Fertility, Social Mobility, and Long Run Inequality*

Juan Carlos Cordoba†, Xiying Liu‡and Marla Ripoll§

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Abstract

Dynastic altruistic models with endogenous fertility have been shown to be unable to generate enough intergenerational persistence. Using a Bewley model of incomplete markets and endogenous fertility we show that it is possible to recover persistence. Key necessary ingredients for our result include exponential child discounting, discrete number of children, and diminishing costs of child rearing. In addition, replicating the negative income-fertility relationship in the data requires an intergenerational elasticity of substitution that is larger than one. Our analysis provides a unified framework of analysis for long-run inequality that incorporates fertility choices.

Keywords: fertility, altruism, inequality, persistence

1 Introduction

During the last two decades the study on inequality has significantly advanced thanks to the development a fairly unified and tractable framework of analysis known as Bewley models.[1]

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†Iowa State University

‡Iowa State University

§University of Pittsburgh

[1] Ljungqvist and Sargent (2012, chapter 18) offers a pedagogical exposition. Some of the contributions in this literature include, among many, Loury (1981), Laitner (1992), Aiyagari (1994), Huggett (1996), Krusell and Smith (1998), Castañeda et al. (2003), and Restuccia and Urrutia (2004). See Cagetti and De Nardi (2008) for a comprehensive survey.

As explained in Aiyagari (1994), these models build upon the standard growth model of Brock and Mirman (1972) by incorporating precautionary saving motives and liquidity constraints. The connection with the standard growth model is very appealing because a single unified framework can be used to study issues of long term growth, business cycles –as in Kydland and Prescott (1982),– and inequality. Implicit in this framework is the idea of dynastic altruism: either individuals are infinitely lived or, more realistically, lives are finite but individuals care about the welfare of their descendants. Dynastic altruism is an important conceptual benchmark because it brings certain level of efficiency, if not full efficiency, to the resulting allocations.

This fairly unified framework, however, seems to fall apart when serious consideration is given to fertility decisions. In particular, Becker and Barro (1988) and Barro and Becker (1989) introduce optimal fertility choices within the optimal growth model and find that some of the most appealing conclusions obtained under the exogenous fertility assumption are seriously altered.[2] On the specific issue of inequality, the optimal fertility choice tends to eliminate any inequality and any persistence of inequality, a result highlighted by Bosi et al. (2011) in the context of a deterministic Barro-Becker model. In contrast, the version of the model with exogenous fertility predicts that any initial inequality is highly persistent, as shown by Chatterjee (1994). An analogous result is obtained using Bewley style models. While Bewley models with infinitely lived agents, as in Aiyagari (1994), or with exogenous fertility, as in Castañeda et al. (2003), predict significant and persistence inequality, the analogous version with endogenous fertility predicts lack of persistence and possibly no inequality (Alvarez, 1999). Section 2 derives and discusses in more detail these results.

The key possibility introduced into the growth model when allowing endogenous fertility is that richer individuals can use family size as a way to obtain higher welfare, an extensive margin, instead of providing more consumption to each descendant, the intensive margin.

[2] Cordoba and Ripoll (2012) discuss some of the counterfactual predictions of the Barro-Becker model. For instance, this model predicts a negative association between individual consumption and individual income. This prediction runs counter to standard consumption theory and a variety of evidence suggesting a positive association between lifetime income and lifetime consumption.

This turns out to be the optimal solution and, as a result, there is no inequality after the original generation. Although inequality can be recovered when markets are incomplete, Alvarez finds an implausible lack of persistence result, or lack of memory, in this case: there is no persistence in economic status after controlling for innate ability. In other words, social mobility is perfect. Jones et al. (2013) find an analogous result, which they call the "resetting" property, in the context of an optimal contract with private information. We derive a version of these results in Section 2 below.

Due to some arguably unrealistic predictions of existing altruistic models with endogenous fertility –namely lack of inequality, lack of persistence and/or a positive response of fertility to income– most of the existing literature on inequality either: *(i)* abstracts from endogenous fertility decisions; or *(ii)* departs from the assumption that parents are purely altruistic and exhibit instead certain type of warm altruism (e.g., De la Croix and Doepke, 2003; Sholz and Seshadri, 2009). Both approaches are convenient for multiple purposes but unsatisfactory for others. For example, by ignoring issues of fertility the recent literature on inequality is silent about the documented strong association between fertility, inequality and poverty, an association that has been used to support family planning programs around the world (e.g., Chu and Koo, 1990). Furthermore, warm altruism is unsatisfactory when addressing issues of policy evaluation and optimal policy design because it introduces, by assumption, inefficiencies at the household level (Kaplow and Shavell, 2001).

Another determinant of inequality is fertility. An older literature on the topic, one that mostly abstracts from savings, inter vivos transfers and bequests, shows that systematic differences in fertility rates among income groups affect the observed distribution of incomes. This literature include authors such as Lam (1986, 1997), and Chu and Koo (1990).

This paper revisits the relationship between fertility, savings and long run inequality in economies populated by altruistic individuals. Since pure altruism is at the core of modern macroeconomics, a field that builds extensively on the dynastic model, it is natural to wonder if pure altruism is ultimate inconsistent with key stylized facts regarding the distribution of wealth and income, as well as evidence of fertility declining with income (Jones and Tertilt, 2008).[3] We consider various ways to recover inequality and asset persistence as well as conditions to replicate a negative fertility-income relationship. We are able to show that, under very natural conditions, pure altruism can generate the degree of inequality, persistence as well as the negative fertility income relationship suggested by the data. To the extent of our knowledge, our model is the first altruistic model to get these predictions right. Our analysis implies that altruism is ultimately consistent with empirical evidence of fertility and inequality, and it provides tools for researchers and policy makers to fully incorporate considerations of fertility and family size into the analysis of inequality.

2 The basic model of dynastic altruism

2.1 Preliminaries

The following is a version of the model studied by Alvarez (1999). An individual lives for two periods, one as a child and one as an adult. Children do not consume. Adults have earning ability ω and receive parental transfers b. We also refer to b as bequest. Lifetime resources are given by $(1+r)\,b+\omega$ where r is a risk free interest rate. Resources can be used to consume, c, or to pay for the cost of raising children. The cost of children includes a time cost, $\Lambda(n)$, and a good cost, nb'. Normalizing total parental time to one, there is maximum feasible number of children, $\overline{n}$, satisfying $\Lambda\left(\overline{n}\right)=1$. Earning abilities are random and drawn from distribution $F(\omega|\omega_{-1})$, where ω_{-1} is the ability of the parent. Individuals know their own earning ability but not the ability of their children.

Preferences are of the form $U(c)+\int_0^n E\left[V_i|\omega\right]\phi_i di$ where $U(c)$ is the utility flow derived from consumption, $E\left[V_i'|\omega\right]$ is expected lifetime utility of child i, $\phi_i \geq 0$ is the weight that the parent places on the welfare of child i, and n is the mass of children. These preferences are appealing because they describe parents as social planners at the house level. Since weights are non-negative, children are goods to parents only if $V_i \geq 0$. This imposes the restriction

[3] Cordoba and Ripoll (2014) address other issues of altruistic models of endogenous fertility besides inequality.

$U(c) \geq 0$. We focus on the CRRA case, $U(c) = \frac{c^{1-\sigma}}{1-\sigma} + A$, where $1/\sigma$ is the elasticity of intergenerational substitution (EGS), a parameter that controls the willingness to substitute consumption between parents and children. As discussed in Cordoba and Ripoll (2014), the EGS is conceptually and quantitatively different from the elasticity of intertemporal substitution (EIS). A positive constant A ensures a positive utility flow in the low curvature case, $\sigma > 1$.

We make assumptions below to guarantee that the optimal V' is a concave function of transfers so that symmetric treatment of children, $V_i' = V$, is optimal.[4] In this case, $\int_0^n E\left[V_i'|\omega\right]\phi_i di = \Phi(n)E\left[V_i'|\omega\right]$ where $\Phi(n) = \int_0^n \phi_i di$ is the weight parents place on their n children. Notice that $\Phi'(n) = \phi_n > 0$. In order to keep utility bounded, it is necessary to assume that parents put more weight on themselves than on all their potential children, $1 > \Phi(\overline{n})$. Assuming further that ϕ_i decreases with i implies that $\Phi(n)$ is concave. Let $\xi(n) = \Phi'(n)\frac{n}{\Phi(n)}$ be elasticity of $\Phi(n)$ with respect to n, an elasticity that plays a central role in fertility choices.

Two functional forms for $\Phi(n)$ are explored below: hyperbolic and exponential child discounting. Hyperbolic discounting is the most common in the literature (e.g., Becker and Barro, 1988). It takes the form $\phi_i = \beta\left(1-\epsilon\right)i^{-\epsilon}$, $0 < \epsilon < 1$, which implies $\Phi(n) = \beta n^{1-\epsilon}$ and a constant elasticity $\xi(n) = 1 - \epsilon$. The restriction $0 < \epsilon < 1$ is required for marginal weights to be positive and decreasing. Alvarez (1999) also considers the case $\epsilon > 1$ combined with a negative utility function so that parental utility increases with the number of children. For completeness, we consider this case below but notice that it implies negative marginal weights, $\phi_i < 0$, so that parents are not altruistic toward all their children.

Exponential child discounting takes the form $\phi_i = \beta\mu e^{-\mu i}$, $\mu > 0$, which implies $\Phi(n) = \beta\left(1 - e^{-\mu n}\right)$ and a decreasing elasticity $\xi(n) = \frac{\mu n}{e^{\mu n}-1}$ which goes from 1 when $n = 0$ to 0 when $n = \infty$. This type of discounting is the natural counterpart of exponential *time* discounting but applied to individuals. It has the convenient property that $\Phi(\infty) = \beta$ so that $\beta < 1$

[4] If parents observe children's ability, then equal division is not necessarily optimal. However, it may be optimal for strategic reasons as in Bernheim and Severinov (2003).

ensures the boundedness of parental utility for any positive fertility.

2.2 Recursive formulation

The following is a recursive formulation of the individual's problem:[5]

$$V(b;\omega) = \underset{\overline{n}\geq n\geq 0,\ \overline{b}\geq b'\geq 0}{Max} \left\{U\left((1+r)\,b+\omega-nb'-\Lambda\left(n\right)\omega\right)+\Phi(n)E\left[V(b';\omega')|\omega\right]\right\}.$$

This problem is not a standard discounted dynamic programming problem due to the endogeneity of the discount factor, $\Phi(n)$, and the non-convexity introduced by the term nb'. As a result standard properties, such as strict concavity of the value function, need to be established. Some properties of the problem are well-known for specific functional forms $U(c)$ and $\Phi(n)$, as in Alvarez (1999). We assume the problem is well-behaved and check numerically that this in fact the case.

Let $n = N(b,\omega)$ and $b' = B(b,\omega)$ be the optimal solution rules. The optimality conditions for n and b', and the Envelope condition for b are, respectively,

$$b' + \omega\Lambda'(n) = \Phi'(n)\frac{E\left[V(b';\omega')|\omega\right]}{U'\left(c\right)}, \tag{1}$$

$$U'\left(c\right) \geq \frac{\Phi(n)}{n}E\left[V_b(b';\omega')|\omega\right],\ \text{with equality if } b'\geq 0,\ \text{and} \tag{2}$$

$$V_b(b;\omega) = (1+r)\,U'(c). \tag{3}$$

The conditions above assume an interior solution for fertility but allow a general solution for transfers. Corner solutions for fertility are discussed below. The left hand side of equation (1) is the marginal cost of a child, including goods and time costs, while the right hand side is the marginal benefit of the n child to a parent. Term $\frac{E[V(b';\omega')|\omega]}{U'(c)}$ is the expected welfare of the child measured in units of parental consumption, while $\Phi'(n) = \phi_n > 0$ is the marginal weight of the n child.

[5] The formulation of the problem introduces upper bounds for n and b', denoted $\overline{n}$ and $\overline{b}$. The upper bound b' is sufficiently large so that it does not bind. A natural upper bound for n is $1/\lambda$.

The optimal condition for bequests can be written, using the last two equations, as

$$U'(c) \geq \frac{\Phi(n)}{n}(1+r)\,E\left[U'\left(c'\right)\right]. \tag{4}$$

This version of the Euler Equation describes optimal intergenerational consumption smoothing. An important difference with the traditional Euler Equation is that the average degree of altruism, $\widehat{\beta}(n) \equiv \frac{\Phi(n)}{n}$, takes the place of the discount factor. As a result, family size plays a key role in determining intergenerational savings, and in particular, larger families have less incentives to save since $\widehat{\beta}'(n) < 0$.

Given the policy functions, the wealth-ability distribution can be computed recursively as:

$$p_{t+1}(b', \omega') = \frac{1}{\overline{n}_t} \sum_{\omega} \sum_{\{b: b'=b(b,\omega)\}} p_t(b,\omega) n(b,\omega) F(\omega'|\omega)$$

where $\overline{n}_t = \sum_{\omega,b} p_t(b,\omega)n(b,\omega)$ is average population growth.

Finally, define (lifetime) earnings and income as $e = \omega(1-\Lambda(n))$ and $i = \omega + rb$. The model does not offer a measure of wealth easily comparable with observed measures of wealth in the data. Variable b' are transfers from parents to children during adulthood and is a measure of dynastic wealth, excluding any life cycle component. Nonetheless, the quantitative exercise we present here will provide insights into the ability of endogenous fertility models to recover certain level of persistence of b. We now discuss two properties of the model regarding persistence and the relationship between fertility, ability and bequests.

2.3 Persistence

The most common functional forms of the dynastic altruism model assumes a constant marginal cost of raising children and hyperbolic child discounting. Proposition 1 states that under those assumption the optimal bequest policy is independent of b and therefore there is no endogenous persistence of inequality.

Proposition 1. Suppose $\Lambda'(n) = \lambda$ and $\xi(n) = \xi$. Then $b' = B(b,\omega) = B(\omega)$.

Proof. Combining (1) and (2) yields:

$$b' + \omega\Lambda'(n) \leq \xi(n)\,\frac{E\left[V(b';\omega')|\omega\right]}{E\left[V_b(b';\omega')|\omega\right]} \text{ with equality if } b' > 0. \tag{5}$$

Under the stated assumptions, condition (5) is independent of n, and therefore the condition fully describes the solution of b': either $b' = 0$ or b' solves equation (5) with equality. Since (5) does not depend on b, the optimal solution takes the form $b' = B(\omega)$.

Proposition 1 states that if the marginal cost of children is constant and the parental weight is an isoelastic function of the number of children, then the optimal bequest policy is independent of b. This result was first obtained by Barro and Becker (1989) for the determinist case, and later extended by Alvarez (1999) to the stochastic case. Our derivation is novel and more direct.[6] We call this result the lack of (endogenous) persistence property. Proposition 1 is particularly important because it remains the most popular, if not the exclusive, formulation of the Barro-Becker model.

Figure 1 illustrates some implications of the lack of persistence property for the deterministic case. Figure 1.a. shows, for given ω, the policy function $b' = B(b)$ for the case of exogenous fertility. The figure assumes for simplicity, but it is not essential, $(1+r)\epsilon = 1$ and $n = 1$. In that case, $b' = b$ is the optimal policy. Thus, if the initial distribution of wealth is described by a vector $\overrightarrow{b}_0$ then financial inequality is perfectly persistent as $\overrightarrow{b}_t = \overrightarrow{b}_0$ for all t. Figure 1.b. shows the policy function for the case of endogenous fertility. In that case, $b' = b^*$ regardless of b. As a result, any initial inequality disappears after one generation, a point made transparent in Bosi et al. (2011). The deterministic altruistic model predicts no persistence of economic status.

Figure 2 illustrates analogous results for the stochastic case. Figure 2a shows the case of exogenous fertility with $(1+r)\epsilon < 1$ and $n = 1$. The figure follows Alvarez (1999). In this case, there is inequality even in the long run and endogenous persistence of wealth:

[6] Our derivation uses the household problem, while Alvarez derive the result by aggregating at the dynasty level. His derivation requires to assume that all children have the same ability $\omega\prime$, while our derivation does not impose this assumption.

conditional on ability, richer parents provide more assets to their children except in the region where $b' = B(b, \omega) = 0$. Figure 2a illustrates the endogenous fertility case: conditional on ability, richer parents do not leave more assets to their children. Economic status is not persistent beyond any persistence that comes from the exogenous persistence of abilities. Whether this channel of pure exogenous persistence is enough to account for the empirical evidence on persistence is a quantitative question. We explore this possibility in the next section and conclude that some important degree of endogenous persistence is needed.

Persistence weakens when fertility is endogenous because richer parents can use family size as a way to increase welfare, the extensive margin, instead of providing more consumption to each descendant, the intensive margin. For the functional forms originally used by Barro and Becker (1989) all (endogenous) persistence disappears.

Proposition 1 suggests that the lack of persistence is an special result obtained for specific but popular functional forms. Equation (5) suggests two ways to recover persistence: an increasing marginal cost of raising children or a decreasing elasticity of altruism. Both alternatives either make more costly or less attractive the use of the family size margin. The second alternative is more appealing since the evidence suggests that the marginal cost of raising children decreases with the number of children due to learning by doing. A third channel is to allow a discrete number of children which limits the extent to which parents can use the family size margin.

2.4 The fertility-ability-bequest relationship

Consider now the ability of the model to generate a negative relationship between fertility and earnings consistent with the empirical evidence. In the context of a deterministic model, Cordoba and Ripoll (2014) have shown that such pattern can only be obtained if the EGS is larger than one, and Cordoba and Liu (2014) find the same result in the context of an stochastic model with no savings. It turns out that $EGS > 1$ is also required in the current model. To see this, it is convenient to rewrite (1) as:

$$\left(b' + \omega\Lambda'(n)\right) U'\left(c\right) = \frac{b' + \omega\Lambda'(n)}{c^{\sigma}} = \Phi'(n)E\left[V(b';\omega')|\omega\right].$$

Notice first that the marginal benefit of having a child, the right hand side of the expression, typically increases with the ability of the parent for two reasons: first, since abilities are intergenerational persistent, the ability of the child also increases; second, since abilities are typically mean reverting, the ability of the child does not increase as much as the ability of the parent inducing parental transfers to increase.

For fertility to decrease with ability, ω, the marginal cost must increase more than the marginal benefit. The marginal cost tend to increase both because the time cost increases, $\omega\Lambda'(n)$, and also because transfers are expected to increase. However, parental consumption also increases with ability which reduces the marginal utility of consumption and lowers the cost of raising children. In other words, the diminishing marginal utility of parental consumption makes children more valuable. If σ is sufficiently large this effect would dominate and fertility will increase with ability. Therefore, a negative fertility-earnings relationship requires a low σ, or high EGS. How high? For this purpose, consider the case of poor individuals, those who are constrained and do not leave any transfers. Their marginal cost of raising children is:

$$\frac{\omega\Lambda'(n)}{c^{\sigma}} = \frac{\omega^{1-\sigma}\Lambda'(n)}{\left(1-\Lambda\left(n\right)\right)^{\sigma}}.$$

For the marginal cost to increase with ability for poor individuals the condition $\sigma < 1$ is needed.

3 Calibration

We now explore the quantitative predictions of various calibrated versions of the dynastic altruistic model. For this purpose we run a horse race between six different versions of the model that differ in: *(i)* whether fertility is endogenous or exogenous; *(ii)* the type of child discounting assumed, either hyperbolic or exponential; *(iii)* the curvature of the utility function, either $EGS > 1$ or $EGS < 1$; and *(iv)* in the marginal cost of raising children, either constant or decreasing. We document weaknesses and strengths of each model, and conclude that overall a model with exponential discounting, $EGS > 1$, and diminish cost of children is the most promising.

3.1 Calibration targets

Our calibration strategy is analogous to the one used by Castañeda et al. (2003). Key parameters are chosen to match specific aspects of Lorenz curves for earnings and wealth. The performance of each model is then assessed along various dimensions, in particular regarding their ability to generate realistic persistence as well as to match other features of the Lorenz curves beyond the matching targets.

When comparing to the existent literature, it is important to keep in mind three aspects of our problem that make the calibration non-standard. First, the earning process is not annual but life time. Second, the curvature of the utility function does not reflect the intertemporal elasticity of substitution. In fact, typical calibrations set $\sigma > 1$ but, as discussed in the previous section, a negative fertility-earnings relationship requires $\sigma < 1$. Third, discount factors are family specific and depend on fertility rates. The following are the models considered:

1. Model 1. Exogenous fertility with $EGS = 2/3 < 1$.

2. Model 2. Exogenous fertility with calibrated EGS.

3. Model 3. Alvarez's (1999) model with continuous number of children.

4. Model 4. Alvarez's (1999) model with discrete number of children.

5. Model 5. Exponential child discounting with constant costs of raising children.

6. Model 6. Exponential child discounting and decreasing costs of raising children.

The models require the Markov chain $F(\omega|\omega_{-1})$, the functional form $\Lambda(n)$, and parameters r, σ, β, ϵ and μ. Models 1 to 4 assume hyperbolic child discounting while Models 5 and 6 consider exponential child discounting. A common interest rate, $r = 2$, is assumed for all models.[7] For the labor endowments shocks we approximate the first-order autoregressive process,

$$\ln \omega' = \rho \ln \omega + e, \; e \sim N(0, \sigma_\omega^2),$$

by a 15 states Markov using the Tauchen Method. The coefficient ρ is the intergenerational persistence of abilities and it will be set to match the intergenerational persistence of (log) earnings. The variance σ_ω^2 is calibrated for each model to match the Gini coefficient of labor earnings.

Consider now the calibration of parameters specific to each model. Models 1 and 2, the exogenous fertility models, only require the extra parameters β and σ since $n = 1$ is assumed. β is identified by targeting the earnings-income correlation. The justification for this target is that β determines the amount of savings, and therefore the earnings-income correlation. For example, the correlation is 100% when there are no savings at all or close to zero if savings are infinite. Model 1 sets $\sigma = 1.5$ as in Castañeda et al. (2003), a standard value in the literature. Model 2 calibrates σ to match the Gini coefficient of bequests. This is because σ controls the degree of precautionary savings and therefore affects the concentration of bequests. The identification of β and σ in all models, except Model 1, is simultaneous because both parameters affect savings, and therefore the correlation of earnings with income and the concentration of bequests.

Models 3 to 6 require to specify a technology for raising children. We use the function $\Lambda(n) = \lambda \left[(n+\kappa)^\theta - \kappa^\theta \right]$, $0 < \theta \leq 1$. Notice that $\Lambda(n) = 0$ and $\Lambda'(n) = \theta\lambda (n+\kappa)^{\theta-1}$. A constant marginal cost is obtained when $\theta = 1$, and decreasing when $\theta < 1$. Parameter κ allows to bound the marginal cost of the first (dn) children. We assume a constant marginal cost, λ, for models 3 to 5. To calibrate λ, existent evidence about time costs of raising

[7] A net return of 2 is obtained if annual returns are 4.5% for 25 years, or 3.73% for 30 years. 25 or 30 years could be considered the midpoint of adult life.

children can be used. However, as discussed in Cordoba and Ripoll (2014), the evidence suggests a wide range of possible values for λ. We decided to calibrate λ within each model and then discuss whether the estimates are plausible or not. The target used to identify λ is the average fertility. λ, turns out, have a strong effect on savings too, because it affects the demand for children, and therefore, the calibration of (β, σ, λ) is simultaneous. To calibrate parameters κ and θ, required for Model 6, we use information about how the cost of raising children changes with number of children.

Finally, the curvature parameters of the altruistic functions $\Phi(n)$, ϵ and μ, are calibrated by targeting the coefficient of variation of fertility. The parameters were chosen to minimize the sum of square errors of the model relative to the targets.

3.2 Computing moments in the data

We use a number of data sources to compute the calibration targets, as well as other moments in the data to evaluate the performance of the different models.

3.2.1 Panel Survey of Income Dynamics

The Panel Survey of Income Dynamics (PSID) is one of our main data sources. Using data from 1968 to 2011, we are able to obtain and link detailed life cycle observations for two generations of parents and their children who have already grown into adults. As it is well known, this is the only available longitudinal data set in which this can be achieved. We use the PSID to compute the persistence, Gini coefficients, and coefficients of variation of wage earnings, income and wealth, as well as the correlation among these variables.

Although, as discussed in the literature, one of the disadvantages of the PSID is that it does not represent well the very rich, it is the best data set for our purpose for three reasons. First, it is the only data set that allows for linking parents and children, as discussed above. Second, because in our model adults live for only one period, measuring earnings, income and wealth requires that we capture the whole lifetime, not just one observation of a specific year. Alternative data sets such as the Survey of Consumer Finances (SCF) provide a better sampling of the very rich, but its cross-sectional nature would not allow us to measure lifetime statistics for individuals. Last, since our purpose is to compare the extent to which different versions of our model can recover intergenerational persistence, we can still provide a ranking of how these models compare for a given set of targets. Intergenerational persistence can only be computed using PSID data, so even if our Gini coefficients were different from those measured using the SCF, our exercise is still informative of our main purpose.

We follow the methodology in Lee and Solon (2009) in order to exploit all available observations for parents and children over the lifecycle. As in Lee and Solon (2009) we: *(i)* exclude any children born before 1952 to avoid over-representing children who left home at a late age; *(ii)* use income observations no earlier than age 25 to more meaningfully capture long-run income; *(iii)* measure children's adult income in the household in which they have become head of head's spouse; *(iv)* use only the Survey Research Center component of PSID and exclude the sample of the Survey of Economic Opportunities, or "poverty sample" due to representation concerns; and *(v)* exclude income observations imputed by major assignments. The result is an unbalanced panel that uses all available years for each individual.

We use the same econometric specification as in Lee and Solon (2009), except that we update the observations until 2011, while their final year was 2000. Their estimation equation is given by:

$$\begin{aligned} y_{ict} = {} & \alpha' D_t + \beta_t X_{ic} + \gamma_1 A_{ic} + \gamma_2 A_{ic}^2 + \gamma_3 A_{ic}^3 + \gamma_4 A_{ic}^4 + \delta_1 (t - c - 40) + \delta_2 (t - c - 40)^2 \\ & + \delta_3 (t - c - 40)^3 + \delta_4 (t - c - 40)^4 + \theta_1 X_{ic} (t - c - 40) + \theta_2 X_{ic} (t - c - 40)^2 \\ & + \theta_3 X_{ic} (t - c - 40)^3 + \theta_4 X_{ic} (t - c - 40)^4 + \varepsilon_{ict} \end{aligned}$$

where y_{ict} is the log of family income for individual i in cohort c and time t; D_t is a vector or year dummies; X_{ic} is parental log income measured as the average of log family income over the three years the child was 15 to 17 years old; β_t is a time-varying intergenerational elasticity; A_{ic} is the parental age at the time in which parental income is observed; and $(t - c - 40)$ is the child's age at the time in which the child's income is observed. The latter implies a normalization such that β is the intergenerational income elasticity at age 40.

For calibration purposes, we only need a single β value, so we eliminate the time variation of this coefficient. However, controlling for age for both parents and children, as well as for time effects, allows our estimation of intergenerational persistence β to include the whole lifetime profile of income of each child who has grown to form his / her own household.

We use the methodology above to compute the intergenerational persistence of both income and wage earnings. Income is measured as PSID variable "total family money", which includes wage earnings of all family members, as well as any other money received by all members of the household. Measuring wage earnings using the PSID is more complicated, as there is not a single variable including wage earning for all family members. Earnings are constructed summing the labor earnings of the head and head's wife, taking into account that after 1994 labor earnings coming from own businesses are reported separately from those coming from employment.

We estimate the specification above separately for sons and daughters, and also for all children including a dummy for daughters. For the case of income, we obtain $\beta_{income} = 0.5319$ (standard deviation of 0.0126) for the later regression, and the equivalent for labor earnings is $\beta_{earnings} = 0.267$ (s.d. of 0.010).

The PSID provides data on family wealth starting only in 1984. Although the methodology of Lee and Solon (2009) described above could in principle be used to compute the persistence of wealth, we instead follow the methodology in Mulligan (1997) for two reasons. First, if parental wealth is measured when the child is between ages 15 and 17, the oldest cohort that could be included is the one from 1969. This means that even for the oldest possible cohort, wealth data after age 25 would only be available until these individuals turned 42 in 2011, relatively earlier in their life cycle. Regressions following Lee and Solon (2009) would then be heavily biased towards the early part of individual's life cycle, partially defeating the purpose of exploiting the whole life cycle information of parents and children. Second, in contrast with income and earnings, wealth is a stock, so the methodology used in Mulligan (1997) should be good enough to estimate intergenerational persistence of wealth.

He measures the average wealth over a five-year period for the parent (head of household) and the child, as well as their average age during that interval. He then regresses the log of the average wealth of the child onto the log of the average wealth of the parent and second-degree polynomials on the average ages of the parent and the child. Given the information available in the PSDI we used this methodology for year intervals 1984-1989, 1994-1999 and 2004-2009. Regressions were run separately for sons, daughters, and for all with a dummy for daughter. The intergenerational elasticity of wealth varies slightly across specifications and interval years. For calibration purposes we used the regression including all children for the latest year, which estimates an intergenerational elasticity of wealth $\beta_{wealth} = 0.3993$ (s.d. of 0.017).

In addition to intergenerational persistence, we use PSID data to compute Gini coefficients of earnings, wealth and income. In order to exploit the panel structure of the data, we control for time and age effects before computing Gini coefficients. In particular, they are computed over the residuals of the following regression:

$$y_{ict} = \alpha' D_t + \delta_1(t-c-40) + \delta_2(t-c-40)^2 + \delta_3(t-c-40)^3 + \delta_4(t-c-40)^4 + \varepsilon_{ict}$$

where y_{ict} is the income, earnings or wealth of individual i in cohort c and time t. Although we computed year-specific Gini coefficients for each variable, in our calibration we only use the Ginis computed over the whole sample of years. We obtained a Gini for income and for earnings of around 0.4, and a Gini for wealth of 0.7632.

We also computed the income - earnings correlation over the residuals of the regression above for each of these two variables. We obtained a correlation of about 0.88. The coefficients of variation for income, earnings, and wealth are given by 1.07913, 1.1855, and 4.02 respectively. Last, the average wealth to average income ratio is 4.022, and the income - wealth correlation is estimated to be 0.3379.

3.2.2 Child Development Survey and USDA

An important set of calibration targets for the model includes the cost of raising children, particularly the time costs. Given that adults in our model live for one period, our target for the model with constant costs of raising children will be the time costs of raising a child as a fraction of lifetime parental income. Using the 1997 Child Development Supplement of the Panel Survey of Income Dynamics, Folbre (2008) estimates the time costs of raising a child by incorporating both primary and secondary time parents spend with children. She concludes that the average amount of both "active" and "passive" parental-care hours per child (not including sleep) is 41.3 per week for a two-parent household with two children ages 0 to 11. Passive care corresponds to the time the child is awake but not engaged in activity with an adult, while active parental care measures the time the child is engaged in activity with at least a parent. In addition to reporting hours spent in child care, Folbre (2008) discusses two alternative ways of computing the monetary value of these hours: one uses a child-care worker's wage and the other the median wage. Folbre (2008) combines this information with the estimates of the goods costs of raising children by the United States Department of Agriculture (USDA, 2012). The latter include direct parental expenses made on children through age 17 such as housing, food, transportation, health care, clothing, child care, and private expenses in education. Folbre concludes that when child-care worker's wages are used to value the hours spend in raising children, then the time cost of raising children is on average around 60% of the total costs (see Table 7.3, p. 135). In addition, since the median wage is around the double of a child-care worker's wage, then using the former time valuation the time cost of raising children increases to 75% of the total costs.

The USDA (2012) computes the present value of the goods costs of raising children ages 0 to 17 for families with low, medium and high income. Using these estimates together with Folbre's scenarios, we can compute the time costs of raising a child as a fraction of lifetime parental income for the average family in each of these income brackets. Since most families in the United States are in the low and middle income brackets, we use the average of these two to compute our target. Specifically, the average family in the USDA (2012) income bracket has an annual income of \$43,625 in 2011, which corresponds to a lifetime income \$1,217,250.[8] Using the most conservative estimate in Folbre (2008) the present value of the time costs of raising a child for this low-income family is \$214,576, about 17.6% of lifetime household income. In the case of the middle-income bracket, the average annual household income is \$81,140, lifetime income is \$2,264,016, and the time cost of raising a child \$297,656, or 13.1% or lifetime income. We use as a calibration target a time cost of raising a child that is on average 15.4% of lifetime household income.

Last, Folbre (2008, Table 6.4) suggests that average relative cost per child goes from 150, 100 to 85 for the first, second and third child (index normalized to 100 for second child). This information is used to calibrate the cost function with decreasing costs of raising children.

3.2.3 Census

The last set of calibration targets in our model include average fertility and the elasticity of fertility with respect to lifetime income. Although the Childbirth and Adoption History module of the PSID includes a measure of total children born that can be used to approximate completed fertility when measured around age 45, this variable is only available starting in 1985. Unfortunately once this information is merged with the income and wealth panel observations, the sample of individuals for which completed fertility is known is too small, under 3,000 observations, to be a representative sample.

Rather than using the PSID to compute average fertility and the income elasticity of fertility, we rely on estimates already available from Jones and Tertilt (2008). They use US Census data as far back as the 1826 cohort to estimate an income elasticity of fertility of about -0.38. Their analysis is distinct in that they construct a more refined measure of lifetime income by using occupational income and education. Lifetime income and fertility are measured for several cross-sections of five-year birth cohorts from 1826-1830 to 1956-1960. They conclude that most of the observed fertility decline in the US can be explained

[8] This computation uses an interest rate of 2% per year and assumes a 40-year working lifespan.

by the negative fertility-income relationship estimated for each cross-section, together with the outward shift of the income distribution over time. The estimated income elasticity is robust to the inclusion of additional controls such as child mortality and the education of husband and wife, suggesting a strong negative correlation between income and fertility. For the latest cohorts in their data, 1956-1960, the income elasticity of fertility is estimated to be -0.22, and the average fertility is 1.8 children per household. Finally, the coefficient of variation of fertility for the latest cohorts is about 0.6.

4 Results

Table 1 shows the calibrated parameters, Table 2 presents the data targets and their model counterparts, and Table 3 reports additional moments besides the matching targets in order to evaluate the performance of different models. Notice that not all models can match all targets. We now discuss the various findings.

Model 1 is a traditional intergenerational Bewley model with exogenous fertility and with $EGS = 0.66$, which corresponds to the standard value of the EIS in quantitative macro models. Model 1 serves as a baseline for comparison. Consistent with similar models in the literature, e.g. Aiyagari (1994), it predicts lower concentration of wealth than in the data as reflected by a relatively low Gini coefficient of bequest (Table 1). Model 2 delivers, by construction, more concentration by reducing σ from 1.5 to 0.79, which decreases risk aversion and the need for precautionary savings. As a result, more individuals become constrained, that is, endow their children with zero bequests. Although Model 2 delivers significantly more concentration than Model 1, it is not enough to match the target.[9]

The calibrated $EGS = 1/\sigma$ in Model 2 is 1.26 which describes individuals much more willing to substitute consumption intergenerationally than intertemporally since common estimates of the EIS are below one. This result is robust to allowing endogenous fertility.[10]

[9] More concentration can be obtained, for example, by changing the earning process as in Castañeda et al. (2003).

[10] Expected utility models do not distinguish between risk aversion and aversion to deterministic fluctuations. Our interpreation of a low σ relative to the typical σ means that parents are less risk averse to gambles on

Another robust result across models is the amount of altruism toward the first child, $\Phi(1)$, which is in the range $0.25 - 0.31$. A common feature of exogenous fertility models is that they predict significant persistence in all outcomes such earnings, income, bequests, and consumption. Although abilities have a 0.267 exogenous persistence, other variables exhibit between 0.73 to 0.83 persistence.[11]

Model 3 is Alvarez's (1999) model, an intergenerational Bewley-Barro-Becker model.[12] Matching all targets is particularly difficult when fertility is endogenous, and the parameters chosen are the ones that minimize the sum squares errors. The calibrated time cost of a child, $\lambda = 0.4$, is concerning but required in order to avoid even more concentration of bequests, which is already high, as more individuals would increase their fertility if the cost is lowered at the expense of reducing bequests per child. Figure 3 shows the policy functions and the predicted relationship between average fertility and both bequests and abilities, and Figure 4 shows Lorenz curves for various variables. The predicted elasticity between fertility and income is -0.22, as in the estimates obtained by Jones and Tertilt (2008), and 0.21 between fertility and bequest (see Table 3).

The most striking difference between the exogenous fertility models and Alvarez's endogenous fertility model is in the degree of persistence of all variables, but particularly of bequests and earnings, which exhibit practically zero persistence when fertility is endogenous. The lack of persistence of earnings, in spite of the 50% persistence of abilities, is explained by a negative correlation in labor supply. Thus, for example, a low ability individual would have more children, lower labor supply, and endow each child with low bequests, so that their relatively asset-poor children would have fewer children of their own and work more. Model 4 restricts fertility to be a discrete number which prevents parents from fully utilizing children as a saving device. The model performs slightly better but the fundamental issue of lack of persistence remains. The last two models assume discrete fertility only.

their children's earnings than to gambles on their own earnings. Cordoba and Ripoll (2014) calibrate the EGS and the EIS in a model with no risk and also obtain $EGS = 1/\sigma > 1$.

[11] Persistence is calculated as the coefficient in a regression between the log of the outcome of the children againts the log of the outcome of the parent.

[12] A continuous number of children is approximated by setting the change in the number of children to 1/30.

Models 5 assumes exponential child discounting, instead of hyperbolic, and maintains the assumption of a constant marginal cost of children. The calibrated time cost of a child is even larger than for Alvarez's model, $\lambda = 0.45$, which is problematic. However, the model now predicts significant amount of persistence of all variables, including earnings and bequests. It also reduces significantly the elasticity of fertility with respect to bequest.

Finally, Model 6 assumes exponential child discounting and diminishing marginal time costs of raising children. Similar to the other models, targets cannot be exactly matched, with the main issue being the low predicted value for coefficient of variation of fertility. The calibrated time cost function for children implies that the marginal time cost of one child is $\theta \cdot \lambda = 0.31$.[13] Figure 5 shows the policy functions and the predicted relationship between average fertility and both bequests and abilities, and Figure 6 shows Lorenz curves for various variables. Jumps up and down in the policies occur because children only come in discrete numbers. The predicted elasticity of fertility to income is -0.23, consistent with empirical evidence, and 0.09 between fertility and bequest, much lower than in hyperbolic case (see Table 3). The key feature of the model is that it predicts similar levels of persistence of bequest as exogenous fertility models, or around 78%, and significantly increases the persistence of earnings.

5 A policy experiment: estate taxes

To illustrate the importance of taking into account fertility decisions, we now conduct a policy experiment and compare the implications according to Model 2, the exogenous fertility model, and Model 6, our preferred model of endogenous fertility. For this purpose consider the long term effect of introducing a 10% estate tax used to finance some exogenous government expenditures. Furthermore, we don't consider changes in the interest rate so that the results correspond either to partial equilibrium or to a small open economy. Results are reported in Table 4. Consider first the effects of the policy in the economy with exogenous fertility.

[13] The calibrated value for $\kappa = 0$. A value of $\kappa > 0$ helps improve the predictions of the model when a continuous number of children is allowed.

The policy significantly reduces steady state bequests, incomes and consumption, but do not affect earnings. The policy reduces inequality, as measured by standard deviations, although Gini coefficients remain unchanged. These results are consistent with the ones reported by Castañeda et. al. (2003). They also report no changes in social mobility, as measured by the fraction of household that remain in same quantile after 5 years, but we find significant reduction in our persistence parameters meaning that higher estate taxes increase social mobility.

The predictions of the endogenous fertility model differ significantly. This is because an increase in estate taxes reduces the welfare of children and therefore the incentives to have children. As a result, fertility rates fall, and labor supply as well as labor earnings increase, instead of being constant or decrease. Consumption and income fall but to much less extent than the exogenous case. Inequality of bequests decrease with the tax, but other measures of inequality increase such standard deviations and Gini coefficients for earnings, consumption and income. Social mobility in income, bequests and consumption increase but decrease in earnings.

6 Conclusions

We have shown that models of endogenous fertility by dynastic altruistic parents can replicate similar persistence as analogous models with exogenous fertility. Introducing endogenous fertility considerations is important for policy evaluation because the decision to have or not children, and how many, affect most long term economic variables like consumption, savings, income or labor supply. We show, for example, that the long run effects of estate taxes are substantially different when fertility is endogenous.

We recover realistic levels of persistence by combining three novel elements into an otherwise standard Bewley model. An intergerational elasticity of substitution larger than 1, as opposed to the typical intergenerational elasticity of substitution less than 1, exponential child discounting instead of hyperbolic discounting, and increasing returns in child rearing.

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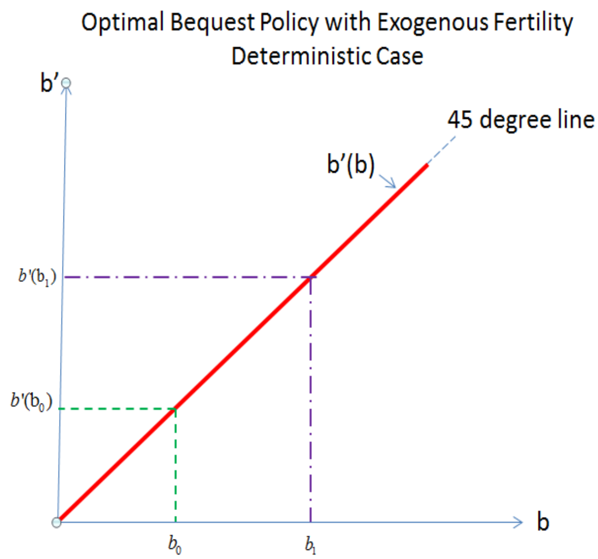


Figure 1.a.

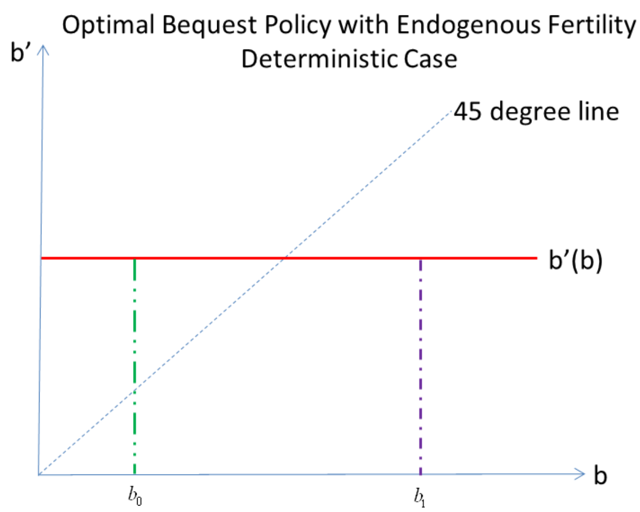


Figure 1.b.

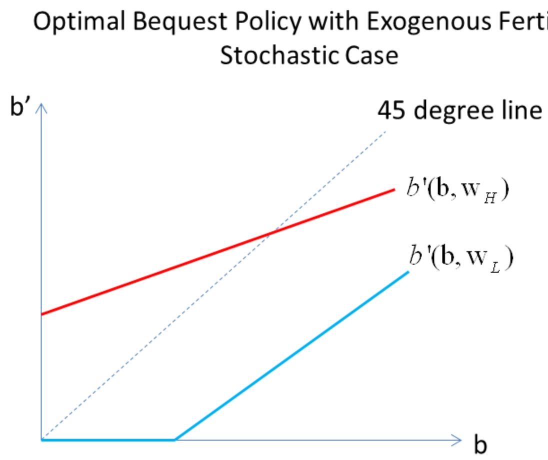


Figure 2.a.

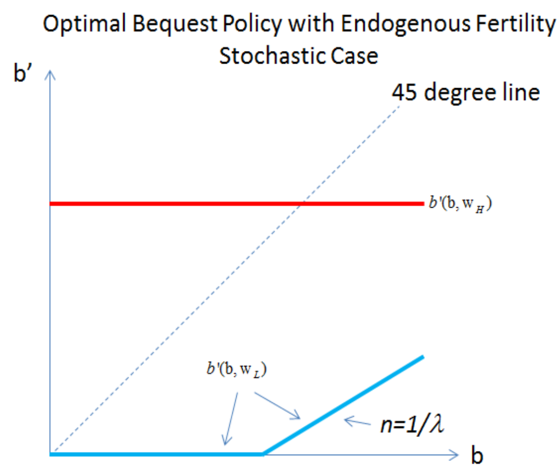


Figure 2.b.

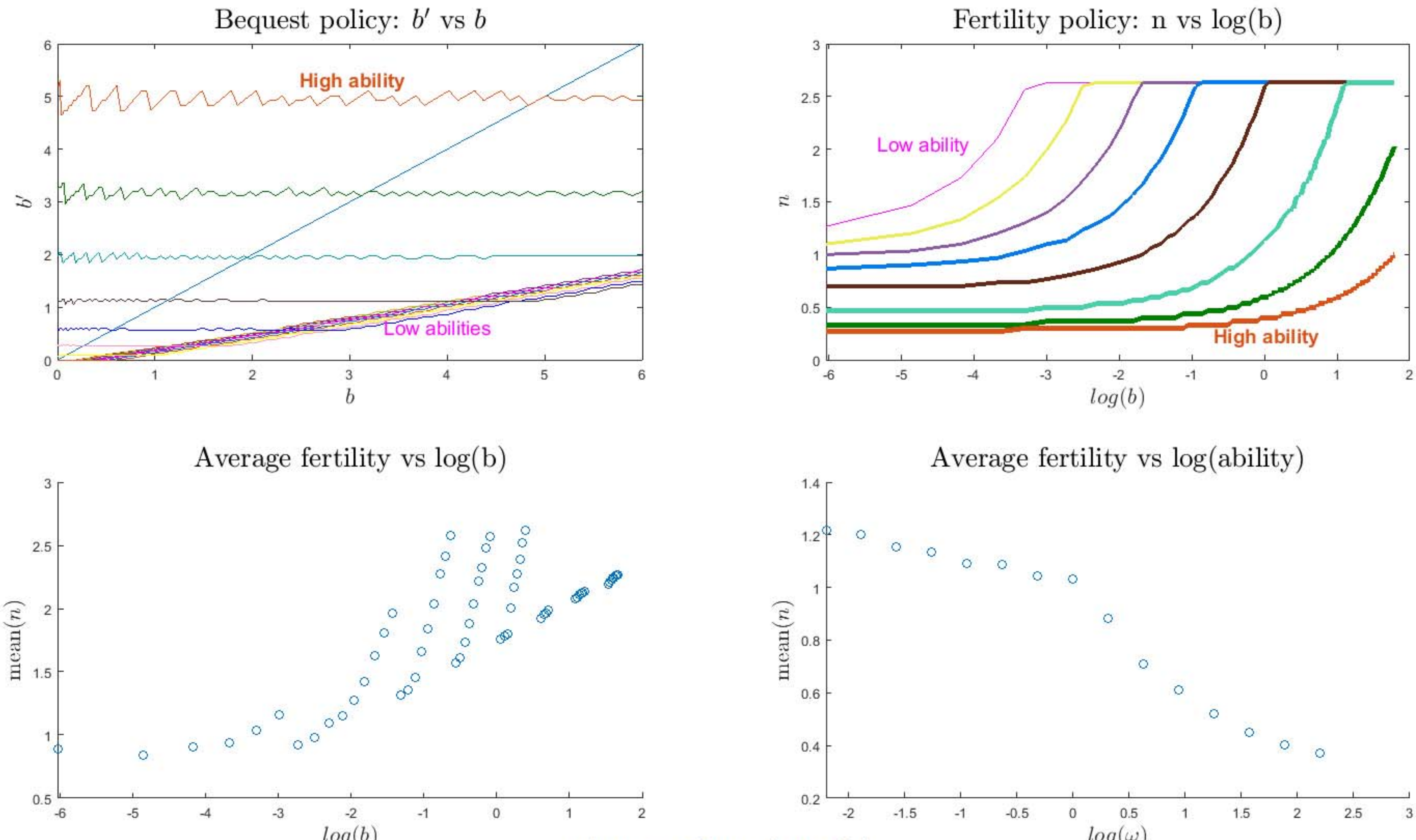
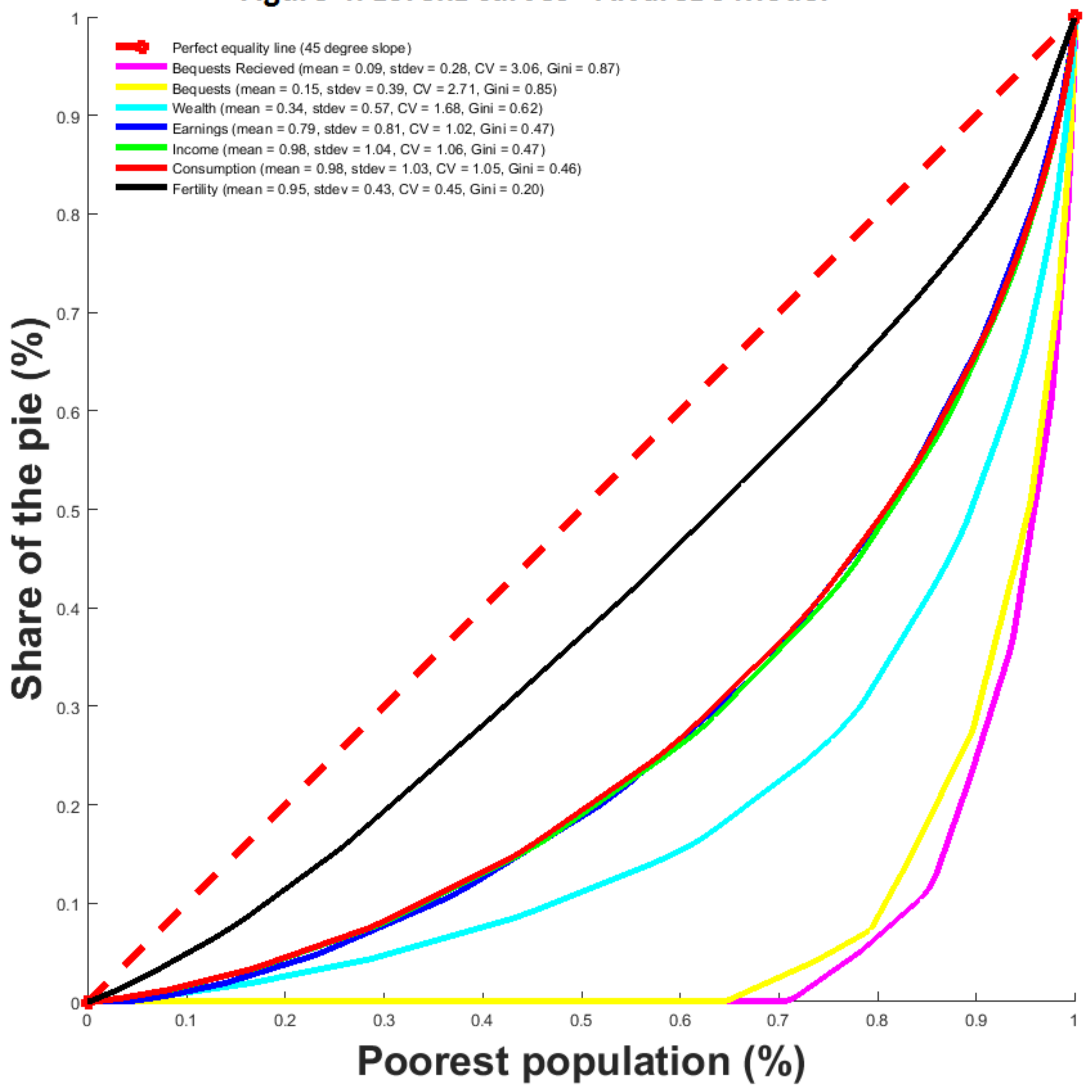


Figure 3: Alvarez's Model

Figure 4: Lorenz curves - Alvarez's Model



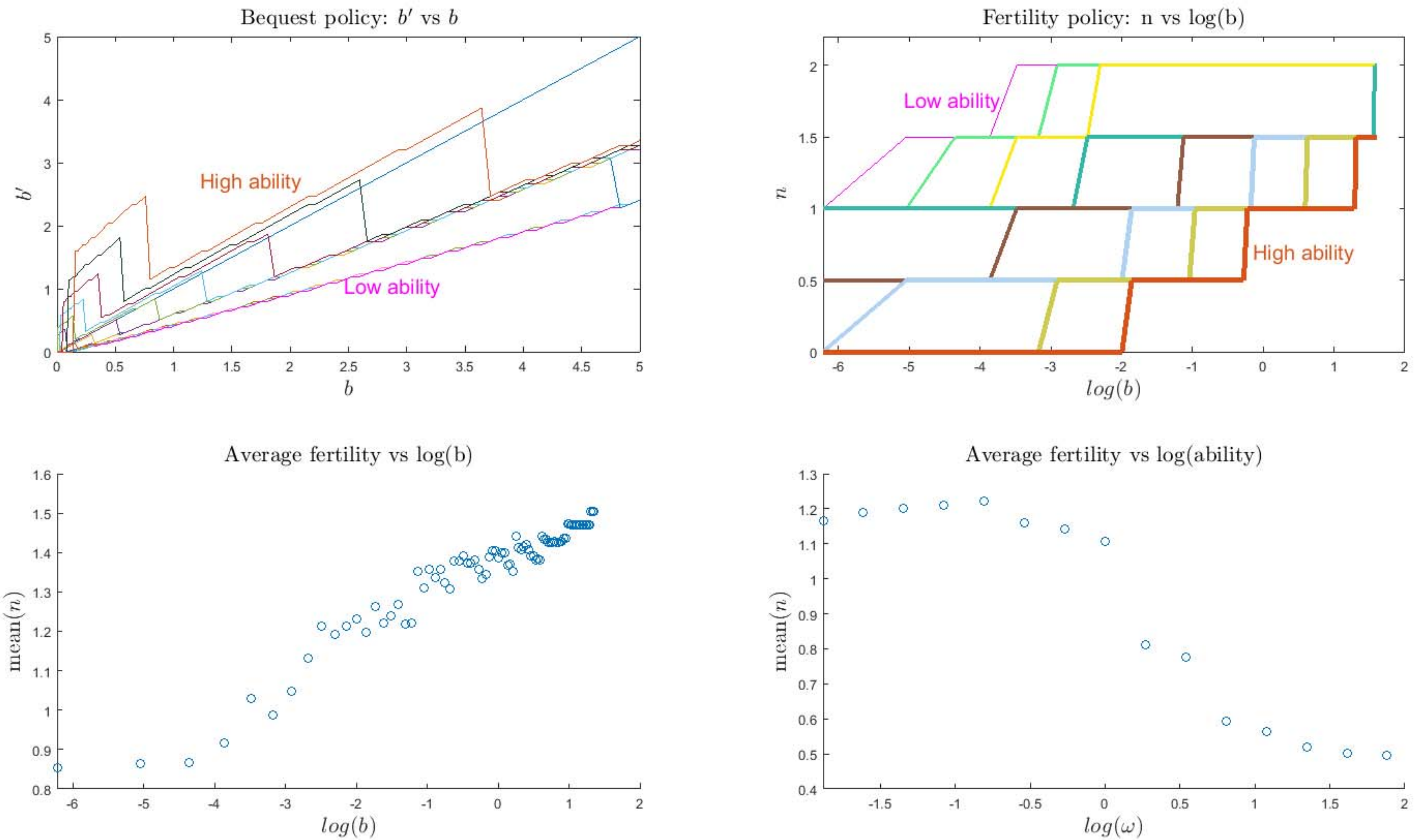


Figure 5: Model with exponential child discounting and diminishing marginal costs

Figure 6: Lorenz curves - Exponential child discounting and diminishing marginal costs

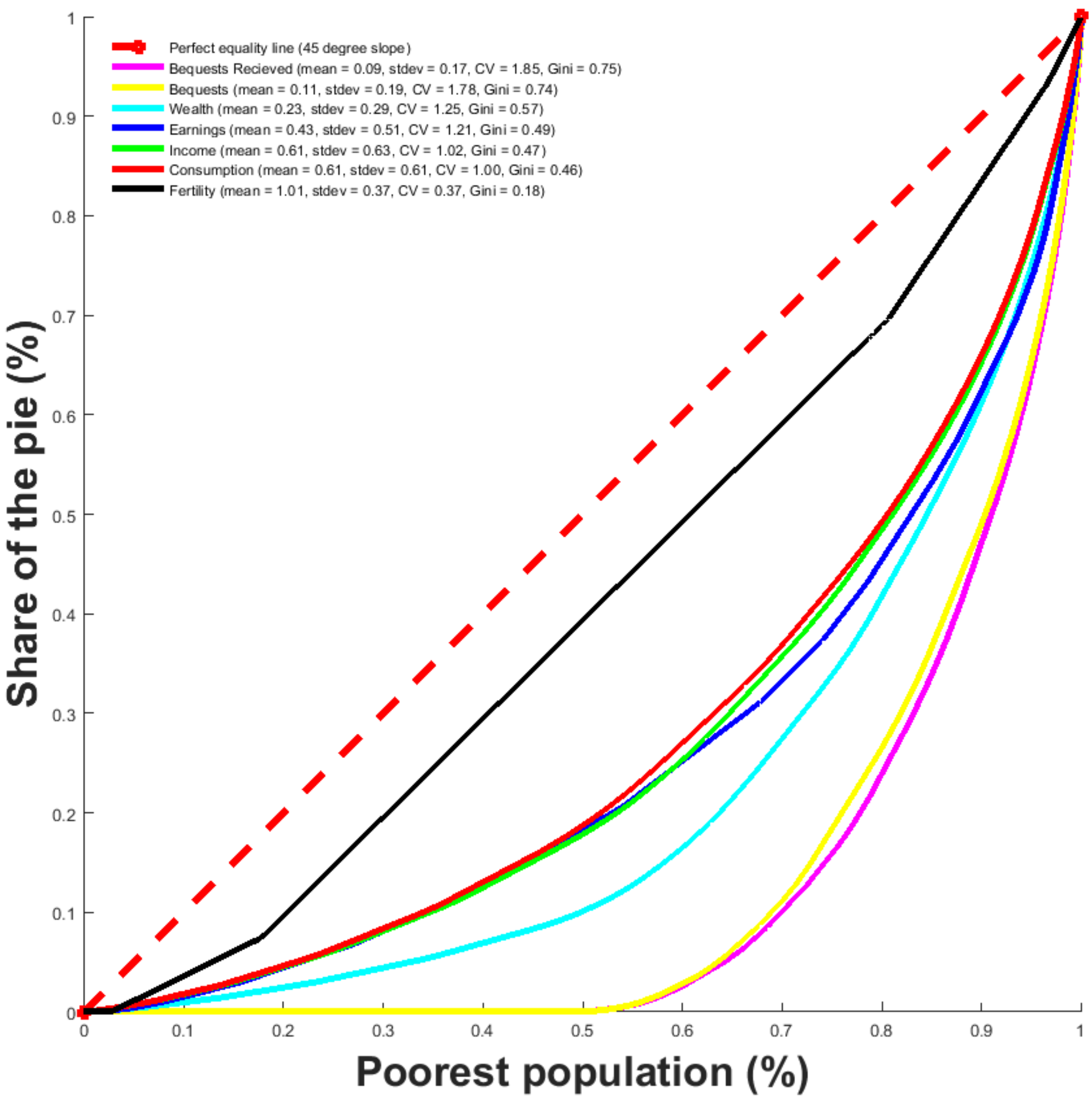


Table 1: Targets

Target	Data	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
An annual interest rate of 4.5 for 25 years	2.00	2.000	2.000	2.000	2.000	2.000	2.000
Gini bequest	0.76	0.620	0.710	0.850	0.760	0.760	0.740
Gini earnings	0.40	0.400	0.410	0.470	0.480	0.460	0.490
Persistence of abilities	0.50	0.500	0.500	0.500	0.500	0.500	0.500
correlation earnings-income	0.88	0.880	0.890	0.839	0.843	0.909	0.834
average fertility	1.00	------	------	0.950	0.940	0.850	1.010
coefficient of variation fertility	0.60	------	------	0.450	0.510	0.300	0.370
cost two children/cost one child	1.33	------	------	------	------	------	1.370
cost three children/cost two children	1.28	------	------	------	------	------	1.200

Model 1: Exogenous fertility (EGS=2/3), Model 2: Exogenous fertility (EGS calibrated), Model 3: Alvarez continuous, Model 4: Alvarez discrete, Model 5: CLR exponential with constant cost, Model 6: CLR exponential with decreasing cost.

Table 2: Parameter Values

Parameter	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
r = Interest rate	2.00	2.00	2.00	2.00	2.00	2.00
σ = curvature utility function	1.50	0.79	0.72	0.69	0.68	0.68
β = discount factor	0.25	0.28	0.25	0.25	0.33	0.46
ε=curvature hyperbolic discounting	---	---	0.57	0.57	---	---
μ = curvature exponential discounting	---	---	---	---	1.36	1.12
$\Phi(1)$ = altruism toward first child	0.25	0.28	0.25	0.25	0.25	0.31
λ = parameter cost of children	---	---	0.38	0.38	0.45	0.68
η = parameter cost of children	---	---	---	---	---	0.00
θ = elasticity cost of children	---	---	---	---	---	0.45
$\Lambda(1)$ = time cost of one child	---	---	0.38	0.38	0.45	0.31
ρ_ω = persistence log ability	0.50	0.50	0.50	0.50	0.50	0.50
σ_ω = dispersion log ability	0.65	0.62	0.64	0.64	0.64	0.54

Model 1: Exogenous fertility (EGS=2/3), Model 2: Exogenous fertility (EGS calibrated), Model 3: Alvarez continuous, Model 4: Alvarez discrete, Model 5: CLR exponential with constant cost, Model 6: CLR exponential with decreasing cost.

Table 3: Performance

	Data	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Persistence of earnings	0.27	0.50	0.50	0.01	0.09	0.41	0.26
Persistence income	0.53	0.73	0.73	0.57	0.59	0.61	0.67
Persistence bequest	0.40	0.76	0.83	0.01	0.02	0.46	0.78
Persistence consumption	0.70	0.77	0.75	0.56	0.59	0.64	0.69
Persistence fertility	---	---	---	-0.08	-0.16	-0.10	0.01
Gini income	0.40	0.41	0.46	0.47	0.47	0.47	0.47
Gini consumption	---	0.39	0.48	0.46	0.47	0.46	0.46
Gini fertility	---	---	---	0.20	0.24	0.14	0.18
Coefficient variation earnings	1.19	0.86	0.84	1.02	1.00	0.99	1.21
Coefficient variation income	1.08	0.82	0.85	1.06	1.04	1.00	1.02
Coefficient variation bequest	4.02	1.37	1.65	2.71	2.03	2.04	1.78
Correlation earnings-bequest (e,b)	---	0.69	0.68	0.91	0.89	0.90	0.46
Correlation income-bequest (i,b)	0.34	0.94	0.92	0.89	0.92	0.96	0.72
Income elasticity of fertility	-0.22	---	---	-0.22	-0.29	-0.23	-0.23
Bequest elasticity of fertility	---	---	---	0.21	0.19	0.05	0.09

Model 1: Exogenous fertility (EGS=2/3), Model 2: Exogenous fertility (EGS calibrated), Model 3: Alvarez continuous, Model 4: Alvarez discrete, Model 5: Exponential altruism with constant cost, Model 6: Exponential altruism with decreasing cost.

Table 4: Estate Taxation

	Exogenous Fertility Model			Endogenous Fertility Model		
	Benchmark	+10% Estate Tax	% change	Benchmark	+10% Estate Tax	% change
Mean earnings	0.81	0.810	0.0%	0.43	0.52	20.9%
Mean income	1.07	0.910	-15.0%	0.61	0.57	-6.6%
Mean bequest	0.13	0.060	-53.8%	0.11	0.04	-63.6%
Mean consumption	1.07	0.910	-15.0%	0.61	0.57	-6.6%
Mean fertility	1.00	1.000	0.0%	1.01	0.87	-13.9%
Std earnings	0.69	0.690	0.0%	0.51	0.68	33.3%
Std income	0.92	0.800	-13.0%	0.63	0.69	9.5%
Std bequest	0.22	0.130	-40.9%	0.19	0.10	-47.4%
Std consumption	0.88	0.750	-14.8%	0.61	0.69	13.1%
Std fertility	0.00	0.000	---	0.37	0.39	5.4%
Persistence of earnings	0.53	0.52	-1.9%	0.26	0.30	16.9%
Persistence income	0.73	0.64	-12.3%	0.66	0.59	-10.6%
Persistence bequest	0.85	0.73	-14.1%	0.78	0.63	-19.2%
Persistence consumption	0.76	0.66	-13.2%	0.68	0.60	-11.8%
Persistence fertility	---	---	---	0.00	-0.16	---
Gini income	0.42	0.42	0.0%	0.47	0.52	10.6%
Gini consumption	0.40	0.40	0.0%	0.46	0.52	13.0%
Gini fertility	---	---	---	0.18	0.21	16.7%
Coefficient variation earnings	0.84	0.84	0.0%	1.21	1.30	7.4%
Coefficient variation income	0.86	0.87	1.2%	1.02	1.21	18.6%
Coefficient variation bequest	1.66	2.35	41.6%	1.78	2.85	60.1%
Correlation earnings-bequest (e,b)	0.70	0.79	12.9%	0.46	0.19	-58.7%
Correlation income-bequest (i,b)	0.94	0.91	-3.2%	0.72	0.31	-56.9%
Income elasticity of fertility	---	---	---	-0.22	-0.15	-31.8%
Bequest elasticity of fertility	---	---	---	0.09	0.11	25.0%