma}}}
\end{aligned}
$$

and then

$$
\begin{aligned}
\frac{\partial^2}{\partial q_i \partial q_j} \log \nu_k = \sigma \frac{1}{q_i} \frac{\partial}{\partial q_i} \log \nu_k \mathbf{1}\left[i=j\right] \\
- (\sigma-\rho) \frac{\alpha_j q_j^{-\sigma}}{\sum_{r\in S_l} \alpha_r q_r^{1-\sigma}} \frac{\partial}{\partial q_i} \log \nu_k \mathbf{1}\left[j,i\in S_l\right] \\
+ (1-\rho) \frac{\partial}{\partial q_j} \log \nu_k \frac{\partial}{\partial q_i} \log \nu_k
\end{aligned}
$$

Now take $i,j \in \Omega_k^2$, then the matrix we are interested in is

$$
\begin{aligned}
\frac{1}{\frac{\partial}{\partial q_i}\nu_k} \frac{\partial^2}{\partial q_i \partial q_j} \log \nu_k &= \frac{1}{\nu_k \frac{\partial}{\partial q_i} \log \nu_k} \frac{\partial^2}{\partial q_i \partial q_j} \log \nu_k \\
&= \frac{1}{\nu_k}\left[\frac{\sigma}{q_i}\mathbf{1}\left[i=j\right] - (\sigma-\rho) \frac{\alpha_j q_j^{-\sigma}}{\sum_{r\in S_l} \alpha_r q_r^{1-\sigma}} \mathbf{1}\left[i,j\in S_l\right] + (1-\rho) \frac{\partial}{\partial q_j} \log \nu_k\right]
\end{aligned}
$$

and

$$
\sum_{j\in\Omega_k^2} q_j \frac{1}{\frac{\partial}{\partial q_i}\nu_k} \frac{\partial^2}{\partial q_i \partial q_j} \log \nu_k = \frac{1}{\nu_k}\left[\sigma - (\sigma-\rho) \frac{\sum_{j\in S_l\cup\Omega_k^2} \alpha_j q_j^{1-\sigma}}{\sum_{r\in S_l} \alpha_r q_r^{1-\sigma}} + (1-\rho) \sum_{j\in\Omega_k^2} q_j \frac{\partial}{\partial q_j} \log \nu_k\right]
$$

As we see the above is not constant for all i's since it is possible that some $i \in \Omega_k^2$'s belong to two different S_l's. It, however, shows that the groups of imports that are in the same partition must have the same tariffs.