# Heterogeneity in the Credit Card Market

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**Abstract**

A bare-bones model of precautionary savings and credit-card borrowing, lending, and default, is constructed to confront facts on contract terms, usage, and performance of anonymized individual credit card accounts. Accounts are distinguished by income and creditworthiness of the account holder at the time of origination of the account. It is found that contract interest rates decline with creditworthiness and income while the credit-limit-to-income (at origination) ratios decline with income but rise with creditworthiness. Utilization rates and delinquency rates decline sharply with creditworthiness and income. If individuals differ by discount factors and default costs, the model can account for almost all of these patterns. The model underpredicts the interest rate offered to relatively creditworthy borrowers but does predicts large spreads between interest rates and default frequencies, as observed, despite modest monopoly power of card companies.

Keywords: Credit cards, credit score, credit limits, interest rates, delinquency, utilization

JEL Codes: (To be added)

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## 1 Introduction

The quantitative macro literature on unsecured consumer credit has sought to understand the reasons why U.S. consumers borrow unsecured at high interest rates and default on these debts with relatively high frequency. These attempts have generally been disciplined by publicly available aggregate and cross-sectional debt and default statistics. But quantitative models formulated to explain aggregate and cross-sectional facts make many predictions about behavior of individual over time. Ideally, we would like to confront these individual-level model implications with individual-level time series data (a panel) to better understand the drivers of unsecured consumer borrowing and default.

In this paper, we take a step in this direction by turning to a relatively new confidential administrative data that follows anonymized individual credit card accounts over time. This is data collected by the Board of Governors of the Federal Reserve System in pursuance of the annual comprehensive capital analysis and review (CCAR) of large U.S. bank holding companies, as required by the 2010 Dodd-Frank Wall Street Reform and Consumer Protection Act. One part of this data, called Y-14M, reports on the terms, usage, and performance of individual credit card accounts originated and managed by the reporting banks.

To begin, we document patterns in newly originated credit accounts with respect to contract terms, namely, interest rates and credit limits; the utilization rate of accounts, i.e., the amount borrowed as a fraction of the account credit limit; and the frequency of serious delinquency which occurs if an account is 120 days past due on payments. We distinguish accounts by creditworthiness of the account holder at the time of origination and, separately, by the income of the account holder at origination. We find that interest rates decline and the credit-limit-to-income (at origination) ratios rise with creditworthiness. We find that interest rates decline with income while credit limits rise, but the credit-limit-to-income ratios decline with income. Utilization rates and delinquency rates decline strongly with creditworthiness and income.

Our goal is to investigate if a bare-bones model of credit cards can generate the patterns we observe in contract terms, usage and performance of card accounts by income and creditworthiness. To this end we formulate a model of precautionary savings and credit card borrowing, lending and default. A credit card is a lending contract wherein the card company commits to provide funds on demand up to a specified limit at some interest rate. These terms are fixed for as long as the individual keeps the card and does not default. Default is costly for both individuals and banks. We assume directed search in the card market: card companies post contracts and individuals search for the most attractive one.

Our first finding is that the model with only income heterogeneity can deliver the patterns we observe when card accounts are distinguished by income at origination, but it cannot account for the fact that creditworthiness and credit-limit-to-income ratios are positively related — where creditworthiness is defined, following industry practice, as the probability of a serious delinquency within a two-year horizon. If individuals differ only by income, there is a tight positive association between income and creditworthiness and, so, the creditworthiness and credit-limit-to-income ratio relationship inherits the negative relationship of these ratios to income.

The model thus calls for some other source or sources of heterogeneity. A natural candidate is differences in discount factors since prior research has linked discount factors, or patience, to creditworthiness: all else the same, more patient individuals are less likely to borrow and default and hence are more creditworthy. However, a second finding of our paper is that differences in patience, while improving the fit of the model along some dimensions, does not help with getting the relationship between credit-limit-to-income ratios and creditworthiness right. The logic of the model is that, all else constant, patient individuals want cards with smaller limits and lower interest rates (why this is the case is explained later in the paper). And since more patient people are also the more creditworthy, the equilibrium relationship between creditworthiness and credit-limit-to-income ratios remains negative.

Lenders perceive creditworthy people as capable of carrying more debt relative to their incomes. This can be an equilibrium outcome if creditworthy people have higher default costs. This motivates our baseline specification wherein we let individuals differ with respect to both discount factors and default costs. A third finding is that the model can explain almost all the patterns we document if people discount the future quite heavily and discount factors and default costs are positively related.

In the data, spread of credit card interest rates over the risk-free rate exceed default probabilities. The fourth finding of the paper is that the model can replicate this fact even though card companies earn only modest pure profits on card contracts. The "excess" spread is explained by the fact that people repay small debts but default on large debts. Since the same interest rate applies to debts of all sizes below the credit limit, the spread exceeds the default probability for card companies to (roughly) breakeven.

In the data, interest rates decline slightly with creditworthiness. In contrast, in the model interest rates decline sharply with creditworthiness. Thus the model underpredicts the interest rate on contracts offered to individuals who are relatively creditworthy. We show that this discrepancy cannot be explained by market power of card companies or by costs of servicing a credit account. Instead, our model suggests that card companies charge high rates to creditworthy individuals because card companies earn very little on good customers in this group but lose a lot on bad customers.

## 2 Connections to the Literature

(To be added)

## 3 Credit Card Terms, Usage, and Performance

We begin with some basic facts about the terms of credit card contracts, the usage of cards and their performance. Our data source is confidential supervisory data, maintained by the Board of Governors of the Federal Reserve System. The data tracks the terms, usage and performance of credit card accounts issued by the 40 largest U.S. banks at a monthly frequency beginning in 2012. Since its inception, the data has covered about 80 percent of all U.S. credit card accounts in existence at a point in time.

The facts we present pertain to a 1-in-500 random sample of all new, revolving, general purpose, and unsecured credit-card accounts originated between June 2012 and May 2013 with one primary account holder.[1] Each newly originated account is followed for a period of 24 months.

For this set of newly originated credit card accounts, the contract terms we focus on are the interest rates (henceforth $R$) levied on revolving balances and ratio of credit limit to income-at-origination (henceforth $CLY$). We measure these contract terms as they exist 18 months following origination. We do not focus on interest rates and credit limits at origination because card com-

[1] Following the initial random selection, we excluded accounts for which reported borrower income was less than \$1000 (most likely, students) and credit-score-at-origination was less than 100 (most likely, a reporting error). We also excluded accounts for which credit-limit-at-origination was less than 100. Following these exclusions, we also eliminated originations with income-at-origination in the top 7 percent or in the bottom 7 percent of the (reported) income distribution. This last step was taken to align the income data with the estimated earnings process (from PSID) used in the calibration of the model.

panies often offer promotional rates and tighter limits at origination and we are not modeling this initial promotional phase.

The vast majority of credit card accounts originated over this period, 85 percent, have adjustable rate APRs — meaning, the interest rate to be charged on revolving balances is a contractually specified spread ("margin") over some market interest rate, most commonly the prime rate. Note, however, that over the period 2011-2015, the bank prime rate was constant at 3.25 percent so adjustable APRs behaved like fixed APRs.

Table 1:
Contract Terms by Creditworthiness and Income Quintiles

| Description | I | II | III | IV | V |
|---|---|---|---|---|---|
| APR ($R$) by Score Bin (in %) | 21.24 | 18.61 | 17.33 | 16.18 | 16.88 |
| APR ($R$) by Income Bin (in %) | 18.91 | 18.32 | 18.14 | 17.67 | 17.22 |
| Credit Limit to Income ($CLY$) by Score Bin | 0.04 | 0.08 | 0.11 | 0.14 | 0.14 |
| Credit Limit to Income ($CLY$) by Income Bin | 0.13 | 0.11 | 0.10 | 0.09 | 0.08 |

**Notes**: Estimates are derived from a bin-scatter regression. The APR is at annual frequency and credit limits are expressed as a ratio of annual income.

Accounts vary in terms of these contract terms and a portion of this variation is strongly related to creditworthiness and income of the account holder. Table 1 presents the systematic variation in contract terms along these two dimensions. In each case, we rank accounts by credit-score-at-origination or income-at-origination and report the mean for quintiles of $R$ and $CLY$, controlling for time fixed effects. $R$ is decreasing in creditworthiness and income, while $CLY$ is increasing in credit-score-at-origination but decreasing in income-at-origination. This last pattern results from the fact that credit limits rise proportionately less than income-at-origination.

Usage of the credit granted by a card is measured by the utilization rate 18 months following origination. The utilization rate is the ratio of balances carrying non-promotional interest rates to the credit limit operative at the time. Accounts are ranked in the same way as in Table 1 — by income and credit score at origination. As shown in Table 2, the utilization rate is declining in income and creditworthiness: Lower-income and less creditworthy account holders use their accounts more intensively.

Turning to performance, we study the frequency of default. An account is in default if any of the following has occurred within 24 months of origination: (i) the account is delinquent for 120

Table 2:
Utilization Rate by Creditworthiness and Income Quintiles

| Description | I | II | III | IV | V |
|---|---|---|---|---|---|
| Debt-to-Credit Limit By Score Bin | 0.53 | 0.31 | 0.16 | 0.07 | 0.02 |
| Debt-to-Credit Limit By Income Bin | 0.30 | 0.26 | 0.22 | 0.19 | 0.14 |

**Notes**:

Table 3:
Default Freq in % by Creditworthiness and Income Quintiles

| Description | I | II | III | IV | V |
|---|---|---|---|---|---|
| By Score Bin | 7.9 | 5.7 | 3.0 | 1.3 | 0.7 |
| By Income Bin | 5.4 | 4.9 | 3.3 | 2.9 | 2.1 |

**Notes**: All frequencies refer to a 2-year horizon

days or more, (ii) the account is in a workout, debt-waiver or debt cancellation program and (iii) the account holder has filed for bankruptcy. A default is a serious credit event for a card company since recovery on defaulted debts is quite low.

Table 3 reports the default frequency of accounts by quintiles of credit-score-at-origin and income-at-origin. The default frequency is 7.9 percent at the lowest credit score quintile and declines sharply as credit-score-at-origination improves. There is a similar declining pattern with regard to income-at-origination but the fall-off is not as marked: Even at the highest income quintile, the frequency of default is a still substantial 2.1 percent. One noteworthy fact is that default is driven primarily by (i); in particular, the frequency of bankruptcy is about one-tenth of the overall default frequency.

Tables 1-3 are the facts we focus on in this paper. In the next section, we construct a bare-bones model of precautionary savings and credit-card borrowing, lending and default to understand the ingredients needed — in terms of preference and default costs heterogeneity on the one hand and credit card search costs on the other — to account for these patterns.

## 4 Model

### 4.1 Environment

Time is discrete. There are $i \in \mathbb{I}$ types of people in the economy who differ in their discount factor and/or default costs. We assume equal measures of each type. Individuals have CRRA (per-period) utility function with a common intertemporal elasticity of substitution $1/\gamma$ and an individuals survive from period to the next with constant probability $\nu \in (0, 1)$.

An individual's income is stochastic and given by $y + m$, where $y$ is the persistent component of income and $m$ is the transitory component. The persistent component is distributed $F(y'|y)$ and the transitory component is distributed uniformly on support that depends on the persistent component of earnings: $[-\lambda y,\ \lambda y]$, $\lambda > 0$. Thus we permit the transitory shock to be negative, meaning that an individual's income can temporarily fall below her persistent income level $y$.

Individuals can borrow via credit cards. A credit card is a bilateral contract between a card company and an individual that allows the individual to borrow up to $\underline{a} < 0$ at a (gross) interest rate $R$. Credit cards can differ with regard to these terms and we use $\omega = (\underline{a}, R)$ to denote the contract terms of a card. We use $\Omega \subset \mathbb{R}^2$ to denote the space of all possible contracts. An individual can hold at most one card and a card is forever associated with one set of contract terms.[2] All individuals can save at the common risk-free (gross) interest rate $R_f$.

A person with a balance on her credit card has the option to default. If she does, she loses her card and (i) she cannot borrow or save in the period of default, (ii) she gives some fraction $0 < 1 - \phi^i \leq 1$ of her transitory income in excess of $-\lambda y$ to her creditors, and (iii) with probability $(1 - \delta)$ she is shut out of the credit card market in the following period. And, conditional on being shut-out, she continues in that state with probability $(1 - \delta)$.

A person who is not shut out of the credit card market but does not have a card searches for one with probability $\mu > 0$. Search happens at the start of a period before the transitory shock $m$ is realized. Since there is complete information, she can only search for contracts that are being offered to people of her type $i$, her asset position $a$, and her persistent earnings $y$. But within this "market", there are potentially many submarkets offering different contract terms $\omega$ and card

[2]This is a simplification as credit card companies do change the interest rates and credit limits of customers over time. As of 2009, these contractual changes must comply with restrictions imposed by the CARD Act. Our assumption that terms never change sidesteps this institutional detail.

companies commit to honoring the terms of a contract if accepted. These assumptions makes our search environment one of directed search. The probability of encountering a contract $\omega$ is given by the function $f^i(\omega; a, y)$ and is an equilibrium object.

### 4.2 Individual's Decision Problem

Let $h^i(a, y, m)$ denote the value of an individual who is not shut-out of the card market but failed to get a card in the current period. This could be because she did not search or she searched but failed to make contact. In this situation, she can only save. Let $S^i(a, y)$ denote the *ex-ante* value of a type $i$ person in state $(a, y)$ who is without a card but searching for one. The qualifier *ex-ante* means that this is an expected value prior to the realization of the transitory income shock $m$. Then,

$$
\begin{aligned}
h^i\left(a, y, m\right) = \max_{a'} \frac{c^{1-\gamma}}{1-\gamma} + \nu\beta^i \mathbb{E}_{y'|y}\{\mu S^i\left(a', y'\right) + (1-\mu)H^i(a', y'^i)\} \\
c = y + m + R_f a - a' \\
a' \geq 0,
\end{aligned}
$$

where $H^i(a, y) = \mathbb{E}_m h^i(a, y, m)$.

Let $x^i(a, y, m)$ denote the value of an individual who is without a card and is excluded from the credit card market following a default. Such a person is also limited to saving and her value is

$$
\begin{aligned}
& x^i\left(a, y, m^i\right) = \\
& \max_{a'} \frac{c^{1-\gamma}}{1-\gamma} + \nu\beta^i \mathbb{E}_{y'|y}\left[\delta\left[\mu S^i(a', y'^i(a', y'^i)\right] + (1-\delta)\, X^i\left(a', y'^i\right)\right] \\
& c = y + m + R_f a - a' \\
& a' \geq 0.
\end{aligned}
$$

where $X^i(a, y)$ is *ex-ante* value of an individual in the shut-out state and is equal to $\mathbb{E}_m x^i(a, y, m)$.

Let $v^i\left(a, y, m; \omega\right)$ denote the value of an individual in state $(a, y, m)$ who has a credit card with terms $\omega$ and who chooses to make payments (if any) on her card. Then, $v^i$ solves the following

Bellman equation:

$$v^i\left(y,m,a;\omega\right)=\max_{a'}\left\{\frac{c^{1-\gamma}}{1-\gamma}+\nu\beta^i\mathbb{E}_{y'|y}W^i(a',y'^i)\right\}$$
$$c=y+m+Pa-a',\ \text{where}$$
$$P=\begin{cases}R \text{ if } a<0\\ R_f \text{ if } a\geq 0\end{cases},$$
$$a'\geq \underline{a},$$

Here $W^i(a',y'^i)$ is the individual's *ex-ante* continuation value in the state $(a',y')$. Let $A^i\left(a,y,m;\omega\right)$ and $C^i\left(a,y,m,;\omega\right)$ denote the associated (optimal) asset and consumption decision rules.

Let $v^i_{\text{DEF}}\left(y,m\right)$ denote the value of default. Then,

$$v^i_{\text{DEF}}\left(y,m\right)=$$
$$\frac{c^{1-\gamma}}{1-\gamma}+\nu\beta^i\mathbb{E}_{y'|y}\left[\ \delta\left[\mu S^i(0,y'^i(0,y')\right]+(1-\delta)\,X^i\left(0,y'^i\right)\ \right]$$
$$c=y+\underline{m}(y)+\phi^i\cdot[m+\lambda y].$$

In the event of default, the individual retains some fraction of her transitory income and pays the rest to her creditors. Since the support of $m$ as $[-\lambda y,\lambda y]$, the individual pays zero if $m$ is at its lower support and pays $2(1-\phi^i)\lambda y$ if it is at its upper support.

We now specify the value $W^i(a,y;\omega)$. When an individual enters a period holding a credit card with no debt on it, she can get exogenously separated from her card with probability $\xi$, or, conditional on not getting exogenously separated, she can choose to separate from the card at a utility cost $k$ drawn from $U(0,\bar{k})$. The separation (exogenous or by choice) occurs before the transitory shock $m$ is realized. If she enters the period with debt then she cannot separate from her card but she can either repay her debt and retain her card or she can default and lose her card. Then,

$$W^i(a,y;\omega)=\begin{cases}\mathbb{E}_m\left[\max\left\{v^i(a,y,m;\omega),v^i_{\text{DEF}}(y,m)\right\}\right] & \text{if } a<0\\ \xi L^i(a,y)+(1-\xi)\mathbb{E}_k\max\left\{V^i(a,y;\omega),L^i(a,y)-k\right\} & \text{if } a\geq 0.\end{cases}$$

Here $L^i(a,y)$ stands in for $\mu S^i(a,y)+(1-\mu)H^i(a,y)$ and $V^i(a,y;\omega)$ solves the recursion $V^i(a,y;\omega) = \mathbb{E}_m v^i(a,y,m;\omega)$.

Finally, the recursion that solves for $S^i(a,y)$ is:

$$S^i(a,y) = \max\Big\{\max_{\omega\in\Omega}\{f^i(a,y;\omega)) \cdot [V^i(a,y;\omega) - H^i(a,y)] + H^i(a,y)\}, H^i(a,y)\Big\}$$

The inner max chooses over $\omega$ for the best contract. While the domain of choice is indicated as $\Omega$, we may assume without loss of generality that the choice is over only those contracts for which $f^i(a,y;\omega)$ is strictly positive. The outer max recognizes that the individual always has the option to not engage in search and get the value $H^i(a,y)$.

Denote the probability of a person separating from her card as $\zeta^i(a,y;\omega)$ and the default decision rule for a person as $D^i(a,y,m;\omega)$. Then,

$$\zeta^i\left(a,y;\omega\right) = \begin{cases} \xi + (1-\xi)\cdot \Pr[V^i\left(a,y;\omega\right)) < L^i\left(a,y\right) - k] & \text{if } a \geq 0 \\ 0 & \text{if } a < 0. \end{cases}$$

And,

$$D^i(a,y,m;\omega) = \begin{cases} \mathbb{1}\{v^i\left(a,y,m;\omega\right) < v^i_{\text{DEF}}\left(y,m\right)\} & \text{if } a < 0 \\ 0 & \text{if } a \geq 0, \end{cases}$$

where $\mathbb{1}\{\cdot\}$ is an indicator function that is 1 if the expression in $\{\cdot\}$ is true (and 0 otherwise).

### 4.3 Credit Card Companies Decision Problem

All card companies have access to funds at the risk-free interest rate $R_f$. Denote the value to a company of a credit card contract $\omega$ held by a person of type $i$ in state $(a,y,m)$ as $\pi^i(a,y,m;\omega)$. Then,

$$\begin{aligned}\pi^i(a,y,m;\omega) = \big[1 - D^i(a,y,m;\omega)\big] \times \Big(&-\min\{0,a\}\cdot R(\omega) + \min\{0, A^i(a,y,m;\omega)\} \\ &+ \frac{\nu}{R_f}\mathbb{E}_{(y',m'|y)}\left[1 - \zeta^i(A^i(a,y,m;\omega),y')\right]\Pi^i(A^i(a,y,m;\omega),y')\Big) \\ &+ D^i(a,y,m;\omega) \times [1-\phi^i](m+\lambda y),\end{aligned}$$

where $\Pi^i(a, y; \omega) = \mathbb{E}_m \pi^i(a, y, m; \omega)$. If the individual is carrying a balance on the card and does not default the card company receives the (gross) interest payment $a \cdot R(\omega)$ minus the individual's new borrowing $A^i(a, y, m; \omega)$. If the individual is not carrying any balances, the company does not receive any funds but might potentially transfer funds to the individual if she chooses to borrow. Regardless, the company also gets the expected continuation value of the contract discounted by the risk-free rate. The expected continuation value takes into account that the card holder survives with probability $\nu$ and that the company might lose the contract with probability $\zeta^i(a', y')$. If the individual defaults on the credit card, the credit line is closed and the credit card company gets $[1 - \phi^i](m + \lambda y)$.

A card company chooses the profit-maximizing contract $\omega$ for people of type $i$ in state $(a, y)$ and, given profit associated with the best contract, chooses the measure of contracts to post at the cost of $\tau > 0$ per post. In making these choices, it takes the contact probability function $q^i(a, y; \omega)$ as given. The profit-maximizing contract solves:

$$\max_{\omega \in \Omega} q^i(a, y; \omega) \cdot \Pi^i(a, y; \omega)$$

Let $\omega^{i\,*}(a, y; q^i)$ be a contract that attains the maximum, let $\Pi^{i\,*}(a, y; q^i)$ denote $\Pi^i(a, y; \omega^{i\,*}(a, y; q^i))$, and let the net expected profit from posting a single $\omega^{i\,*}(a, y; q^i)$ contract be

$$\eta^{i\,*}(a, y; q^i) = q^i(a, y; \omega^{i\,*}(a, y; q^i)) \cdot \Pi^i(a, y; \omega^{i\,*}(a, y; q^i)) - \tau.$$

Let $e^i(a, y; q^i)$ be the measure of contracts $\omega^{i\,*}(a, y; q^i)$ posted to match with a type $i$ person in state $(a, y)$. Then,

$$e^i(a, y; q^i) = \begin{cases} 0 & \text{if } \eta^{i\,*}(a, y; q^i) < 0 \\ \text{indeterminate} & \text{if } \eta^{i\,*}(a, y; q^i) = 0 \end{cases}$$

### 4.4 Contact Probabilities and Market Tightness

We follow den Haan, Ramey, and Watson (2000) and assume that the matching function is

$$M\left(B^i(y, a; \omega), E^i(y, a; \omega)\right) = \frac{B^i(y, a; \omega) \cdot E^i(y, a; \omega)}{\left[B^i(y, a; \omega)^\alpha + E^i(y, a; \omega)^\alpha\right]^{1/\alpha}}, \quad \alpha \geq 0,$$

where $B^i(a, y; \omega)$ denotes the mass of individuals of type $i$ in state $(a, y)$ who are searching in the submarket $\omega$ and $E^i(a, y; \omega)$ denotes the mass of contact attempts made by the totality of card companies in the same submarket.

With this matching function, the probability that a credit card company will successfully contact a type $i$ customer in state $(a, y)$ is:

$$q^i(a, y; \omega) = \frac{M\left(B^i(\cdot), E^i(\cdot)\right)}{E^i(\cdot)} = \frac{1}{\left(1 + \theta^i(a, y; \omega)^{\alpha}\right)^{1/\alpha}} = q(\theta^i(a, y; \omega)),$$

where

$$\theta^i(a, y; \omega) = \frac{E^i(a, y; \omega)}{B^i(a, y; \omega)}.$$

The ratio $\theta^i(a, y; \omega)$ can be interpreted as the "tightness" — from the perspective of card companies — of the submarket. A high value means stiff competition for customers and a low probability of a successful contact. On the other side, the probability that an individual of type $i$ in state $(a, y)$ in the submarket $\omega$ will successfully contact a card company is

$$f^i(a, y; \omega) = \frac{M\left(B^i(\cdot), E^i(\cdot)\right)}{B^i(\cdot)} = \frac{\theta^i(a, y; \omega)}{\left(1 + \theta^i(a, y; \omega)^{\alpha}\right)^{1/\alpha}} = f(\theta^i(a, y; \omega)).$$

This probability is increasing in market tightness: In a tight market one can get a credit card quickly.

## 5 Equilibrium

Since there is no interaction between types, equilibrium can be described in terms of equilibrium for each type $i \in \mathbb{I}$. An equilibrium for type $i$ is a pair of functions $S^{i\,*}(a, y)$ and $\theta^{i\,*}(a, y; \omega) \geq 0$ such that

$$S^{i\,*}(a, y) = \tag{1}$$
$$\max\left\{\max_{\omega \in \Omega}\left\{f(\theta^{i\,*}(a, y; \omega)) \cdot [V^{i*}(a, y; \omega) - H^{i*}(a, y)] + H^{i*}(a, y)\right\}, H^{i*}(a, y)\right\}$$

$$q^{i\,*}(a, y; \omega) = \begin{cases} 1/\left[1 + f^{-1}\left(\frac{S^{i\,*}(a,y) - H^{i\,*}(a,y)}{V^{i*}(a,y;\omega) - H^{i*}(a,y;)}\right)\right] & \text{if } V^{i*}(a, y; \omega) > H^{i*}(a, y) \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

$$q^{i\,*}(a,y;\omega)\Pi^{i*}(a,y;\omega) \leq \tau \text{ for all } \omega \in \Omega \tag{3}$$

$$\big[q^{i\,*}(a,y;\omega)\Pi^{i*}(a,y;\omega) - \tau\big]\theta^{i\,*}(a,y;\omega) = 0 \text{ for all } \omega \in \Omega \tag{4}$$

Equation (1) asserts that equilibrium values of $S^i$ and $\theta^i$ must be consistent with individual optimization. The asterisks on $V^i$, $H^i$ and $\Pi^i$ indicate that these functions implicitly depend on $S^{i*}(a,y)$.

Equation (2) is a key equilibrium condition in our directed search environment. The top branch asserts that for any contract $\omega$ offered to people of type $i$ in state $(a,y)$ that delivers more utility than $H^{i\,*}(a,y;S^{i\,*})$, the market tightness must be such that the ex-ante value of searching for that contract is the same as the equilibrium ex-ante value of active search, $S^{i\,*}(a,y)$. The bottom branch asserts that any contract that delivers less utility than $H^{i\,*}(a,y)$ don't attract any customers and, so, $q^{i\,*}(a,y;\omega)$ is zero (in effect, market tightness for such a contract is infinite).

Equation (3) asserts that all feasible contracts earn nonpositive expected net profits.

Equation (4) is the complementary slackness condition that ensures consistency with free entry of lenders in any submarket: if equilibrium market tightness for a contract is strictly positive, the contract must earn zero expected net profits and if a contract generates negative expected net profits, its equilibrium market tightness must be 0. The conditions (3) and (4) together assert that contracts actually offered in equilibrium maximize expected net profits.

The equilibrium measure of contactable people, $B^{i\,*}(a,y;\omega)$, and the aggregate measure of posted contracts, $E^{i\,*}(a,y;\omega)$, do not appear directly in these equilibrium equations because these quantities effect equilibrium outcomes only through $\theta^{i\,*}(a,y;\omega)$. They can be backed out once equilibrium market tightness is determined. By assumption, the only people who are in a position to be contacted by card companies are people with nonnegative assets. Furthermore, the fact that anyone with assets $a \geq 0$ can get separated from their cards with positive probability ensures that the steady-state measure of $B^{i\,*}(a,y;\omega)$ is strictly positive for all $y$ and all $a \geq 0$ and 0 otherwise. The aggregate measure of posts is then given by

$$E^{i\,*}(a,y;\omega) = B^{i\,*}(a,y;\omega) \cdot \theta^{i\,*}(a,y;\omega). \tag{5}$$

Table 4:
Income Process Parameters

| Parameter | Description | Value |
|---|---|---|
| $\rho$ | Autocorr. of log of persistent component | 0.91 |
| $\sigma_\varepsilon$ | Var. of innovation to log of persistent component | 0.043 |
| $\sigma_z$ | Var. of log transitory shock | 0.042 |

**Notes**: Source: Floden and Lindé (2001), Table ? p. All parameters are at annual frequency.

At the company level, the equilibrium measure of contact attempts is either 0 (if expected net profitability is negative or it is indeterminate.[3]

# 6 Quantitative Analysis

At a broad level, card contracts offered must reflect the card companies' assessment of the expected behavior of their intended customers. The goal of the quantitative analysis is to learn the ways in which customers must differ in order for the equilibrium of the model to match what we see the card companies doing. Given this objective, it is natural to focus on new card originations as we expect contract terms to be tied to the perceived characteristics of customers most closely when a card is first issued.[4]

## 6.1 Preliminaries

We calibrate the earnings process, we use the estimates provided in Floden and Lindé (2001). These authors estimate an earnings process for annual earnings given by $y(t)z(t)$, where $y$, the persistent component of earnings, is an AR1 process in logs and $z$, the transitory component, is i.i.d. with $\ln(z)$ having standard deviation $\sigma_z$. Their estimates are reported in Table 4.

Our model period is a quarter and we assume that income is the sum of the persistent and transitory components: $y + m$. We assume that $\ln(y_t) = \rho \ln(y_{t-1}) + \varepsilon_t$ and that $m$ is uniformly distributed with support $[-\lambda y, \ \lambda y]$, given $y$. Note that this latter assumption is equivalent to

[3] As noted earlier, only a fraction $\mu$ of people who are without a card and not shut-out actively search for a card. We assume that credit card companies can direct their contact attempts toward this group and set $B^{i*}$ to be the measure of this group. If we assumed instead that card companies can only distinguish people by their type and state and cannot tell if they are actively searching, $B^{i*}$ would be set the measure of people who are without a card and not shut out of the market. These alternatives differ with respect to the implied equilibrium measure of contact attempts. Since an individual card company does not care how many offers it sends out (each attempt earns zero net profits in expectation), either interpretation of $B^{i*}$ is valid.

[4] As time passes, the circumstances of the account holders might stray far from what was expected at origination.

assuming that $z$ is uniformly distributed with support $[(1-\lambda),\ (1+\lambda)]$. We discretize $y$ into 3 levels and assume that it follows a first-order Markov process. We pick $\lambda$ and the values of the Markov transition matrix such that the model earnings generate a $\rho$, $\sigma_\varepsilon$ and $\sigma_z$ at annual frequencies that matches what is reported in Table 4.

Table 5 displays the other parameters whose values are also set independently. The (real) risk-free (gross) rate $R_f$ was set at 1.01 percent annualized and the probability of survival, $\nu$, at 0.82 percent annualized. The CRRA parameter $\gamma$ was set at 2, which is conventional in macroeconomics.

The cost of posting a contract, $\tau$, is set to 0.001, a value low enough to ensure that a profitable credit card exists for all $\{i, a, y\}$ triples. For similar reasons, the probability of search following a separation, $\mu$, is set at 0.25: Delays in getting a new card creates a lock-in effect for existing cards which increases profits.

The entry probability following default, $\delta$, is set so that the average period of exclusion from the credit card market following default is 6 years. Since we assume that a person without a card does not look for a card for about a year on average, the effective exclusion from the credit card market following a default is about 7 years.

The separation parameters, $\xi$ and $\kappa$, are set at low values. The purpose of this feature of the model is to make the decision to look for a better card a probabilistic function of $a$, $y$, and $\omega$ as opposed to a deterministic function. These small values are sufficient for this purpose.

Finally, the elasticity of the matching function $\alpha$ is set to 1.

These parameter choices will be held fixed in the subsequent analyses. The parameters that will vary across the different economies studied below are the number of types and $\beta^i$ and $\phi^i$ for each type.

In order to address the facts discussed earlier, we need a model analog of a credit score. Consistent with real-world credit scores, the model credit score is defined to be the probability of a default within the next two years (i.e., 8 model periods). This probability can be computed recursively as described in Appendix A.

The model we build is of an individual. However, the Y-14M collects account level data and it is not possible to merge accounts belong to the same individual to get a complete picture of his or her credit card indebtedness. To take account of this, we scale up the credit-limit-to-income

Table 5:
Parameters Set Independently

| Parameter | Description | Value |
|---|---|---|
| $\lambda$ | Bounds for transitory shock | 0.67 |
| $R_f$ | Gross risk-free rate | 1.0025 |
| $\nu$ | Probability of survival | 0.998 |
| $\gamma$ | Curvature of CRRA utility function | 2.0 |
| $\delta$ | Entry rate after default | 1/28 |
| $\tau$ | Cost of posting a contract | 0.001 |
| $\mu$ | Probability of searching for a card | 0.50 |
| $\xi$ | Probability of exogenously separating from card | 0.005 |
| $\kappa$ | Upper bound on cost of separation | 0.005 |
| $\alpha$ | Elasticity of the matching function | 1.00 |

**Notes**: All rates are at quarterly frequency.

ratios in Table 1 by a factor of 2.2 which is the average number of cards that individuals in the lowest quintile of credit scores possess, if they possess any card at all.[5] We use 2.2 as the scaling factor rather than the average (which is 2.7) because we wanted to avoid overestimating the debt capacity of lowest credit score people.[6] With regard to the interest rate and utilization rates of card accounts, we proceed on the assumption that the average interest rate and utilization rates for the score and income bins are a good proxy for the individual-level averages for people in the corresponding score and income bins.

All estimation results presented in subsequent (sub) sections stem from minimizing the following objective function:

$$10 \times \sum_{j=1}^{5} \Big(\text{default probability}_{\text{ data, score}_j} - \text{default probability}_{\text{ model, score}_j}\Big)^2 \; +$$
$$0.1 \times \sum_{j=1}^{5} \Big(\text{credit-limit-to-income ratio}_{\text{ data, score}_j} - \text{credit-limit-to-income ratio}_{\text{ model, score}_j}\Big)^2$$

Thus, the estimation locates the $\{\beta^i, \phi^i\}_{i\in\mathbb{I}}$ pairs that minimizes the weighted sum of squared deviations between the data and model default probabilities and credit-limit-to-income ratio across credit score quintiles.[7] The reason for choosing these moments over others is two-fold. First, these

[5]This information comes from the FRBNY/Equifax CCP dataset

[6]Another option would have been to adjust target credit-limit-to-income ratios differently for different credit score bins. But CCP has no data on income of the individuals and this approach would have made it difficult to decide on the credit limit targets when bins are sorted with respect to income.

[7]The weights are chosen to equalize the the order of magnitude of the two sums.

moments are informative about a person's type: Holding fixed the income process, the default cost is a key determinant of an individual's debt capacity and hence of her credit limit, and the discount factor is a key determinant of her propensity to borrow and default. Second, we use the variation across score quintiles rather than income quintiles because in the data default frequencies and credit limits are related more tightly to credit scores than to income (at origination). Details of the computation and estimation procedures are given in Appendix (to be added)

### 6.2 One Type

We start off our quantitative analysis with one type of individual only, i.e., all individuals have common $\beta$ and $\phi$. For this investigation, we discretize the income process (in Table 4) with a Markov process with 5 states. Table 6 displays the performance of the model with respect to (two-year) default frequencies sorted by income. In the data, income is income at origination and in the model it is income of those individuals who successfully acquire a credit card in a period. For this comparison, average income in each quintile is normalized by the average income in the middle (third) quintile for both the data and the model.

First note that the relative incomes in the model bins is quite close to that in the data. This is not a given since incomes in the model is determined by the income process in Table 4, estimated on data from a completely different source. Furthermore, the set of people looking for a new card has endogenous components: Some people choose to look for a better card and others look for a new card after being shut out of the credit card market following a default.[8]

[8] The concordance is helped by the fact that our data sample considers only individuals in 7th to 93rd percentiles of incomes-at-origination; had we not "winsorized" the sample with respect to income, the model distribution would be less dispersed than the data.

Table 6:
Income and Default Frequency by Income Quintiles: Data and Model

| Description | I | II | III | IV | V |
|---|---|---|---|---|---|
| Income, Data | 0.48 | 0.74 | 1.00 | 1.36 | 2.05 |
| Income, Model | 0.47 | 0.72 | 1.00 | 1.38 | 1.94 |
| Default Frequency in %, Data | 5.40 | 4.88 | 3.29 | 2.88 | 2.06 |
| Default Frequency in %, Model | 6.72 | 4.83 | 4.07 | 3.71 | 2.95 |

**Notes**: All frequencies refer to a 2-year horizon

Turning to the default frequencies, the model outcome is quite close to the data. Our estimation strategy does not target these moments, so the fit is noteworthy. The pattern of declining default frequencies with respect to income has a simple explanation in the model, namely, mean reversion in incomes. A high-income person expects her income to decline in the future which induces her to borrow less or save. In effect, temporarily high income makes an individual patient and, therefore, she is less likely to default over a two year horizon. The opposite is true for a low-income individual who expects her future income to rise.

The relationship between income and credit-limit-to-income ratio is qualitatively similar to the data in that the ratio declines with income, but the decline is steeper in the model. Why does the credit limit ratio decline with income in the model? One reason is that the credit card contract commits to a credit limit and an interest rate, regardless of future income states. To see why this matters, consider the case where there is no transition between income states, i.e., a person's income is permanent. In this case, as shown in Table 7, the credit-limit-to-income ratio is roughly constant across income quintiles. Since support of the (uniform) distribution of the $m$-shock is proportional to $y$ and the (fixed) cost of sending a credit offer is small, the homotheticity of preferences implies that household's decision rule is almost scale-independent. Consequently, the credit limit ratios become essentially constant with respect to income. But with positive probability of transitions, a uniform credit limit *ratio* is likely to be very sub-optimal for card companies. If the credit limit-to-income ratio was identical across persistent income levels, an individual holding a credit card designed for a person with, say, double her income would have a strong incentive to default: Her expected default cost would be half as large and she might be tempted to borrow all the way to her large limit and default. In other words, the contract needs to ensure that the temptation to default does not increase too much when there is a drop in the persistent component of income. This constrains the credit limit of high-income individuals.

A second force that works to constrain the credit limit of high-income individuals is that they are likely to borrow less. Consequently, a credit card with a high limit and a high interest rates is not attractive to them; they would prefer a card with a smaller limit and lower interest rate.[9] To verify that greater patience does indeed lower credit limits, Table 7 also reports credit limit ratios if the discount factor is raised in a model where income levels are permanent. Observe that ratios are systematically lower in comparison to column 3. In sum, these two reasons work to make credit

[9]The situation here is similar to that in insurance: the good risks prefer contracts that have high deductibles and low premium.

Table 7:
Ratio of Credit Limit to Income by Income Bins
Data and Model

| Description | I | II | III | IV | V |
|---|---|---|---|---|---|
| Data | 1.13 | 0.94 | 0.90 | 0.80 | 0.73 |
| Model | 1.19 | 0.91 | 0.74 | 0.64 | 0.49 |
| Model, No transition | 1.02 | 1.00 | 1.03 | 1.04 | 1.03 |
| Model, No transition and high patience | 0.93 | 0.93 | 0.93 | 0.93 | 0.94 |

**Notes**: Income refers to income at origination both in the data and in the model. The data row is the same (ignoring rounding) as the corresponding row in Table 1 multiplied by 2.2 and expressed as ratio to quarterly (as opposed to annual) income.

limits rise less than proportionately with income and, hence, for credit limit-to-income ratio to fall with income.

We now turn to the patterns with respect to quintiles of credit scores. Table 8 reports the (two year) default frequency across score quintiles in the data and the model. Even though these are moments that our estimation is designed to target, the match between model and data is quite imperfect. While the default frequency is declining across score quintiles (as it must), the model underpredicts default frequencies for the bottom two quintiles and overpredicts for the top three quintiles. The overprediction for the top quintile is particularly severe. To understand why the model struggles to get these moments, it is helpful to compute the average income in each of the score quintiles. In the model, there is a very sharp rise in average incomes across the score quintiles. In light of our discussion of how the default frequency is affected by a person's permanent component of income, this is to be expected: people with low scores and high default probability must be people with low $y$ and those with high scores and low default probability must be people with high $y$. In contrast, the link between credit scores and income at origination is much weaker in the data — income does rise with score quintiles but the rise is not as steep: the average income of individuals in the highest score quintile is only 1.4 times the average income of people in the bottom score quintile, whereas in the model this factor is 3.4. One plausible explanation for this mismatch is that there are factors other than income that also affects credit scores and these factors are not strongly correlated with income.

Table 8:
Income and Default Frequencies by Score Quintiles: Data and Model

| Description | I | II | III | IV | V |
|---|---|---|---|---|---|
| Income, Data | 0.83 | 0.94 | 1.00 | 1.06 | 1.15 |
| Income, Model | 0.44 | 0.85 | 1.00 | 1.26 | 1.49 |
| Default Freq in %, Data | 7.85 | 5.72 | 3.00 | 1.31 | 0.65 |
| Default Freq in %, Model | 7.05 | 5.01 | 4.25 | 3.51 | 2.41 |

**Notes**: All default frequency are for 2-year horizon

Table 9:
Credit Limit to Income Ratio by Score Quintiles: Data and Model

| Description | I | II | III | IV | V |
|---|---|---|---|---|---|
| Data | 0.37 | 0.72 | 0.98 | 1.23 | 1.21 |
| Model | 1.17 | 0.82 | 0.75 | 0.65 | 0.58 |

**Notes**: Income refers to income at origination both in the data and in the model. The data row is the same (ignoring rounding) as the corresponding row in Table 1 multiplied by 2.2 and expressed as ratio to quarterly (as opposed to annual) income.

This conclusion is reinforced when we examine model predictions regarding credit scores and the credit-limit-to-income ratio. As shown in Table 9, there is now a qualitative mismatch, not just a quantitative one. In the data, the credit limit ratio is increasing with credit scores while in the model it is *decreasing.* As explained earlier, the credit limit ratio is a decreasing function of $y$ since card companies are constrained on how generous a credit limit they can offer to high-income individuals when there is a chance that an individual's income might fall in the future. And, if individuals can differ only in their incomes, high-income individuals will have low probability of default and high credit scores. The combination of these two features implies that in the model the credit limit-to-income ratio *must* decline with credit scores.

In summary, the model with only one type of person in the economy can match the relationship of income at origination to default and credit-limit-to-income ratios reasonably well, but fails to match the relationship of these variables to credit scores. The lesson is that the model is missing some dimension of heterogeneity besides income. In the context of our model, the dimensions of heterogeneity that are most natural are discount factors and default costs. The former is a key determinant of default frequency (impatience leads to debt and the possibility of default) and the latter is a key determinant of debt capacity and, hence credit limits. Our framework incorporates heterogeneity in in $\beta$ and $\phi$ and we assume that this variation is independent of $y$, i.e., the earnings process does not depend on type.

### 6.3 The Baseline Model

#### 6.3.1 Parameter Estimates

In the baseline model we postulate 3 types of equal measure. Since any joint distribution of discount factors and default costs can be approximated by appropriate choices of($\beta^i$, $\phi^i$) even if each type is of equal measure, the latter assumption is not restrictive. Of course, the fit of the model will improve with the number of types but having more than 3 types makes the estimation of the model quite time consuming.

The estimated parameters are shown in Table 10. The estimation implies that people are quite impatient, with the lowest type discounting the future at a 20 percent annual rate and the highest type at a 6 percent annual rate. And the costs of default are estimated to be positively related to

Table 10:
Estimate of Type Parameters

| Type | Patience, $\beta$ | Default Cost, $(1-\phi)$ |
|---|---|---|
| Type 1 | 0.80 | 0.29 |
| Type 2 | 0.87 | 0.75 |
| Type 3 | 0.94 | 0.98 |

**Notes**:

Table 11:
Income, Default Frequency and Credit Limit Ratios by Score Quintiles
Data and Model

| Description | I | II | III | IV | V |
|---|---|---|---|---|---|
| Income, Data | 0.83 | 0.94 | 1.00 | 1.06 | 1.15 |
| Income, Model | 0.60 | 0.99 | 1.00 | 1.03 | 1.08 |
| Default Freq, % Data | 7.85 | 5.72 | 3.00 | 1.31 | 0.65 |
| Default Freq, % Model | 8.55 | 5.19 | 3.46 | 2.18 | 0.80 |
| Credit Limit Ratio, Data | 0.37 | 0.72 | 0.98 | 1.23 | 1.21 |
| Credit Limit Ratio, Model | 0.36 | 0.64 | 1.05 | 1.23 | 1.19 |

**Notes**: All frequencies refer to a 2-year horizon. The data row for credit limit ratio is the same as the corresponding row in 1 multiplied by 2.2 and expressed as ratio of quarterly (as opposed to annual) income.

patience: Highest types must pay 98 percent of their transitory income to creditors upon default, whereas the lowest type must pay only 29 percent.

### 6.3.2 Fit of the Model to Targeted Moments

Table 11 reports the two main targeted data moments and their model counterparts. Default frequencies fall sharply with score quintiles and credit-limit-to-income-ratios rise with score quintiles. Furthermore, the model moments are quite close to their data counterparts. The table also reports the average income in each of the score bins and correspondence between model and data is quite good. Thus, even with just three types the fit of the model improves greatly.

How does heterogeneity in discount factors and default costs help to fit the patterns? The heterogeneity in discount factor results in heterogeneity in default frequency. If some highest discount factor individuals are searching for and acquiring new cards, they will likely show up in the top quintiles and help reduce the default frequencies of those quintiles. Combined with the heterogeneity in default frequencies that come from differences in income (high-income individuals are less likely to default), discount factor heterogeneity helps produce the *sharply* declining pattern of default frequencies with score quintiles.

But when the highest credit score individuals are also the most patient, it becomes even harder to match the positive relationship between credit score quintiles and credit-limit-to-income ratios. Recall that this ratio declines with higher income and higher discount factors. The estimation overcomes this challenge by permitting heterogeneity in default costs and attributing the highest default cost to the most patient individuals: In equilibrium, with high default costs highly patient individuals are offered high credit limits.

To verify this explanation, Table 12 reports the distribution of the three types among the population of people who acquire new cards in any period. The least patient type (Type 1) is mostly concentrated in the bottom three quintiles and the most patient type (Type 3) is entirely concentrated in the top two quintiles. Those with the middle level of patience concentrate in the third and fourth quintiles and occasionally stray into the second and fifth quintiles.

The table also reports the prevalence of the three types among the people who are getting new cards. While there is an equal measure of the three types in the population, representation among those who get new cards is not uniform: The lowest type is the most prevalent and the highest type is the least prevalent. The reason is selection: The most impatient type are more likely to drop their existing card and search for a new one when their income improves since they are more inclined to borrow and the credit limit rises with income. To a lesser degree, the same is true of the middle patience type. Also, among the group that is searching and obtaining new cards are individuals who defaulted in the past and the most impatient type is over-represented in that set as well.

Table 12:
Model Distribution of Types Across Score Quintiles

| Description | I | II | III | IV | V | Fraction of New Holders |
|---|---|---|---|---|---|---|
| Type 1 | 0.48 | 0.36 | 0.12 | 0.03 | 0.00 | 0.42 |
| Type 2 | 0.00 | 0.14 | 0.42 | 0.35 | 0.09 | 0.35 |
| Type 3 | 0.00 | 0.00 | 0.00 | 0.26 | 0.74 | 0.23 |

**Notes**: Model outcomes.

#### 6.3.3 Equilibrium Contracts

In the baseline model, we allow card companies to distinguish between 6 different wealth levels, $\{0, x, 2x, 3x, 4x, 5x\}$ where $x = 0.52$, and we permit only 3 (persistent) income levels. Since there are 3 types, the total number of distinct contracts offered is 54. Tables 13 and 14 display the interest rates and credit-limits for each of these contracts.

Focusing on the contract offered to individuals with little or no assets, the top panel shows that holding fixed type, card interest rate is increasing in income and, holding fixed income, it is decreasing in type, meaning that more patient types are offered lower interest rates. The bottom panel shows that holding fixed type, the credit-limit-to-income-at-origination ratio is decreasing in income, and holding fixed income, it is increasing in type. Both panels show that contract terms are quite insensitive to financial wealth, holding fixed type and income; if terms change, higher wealth is generally associated with lower interest rates and higher credit limit ratios.

Holding fixed income, the pattern across types is intuitive: Since default costs and patience are both increasing in type, the former pushes credit limits to increase with types and the latter pushes

Table 13:
Model Equilibrium Contract Gross Interest Rates

| | Type 1 | | | Type 2 | | | Type 3 | | |
|---|---|---|---|---|---|---|---|---|---|
| Assets/Income | 0.50 | 1.00 | 1.98 | 0.50 | 1.00 | 1.98 | 0.50 | 1.00 | 1.98 |
| 0.00 <= 0.52 | 1.21 | 1.21 | 1.22 | 1.10 | 1.11 | 1.13 | 1.09 | 1.09 | 1.09 |
| 0.52 <= 1.03 | 1.19 | 1.20 | 1.21 | 1.10 | 1.11 | 1.13 | 1.09 | 1.09 | 1.09 |
| 1.03 <= 1.57 | 1.18 | 1.19 | 1.20 | 1.10 | 1.11 | 1.13 | 1.09 | 1.09 | 1.09 |
| 1.57 <= 2.07 | 1.17 | 1.19 | 1.20 | 1.10 | 1.11 | 1.13 | 1.09 | 1.09 | 1.10 |
| 2.07 <= 2.60 | 1.17 | 1.19 | 1.20 | 1.10 | 1.11 | 1.13 | 1.09 | 1.09 | 1.10 |
| 2.60 <= − | 1.17 | 1.19 | 1.19 | 1.10 | 1.11 | 1.13 | 1.09 | 1.09 | 1.09 |

**Notes**: Model outcomes. Asset and Income refer to levels at origination

Table 14:
Model Equilibrium Contract Credit Limit to Income Ratio

| | Type 1 | | | Type 2 | | | Type 3 | | |
|---|---|---|---|---|---|---|---|---|---|
| Assets/Income | 0.50 | 1.00 | 1.98 | 0.50 | 1.00 | 1.98 | 0.50 | 1.00 | 1.98 |
| 0.00 <= 0.52 | −0.41 | −0.31 | −0.20 | −1.83 | −1.28 | −0.80 | −2.20 | −1.36 | −0.70 |
| 0.52 <= 1.03 | −0.46 | −0.32 | −0.21 | −1.87 | −1.28 | −0.80 | −2.26 | −1.36 | −0.70 |
| 1.03 <= 1.57 | −0.49 | −0.32 | −0.20 | −1.89 | −1.27 | −0.80 | −2.28 | −1.34 | −0.70 |
| 1.57 <= 2.07 | −0.50 | −0.32 | −0.20 | −1.89 | −1.26 | −0.79 | −2.28 | −1.30 | −0.69 |
| 2.07 <= 2.60 | −0.50 | −0.32 | −0.20 | −1.90 | −1.25 | −0.79 | −2.28 | −1.30 | −0.69 |
| 2.60 <= − | −0.50 | −0.32 | −0.20 | −1.90 | −1.25 | −0.79 | −2.28 | −1.30 | −0.68 |

**Notes**: Model outcomes. Asset and Income refer to levels at origination

interest rates to decrease with types. Holding fixed type, the pattern with respect to income results from the same forces that were discussed for the model with a single type: Higher income increases credit limits but less than in proportion to income and so the credit limit ratio declines with income. It is more surprising that interest rates rise with income, given type. This is because higher income individuals have higher limits and, so, when they default they do so on larger debts. The interest rate on the card reflects this (more on this below). The lack of sensitivity of contract terms to wealth is a consequence of the long-run behavior of consumers, given income and type, being independent of initial wealth levels (due to impatience). Since a card is long-duration contract, its performance is heavily influenced by long-run behavior of cardholders and, consequently, its terms are not very sensitive to initial wealth levels.

### 6.4 Fit with Income-Based Patterns

Table 15 reports average income, default probabilities, and credit-limit-to-income ratios across income quintiles for the the data and the model. There is a perfect qualitative match and a reasonable quantitative match between the data and the model: The default probabilities and the credit-limit-to-income ratios decline with income both in the data and in the model. Quantitatively, the model default frequencies tend to exceed data frequencies and the decline in the credit-limit-ratio is steeper in the model than in the data.

Table 15:
Income, Default Frequency and Credit Limit Ratios by Income Quintiles
Data and Model

| Description | I | II | III | IV | V |
|---|---|---|---|---|---|
| Income, Data | 0.48 | 0.74 | 1.00 | 1.36 | 2.05 |
| Income, Model | 0.50 | 0.87 | 1.00 | 1.59 | 1.98 |
| Default Freq, % Data | 5.40 | 4.88 | 3.29 | 2.88 | 2.06 |
| Default Freq, % Model | 5.60 | 4.45 | 4.06 | 3.29 | 2.78 |
| Credit Limit Ratio, Data | 1.13 | 0.94 | 0.90 | 0.80 | 0.73 |
| Credit Limit Ratio, Model | 1.36 | 1.01 | 0.90 | 0.67 | 0.52 |

**Notes**: All frequencies refer to a 2-year horizon. The data row for credit limit ratio is the same as the corresponding row in 1 multiplied by 2.2 and expressed as ratio of quarterly (as opposed to annual) income.

# 7 Behavior of Card Holders

The market arrangement in our model resembles an Aiyagari-style model with a borrowing constraint and idiosyncratic income shocks. However, there are also important differences. Since people can default on their debts, the borrowing rate exceeds the risk-free saving rate. And, the $(\underline{a}, R)$ pair that a type $i$ has access to at the current time is a function of her $(y, a)$ at the time she accepted the contract. Following acceptance, an individual can search for a card with better terms if her circumstances improve, but she is not obliged to give up her current card if her circumstances deteriorate.

## 7.1 Utilization

Given the discount factors of each type and the borrowing interest rate they face, every type would want to borrow if there was no income uncertainty.[10] But uncertainty in income provides a reason to accumulate precautionary balances.

Table 16 reports the steady-state average assets of different types with different persistent income levels. When persistent income level is at the lowest level, all types are, on average, are

[10]By this we mean that the product of the highest gross borrowing interest a type can face and the type's discount factor is always strictly less than 1. Thus, if income were constant, every type would have an incentive to accumulate debt.

Table 16:
Average Assets By Income and Type

| Income, $y$ | Types | | |
|---|---|---|---|
| | 1 | 2 | 3 |
| Low | $-0.03$ | $-0.50$ | $-0.50$ |
| Middle | 0.08 | $-0.44$ | 0.12 |
| High | 0.37 | 0.02 | 1.74 |

**Notes**: Model Steady State

Table 17:
Utilization Rate By Income and Type

| Income, $y$ | Types | | |
|---|---|---|---|
| | 1 | 2 | 3 |
| Low | 0.46 | 0.73 | 0.58 |
| Middle | 0.32 | 0.54 | 0.25 |
| High | 0.21 | 0.31 | 0.06 |

**Notes**: Model Steady State

indebted. But for each type, the average asset position is increasing in $y$, as the utility cost of accumulating precautionary balances declines.

It is striking that for a given income level, the variation with respect to type is not generally monotone. For the middle income individuals, the middle type is, on average, indebted whereas the low and high types are not. The positive asset position of low types is a consequence of their tight credit limit; these individuals save to self-insure even though they are very impatient. Although the middle type is more patient than the low type, they face a much more relaxed credit limit and a lower interest rate — both factors attenuate the precauationary savings motive and they end up indebted, on average.

Table 17 reports the average utililization rate on cards across (persistent) incomes and types in steady state. The utilization rate is zero for people who are holding positive assets. For a given type, the utilization rate is decreasing in income. For a given income, the utilization rate peaks for the middle type. The utilization rate of the lowest type is generally higher than the utilization rate of the highest type, except when income is low.

### 7.2 Default

In the event of a default, the cardholder's debt is erased, she pays $(1-\phi^i)$ of the excess of $m$ over $-\lambda y$ to the card company, and she is excluded for some random length of time from the credit card market. Hence her consumption in the period of default is $y - \lambda y + \phi^i \cdot [m + \lambda y]$. If $m = -\lambda y$, the term in square brackets is zero and she does not pay anything to the card company; if $m = \lambda y$, she pays the maximum possible (given $y$) which is $(1-\phi^i)2\lambda y$; for intermediate values of $m$, her cost is somewhere in between.[11]

Consider a type 2 individual with middle $y$ who holds a card. If this individual has debt and draws a low $m$, she has can buffer her consumption against the low $m$ by shedding her debt at a low cost, i.e., defaulting. On the other hand, if her debt is well below her card limit, she can also buffer her consumption against the low $m$ by borrowing more. We would expect default to be more likely in situations where the second option is unavailable or its scope is limited because her utilization rate is close to 1 or at 1.

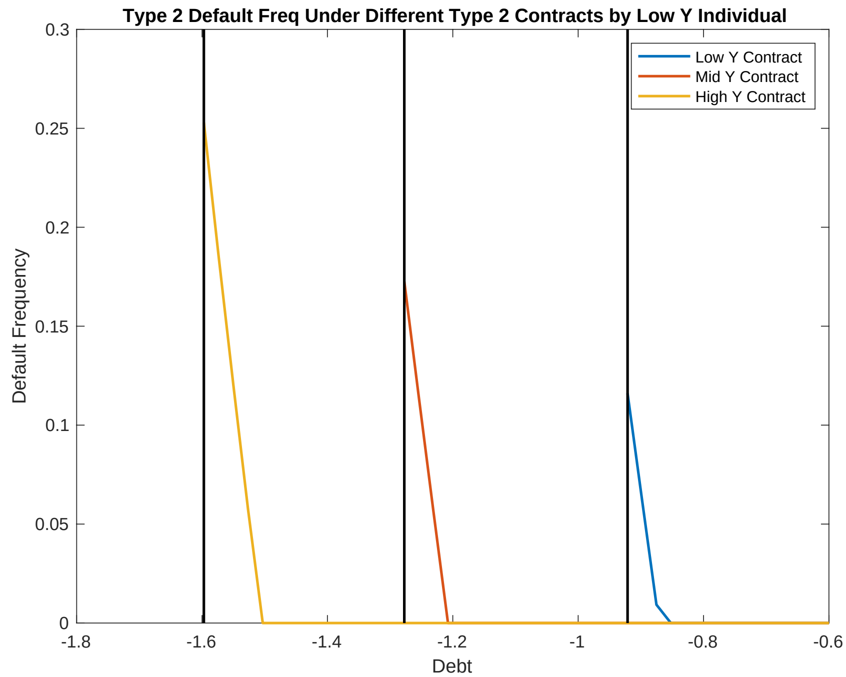
Figure 2 plots a low-$y$-type-2 individual's default frequency at different debt levels for three different contracts she might hold in equilibrium — contracts offered to type 2 individuals with low, medium or high income and little or no assets at origination.[12] The solid vertical lines are drawn at the credit limit of each of these three contracts. Observe that for each type of contract, default frequency is zero until the debt level gets close to the credit limit. Thus, the individual never defaults unless her utilization rate is close to 1. Evidently, if the individual can borrow on her card, she prefers that over defaulting. The plot would look similar if we focused on Type 1 or Type 3 individuals.

Looking across contracts, the default frequency is increasing in the credit limit but remains well below 1. That said, it is true that an individual's debt level *per se* is not informative about the likelihood of default: an individual with low y will never default on a debt level of 1 ( the point $-1$ on the $x$-axis) if she has the mid or high $y$ contract, but she may default on debt levels *lower*

[11] We could modeled a fixed cost of default that depends on $y$ but there are two drawbacks to this. The reason people default for low $m$ is that the default cost is close to zero for low $m$. Once a fixed cost is added, it will make it harder induce the high default rates without making individuals even more impatient. Another possibility is a utility cost of default that has an extreme value distribution for tractability. This comes at the cost of introducing a utility benefit or cost of borrowing that might distort behavior. And we will have to introduce some income cost of default anyway in order to account for recoveries (we are focusing on delinquencies rather than bankruptcies and recoveries are important for delinquencies).

[12] As noted earlier, the contracts do not depend much on the level of assets at origination.

Figure 1:
Model Default Probability



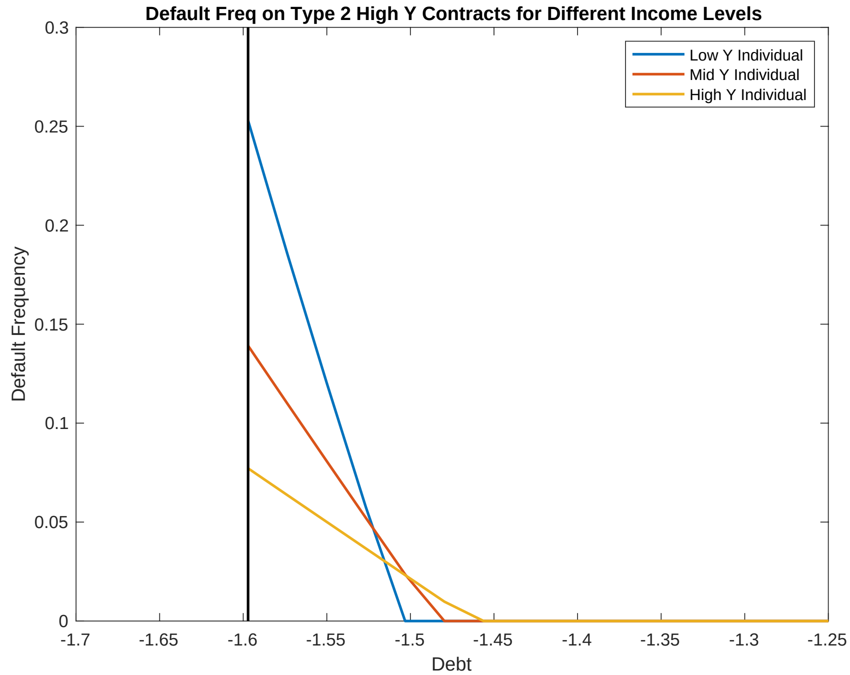

Source: Authors' calculations.

than 1 if she has the low $y$ contract. What matters for default is the cardholder's proximity to the credit limit, i.e., her utilization rate.

Figure 2 compares the default behavior a type 2 individual who holds a card offered to individuals with high income and zero assets when her income level is low, medium or high. Observe that the set of debt levels on which a low $y$ individual shows positive probability of default is a strict *subset* of the debt levels on which the mid $y$ individuals shows a positive probability of default and that set, in turn, is a strict subset of the set for which a high $y$ individual shows positive probability of default. In this sense, the low $y$ individual is the most reluctant to default. On the other hand, default probabilities rise fastest for the low $y$ individuals and the slowest for high $y$ individuals.

What explains these patterns? A card with a limit meant for a high $y$ individual is very valuable to the low $y$ individual, since she is unlikely to get such generous terms on her new card subsequent to a default. This makes her reluctant to default. On the other hand, a given debt level is more onerous to service if (mean) income is low versus when it is high. On this account, a low $y$ individual is more likely to default than a high $y$ individual. The pattern of default probabilities

Figure 2:
Model Default Probability Across Incomes




Source: Authors' calculations.

shown in Figure 2 mixes these two forces. Similar patterns emerge if the graphs for type 1 or type 3 individuals are plotted.

Table 18 displays the incidence of default across income and types in steady state. Default is most prevalent among people with low income (last column) and among low types (last row). That said, the difference between the low and middle type is not pronounced — for each level of income, the incidence of default among the middle types is only slightly less than the incidence among the low types. In contrast, for each income level, the incidence of default among the high types is substantially lower than for the other two types.

### 7.3 Separation

Who chooses to drop their card and look for a new one? Figure 3 shows the separation rates over time on cards offered to different (persistent) income groups in the lowest asset class level, i.e., to people with assets at origination between 0 to 0.52. At any horizon, the separation rate is decreasing in income. There are two reasons for this: First, for any type, the contract offered to a higher income type is more desirable because it has higher credit limits. Thus, when an individual transitions from a lower to a higher income, she gives up her current card and searches for a better

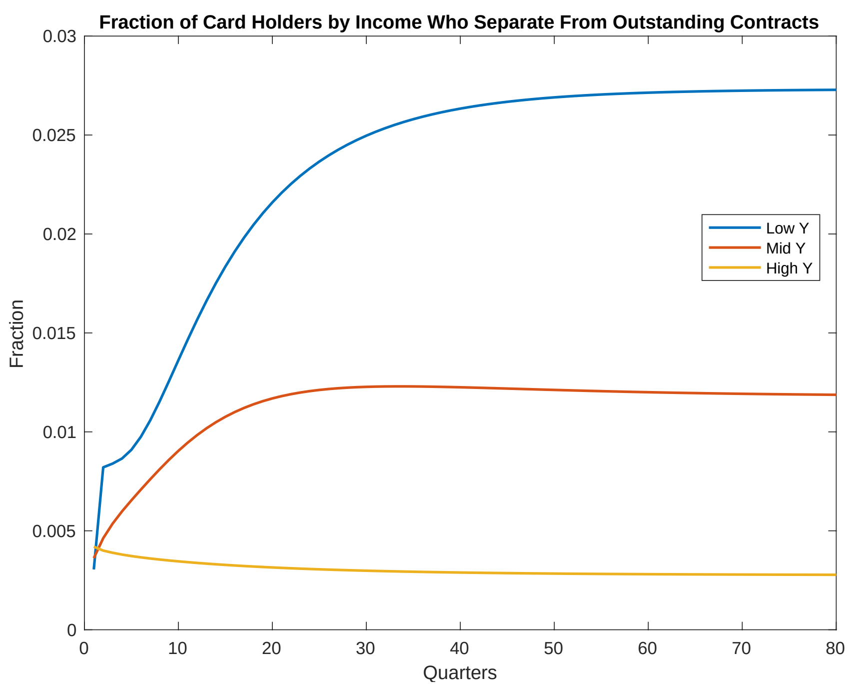
Table 18:
Share Defaulting By Income and Type

| Income, $y$ | Types | | | Incidence by Income |
|---|---|---|---|---|
| | 1 | 2 | 3 | |
| Low | 0.20 | 0.20 | 0.12 | 0.52 |
| Middle | 0.16 | 0.15 | 0.06 | 0.37 |
| High | 0.06 | 0.04 | 0.01 | 0.11 |
| Incidence by Type | 0.42 | 0.39 | 0.19 | 1.00 |

**Notes**: Model Steady State

card. In contrast, when an individual transitions from higher to lower income, she has no incentive to drop her card and look for a better one. This asymmetry is one reason for the ordering of the three lines. Another reason is that default rate is decreasing in income (see Table 18) which elevates the separation rate (which also includes separation as a result of default) of the lower income people.

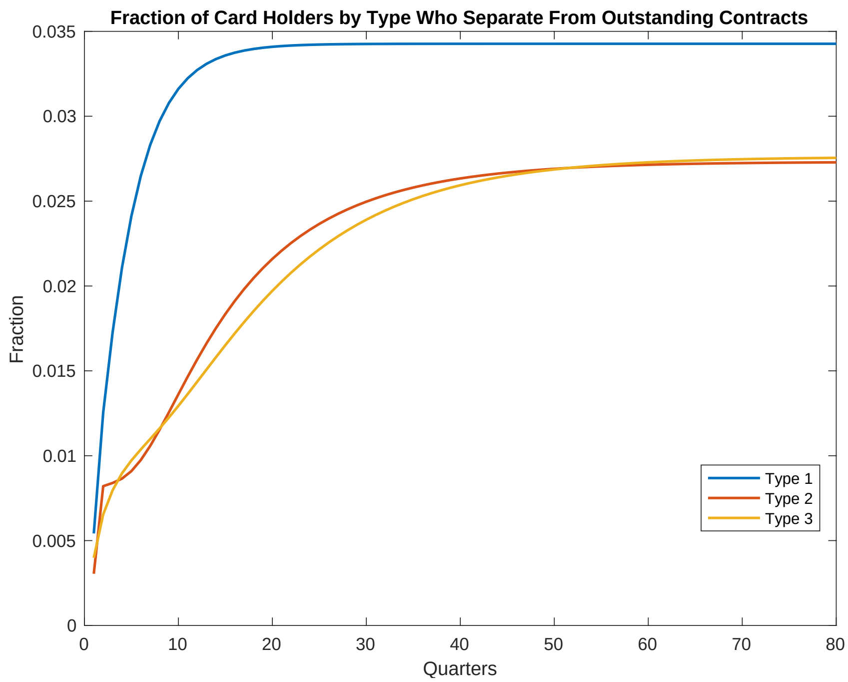
Figure 3:
Model Separation Rates by Incomes




Notes: All contracts are for the lowest asset level class.

The separation rates for low and middle income contracts is rising over time. This is due to the dependence of separation rate on the asset level. If the asset level is low, separation rate is low even if there is a transition to higher income because the insurance value of the current current exceeds the cost of being without a card for a few periods. Given that we are looking at the behavior of zero-assets-at-origination contracts, the separation rate rises over time as some of the people accumulate assets which encourages switching. For the highest income individuals the pattern is opposite: the separation rate goes down with time. For this group the endogenous separation is zero and exogenous separation can only happen with positive assets and as some of the individuals dis-save, the separation rate goes down.

Figure 4:
Model Separation Rates by Types




Notes: All contracts are for the lowest asset level class.

Figure 4 shows the separation rates over time for people of different types on cards in the lowest asset-at-origination class. The lowest type is most prone to separation at all horizons, with the the other two types showing lower but similar separation rates. The main reason for this is that lowest type is also most prone to default. The fact that separation rates rise over time is simply a consequence of debt accumulation: As time passes, the indebted fraction rises and this contributes to raising separation rates over time.

## 8 Behavior of Card Companies

A card company chooses the interest rate and credit limit on a contract offered to a person of a particular type, income and wealth level. Given these terms, it extends loans on demand until the contract expires because of a default or because the individual gives up the card to search for a better one. In choosing these contract terms, a card company takes as given the outside option of the individual which, in turn, depends on the contract terms being offered by other card companies and how quickly the individual can expect to get another offer.

In a directed search market, the market tightness adjusts to make the ex-ante value from a contract always equal to the individual's outside option. Given $R$, this equality defines a locus of $(q, L)$ pairs that are equally attractive to the individual but generates varying levels of ex-ante profits for the card company. Starting from $(q, L)$ combination that is on the locus, increasing $L$ can increase ex-ante profits for two reasons: A higher $L$ generates more revenues from borrowing and hence a higher $\Pi$ and higher $L$ will also increase the value the individual places on the card which then generates a higher $q$. However, increases in $L$ eventually decrease $\Pi$ because loss conditional on default rises with $L$: recall that at the point of default utilization rate on the card is at 1 or close to 1. Thus, for a given $R$ there is a value of $L$ that maximizes ex-ante profits while giving individuals their outside option. This is the content of the first-order optimality condition with respect to $L$.

This logic helps us understand why credit-limit-to-income ratios are increasing in types, as seen in Table 14. For card companies, the only cost of increasing $L$ is the greater loss incurred in default. All else the same, the likelihood of default on a card with limit $L$ will be lower for more patient types since their default costs are higher and, therefore, it is optimal to offer a higher credit limit to a more patient type.

Similarly, given $L$, there is a locus of $(q, R)$ combinations that give the individual her outside option. Starting on the locus and increasing $R$ can increase $\Pi$ if the increase in $R$ does not discourage borrowing too much. Of course, a higher $R$ is welfare-reducing for borrowers which means that market tightness must fall to keep the individual indifferent with respect to her outside option. So, the increase in $\Pi$ will be partially offset by a decrease in $q$. Eventually, though, increases in $R$ will discourage borrowing and both $\Pi$ and $q$ will decline and, hence, ex-ante profits will decline as well. Thus given $L$, there will be a value of $R$ that maximizes ex-ante profits. This is the content of the first-order condition with respect to $R$.

It is possible that for some market the best $(L, R)$ combination implies negative expected net profits. In that case, the company will not offer any contracts in that market.

This first-order logic does not readily illuminate the pattern of interest rates we see in Table 13. To better understand these patterns, it is helpful to consider the following stylized two-period example. Imagine a card company has committed to a credit limit $L$ and an interest rate $R$. Assume that the opportunity cost of its fund is the (gross) risk-free $R_f$. The card company expects the cardholder to borrow $l$ in the first period. In the second period, there are two possibilities. In the good state of the world, which occurs with probability $p$, the card-holder repays $Rl$. In the bad state of the world, which occurs with probability $1-p$, the card-holder borrows all the way to the limit, defaults, and pays nothing on the defaulted debt. The expected (second-period) profits of the card company, $\Pi$, can be expressed as $\Pi = -lR_f + p \cdot Rl - (1-p)[L - Rl]$. This identity can be written as

$$[R - R_f]\frac{l}{L} = (1-p) + \frac{\Pi}{L}.$$

The l.h.s of this identity is the product of the contract spread $R-R_f$ (the excess of the contract rate over the risk-free rate) and the utilization rate $l/L$. The r.h.s. is sum of the probability of default $(1-p)$ and the profit rate expressed in units of loan commitment $L$. For some given competitively determined profit rate, this identity shows that the contract spread depends negatively on the utilization rate $l/L$ and positively on the probability of default $(1-p)$ The negative dependence on the utilization arises because $l$ is what is repaid with interest and lower $l$ is the higher must the spread be to generate the same profit rate. In Table 13, we find that for all types the interest rate is rising with income. This seems initially puzzling because default frequency is declining in income. However, it is also the case that for any type, the utilization rate is declining in income and the negative dependence of contract spread on the utilization rate explains why interest rates rise with income.

The pattern of declining interest with types, given income and wealth, of course has a ready explanation: more patient types default less frequently and given some competitively determined profit rate, we would expect interest rates to decline in types. The reason for why interest rates are insensitive to wealth or fall very slightly was mentioned earlier: the pricing of the card reflects long run behavior and this is relatively insensitive to wealth at origination.

Finally, there is one aspect of contract interest rates that is important and interesting. For the moment assume that $\Pi = 0$. Then the contract spread that solves the above equation is the breakeven (zero-profit) contract spread. If the utilization rate $l/L$ less than 1, the breakeven contract spread $[R - R_f]$ will *exceed* the probability of default. This divergence occurs because

Table 19: Spreads, Default Frequencies and Utilization Rates

| Type 2, Lowest Asset Class | Spread | Util Rate @ 10 yr | Def Freq @ 10 yr | Spread x Util Rate |
|---|---|---|---|---|
| Low Y | 0.10 | 0.6669 | 0.0668 | 0.0667 |
| Med Y | 0.11 | 0.5876 | 0.0692 | 0.0646 |
| High Y | 0.12 | 0.4871 | 0.0625 | 0.0585 |

**Notes**: Model values (annual frequency).

repayment occurs on small debts $l$ but default occurs on large debts $L$. In the macro consumer default literature, the typical market arrangement is such that $l$ is always equal to $L$ and the utilization rate is 1. In this case, the breakeven contract spread is exactly equal to the default probability (assuming risk-neutral lenders, of course). But with a credit card contract, the card company must factor in the fact that repayments occur on debts that are likely to be smaller than the debts on which defaults occur. Since the same interest rate applies to all debts, the breakeven contract spread will need to exceed the default probability and the amount by which it needs to exceed it will be larger the smaller is the utilization rate (i.e., the smaller is the debts on which repayments occur). Of course, if there are monopoly profits, i.e., $\Pi/L > 0$, contract spreads will be higher still. Thus, the "loan commitment" nature of a credit card contract, as well as monopoly power, play a role in elevating credit card interest rates above default probabilities.

Table 19 reports the contract spread, average default frequency and average utilization rate on the 0-asset contract offered to the middle type at a 10 year horizon. Equilibrium contract spread substantially exceeds long run default probability but when it is scaled by the utilization rate, the excess goes away.

How much of the contract spread results from monopoly power? Table 20 gives one measure of this. For each contract, it reports that excess of the equilibrium interest rate over the interest rate that would give zero profits to the card company, holding the terms for all the other contracts fixed. The excess is largest for the contract offered to the the most impatient, low income, and lowest asset individuals. But this maximum excess is only 7 percent, implying a difference of 2 percentage points. More generally, the equilibrium contract terms do not incorporate a significant monopoly premium.

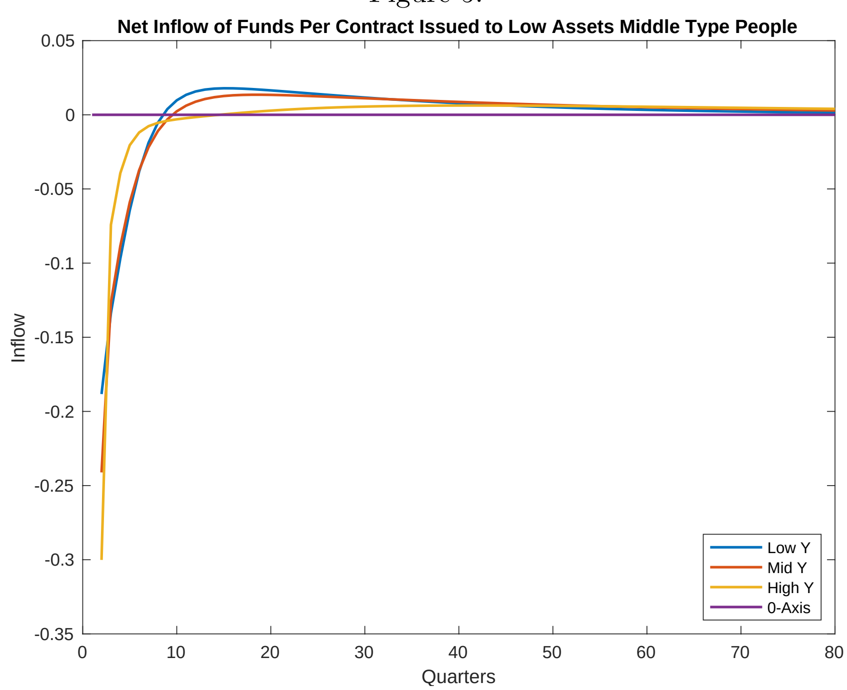
We turn now to examining how the baseline model performs with respect to interest rate on credit card contracts. Table 21 reports the APR on (newly originated) card by score quintiles along with the utilization rate. As noted earlier, interest rates decline with credit scores but the decline

Table 20:
Percentage Excess of Interest Rate Over Zero-Profit Interest Rates

| | Type 1 | | | Type 2 | | | Type 3 | | |
|---|---|---|---|---|---|---|---|---|---|
| Asset Class/Income | 0.50 | 1.00 | 1.98 | 0.50 | 1.00 | 1.98 | 0.50 | 1.00 | 1.98 |
| Lowest | 0.07 | 0.05 | 0.03 | 0.03 | 0.02 | 0.01 | 0.03 | 0.02 | 0.02 |
| Medium-Low | 0.04 | 0.03 | 0.02 | 0.01 | 0.01 | 0.01 | 0.01 | 0.00 | 0.01 |
| Medium | 0.03 | 0.02 | 0.02 | 0.01 | 0.01 | 0.00 | 0.01 | 0.00 | 0.03 |
| Medium-High | 0.02 | 0.02 | 0.02 | 0.01 | 0.00 | 0.00 | 0.01 | 0.00 | 0.00 |
| High | 0.02 | 0.02 | 0.01 | 0.01 | 0.00 | 0.00 | 0.01 | 0.00 | 0.00 |
| Highest | 0.02 | 0.02 | 0.01 | 0.01 | 0.00 | 0.00 | 0.01 | 0.00 | 0.00 |

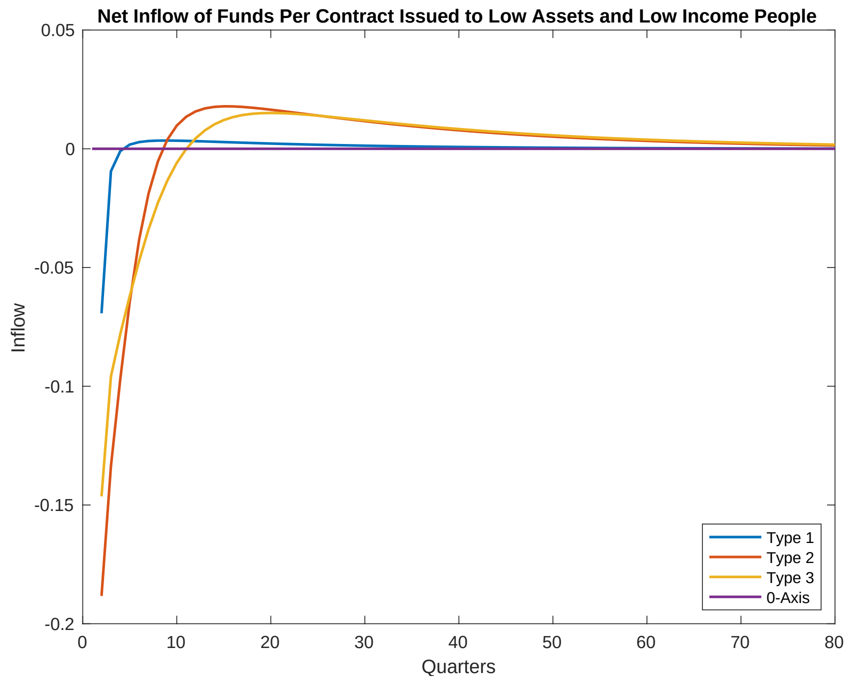
**Notes**: Model values. Asset and Income refer to levels at origination

Figure 5:




Notes: Model values. All contracts are for the lowest asset level class.

Figure 6:




Notes: Model values. All contracts are for the lowest asset level class.

is not as sharp as the decline in default frequencies. The model counterparts for the bottom two quintiles are close to the data, but there is substantial discrepancies between model and data for the top three quintiles: model interest rates drop off much more steeply than in the data. This discrepancy is then reflected in utilization rates which are much higher in the model for the top 3 quintiles than in the data.

As noted in the introduction, the fact that credit card APRs are generally far in excess of near-term default probabilities suggests monopoly power on part of card companies. Our directed search framework permits the exercise of monopoly power but because card companies must commit to

Table 21:
Utilization Rates and APR by Score Quintiles
Data and Model

| Description | I | II | III | IV | V |
|---|---|---|---|---|---|
| Utilization Rate, Data | 0.53 | 0.32 | 0.16 | 0.07 | 0.02 |
| Utilization Rate, Model | 0.38 | 0.38 | 0.44 | 0.37 | 0.17 |
| APR in %, Data | 21 | 19 | 17 | 16 | 17 |
| APR in %, Model | 21 | 18 | 14 | 12 | 10 |

**Notes**:

Table 22:
Utilization Rates and APR by Income Quintiles
Data and Model

| Description | I | II | III | IV | V |
|---|---|---|---|---|---|
| Utilization Rate, Data | 0.30 | 0.26 | 0.22 | 0.19 | 0.14 |
| Utilization Rate, Model | 0.50 | 0.39 | 0.35 | 0.28 | 0.23 |
| APR in %, Data | 19 | 18 | 18 | 18 | 17 |
| APR in %, Model | 14 | 15 | 15 | 15 | 16 |

**Notes**:

contract terms ahead of meeting a customer, the competition to acquire customers blunts that power.

Another important reason for the divergence is the nature of the card contract itself. This logic helps us understand the proximate cause of the underprediction of interest rates for the top 3 score quintiles. Table 21 displays the average utilization rate by score quintiles. Observe that the utilization rate for the bottom two quintiles are quite close to the data and, correspondingly, the interest rates are also close to the data. In contrast, for the top three quintiles, the utilization rates exceeds the utilization rate in the data quite substantially. Correspondingly, the model interest rates are lower than the interest rates in the data. The discrepancy between the model and data utilization rates for the top quintile is very large and so is model underprediction of the interest rate.

Table 22 reports utilization rate and contract interest rates. The gaps between the model predictions and the data are more striking. First, the model utilization is declining in income as it is in the data, but the actual rate is substantially higher in the model. For interest rates, the magnitudes are in rough agreement but the patterns diverge: In the data, interest rates decline with income and in the model they increase with income. As already explained, equilibrium card contracts have interest rates increasing with income, for all type and financial wealth combinations. Recall that higher income individuals are less likely to default, they are also less likely to utilize their card. The low utilization rate is force for higher interest rates (as explained above) and this force wins out.

## 9 "Targeted" Search and Costly Contracts

In this section, we investigate the sensitivity of equilibrium outcomes with respect to model assumptions along two different directions.

First, we drop the assumption of directed search and assume that search is "targeted" as in Gajen. In this set up, the contract terms are determined via Nash bargaining after contact is made. While all parties have rational expectations about the contract terms to expect upon contact, these terms are not committed to in advance by card companies. Given that the individual and the card company are in a bilateral monopoly situation post contact, the determination of contract terms will be affected by the bargaining power of the customer vis a vis that card company.

In the first experiment with targeted search, we estimate the bargaining weight of companies by giving some weight to the pattern of interest rates across score quintiles. This gave a bargaining weight of about 1/3 for companies and of 2/3 for individuals. The implied pattern of interest rates by score quintiles is quite similar to the baseline model, in particular, the targeted search model does not help in reducing the gap between observed interest rates and model interest rates for the top quintile. In two other experiments, we gave companies essentially all bargaining power and individuals all bargaining power. In the former case, the interest rates rise substantially for the bottom 3 quintiles and exceed the observed interest rates for these groups. For the fourth quintile the match between model and data is very good but the gap for the top quintile closes by only 1 percentage point. In the final experiment, the pattern of interest rates is again quite close to the baseline model, reiterating our earlier demonstration that equilibrium contracts do not imply significant monopoly power of card companies.

The second dimension along which we examine sensitivity is the flow cost of maintaining a credit card account. Adding a flow cost increases contract interest rates but it does so more for the lowest score quintile than for the top quintile.

Table 23:
APR by Score Quintiles in Targeted Search and Flow Cost Models Search

| | Score Quintiles | | | | |
|---|---|---|---|---|---|
| Description | I | II | III | IV | V |
| APR in %, Data | 21 | 19 | 17 | 16 | 17 |
| APR in %, Baseline | 21 | 18 | 14 | 12 | 10 |
| APR in %, Est. Company Wght 0.32 | 21 | 17 | 15 | 12 | 10 |
| APR in %, Company Wght 0.97 | 32 | 44 | 26 | 17 | 11 |
| APR in %, Company Wght 0.10 | 22 | 21 | 15 | 12 | 10 |
| APR in %, Flow Cost 0.32 per qtr | 28 | 21 | 16 | 14 | 12 |

**Notes**: Model values.

## 10 Conclusion

(To be added)

## References

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## Appendix A Computation of Credit Score

Let $j = 1, 2, 3, \ldots, 8$ denote the time period that is $j$ periods in the future ($j = 1$ is the next period). We initialize the credit score with 1 at the end of 8 periods at all the nodes the individual might end up in, and find its current credit score with backward induction. The credit score when the person holds a credit card is given by: Denote probability of death with $g$

$$k^i_{cc}\left(a, y; \omega, t\right) = (1-g) * \left(\begin{array}{c} \zeta^i\left(a, y; \omega\right) * \left[\begin{array}{c} f * \mu * E_{m,y'} k^i_{cc}\left(a'\left(a, y, m; \widetilde{\omega}\left(a, y\right)\right), y'; \widetilde{\omega}\left(a, y\right), t+1\right) \\ +\left(1 - f * \mu\right) E_{m,y'} k^i_{ncg}\left(a'\left(a, y, m\right), y'; t+1\right) \end{array}\right] \\ +\left(1 - \zeta^i\left(a, y; \omega\right)\right) * \left(E_{m,y'}\left[\left(1 - D\left(a, y, m\right)\right) * k^i_{cc}\left(a'\left(a, y, m\right), y'; \omega, t+1\right)\right]\right) \end{array}\right)$$

$$\begin{aligned} k^i_{ncg}\left(a, y; t\right) &= g * 1 \\ &\quad (1-g) * \left(\begin{array}{c} f * \mu * E_{m,y'} k^i_{cc}\left(a'\left(a, y, m; \widetilde{\omega}\left(a, y\right)\right), y'; \widetilde{\omega}\left(a, y\right), t+1\right) \\ +\left(1 - f * \mu\right) E_{m,y'} k^i_{ncg}\left(a'\left(a, y, m\right), y'; t+1\right) \end{array}\right) \end{aligned}$$

$$\begin{aligned} k^i_{ncb}\left(a, y; t\right) &= g * 1 \\ &\quad (1-g) * \left(\begin{array}{c} \delta * f * \mu * E_{m,y'} k^i_{cc}\left(a'\left(a, y, m; \widetilde{\omega}\left(a, y\right)\right), y'; \widetilde{\omega}\left(a, y\right), t+1\right) \\ +\delta * \left(1 - f * \mu\right) * E_{m,y'} k^i_{ncg}\left(a'\left(a, y, m\right), y'; t+1\right) \\ +\left(1 - \delta\right) E_{m,y'} k^i_{ncb}\left(a'\left(a, y, m\right), y'; t+1\right) \end{array}\right) \end{aligned}$$

## Appendix B Finding the default threshold:

When the country defaults, the m-shock reduces to $c(m) = \underline{m} + s * \left(m - \underline{m}\right)$.

$$c(m) = \left(1 - s\right)\underline{m} + sm$$

Say, in the case of no default, the country chooses the asset level $a_k$ between the m-thresholds $[m_k, m_{k+1}]$ Now we will need to compare the utility under default and no-default: Denote the utility under repayment with:

$$-\frac{1}{c\left(a, a_k\right) + m} + W$$

Denote the utility under default with

$$-\frac{1}{c_d + sm} + W_d$$

where $c_d = c(0,0) + (1-s)\,\underline{m}$ There are some conditions where there might be a dominant strategy. These are: (1) If $W > W_d$ and $c(a, a_k) + m_k > c_d + sm_k$ then for all $m \in [m_k, m_{k+1}]$ the individual chooses to not-default. (2) If $W_d > W$ and $c_d + sm_{k+1} > c(a, a_k) + m_{k+1}$ then for all $m \in [m_k, m_{k+1}]$ the individual chooses to default. (3) Else, we need to find the two m-thresholds where the two values are identical.

$$-\frac{1}{c(a, a_k) + m} + W = -\frac{1}{c_d + sm} + W_d$$

$$-(c_d + sm) + (W - W_d) * (c(a, a_k) + m) * (c_d + sm) = -(c(a, a_k) + m)$$

$$\frac{c(a, a_k) - c_d}{(W - W_d)} + \frac{(1-s)}{(W - W_d)} m + sm^2 + smc(a, a_k) + c_d m + c_d c(a, a_k) = 0$$

$$sm^2 + \left(sc(a, a_k) + c_d + \frac{(1-s)}{(W - W_d)}\right) m + c_d c(a, a_k) + \frac{c(a, a_k) - c_d}{(W - W_d)} = 0$$

As seen, this is a quadratic equation. If $W > W_d$, then the person prefers to default between these

two thresholds, and otherwise does not default. If $W < W_d$, then the person prefers to not-default between the two points, otherwise default. this way, the behavior of the person between $[m_k, m_{k+1}]$ is found.