# Social Security and Trends in Inequality

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**Abstract**

Recent influential work finds large increases in inequality in the U.S., based on measures of wealth concentration that notably exclude the value of social insurance programs. This paper revisits this conclusion by incorporating Social Security retirement benefits into measures of wealth inequality. Wealth inequality has not increased in the last three decades when Social Security is accounted for. When discounted at the risk-free rate, real Social Security wealth increased substantially from \$5.6 trillion in 1989 to just over \$42.0 trillion in 2016. When we adjust for systematic risk coming from the covariance of Social Security returns with the market portfolio, this increase remains sizable, growing from over \$4.6 trillion in 1989 to \$34.0 trillion in 2016. Consequently, by 2016, Social Security wealth represented 58% of the wealth of the bottom 90% of the wealth distribution. Redistribution through programs like Social Security increases the progressivity of the economy, and it is important that our estimates of wealth concentration reflect this.

*Keywords*: Social Security, Inequality, Top Wealth Shares
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# 1 Introduction

It is widely believed that wealth inequality in the United States is on the rise. This belief is supported by several studies which, though they differ in their methodology, all use Piketty (2013)'s definition of wealth: the market value of all assets owned by households, net of debt. This paper builds on past work to broaden the definition of wealth to include the value of Social Security retirement benefits. In doing so, we illustrate how the "marketable wealth" concept is incomplete, and it leads to misconceptions about both the level of and recent trends in wealth concentration. Social Security wealth has grown more than three-fold in the last three decades. As such, by 2016, for the bottom 90%, Social Security wealth exceeds marketable wealth. Its exclusion thus dramatically overstates the growth of wealth inequality.

This fact is well-illustrated by a simple back-of-the-envelope exercise. Piketty et al. (2018) report household wealth, excluding Social Security, totaled \$79 trillion[1] in 2016. The Social Security Administration (SSA) estimates that aggregate Social Security wealth was \$33 trillion in the same year. We can naively assume that the top 1% receives a disproportionate share (10%) of total Social Security wealth. Social Security is progressive so this assumption vastly overstates the share of Social Security wealth at the top. Even so, including Social Security decreases the top 1% wealth share in 2016 by 9 percentage points relative to prior estimates.

If we are to include Social Security into top wealth estimates in a more sophisticated manner, we must know both the aggregate size of the Social Security program, and how Social Security wealth accrues across the marketable wealth distribution. This paper derives estimates of the stock and distribution of Social Security wealth by simulating households' future benefits and payroll taxes, relying on data from the Survey of Consumer Finances (SCF). Importantly, our focus is on Social Security's old-age retirement program, and we exclude disability insurance, which would lead to an even larger reduction in top wealth shares.

For retirees, the SCF reports the Social Security benefits that recipients will receive until death. For workers who are still in the labor force, we simulate earnings trajectories by relying on previous empirical work that provides a labor market income process that matches many moments of the SSA administrative panel data (Guvenen et al., 2019a,b). We then apply the

[1]Unless noted otherwise, all dollar estimates are in 2018 dollars.

Social Security benefit and tax formulas to construct estimates of future Social Security benefits that these households will accrue net of the taxes that they will pay. To validate our simulation, we show that we can match the aggregate estimate of Social Security wealth of the SSA, and that our estimates correctly match actual retirement-age benefits reported in the SCF. Finally, we assign Social Security wealth to different centiles of the wealth distribution based on the relationship between Social Security and marketable wealth for retired workers, readily observable in the SCF.

Computing the present value of Social Security wealth also requires choosing an appropriate discount rate. We first offer a risk-free valuation of Social Security wealth using the treasury market yield curve. The top 10% and top 1% "marketable wealth" share (excluding Social Security) grew by 10 percentage points between 1989 and 2016, in line with estimates from past work (Piketty et al., 2018, Smith et al., 2019). Once Social Security wealth is included, this trend goes negative: rather than rising, the top 10% and top 1% wealth shares drop by 5.6 and 0.2 percentage points, respectively.

Nevertheless, discounting should reflect the risks associated with the Social Security program (Geanakoplos et al. (1999)). As such, our second set of results account for the labor market risk inherent in pay-as-you-go systems. Social Security is wage-indexed, so future benefits are directly tied to economic growth. Given the cointegration between the labor and stock markets (Benzoni et al. (2007)), it is important to adjust for the market beta of future Social Security payouts (Catherine, 2019, Geanokoplos and Zeldes, 2010). Our risk-adjustment decreases the stock of Social Security wealth by nearly 20 percent. This has a disproportionate impact on young workers who are most exposed to long-run systematic labor market risk. These workers are nearly always in the bottom 90% of the wealth distribution, and so adjusting for labor market risk decreases the Social Security wealth of this group. On the other end of the distribution, this adjustment barely impacts the Social Security wealth of those in the top 10%, because they are significantly older. Even after this correction, we find that inequality trends are substantially attenuated relative to past estimates that exclude Social Security; from 1989 to 2016, the top 10% wealth shares decrease by 3.0 percentage points. The top 1% shares increase, but only by 1.2 percentage points.

Why does Social Security have such a dramatic effect on inequality trends? One reason is that Social Security wealth increased more than three-fold between 1989 and 2016. This increase can be attributed to at least three components. First, Social Security expanded in scope over our sample period, as the share of earnings subject to Social Security payroll taxes increased from a maximum of 1.5 times average annual earnings to 2.5 times. Second, real interest rates have fallen, increasing the value of future retirement benefits. Finally, the U.S. population is aging and living longer. The share of workers that is near retirement age and for whom Social Security wealth is at its peak (because they have paid in fully to the fund, but have yet to receive any benefits) grew by nearly 50 percent. Moreover, life expectancy increased by nearly 4 years.

It is challenging to provide a convincing rationale for excluding Social Security in the study of wealth concentration. Some argue that the value of Social Security wealth is unknown, given labor market risk, policy uncertainty, and the lack of readily observable market valuations (Zucman (2019)). But income sources that are capitalized for inclusion in top wealth estimates – like private business income – are also subject to substantial uncertainty in valuation.

It is also conceptually strange to ignore the impact of Social Security wealth in estimates of wealth concentration, since the traditional life-cycle framework implies a one-for-one reduction in personal wealth accumulation as the present value of future Social Security benefits rise (Feldstein (1974, 1977)). Feldstein (1979) provides an early review of the empirical evidence that confirms the life-cycle model, finding that large Social Security benefits displace private saving. In more recent work, Scholz et al. (2006) find near-perfect substitution between Social Security benefits and private wealth accumulation, adding to a long literature confirming that pension wealth and private wealth are substitutes (e.g., Attanasio and Brugiavini (2003) and Attanasio and Rohwedder (2003)).[2]

The implication of the life-cycle model – and the empirical evidence – is that in a counterfactual world without Social Security, private wealth would rise by the present value of expected Social Security benefits. Recent studies of trends in wealth inequality implicitly assume away this counterfactual by ignoring Social Security wealth, which unsurprisingly distorts inequality trends. More generally, a singular focus on marketable wealth when measuring inequality is erroneous,

[2]It is worth noting that there is evidence that retirement saving through employer-provided retirement accounts does not displace private wealth accumulation by passive savers Chetty et al. (2014). In addition to being a different context than Social Security, these estimates are based on short-run responses, and in the longer-run, there is evidence that employees do in fact offset these wealth increases by saving less in the future Choukhmane (2018).

insofar as changes over time in the size of the social safety net affect private wealth accumulation. Perversely, tax reforms like wealth taxation to fund additional transfers or increase the generosity of existing programs could cause an increase in measured wealth inequality.

We are left to conclude that prior studies of top wealth exclude Social Security because existing approaches to measuring wealth concentration make its inclusion complex. Extrapolating from estimates of Social Security benefits to the overall size of the program and its distribution across centiles of wealth is nontrivial and requires a careful study of the trajectory of workers' earnings, a task that this paper undertakes. We thus contribute to the literature by showing how to sensibly value progressive programs like Social Security, and also by considering the consequences of this broader definition of wealth on the evolution of top wealth shares. To be sure, this is an incomplete undertaking: we too exclude important components of wealth from our estimates, for example, the provision of public healthcare benefits. It is our hope that this paper represents a first step toward a broader wealth concept that will enable proper measurement and analysis of inequality trends.

**Related Literature** Narrowly defined marketable wealth (Saez and Zucman (2016)) understates the wealth of workers and consequently overstates inequality substantially. It also ignores a long literature that documents the importance of Social Security for the distribution of income and wealth. For instance, (Wolff, 1992, 1996) shows that the inclusion of pension and Social Security wealth impacts both the level of and changes in measured wage inequality. Gustman et al. (1999) investigate the importance of pension and Social Security wealth for those nearing retirement, showing that it accounts for half–or more–of the total wealth of all those below the 95th percentile of the wealth distribution. Poterba (2014) also sheds light on the importance of Social Security to the elderly, documenting that for people over age 65, this stream of cash flows accounts for more than half of total income for the bottom three quartiles of the income distribution. Outside of the US, evidence confirms that ignoring the effects of redistributive pension programs inflates measured wage inequality (Domeij and Klein, 2002).

Based on the insights of this past literature, we augment our definition of wealth to include Social Security benefits that workers accrue. In essence, we update and extend Feldstein (1974), who relied on survey data to show that in 1962, the ownership of total wealth, inclusive of Social

Security, was much less concentrated than the ownership of market wealth. We show this pattern remains true across cohorts, and the differences between the "market wealth" and "total wealth" series are of growing importance over time. We thus contribute to the literature by documenting the sizeable impact of Social Security on trends in wealth inequality. Our exercise confirms Weil (2015) who suggests that the concept of market wealth is incomplete and overstates inequality by ignoring transfer wealth, which is both large and, unlike market wealth, not skewed to the top of the distribution. A similar point has been made by Auten and Splinter (2019) in the context of income inequality, highlighting that including government transfer programs decreases top income shares.

Finally, our work is related to an extensive literature on the magnitude and beneficiaries of redistribution through Social Security. Because the Social Security benefit formula replaces a greater fraction of the lifetime earnings of lower earners than higher earners, it is generally thought of as progressive. Past work documents how much of the intracohort redistribution in the United States is related to factors beyond income: for example, benefits are transferred from those with low life expectancies to those with higher, and from single workers to non-working spouses ((Feldstein and Liebman, 2002, Gustman and Steinmeier, 2000, 2001, Liebman, 2002)).

The remainder of our paper is organized as follows. Section 2 describes the Social Security program and changes to the benefit calculation over time, as well as U.S. demographics, that contribute to the system's recent growth. Section 3 describes our data sources. Section 4 lays out our approach to estimating Social Security wealth and its distribution. We present our results on the distribution of household wealth, inclusive of Social Security and discounted at the risk-free rate, in Section 5. Results for our risk-adjusted valuation are presented in Section 6. Section 7 compares our estimates to previous studies and provides counterfactual estimates on the growth of Social Security and top wealth shares under alternative assumptions. Section 8 concludes.

# 2 Social Security and the Distribution of Wealth

We hypothesize that Social Security may impact inequality trends for two reasons. First, the stock of Social Security wealth is large: today the SSA estimates it totals $33 trillion, or over 40 percent of marketable wealth, and it is the primary source of income for the vast majority of retired American households.[3] Second, Social Security wealth is more progressively distributed than marketable wealth. Figure 1 shows the distribution of retirement benefits by decile of net worth. They are larger for wealthier retirees, who receive greater benefits because they paid more into Social Security over their lifetimes. But, compared to the distribution of capital income, these are minor differences. Among recent retirees, the top decile receives less than 15 percent of Social Security benefits, and nearly 50 percent of income from capital.

[Insert Figure 1 about here]

Importantly, Social Security rose in value by over 200 percent in real terms between 1989-2016. Given its progressivity, this growth is likely to have important implications for the measurement of wealth concentration. We discuss three elements of Social Security's recent growth: changes in the scope of the program; the interest rate environment; and population aging, which boosts the share of U.S. citizens who receive Social Security benefits.

## 2.1 Computation of Social Security Benefits

Social Security was introduced in the U.S. in the aftermath of the Great Depression as a response to concerns that lost wages due to death, disability, or retirement left individuals vulnerable later in life (Perkins (1962)). The hope was that a broad program of social insurance would help smooth the volatility of income and provide support in old age as individuals transitioned out of the labor force. Today, Social Security provides the majority of income to most elderly Americans: nearly 90 percent of individuals above the age of 65 receive Social Security benefits, and for over half of beneficiaries, these benefits represent 50 percent or more of their total income (SSA 2019).

[3] Note that researchers come to different estimates about the share of retirees who receive most or all of their income from Social Security. Biggs (2020) discusses these differences. But there is general agreement that Social Security plays a large role in maintaining living standards in retirement.

Social Security benefits are funded by payroll taxes on employers and employees. Since its inception, it has been a largely unfunded program, such that contributions of those paying into Social Security as workers are contemporaneously distributed to retirees (Social Security Bulletin 2005). Today, there is a 12.4% payroll tax, and contributions are capped at an upper limit: in 2019, top earners paid Social Security taxes on only their first $132,900 of annual earnings.

Benefits are determined based on individuals historical earnings. First, average taxable earnings[4] are adjusted by indexing average wages in each year the individual worked by average wages in the year she turns 60.[5] Practically, wage indexing in this manner adjusts workers benefits for both inflation and real wage growth. After the earnings in each year have been indexed, the best 35 years are kept and averaged to determined an individuals' average indexed yearly earnings ("AIYE").

The Social Security formula determines benefits in a progressive manner as a function of the AIYE. The formula replaces a higher share of earnings for lower-wage workers. In 2019, an individual who retires at the full retirement age will receive first year benefits as the sum of:

1. 90% of the share of the AIYE below the first bend point ($11,112);
2. 32% of the AIYE between the first and second bend point ($66,996);
3. and 15% of the AIYE above the second bend point

These "bend points" are different for each calendar year of attainment of full retirement age. Early retirement (e.g., at age 62) reduces benefits and delayed retirement increases benefits up to age 70.

## 2.2 Valuing Social Security

Figure 2 illustrates how the stock of Social Security wealth has changed over time according to SSA's annual reports. We graph data reported annually by the SSAs Office of the Chief Actuary, which estimates the theoretical "transition cost" for the program, intended to provide a rough

[4] Average wages below the maximum taxable earnings (or Social Security wage base), above which earnings are neither taxed nor taken into account in the computation of benefits.

[5] N.B.: Earnings after age 60 can increase benefits, but they enter in nominal terms.

estimate of the cost of terminating the program for all new entrants. The closed-group transition cost estimates reported reflect the present value of expected benefits that will accrue to those currently contributing to Social Security, net of their expected payroll tax payments and current Social Security reserves. SSA transition costs include a 100-year projection period that is intended to capture the lifetime of all current participants included in the valuation exercise.[6]

[Insert Figure 2 about here]

The total value of Social Security benefits owed to workers net of the taxes they will pay into the Social Security program in their lifetimes–has more than doubled in real terms over the last three decades. By 2019, this total was over $33 trillion or 42% of estimates of household net worth, excluding Social Security (Piketty et al., 2018). We next discuss the causes of this recent growth.

### 2.2.1 Policy shifts in Social Security

Over time there have been several concerns about the financial stability of the U.S. old-age retirement program. In response, policymakers have repeatedly increased the maximum salary subject to the payroll tax. In the short-run, these changes provide additional funding for Social Security beneficiaries through progressive tax increases on those with highest earnings. Nevertheless, raising the tax ceiling also increases what Social Security must pay these beneficiaries in the future.

Over the last 70 years, maximum taxable earnings increased by three times as much as wages grew. This means that the scope of Social Security, and its relative importance for retirement savings, grew substantially. These policy choices increased the benefits owed to middle and upper-income individuals later in life, with important implications for patterns of wealth accumulation during labor market years (Gustman et al. (2012)).

[Insert Figure 3 about here]

Figure 3 reports the evolution of the relationship between the maximum taxable earnings base and average earnings in the United States from 1961 onward. The share of the working-age

[6]The SSA also reports open-group transition cost estimates, which includes future cost and future scheduled tax income for those not currently covered by the program (e.g., those under 15 years of age). Because our focus is on the Social Security wealth of workers in the labor force during our sample period, the closed group estimates are the appropriate benchmark.

population whose earnings fall below the maximum base increased from 73 to 94 percent over this period;[7] with the result being that Social Security benefits replace a greater share of lifetime benefits today than they did previously for the upper middle-class. The ratio of maximum taxable earnings to average annual wages was less than 1.5 until 1970; by the 1980s, when the tax cap was automatically indexed to changes to wage growth, this ratio stabilized at around 2.5.

The oldest workers in our sample entered the labor force in 1959. Top earners in this group saw their contributions to Social Security increase by over 90 percent (relative to average wages) during their time in the labor force. The youngest cohort reaching retirement that we observe entered the labor force in 1986, by which time the maximum contributions (relative to average wages) were stable.

### 2.2.2 Decline in Interest Rates

We define the expected Social Security wealth of current program participants as the present value of benefits net of the present value of payroll taxes to be paid, discounted using the average nominal yield curve in each survey year.[8]

To illustrate, consider an individual who was 40 years old in 1989. By this point, he had spent 20 years paying Social Security payroll taxes. He will spend 25 more years working before he begins receiving Social Security benefits at age 65, for 20 years (his assumed life expectancy is 85). The present value of his aggregate Social Security wealth is thus:

$$\begin{aligned} \mathrm{PV}_{\text{SS Wealth}} = & \frac{\text{Benefit}}{(1+r_{25})^{25}} + \frac{\text{Benefit}}{(1+r_{26})^{26}} + \dots \\ & - \text{Tax}_{1989} - \frac{\text{Tax}_{1990}}{(1+r_2)^2} - \dots - \frac{\text{Tax}_{2014}}{(1+r_{25})^{25}}, \end{aligned} \tag{2.1}$$

where $r_i$ represents the annualized zero-coupon spot rate $i$ periods into the future; and benefits are benchmarked to economy-wide average annual earnings presently, but to prices post-retirement, thus reflecting both the trajectory of inflation and wage growth. The payroll tax contribution is

[7] *Social Security Administration, Annual Statistical Supplement*, 2018, Table 4.B1

[8] As we discuss below, this is somewhat different from the SSA, which discounts cashflows based on its projection of future interest rates. Either set of assumptions show a large increase in Social Security wealth over our period.

the relevant percentage of each year's labor market earnings, up to the maximum taxable earnings cap, thus reflecting the trajectory of inflation but independent of economy-wide wage growth.[9]

[Insert Figure 4 about here]

Figure 4 traces out the evolution of the market yield curve over our sample period (1989-2016) following Gürkaynak et al. (2007). During this period, the average nominal yield curve fell by 70 percent. This mechanically increases the value of future retirement benefits.

Unsurprisingly, the choice of discount rate matters substantially to the valuation of aggregate Social Security wealth. In its own estimates, the SSA discounts cash flows based on its own projection of future interest rates, informed by historical yield curve movements in prior business cycles. Over our sample period, the SSA projected nominal yield curve fell by an average of 2 percentage points. Over the same period, the market yield curve fell by three times that amount. Our analysis thus calls into question the soundness of the SSA assumptions: interest rates fell by more historically and are projected to be lower for longer than the SSA suggests. This results in a mis-estimation of the stock of Social Security wealth.

#### 2.2.3 Demographic changes

There is a direct link between the level of Social Security wealth and the age distribution. Social Security wealth peaks around retirement, when individuals have paid in maximally to the program, yet accrued benefits have yet to be disbursed.

Figure 5 shows the age distribution at the beginning and end of our analysis period (1989-2016). The share of the population near retirement age and for whom Social Security wealth is at its peak (those between the ages of 50-70) has increased by over 75 percent.

[Insert Figure 5 about here]

In the coming years, the age pyramid will shift further: the share of the US population over the age of 65 has risen from 12.5% to 16.9%, and it is projected to grow to 22% by 2050, as the Baby Boomer generation moves to claim Social Security payments. Barring other contemporaneous

[9] Practically, this means that Social Security benefits, but not the tax payments paid in to fund Social Security, are exposed to aggregate labor market risk, as discussed in Section 6.

changes, population aging will decrease the value of Social Security wealth, since a growing share of accrued benefits will be paid out to new retirees. As a result of this demographic shift, Social Security will be the primary source of income for an even larger swath of the population.

# 3 Data

## 3.1 Survey of Consumer Finances

We use the Survey of Consumer Finances (SCF) to estimate the share of Social Security wealth held by each centile of the marketable wealth distribution. The SCF samples households every three years and makes a public dataset available for researchers.

The ideal dataset for this exercise would provide insight into not only marketable wealth, but also on individual's historical earnings and Social Security income. To the best of our knowledge, such a dataset does not exist. Unlike alternatives that provide comprehensive data on wealth or income, the SCF provides information on the wealth distribution for all ages and disaggregated data on Social Security benefits for retirees, from which we can extrapolate to estimate both the stock and distribution of Social Security wealth across cohorts.

Further, the SCF gives the age or year of retirement for each participant. This is useful in that it allows us to compute full-retirement benefits by taking into account benefit discounts and credits for early and late retirement, respectively. It is worth noting that the baseline SCF estimates exclude certain categories of wealth, for example defined benefit pension wealth, which is reported in the Federal Reserve's Distributional National Accounts.[10].

Finally, we add data from the Forbes 400 to the top 0.01% of the wealth distribution in the SCF. The SCF does not survey extremely high net worth individuals per agreement with the U.S. Treasury and, as such, their wealth is excluded when only considering raw SCF net worth. We obtain this data from the replication code of Saez and Zucman (2016) and supplement with 2016 data from Forbes.

[10]Consequently, the constituents of top wealth in our SCF estimates differ somewhat from past work that relies on different data sources. We attempt to incorporate our Social Security wealth estimates into wealth concentration measures based on alternative estimates in Figure 16.

## 3.2 Other sources

**Mortality** Data on mortality come from the Human Mortality Database (HMD) operated by the University of California, Berkeley and the Max Planck Institute for Demographic Research. The HMD provides data on life expectancy and conditional survival probabilitiesby gender from 1933-2017.

**Yield curve** We use yield curve data from the Federal Reserve Board of Governors who broadly follow the methods of Gürkaynak et al. (2007). These data provides an estimate of the zero-coupon yield curve using off-the-run Treasury coupon securities for horizons up to 30 years. This series indicates the rate of return investors require to hold government debt and is often thought of as the nominal risk-free rate of return. Since we need interest rates at horizons greater than 30 years, we extend this series beyond 30 years by repeatedly applying the 29-to-30 year forward rate to the annualized spot rate at 30 years[11], under the assumption that this forward rate represents the long-run interest rate on nominal government claims. In addition, we use data from Gürkaynak et al. (2008) on the implied real yield curve from Treasury Inflation Protected Securities (TIPS) to test the sensitivity of our results to using alternative discount rate assumptions.

**Social Security reports** We also use inflation, wage, and discount rate projections from the SSA. These data are gathered manually from annual reports. We use the wage growth and inflation data in our simulation and capitalization exercises, and the yield curve data for validation.

[11]For example, the annualized spot rate at $30+h$ is given by $r_{t,t+30+h} = \left((r_{t+29,t+30})^{h}(r_{t,t+30})^{30}\right)^{\frac{1}{30+h}}$.

# 4 Methodology

Our exercise in this paper is to trace out how the inclusion of Social Security wealth impacts trends in wealth concentration in the last three decades. Specifically, we construct estimates of Social Security wealth, and distribute this wealth between the top decile of marketable wealth and the rest of the population.

## 4.1 Social Security wealth of retired workers

For retired workers, the computation of Social Security wealth is relatively straightforward. For each SCF survey year and each retiree, we observe both the stock of marketable wealth and Social Security benefits. We define Social Security wealth as its present value, which we obtain by projecting expected benefits and discounting them using the nominal market yield curve. Specifically, Social Security wealth is:

$$\text{Social Security Wealth}_{it} = \sum_{s=t}^{T} \left( \prod_{k=t}^{s-1} (1 - m_{itk}) \right) \frac{\text{Benefits}_{it}}{(1 + r_{t,s})^{s-t}} \frac{\mathbb{E}[\text{CPI}_s]}{\text{CPI}_t} \tag{4.1}$$

where benefits are observed in the data, $r_{t,s}$ is the prevailing nominal rate of return at time $t$ for a risk-free zero coupon paid in year $s$, $m_{itk}$ is the expected mortality rate of the agent in year $k$, which depends on his sex and life table from current year $t$, and $\frac{\mathbb{E}[\text{CPI}_s]}{\text{CPI}_t}$ is the expected change in the consumer price index.

We also include survivor benefits in this calculation. Survivor benefits are paid to the surviving spouse and can represent up to 100% of the benefits of the deceased husband or wife. Actual survivor benefits are added to the benefits of the surviving spouse up to a family maximum which depends on the benefits of the deceased. Survivor benefits represent only 16% of old-age benefits and are determined by a relatively complicated formula. We provide more details on the computation of their present value in Appendix A.5.

## 4.2 Social Security wealth of non-retired workers

For workers still in the labor force, estimating Social Security wealth is more complicated. Social Security wealth depends on yet unrealized earnings trajectories, which we simulate in two steps.

First, we use previous empirical work based on SSA administrative panel data by Guvenen et al. (2019a) and Guvenen et al. (2019b) to simulate past and future earnings. Specifically, we borrow the labor income process estimated in Guvenen et al. (2019a) because it matches many moments of the cross section and individual labor market earnings dynamics. We also use summary statistics from Guvenen et al. (2019b) to calibrate the average lifetime earnings trajectory of each cohort×gender. This procedure allows us to simulate granular panel data that looks like the actual historical and expected future earnings trajectories. Second, we apply the Social Security benefit and tax formulas to construct estimates of future benefits[12] that households will accrue, net of the taxes that they will pay. Both steps are detailed in the sections that follow.

### 4.2.1 Simulating past and future earnings trajectories

We use essentially the same flexible labor income process as the one estimated in Guvenen et al. (2019a). Specifically, we assume that worker's $L_{it}$ earnings are the product of the national wage index $L_{1,t}$ and a idiosyncratic component $L_{2,it}$:

$$L_{it} = L_{1,t} \cdot L_{2,it}. \tag{4.2}$$

The idiosyncratic component $L_{2,it}$ evolves as follows:

[12] Note that we abstract from modeling spousal benefits in the simulation which would require us to take a stand on the household formation process. However, benefits paid to spouses make up a small proportion of benefits according to the SSA, roughly 3.8%. Including these would likely have little quantitative effect on our results.

$$\text{Level of idiosyncratic earnings:} \quad L_{2,it} = (1-\nu_t^i)e^{\left(g(t)+\alpha^i+\beta^i t+z_t^i+\varepsilon_t^i\right)} \tag{4.2.1}$$

$$\text{Persistent component:} \quad z_t^i = \rho z_{t-1}^i + \eta_t^i \tag{4.2.2}$$

$$\text{Innovations to AR(1):} \quad \eta_t^i \sim \begin{cases} \mathcal{N}(\mu_{\eta,1}, \sigma_{\eta,1}^2) & \text{with prob. } p_z \\ \mathcal{N}(\mu_{\eta,2}, \sigma_{\eta,2}^2) & \text{with prob. } 1-p_z \end{cases} \tag{4.2.3}$$

$$\text{Initial condition of } z_t^i\text{:} \quad z_0^i \sim \mathcal{N}(0, \sigma_{z,0}^2) \tag{4.2.4}$$

$$\text{Transitory shock:} \quad \varepsilon_t^i \sim \begin{cases} \mathcal{N}(\mu_{\varepsilon,1}, \sigma_{\varepsilon,1}^2) & \text{with prob. } p_\varepsilon \\ \mathcal{N}(\mu_{\varepsilon,2}, \sigma_{\varepsilon,2}^2) & \text{with prob. } 1-p_\varepsilon \end{cases} \tag{4.2.5}$$

$$\text{Nonemployment duration:} \quad \nu_t^i \sim \begin{cases} 0 & \text{with prob. } 1-p_\nu(t, z_t^i) \\ \min\{1, \exp\{\lambda\}\} & \text{with prob. } p_\nu(t, z_t^i) \end{cases} \tag{4.2.6}$$

$$\text{Prob. of Nonemp. shock:} \quad p_\nu^i(t, z_t) = \frac{e^{\xi_t^i}}{1+e^{\xi_t^i}}, \text{ where } \xi_t^i = a + bt + cz_t^i + dz_t^i i \tag{4.2.7}$$

The persistent component of earnings $z_i$ follows an AR(1) process with innovations drawn from a mixture of Normal Distributions. Transitory shocks $\varepsilon_i$ are also drawn from a normal mixture and fully mean revert within the year. Workers can also experience a non-employment shock with some probability $p$ that can vary with any mixture of age, income, and sex, and whose duration is exponentially distributed.

In equation 4.2.1, $g(t)$ is a quadratic polynomial of age that captures the life-cycle profile of earnings common to all workers. The vector $(\alpha_i, \beta_i)$ determines heterogeneity in the level and growth rate of earnings and is drawn from a multivariate normal distribution with zero mean and correlation coefficient $\text{corr}_{\alpha\beta}$. Heterogeneity in initial conditions of the persistent process is captured by $z_0$. The final component of the earnings process is the nonemployment shock (equation

4.2.6), which is realized with probability $p_\nu$ in each period. The duration $\nu_t$ reflects the duration of full-year nonemployment (zero annual income).

### 4.2.2 Future Social Security cash flows

**Taxes** Contributions depend on workers' earnings below the Social Security wage base ($\mathrm{SSWB}_t$) and the level of the tax rate. Specifically, future Social Security contributions of worker $i$ will be:

$$\mathrm{Taxes}_{it} = \text{Tax Rate} \times \min\left\{L_{it}, \mathrm{SSWB}_t\right\} \tag{4.3}$$

**Benefits** For simplicity, we assume that workers all retire at full-retirement age of 66. Yearly benefits at age 66 depend on the workers' average indexed yearly earnings. Only the share of earnings falling below the Social Security wage base of their year are taken into account. Hence, the indexed earnings of year $t$ are:

$$\text{Indexed Earnings}_{it} = \min\left\{L_{it}, \mathrm{SSWB}_t\right\} \frac{L_{1,c+60}}{L_{1,t}} \tag{4.4}$$

where $\frac{L_{1,c+60}}{L_{1,t}}$ is the indexation coefficient. The AIYE is the average of the best 35 years of indexed earnings up to retirement. If we denote $c_i$ the birth year of worker $i$, the benefits paid in his first retirement year are depends on the value of the AIYE relative to the Social Security bends points when he turns 60: $\text{Bend}_{1,c_i+60}$ and $\text{Bend}_{2,c_i+60}$. Specifically, benefits paid if year $t \geq c_i + 66$ are:

$$
\begin{cases}
\text{– if } \text{AIYE}_i < \text{BEND}_{1,c_i+60}: \\
\quad \text{Benefits}_{it} = \dfrac{\text{CPI}_t}{\text{CPI}_{c_i+60}} \times 0.9 \times \text{AIYE}_i, \\
\\
\text{– else, if } \text{Bend}_{1,c_i+60} \leq \text{AIYE}_i < \text{Bend}_{2,c_i+60}: \\
\quad \text{Benefits}_{it} = \dfrac{\text{CPI}_t}{\text{CPI}_{c_i+60}} \Big[ 0.9 \times \text{Bend}_{1,c_i+60} + 0.32 \times (\text{AIYE}_i - \text{Bend}_{1,c_i+60}) \Big], \\
\\
\text{– otherwise}: \\
\quad \text{Benefits}_{it} = \dfrac{\text{CPI}_t}{\text{CPI}_{c_i+60}} \Big[ 0.9 \times \text{Bend}_{1,c_i+60} + 0.32 \times (\text{Bend}_{2,c_i+60} - \text{Bend}_{1,c_i+60}) \\
\qquad\qquad + 0.15 \times (\text{AIYE}_i - \text{Bend}_{2,c_i+60}) \Big],
\end{cases}
\tag{4.5}
$$

where $\frac{\text{CPI}_t}{\text{CPI}_{c_i+60}}$ is an adjustment for the increase in the cost-of-living index since the retiree turned 60.

In practice, workers can start receiving benefits as early as age 62 or as late as age 70. However, this option is relatively fairly priced as retiring earlier (later) reduces (increases) benefits in a proportion consistent with life expectancy at retirement, such that overall the total present value of benefits remains the same (Auerbach et al., 2017).

### 4.2.3 Present value

Social Security wealth is defined as the present value of future benefits net of future taxes, including cash flows in the present year. Hence, the Social Security wealth of a worker from cohort $c$ in year $t$ is:

$$
\begin{aligned}
\text{Social Security Wealth}_{it} = & \sum_{s=c+66}^{T} \left( \prod_{k=t}^{s-1} (1 - m_{itk}) \right) \frac{\mathbb{E}\left[\text{Benefits}_{it}\right]}{(1 + r_{ts})^{s-t}} \\
& - \sum_{s=t+1}^{c+65} \left( \prod_{k=t}^{s-1} (1 - m_{itk}) \right) \frac{\mathbb{E}\left[\text{Taxes}_{it}\right]}{(1 + r_{ts})^{s-t}}
\end{aligned}
\tag{4.6}
$$

where $T$ is the maximum age and expectation terms take mortality into account.

## 4.3 Calibration

**Income process** We calibrate the labor income process described in equations (4.2.1) to (4.2.7) using estimates from Guvenen et al. (2019a) listed in Appendix Table C.1. These parameters are estimated by SMM and match a large variety of moments from the SSA administrative data, both in terms of the cross-section of earnings and its evolution over the life-cycle, and also moments relative to the dynamics of labor income. One important caveat is that this process is estimated only for males, though Guvenen et al. (2019a) note that their "analysis for women found qualitatively similar patterns."

Because we are interested in allocating Social Security entitlements by wealth decile overall, across both sexes, we calibrate $g(t)$ to follow a quadratic form that reflects how the earnings profiles of men and women evolve for each cohort in our sample.

To estimate these $g(t)$, we rely on data in Guvenen et al. (2019b), who report detailed earnings statistics for age and year separately for men and women. Specifically, for each cohort $c$ and gender $g$, we estimate life cycle earnings profile by running the following OLS regression:

$$\hat{g}_{cg}(t) = \ln\left(\frac{\text{Cohort Earnings}_{cgt}}{L_{1,t}}\right) = \alpha_{cg} + \beta_{cg,1} \times \text{Age}_{ct} + \beta_{cg,2} \times \text{Age}_{ct}^2 + \beta_{cg,3} \times \text{Age}_{ct}^3 + \epsilon_{gct} \quad (4.7)$$

based on average earnings in a year (Cohort Earnings$_{cgt}$) and average economy-wide earnings in that same year ($L_{1,t}$) for each cohort, by gender. Guvenen et al. (2019b)'s data is for individual labor income histories from 1957 to 2013. Our data includes workers who enter the labor force from 1949-2016, so it is broader than their sample. For cohorts where there is insufficient labor market data to estimate $g(t)$ directly, we rely on estimates for nearby cohorts, whose earnings trajectories follow similar paths. In our simulation, we use the predicted values derived from equation (4.7) for each gender and cohort as our calibration for $g(t)$, from which we subtract half the variance generated by idiosyncratic shocks and heterogeneity in income profiles to adjust for Jensen's inequality.

**Social Security parameters** Over our sample period, Social Security parameters have scaled up nearly perfectly with the level of the wage index. We assume that this pattern will persist in the future. Hence, we assume that the Social Security wage base will remain 2.5 times the wage index ($\text{SSWB}_t = 2.5 \times L_{1,t}$), and the bend points of the benefits formula will remain .21 and 1.25 the wage index ($\text{Bend}_{1,t} = 0.21 \times L_{1,t}$ and $\text{Bend}_{2,t} = 1.25 \times L_{1,t}$). Part of our simulation covers historical years before the 1980's when the Social Security wage base was lower, so we use actual historical values of the Social Security wage base to simulate covered earnings preceding any SCF survey year. We assume that Social Security respectively covers 90% and 80% of the male and female populations. We estimate these coverage ratios by looking at the share of male and female above 70 years old that do not receive any benefit (Appendix Figure C.2).

**Macroeconomic assumptions** Note that since Social Security benefits are inflation-indexed, the real yield curve is the correct discount rate to use for people 62 and older. This point is made forcefully by Blocker and Vallenas (2019) who suggest that the Treasury Inflation Protected Securities (TIPS) term structure should be used to price Social Security. Unfortunately, TIPS data are only available from 1999 onward, so we elect against using the TIPS yield curve to preserve consistency in our methodology.

In our baseline specification, we use the nominal yield curve for Treasury notes adjusting for projected inflation from the SSA reports to place the series in real terms, as there is not a single dataset that provides long term inflation projections throughout our entire sample.[13]

Further, we elect not to use the SSA interest rate projections in our baseline estimates. Appendix Figure C.1 compares historical yield curves based on Treasury estimates to SSA projections. The differences are stark in both level and trend. SSA projected interest rates in 1989 were lower than the rates implied by the market yield curve; and by 2016 they were higher. These are not minor discrepancies: the absolute value of the difference between the two series averages around 200 basis points. Between 1989-2016, the SSA projected that the two-year interest rate fell by 400 basis points. The market yield curve implies that this decrease was around 700 basis points, with short-term rates near zero today. The reason for this difference is that the SSA projections are derived from historical interest rate trajectories for the last five economic cycles, rather than

[13]TIPS and inflation expectations data from the Fed are available from 1999-present, but not earlier. Breakeven inflation rates are also available from the late 1999s on, but nothing going back to 1989. Further, the Michigan Survey of Consumers has inflation expectations going back to 1974, but these only give insights into short-term inflation expectations. None of these alternate sources provide the time series of data required by our methodology.

current rates on outstanding Treasury notes.

Finally, we use labor income growth coming from the SSA reports to adjust expected benefits, as Social Security benefits grow with aggregate real wages.

### 4.4 Aggregate estimates of Social Security wealth

We combine the simulated and the SCF data by assigning simulated average Social Security wealth to SCF respondents by survey, age, and gender. We then create aggregate age cohort Social Security wealth by taking a weighted sum of these means. These aggregated totals are used to assign wealth to the top 10% and bottom 90% of the marketable wealth distribution in Section 4.6.

For respondents from 62 to 65, the simulated data and SCF overlap. For those whose benefits are reported in the SCF, we rely on these estimates. For individuals without explicit benefits, we fill in the average simulated Social Security wealth, adjusting for the share of the population that will never receive benefits. We then aggregate both the SCF and the simulation estimates to account for the fact that the SCF respondents not currently receiving benefits will receive benefits in the future.

Further, for SCF respondents between 66 and 69 there is the same issue, but no overlap of simulated data and survey data. For these individuals, we backfill[14] average benefits and wealth from the succeeding survey for respondents from 70 to 73 years of age, adjusting for inflation and the number of respondents that will not receive benefits.

### 4.5 Validation

Our simulation values individuals' Social Security claims based on estimation of both historical and future labor market earnings. The former (historical earnings) are unknown to us because of data limitations: the SCF provides snapshots of only current earnings in each of the years that the survey is conducted.[15] The latter (future earnings) are unknown broadly, because there is

[14]For 2016, we cannot backfill, so we fill in directly average benefits and wealth for 70-73 year olds within the same survey.

[15]While this data is unavailable in the SCF, other sources contain this information, like the the Health Retirement Study. But this is limited as well in coverage, studying only older Americans (Gustman et al., 1999).

uncertainty in future earnings – in growth and changes in earnings, and in employment spells. It is implausible for us to precisely estimate the labor market outcomes and thus Social Security claims for specific individuals. If in aggregate–across different deciles of wealth–our simulation estimates of the evolution of Social Security wealth and its distribution across different definitions of top wealth, this will provide confidence in the trends that we document.

**Matching observed benefits at retirement age** The first validation of our simulation comes from comparing our simulation of full retirement age benefits to observed retirement benefits in the SCF. In Figure 6, for each SCF survey we report mean Social Security benefits at full retirement age observed for pensioners between 62 and 67 years old.[16] We also use the model simulations to project the benefits of retirees of the same age in each year. The data are aligned nearly perfectly for male and female retirees in our sample in each year of the SCF.

[Insert Figure 6 about here]

**Matching SSA estimates of aggregate Social Security wealth** As reported in Figure 2, the SSA estimates the aggregate stock of Social Security wealth each year. It calculates the present value of benefits to current participants, net of the present value of payroll taxes. Our goal in this paper is not to replicate the SSA estimates of Social Security wealth, as we disagree with the SSA actuaries' assumptions regarding the level and slope of the yield curve. In our risk-free valuation, we discount cash flows using Treasury estimates of the off-the-run yield curve based on a large set of outstanding Treasury notes and bonds, reported daily Gürkaynak et al. (2007). However, if we chose to use the same discount rates, we should be able to match the SSA's estimates.

Figure 7 reflects the results of this validation exercise. The evolution of aggregate Social Security wealth reported by the SSA tracks our estimates, giving us confidence in our simulated estimate of workers' lifetime earnings histories, from which we derive their Social Security wealth. Note that in this exercise, we only report 85% of the SSA estimation because 15% of Social Security cash flows and revenues can be attributed to the disability insurance program, outside of the scope

[16] For those who retire before or after full retirement age, we use the Social Security rules to determine what their full retirement age benefits would have been.

of our study. [17]

[Insert Figure 7 about here]

For comparison, we also include our estimate of aggregate Social Security wealth discounting based on the market-implied yield curve. The deviations between discounting based on SSA projections and Treasury reported rates is fairly small in the first decade of our sample, but it grew substantially in the last 15 years. SSA-implied aggregate Social Security wealth was nearly $30 trillion in 2016 and nearly $40 trillion when using market rates.

**Using the real yield curve to validate inflation expectations** Finally, because we discount future cash flows using the nominal yield curve, our findings are sensitive to our projections for the consumer price index, which we take from SSA annual reports. To make sure that our results are not driven by these assumptions, we also conduct our valuation exercise by discounting future cash flows using real yield curve implied by TIPS prices and assuming no inflation. This exercise can only be done for the 1999-2016 period. As reported in Figure 7, this alternative methodology implies a faster increase in aggregate Social Security wealth than ours, and a greater value for 2016. Hence, we feel confident that our findings are not driven by incorrect inflation forecasts.

## 4.6 Assigning Social Security Wealth to the Top

Our goal is to understand how trends in inequality documented in prior work are impacted by the large and growing stock of Social Security wealth. To do so, we must determine the distribution of Social Security wealth. The appropriate approach depends on whether households have already claimed their retirement benefits, or if instead they are still in the labor force.

In the first case, we observe benefits and compute the Social Security wealth of retirees following the procedure described in Section 4.1. Hence, for this part of our sample, we can precisely estimate the share of Social Security wealth that is captured by each centile of the overall marketable wealth distribution.

[17] Note that, in principle, our estimates should not match the SSA's aggregate valuation perfectly because the later includes future workers as young as 15 whereas we restrict our analysis to independent adults. For teenagers, Social Security wealth was negative in 1989 because of high interest rates, but positive in 2016.

For households which are not retired, our simulation exercise only produces an estimate of aggregate Social Security wealth by gender, year and age. Therefore, we need to make assumptions regarding how this Social Security wealth is split between the top 10% and the bottom 90% of the overall marketable wealth distribution. We rely on two relationships readily observable in the SCF: first, the share of Social Security wealth that accrues to each centile of the retired population; and second, the position in the cohort marketable wealth distribution required to reach the top 10% of the overall distribution.

[Insert Figure 8 about here]

An example is illustrative of our approach. According to our simulation, in 2016, 60 year-olds had $1.1 trillion in Social Security wealth. As Panel C of Figure 8 shows, to be in the top 10% of the overall marketable wealth distribution, a 60-year old household only needs to be in the 80th percentile of the marketable wealth distribution within its cohort.[18]

Panel B shows that within the population of young retirees (65-75), those below the 80th percentile hold 75% of the Social Security wealth of this age group. We assume this relationship for retirees holds for 60 year-olds as well. As Panel C shows, we assign 75% of the Social Security wealth of 60 year-olds to those below the 80th percentile, and 25% to those above. Since those above the 80th percentile in the 60 year-old cohort are in the top 10% of the overall population, this means that 25% of 60 year-olds' Social Security wealth should be assigned to the top 10%. Hence, we allocate $275 billion ($1.1 trillion x .25) of Social Security wealth to the top 10%. By repeating this exercise for all cohorts, we determine the overall amount of simulated Social Security wealth owned by the top 10% and bottom 90% in 2016. We use the same procedure for other survey years.

In this exercise, our key assumption is that the share of Social Security wealth that accrues to different centiles of the marketable wealth distribution is constant across ages. This is how we use the relationship that we observe in the SCF for retirees (Panel A) to allocate Social Security wealth for cohorts still in the labor force.

[18] By way of contrast, for a 20-year old to be in the top 10% of the overall distribution, he needs to be in the 99th percentile of his cohort. The mechanical relationship between age and wealth accumulation suggests the importance of intra-cohort estimates of inequality.

To be sure, the relationship between marketable and Social Security wealth is likely not constant across ages. However, there are several reasons why assuming the reverse is sensible for our exercise. First, among current workers, Social Security wealth is concentrated in those near-retirement, who are nearly through paying into Social Security and have yet to claim their benefits (Figure 10). For these cohorts, relying on the relationship between marketable wealth and Social Security wealth observed for retirees only a few years older is very reasonable. Second, for younger cohorts, where our assumption is most tenuous, the consequences are quantitatively irrelevant. It is true that we are limited in how well we are able to assign Social Security wealth across centiles of the marketable wealth distribution for younger workers. But because their chances of being in the top 10% of the overall population are negligible, this is inconequential to our understanding of top wealth shares.

If anything, our assumption is conservative and overstates the share of Social Security wealth that accrues to the top 10%. This is because the value of Social Security is low and perhaps even negative for the wealthiest individuals in younger cohorts. Social Security is progressive, and so it offers higher replacement rates to low earners. Though high earners who recently retired have more Social Security wealth than low earners, each dollar has been bought at a higher price. At retirement, this price is sunk and does not change their Social Security wealth. However, for younger cohorts, a large fraction of this cost remains to be paid, which reduces the net present value of Social Security disproportionately more for wealthy households.

## 5 Risk-free valuation

This section compares the level and trends of top wealth concentration under alternative specifications, both including and excluding Social Security wealth. We define top wealth shares based on the top 10% and top 1% of the population by measures of marketable wealth, using Saez and Zucman (2016)'s definition of wealth. This allows for comparison of how previously documented inequality trends are impacted by the inclusion of Social Security, a large and progressively distributed source of wealth.

## 5.1 Market yield curve valuation

### 5.1.1 The Level of Top Wealth

Figure 9 reflects our baseline specification, which studies the impact of Social Security wealth on trends in top wealth shares based on market discount rates.

Panel A focuses on the top 10%. The top 10% wealth share (excluding Social Security) grew by 10 percentage points between 1989-2016. This is in line with top wealth estimates from others: for example, Piketty et al. (2018) report a 9 percentage point increase in top wealth over this period.

Once Social Security wealth is considered, this trend is reversed. Rather than rising, the top 10% wealth share falls by 5.6 percentage points over this period. Given the progressive nature of Social Security and its recent growth, the impact of Social Security wealth on wealth concentration is not surprising. But our estimates demonstrate how conclusions on inequality trends that exclude the old age retirement program are incomplete and precipitate misconceptions about both level and trends in inequality.

[Insert Figure 9 about here]

Panel B of Figure 9 shows the impact of Social Security wealth on top 1% wealth shares. When Social Security wealth is excluded, the top 1% share has grown by ten percentage points over our sample period. Once it is included, the top 1% share remains roughly flat.

### 5.1.2 Distribution of Social Security wealth

Figure 10 reports the shares of total wealth separately for marketable and Social Security wealth for the bottom 90% and the top 10% by age group. Two facts are striking. First, the Social Security wealth of the bottom 90% has increased dramatically. In 1989, Social Security was only 17.2% of total bottom 90% wealth. By 2016, this share had nearly quadrupled. Second, Social Security is fairly evenly distributed across the wealth distribution (Figure 1). However, it is unevenly distributed across age groups. While it is less relevant for the top 10%, Social Security's share of wealth for the bottom 90% peaks for age groups those nearing retirement.

[Insert Figure 10 about here]

# 6 Risk-adjusted valuation

Overlapping-generation models tell us that the rate of return of pay-as-you go systems is the sum of the growth rates of the population and per capita earnings (Samuelson (1958)). For U.S. Social Security, the relationship between returns on contributions and the long-run growth in per capita earnings is explicitly achieved through wage-indexation. Therefore, Social Security participants are exposed to long-run macroeconomic risk. For this reason, Geanokoplos and Zeldes (2010) and Catherine (2019) argue that Social Security cash flows should not be discounted at the risk-free rate. These studies respectively find that, after adjusting for systematic risk, the market and private values of Social Security obligations is 19% and 37% lower than the sum of future cash flows discounted at the risk-free rate.

In this section, we try to determine what the market value of Social Security claims would be if they could be sold to diversified investors. To take systematic risk into account, we assume that future Social Security cash flows perfectly scale up with the level of per capita earnings in the economy, which seems consistent with the data. Indeed, over our sample period, the values of the Social Security wage base and bend points have been growing at the same rate as earnings. In section 4.2.2, we have shown that tax payments are proportional to the level of the wage index ($L_{1,t}$) whereas benefits are proportional to the value of the wage index the year a worker turns 60 ($L_{1,c+60}$). Because he does not care about idiosyncratic risk, a fully diversified investor would discount each tax payment and each benefit he will receive like a security paying a single coupon in the year that the cashflow is realized, which is also indexed on the value of $L_1$ in the same year. Therefore, we want to determine the expected return for such a security and use it to discount future Social Security cash flows.

## 6.1 Model

At what rate should we discount a cash flow that is proportional to the average level of earnings ($L_{1,t+n}$) in $n$ years? To answer this question, we follow Geanokoplos and Zeldes (2010) and Catherine (2019) by assuming that the stock and labor markets are cointegrated. Cointegration between dividends and earnings is documented in Benzoni et al. (2007) and would be expected in an economy where the shares of labor and profits are stable over long periods. Specifically, we assume that the log of $L_1$ evolves as follows:

$$dl_{1,t} = \left( (\phi - \kappa) y_t + \mu - \delta - \frac{\sigma_l^2}{2} \right) dt + v_1 dz_{1,t}, \tag{6.1}$$

where $\mu - \delta$ determines the unconditional log aggregate growth rate of earnings and $v_1$ its volatility. On the other hand, log stock market gains follow:

$$ds_t = \left( \mu + \phi y_t - \frac{\sigma_s^2}{2} \right) dt + \sigma_s dz_{2,t}, \tag{6.2}$$

where $\mu$ and $\sigma_s$ represent expected stock market log returns and their volatility. The state variable $y_t$ keeps track of whether the labor market performed better or worse than the stock market relative to expectations. Specifically, $y_t$ evolves as follows:

$$dy_t = -\kappa y_t + \sigma_l dz_{1,t} - \sigma_s dz_{2,t}, \tag{6.3}$$

where $\kappa$ determines the strength of the cointegration. If the two markets are cointegrated, $y_t$ should mean revert to zero. Mean reversion takes two forms. If stock markets gains are caused by higher long run economic growth, wages will catch up. If stock market returns have nothing to do with future economic growth, we should expect them to mean revert. The parameter $\phi$ controls the fraction of the mean reversion in $y_t$ caused my mean reversion in stock market returns.

In Appendix B, we show that the market beta of a security delivering a single coupon proportional to $L_{1,t+n} = e^{l_{1,t+n}}$ is:

$$\beta_t^{L_1,n} = \left( 1 - \frac{\phi}{\kappa} \right) \left( 1 - e^{-\kappa n} \right) \tag{6.4}$$

and we demonstrate that, under the no-arbitrage condition, the expected return on this security is:

$$\mathrm{E}_t \left[ r_t^{L_1,n} \right] = \beta_t^{L_1,n} (\mu - r) + r \tag{6.5}$$

where $r$ is the risk-free rate. Note that, assuming policy risk away, any Social Security payment proportional to $L_{t+n}$ would deliver the same expected return if it were publicly traded, as all other sources of risk are purely idiosyncratic.

Given the discrete nature of our exercise, we approximate our continuous time results in discrete time by assuming that the discount factor for a Social Security cash flow proportional to the wage index in year $n$ and paid in year $k > n$ is:

$$\text{Discount Factor}_{t,n,k} \approx \left[ \prod_{s=t}^{n} \left(1 + \beta_s^{L_1,n} (\mu - r) + r_{ts}\right) \prod_{s=n+1}^{k} (1 + r_{ts}) \right]^{-1}, \tag{6.6}$$

and the risk-adjusted present value of Social Security is:

$$\begin{aligned} \text{Adj. Social Security Wealth}_{it} = & \sum_{s=c_i+66}^{T} \left( \prod_{k=t}^{s-1} (1 - m_{itk}) \right) \mathbb{E}\left[\text{Benefits}_{it}\right] \times \text{Discount Factor}_{t,c_i+60,s} \\ & - \sum_{s=t+1}^{c+65} \left( \prod_{k=t}^{s-1} (1 - m_{itk}) \right) \mathbb{E}\left[\text{Taxes}_{it}\right] \times \text{Discount Factor}_{t,s,s} \end{aligned} \tag{6.7}$$

where real benefits are indexed on the level of $L_1$ in the year in which the worker turns 60.

We calibrate our model as in Benzoni et al. (2007) who estimate $\kappa$ and $\phi$ using US macroeconomic data from 1929 to 2004. Specifically, we set $\kappa = .16$ and $\phi = .08$ which, at the limit ($n = \infty$) implies a market beta of 0.5 for distant Social Security cash flows. We assume a constant equity premium of $\mu - r = 0.06$.

## 6.2 Aggregate Social Security wealth

Figure 11 reports aggregate Social Security wealth with and without adjusting for systematic labor market risk. In line with previous studies, we find that adjusting for systematic risk leads to a large reduction in the net present value of Social Security.

[Insert Figure 11 about here]

## 6.3 The level of top wealth

Regarding the evolution of wealth inequalities, Figure 12 now suggests that the share of top 10% decreased by 3.0 percentage points and that top 1% shares have increased by 1.2 percentage points. This finding differs from our baseline risk-free specification because Social Security wealth is smaller, and therefore plays a lesser role in the evolution of wealth inequality. Still, top wealth shares remain substantially attenuated relative to prior work that estimates, for example, an increase in top 1% shares of 10 percentage points (Piketty et al. (2018)).

[Insert Figure 12 about here]

## 6.4 Distribution of Social Security wealth

Figure 13 reports the shares of total wealth separately for marketable and Social Security wealth for the bottom 90% and the top 10% by age group. Although adjusting Social Security wealth for labor market risk decreases the value of the program, the changes relative to the beginning of the sample period remain stark: Social security grows from representing 14.2% of the wealth of the bottom 90% of the distribution to over 57.7% by 2016. As evident when comparing with Figure 10, the risk-adjusted results primarily decrease Social Security wealth for younger workers, who are only rarely in the top 10%. This is because the cointegration of labor market income and stock returns is a long-run relationship. By the time older workers are retired or nearing retirement, they are no longer exposed to systematic risk. Consequently, this adjustment decreases the wealth of the bottom 90%, with only a small impact on Social Security wealth of the top of the distribution.

[Insert Figure 13 about here]

# 7 Discussion

Our baseline specification demonstrates that recent increases in inequality are muted when accounting for the stock of and simultaneous growth in Social Security wealth. In this section, we show how differences in life expectancy across the marketable wealth distribution, demographic patterns, and the interest rate environment would impact our estimates. We also compare the trends we document to alternative specifications of top wealth: for example, those that account for heterogeneous returns within asset classes when mapping income flows from the SCF to wealth estimates.

## 7.1 Differences in life expectancy

Chetty et al. (2016) notes a relationship between life expectancy and where individuals fall on the income (or wealth) distribution. These differences are large: average life expectancy for men in the top 1% by income is nearly 10 years longer than average life expectancy for the bottom

10% (Chetty et al., 2014). Because those at the top of the lifetime income distribution live longer on average, their stream of cashflows is longer, and their total Social Security wealth will be understated by using cohort average life expectancy. Similarly, we will overstate the Social Security wealth of those at the bottom of the distribution, who life for less long on average.

Therefore, we adjust for these differences in life expectancy using data from the Health Inequality Project (HIP), by adjusting the survival probabilities of SCF respondents receiving Social Security retirement benefits using their centile of Social Security benefits. Social Security benefits, by construction, reflect the lifetime wage incomes of recipients and are therefore a reasonable counterpart to the lifetime earnings measures employed in Chetty et al. (2016) and Chetty et al. (2014). [19] In doing this we effectively make high income retirees younger and low income retirees older, a procedure which we outline in detail in Appendix Section A.3. We then perform the assignment procedure laid out in Section 4.6 to reassign wealth based on the adjusted life expectancies.

Figure 14 shows the distribution of Social Security wealth for the top 10% and bottom 90%, both with, and without, adjusting for differences in life expectancy. These differences increase the average stock of Social Security wealth that accrues to the top wealth decile by approximately 6.4% percent in 2016. Surprisingly, the Social Security wealth of the bottom 90% increases as well, by roughly 4.6% 2016. This is due to an increase in the benefit-weighted average life expectancy of beneficiaries in the bottom 90%. [20]

The effect of the life expectancy adjustment is to increase the Social Security wealth at both the top and bottom of the wealth distribution. Though the increase in aggregate Social Security wealth goes disproportionately to the wealthy, it remains, nonetheless, more equally distributed than marketable wealth. As such, Appendix Figure C.3 shows that top wealth shares are reduced slightly by this adjustment. Since the quantitative differences are small, we elect to exclude this adjustment from the baseline specification. Our conclusions are unchanged by its inclusion.

[19] Indeed, when ranking this way, we find that the gap between the top and bottom deciles of earners ranked by Social Security income increases by $86,000, which is in line with the $130,000 figure reported by Chetty et al. (2016)

[20] We also do this exercise using within-cohort wealth centiles and find that accounting for this channel is of little quantitative importance.

[Insert Figure 14 about here]

## 7.2 Aging of U.S. population

In the last three decades, the age pyramid has shifted. The share of the total population with peak Social Security wealth – those near retirement, who have paid into Social Security fully, but have yet to receive any benefits – has grown by nearly 50 percent.

Although Social Security is fairly equally distributed across the age distribution; it is concentrated in precisely those individuals who are near retirement age or newly retired. That is why for the bottom 90% (for whom Social Security is over half of total wealth by the end of their labor market years), people between 50 to 70 years of age hold more than 20% of the total wealth of this group (Figure 10, Panel A).[21]

As a counterfactual, we assume that the age pyramid had not shifted over the three decades we study. Instead, we arrive at an alternative age pyramid by taking the average distribution of ages across our sample period.

Table 1 shows the relative contribution of different factors to the observed increase in Social Security wealth from 1989 to 2016. Log aggregate Social Security wealth would have been 0.260 lower by 2016 discounting by the market implied yield curve, and this jumps to 0.308 after the risk adjustment is implemented.[22]

## 7.3 Interest rates

The change in the interest rate environment is the main contributor to the growth in Social Security wealth, as Table 1 shows. Under the market specification, the change in the interest

[21] As Figure 10 shows, wealth for the top 10% is also concentrated in these age groups; but this concentration is not driven by Social Security, which is relatively inconsequential for the top of the wealth distribution. Overall, this exercise illustrates the importance of considering within-cohort inequality measures. Aggregate trends are influenced by the age distribution.

[22] As a caveat, the order in which these effects are examined matters for their overall contribution. The change in the age distribution matters relatively more under a high rate regime than under a low one. For example, the relative importance falls of changes in the age distribution is lower when adjusting the yield curve first, under the market implied specification. However, under the risk adjusted specification, the ordering matters far less.

rate environment is responsible for a 1.093 in log aggregate Social Security wealth from 1989 to 2016. However, the importance of this channel is dampened under the risk adjusted specification, accounting for only 0.981 of the increase.

We perform a similar counterfactual exercise with respect to the market yield curve that is our preferred discount rate. As Figure 4 shows, the yield curve shifted significantly across all horizons in the last three decades. In Figure C.4, we consider how aggregate Social Security wealth would have looked if instead of following the actual evolution of interest rates over time, the yield curve had been constant, at its average level over our sample period.

If the yield curve was constant at the average of the 1989 to 2016 period; instead of changing over time, the increase in Social Security wealth over our sample period would have been much less stark: Rather than rising by more than seven times (in real terms), the stock of Social Security wealth would have roughly doubled. By 2016, the stock of Social Security wealth would have been more than \$11 trillion lower than our estimates suggest. Moreover, the aggregate value in 1989 would have been \$8.4 trillion higher in 2018 dollars, as the mean yield curve is significantly lower than the market implied yield curve in 1989.

As Figure 12 shows, relative to the benchmark models which exclude Social Security from wealth entirely, the end-of-sample top wealth shares would still be much lower than prior work suggests. This is because even \$28 trillion of progressively distributed Social Security wealth has a significant impact on inequality trends. The main difference between the market yield curve assumption; and fixing the yield curve at the sample average is that rather than the top 10% wealth share falling by around 5.6 percentage points over our sample period, it instead rises by 4 percentage points, as shown in Figure C.5. Even in this counterfactual world, the increase in top wealth shares would have been only half as steep as recent estimates that exclude Social Security entirely.

## 7.4 Social Security funding gap

What is the appropriate way to adjust Social Security wealth estimates for the trust fund's projected insolvency? Fortunately, the SSA provides benchmark estimates of the extent to which the fund's bankruptcy will impair obligations to future beneficiaries under three scenarios: low cost (alternative I), intermediate (alternative II), and high cost (alternative III) assumptions. Appendix Figure C.6 reports the proportion of payable benefits under the SSA's 2016 cost assumptions for each of these scenarios.

To understand the impact of insolvency risk on our inequality estimates, we collect annual data from the SSA on the year that the trust fund is projected to run out, as well as on the total revenue generated from Social Security payroll taxes, and the total obligations to beneficiaries. Once the Social Security fund is extinguished (estimated to be between 2030-2035), benefits paid in a year must be less than taxes collected going forward.

Even assuming maximal cuts to expected Social Security benefits (alternative III), the bottom 90% wealth share is attenuated only slightly, by 4.2 percentage points in 2016 relative to the baseline of fully paid benefits under the risk-adjusted valuation (Figure 15). This limited impact has two dimensions. First, for early cohorts, the impact of the fund's extinguishment is zero. Second, even for cohorts impacted, 60% of expected Social Security benefits represent a sizable sum relative to their marketable wealth. Progressive tax reforms to ensure benefits are not impaired may make this alternative unlikely. But even assuming policy risk is realized, future impairment of Social Security benefits does not detract substantially from the importance this stock of wealth has for all but the very top of the wealth distribution.

[Insert Figure 15 about here]

## 7.5 Adjusting previous studies on wealth inequality

Figure 16 illustrates the impact of Social Security wealth on top wealth share estimates. We begin in Panel A by presenting four existing estimates of the evolution of the top 1% share: from the Survey of Consumer Finances; Saez and Zucman (2016) which capitalizes income tax returns using a constant rate of return within an asset class; Smith et al. (2019) which adjusts the Saez and Zucman (2016) estimates to account for heterogeneous returns; and Batty et al. (2019) which relies on data from the Distributional Financial Accounts series. There are significant discrepancies between existing top wealth share estimates (Kopczuk, 2015); however, they all show an upward trend for the top 1% share over our sample period.

[Insert Figure 16 about here]

Once Social Security wealth is included (Panel B), this trend disappears.[23] Given Social Security's progressivity, it is unsurprising that its inclusion has the effect of scaling down the top 1% estimates. Without Social Security, top 1% wealth shares range from 25% to 45% of total wealth. With Social Security included, this share drops from 20% to 30%. What is even more striking than the level effect is the impact on inequality trends: depending on the series, there remains either only a minimal increase in top 1% shares over our sample period, or even a decrease.

We can also examine the impact of the inclusion of Social Security wealth on the evolution of alternative definitions of top wealth. Figure 17 shows how top 1%, 0.1%, and top 0.01% wealth shares evolve under Smith et al. (2019)'s assumption of heterogeneous returns with asset classes. When Social Security is excluded, top 1% shares have risen from 24.5% to 28.2% since 1989 to 2013, top 0.1% shares have risen from 9.7% to 13.2%, and top .01% shares have risen from 3.7% to 6.3%. When Social Security is included, top wealth shares decline under each definition. The difference in inequality trends is striking: there is little difference in top wealth estimates with and without Social Security in 1989, but these have expanded significantly over the last three decades, such that the gap between the series is large and growing. The trends in wealth shares from 1989 to 2016 for the top 1%, 0.1%, and 0.01% are reduced by 5.5%, 2.8%, and 1.4%, respectively.

[Insert Figure 17 about here]

# 8 Conclusion

Prior studies find large recent increases in U.S. wealth inequality based on measures of wealth concentration that exclude Social Security. This paper builds on past work by incorporating Social Security into inequality estimates. We find that wealth inequality has not increased once the old age retirement program is accounted for.

We document that Social Security wealth has risen: In 1989, Social Security represents 17.2% of the wealth held by the bottom 90% of the wealth distribution. By 2016, this share had grown to 63.4%. Even after adjusting for systematic risk, Social Security rose from only 14.2% of the total wealth of the bottom 90%to 57.7%.

[23]We choose for this illustration our risk-adjusted Social Security estimates, an adjustment for long-run labor market risk that decreases the size of the old-age fund.

This rise is contemporaneous with an increase in marketable wealth inequality. However, the life-cycle model and much empirical evidence suggest that Social Security and private wealth are substitutes (Feldstein (1974)). As such, a narrow definition of wealth paints an incomplete picture of inequality trends. Our most conservative estimates suggest that between 1989 and 2016 the top 10% share declined by 3.0 percentage points and the top 1% share increased only slightly by 1.2 percentage points. This differs drastically from recent work that excludes Social Security and finds the top 10% and 1% shares rose by over 10 percentage points over this period.

The inequality estimates in this paper are still overstated, because we exclude programs like disability insurance and Medicare, which constitute a larger share of the wealth of the bottom of the distribution than the top. Overall, this paper makes the point that public transfer programs like Social Security make the U.S. economy more progressive, and it is important for inequality estimates to reflect this. Much more work is needed to arrive at a fuller understanding of wealth concentration in America.

# References


**Attanasio, Orazio P. and Agar Brugiavini**, "Social Security and Households' Saving*," *The Quarterly Journal of Economics*, 08 2003, *118* (3), 1075–1119.

_ **and Susann Rohwedder**, "Pension Wealth and Household Saving: Evidence from Pension Reforms in the United Kingdom," *American Economic Review*, December 2003, *93* (5), 1499–1521.

**Auerbach, Alan J, Kerwin K Charles, Courtney C Coile, William Gale, Dana Goldman, Los Angeles, Ronald Lee, Charles M Lucas, Peter R Orszag, Louise M Sheiner, Bryan Tysinger, David N Weil, Justin Wolfers, and Rebeca Wong**, "How the Growing Gap in Life Expectancy May Affect Retirement Benefits and Reforms," *The Geneva Papers on Risk and Insurance - Issues and Practice*, 2017, *42* (3), 475–499.

**Auten, Gerald and David Splinter**, "Income Inequality in the United States: Using Tax Data to Measure Long-term Trends," 2019.

**Batty, Michael, Jesse Bricker, Joseph Briggs, Elizabeth Holmquist, Susan McIntosh, Kevin Moore, Eric Nielsen, Sarah Reber, Molly Shatto, Kamila Sommer, Tom Sweeney, and Alice Henriques Volz**, "Introducing the Distributional Financial Accounts of the United States," *Federal Reserve Board Finance and Economics Discussion Series*, 2019, (005).

**Benzoni, Luca, Pierre Collin-Dufresne, and Robert S. Godlstein**, "Portfolio Choice over the Life-Cycle when the Stock and Labor Markets Are Cointegrated," *Journal of Finance*, October 2007, *62* (5), 2123–2167.

**Biggs, Andrew**, "Factcheck: Do 40% Of Retirees Rely On Social Security For Their Entire Income?," *Forbes Online*, 2020.

**Catherine, Sylvain**, "Labor Market Risk and the Private Value of Social Security," *Working paper*, 2019.

**Chetty, Raj, John N Friedman, Soren Leth-Peterson, Torben Heien Nielsen, and Tore Olsen**, "Active vs Passive Decisions and Crowd-out in Retirement Savings Accounts: Evidence from Denmark Faculty Research," *Quarterly Journal of Economics*, 2014, *121* (4), 1311–1346.

**_ , Michael Stepner, Sarah Abraham, Shelby Lin, Benjamin Scuderi, Nicholas Turner, Augustin Bergeron, and David Cutler**, "The association between income and life expectancy in the United States, 2001-2014," *JAMA - Journal of the American Medical Association*, 2016, *315* (16), 1750–1766.

**Choukhmane, Taha**, "Default Options and Retirement Saving Dynamics," 2018.

**Domeij, David and Paul Klein**, "Private Pensions: To What Extent Do They Account for Swedish Wealth Inequality?," *Review of Economic Dynamics*, 2002, *5* (3), 503–534.

**Feldstein, Martin**, "Social Security, Induced Retirement, and Aggregate Capital Accumulation," *Journal of Political Economy*, 1974, *82* (5), 905–926.

_ , "The Effect of Social Security on Saving," *NBER Working Paper Series*, 1979, *No. 334*, 1–29.

**_ and Jeffrey B. Liebman**, "The Distributional Aspects of an Investment-Based Social Security System," in "The Distributional Aspects of Social Security and Social Security Reform" 2002.

**Feldstein, Martin S**, "Does the United States Save too Little?," *American Economic Review*, February 1977, *67* (1), 116–121.

**Geanakoplos, John, Olivia S. Mitchell, and Stephen P. Zeldes**, "Social Security Money's Worth," in Olivia S. Mitchell, Robert J. Meyers, and Howard Young, eds., *Prospects for Social Security Reform*, University of Pennsylvania Press, 1999, chapter 5, pp. 80–151.

**Geanokoplos, John and Stephen P. Zeldes**, "Market Valuation of Accrued Social Security Benefits," 2010, pp. 213–233.

**Gürkaynak, Refet, Brian Sack, and Jonathan Wright**, "The U.S. Treasury Yield Curve: 1961 to the Present," *Journal of Monetary Economics*, 2007, *54* (8), 2291–2304.

**_ , _ , and _** , "The TIPS Yield Curve and Inflation Compensation," *Finance and Economics Discussion Series*, 2008, *2008* (5).

**Gustman, Alan L. and Thomas L. Steinmeier**, "Retirement in DualCareer Families: A Structural Model," *Journal of Labor Economics*, 2000, *18* (3), 503–545.

**_ and _** , "How Effective is Redistribution Under the Social Security Benefit Formula?," *Journal of Public Economics*, 2001, *82.*

**Gustman, Alan L, Olivia S Mitchell, Andrew A Samwick, and Thomas L Steinmeier**, "Pension and Social Security Wealth in the Health and Retirement Study," *Wealth, Work, and Health: Innovations in Survey Measurement in the Social Sciences*, 1999, pp. 150–208.

**Gustman, Alan L., Thomas L. Steinmeier, and Nahid Tabatabai**, "The Growth in Social Security Benefits Among the Retirement-Age Population from Increases in the Cap on Covered Earnings," *Social Security Bulletin*, 2012, *72* (2), 49–61.

**Guvenen, Fatih, Fatih Karahan, Serdar Ozkan, and Jae Song**, "What Do Data on Millions of U.S. Workers Reveal About Life-Cycle Earnings Risk?," *SSRN Electronic Journal*, 2019.

**_ , Greg Kaplan, Jae Song, and Justin Weidner**, "Lifetime Incomes in the United States Over Six Decades," *NBER Working Paper Series*, 2019, (23371), 76.

**Kopczuk, Wojciech**, "What do we know about the evolution of top wealth shares in the United States?," *Journal of Economic Perspectives*, 2015, *29* (1), 47–66.

**Liebman, Jeffrey B.**, "Redistribution in the Current U.S. Social Security System," in Martin Feldstein and Jeffrey B. Liebman, eds., *The Distributional Aspects of Social Security and Social Security Reform*, number January 2002, pp. 11–41.

**Perkins, Frances**, "The Roots of Social Security," 10 1962. Speech delivered at Social Security Administration Headquarters, Baltimore, Maryland, October 23, 1962.

**Piketty, Thomas**, *Capital in the 21st Century*, Cambridge: Harvard University Press, 2013.

_ , **Emmanuel Saez, and Gabriel Zucman**, "Distributional National Accounts: Methods and Estimates for the United States," 2018.

**Poterba, James M.**, "Retirement Security in an Aging Population," *American Economic Review*, May 2014, *104* (5), 1–30.

**Saez, Emmanuel and Gabriel Zucman**, " Wealth Inequality in the United States since 1913: Evidence from Capitalized Income Tax Data *," *The Quarterly Journal of Economics*, 02 2016, *131* (2), 519–578.

**Samuelson, Paul A.**, "An Exact Consumption-Loan Model of Interest with or without the Social Contrivance of Money," *Journal of Political Economy*, 1958, *66*, 467–467.

**Scholz, John Karl, Ananth Seshadri, and Surachai Khitatrakun**, "Are Americans Saving Optimally for Retirement?," *Journal of Political Economy*, 2006, *114* (4), 607–643.

**Smith, Matthew, Owen Zidar, and Eric Zwick**, "Top Wealth in the United States: New Estimates and Implications for Taxing the Rich," 2019.

**W., Kotlikoff Laurence J. Ross Stephen A. Blocker Alexander and Sergio Villar Vallenas**, "The True Cost of Social Security," *Tax Policy and the Economy*, 2019, *33.*

**Weil, David N**, "Capital and Wealth in the 21st Century," *The American Economic Review: Papers and Proceedings*, 2015, *105.*

**Wolff, Edward N.**, "Changing Inequality of Wealth," *American Economic Review*, 1992, *82* (2), 552–558.

_ , "International comparisons of wealth inequality," *Review of Income and Wealth*, 1996, *42* (4), 433–451.

**Zucman, Gabriel**, "Global Wealth Inequality," *Annual Review of Economics*, 2019, *11* (1), 109–138.

**Figure 1: Distribution of Social Security Benefits and Capital Income for around retirement**

This figure shows the distribution of Social Security retirement benefits and capital income in the SCF for respondents between 62 and 76 years of age in 2016. Social Security benefits are adjusted such that they reflect retirement benefits at the full retirement age for a given survey-year. Capital income is total income reported in the SCF less wage income, Social Security income, and government transfers. The horizontal axis is the non-Social Security wealth decile computed based on the SCF population aged 62 to 76.

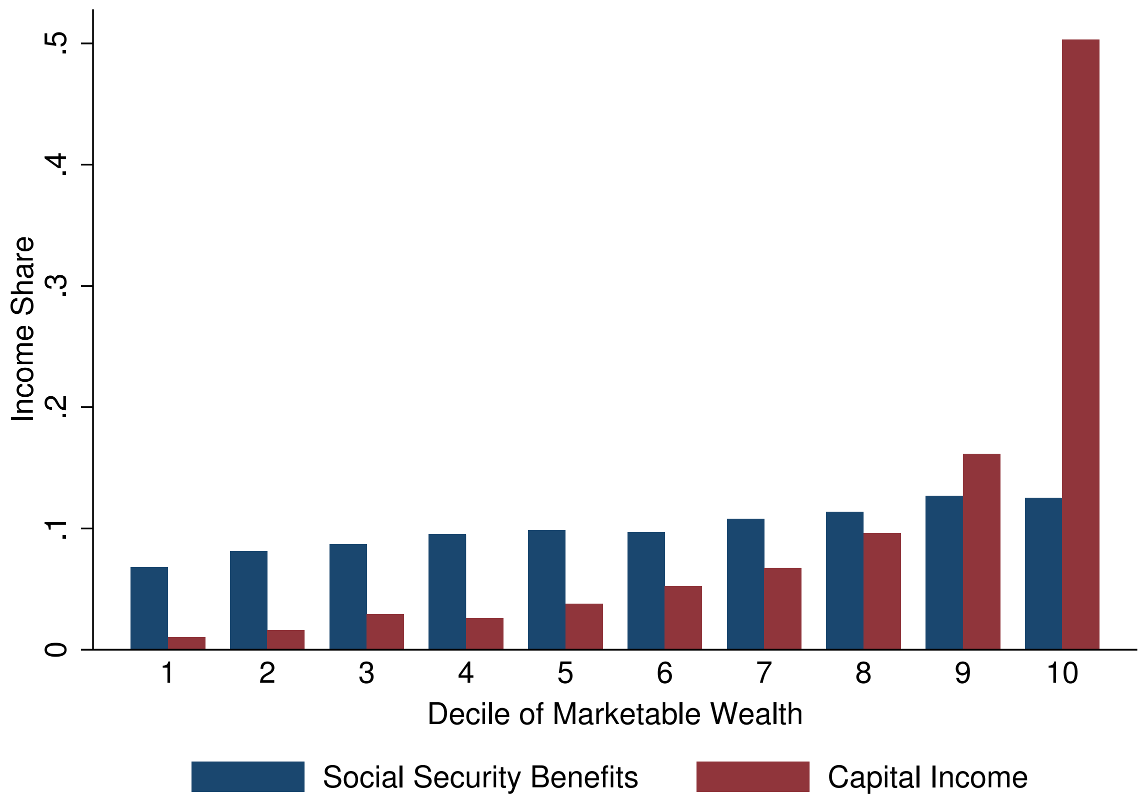


**Figure 2: Value of accrued benefits based on SSA estimates**

This figure represents the value of accrued benefits for the Old-Age, Survivor and Disability Insurance programs based on actuarial estimates from the Social Security Administration. We define accrued benefits as the sum of the "closed-group transition cost" and the value of the Social Security Trust Fund. The "closed-group transition cost" refers to the present value of expected future benefits to current Social Security participants minus the present values of their future expected payroll tax payments, minus the value of the Trust fund. SSA actuaries estimate the "closed-group transition cost" every year using a 100-year projection. Details are available in SSA's *Actuarial Note #2019.1.*

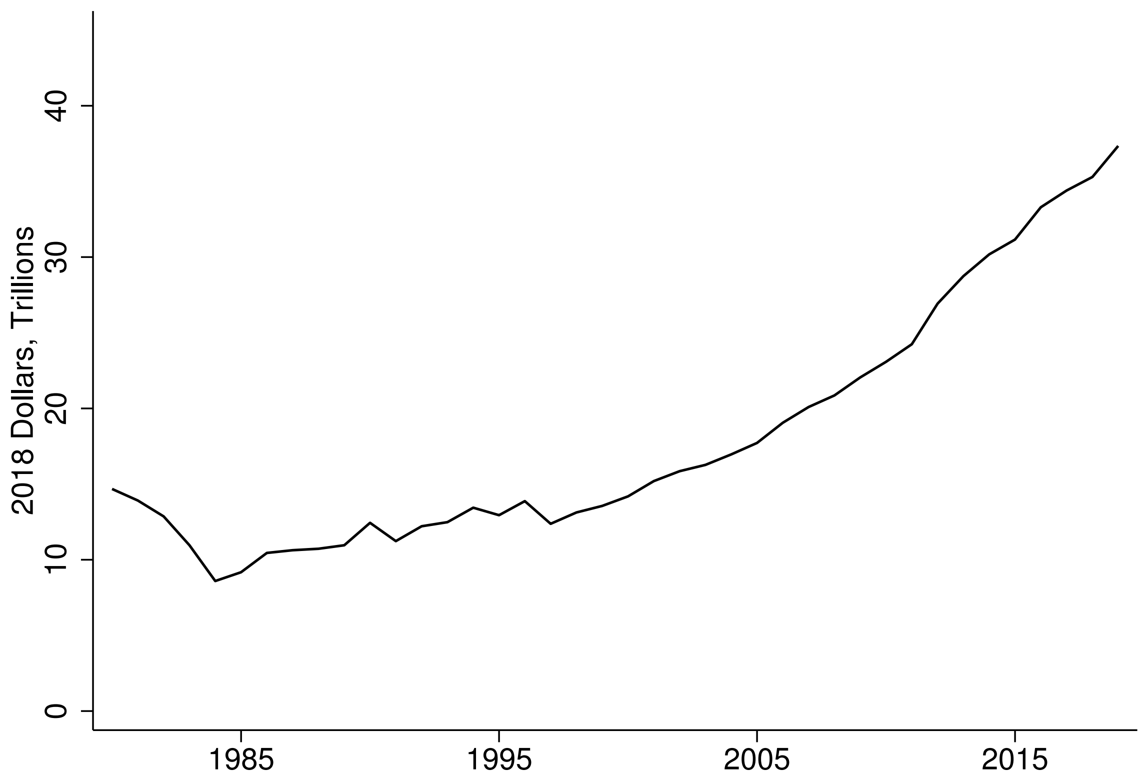


**Figure 3: Social Security Benefits and Wages: 1961-2018**

This figure shows the differential growth of the base of revenue supporting Social Security benefits compared to the Wage Index used by the SSA and median personal wage coming from the US Census Bureau. Panel A shows the nominal levels for each series. Panel B shows the benefits base series divided by both the wage index (dashed, red line) and the median wage (dotted, blue line).

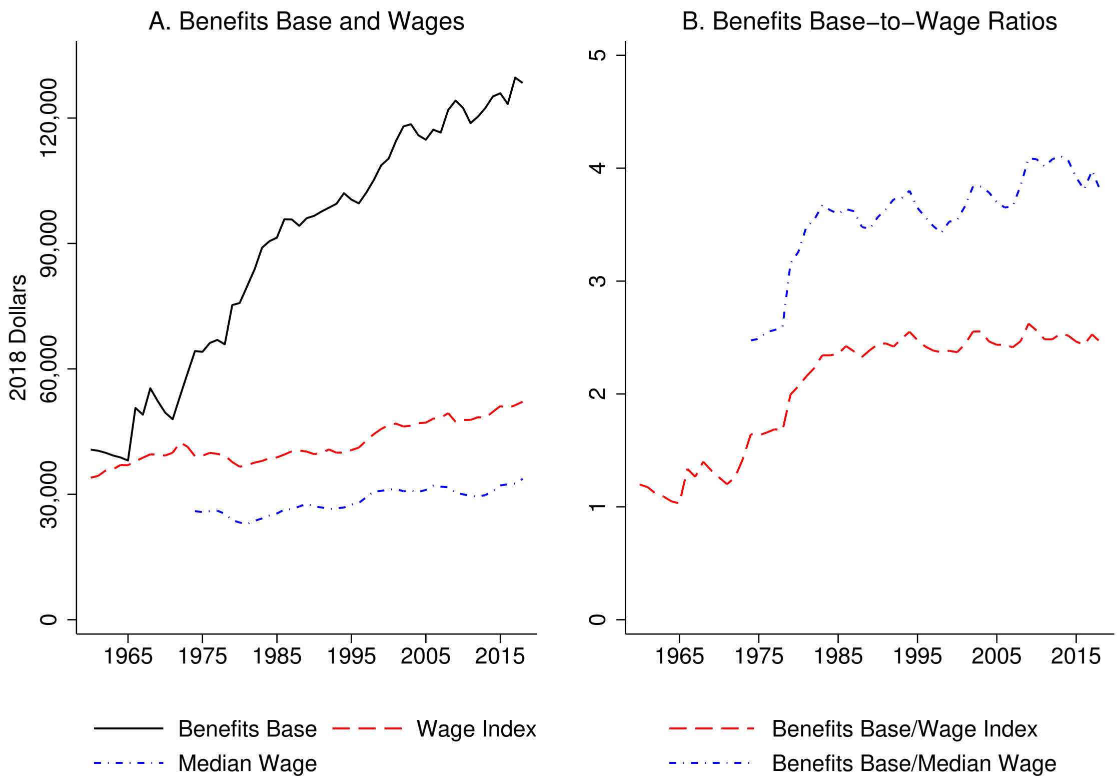


**Figure 4: Market Implied Yield Curve**

This figure shows the annualized zero coupon rates taken from Gurkaynak, Sack, and Wright (2006) from 1-48 years for 1989, 2001, and 2016 – the beginning, middle, and end of the SCF time series. The data are extended beyond 30 years by applying the 29 to 30 year forward rate to the annualized spot rate at 30 years, under the assumption that this forward rate represents the long-run interest rate on nominal government claims.

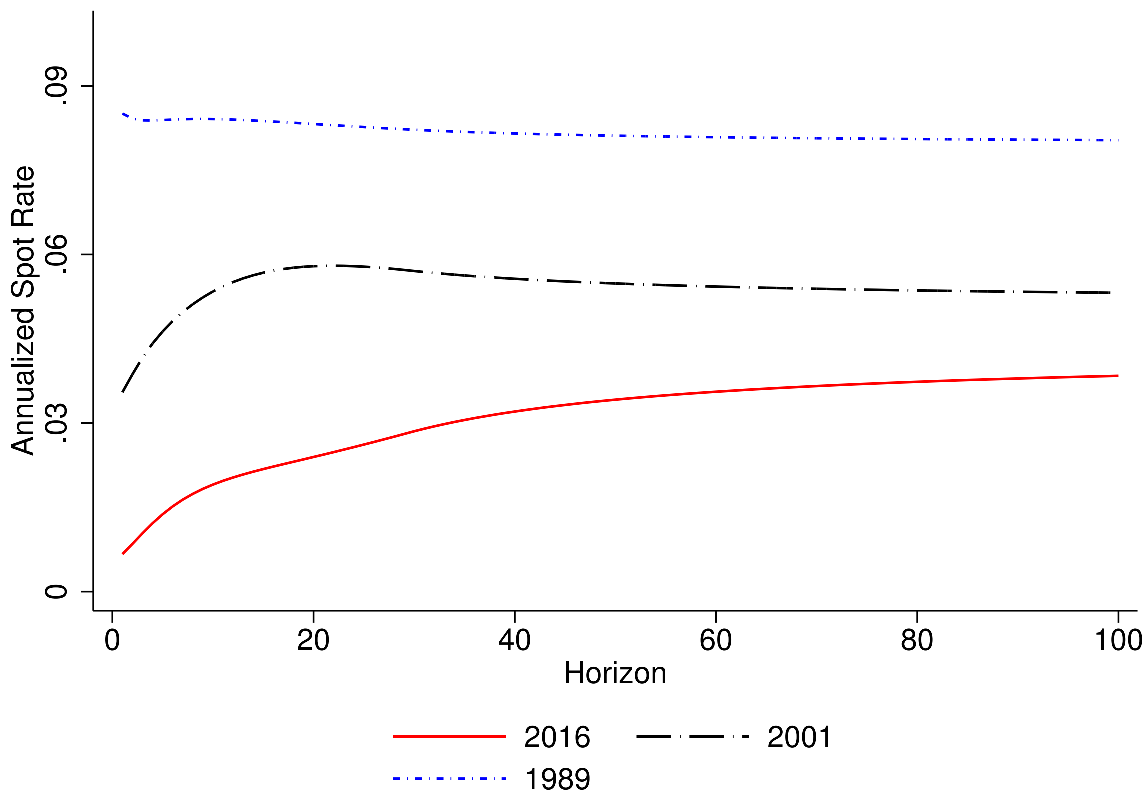


**Figure 5: Age Distribution in the United States**

This graph shows the age distribution in the United States in 1989 and 2016. Data are from the United Nations, Department of Economic and Social Affairs, Population Division. The data are represented as a proportion of total population in each year.

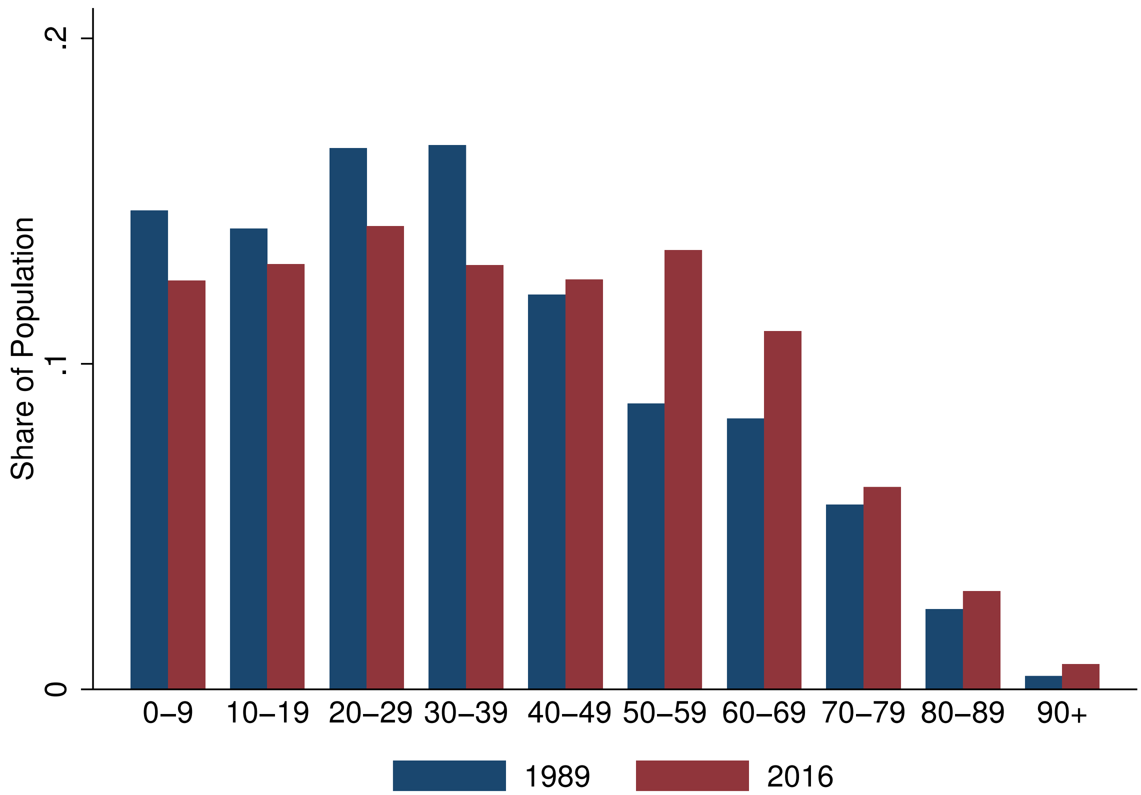


**Figure 6: Simulated and actual full-retirement-age benefits**

This figure reports mean Social Security benefits at full retirement age predicted by the model and observed in the SCF data, by gender and survey, and conditional on receiving benefits. Individuals not receiving benefits are not included. For each SCF survey, we report the mean "full retirement age equivalent" benefits observed in the next survey for pensioners between 62 and 67 years old. Panel A represents mean benefits for men and Panel B represents mean benefits for women. Because pensioners retire at different ages, we use Social Security rules to compute the benefits they would receive had they claimed their benefits at full retirement age, a process described in Appendix Section A.2. Benefits are reported in nominal dollars of the survey year.

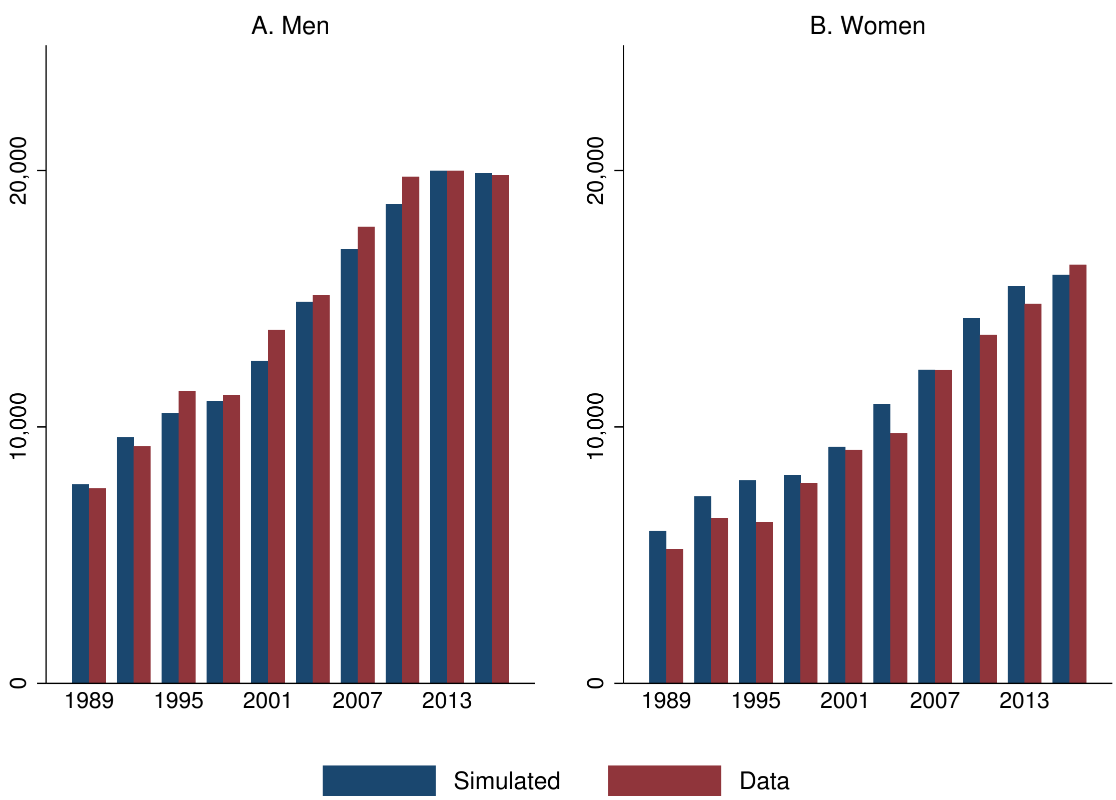


**Figure 7: Aggregate Social Security Wealth under Alternative Yield Curve Assumptions**

This figure shows the present value of Social Security calculated using our methodology compared to what is reported by the SSA. The solid black and dotted blue lines show the present value of Social Security, calculated using the methodology outlined in Section 4 using the market implied yield curve and the SSA yield curve, respectively. Each series are also adjusted for inflation by placing them in 2018 dollars. The red line comes from the SSA reports, with disability benefits removed. To remove disability benefits, we assume that they represent a constant 15% of total Social Security wealth, which is supported by SSA reports.

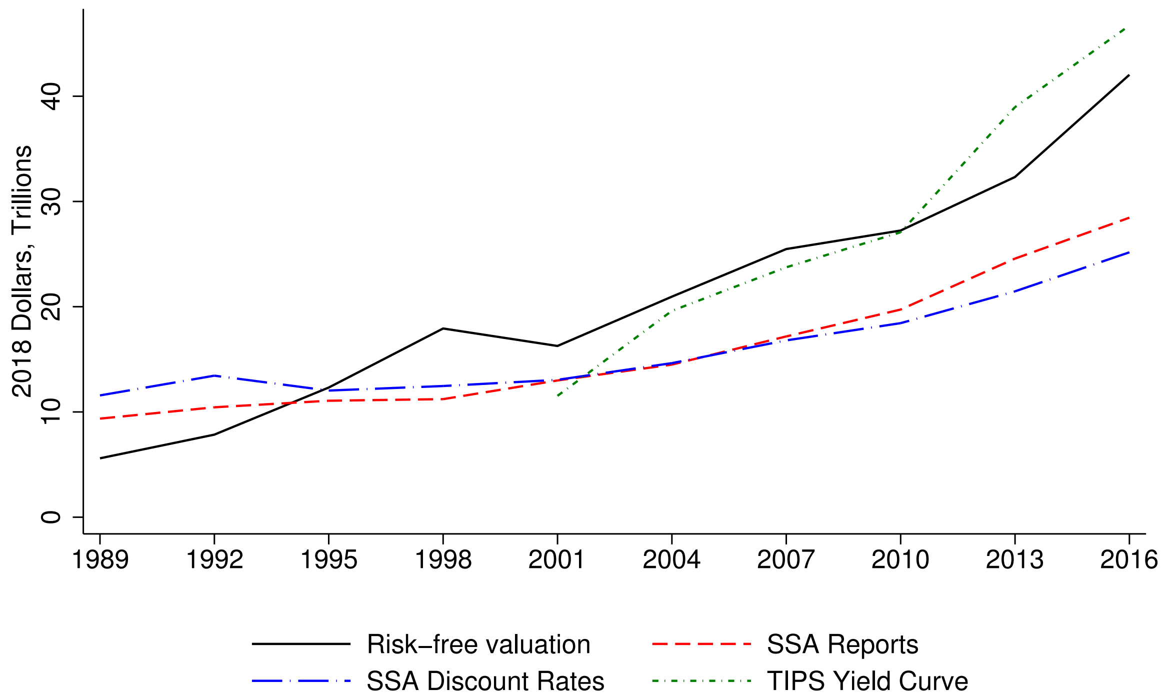


**Figure 8: Shares of Social Security Wealth and Top 10% Cutoffs by Age**

This figure describes how Social Security wealth is allocated by wealth centile. Panel C shows the within-cohort centile of marketable wealth above which an household needs to be to be in the overall top 10% of marketable wealth in 2016. Panel A shows the cumulative share of Social Security wealth by centile of marketable wealth among recent retirees, for which benefits are observable in the SCF. Panel B represent the share of a cohort's aggregate Social Security wealth going to the bottom 90%, and is obtained by successively applying the functions of Panel C and A, under the assumption that the cumulative distribution of Social Security wealth oberved among recent retirees is valid for younger cohorts.

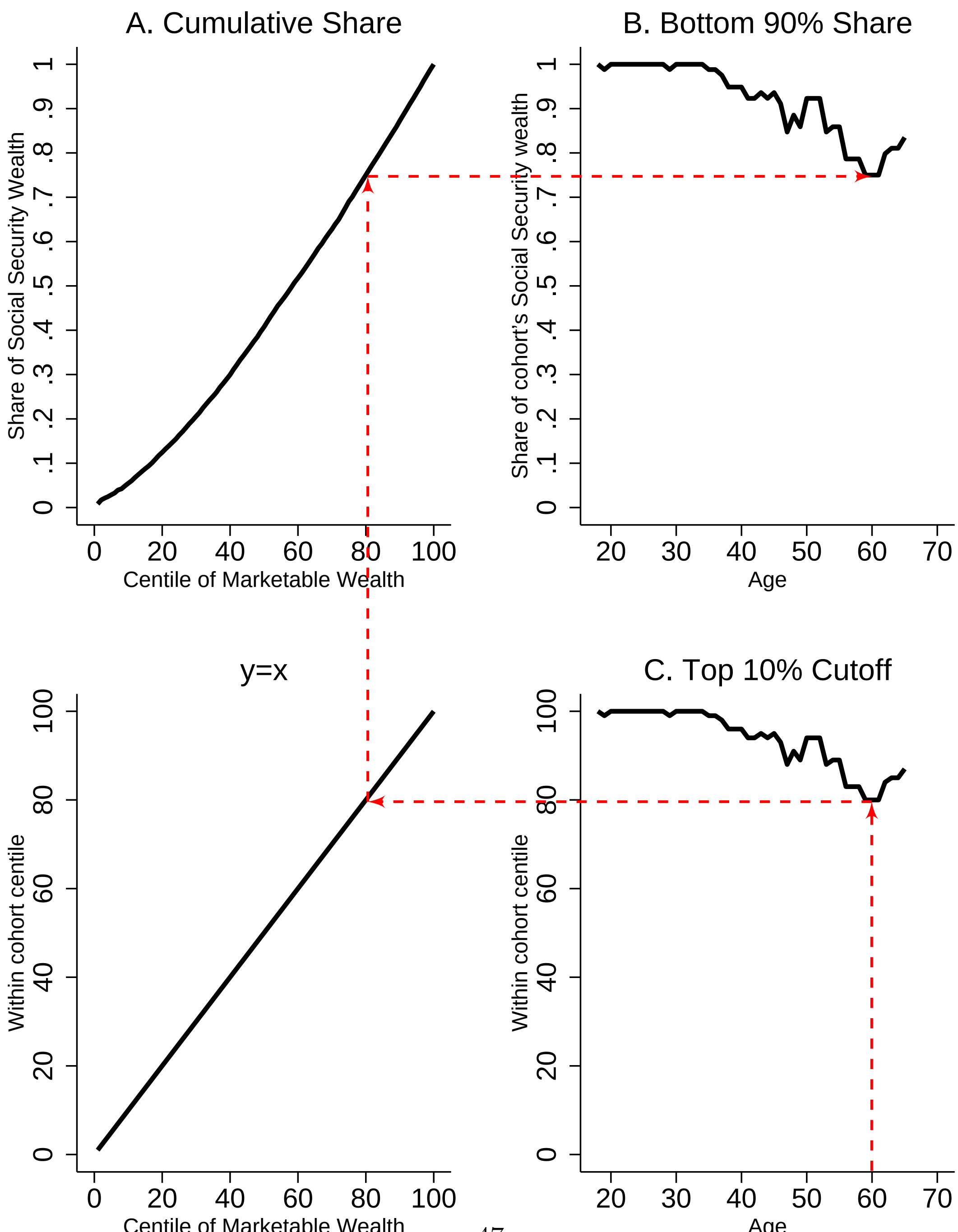


**Figure 9: Top 10% and Top 1% Wealth Shares – Risk-Free Valuation**

This figure shows the top 10% (Panel A) and top 1% (Panel B) wealth shares with and without Social Security included. The solid, black line includes Social Security. The dashed, blue line described the series without Social Security wealth included. Wealth centiles are calculated at the household level based on non-Social Security wealth. Social Security wealth is calculated using the procedure described in Section 4.

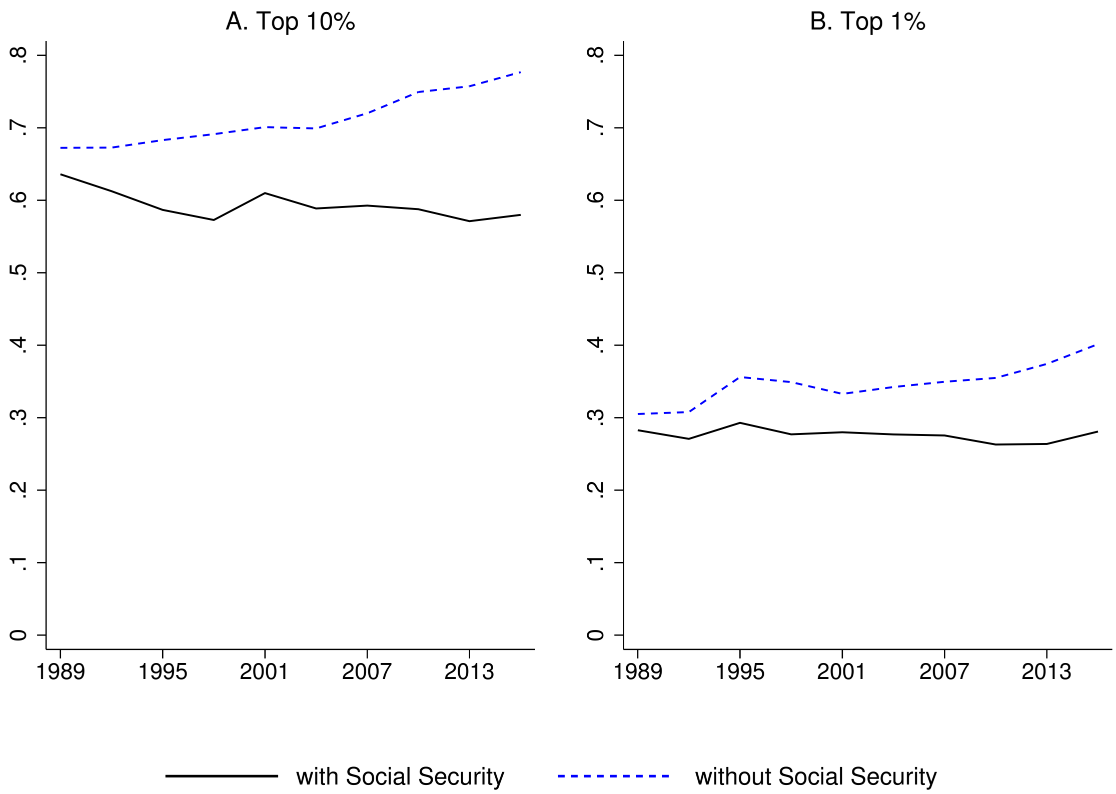
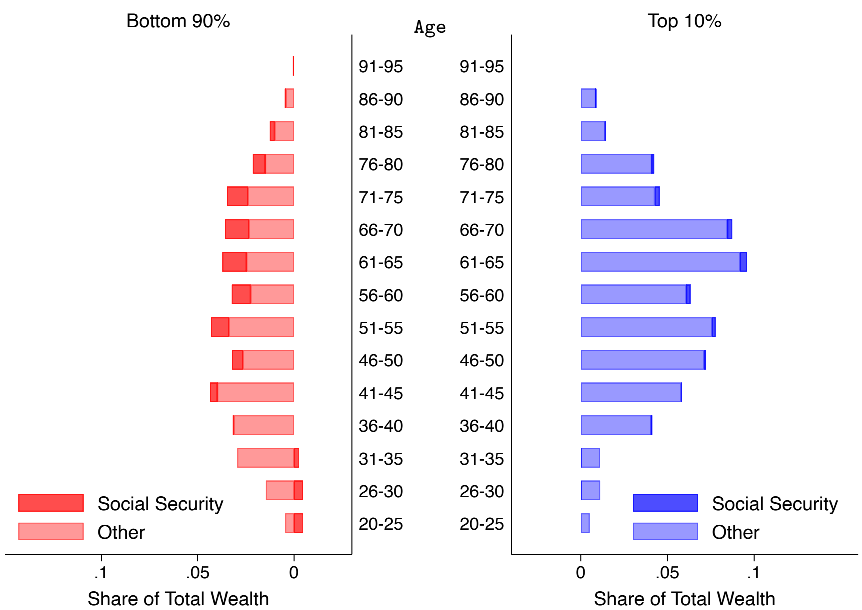


**Figure 10: Total Wealth Distribution by Age – Risk-Free Valuation**

This figure plots the shares of total wealth by age group for Social Security and non-Social Security wealth for 2016 and 1989. The shares of total wealth for the bottom 90% and top 10% of the wealth distribution are in red and blue, respectively. Social Security wealth is represented by the darker hue, whereas non-Social Security wealth by the lighter hue. Wealth figures are capitalized using the market implied real yield curve as described in Section 4.

### A. 1989




### B. 2016

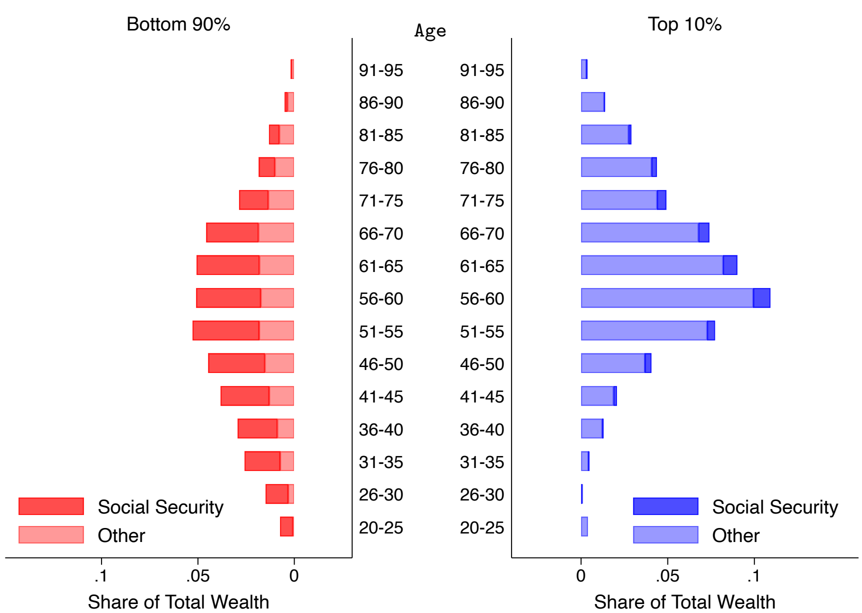


**Figure 11: Total Social Security Wealth – Risk-Adjusted Valuation**

This figure shows the present value of Social Security under two different discounting specifications. The first, represented by the solid, black line, shows the present value of Social Security, calculated using the methodology outlined in Section 4 using the market implied yield curve to discount cashflows coming from Social Security benefits. The second, represented by the dashed, blue lines, discounts cashflows at the market rate, but adds a risk adjustment through the procedure outlined in Section 6. Each series are also adjusted for inflation by placing them in 2018 dollars.

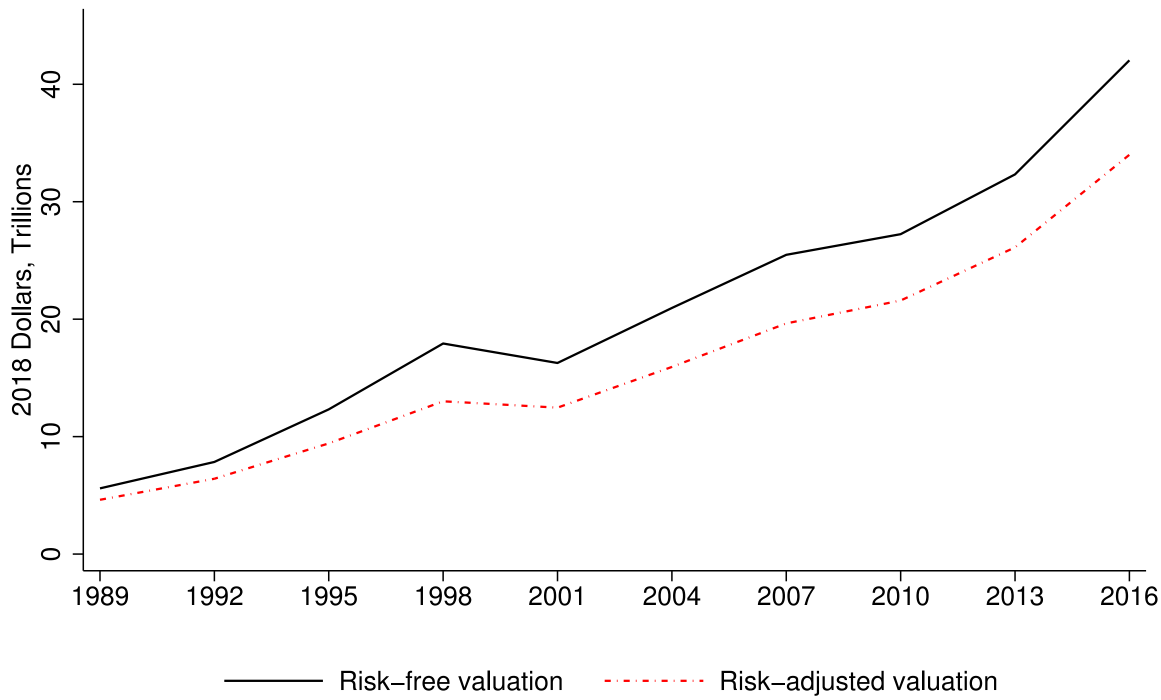
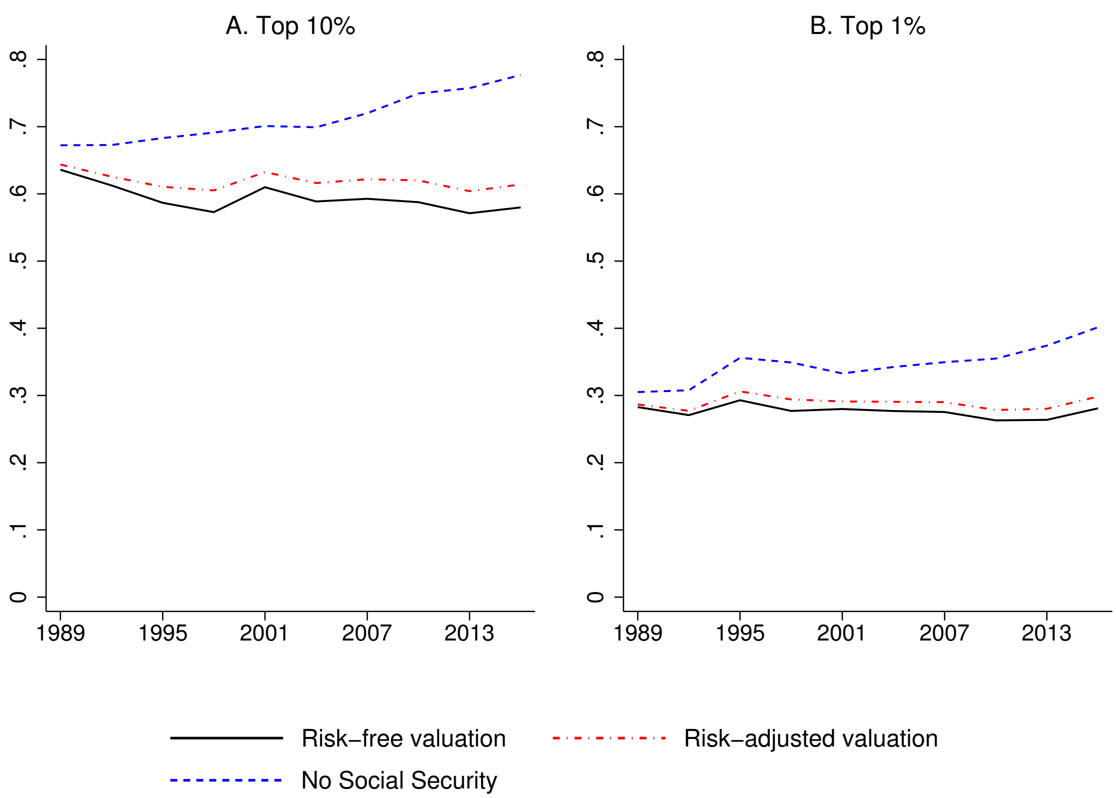


**Figure 12: Top 10% and Top 1% Wealth Shares – Risk-Adjusted Valuation**

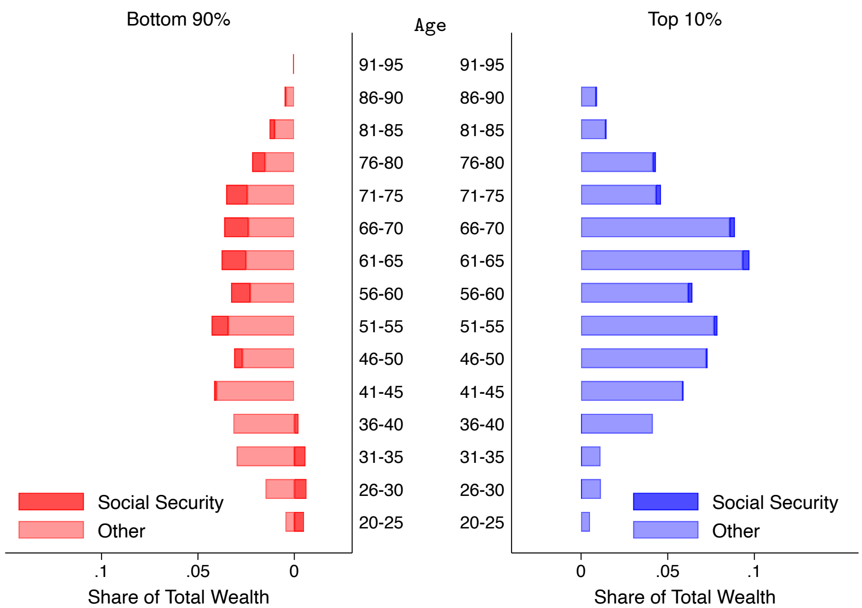
This figure shows the top 10% (Panel A) and top 1% (Panel B) wealth shares with and without Social Security included under the market implied yield curve and the risk adjusted specification. The solid, black line describes the series with Social Security wealth included and discounting at the market implied rate with no risk adjustment. The dashed, blue line describes the series with Social Security wealth included and discounting at market implied rate with a risk adjustment as outlined in 6. The dotted red line shows the series without Social Security wealth included. Wealth centiles are calculated at the household level based on non-Social Security wealth. Social Security wealth is calculated using the procedure described in Section 4.



### Figure 13: Total Wealth Distribution by Age – Risk-Adjusted Valuation

This figure plots the shares of total wealth by age group for Social Security and non-Social Security wealth for 2016 and 1989. The shares of total wealth for the bottom 90% and top 10% of the wealth distribution are in red and blue, respectively. Social Security wealth is represented by the darker hue, whereas non-Social Security wealth by the lighter hue. Wealth figures are capitalized using the market implied real yield curve as described in Section 4 and adjusted for aggregate labor income risk as described in Section 6.

**A. 1989**




**B. 2016**

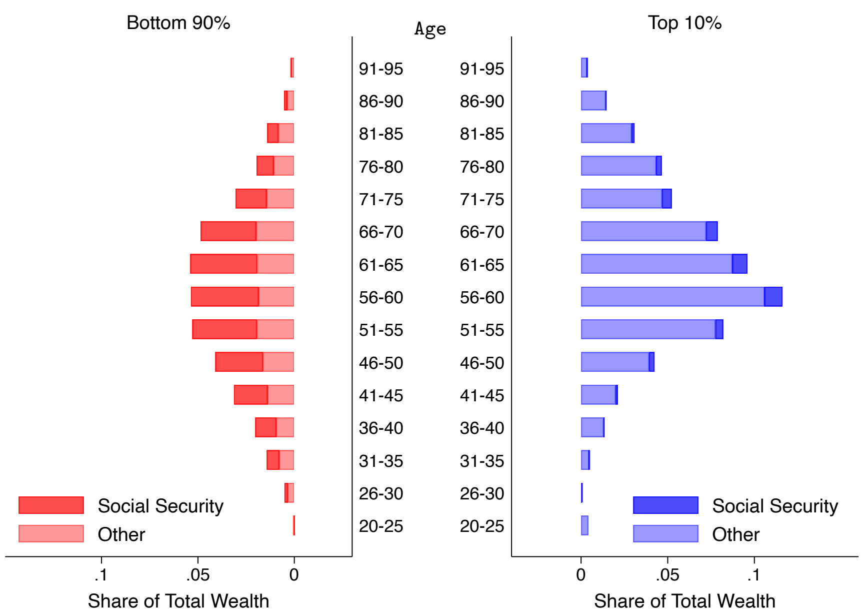


**Figure 14: Adjusting for Differential Life Expectancy**

This figure shows per capita Social Security wealth for each person in the SCF, applying population weights, for people in the top 10% (Panel A) and bottom 90% (Panel B) of the non-Social Security wealth distribution. The values in blue adjust for the differential life expectancy across income centiles using data from the Health Inequality Project (HIP). The values in red do not incorporate this adjustment. This procedure for this adjustment is outlined in Section 4.

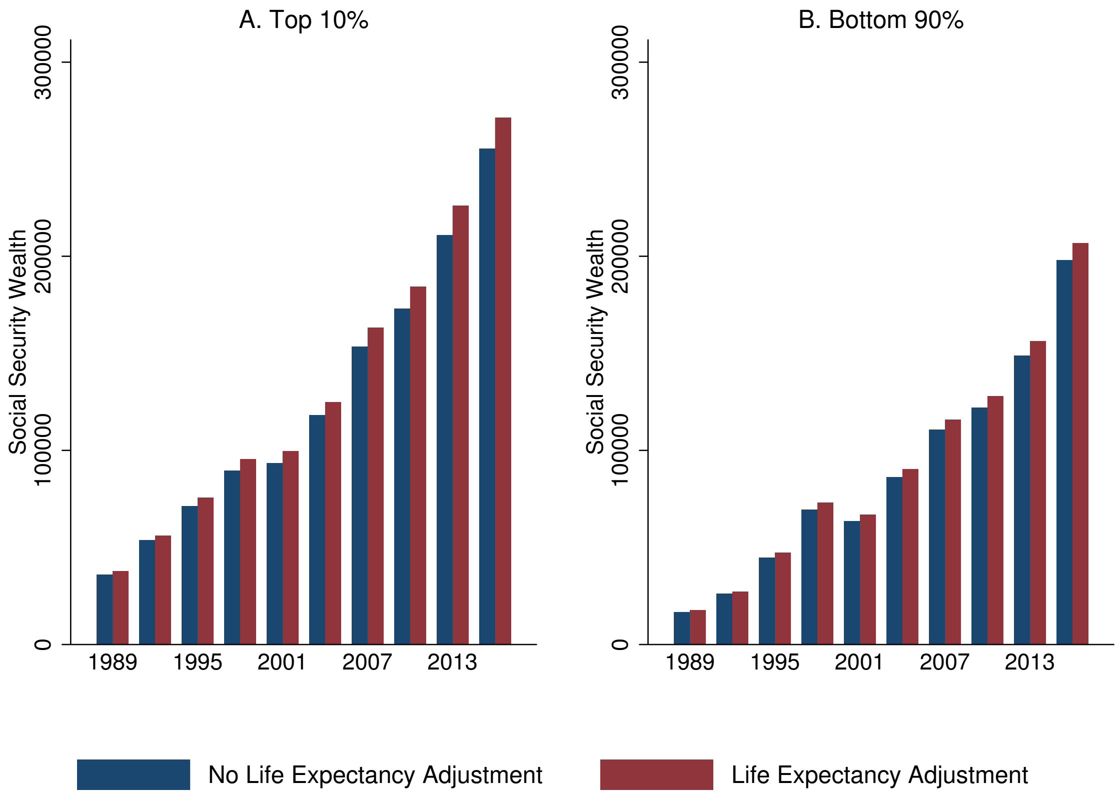


**Figure 15: Top Wealth Shares: Funding Gap Adjustment**

This figure presents top 10% (Panel A) and 1% (Panel B) wealth shares under four specifications. The first specification is wealth shares of only non-Social Security assets in SCF and is shown by the dashed, blue line. The second specification, shown by the solid, black line, is the risk-adjusted specification, assuming that Social Security benefits are not reduced (Alternative I). The third specification, shown by the dashed and dotted, green line, is the risk-adjusted specification, assuming that Social Security benefits are reduced under the SSA's intermediate assumptions (Alternative II). The fourth specification, shown by the long-dashed and dotted, red line, is the risk-adjusted specification, assuming that Social Security benefits are reduced under the SSA's high cost assumptions (Alternative III).

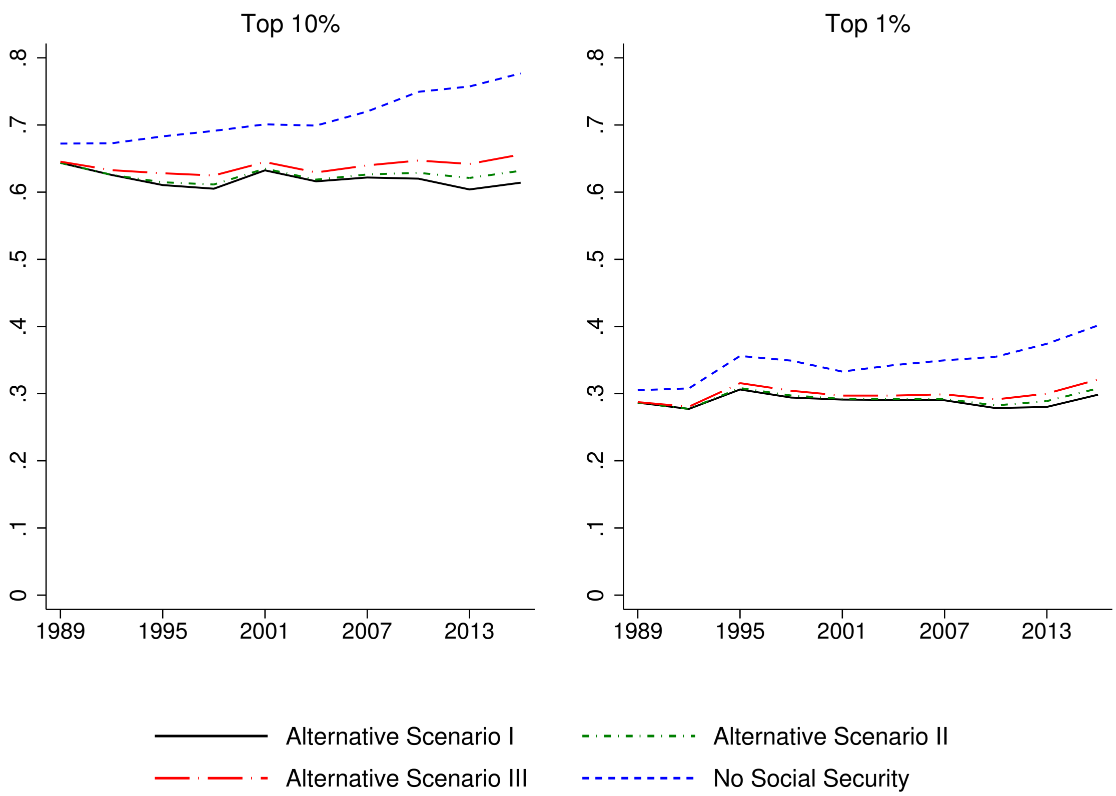


**Figure 16: Social Security Wealth and Other Measures**

This figure shows the wealth shares for the top 1% from other studies before (Panel A) and after (Panel B) adding aggregate Social Security wealth. The solid, black line represents top 1% shares in the SCF. The dashed, blue line shows top 1% shares from Saez and Zucman (2016); the dashed and dotted, red line shows top 1% shares from Smith et al. (2019); and the long-dashed, green line shows top 1% shares from Batty et al. (2019). All series with Social Security wealth included are calculated using the methodology outlined in Section 4 and are risk-adjusted using the procedure detailed in Section 6.

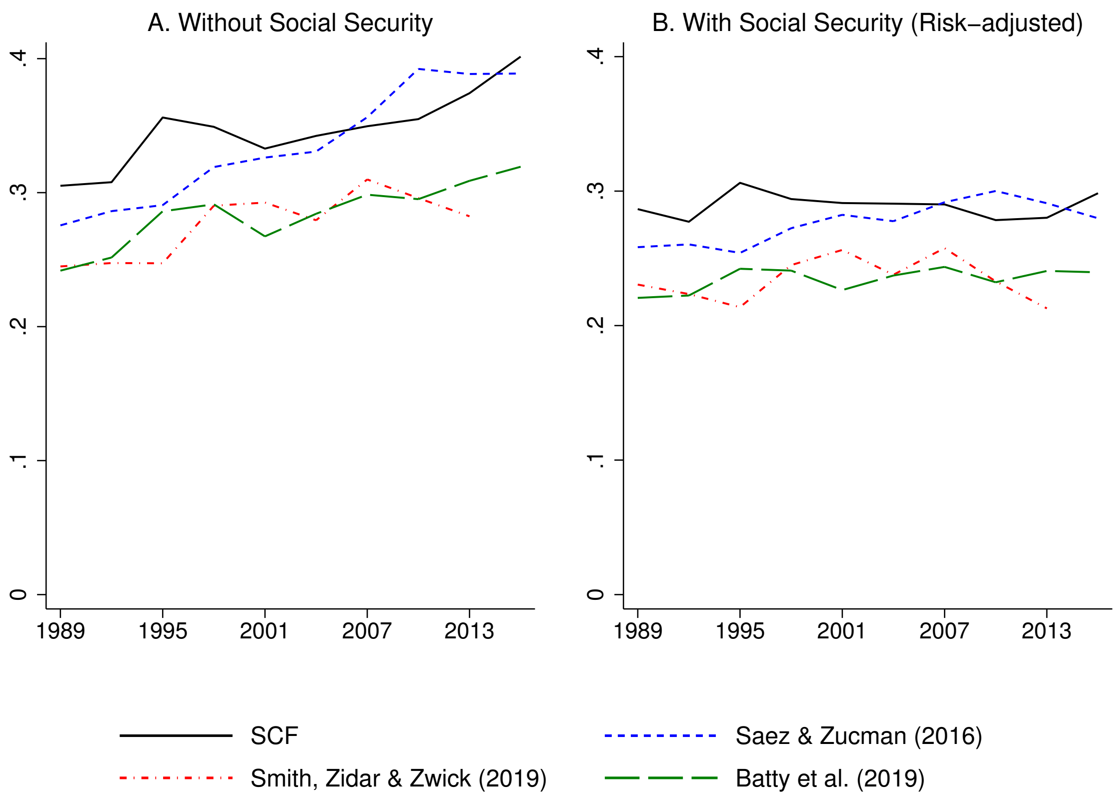


**Figure 17: Adding Social Security to Smith et al. (2019)**

In this figure, we add Social Security wealth to the top wealth shares estimates in Smith et al. (2019). Panel A shows top 1% shares, Panel B top 0.1% shares, and Panel C top 0.01% shares. The dashed, blue lines show estimates without Social Security wealth included and the solid, black lines show estimates with Social Security wealth included.

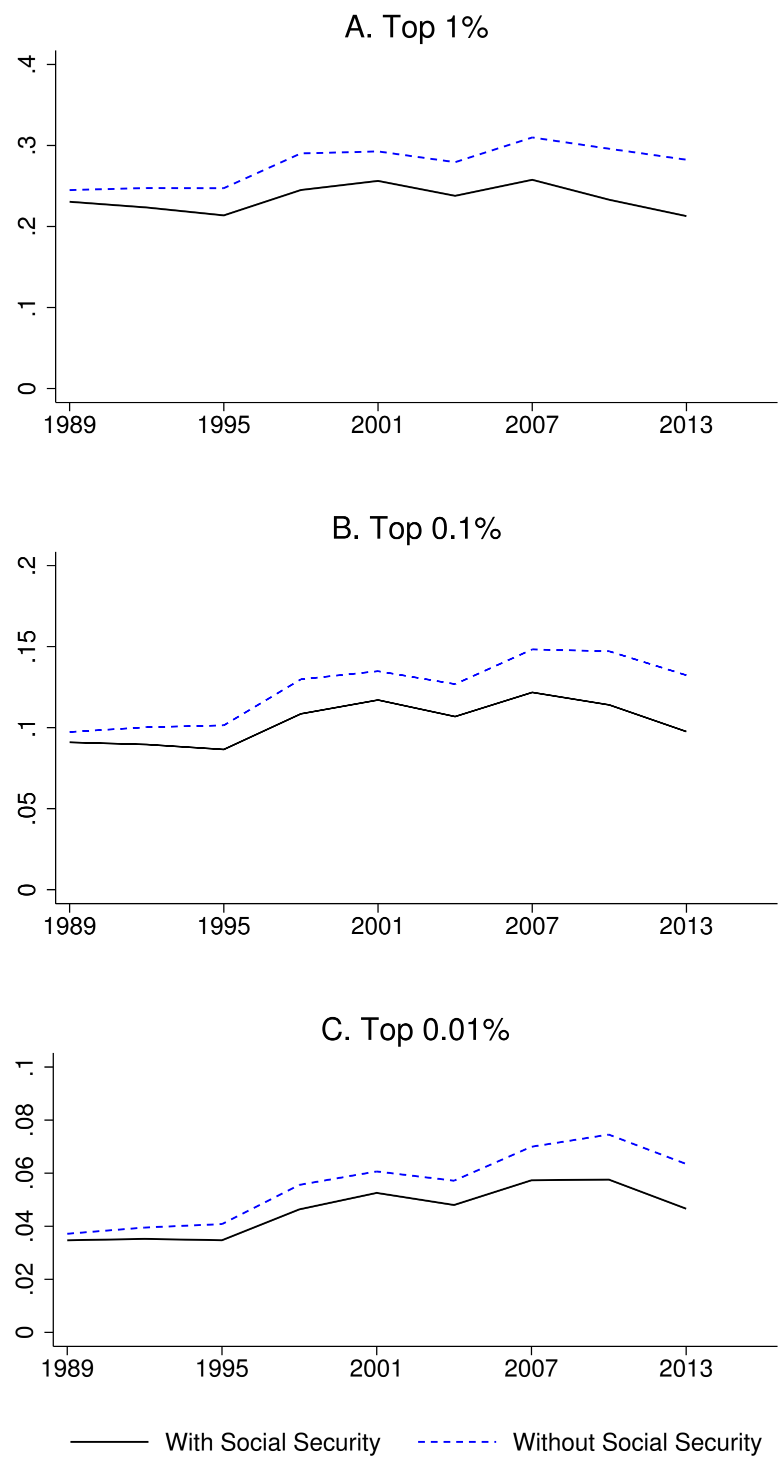


**Table 1: Decomposing the Increase in Social Security wealth**

This table shows the relative contribution of different effects on per capita Social Security wealth. The first row is computed by subtracting log per capita Social Security wealth in 2016 under the 1989 age distribution, yield curve, and survival probabilities from log per capita Social Security wealth in 1989. The second row is computed by subtracting the log per capita in 2016 under the 1989 age distribution and yield curve from log per capita Social Security wealth per capita in 2016 under the 1989 age distribution, yield curve, and survival probabilities. The third row is computed by subtracting log per capita Social Security wealth in 2016 under the 1989 yield curve from Social Security wealth in 2016 under the 1989 age distribution and yield curve. The fourth row is calculated as the difference between log per capita Social Security wealth in 2016 and log per capita Social Security wealth in 2016 under the 1989 yield curve. All calculations described above were performed after adjusting for inflation. The total log per capita wealth change is given by $\log(W_s^{2016}) - \log(W_s^{1989})$ where both terms are calculated under the 2016 and 1989 populations, life expectancies, benefit policies, and yield curves, respectively. Next, log population growth from 1989 to 2016 is added in line 6, which is added to the per capita total, to yield the total change in aggregate Social Security wealth in line 7.

| | Valuation method | |
|---|---|---|
| | Risk-free | Risk-adjusted |
| Social Security Expansion & Other | 0.121 | 0.180 |
| Life Expectancy | 0.290 | 0.285 |
| Shift in Age Distribution | 0.260 | 0.308 |
| Change in Yield Curve | 1.093 | 0.981 |
| Log Total per Capita | 1.764 | 1.754 |
| Population Growth | 0.303 | 0.303 |
| Log Total | 2.067 | 2.057 |

# INTERNET APPENDIX

In this section, we give a detailed account of the methodology described in Section 4.

# A Data appendix

## A.1 Social Security in the SCF

Data on Social Security benefits is given in the raw SCF which can be found on the Federal Reserve Board of Governors' webpage. The SCF interviews households, meaning that for couples their will be two variables reported for each household. As a rule, when there are two potential respondents the SCF takes the man in the couple to be the head of the household, and his information will be given first. For the remainder of this description, I will state the head of household variables title first, and then the second member variables second (when applicable).

The first step is locating Social Security benefits, variables X5306 and X5311. Note that these variables can be at different frequencies and must be adjusted into annual totals. This can be done by using X5307 and X5312 which describe at what frequency benefits are paid. For information on what each variable value means, see the SCF survey documentation.

Next, we locate the type of payment each member of the household is receiving, given by X5304 and X5309. These fall into three categories: retirement, survivor, and disability. For this exercise, we only focus on the first two types of benefits. This information is enough to compute the annual Social Security income received by each household in the SCF. To compute Social Security wealth, the age and biological sex of both members of the household are also needed. The head of household value for each of these are in the SCF produced extracts. In the raw data, the second member of the household's age is given by X19 and biological sex by X103.

## A.2 Full-retirement-age equivalent benefits

To validate the model, we compute full retirement age benefits using the observed benefits in the data. To do this, we need to know when each respondent retired, as Social Security benefits are a function of the retirement age. Further, this function is time varying; the full retirement age and discounts and credits applied for early or late retirement with birth cohort. The rules for doing this are on the SSA's webpage[24][25].

[24] The link for early/late retirement discounts/credits is https://www.ssa.gov/OACT/quickcalc/early_late.html.

[25] The link to get the age of full retirement is https://www.ssa.gov/OACT/ProgData/nra.html.

From these rules, the retirement age, and the birth cohort, we receive discount or credit from the full retirement amount. For example, if the full retirement age is 66 and the person retires at 63, the the discount would be $(1 - \frac{5}{9} \times 36)$ based on the SSA rules, meaning that the full retirement benefit in this case would be

$$\text{Full Ret. Benefit}_i = \frac{\text{Benefit}_i}{(1 - \frac{5}{9} \times 36)}.$$

Similarly, if the individual retires late, her full retirement benefits are a reduced share of the observed benefits. This reduction depends on the birth cohort and age of retirement.

We apply these rules using the X5305 and X5310 variables in the SCF which denote the number of years the respondent has been receiving benefits. This allows us to calculate the retirement age and the appropriate adjustments to ascertain the full retirement age benefits.

## A.3 Adjusting Life Expectancy for Mortality Differences by Income Centile

Using data on differential mortality by income centile from the Health Inequality Project (HIP) we construct life expectancy spreads for each biological sex, which is the difference in life expectancy for lifetime income centile $i$ and the average life expectancy for that sex during that year.[26] For example, if the average life expectancy is 75 for a men in a given year, and the life expectancy for someone in the 100th lifetime income centile is 80, then the life expectancy spread is 5 years. Note that these spreads can be (and are) negative for the lowest income centiles.

We then take these spreads and subtract them from the age of each SCF respondent and calculate the effective mortality age in the data. Note that we only do this for people under the age of 40 as the HIP data calculates differential mortality from 40 years of age. This may leave out mortality earlier than 40 years of age. This may have some effect on the results using the market yield curve specification, particularly in the post-crisis period, but it would depend on whether early mortality is correlated with having an insufficient income history to receive benefits, a counterfactual that would be very difficult to assess. For the other specifications this will likely not have a large effect, as Social Security wealth is near zero or negative before 40 years of age.

In additional, the HIP data only covers the period of 2001-2014. For this reason, we extend the 2014 results to 2016 and the 2001 results back to 1989. While this is not perfect, it is the best we are able to do with these data limitations. However, the life expectancy differences have little quantitative impact on the wealth shares. This is for two reasons: 1) the Social Security benefit-weighted life expectancy of the bottom 90% increases when accounting for differential life expectancy, and 2) the additional wealth received by the top 10% (roughly $16,000) does not increase their aggregate wealth by much. As such, Figure C.3 shows that this adjustment actually increases measured inequality, as the bottom 90% receive more Social Security wealth under the adjustment.

[26] This data is available for 40 year-olds annually from 2001 onward. For prior years, we backfill this data. We assume that the relationship between income centile and life expectancy for 40 year-olds holds at all ages.

As mentioned in Section 7.1, we proxy for the lifetime income centile using the Social Security retirement benefits centile. We do this for two main reasons; first, Social Security benefits are a monotonically increasing function of lifetime wage earnings, and thus serve as a nearly ideal proxy, and second, the SCF does not have information on lifetime earnings. Using Social Security benefits centiles are slightly thrown off by income coming from capital. To assure our results are robust to this, we also do this exercise using within-age cohort wealth centiles. There is some difference, as the benefits-weighted average of the bottom 90% decreases under this assumption, and the Social Security wealth of the top 10% increase by nearly $20,000. However, the quantitative implications are unchanged; top wealth shares are nearly unaffected by this adjustment.

## A.4 SSA Yield Curve Assumptions

Figure C.1 shows the differences in the yield curve assumptions implied from Treasuries notes from Gurkaynak, Sack, and Wright (2006) and the assumptions used by the SSA to compute the present value of Social Security obligations. As discussed above, this is due to the SSA estimating based on the last 5 economic cycles and not using market implied rates.

However, there has been a large literature in macroeconomics and finance that suggests that low interest rate regimes are to be the norm going forward. If this is the case, it is not reasonable to assume a reversion to the relatively high interest rates observed in the past. The Board of Governors of the Federal Reserve seems to agree. In the projections they release after quarter end FOMC meetings, the median long-run nominal interest rates are expected to be around 3-3.5% with an upper bound around 4%. These are significantly lower than the over 5% long-run nominal rates suggested by the SSA.

## A.5 Capitalizing Implied Survivor Benefits

The SSA provides benefits to the widows of retirement benefit recipients upon their passing. We account for this in our capitalization by calculating the conditional probability of a respondent's spouse being alive given that the respondent is deceased, under the assumption that the survival probabilities of the spousal pair are uncorrelated.

Further, we also account for the maximal benefits a spouse can receive under SSA rules. Uses a series of three bend points and the national wage index to determine the maximal benefits a family can receive[27]. We adjust our survival benefit calculations such that the received benefits do not exceed the maximal family threshold.

Once the maximum benefit is calculated, the implied wealth coming from survivor benefits is given by

[27] This is detailed here: https://www.ssa.gov/OACT/COLA/familymax.html

$$\text{Implied Survivor Benefits}_{it,s} = \max\left\{\min\left\{\text{Max. Family Benefits} - \text{Spouse Benefits}, \text{Benefits}_{it}\right\}, 0\right\}$$
$$\times \sum_{h=0}^{\infty} \frac{(1+\pi_{t,t+h}) \prod_{k=t}^{h-1} m_{i,t+k}(1-m_{i,t+k}^{spouse})}{1+i_{t,t+h}}$$

## A.6 Finding the Zeros: Proportion of People with No Benefits

While the vast majority of retirees receive some form of Social Security benefits, there are a not insignificant fraction of retirees with insufficient work history to receive benefits. When assigning Social Security benefits to non-retirees, we must take this into account. This requires a reasonable estimation of the proportion of people in each cohort that do not receive benefits.

We estimate this using a Deaton-Paxson regression for each biological sex, which is a constrained regression of the following form

$$\log(Pr(\text{No Benefits}))_{t,a,c} = \gamma_t + \eta_a + \delta_c + \varepsilon_{t,a,c} \tag{A.1}$$

subject to

$$\sum_{1989}^{2016} \gamma_t = 0 \tag{A.2}$$
$$\sum_{1989}^{2016} \gamma_t (t - 2002.5) = 0 \tag{A.3}$$
$$\eta_{72} = 0. \tag{A.4}$$

The coefficients of interest are the cohort fixed effects where this empirical set-up allows us to adjust for survey specific sampling error and age specific effects. The fitted values by cohort are shown in Figure C.2, where the average number of zero Social Security income respondents is shown to be 10% for men and 20% for women. In the paper, we use these figures in determining aggregate Social Security wealth.

# B Calculating the Market Beta of Aggregate Labor Income

Consider the following exogenous system of stochastic processes

$$
\begin{aligned}
dy_t &= -\kappa y_t dt + \begin{bmatrix} \nu_1 \\ -\nu_3 \end{bmatrix}^T dz_t, \\
ds_t &= \left(\mu - \frac{\sigma^2}{2} + \phi y_t\right) dt + \begin{bmatrix} 0 \\ \sigma \end{bmatrix}^T dz_t, \\
l_t &= y_t + s_t - \delta t, \\
d\pi_t &= -r\pi_t dt - \begin{bmatrix} 0 \\ \lambda \end{bmatrix}^T \pi_t dz_t,
\end{aligned}
$$

where $l_t = \log L_t$ is log wage, $s_t = \log S_t$ is log stock price, $\pi_t$is the state-price density, $\lambda \equiv \frac{\mu - r}{\sigma}$, and $z_t = \begin{bmatrix} z_{1,t} & z_{3,t} \end{bmatrix}^T$.

We want to find the beta at time $t$ on a "wage strip", which is a security that pays out $L_{t+n}$ at $t+n$. In other words, we want to find the following:

$$
\beta_t^{L,n} = \frac{Cov_t\left(r_t^m dt, r_t^{L,n} dt\right)}{Var_t\left[r_t^m dt\right]},
$$

where $r_t^m$ is the instantaneous return on the market defined as

$$
r_t^m dt = \frac{dS_t}{S_t} = ds_t + \frac{1}{2}\left(ds_t\right)^2 = \left(\mu + \phi y_t\right) dt + \begin{bmatrix} 0 \\ \sigma \end{bmatrix}^T dz_t,
$$

and $r_t^{L,n}$ is the instantaneous return on the wage strip defined as

$$
r_t^{L,n} dt = \frac{dP_t^{L,n}}{P_t^{L,n}},
$$

where $P_t^{L,n}$ is the price of the wage strip. So, we first need to find this price.
Note that by no-arbitrage the price of the wage strip equals the following:

$$
P_t^{L,n} = \mathbb{E}_t\left[\frac{\pi_{t+n}}{\pi_t} L_{t+n}\right] = \mathbb{E}_t\left[\exp\left\{\tilde{\pi}_{t+n} - \tilde{\pi}_t + l_{t+n}\right\}\right], \tag{*}
$$

where $\tilde{\pi}_t \equiv \log \pi_t$.

By Ito's lemma,the process for $\tilde{\pi}_t$ is

$$
\begin{aligned}
d\tilde{\pi}_t &= \frac{d\pi_t}{\pi_t} - \frac{1}{2}\left(\frac{d\pi_t}{\pi_t}\right)^2 = \left(-r - \frac{1}{2}\lambda^2\right) dt - \begin{bmatrix} 0 \\ \lambda \end{bmatrix}^T dz_t \\
\Rightarrow \tilde{\pi}_t &= \left(-r - \frac{1}{2}\lambda^2\right) t - \begin{bmatrix} 0 \\ \lambda \end{bmatrix}^T z_t
\end{aligned}
$$

Hence, to solve (*) we are left with finding $l_{t+n}$, which is equivalent to solving for $y_t$ and $s_t$.

It can be easily verified by applying Ito's lemma that the solution for $y_t$ is as follows

$$y_t = e^{-\kappa t}\left(y_0 + \begin{bmatrix}\nu_1\\ -\nu_3\end{bmatrix}^T \int_0^t e^{\kappa s}dz_s\right)$$

Now, to find $s_t$, we introduce a new variable $\tilde{s}_t$ defined as

$$\tilde{s}_t = s_t + \frac{\phi}{\kappa}y_t,$$

then we have

$$d\tilde{s}_t = ds_t + \frac{\phi}{\kappa}dy_t = \left(\mu - \frac{\sigma^2}{2}\right)dt + \begin{bmatrix}\frac{\phi}{\kappa}\nu_1\\ \sigma - \frac{\phi}{\kappa}\nu_3\end{bmatrix}^T dz_t$$

$$\Rightarrow \tilde{s}_t = \left(\mu - \frac{\sigma^2}{2}\right)t + \begin{bmatrix}\frac{\phi}{\kappa}\nu_1\\ \sigma - \frac{\phi}{\kappa}\nu_3\end{bmatrix}^T z_t$$

So now we are ready to find $s_t$:

$$s_t = \tilde{s}_t - \frac{\phi}{\kappa}y_t = \left(\mu - \frac{\sigma^2}{2}\right)t + \begin{bmatrix}\frac{\phi}{\kappa}\nu_1\\ \sigma - \frac{\phi}{\kappa}\nu_3\end{bmatrix}^T z_t - \frac{\phi}{\kappa}e^{-\kappa t}\left(y_0 + \begin{bmatrix}\nu_1\\ -\nu_3\end{bmatrix}^T \int_0^t e^{\kappa s}dz_s\right)$$

and $l_t$ then equals to

$$l_t = y_t + s_t - \delta t = \left(\mu - \frac{\sigma^2}{2} - \delta\right)t + \begin{bmatrix}\frac{\phi}{\kappa}\nu_1\\ \sigma - \frac{\phi}{\kappa}\nu_3\end{bmatrix}^T z_t + \left(1 - \frac{\phi}{\kappa}\right)y_t,$$

so we can return to solving (*).

Plugging everything back into the exponent of (*), we obtain

$$\begin{aligned}
\tilde{\pi}_{t+n} - \tilde{\pi}_t + l_{t+n} &= \left(-r - \frac{1}{2}\lambda^2\right)(t+n) - \begin{bmatrix}0\\ \lambda\end{bmatrix}^T z_{t+n} - \left(-r - \frac{1}{2}\lambda^2\right)t + \begin{bmatrix}0\\ \lambda\end{bmatrix}^T z_t \\
&+ \left(\mu - \frac{\sigma^2}{2} - \delta\right)(t+n) + \begin{bmatrix}\frac{\phi}{\kappa}\nu_1\\ \sigma - \frac{\phi}{\kappa}\nu_3\end{bmatrix}^T z_{t+n} + \left(1 - \frac{\phi}{\kappa}\right)y_{t+n} \\
&= \left(-r - \frac{1}{2}\lambda^2\right)n + \left(\mu - \frac{\sigma^2}{2} - \delta\right)(t+n) + \begin{bmatrix}0\\ \lambda\end{bmatrix}^T z_t + \begin{bmatrix}\frac{\phi}{\kappa}\nu_1\\ \sigma - \frac{\phi}{\kappa}\nu_3 - \lambda\end{bmatrix}^T z_{t+n} \\
&+ \left(1 - \frac{\phi}{\kappa}\right)y_{t+n}
\end{aligned}$$

Note that all components inside the exponent in (*) are normal variables, hence, we can rewrite the equation as

$$P_t^{L,n} = \exp\left\{\mathbb{E}_t\left[\tilde{\pi}_{t+n} - \tilde{\pi}_t + l_{t+n}\right] + \frac{1}{2}Var_t\left[\tilde{\pi}_{t+n} - \tilde{\pi}_t + l_{t+n}\right]\right\}, \qquad (**)$$

which leaves us with finding the two components in the exponent.

Also note how we can express $y_{t+n}$ via $y_t$:

$$y_{t+n} = e^{-\kappa(t+n)} \left( y_0 + \begin{bmatrix} \nu_1 \\ -\nu_3 \end{bmatrix}^T \int_0^{t+n} e^{\kappa s} dz_s \right) = e^{-\kappa n} \left( y_t + \begin{bmatrix} \nu_1 \\ -\nu_3 \end{bmatrix}^T \int_t^{t+n} e^{\kappa(s-t)} dz_s \right)$$

Finding $\mathbb{E}_t \left[\tilde{\pi}_{t+n} - \tilde{\pi}_t + l_{t+n}\right]$:

$$\begin{aligned}
\mathbb{E}_t \left[\tilde{\pi}_{t+n} - \tilde{\pi}_t + l_{t+n}\right] &= \left(-r - \frac{1}{2}\lambda^2\right) n + \left(\mu - \frac{\sigma^2}{2} - \delta\right)(t+n) + \begin{bmatrix} \frac{\phi}{\kappa}\nu_1 \\ \sigma - \frac{\phi}{\kappa}\nu_3 \end{bmatrix}^T z_t + \left(1 - \frac{\phi}{\kappa}\right) e^{-\kappa n} y_t \\
&= \left(\mu - \frac{\sigma^2}{2} - \delta\right) t - \left(\frac{1}{2}(\lambda - \sigma)^2 + \delta\right) n + \left(1 - \frac{\phi}{\kappa}\right) e^{-\kappa n} y_t + \begin{bmatrix} \frac{\phi}{\kappa}\nu_1 \\ \sigma - \frac{\phi}{\kappa}\nu_3 \end{bmatrix}^T z_t
\end{aligned}$$

Finding $Var_t \left[\tilde{\pi}_{t+n} - \tilde{\pi}_t + l_{t+n}\right]$:

$$\begin{aligned}
Var_t \left[\tilde{\pi}_{t+n} - \tilde{\pi}_t + l_{t+n}\right] &= Var_t \left[ \begin{bmatrix} \frac{\phi}{\kappa}\nu_1 \\ \sigma - \frac{\phi}{\kappa}\nu_3 - \lambda \end{bmatrix}^T z_{t+n} + \left(1 - \frac{\phi}{\kappa}\right) e^{-\kappa(t+n)} \begin{bmatrix} \nu_1 \\ -\nu_3 \end{bmatrix}^T \int_t^{t+n} e^{\kappa s} dz_s \right] \\
&= \left( \left(\frac{\phi}{\kappa}\nu_1\right)^2 + \left(\sigma - \frac{\phi}{\kappa}\nu_3 - \lambda\right)^2 \right) n + \left(1 - \frac{\phi}{\kappa}\right)^2 \left(\nu_1^2 + \nu_3^2\right) \frac{1}{2\kappa} \left(1 - e^{-2\kappa n}\right) \\
&+ 2\left(1 - \frac{\phi}{\kappa}\right) \left(\frac{\phi}{\kappa}\nu_1^2 + \frac{\phi}{\kappa}\nu_3^2 - \sigma\nu_3 + \lambda\nu_3\right) \frac{1}{\kappa} \left(1 - e^{-\kappa n}\right)
\end{aligned}$$

So, the solution for $P_t^{L,n}$ is

$$\begin{aligned}
P_t^{L,n} = \exp \Bigg\{ & \left(\mu - \frac{\sigma^2}{2} - \delta\right) t - \left(\frac{1}{2}(\lambda - \sigma)^2 + \delta\right) n + \left(1 - \frac{\phi}{\kappa}\right) e^{-\kappa n} y_t + \begin{bmatrix} \frac{\phi}{\kappa}\nu_1 \\ \sigma - \frac{\phi}{\kappa}\nu_3 \end{bmatrix}^T z_t \\
& + \frac{1}{2} \left( \left(\frac{\phi}{\kappa}\nu_1\right)^2 + \left(\sigma - \frac{\phi}{\kappa}\nu_3 - \lambda\right)^2 \right) n + \frac{1}{2} \left(1 - \frac{\phi}{\kappa}\right)^2 \left(\nu_1^2 + \nu_3^2\right) \frac{1}{2\kappa} \left(1 - e^{-2\kappa n}\right) \\
& + \left(1 - \frac{\phi}{\kappa}\right) \left(\frac{\phi}{\kappa}\nu_1^2 + \frac{\phi}{\kappa}\nu_3^2 - \sigma\nu_3 + \lambda\nu_3\right) \frac{1}{\kappa} \left(1 - e^{-\kappa n}\right) \Bigg\} \\
= \exp \Bigg\{ & \left(\mu - \frac{\sigma^2}{2} - \delta\right) t + \left(1 - \frac{\phi}{\kappa}\right) e^{-\kappa n} y_t + \begin{bmatrix} \frac{\phi}{\kappa}\nu_1 \\ \sigma - \frac{\phi}{\kappa}\nu_3 \end{bmatrix}^T z_t - \left(\delta - \frac{1}{2}\frac{\phi^2}{\kappa^2}\left(\nu_1^2 + \nu_3^2\right) + \frac{\phi}{\kappa}\nu_3(\sigma - \lambda)\right) n \\
& + \left(1 - \frac{\phi}{\kappa}\right)^2 \left(\nu_1^2 + \nu_3^2\right) \frac{1}{4\kappa} \left(1 - e^{-2\kappa n}\right) + \left(1 - \frac{\phi}{\kappa}\right) \left(\frac{\phi}{\kappa}\left(\nu_1^2 + \nu_3^2\right) - \nu_3(\sigma - \lambda)\right) \frac{1}{\kappa} \left(1 - e^{-\kappa n}\right) \Bigg\} \\
\equiv \exp & \left\{ at + b + cy_t + d^T z_t \right\},
\end{aligned}$$

where

$$
\begin{aligned}
a &\equiv \mu - \frac{\sigma^2}{2} - \delta \\
b(n) &\equiv -\left(\delta - \frac{1}{2}\frac{\phi^2}{\kappa^2}\left(\nu_1^2+\nu_3^2\right) + \frac{\phi}{\kappa}\nu_3\left(\sigma-\lambda\right)\right) n + \left(1-\frac{\phi}{\kappa}\right)^2\left(\nu_1^2+\nu_3^2\right)\frac{1}{4\kappa}\left(1-e^{-2\kappa n}\right) \\
&\quad + \left(1-\frac{\phi}{\kappa}\right)\left(\frac{\phi}{\kappa}\left(\nu_1^2+\nu_3^2\right) - \nu_3\left(\sigma-\lambda\right)\right)\frac{1}{\kappa}\left(1-e^{-\kappa n}\right) \\
c(n) &\equiv \left(1-\frac{\phi}{\kappa}\right)e^{-\kappa n} \\
d &= \begin{bmatrix} \frac{\phi}{\kappa}\nu_1 \\ \sigma - \frac{\phi}{\kappa}\nu_3 \end{bmatrix}
\end{aligned}
$$

Now, to find the return on the wage strip, we need to differentiate its price. To do that, we can rewrite the price as follows:

$$
P_t^{L,n} = \exp\left\{p_t^{L,n}\right\},
$$

where

$$
p_t^{L,n} = at + b(n) + c(n)y_t + d^T z_t
$$

Therefore, by Ito's lemma we have (note that $dn = -dt$)

$$
\begin{aligned}
dp_t^{L,n} &= adt - b'(n)dt - c'(n)y_t dt + c(n)dy_t + d^T dz_t \\
&= \left(a - b'(n) - c'(n)y_t - \kappa c(n)y_t\right)dt + \left(c(n)\begin{bmatrix}\nu_1 \\ -\nu_3\end{bmatrix} + d\right)^T dz_t,
\end{aligned}
$$

where

$$
\begin{aligned}
b'(n) &= \frac{1}{2}\frac{\phi^2}{\kappa^2}\left(\nu_1^2+\nu_3^2\right) - \frac{\phi}{\kappa}\nu_3\left(\sigma-\lambda\right) - \delta + \left(1-\frac{\phi}{\kappa}\right)^2\left(\nu_1^2+\nu_3^2\right)\frac{e^{-2\kappa n}}{2} \\
&\quad + \left(1-\frac{\phi}{\kappa}\right)\left(\frac{\phi}{\kappa}\left(\nu_1^2+\nu_3^2\right) - \nu_3\left(\sigma-\lambda\right)\right)e^{-\kappa n} \\
&= \frac{1}{2}\left(\nu_1^2+\nu_3^2\right)\left(\frac{\phi}{\kappa}+c\right)^2 - \nu_3\left(\sigma-\lambda\right)\left(\frac{\phi}{\kappa}+c\right) - \delta \\
c'(n) &= -\kappa\left(1-\frac{\phi}{\kappa}\right)e^{-\kappa n} = -\kappa c(n)
\end{aligned}
$$

Then, the return on the wage strip equals

$$
\begin{aligned}
r_t^{L,n}dt &= \frac{dP_t^{L,n}}{P_t^{L,n}} = dp_t^{L,n} + \frac{1}{2}\left(dp_t^{L,n}\right)^2 \\
&= \left(a - b'(n) + \frac{1}{2}\left(c\nu_1 + \frac{\phi}{\kappa}\nu_1\right)^2 + \frac{1}{2}\left(\sigma - \frac{\phi}{\kappa}\nu_3 - c\nu_3\right)^2\right)dt + \begin{bmatrix} c\nu_1 + \frac{\phi}{\kappa}\nu_1 \\ \sigma - \frac{\phi}{\kappa}\nu_3 - c\nu_3\end{bmatrix}^T dz_t
\end{aligned}
$$

Therefore, the expected return is

$$\begin{aligned}\mathbb{E}_t\left[r_t^{L,n}\right] &= a - b'(n) + \frac{1}{2}\left(c + \frac{\phi}{\kappa}\right)^2 \nu_1^2 + \frac{1}{2}\left(c - \frac{\sigma}{\nu_3} + \frac{\phi}{\kappa}\right)^2 \nu_3^2 \\ &= \mu - \nu_3\lambda\left(\frac{\phi}{\kappa} + c\right) = \mu - (\mu - r)\frac{\nu_3}{\sigma}\left(\frac{\phi}{\kappa} + c\right)\end{aligned}$$

And the beta is

$$\beta^{L,n} = \frac{Cov_t\left(r_t^m dt, r_t^{L,n} dt\right)}{Var_t\left[r_t^m dt\right]} = \frac{\sigma - \frac{\phi}{\kappa}\nu_3 - c\nu_3}{\sigma} = 1 - \frac{\nu_3}{\sigma}\left(\frac{\phi}{\kappa} + c\right)$$

Also, let's see if the CAPM holds in this economy:

$$\beta^{L,n}\mathbb{E}_t\left[r_t^m - r\right] = \left(1 - \frac{\nu_3}{\sigma}\left(\frac{\phi}{\kappa} + c\right)\right)\left(\mu - r + \phi y_t\right) = \left(1 - \frac{\nu_3}{\sigma}\left(\frac{\phi}{\kappa} + c\right)\right)\left(\mu - r + \phi y_t\right)$$

Compare this to the risk-premium of the return on the wage strip:

$$\mathbb{E}_t\left[r_t^{L,n} - r\right] = \mu - (\mu - r)\frac{\nu_3}{\sigma}\left(\frac{\phi}{\kappa} + c\right) - r = \left(1 - \frac{\nu_3}{\sigma}\left(\frac{\phi}{\kappa} + c\right)\right)(\mu - r)$$

So the CAPM only holds when $y_t$ happen to be zero.

Finally, note that if we assume that $\nu_3 = \sigma$, then the results reduce to

$$\begin{aligned}\beta_t^{L,n} &= \left(1 - \frac{\phi}{\kappa}\right)\left(1 - e^{-\kappa n}\right) \\ \mathbb{E}_t\left[r_t^{L,n}\right] &= \left(1 - \frac{\phi}{\kappa}\right)\left(1 - e^{-\kappa n}\right)(\mu - r) + r\end{aligned}$$

while the discount rate remains unchanged as it does not depend on $\nu_3$.

So, when $n \to \infty$, the beta converges to $1 - \frac{\phi}{\kappa} = 1 - \frac{0.08}{0.16} = 0.5$.

# C Additional figures

**Figure C.1: Market Implied and Social Security Administration Yield Curve Estimates**

The figure presents the differences between the yield curves implied by treasury markets and those used in SSA reports. The SSA implied annualized spot rates are in red for 1989 (dashed) and 2016 (solid) for a 1-79 year horizon. The market implied yield curve is in blue for 1989 (dashed) and 2016 (solid) for the same 79 year horizon. The market series is extended by extrapolating the 29-to-30 year forward rate into the future.

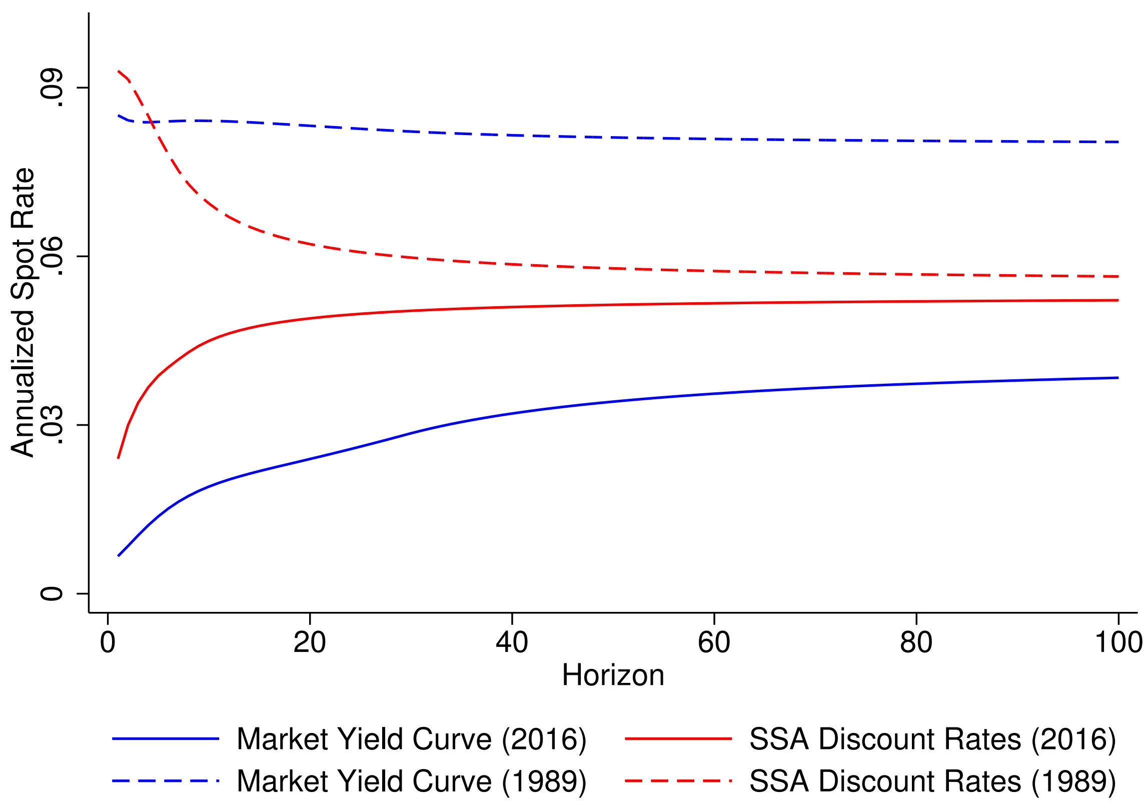


**Figure C.2: Zero-Social Security Income Estimates: Deaton-Paxson Regressions**

This figure shows the results for the Deaton-Paxson regressions outlined in Section A.6. The solid blue and red lines represent the estimated proportion of male and female respondents, respectively, not receiving benefits after adjusting for survey-year and age specific fixed effects in a constrained. The dashed blue and red lines represent the mean proportion not receiving benefits for the 1929-1953 birth cohorts.

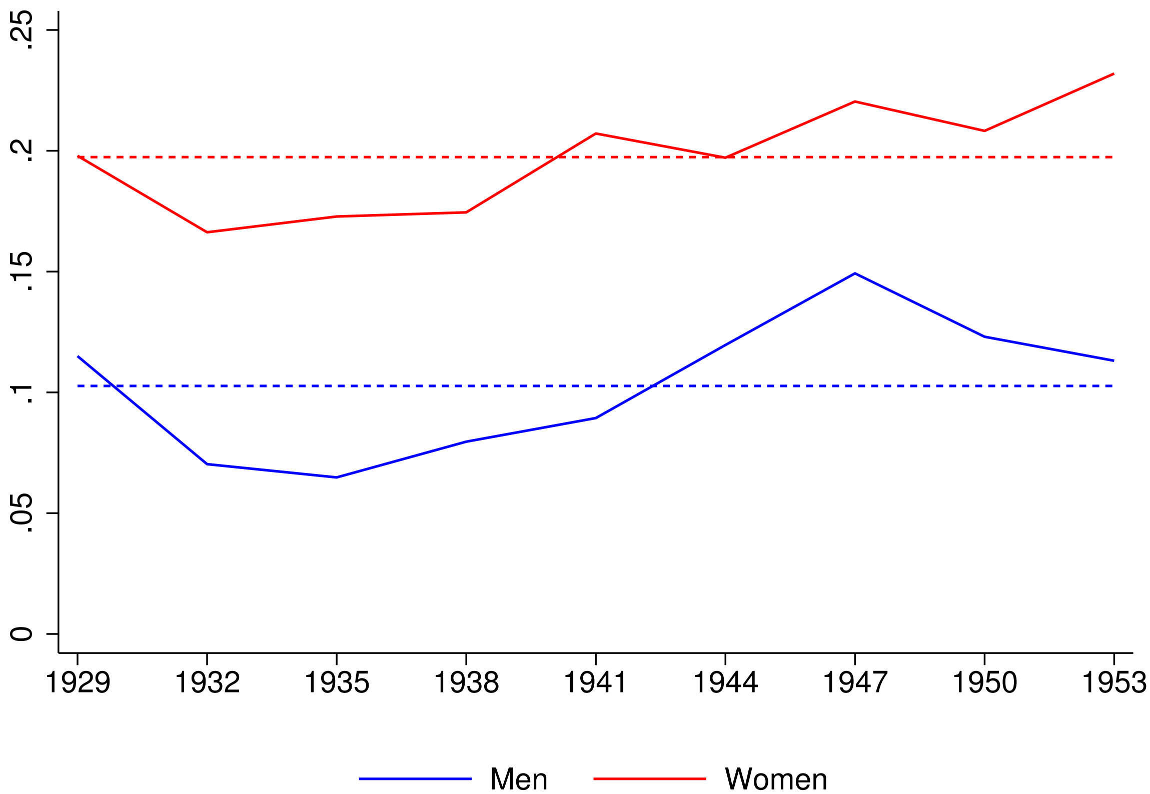


**Figure C.3: Top Wealth Shares: Life Expectancy Adjustment**

This figure shows the top wealth shares with and without adjusting for differences in life expectancy among income centiles using HIP data. The solid, black line shows the top wealth shares without adjusting for life expectancy differences. The dashed, blue line shows the top wealth shares with this adjustment. The methodology for constructing life expectancy adjustments is described in Section A.3

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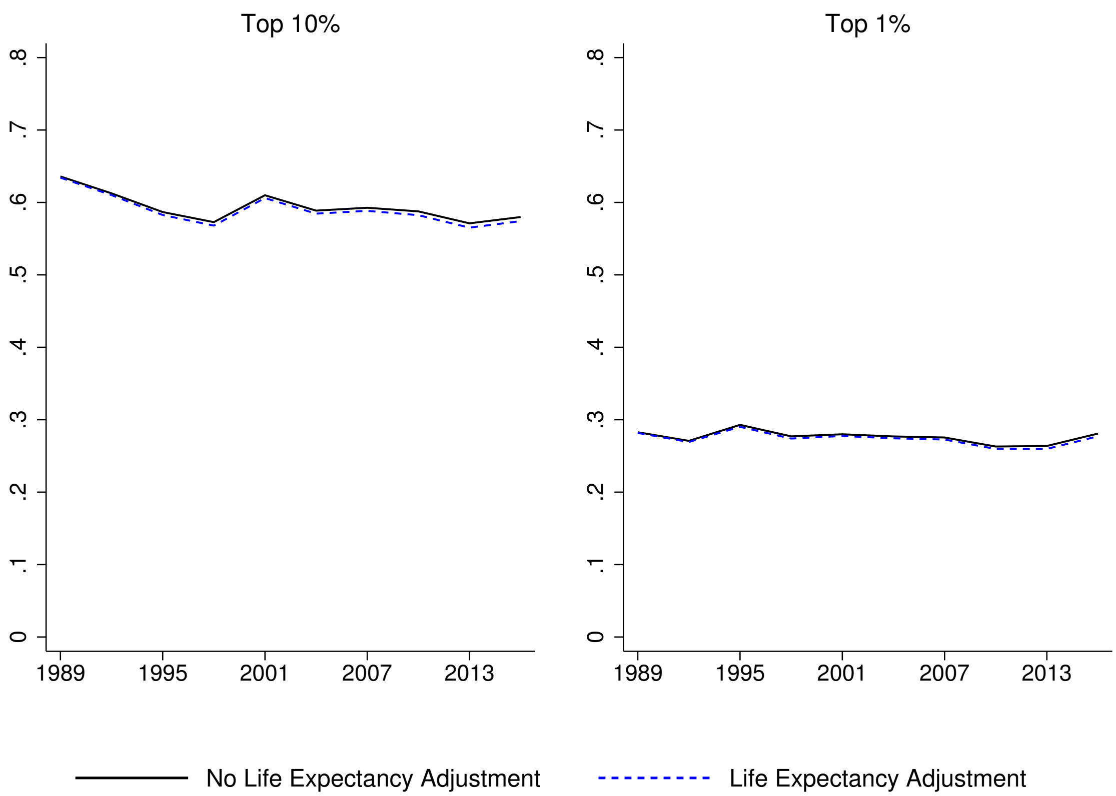

**Figure C.4: Total Social Security Wealth: Fixed Yield Curve**

This figure shows the present value of Social Security under two different yield curve specifications. The first, represented by the solid, black line, shows the present value of Social Security, calculated using the methodology outlined in Section 4 using the market implied yield curve to discount cashflows coming from Social Security benefits. The second, represented by the dashed, blue lines, uses the average yield curve from 1989-2016 to discount the cashflows, respectively. Each series are also adjusted for inflation by placing them in 2018 dollars.

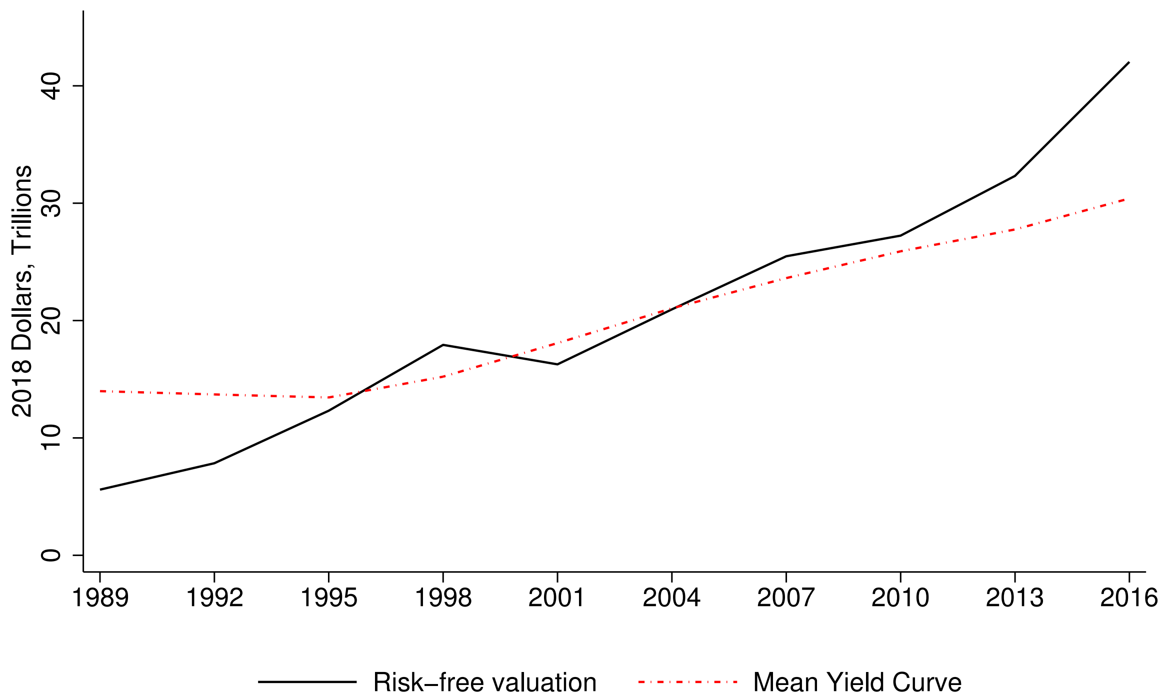

**Figure C.5: Top 10% and Top 1% Wealth Shares: Fixed Yield Curve**

This figure shows the top 10% (Panel A) and top 1% (Panel B) wealth shares with and without Social Security included under the market implied yield curve and the average yield curve from 1989-2016. The solid, black line describes the series with Social Security wealth included and discounting at the market implied rate. The dashed, blue line describes the series with Social Security wealth included and discounting at the average yield curve from 1989-2016. The dotted red line shows the series without Social Security wealth included. Wealth centiles are calculated at the household level based on non-Social Security wealth. Social Security wealth is calculated using the procedure described in Section 4.

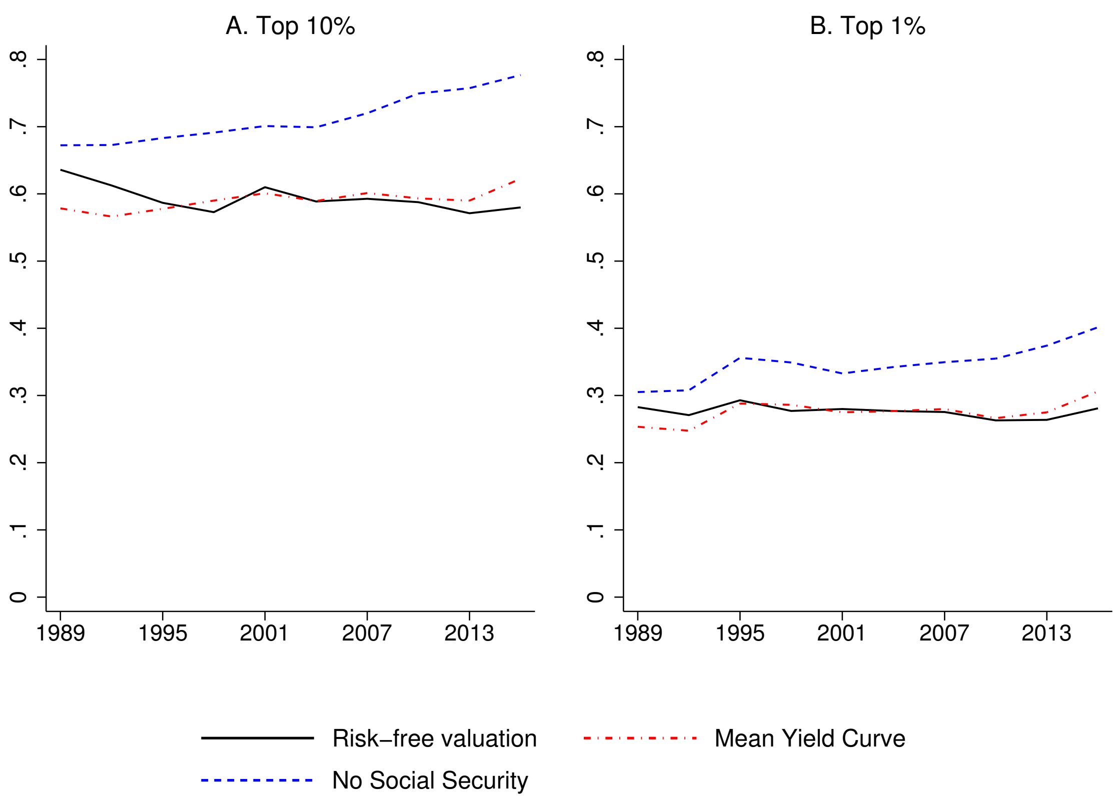

**Figure C.6: Funding Gap: Payable Benefits Under 2016 SSA Projections**

This figure shows the proportion of payable benefits under the SSA's different funding gap assumptions. The solid, black line represents Alternative I under which all scheduled benefits can be paid in full. The dashed, green line represents Alternative II, the SSA's intermediate assumptions, under which scheduled benefits will be cut. The long-dashed and dotted, red line represents Alternative III, the SSA's high cost assumptions, under which scheduled benefits will be cut. Benefits for horizons greater than 75 years are assumed to be the same as the 75th year benefits.

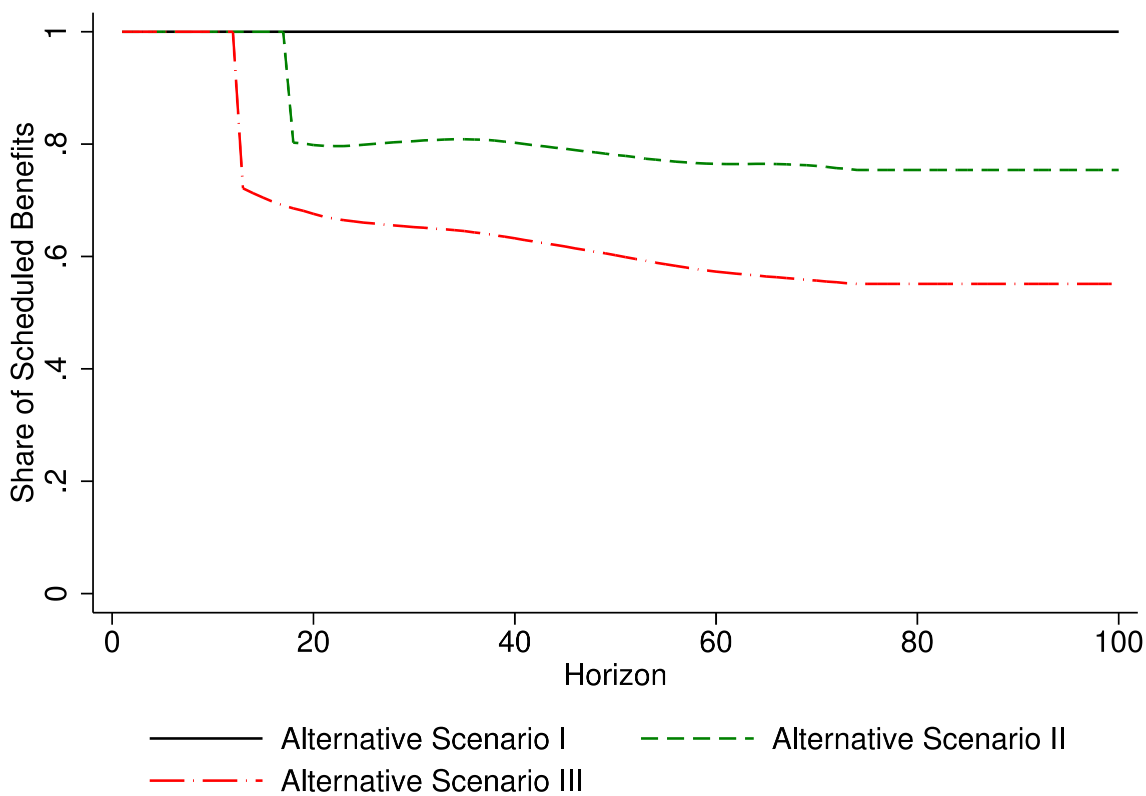

**Table C.1: Calibration of labor income process**

| Parameter | Value | Parameter | Value |
|---|---|---|---|
| $\rho$ | 0.958 | $\sigma_{\varepsilon,2}$ | 0.063 |
| $p_z$ | 21.9% | $\sigma_\alpha$ | 0.298 |
| $\mu_{\eta,1}$ | -0.147 | $\sigma_\beta \times 10$ | 0.185 |
| $\sigma_{\eta,1}$ | 0.457 | $\text{corr}_{\alpha\beta}$ | 0.976 |
| $\sigma_{\eta,2}$ | 0.139 | $a_\nu \times 1$ | -3.2740 |
| $\sigma_{z_1,0}$ | 0.667 | $b_\nu \times t$ | -0.8935 |
| $\lambda$ | 0.001 | $c_\nu \times z_t$ | -4.5692 |
| $p_\varepsilon$ | 12.6% | $d_\nu \times t \times z_t$ | -2.9203 |
| $\mu_{\varepsilon,1}$ | 0.236 | $a_{z_1} \times 1$ | 0.2191 |
| $\sigma_{\varepsilon,1}$ | 0.343 | | |