Discussion of
Monfort, Pegoraro, Renne, and Roussellet

"Staying at Zero with Affine Processes: A New Dynamic Term Structure Model"

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(With special thanks to Leo Krippner and Minchul Shin)

Volatility Institute
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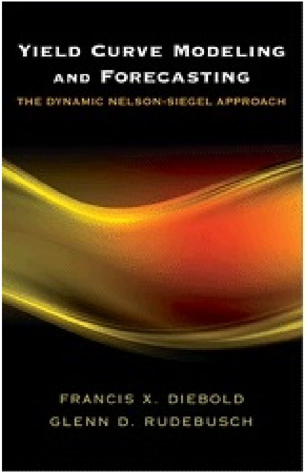
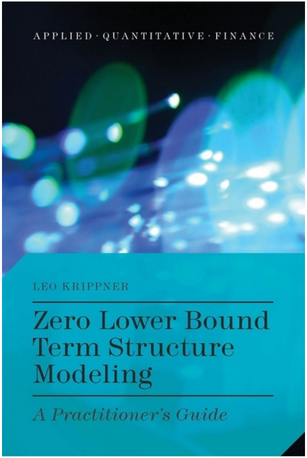
Favorite Books





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Lots of Constrained Series in Finance

"Soft" barriers:

- Exchange rate target zones
- Inflation corridors

"Hard" barriers:

- Volatilities: e.g., asset returns
- Durations: e.g., intertrade
- Rare-event counts: e.g., bankruptcies
- Nominal bond yields

Lots of Associated Constrained Processes in Financial Econometrics

- Vols: GARCH, stochastic volatility, and more
- Durations: ACD and more
- GAS and MEM
  (Creal, Koopman, and Lucas, 2013; Harvey, 2013)

What About Bond Yields?

Duffie-Kan (1996) Gaussian affine term structure model (GATSM):

State x_t is an affine diffusion under the risk-neutral measure:

$$dx_t = K(\theta - x_t)dt + \Sigma \, dW_t$$

Instantaneous risk-free rate r_t is affine in x_t:

$$r_t = \rho_0 + \rho_1' x_t$$

Duffie-Kan arbitrage-free result:

$$y_t(\tau) = -\frac{1}{\tau} B(\tau)' x_t - \frac{1}{\tau} C(\tau)$$

– Arbitrage-free
– Simple (closed-form)
– But fails to respect the ZLB

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Constrained Processes for Bonds

- Square root: $dx_t = k(\theta - x_t)\,dt + \sigma\sqrt{x_t}\,dW_t$
  (Cox, Ingersol and Ross, 1976)
- Others: lognormal, quadratic
- Autoregressive gamma ($ARG(1)$)
  (Gourieroux and Jasiak, 2006)
- $ARG0(1)$
  (MPRR, 2015)

$ARG(1)$

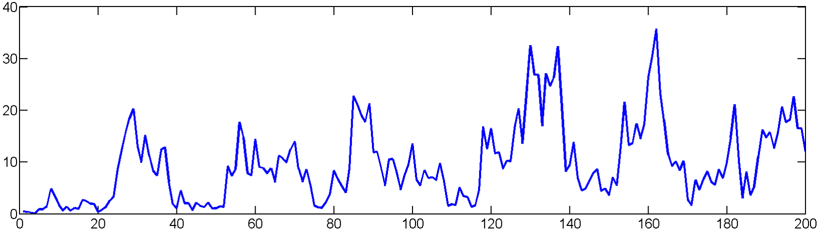
x_t is an $ARG(1)$ process if
$x_t | x_{t-1}$ is distributed non-central gamma with:

- Non-centrality parameter βx_{t-1}
- Scale parameter $c > 0$
- Degree of freedom parameter $\delta > 0$

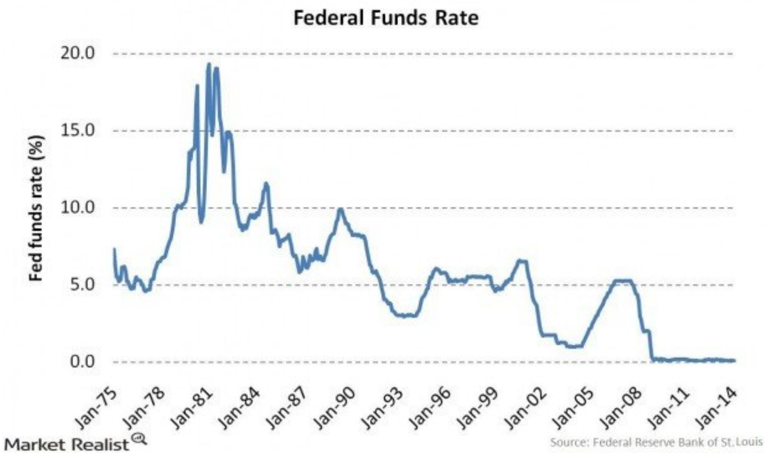
– Non-negative (obvious)

– Diffusion limit is CIR (not obvious)

Simulated $ARG(1)$ Realization



But Alas...


Federal Funds Rate
20.0
15.0
10.0
5.0
0.0
Fed funds rate (%)
Jan-75
Jan-78
Jan-81
Jan-84
Jan-87
Jan-90
Jan-93
Jan-96
Jan-99
Jan-02
Jan-05
Jan-08
Jan-11
Jan-14
Market Realist
Source: Federal Reserve Bank of St. Louis

$ARG0(1)$

If $x_t \sim ARG(1)$, then

$$x_t | z_t \sim Gamma(\delta + z_t, c)$$
$$z_t | x_{t-1} \sim Poisson(\beta x_{t-1})$$

If $x_t \sim ARG0(1)$, then

$$x_t | z_t \sim Gamma(z_t, c)$$
$$z_t | x_{t-1} \sim Poisson(\alpha + \beta x_{t-1})$$

- $ARG0$ takes $\delta \to 0$, which makes $x_t = 0$ a mass point. (As $\delta \to 0$, $G(\delta, c) \to$ Dirac's delta.)
- Introduces α, which governs probability of escaping the ZLB. (Note that $\alpha = 0 \implies x_t = 0$ is an absorbing state.)

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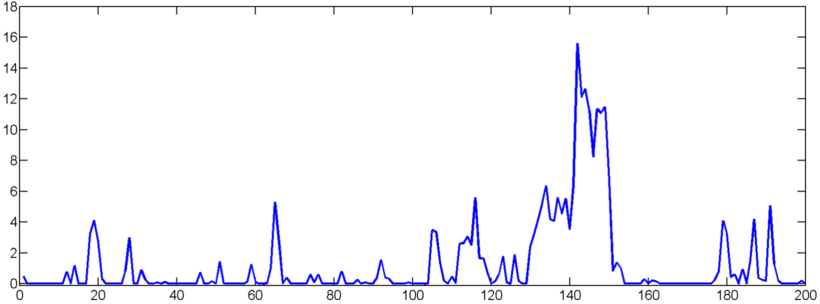
Simulated $ARG0(1)$ Realization



$ARG0$ Approach

$$x_t | z_t \sim Gamma(z_t, c)$$
$$z_t | x_{t-1} \sim Poisson(\alpha + \beta x_{t-1})$$

1. Arbitrage-free
2. Simple (closed-form)
3. Respects the ZLB

End of story?

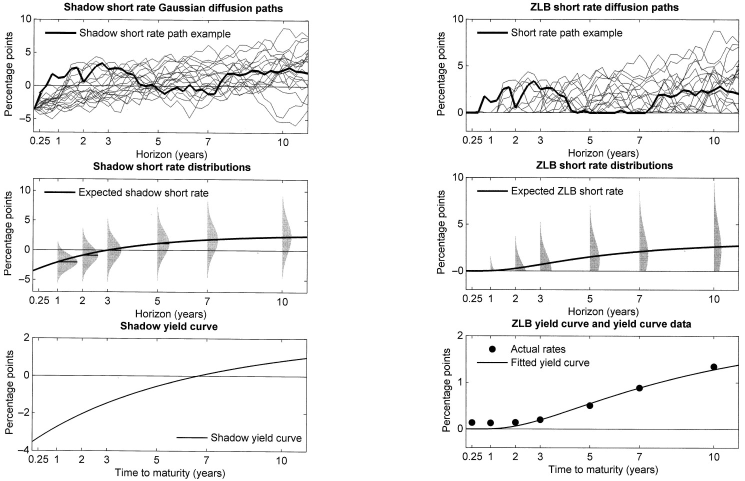
Shadow-Rate Approach (Shadow/ZLB GATSM)

$$x_{s,t} = \mu(1-\rho) + \rho x_{s,t-1} + \varepsilon_t$$

$$x_t = max(x_{s,t}, 0)$$

1. Arbitrage-free
2. Simple (simulation)
3. Respects the ZLB

Shadow Rates and ZLB Rates



Shadow-Rate Approach (Shadow/ZLB GATSM)

$$x_{s,t} = \mu(1 - \rho) + \rho x_{s,t-1} + \varepsilon_t$$

$$x_t = max(x_{s,t}, 0)$$

1. Arbitrage-free
2. Simple (simulation)
3. Respects the ZLB
4. Sample path feature probabilities (e.g., lift-off from ZLB)
5. Sample path integral densities (e.g., effective stimulus)

   But MPRR could also do points 4 and 5...

6. Shadow rate path and shadow yield curve

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Final Thoughts on Relative Performance

Much boils down to:

– Value of the shadow rate path and shadow yield curve

– Views about "simplicity"

My balance tips slightly toward shadow/ZLB GATSM

Interesting question:
With appropriate constraints on the Gamma and Poisson processes, can MPRR "replicate" a shadow/ZLB GATSM, but without the mechanism of shadow short rates and the shadow yield curve?