A Probability-Based Stress Test of Federal Reserve Assets and Income

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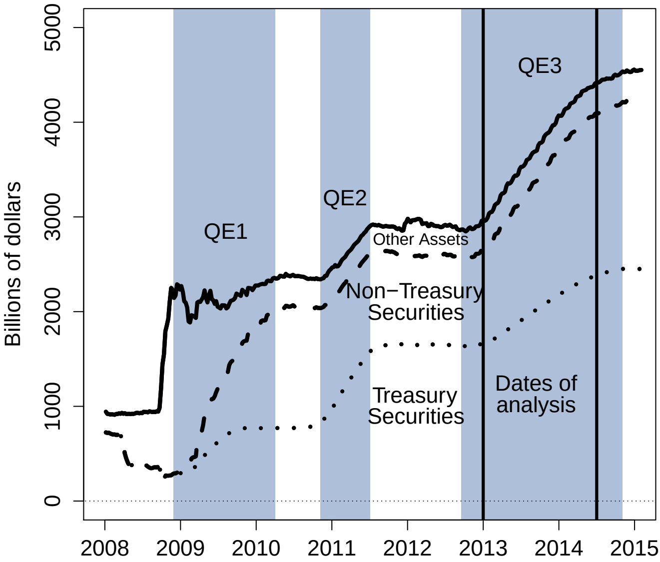
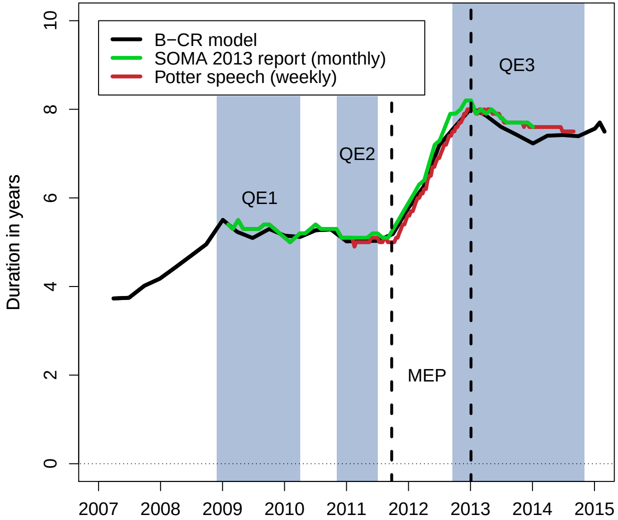
The Seventh Annual Volatility Institute Conference
NYU Stern Business School
April 24, 2015

Federal Reserve Assets

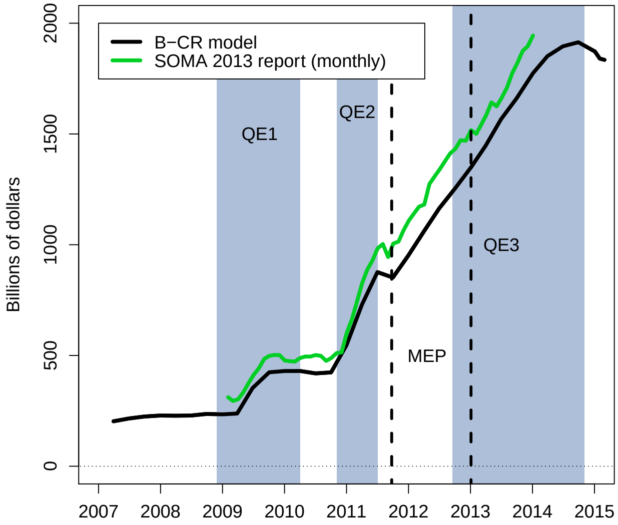


Duration of Fed's Treasury Securities Holdings



- The figure shows the estimated duration of the Fed's Treasury securities holdings.
- Since 2008, the duration has increased markedly.

Fed’s Treasury Securities in Ten-Year Equivalents



- Fed’s Treasury holdings in ten-year equivalents is a measure sensitive to both size and duration.
- The increase implies significant interest rate risk.

Concerns: Capital Losses and Income Shortfalls

Former Fed Governor Mishkin (2010): “Major holdings of long-term securities expose the Fed’s balance sheet to potentially large losses if interest rates rise. Such losses would result in severe criticism of the Fed and a weakening of its independence.”

Former Fed Vice Chairman Kohn (2014): “As long-term rates rise, the Federal Reserve will have mark-to-market losses on its balance sheet. These losses are not a threat to the Federal Reserve’s ability to tighten nor do they have any economic significance, but losses could be used as a political weapon by those who seek to curtail the Federal Reserve’s independence or limit its powers.”

Minutes of March 20, 2013 FOMC meeting: “Some participants were concerned that a substantial decline in remittances might lead to an adverse public reaction or potentially undermine Federal Reserve credibility or effectiveness.”

Fed's Interest Rate Risks

***Balance sheet risk*: Increases in *longer-term* interest rates erode the market value of the Fed's portfolio. (Fed does not mark-to-market, so these would be unrealized losses—unless Fed sells securities.)

***Income risk*: Increases in *short-term* interest rates (including IOER rate) raise cost of funding portfolio and lower net interest income and remittances to Treasury.

We examine how interest rate changes will affect the Fed's assets and income using a **probability-based stress test.** Our goal is to propose a stress-testing methodology better grounded in distribution forecasting as well as present empirical results around our policy question of interest.

What is a Probability-Based Stress Test?

Conventional stress test: How much bank capital remains in specific, arbitrary economic scenarios?

Carpenter et al. (2013) and Greenlaw et al. (2013, GHHM) apply this method to the Fed's balance sheet. They project the Fed's assets and income assuming $\pm$100 basis points parallel shifts in yield curve.

But how to weigh scenarios without probabilities?

Probability-Based Stress Test: We use a dynamic term structure model to generate distributional interest rate forecasts and attach probabilities to specific portfolio outcomes.

Our Results: We find fairly low probabilities of large losses on Fed's portfolio and of large interest income shortfalls as of year-end 2013.

Outline of Presentation

- **Description of shadow-rate term structure model**
- Probability-based stress test of Fed’s portfolio
- Probability-based stress test of Fed’s income
- Conclusion

Standard Gaussian term structure models do not restrict interest rates to be nonnegative!

To account for the ZLB, Black (1995) proposed a shadow rate, s_t, that may be negative, while observed short rate, r_t, is truncated:

$$\boxed{r_t = \max\{s_t, 0\}.}$$

This nonlinearity is computationally burdensome.

Christensen and Rudebusch (2014) combine option-based approximation of Krippner (2013) with arbitrage-free Nelson-Siegel (AFNS) model to deliver a tractable shadow-rate model.

The Arbitrage-Free Nelson-Siegel (AFNS) Model

Proposition: Given the risk-free rate

$$\boxed{r_t = L_t + S_t}$$

and risk-neutral Q-dynamics of factors $X_t = (L_t, S_t, C_t)$

$$\begin{pmatrix} dL_t \\ dS_t \\ dC_t \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \lambda & -\lambda \\ 0 & 0 & \lambda \end{pmatrix} \left[\begin{pmatrix} \theta_1^Q \\ \theta_2^Q \\ \theta_3^Q \end{pmatrix} - \begin{pmatrix} L_t \\ S_t \\ C_t \end{pmatrix} \right] dt + \Sigma dW_t^Q,$$

where Σ is a constant matrix, then zero-coupon yields have the popular Nelson-Siegel factor structure

$$\boxed{y_t(\tau) = \boxed{L_t} + \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau}\right)\boxed{S_t} + \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right)\boxed{C_t} - \frac{A(\tau)}{\tau}.}$$

This defines the AFNS model class derived in Christensen, Diebold, and Rudebusch (2011).

The Shadow-Rate AFNS Model Class (1)

Christensen and Rudebusch (2014) derive shadow-rate AFNS class of models with shadow rate

$$\boxed{s_t = L_t + S_t,}$$

and unchanged AFNS Q-dynamics.

The **instantaneous shadow forward rate** is

$$\boxed{f_t(\tau) = \boxed{L_t} + e^{-\lambda\tau}\boxed{S_t} + \lambda\tau e^{-\lambda\tau}\boxed{C_t} + A^f(\tau),}$$

where $A^f(\tau)$ is an analytical function of model parameters.

The **nonnegative instantaneous forward rate** is

$$\boxed{\underline{f}_t(\tau) = f_t(\tau)\Phi\Big(\frac{f_t(\tau)}{\omega(\tau)}\Big) + \omega(\tau)\frac{1}{\sqrt{2\pi}}\exp\Big(-\frac{1}{2}\Big[\frac{f_t(\tau)}{\omega(\tau)}\Big]^2\Big),}$$

where $\omega(\tau)$ is a deterministic function of model parameters.

The Shadow-Rate AFNS Model Class (2)

The yield-to-maturity is defined the usual way as

$$\boxed{\underline{y}_t(\tau) = \frac{1}{\tau}\int_t^{t+\tau} \underline{f}_t(s)ds}$$

$$= \frac{1}{\tau}\int_t^{t+\tau}\Big[f_t(s)\Phi\Big(\frac{f_t(s)}{\omega(s)}\Big) + \omega(s)\frac{1}{\sqrt{2\pi}}\exp\Big(-\frac{1}{2}\Big[\frac{f_t(s)}{\omega(s)}\Big]^2\Big)\Big]ds.$$

This is the measurement equation in the Kalman filter.

Since yields are nonlinear functions of the state variables, we use the standard extended Kalman filter.

The Real-World Dynamics

Christensen and Rudebusch (2014) apply the shadow-rate AFNS model to weekly U.S. Treasury yields (1985-2012).

They favor these model P-dynamics:

$$\begin{pmatrix} dL_t \\ dS_t \\ dC_t \end{pmatrix} = \begin{pmatrix} 10^{-7} & 0 & 0 \\ \kappa_{21}^P & \kappa_{22}^P & \kappa_{23}^P \\ 0 & 0 & \kappa_{33}^P \end{pmatrix} \left(\begin{pmatrix} 0 \\ \theta_2^P \\ \theta_3^P \end{pmatrix} - \begin{pmatrix} L_t \\ S_t \\ C_t \end{pmatrix} \right) dt + \Sigma \begin{pmatrix} dW_t^{L,P} \\ dW_t^{S,P} \\ dW_t^{C,P} \end{pmatrix},$$

where Σ is a diagonal matrix.

This is the transition equation in the Kalman filter.

We use this “B-CR model” for our analysis.

The B-CR Model: Further Details

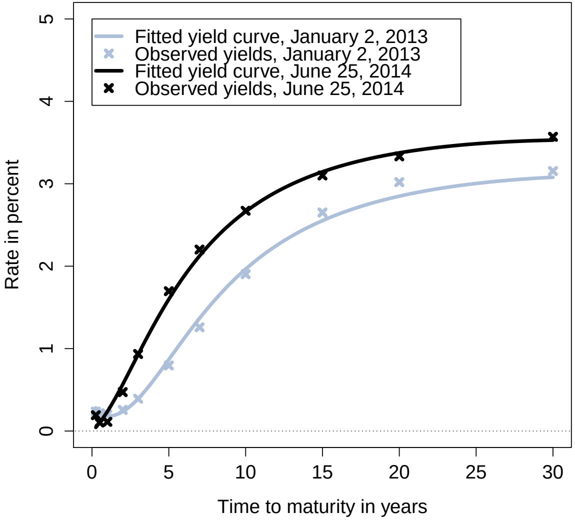
Key strengths of B-CR model:

- It fits yield curve quite well.
- Its short rate forecasts have been accurate and fairly closely match federal funds futures.
- It can replicate the compression in short- and medium-term Treasury yield volatility since 2009.

We estimate B-CR model with daily zero-coupon Treasury bond yields from the Gürkaynak, Sack, and Wright (2007) database.

We use 11 maturities from 3 months to 30 years starting in January 2, 1986. Three estimation sample end dates.

Observed and Fitted Yield Curves (1/13 & 6/14)



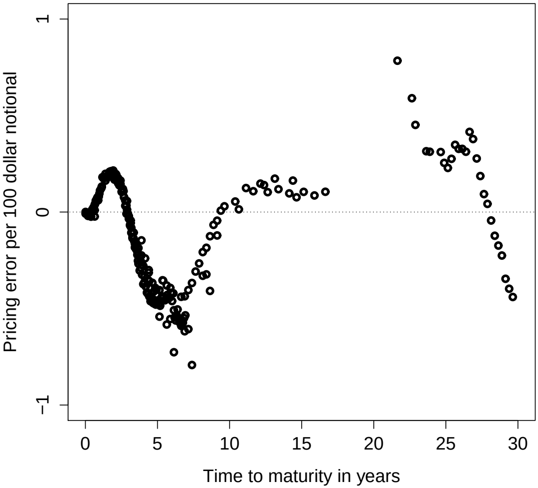
- The model provides close fit to entire cross section of yields and gives good bond pricing.

Test of Model: Value Fed's Treasury Portfolio

Number of Securities	Value in billions as of June 25, 2014		
	Face value	Bloomberg	Model
237	2,284	2,470	2,473

- We consider the future cash flows for all 237 nominal Treasury securities in Fed's portfolio on June 25, 2014.
- We value each payout stream using the model-implied yield curve to obtain the model-implied market value.
- This model value of the entire Treasury portfolio differs from the market value according to Bloomberg bond prices by less than $3 billion.
- This is a strong test of model fit as these raw bond prices were not used in model estimation.

Illustration of Treasury Pricing Performance



- Illustration of pricing errors in dollars per $100 notional across bond maturities.
- As of June 25, 2014, the pricing errors are limited in size.

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P-Based Stress Test of Fed’s Treasury Portfolio

- To examine balance sheet risk, we project the value of the Fed’s **Treasury securities** for three years.
- We analyze two start dates: January 2, 2013, and June 25, 2014.
- We fix Fed’s Treasury portfolio as of each start date.
- We use the B-CR model estimated as of the start date to generate Treasury yield curve projections.

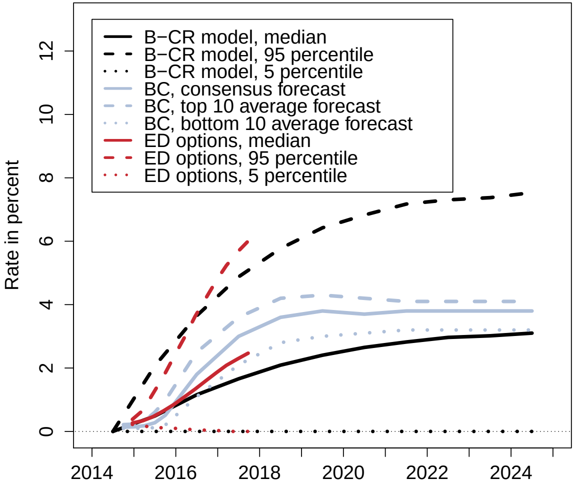
Model-Based Portfolio Value Projections

We use the estimated B-CR model to generate Treasury yield curve projections and assign them probabilities.

Procedure:

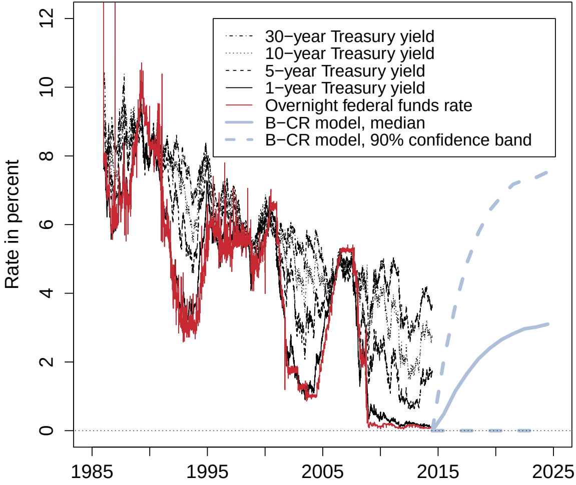
- Estimate parameters and state variables from the B-CR model as of the start date.
- Simulate $N = 10{,}000$ sample paths for the state variables.
- Convert the simulated state variables into full yield curves and calculate the corresponding Treasury portfolio values along each path (i.e. value all future cash flows).
- Examine the resulting portfolio value distributions.

Short Rate Projections (June, 2014)



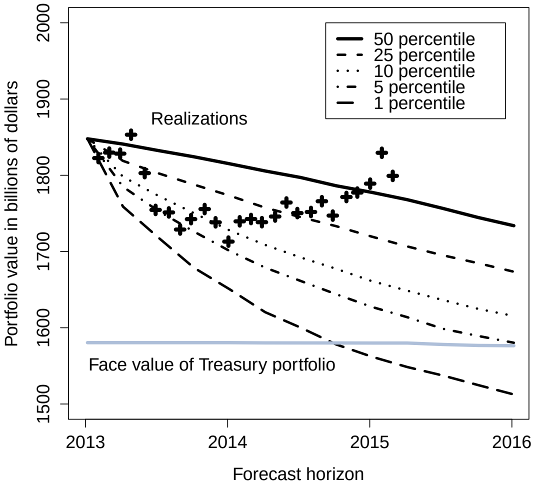
- The figure shows short-rate distributions from the B-CR model, 3-m LIBOR outcomes implied by options on eurodollar futures, and federal funds rate forecasts from the Blue Chip survey.

Short Rate Projections (June, 2014) cont.



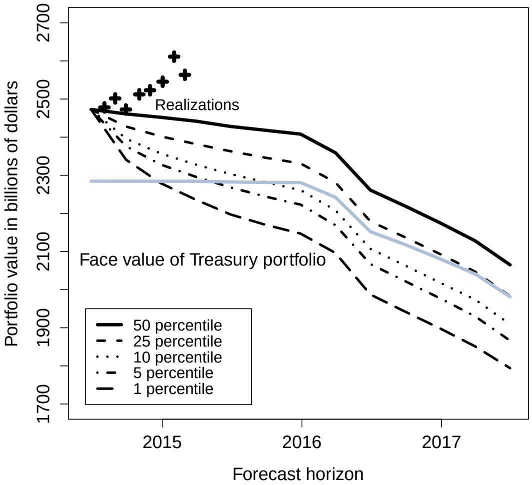
- The figure shows the short-rate distribution from the B-CR model with a comparison to past Treasury yields and overnight federal funds rates.

Treasury Portfolio Valuations (as of Jan. 2, 2013)



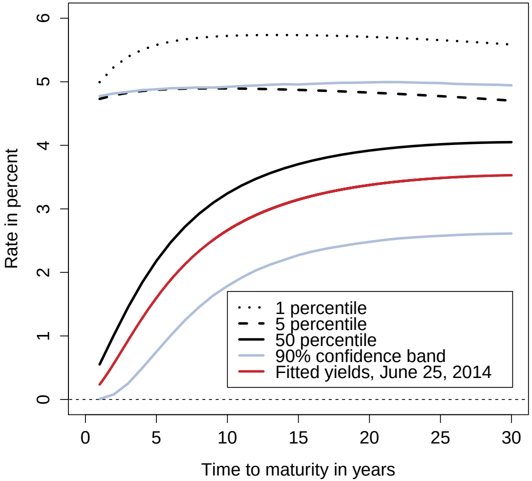
- Shown are lower percentiles of the Fed's Treasury portfolio value distribution from B-CR model projections.
- As of January 2, 2013, the chance of the Treasury portfolio value going below par is less than 5 percent.

Treasury Portfolio Valuations (as of June 25, 2014)



- Shown are lower percentiles of the Fed's Treasury portfolio value distribution from B-CR model projections.
- As of June 25, 2014, the chance of the Treasury portfolio value going below par is less than 25 percent.

Underlying Yield Curves

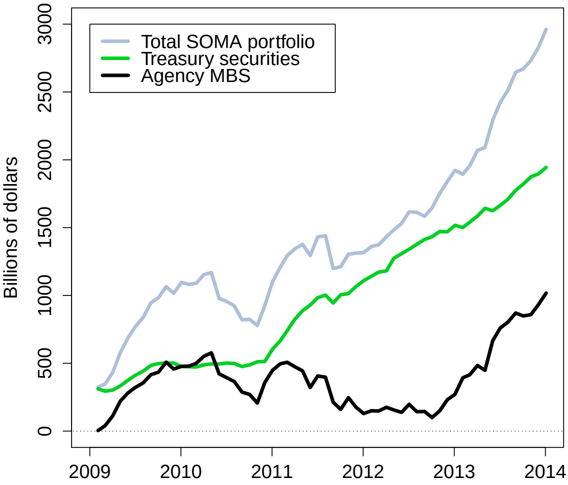


- Projected yield curves that produce the 1^{st}, 5^{th}, and 50^{th} (i.e., median) percentiles of the Treasury portfolio values by mid-2017.

P-Based Stress Test Including Fed's MBS Holdings

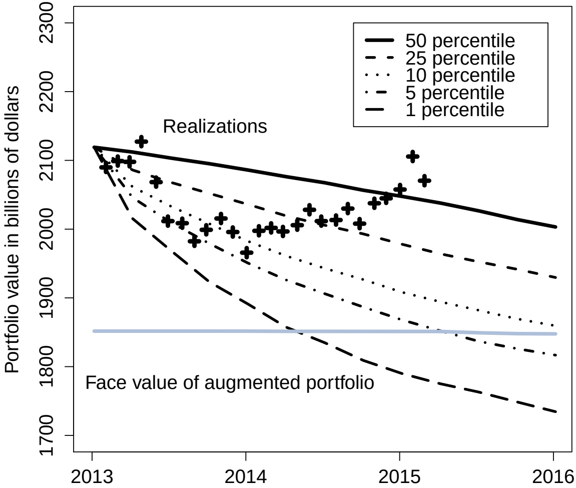
- MBS portfolio is complex, diverse, hard to value.
- We convert Fed's MBS portfolio into ten-year equivalent U.S. Treasury securities.
- As of June 25, 2014, Fed held 68,557 separate MBS, which had a total face value of $1,664 billion and a duration of 5.8 years.
- According to B-CR model, the ten-year Treasury par-coupon yield was 2.6% and duration was 8.8 years.
- Thus, we replace MBS with $1,093 billion of ten-year Treasuries with 2.6% coupon.
- As of Jan. 2, 2013, MBS holdings represented $271.2 billion in ten-year equivalents.

Fed's Securities Portfolio in Ten-Year Equivalents



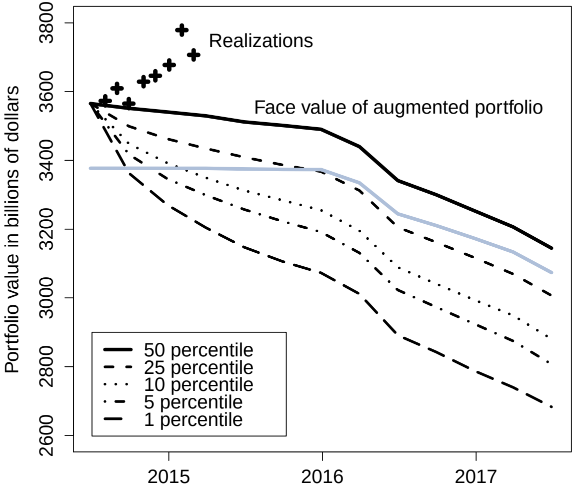
- Fed's portfolio measured in ten-year equivalents—a measure sensitive to both size and duration.

Treasury & MBS Portfolio Valuations (Jan. 2, 2013)



- Shown are lower percentiles of the Fed's Treasury & MBS portfolio value distribution from B-CR model projections.
- As of January 2, 2013, the chance of the Treasury & MBS portfolio value going below par is less than 10 percent.

Treasury & MBS Portfolio Valuations (Jun. 25, 2014)



- Shown are lower percentiles of the Fed's Treasury & MBS portfolio value distribution from B-CR model projections.
- As of June 25, 2014, the chance of the Treasury & MBS portfolio value going below par has increased to 1-in-3.

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P-Based Stress Test of Fed's Income

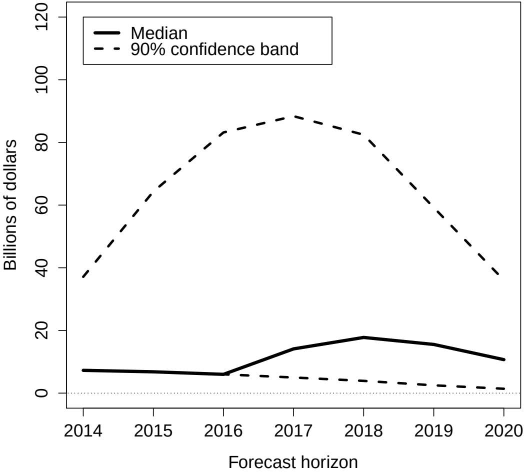
To generate projections of the Fed's income, we use B-CR model and GHHM accounting framework.

We use GHHM baseline assumptions with three changes:

- We assume no securities sales.
- We use asset purchase path through 2014 from NY Fed Primary Dealer Survey of June 2014.
- We set the path for the interest paid on excess reserves (IOER) equal to the short rate from model simulation, but with a minimum of 25 basis points.

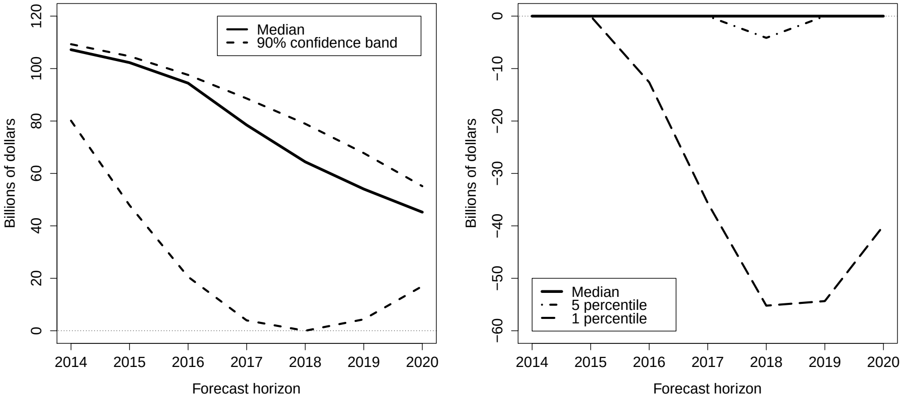
We simulate the B-CR model (estimated as of December 31, 2013) to generate interest rate scenarios through 2020.

Distributional Forecast of Fed's Interest Expenses



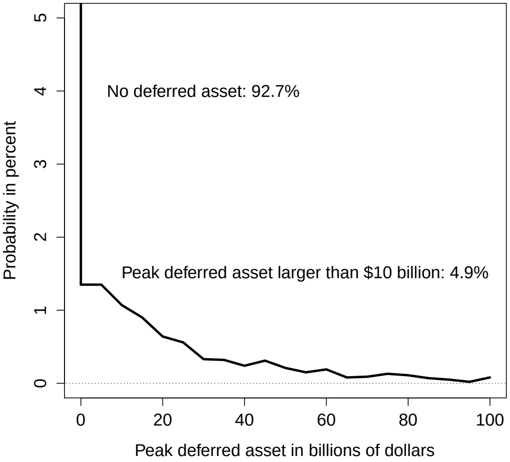
- At longer forecast horizons, the estimated B-CR model allows for significant variation in the overnight rate.
- Thus, the Fed is exposed to significant uncertainty about funding costs and projected interest expenses.

Fed's Payments to the U.S. Treasury



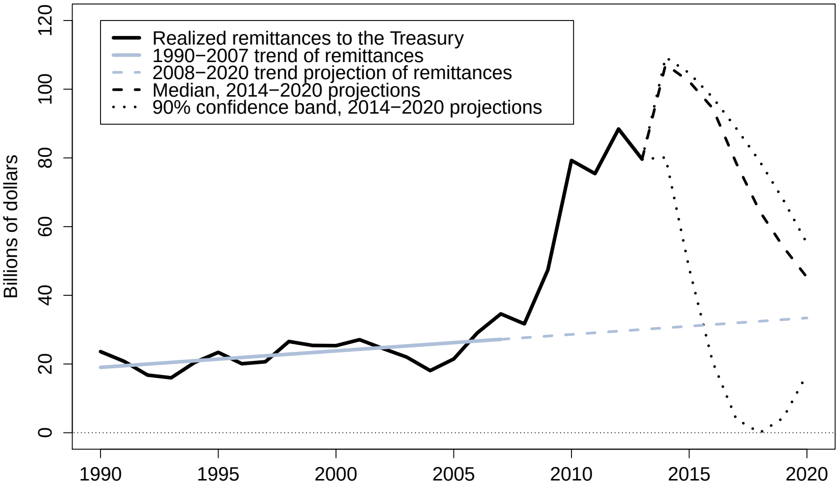
- Left figure shows Fed's remittances to the Treasury, which will vary due to variation in interest expenses.
- Right figure shows about a 5 percent chance that remittances will halt. Then Fed accrues a "deferred asset" that peaks in 2018.

Probability Distribution of Peak Deferred Asset

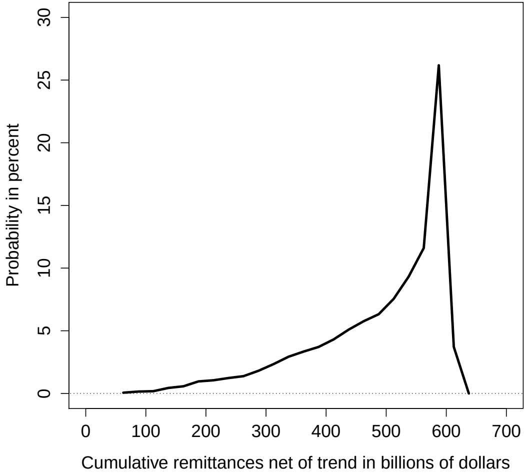


- Distribution shows 92.7% of no deferred asset, and 4.9% chance of a peak deferred asset in excess of $10 billion.

Projected Fed Remittances to U.S. Treasury



Cumulative Remittances to Treasury Net of Trend



- Probability distribution for cumulative Fed remittances 2008-2020 minus projected trend (from 1990-2007). QE boosted remittances above trend.

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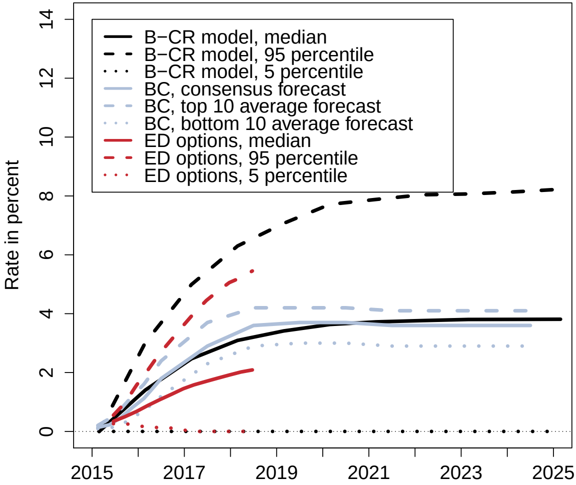
Conclusion

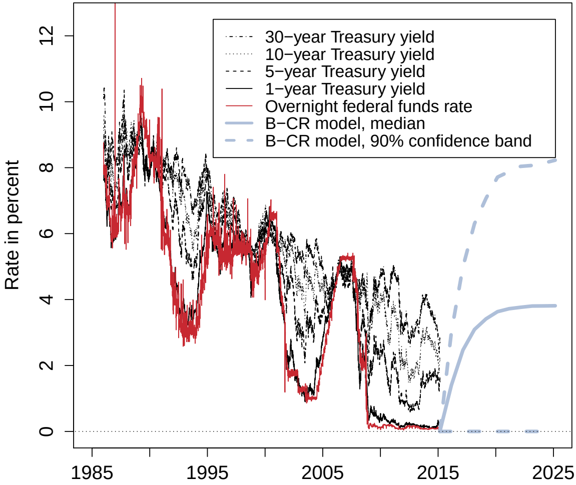
- We introduce probability-based stress tests and argue that attaching likelihoods to adverse outcomes is an important addition to the debate.
- We use a shadow-rate term structure model that respects the zero lower bound to generate Treasury yield curve projections.
- We find that the Fed's potential losses over the next several years are in most cases relatively small.
- We generate comprehensive projections of the Fed's future income and find a very small chance of a temporary halt of the remittances to the Treasury.

Short Rate Projections (February 25, 2015)



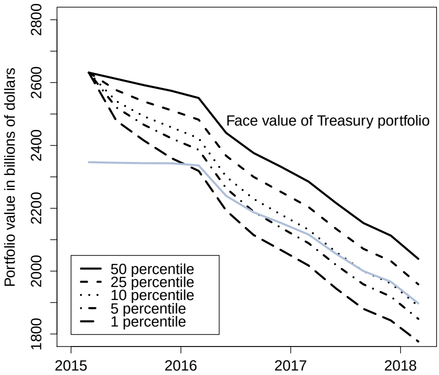
- The figure shows short-rate distributions from the B-CR model and 3-m LIBOR outcomes implied by options on eurodollar futures as of February 25, 2015.
- The Blue Chip ff rate forecasts are from Dec. 2014.

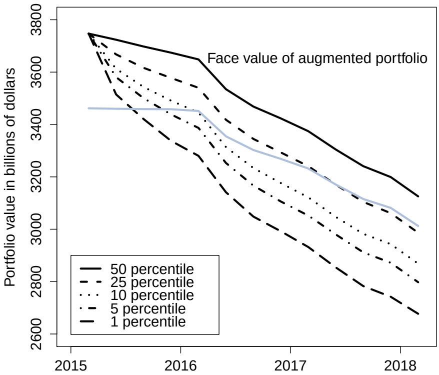
Short Rate Projections (February 25, 2015) cont.



- The figure shows the short-rate distribution from the B-CR model with a comparison to past Treasury yields and overnight federal funds rates as of February 25, 2015.
- Note the considerable model-implied variation.

Update of Portfolio Projections (Feb. 25, 2015)





- Lower percentiles of the Fed's portfolio values from model-based projections as of February 25, 2015.
- Due to recent yield declines, the balance sheet risk has declined slightly—despite the final portfolio expansion and the sizeable dispersion in model-implied yield curves.