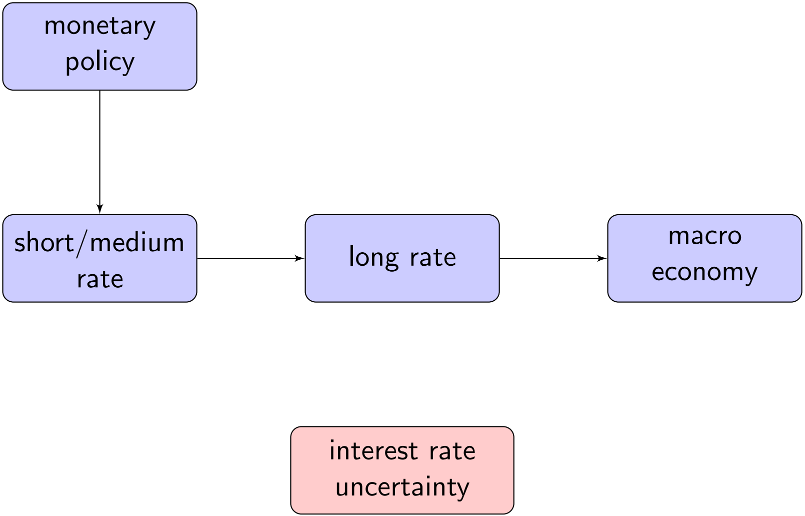
Interest Rate Uncertainty and Economic Fluctuations

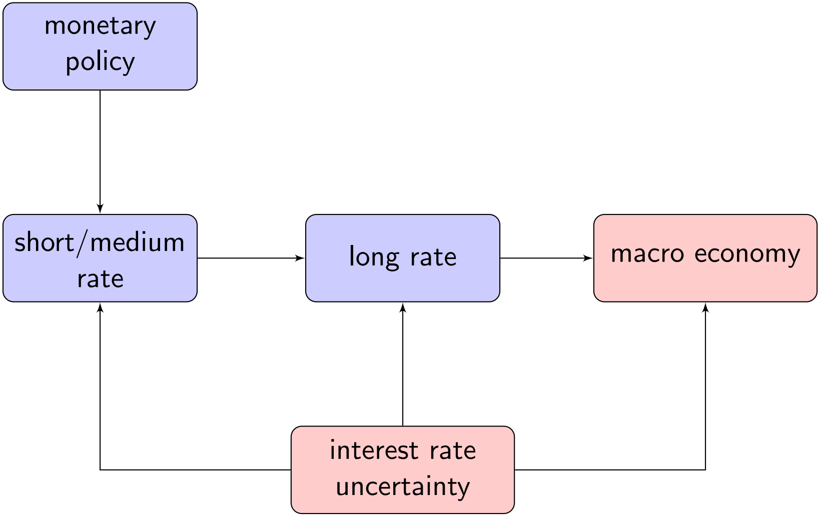
Drew D. Creal
Chicago Booth

Jing Cynthia Wu
Chicago Booth & NBER

Monetary policy transmission



Question: interest rate uncertainty → macroeconomy



Literature

Uncertainty

- first moment
  - Uncertainty: *Baker, Bloom, and Davis(2013), Jurado, Ludvigson, and Ng(2013), Bekaert, Hoerova, and Lo Duca(2013)*
- second moment
  - SV in VAR: *Cogley and Sargent (2001, 2005), and Primiceri(2005)*
- This paper: first moment + second moment

Term structure models

- Spanned model: *Dai and Singleton(2000) and Duffee(2002)*
  - does not fit yield volatility
- Unspanned model: *Collin-Dufresne and Goldstein(2002), Collin-Dufresne, Goldstein and Jones(2009)*
  - restrict yield fitting
  - only 1 volatility factor
- This paper: fit both yields and volatility

More Literature

Contribution: a new model for interest rate uncertainty

Contribution to the uncertainty literature:

- jointly model the first and second moments
    - first moment: conditional mean of macro variables
    - second moment: volatility of interest rates

Contribution to the term structure literature:

- introduce multiple volatility factors that fit the data
    - volatility factors and yield factors are distinct

Result highlight: two dimensions of uncertainty

We find

- 2 volatility factors capture the cross section of yield volatility
- We rotate to “short-term” uncertainty and “long-term” uncertainty
- increases in either of them lead higher unemployment rates
- but they interact with inflation in opposite directions.

Outline

1. Model and estimation
2. Economic implication
3. Yield curve fitting

Factors

- m_t : $M \times 1$ Macro factors
- g_t : $G \times 1$ Gaussian yield factors
- h_t : $H \times 1$ yield volatility factors

Dynamics

$$\begin{aligned} m_{t+1} &= \mu_m + \Phi_m m_t + \Phi_{mg} g_t + \Phi_{mh} h_t + \Sigma_m \varepsilon_{m,t+1}. \\ g_{t+1} &= \mu_g + \Phi_{gm} m_t + \Phi_g g_t + \Phi_{gh} h_t + \Sigma_{gm} \varepsilon_{m,t+1} + \Sigma_g D_t \varepsilon_{g,t+1}, \\ h_{t+1} &= \mu_h + \Phi_h h_t + \Sigma_{hm} \varepsilon_{m,t+1} + \Sigma_{hg} D_t \varepsilon_{g,t+1} + \Sigma_h \varepsilon_{h,t+1}. \end{aligned}$$

where the diagonal time-varying volatility is a function of h_t

$$D_t = \operatorname{diag}\left(\exp\left(\frac{\Gamma_0 + \Gamma_1 h_t}{2}\right)\right).$$

h_t enters the model through

- conditional mean: h_t
- conditional variance: D_t

Impulse responses

Bond prices

Short rate

$$r_t = \delta_0 + \delta_1' g_t.$$

Pricing equation

$$P_t^n = \mathbb{E}_t^{\mathbb{Q}} \left[\exp\left(-r_t\right) P_{t+1}^{n-1} \right]$$

under risk neutral dynamics

$$g_{t+1} = \mu_g^{\mathbb{Q}} + \Phi_g^{\mathbb{Q}} g_t + \Sigma_g^{\mathbb{Q}} \varepsilon_{g,t+1}^{\mathbb{Q}}$$

Bond prices

Bond prices are exponentially affine

$$P_t^n = \exp\left(\bar{a}_n + \bar{b}_n' g_t\right)$$

where

$$\begin{aligned}\bar{a}_n &= -\delta_0 + \bar{a}_{n-1} + \mu_g^{\mathbb{Q}\prime} \bar{b}_{n-1} + \frac{1}{2} \bar{b}_{n-1}' \Sigma_g^{\mathbb{Q}} \Sigma_g^{\mathbb{Q}\prime} \bar{b}_{n-1}, \\ \bar{b}_n &= -\delta_1 + \Phi_g^{\mathbb{Q}\prime} \bar{b}_{n-1}.\end{aligned}$$

Yields $y_t^n \equiv -\frac{1}{n} \log P_t^n$ are linear

$$y_t^n = a_n + b_n' g_t$$

with $a_n = -\frac{1}{n}\bar{a}_n$, $b_n = -\frac{1}{n}\bar{b}_n$. SDF

Novel approach

- bond prices identical to Gaussian ATSMs

Tension between fitting the yield curve and volatility

$$y_t^n = a_n + b_n' g_t + b_{n,h}' h_t$$

Spanned models ($b_{n,h} \neq 0$)

- dual role: volatility factors price bonds
- h_t are forced to fit the conditional mean of yields.

Unspanned models/USV ($b_{n,h} = 0$)

- fit volatility better, but only allow one factor
- restrict yield fitting, see Creal and Wu (2015)

Our model ($b_{n,h} = 0$)

- no restriction on fitting yield curve
- multiple volatility factors

Bayesian estimation

Model

- non-Gaussian non-linear state space model
- likelihood not known in closed form

MCMC

- In each step, conditionally linear Gaussian state space model
- Kalman filter: draw parameters not conditioning on the state variables
- forward filtering and backward sampling: draw state variables jointly

particle filter: compute likelihood

Details

Data and factors

Monthly from June 1953 to December 2013

Yields

- Fama-Bliss zero-coupon yields from CRSP
- maturities: 1m, 3m, 1y, 2y, 3y, 4y, 5y

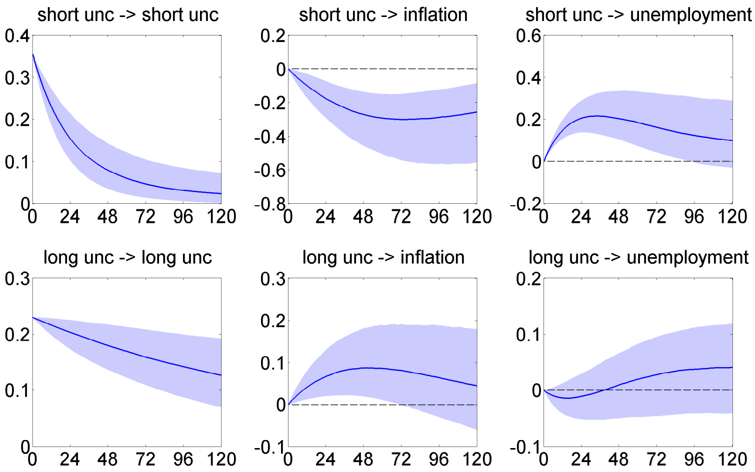
Macro

- FRED

Factors

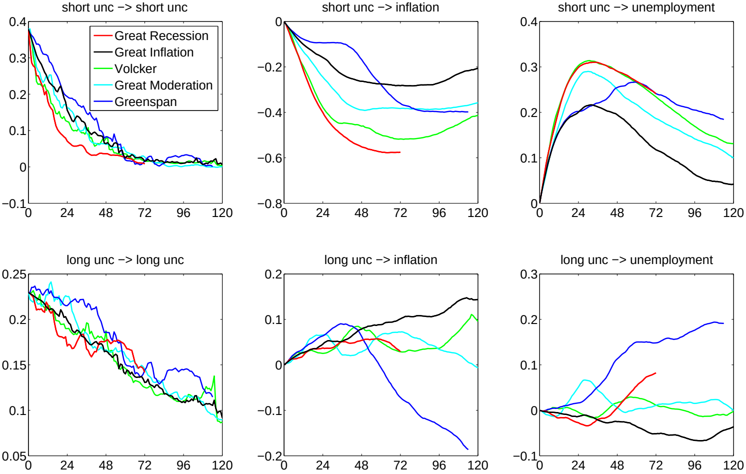
- g_t: 3m, 5y and 1y with errors ▸ Rotation
- h_t: volatility of 3m and 5y
- m_t: inflation and unemployment

Impulse responses



model

Time-varying impulse responses



Uncertainty and recession

$$h_{jt} = \alpha + \beta \mathbb{1}_{recession,t} + u_{jt}$$

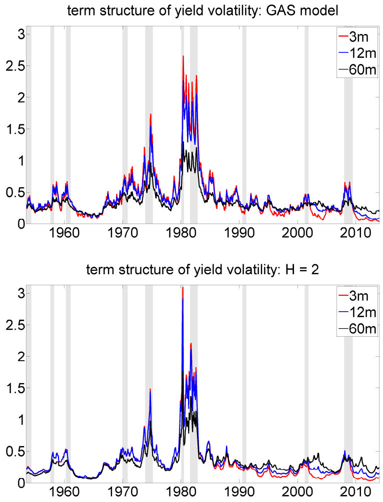
- Coeff: 2.3 for short term; 0.6 for long term
- p-values: 0 for both

Time series of uncertainty

Model specification

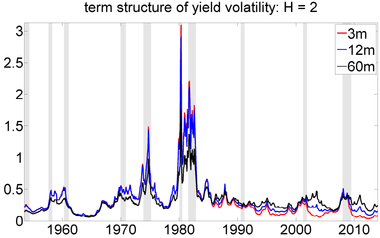
- $M = 0, 2$
- $G = 3$
- $H = 0, 1, 2, 3$

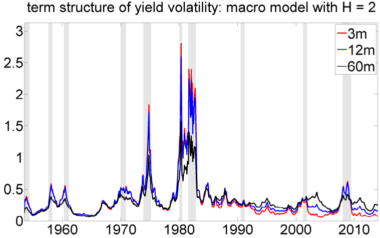
Yield volatilities: how many factors?



BIC chooses $H = 2$ as well.

Yield volatilities: adding macro variables





Cross section of yields

Table: measurement errors

model	$\mathbb{H}_0$	$\mathbb{H}_1$	$\mathbb{H}_2$	$\mathbb{H}_3$	macro
unites	%	ratio			
1m	0.2524	0.9917	1.0170	1.0539	1.0059
3m	0.1283	0.7155	0.6539	0.6196	0.7007
12m	0.1262	0.7726	0.7599	0.7583	0.7995
24m	0.0941	1.0499	0.9904	0.9586	0.9894
36m	0.0781	0.9577	0.8912	0.8489	0.8822
48m	0.1070	0.9103	0.8748	0.8598	0.8804
60m	0.0841	0.9382	0.8644	0.8728	0.9298

Conclusion

We propose a new model

- study the effect of interest rate uncertainty on macro variables
- uncertainty enters both the first and second moments
- the model has multiple volatility factors
- volatility factors evolve separately from yield factors

We find

- 2 volatility factors capture the cross section of yield volatility
- increases in either of them lead higher unemployment rates
- but they interact with inflation in opposite directions.

Literature

Volatility in mean with different applications

- GARCH : *Engle, Lilien, and Robins(1987) and Elder(2004)*
- SV: *Jo (2013)*

Bayesian

- *Chib and Ergashev(2009) and Bauer(2014)*

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Stochastic discount factor

Pricing equation I

$$P_t^n = \mathbb{E}_t^{\mathbb{Q}}\left[\exp\left(-r_t\right) P_{t+1}^{n-1}\right]$$

Pricing equation II

$$P_t^n = \mathbb{E}_t\left[\mathcal{M}_{t+1} P_{t+1}^{n-1}\right].$$

Pricing kernel for any process of h_t under $\mathbb{Q}$.

$$\mathcal{M}_{t+1} = \frac{\exp\left(-r_t\right) p^{\mathbb{Q}}\left(g_{t+1}|\mathcal{I}_t;\theta\right) p^{\mathbb{Q}}\left(h_{t+1}|\mathcal{I}_t;\theta\right)}{p\left(g_{t+1}|\mathcal{I}_t;\theta\right) p\left(h_{t+1}|\mathcal{I}_t;\theta\right)}$$

If we assume the process for h_t is the same under $\mathbb{P}$ and $\mathbb{Q}$

$$\mathcal{M}_{t+1} = \frac{\exp\left(-r_t\right) p^{\mathbb{Q}}\left(g_{t+1}|\mathcal{I}_t;\theta\right)}{p\left(g_{t+1}|\mathcal{I}_t;\theta\right)}$$

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Observed yields

Stack

$$y_t^n = a_n + b_n' g_t$$

for different maturities $n_1, n_2, ..., n_N$ to

$$Y_t = A + Bg_t + \eta_t$$

where $A = (a_{n_1}, \ldots, a_{n_N})'$, $B = (b_{n_1}', ..., b_{n_N}')'$.

State space form I conditional on $h_{0:T}$

Transition equation

$$g_{t+1} = \mu_g + \Phi_{gm} m_t + \Phi_g g_t + \Phi_{gh} h_t + \Sigma_{gm} \varepsilon_{m,t+1} + \Sigma_g D_t \varepsilon_{g,t+1}$$

Observation equations

$$\begin{aligned} m_{t+1} &= \mu_m + \Phi_m m_t + \Phi_{mg} g_t + \Phi_{mh} h_t + \Sigma_m \varepsilon_{m,t+1} \\ h_{t+1} &= \mu_h + \Phi_h h_t + \Sigma_{hm} \varepsilon_{m,t+1} + \Sigma_{hg} D_t \varepsilon_{g,t+1} + \Sigma_h \varepsilon_{h,t+1} \\ Y_{t+1} &= A + B g_{t+1} + \eta_{t+1} \end{aligned}$$

- The volatilities $h_{0:T}$ are known
- Gaussian factors $g_{1:T}$ are latent

State space form II conditional on $g_{1:T}$

Transition equation

$$h_{t+1} = \mu_h + \Phi_h h_t + \Sigma_{hm}\varepsilon_{m,t+1} + \Sigma_{hg} D_t \varepsilon_{g,t+1} + \Sigma_h \varepsilon_{h,t+1}$$

Observation equations

$$\begin{aligned} m_{t+1} &= \mu_m + \Phi_m m_t + \Phi_{mg} g_t + \Phi_{mh} h_t + \Sigma_m \varepsilon_{m,t+1} \\ \hat{g}_{t+1} &= \Gamma_0 + \Gamma_1 h_t + \hat{\varepsilon}_{t+1} \end{aligned}$$

where we define $\tilde{g}_{t+1} = D_t \varepsilon_{g,t+1}$, $\hat{g}_{t+1} = \log\left(\tilde{g}_{t+1} \odot \tilde{g}_{t+1}\right)$.

- Gaussian factors $g_{1:T}$ are observed.
- The volatilities $h_{0:T}$ are latent.
- Approximate the error with mixture of normals using Omori, Chib, Shephard, and Nakajima(2007).

Sketch of MCMC algorithm

- Conditional on $h_{0:T}$, use state space form I
  - Draw θ_g using Kalman filter without depending on $g_{1:T}$
  - Draw $g_{1:T}$ using forward filtering and backward sampling
- Conditional on $g_{1:T}$, use state space form II
  - Draw θ_h using Kalman filter without depending on $h_{0:T}$
  - Draw $h_{0:T-1}$ using forward filtering and backward sampling
- Draw the remaining parameters

Particle filter

- Calculate the likelihood of the model: $p\left(Y_{1:T};\theta\right)$
- Calculate filtered estimates
- We use the mixture Kalman filter, see Chen and Liu (2000)

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Interpretation of Gaussian factors

We rotate the state vector as

$$\begin{pmatrix} y_t^3 \\ y_t^{60} \\ y_t^{12} \\ \vdots \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \vdots \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ & \vdots & \end{pmatrix} g_t + \eta_t$$

The provides an interpretation of the state variables $g_t = (g_{1t}, g_{2t}, g_{3t})'$.

- $g_{1t} = y_t^3$ is the short-term maturity - m.e.
- $g_{2t} = y_t^{60}$ is the long-term maturity - m.e.
- $g_{3t} = y_t^{12}$ is the mid-term maturity - m.e.

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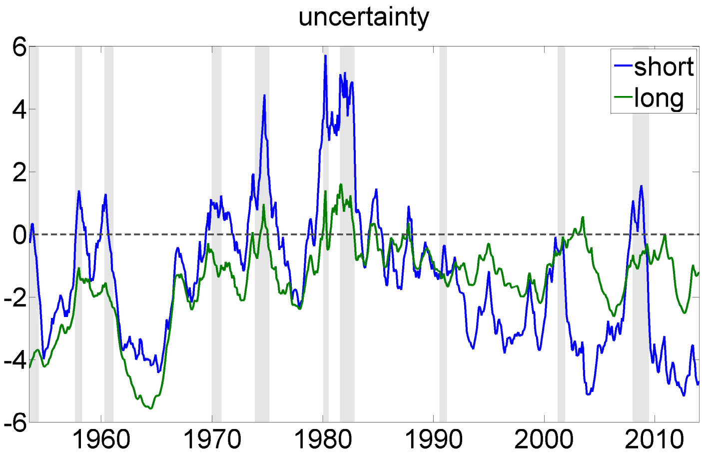
Magnitude of uncertainty



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