Review of Accounting Studies
https://doi.org/10.1007/s11142-021-09627-3

Disagreement about fundamentals: measurement and consequences

Paul Fischer[1] · **Chongho Kim**[2] · **Frank Zhou**[1]

Accepted: 6 July 2021 Published online: 12 August 2021

Abstract

We propose a measure of disagreement, which reflects differences of opinion as opposed to information asymmetry, that can be extracted from sequences of analyst forecasts. Using a Bayesian theoretical framework, we prove that when analysts agree, a regression of an analyst's forecast on the previous forecast issued by another analyst should have a slope coefficient of one. The magnitude of the estimated regression coefficient's deviation from one is then employed as a disagreement measure. We validate the measure using tests tied to predicted relations between disagreement and trading volume and bid-ask spreads. Finally, we employ our measure to test for associations between disagreement and expected returns predicted by antecedent theoretical studies.

Keywords Disagreement · Divergence of opinion · Expected returns · Analyst forecasts

JEL Classification G14 · M41

We thank Russell Lundholm (the editor), two anonymous referees, Henry Friedman (discussant), participants in the Thirteenth Annual Bauer Accounting Research Symposium, the Wharton Spring Accounting Conference, and the Colorado Summer Accounting Conference, and workshop participants at Pennsylvania State University, University of Miami, and Carnegie Mellon University for helpful comments. We appreciate the financial support from the Dean's Research Fund of the Wharton School, the University of Pennsylvania, and New York University and excellent research assistance from Dengsheng Chen, Elisa Saporito, and Saloni Wadhwa. All errors are our own.

✉ Frank Zhou
szho@wharton.upenn.edu

[1] The Wharton School, University of Pennsylvania, 3620 Locust Walk, Philadelphia, PA 19103, USA

[2] Stern School of Business, New York University, 44 West 4th Street, New York, NY 10012, USA

1 Introduction

Models of financial markets and disclosure have generally attributed divergent beliefs to one of two sources: information asymmetry and disagreement (i.e., differences of opinion). Information asymmetries arise because investors are assumed to have access to different information, and they are sustained in equilibrium because of noise trade. Disagreements, or differences of opinion, arise because investors agree to disagree, perhaps because they have different models (i.e., prior beliefs) for processing information or perhaps because of psychological biases. In some contexts, both sources of divergent beliefs have similar implications. For example, increased information asymmetry and greater disagreement should both be associated with increased trading volume. In other contexts, their implications differ. For example, disagreement may be fostered by a disclosure, even though that disclosure reduces information asymmetry. Given that possibility, distinguishing disagreement from information asymmetry and assessing the implications of each is warranted. We contribute to that effort by suggesting and validating a measure of investor disagreement and then applying the measure to test predicted associations between disagreement and expected returns.

A challenge in distinguishing disagreement from information asymmetry empirically is that some obvious measures of belief divergence, such as the dispersion of earnings forecasts, are influenced by both information asymmetry and disagreement. We confront this empirical challenge by exploiting the intuition that, when two parties disagree, the updated beliefs expressed by one will have less influence on the beliefs of the other. To do so rigorously, we adopt a Bayesian framework and consider a sequence of individuals' forecasts of an uncertain outcome (e.g., future earnings). We prove that, if individuals are Bayesian, publicly forecast their expectations of an outcome, and agree as to how information should influence their beliefs about that outcome, then one individual's date t forecast will equal that person's date t expectation of another individual's date $t + 1$ forecast. Notably, this result holds *regardless of* the differences in the information held by the individuals when they make their forecasts. Thus, if there is no disagreement, the regression of one forecast on a previous forecast by a different individual should yield a coefficient of one. We then employ the extent of that coefficient's deviation from one as a basis for our empirical measure of disagreement.[1]

We apply our measurement approach to assess disagreement about a firm's near-term earnings prospects and use analyst EPS forecasts to proxy for investor forecasts within the marketplace. To do so, we regress individual analyst quarterly EPS forecast on the immediately preceding forecast of another analyst for the same

[1]Exploiting forecast deviations from a statistical rule is not unprecedented in the literature. In particular, Lundholm and Rogo (2016) document that the time-series and cross section of individual analyst forecasts violate statistical variance bounds 17% and 8% of the time, respectively, suggesting that some analyst forecasts vary excessively. In a subsequent paper, Lundholm and Rogo (2020) show that firms whose forecasts vary excessively experience higher equity price volatility and lower equity returns.

quarterly earnings. We define our firm-quarter level measure of disagreement using that regression's slope coefficient's deviation from one, standardized by the coefficient's standard error to control for noise in the coefficient estimate. We find that about 67% of firm quarters have slope coefficient estimates that differ significantly from one at the 1% level in a two-tailed test, which is consistent with investor disagreement. Furthermore, those estimates that differ significantly from one are almost all less than one, which, according to our model, is consistent with investors generally believing that others are overreacting to some underlying information.

To validate that our measure reflects investor disagreement, we relate it to two measures that are expected to be influenced by belief divergence: trading volume and bid-ask spread. Because disagreement causes divergent beliefs and divergent beliefs motivate trading, we expect a positive relation between the magnitude (i.e., absolute value) of our measure and trading volume, which mirrors the expected positive relation between information asymmetry and trading volume. With respect to bid-ask spread, we expect a negative relation between disagreement and spreads because trades are more likely to cross within shorter horizons when there is more disagreement, which should manifest in lower inventory holding costs for market makers. In contrast, we expect a positive relation between information asymmetry and bid-ask spreads (due to adverse selection). Consistent with our expectations, the magnitude of our measure is positively associated with volume and negatively associated with bid-ask spreads, which provides some assurance that the measure captures disagreement. Because our measure is based on analyst forecasts, we also construct alternative measures that explicitly address the concern that analyst forecasts are predictably biased. Those alternatives relate positively to our main disagreement measures and volume and negatively to spreads, and their use as the measure of disagreement does not alter our inferences regarding the relation between disagreement and expected returns, as discussed below. These findings indicate that the measurement concern, due to analyst bias, is not significant. Furthermore, we caution that adjusting for analyst bias is likely unwarranted, as evidence indicates that investors do not fully adjust for forecast biases (So 2013; Veenman and Verwijmeren 2018; Lundholm and Rogo 2020), in which case those biases themselves would generate disagreement among investors. Finally, we also control for cross-sectional factors that explain information asymmetry and find that our disagreement measure continues to relate positively to volume and negatively to spread, suggesting that our measure captures a distinct construct from the determinants of information asymmetry.

Having validated the measure of disagreement, we apply it to ascertain whether there is a relation between disagreement and expected returns. Our analysis is motivated by two theoretical arguments. In the first, which is considered by Bloomfield and Fischer (2011) and Banerjee (2011), disagreement and expected returns are linked because disagreement influences investor perceptions of undiversifiable price uncertainty through higher order beliefs (i.e., beliefs about the evolution of beliefs). To illustrate, we offer a formal model in which analyst/investor disagreement is attributable to differential beliefs about the noise in informative signals. The model predicts a positive association between disagreement and expected returns because greater disagreement leads to perceptions of higher future price variability than is warranted by the information.[2] The second argument, which originates with Miller (1977) and is further articulated by Hong and Stein (2007), assumes that adopting a short position is prohibitively costly for many investors. Given this friction, an increase in disagreement causes a higher price level, because the friction results in a price that weights the beliefs of optimists more heavily than those of pessimists. Hence, all else equal, returns will be higher (lower) when disagreement increases (decreases) from one period to another. Consistent with both theories, we find a positive relation between disagreement and returns. Furthermore, consistent with the first, we find a significant association for firms in which the cost of short selling is very low, and, consistent with the second, we find that the association is larger for the firms that are most costly to short.

Our study contributes to the broad literature focused on differential beliefs within capital markets. Differential beliefs have been captured by dispersion of analyst forecasts, which theoretically reflect fundamental uncertainty and information asymmetry in addition to disagreement.[3] Garfinkel and Sokobin (2006) measure differential beliefs, referred to as divergence of opinion in their study, using unexplained trading volume, which Garfinkel (2009), using proprietary limit and market order data, argues better captures divergence of opinion, relative to other nonproprietary proxies (bid-ask spread, price volatility, and forecast dispersion). Garfinkel and Sokobin (2006) and Garfinkel (2009), however, do not distinguish between whether the divergence of opinion is attributable to information asymmetry or disagreement. Our study differs in that we try to capture just one source of differential beliefs – disagreement and, in particular, disagreement about near-term financial performance.

With its focus on differences of opinion, our paper contributes to the substantial literature on subjective beliefs (i.e., investors agreeing to disagree). Much of that work has involved theoretical analyses linking disagreement to observed trading patterns, trading volume, or returns.[4] The work of Banerjee (2011) is closest to ours in the sense that the author tries to distinguish information asymmetry from disagreement. In particular, he employs a model that incorporates both constructs and shows that pure forms of the two constructs – "rational expectations" (information asymmetry), where investors use prices to update their beliefs, and "differences of opinion" (disagreement), where they do not – offer starkly different predictions for the relation between belief dispersion and expected returns, volatility, beta, and return autocorrelation. His empirical analysis suggests that, consistent with the information

[2]More generally, the impact of disagreement on expected returns depends on the nature of the disagreement. For example, Bloomfield and Fischer (2011) argue that, if disagreement arises because investors believe others will overweight noise, which the authors refer to as perceived errors of commission, investors will expect higher price variability and demand higher expected returns. If, on the other hand, disagreement arises because investors believe others will underweight highly informative signals, which Bloomfield and Fischer (2011) refer to as perceived errors of omission, investors will expect lower price variability and demand lower expected returns.

[3]See, for example, Abarbanell et al. (1995), Bamber et al. (1997), Barron et al. (1998), Clement et al. (2003), Johnson (2004), Zhang (2006), and Wang et al. (2017).

[4]See, for example, Harrison and Kreps (1978), Harris and Raviv (1993), Kandel and Pearson (1995), Cao and Ou-Yang (2008), Banerjee et al. (2009), Banerjee and Kremer (2010), Banerjee (2011), and Kondor (2012).

asymmetry construct, investors do rely on prices to update beliefs. He does not, however, empirically rule out the presence of disagreement, just extreme disagreement in which no updating on prices occurs. In lieu of running an empirical horse race between pure forms of information asymmetry and disagreement, we try to identify an empirical metric that homes in on the extent of disagreement, should it exist.

Within the context of our model, we focus on disagreement attributable to investors believing that others are committing information processing errors, which could be attributed to overconfidence or beliefs about the overconfidence of others. Overconfidence is widely studied in behavioral finance and can explain predictable patterns in returns and individual trading behaviors that cannot be easily explained by classical models with rational Bayesian investors.[5] Within this literature, overconfidence leads to systematic errors in beliefs, which results in predictable return patterns that are not fully arbitraged away in equilibrium, due to some economic friction (e.g., high transaction costs for short selling). We have used the more generic term "disagreement" in our analysis, because, consistent with the subjective beliefs literature, we are agnostic as to which set of beliefs, if any, is correct. In addition, unlike studies that focus on predicting returns, we aim to explain contemporaneous returns.

Through linking disagreement about fundamentals with expected returns, our study adds to research focused on the relation between belief differences and expected returns. Diether et al. (2002) document a negative association between expected returns and previous analyst forecast dispersion. They argue that this negative association is consistent with the work of Miller (1977), who predicts that more disagreement implies overvaluation in the presence of short-selling constraints.[6] Banerjee (2011), as discussed above, provides an alternative explanation for the negative relation between forecast dispersion and average returns. Regardless of the source of the relation, as we discussed earlier, forecast dispersion can reflect multiple constructs, which suggests that the results of Diether et al. (2002), Banerjee (2011), and subsequent related studies might be driven by constructs other than disagreement as we define it.

2 Measuring disagreement

Our analysis hinges on identifying an empirical measure of disagreement that can serve as a plausible proxy for distinguishing firms experiencing more disagreement from those experiencing less. The measure we propose relies on the statistical relation

[5] See Daniel and Hirshleifer (2015) for an overview or, for example, Daniel et al. (1998), Odean (1999), Barber and Odean (2001), Gervais and Odean (2001), Scheinkman and Xiong (2003), and Grinblatt and Keloharju (2009).

[6] Sadka and Scherbina (2007) further document that high transaction costs sustain this mispricing and that improvements in aggregate liquidity accelerate the correction of mispricing. Johnson (2004) shows that an increase in idiosyncratic volatility leads to lower expected returns when a firm is levered, which offers a rational alternative explanation for the findings of Diether et al. (2002). Furthermore, Avramov et al. (2009) show that common proxies for short-selling cost do not capture the negative association documented by Diether et al. (2002) and that the association in fact only concentrates on non-investment grade firms during periods of credit rating downgrades.

among analyst forecasts issued in sequence and builds on the intuition that, when there is more disagreement, individuals are less inclined to update their beliefs to align with the observed forecasts of others. We couch that intuition within the context of a simple statistical argument, discussed below.

2.1 Agreement and sequential forecasts

In the empirical domain, forecasts are generally not observed at the same time, which means they are based upon different information. Furthermore, even if forecasts are simultaneous, one could argue that they might still be based upon different information because forecasters generally do not observe the simultaneous forecasts of others. As a consequence, it is difficult to use, say, a simple measure of dispersion in forecasts to reflect disagreement, because that statistic is also influenced by *differences in information*. If we consider a sequence of public forecasts, however, we can tease out a test of agreement that is, in theory, not contaminated by differences in information. This test is then employed to motivate our measure of disagreement.

Our test of agreement is based upon first defining agreement in a statistical sense. Given that definition, we show that, if two individuals agree, the forecast offered by one equals the expectation of the other's subsequent forecast if the latter is aware of the former's forecast. This observation, in turn, suggests that regressing one individual's forecast on the prior forecast of the other should yield an intercept of zero and coefficient of one if the two individuals agree. Deviations from those coefficients should indicate the extent of disagreement.

To formally illustrate the logic of our disagreement measure, consider a setting with two Bayesian agents, A and B, who forecast a firm's earnings, $\tilde{e}$. Furthermore, assume the forecasts offered by each individual equal their expectation of earnings. Let $\tilde{\omega}$ denote information that can be used to update beliefs about $\tilde{e}$ and $g\,(e, \omega; I)$, the joint density function that characterizes the beliefs of agent $I \in \{A, B\}$. With that notation in hand, we define agreement as follows.

Definition 1 A and B agree if and only if $g\,(e, \omega; A) = g\,(e, \omega; B)$ for all $\{e, \omega\}$.

The definition of agreement implies that, given the same information ω, A and B would have the same forecast: $f_A = E\left[\tilde{e}|\omega; A\right] = E\left[\tilde{e}|\omega; B\right] = f_B$, where f_I is the forecast of agent $I \in \{A, B\}$ and $E\left[\tilde{e}|\omega; I\right]$ is the expectation of $\tilde{e}$ conditional on ω for agent $I \in \{A, B\}$.

With agreement defined, assume that A and B forecast in sequence, with A forecasting first and B, after observing A's forecast, forecasting second. If they agree, the law of iterated expectations is easily exploited to show that the common expectation for B's forecast conditional upon A's forecast is simply A's forecast.

Proposition 1 *Let ω_A denote A's information at the time of A's forecast, and $\{f_A, \omega_B\}$ denote B's information at the time of B's forecast. If A and B agree, the common expectation of B's forecast conditional upon A's forecast is A's forecast: $E[\tilde{f}_B|f_A] = f_A$.*

Proof Because the two agents agree, we suppress agent specific arguments in the expectations. By the law of iterated expectations we know that $E[\tilde{f}_B|f_A] = E[E\left[\tilde{e}|f_A, \omega_B\right]|f_A] = E\left[\tilde{e}|f_A\right]$. The observation that $E[\tilde{f}_B|f_A] = f_A$ is completed by noting that $E\left[\tilde{e}|f_A\right] = E\left[\tilde{e}|E\left[\tilde{e}|\omega_A\right]\right] = E\left[\tilde{e}|\omega_A\right] = f_A$. □

Note that the proof that $E[\tilde{f}_B|f_A] = f_A$ does *not* hinge on some relation between the two information sets, ω_A and ω_B. That is, ω_A may or may not be a subset of ω_B, which implies that the relation holds, regardless of whether B knows all, some, or none of the information embedded into A's forecast. What B must know is simply A's forecast and that B would make the same forecast as A, given whatever A has privately observed.

To illustrate, consider a setting where earnings can be represented by $\tilde{e} = \mu + \tilde{a} + \tilde{b}$, where μ is the prior mean and $\tilde{a}$ and $\tilde{b}$ are independent mean zero normally distributed random variables with variance s. Furthermore, assume that A privately observes the realization y_A of a noisy signal $\tilde{y}_A = \tilde{a} + \tilde{\alpha}$ prior to forecasting and that B privately observes A's forecast and the realization y_B of a noisy signal $\tilde{y}_B = \tilde{b} + \tilde{\beta}$ prior to forecasting, where $\tilde{\alpha}$ and $\tilde{\beta}$ are independent mean zero normally distributed random variables with variance σ. In this simple example, A's forecast is $f_A = \mu + \frac{s}{s+\sigma} y_A$, and B's forecast is $f_B = f_A + \frac{s}{s+\sigma} y_B$, which implies that $E[\tilde{f}_B|f_A] = f_A$.

2.2 Measurement

To motivate our measure, we begin by employing the insight that $E[\tilde{f}_B|f_A] = f_A$ if the two individuals agree, and use it to predict the intercept and slope coefficient in the following regression.

$$f_B = \gamma_0 + \gamma_1 f_A + \epsilon, \tag{1}$$

where ϵ is the residual. If the two individuals agree in our theoretical setting, the predicted regression intercept and coefficient are $\gamma_0 = 0$ and $\gamma_1 = 1$, respectively. Otherwise, the regression would not be consistent with A's forecast equaling the expectation of B's forecast.

Consistent with the reasoning to this point, we use analyst quarterly earnings forecasts to empirically estimate γ_1 to proxy for the extent of disagreement at the level of a firm quarter (i.e., the proxy reflects disagreement regarding firm i's quarter t earnings). In particular, we run regressions of the form in Eq. 1 using a sequence of analyst forecasts over six months prior to the end of a particular firm's fiscal quarter and use the deviation of the coefficient from 1, namely $1 - \gamma_1$, as a disagreement measure for that firm quarter. As will be shown below, the situation where $1 - \gamma_1 > 0$ reflects cases in which analysts believe that other analysts are overreacting to information or noise. On the other hand, the situation where $1 - \gamma_1 < 0$ reflects cases in which analysts believe that other analysts are underreacting to information. Hence a larger absolute value of our measure, $|1 - \gamma_1|$, would indicate more disagreement.

To illustrate how the regression captures disagreement within the context of the example in the preceding section, assume the analysts disagree about the precision of their private signals. Specifically, assume A and B still believe that the noise term in their own private signal has variance σ but now believe that the noise term in the other's private signal has a variance of $\sigma + \delta$, where $\delta \in (-\sigma, \infty)$. A positive δ implies that each analyst believes the other is overreacting to his or her private signal, while a negative δ captures the idea that each analyst believes the other is underreacting to the private signal. Given their disagreement, B's forecast conditioned upon A's forecast and y_B is $f_B = \frac{\delta}{s+\sigma+\delta}\mu + \frac{s+\sigma}{s+\sigma+\delta} f_A + y_B$, which implies $E[\tilde{f}_B | f_A] = \frac{\delta}{s+\sigma+\delta}\mu + \frac{s+\sigma}{s+\sigma+\delta} f_A$. The coefficient on f_A in the conditional expectation, $\gamma_1 = \frac{s+\sigma}{s+\sigma+\delta}$, is one if there is no disagreement, that is, $\delta = 0$, and its distance from one, $|1 - \gamma_1|$, would increase with the extent of disagreement, as captured by $|\delta|$.

We use the deviation of the slope coefficient from one as opposed to the deviation of the intercept from zero to construct our disagreement measure for several reasons. First, from a purely theoretical perspective, the intercept conceptually reflects disagreement in a manner that inhibits interpretation. More formally, consider our example with disagreement regarding information processing and further suppose A and B also differ in their beliefs about the prior mean μ: μ_A and μ_B. Then $f_A = \mu_A + \frac{s}{s+\sigma} y_A$ and $f_B = \mu_B - \frac{s+\sigma}{s+\sigma+\delta}\mu_A + \frac{s+\sigma}{s+\sigma+\delta} f_A + y_B$. The intercept γ_0, which is $\mu_B - \frac{s+\sigma}{s+\sigma+\delta}\mu_A$, reflects both disagreement about information processing (δ) and differences in prior means (μ_A and μ_B) in a complex way, impeding tying its numerical value cleanly to either disagreement construct (i.e., $\mu_A - \mu_B$ or δ). Second, from an empirical perspective, the theoretical intercepts will, in a loose sense, be averaged over different analyst forecasts, which further muddies how to link the estimate of the intercept with a construct of disagreement. For example, even if there were vast differences in prior means and beliefs about signal variances, the estimate of the intercept could conceivably be zero if the differences in prior means wash out in the regression process, because the sample includes equal numbers of forecasts from the prior optimists and pessimists. Finally, in our empirical implementation, we employ firm-quarter time-series data to fit the regression and anticipate that the intercept will pick up drift toward the earnings realization in any firm quarter.

2.3 Validation, noise, and bias

Our measurement approach reflects the intuition that individuals who disagree are less likely to be influenced by each other's beliefs and formally relies upon the logic of a traditional Bayesian forecasting framework. Antecedent literature, however, suggests that analyst forecasts might systematically deviate from such a framework. For example, analysts might bias their forecasts to curry favor with firm management (Dugar and Nathan 1995; Morgan and Stocken 2003), to herd with other analysts (Bernhardt et al. 2006)), to depart from the consensus (Liu and Natarajan 2012), or exclude certain items from their forecasts (Bradshaw and Sloan 2002; Brown et al. 2015). In light of potential issues with measurement noise and bias when we empirically apply our approach to sequences of analyst forecasts, we subject our measure to two joint validation tests. These tests not only validate our measure empirically but also conceptually rule out the possibility that our measure is driven by some other factors (e.g., information asymmetry).

We validate our disagreement measure by testing whether it relates to two observable outcomes, trading volume and bid-ask spread, in a theoretically consistent manner. Like any source of belief differentials (e.g., information asymmetry), disagreement should motivate trade and, as a consequence, should be positively associated with trading volume. Hence a valid proxy for the extent of disagreement, $|1-\gamma_1|$ in our theoretical framework, should be positively associated with trading volume. In addition to being increasing in information asymmetry, bid-ask spread is increasing in inventory holding costs, which are attributable to uncertainty that market makers face during their holding period. Disagreement can lower inventory holding costs due to a greater likelihood of orders crossing within a shorter timeframe. Hence a valid proxy for the extent of disagreement should relate negatively to bid-ask spread. We present the results of these validation tests in Section 4.2.

Our measure is subject to two sources of noise: information asymmetry and uncertain analyst forecast incentives. To illustrate, we extend the simple illustrative example with disagreement and assume that A has uncertain incentives, which cause A to process the signal y_A with intentional noise $\tilde{\eta}_1+\tilde{\eta}_2$ (i.e., $f_A=\mu+\frac{s}{s+\sigma}(y_A+\eta_1+\eta_2)$), where $\tilde{\eta}_1$ and $\tilde{\eta}_2$ are independent mean-zero normally distributed random variables with variances v_1 and v_2, respectively, and both are independent of all other random variables. Further, assume that, prior to forecasting, B observes A's forecast and one of A's forecast incentive parameters, η_1, in addition to the realization for $\tilde{y_B}$. In this case, B's forecast is $f_B=\frac{\delta+v_2}{s+\sigma+\delta+v_2}\mu+\frac{s+\sigma}{s+\sigma+\delta+v_2}(f_A-\frac{s}{s+\sigma}\eta_1)+y_B$. It follows that $E[\tilde{f}_B|f_A]=\frac{\delta+v_1+v_2}{s+\sigma+\delta+v_1+v_2}\mu+\frac{s+\sigma}{s+\sigma+\delta+v_1+v_2}f_A$. Hence the magnitude of the coefficient on A's forecast, $\gamma_1=\frac{s+\sigma}{s+\sigma+\delta+v_1+v_2}$, is a function not only of disagreement, the difference between σ and $\sigma+\delta$, but also of information asymmetry, s, and the uncertain incentives, v_1+v_2.

Consider first the noise introduced into the disagreement measure that is attributable to information asymmetry. If $1-\gamma_1>0$ (i.e., $\delta+v_1+v_2>0$), which is theoretically plausible and proves to be the case in almost all of our empirical estimates, then $|1-\gamma_1|$ is decreasing in s. However, if the cross-sectional variation in the measure is primarily attributable to information asymmetry, the empirical estimates of $|1-\gamma_1|$ should have a negative association with both volume and spreads, because greater information asymmetry is expected to motivate more trade by informed traders and to increase market maker bid-ask spreads. Given that observation, our measure for the extent of disagreement, $|1-\gamma_1|$, will fail the joint validation tests requiring that it be positively associated with volume and negatively associated with spreads.

Consider next the noise introduced by the uncertain analyst reporting incentives, which is due to the fact that $1-\gamma_1$ is increasing in v_1+v_2. Furthermore, note that this source of noise would cause the disagreement measure to deviate from zero, even in the absence of disagreement, $\delta=0$. Such noise, however, would not cause the measure $|1-\gamma_1|$ to pass the joint validation tests unless, for some reason, uncertainty about analysts' incentives is positively associated with volume and negatively associated with spreads. We are, however, not aware of any theoretical reason for such an association besides investor disagreement about those incentives.

Analyst bias might also be predictable (i.e., not a source of noise) in a manner that distorts our measure of disagreement. For example, Richardson et al. (2004) provide evidence of a walk-down pattern in forecasts over a forecasting period. That is, forecasts are biased upward in the earlier part of a period, and that bias then declines until it is negative by the end of the period. Assuming earnings are positive, this behavior would lead to a slope coefficient of less than one, even if the analysts were in agreement. To address the issue of analyst bias, in addition to our validation tests, we consider variations of our measure that controls for walk-down bias in Section 4.3.1.

Finally, our discussion of noise and bias in forecasts has implicitly assumed that investors fully adjust for forecast noise or bias. Some studies suggest that this assumption does not hold. For example, Lundholm and Rogo (2020) demonstrate that analyst forecasts often exhibit excessive volatility that cannot be explained by rational Bayesian updating and these forecasts nevertheless significantly impact market returns. In addition, So (2013) and Veenman and Verwijmeren (2018) both show that investors do not fully adjust for the implications of analyst strategic incentives. Obviously, if investors do not adjust for the bias in the forecasts of the analysts they follow, then that bias itself would be a source of disagreement, and the impact of analyst bias would not harm our measure's ability to capture disagreement among investors.

3 Sample and variable measurements

The empirical analysis combines several data sources. We collect analyst earnings forecasts (EPS) from I/B/E/S unadjusted detailed history file. We obtain stock price, bid-ask spread, return, trading volume, and the number of shares outstanding data from CRSP daily and monthly files. Accounting data is from Compustat. Monthly Fama-French three factors and the momentum factors are from WRDS Fama French & Liquidity Factors. We collect management forecasts from I/B/E/S Guidance and institutional ownership from Thomson Reuters 13-F database. The sample period covers from January 1, 2003, to December 31, 2017. We choose 2003 as the starting year because a series of new rules targeting analyst research in the early 2000s, including NASD Rule 2711, NYSE Rule 472, and the Global Analyst Research Settlement, may affect forecast properties (Lehmer et al. 2020).

To construct our disagreement measures, we first process the I/B/E/S data to obtain a sample of quarterly EPS forecasts and adjust all forecasts for stock splits.[7] For each firm quarter, we keep all quarterly EPS forecasts that are issued for that quarter and announced within six months prior to the corresponding quarter-end. We choose six

[7] Analysts routinely exclude nonrecurring items from their forecasts (Bradshaw and Sloan 2002; Brown et al. 2015). According to its data manual, I/B/E/S follows a majority policy where "the accounting basis of each company estimate is determined by the basis used by the majority of contributing analysts. Once the majority basis has been established, contributing analysts in the minority may keep their original estimates, or are also given the opportunity to adjust to the majority basis." This policy implies that any differences in analysts' exclusions within a firm quarter are likely small and therefore unlikely to be the primary source of variation driving our disagreement measure. Nevertheless, differences in analysts' exclusions could be a source of noise.

months, rather than just the quarter itself, to ensure a sufficient number of forecasts for each firm quarter. To measure disagreement, a firm quarter must have at least two analysts issuing forecasts.

Next we run regressions at the firm-quarter level to produce two measures of disagreement for each firm quarter. We regress each analyst forecast on the most recent forecast issued by a different analyst. To run this regression, we require that the two analyst forecasts be issued at least one week apart to allow sufficient time for the second analyst to process information from the first analyst's forecast. When the most recent forecasts are multiple forecasts issued by different analysts, we use the mean of those forecasts. Each regression needs to have at least four observations. The regression model using analyst forecasts of firm i and quarter t is

$$f_{it,k} = \gamma_{0,it} + \gamma_{1,it} g_{it,k} + \epsilon_{it,k}, \tag{2}$$

where $f_{it,k}$ denotes the k^{th} quarterly EPS forecast issued for firm i in quarter t and $g_{it,k}$ denotes the most recent forecast that is issued by a different analyst and is at least one week apart. All analyst forecasts are scaled by the stock price one month prior to the first forecast of quarter t. The scaling has no effect on the slope coefficient, $\gamma_{1,it}$, which is used to construct our disagreement measure.

Recall from Section 2 that the coefficient $\gamma_{1,it}$ should equal one when there is no disagreement. Hence our first disagreement measure is the difference between one and the estimated coefficient scaled by the estimate's standard error:

$$Disagree^c_{it} = \frac{1 - \gamma_{1,it}}{se(\gamma_{1,it})}, \tag{3}$$

where $se(\gamma_{1,it})$ denotes the robust standard error from the regression model Eq. 2. We scale the measure by the standard error, because a larger deviation of γ_1 from one does not necessarily imply greater disagreement when γ_1 is estimated imprecisely. Imprecision can be a potential issue because the firm-quarter regressions often have a small number of observations (the median is 12, and the mean is 15). As the level of imprecision is captured by the standard error of the coefficient estimate, we scale $1-\gamma_1$ by the standard error. We use heteroskedasticity-robust standard errors to allow the variance of the error term to differ across forecasts. This choice captures the possibility that the variance of the error term, which represents the new information that each successive analyst observes and incorporates into their forecasts, can change over time.

The sign of $Disagree^c_{it}$ reflects the nature of disagreement. As discussed in Section 2.2, a positive $Disagree^c_{it}$, or $1 - \gamma_1 > 0$, implies perceptions of overreaction to information, and a negative $Disagree^c_{it}$, or $1 - \gamma_1 < 0$, implies perceptions of underreaction to information. In our analyses, we focus on the extent of disagreement as opposed to the nature of disagreement, so we primarily employ the unsigned disagreement measure, $|Disagree^c_{it}|$.

To reduce the chance of falsely identifying cases of high disagreement because of noise in the measure, we create an alternative measure that is discrete and based on whether the estimated measure is statistically significant at the 1% level in a two-tailed test. Our discrete measure is captured by a simple indicator variable,

$Disagree^d_{it}$, which takes on the value of one if $Disagree^c$ exceeds 2.58, -1 if $Disagree^c$ is below -2.58, and zero otherwise:

$$Disagree^d_{it} = \begin{cases} 1 & \text{if } \frac{1-\gamma_{1,it}}{se(\gamma_{1,it})} \geq 2.58 \\ 0 & \text{if } -2.58 < \frac{1-\gamma_{1,it}}{se(\gamma_{1,it})} < 2.58 \\ -1 & \text{if } \frac{1-\gamma_{1,it}}{se(\gamma_{1,it})} \leq -2.58. \end{cases} \quad (4)$$

The interpretation of this measure is straightforward. As we cannot conclude that γ_1 statistically differs from one when the statistic in Eq. 4 falls between -2.58 and 2.58, we classify the firm quarter as having *no* disagreement.

4 Results: disagreement measures

4.1 Descriptive statistics

The distribution of our continuous disagreement measure, $Disagree^c$, is graphically represented in Fig. 1 and summarized in the first row of Table 1. The distribution suggests that disagreement is widespread within the sample, with the vast majority of estimates, about 95% (untabulated), being positive, which is consistent with analysts believing that others are generally overreacting to some information. Although the

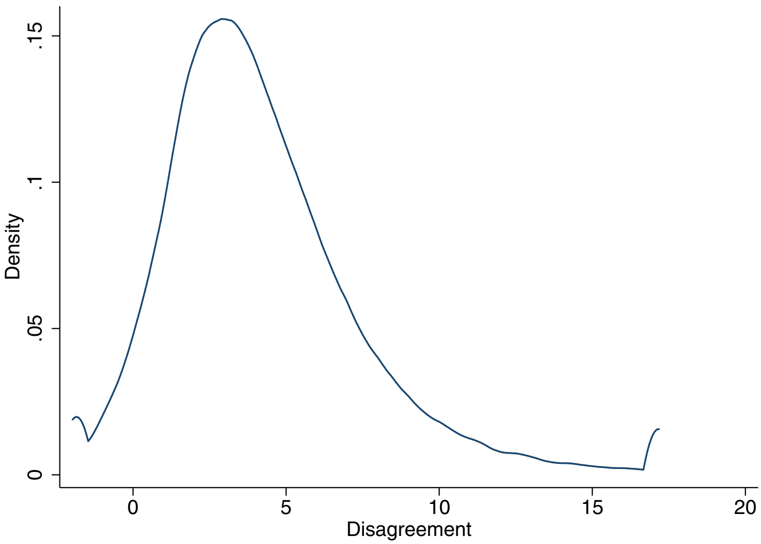

Fig. 1 The density of disagreement. This figure plots the density of our disagreement measure. To measure disagreement, we collect analyst quarterly EPS forecasts within six months before the fiscal quarter end date and regress an analyst forecast on the most recent forecast issued by a different analyst. The two analyst forecasts need to be issued at least one week apart. Each regression needs to have at least four observations. The regression model is $f_{it,k} = \gamma_{0,it} + \gamma_{1,it} g_{it,k} + \epsilon_{it,k}$, where $f_{it,k}$ denotes the k^{th} quarterly EPS forecast issued for firm i in year t and $g_{it,m}$ denotes the most recent forecast issued by a different analyst. Disagreement in this figure is defined as $\frac{1-\gamma_{1,it}}{se(\gamma_{1,it})}$, as shown in Eq. 3

Table 1 Summary statistics

Variables	N	Mean	Std	1%	5%	25%	50%	75%	95%	99%
$Disagree^c$	148,840	4.216	3.281	−1.975	−0.072	2.055	3.679	5.749	10.304	17.181
$Disagree^d$	148,840	0.670	0.470	0.000	0.000	0.000	1.000	1.000	1.000	1.000
$Volume$	148,840	0.907	0.886	0.000	0.000	0.338	0.707	1.214	2.622	4.708
$Spread$	124,169	0.185	0.271	0.012	0.019	0.051	0.101	0.195	0.651	1.826

This table presents the summary statistics of the two disagreement measures along with trading volume ($Volume$) and bid-ask spread ($Spread$). To measure disagreement, we collect analyst quarterly EPS forecasts for a firm quarter within six months prior to the end of quarter t and regress an analyst forecast on the most recent forecast issued by a different analyst for the same firm quarter. The two analyst forecasts need to be issued at least one week apart. The regression is performed at the firm quarter level. Each regression needs to have at least four observations. The regression model is $f_{it,k} = \gamma_{0,it} + \gamma_{1,it} g_{it,k} + \epsilon_{it,k}$, where $f_{it,k}$ denotes the k^{th} quarterly EPS forecast issued for firm i in quarter t and $g_{it,k}$ denotes the most recent forecast issued by a different analyst. Disagreement measures are based on $\gamma_{1,it}$ and are defined in Eqs. 3 and 4. $Volume$ represents the daily trading volume divided by the total daily number of shares outstanding, averaged over the six-month window prior to the end of quarter t. $Spread$ represents the daily bid-ask spread divided by the daily mid-point of bid and ask prices, averaged over the six-month window prior to the end of quarter t. $Volume$ and $Spread$ are multiplied by 100

nature of disagreement seems similar across quarters, in the sense that the measure is generally positive, there is substantial variation in the estimates. This variation can be seen visually in Fig. 1 as well as through the size of the standard deviation reported in Table 1, which equals 3.28. In untabulated analyses, we further investigate the source of this variation. We find that the within-year standard deviation of $Disagree^c$ is stable across years, hovering around 3.27 on average and ranging from 3.05 to 3.45. Examining within-firm variation, we find the average within-firm standard deviation of $Disagree^c$ is 2.93 with significant differences across firms, ranging from 0 to 13.55. Similar patterns exist for the discrete disagreement. Overall, disagreement varies both over the cross section and within a firm across time. The cross-sectional variation in disagreement, on average, seems greater than the variation along the within-firm time series dimension.

The distribution of the discrete disagreement measure, $Disagree^d$, aligns with that of the continuous measure. About 67% of our continuous measure estimates are significantly positive (i.e., $Disagree^d = 1$); 33% are insignificantly different from zero (i.e., $Disagree^d = 0$); and none are significantly negative (i.e., $Disagree^d = -1$). Hence disagreement appears to largely reflect cases in which analysts believe others are overreacting to some information.

4.2 Measure validation

We validate our two measures by assessing whether they relate to two outcomes, trading volume and bid-ask spread, in a manner consistent with their reflecting disagreement. As discussed in Section 2.3, if our measures primarily reflect disagreement, the extent of disagreement captured by our measure, $|Disagree|$, should relate positively to volume but negatively to spreads. We defer the discussions of specific sources of measurement errors (e.g., analyst forecast bias or information asymmetry) and the relevant empirical tests allaying these concerns to Section 4.3.

To be consistent with the measurement window of the disagreement measures, we construct trading volume and bid-ask spread using the six-month window preceding the fiscal quarter end date. Trading volume ($Volume$) is the daily trading volume divided by the total daily number of shares outstanding, averaged over the six-month window. Bid-ask spread ($Spread$) is the average daily bid-ask spread over the six-month window. The regressions of the validation tests take the following form.

$$Volume_{it}(Spread_{it}) = \alpha_0 + \alpha_1 |Disagree_{it}| + \epsilon_{it}. \tag{5}$$

The coefficient of interest is α_1, which should be positive (negative) when the dependent variable is $Volume$ ($Spread$) if our measures capture disagreement. Standard errors are clustered at the firm level to allow for arbitrary correlation in the error terms within a firm. In untabulated analyses, we also include year fixed effects, which allow us to evaluate our construct validity over the cross-section within a year. The inclusion of year fixed effects is inconsequential for our inferences.

The results from the validation tests, presented in Table 2, are consistent with our measures capturing disagreement. Specifically, our measures are positively associated with trading volume, as shown in columns (1) and (2). The explanatory power is small but consistent with the magnitude in prior studies that examine other

Table 2 Measure validation

Variables	(1) Volume	(2) Volume	(3) Spread	(4) Spread
$\lvert Disagree^c \rvert$	0.010***		−0.008***	
	[0.001]		[0.000]	
$\lvert Disagree^d \rvert$		0.029***		−0.057***
		[0.008]		[0.002]
Observations	148,840	148,840	124,169	124,169
R-squared	0.001	0.000	0.010	0.008

This table presents results from regressing trading volume and bid-ask spread on the disagreement measures. To measure disagreement, we collect analyst quarterly EPS forecasts for a firm quarter within six months prior to the end of quarter t and regress an analyst forecast on the most recent forecast issued by a different analyst for the same firm quarter. The two analyst forecasts need to be issued at least one week apart. The regression is performed at the firm quarter level. Each regression needs to have at least four observations. The regression model is $f_{it,k} = \gamma_{0,it} + \gamma_{1,it} g_{it,k} + \epsilon_{it,k}$, where $f_{it,k}$ denotes the k^{th} quarterly EPS forecast issued for firm i in quarter t and $g_{it,k}$ denotes the most recent forecast issued by a different analyst. The disagreement measures are based on $\gamma_{1,it}$ and are defined in Eqs. 3 and 4. The regression model is:

$$Volume_{it}\ (Spread_{it}) = \alpha_0 + \alpha_1 |Disagree_{it}| + \epsilon_{it}.$$

$|Disagree|$ represents the absolute value of the corresponding disagreement measure. $Volume$ represents the daily trading volume divided by the total daily number of shares outstanding, averaged over the six-month window prior to the end of quarter t. $Spread$ represents the daily bid-ask spread divided by the daily mid-point of bid and ask prices, averaged over the six-month window prior to the end of quarter t. $Volume$ and $Spread$ are multiplied by 100. The standard error is clustered at the firm level factors influencing trading volume (e.g., Bamber et al. 1997; Chae 2005; Goetzmann and Massa 2005). We also document a negative relation between our measures and bid-ask spread in columns (3) and (4).

In addition to the validation tests above, we construct a more focused validation test using trading volume around earnings announcements. When investors disagree over the interpretation of public signals, they trade with each other even when prices *do not change* (e.g., Kandel and Pearson 1995). We follow this idea and examine the relation between disagreement over quarter $t + 1$ earnings and the average three-day trading volume centered around the earnings announcement for quarter t earnings (which happens during quarter $t + 1$), restricting the sample to earnings announcements with absolute cumulative stock return of the three-day window within 25 basis points. Consistent with our measure capturing disagreement, we find that greater disagreement over the earnings of quarter $t + 1$ is associated with larger trading volumes around the quarterly earnings announcements of quarter t, even when prices do not change around the announcements (untabulated). The positive association between disagreement and trading volume in a setting with no significant price reaction to a news event suggests that the more general result on trading volume in Table 2 is not driven by our measures capturing cross-sectional differences in the flow of news about the firm over the six-month measurement period.

We also perform another robustness test to address the concern that our results on trading volume might simply reflect smaller adverse selection driven by lower bid-ask spread, which decreases transaction costs. We find that both our disagreement measures are positively associated with trading volume after controlling for contemporaneous bid-ask spread or calendar quarter fixed effects, suggesting that the effects on trading volume do not simply reflect lower bid-ask spread or unobserved time trends in volume (untabulated). Section 4.3.2 provides further evidence on whether our measures continue to be associated with greater trading volume and lower bid-ask spread, after controlling for variables associated with information asymmetry.

4.3 Analyses of measurement errors

In addition to the validation tests using volume and spreads, we conduct two sets of analyses to further address concerns that our measures primarily capture constructs other than disagreement. The first set constructs alternative measures of disagreement to allay concerns that our measures reflect analyst forecast biases as opposed to disagreement in the marketplace. The second controls for a series of variables that relate to firms' information environments and aims to allay concerns that our measures reflect information asymmetry as opposed to disagreement. We do not find that these measurement concerns are significant.

4.3.1 Analyst forecast bias

We construct three alternative measures to address the concern that our primary measures reflect the biasing activities of analysts. The first measure controls for the possibility that an analyst's forecast is systematically more or less biased than other analysts' forecasts. We construct $Disagree^{control}$ from the regression model

$$f_{it,k} = \gamma_{0,it} + \gamma_{1,it} g_{it,k} + \gamma_{2,it} f_{it,k-1} + \epsilon_{it,k}, \tag{6}$$

where $f_{it,k-1}$ is the analyst's own forecast that is issued prior to $g_{it,k}$. An analyst's own previous forecast provides information for the level of her forecast bias. The procedures of constructing disagreement using $\gamma_{1,it}$ are otherwise the same as described in Section 3.

The next two measures address the concern that our disagreement measures jointly capture biased analyst forecasts and the subsequent walk-down of these forecasts, namely that analyst forecasts are relatively more optimistic at the beginning of a quarter and this optimism then declines (Richardson et al. 2004). For the first measure, we construct $Disagree^{rev}$ using the same procedures described in Section 3, except for using quarterly revenue forecasts instead of quarterly earnings forecasts. This measure is less susceptible to bias that could be generated from the walk-down patterns of earnings forecasts, as Bradshaw et al. (2016) show that revenue forecasts do not exhibit the walk-down patterns. A downside to using revenue forecasts is that they do not fully capture analysts' disagreement about earnings, as analysts can also disagree about expenses.

The second measure, $|Disagree|^{res}$, residualizes the unsigned, continuous disagreement measure, as defined in Eq. 3, against the three variables that Bradshaw et al. (2016) identify to explain the walk-down incentives. These variables are investment banking business, optimistic target price, and induced guidance. For each calendar year, we regress $|Disagree^{c}|$ on these three variables, using the model

$$|Disagree^{c}_{it}| = \beta_0 + \beta_1 DXFIN_{it} + \beta_2 TargetPrice_{it} + \beta_3 Guidance_{it} + \epsilon_{it},$$

where $DXFIN_{it}$, $TargetPrice_{it}$, and $Guidance_{it}$ are the aforementioned three factors that determine walk-down incentives and are measured contemporaneously to and using the same window as the disagreement measure (i.e., the six-month window preceding the fiscal quarter-end date). $DXFIN_{it}$ represents the change in external financing during this six-month window.[8] $TargetPrice_{it}$ is the difference between the first target price during the six-month window and the actual stock price on the announcement date of this target price, divided by the announcement date stock price. $Guidance_{it}$ equals one when a management forecast is issued during the six-month window and zero otherwise. We use the coefficient estimates in year y to compute the residual for year $y + 1$, which forms our disagreement measure, $|Disagree|^{res}$.

Table 3 repeats the analyses of Table 2 with the three alternative measures of disagreement, as defined above. We continue to find that greater disagreement increases volume and reduces bid-ask spread, consistent with these measures

[8]Following Bradshaw et al. (2016), $DXFIN_{it}$ is equal to the change in equity plus the change in debt during the six-month window. Change in equity is defined as net stock purchased and issued less dividends ($SSTK - PRSTKC - DV$), divided by assets as of the beginning of the six-month window. Change in debt is defined as the net cash received from the issuance or reduction of debt ($LTIS - DLTR - DLCCH$), divided by assets as of the beginning of the six-month window.

Table 3 Alternative disagreement measures

Panel A: Average daily trading volume			
	(1)	(2)	(3)
Variables	Volume	Volume	Volume
$\lvert Disagree^{control}\rvert$	0.007***		
	[0.001]		
$\lvert Disagree^{rev}\rvert$		0.001	
		[0.001]	
$\lvert Disagree\rvert^{res}$			0.011***
			[0.001]
Observations	83,663	119,948	129,269
R-squared	0.001	0.000	0.002
Panel B: Average daily bid-ask spread			
	(1)	(2)	(3)
Variables	Spread	Spread	Spread
$\lvert Disagree^{control}\rvert$	−0.003***		
	[0.000]		
$\lvert Disagree^{rev}\rvert$		−0.004***	
		[0.000]	
$\lvert Disagree\rvert^{res}$			−0.006***
			[0.000]
Observations	70,109	101,142	107,633
R-squared	0.004	0.004	0.006

This table presents results from regressing trading volume and bid-ask spread on alternative disagreement measures. The regression model is:

$$Volume_{it}(Spread_{it}) = \alpha_0 + \alpha_1 |Disagree_{it}| + \epsilon_{it}.$$

These alternative measures are defined in Section 4.3.1. In Panel A, $Volume$ represents the daily trading volume divided by the total daily number of shares outstanding, averaged over the six-month window prior to the end of quarter t. In Panel B, $Spread$ represents the daily bid-ask spread divided by the daily mid-point of bid and ask prices, averaged over the six-month window prior to the end of quarter t. $Volume$ and $Spread$ are multiplied by 100. The standard error is clustered at the firm level

capturing disagreement. The coefficient on $|Disagree^{rev}|$ is positive but statistically insignificant at the 10% level when the dependent variable is volume. As discussed, revenue forecasts do not include expenses, which can introduce noise to measuring investor disagreement. In untabulated analyses, we find a positive and statistically significant coefficient (at the 1% level) on $|Disagree^{rev}|$ after including year fixed effects, which limit the analyses to within-year variation. The evidence suggests that $|Disagree^{rev}|$ primarily captures cross-sectional variation in disagreement.

In untabulated analyses, we find that the three alternative disagreement measures are all positively correlated with $|Disagree^{c}|$. The correlation between $|Disagree^{c}|$ and the three alternative disagreement measures, namely $|Disagree^{control}|$,

$|Disagree^{rev}|$, and $|Disagree|^{res}$, is 47.5%, 18.9%, and 99.6%, respectively.[9] The finding provides some comfort that the baseline and alternative measures pick up related constructs.

4.3.2 Controlling for predictor variables of volume and spread

The relations we have documented, although consistent with our measures reflecting disagreement, may be attributable to them being correlated with other variables that explain volume and spreads. This section further addresses this concern by controlling for variables that reflect information asymmetry, investor uncertainty, or overall market interest in a firm, which could relate to both our measures and volume and spread. We caveat that this approach is conservative, because some of these variables could foster or be associated with disagreement (e.g., firm size or uncertainty). Thus the coefficient estimates on our disagreement measures are expected to change after including the control variables.

In the regressions involving control variables, we follow studies on liquidity and control for firm size ($Size$), market-to-book ratio (MTB), S&P 500 membership ($S\&P$), and institutional ownership ($Inst$) (e.g., Leuz and Verrecchia 2000; Balakrishnan et al. 2014). We also control for the natural logarithm of firm stock price ($log(Price)$) because of its effects on trading and, by extension, liquidity (e.g., Heflin et al. 2005). We measure these five control variables as of six months prior to the end date of quarter t. In addition to the five control variables, we also include the dispersion of analyst forecasts ($Dispersion$), which we define as the standard deviation of first forecasts issued by analysts during the six-month window prior to the end date of quarter t, divided by the stock price one month prior to the first forecast of the six-month window. We control for forecast dispersion to capture the effects of overall economic uncertainty. As our disagreement measures use analyst forecasts, forecast dispersion is a more suitable control variable for overall uncertainty than stock return volatility, which is another commonly used measure (e.g., Clement et al. 2003; Zhang 2006). Nevertheless, our inferences on the validity of the disagreement measures are unaffected by instead using return volatility (untabulated). Studies have also used forecast dispersion to capture disagreement (e.g., Diether et al. 2002; Banerjee 2011). As dispersion captures multiple constructs, its inclusion in our regression model should not qualitatively change the relation of our measures with volume and bid-ask spread (i.e., the sign and statistical significance of the coefficient estimates) if our measures primarily capture disagreement, as opposed to overall uncertainty.

The results, presented in Table 4, continue to document a positive relation of our measures with volume and a negative relation with bid-ask spread, consistent with them capturing disagreement. The coefficient magnitude changes, relative to Table 2, because many of the control variables are plausibly fostering or associated with disagreement.

[9] $Disagree^{control}$ and $Disagree^{rev}$, which are the underlying signed versions, have correlation coefficients of 49.3% and 19.7%, respectively, with $Disagree^{c}$.

Table 4 Controlling for predictor variables of volume and spread

Variables	(1) Volume	(2) Volume	(3) Spread	(4) Spread
$\lvert Disagree^c \rvert$	0.008***		−0.002***	
	[0.001]		[0.000]	
$\lvert Disagree^d \rvert$		0.033***		−0.014***
		[0.006]		[0.002]
$Size$	−0.048***	−0.047***	−0.065***	−0.065***
	[0.008]	[0.008]	[0.003]	[0.003]
MTB	0.122***	0.122***	−0.003**	−0.003**
	[0.006]	[0.006]	[0.001]	[0.001]
$S\&P$	0.222***	0.223***	0.085***	0.085***
	[0.029]	[0.029]	[0.009]	[0.009]
$Dispersion$	0.240***	0.241***	0.054***	0.054***
	[0.017]	[0.017]	[0.004]	[0.004]
$Inst$	0.895***	0.895***	−0.183***	−0.183***
	[0.025]	[0.025]	[0.015]	[0.015]
$log(Price)$	−0.024	−0.024	−0.043***	−0.043***
	[0.015]	[0.015]	[0.003]	[0.003]
Observations	146,682	146,682	123,021	123,021
R−squared	0.153	0.153	0.330	0.330

This table presents the results from regressing trading volume and bid-ask spread on disagreement after controlling for factors that are associated with information asymmetry or overall uncertainty. $Volume$ represents the daily trading volume divided by the total daily number of shares outstanding, averaged over the six-month window prior to the end of quarter t. $Spread$ represents the daily bid-ask spread divided by the daily mid-point of bid and ask prices, averaged over the six-month window prior to the end of quarter t. $Volume$ and $Spread$ are multiplied by 100. The regression model is:

$$Volume_{it}(Spread_{it}) = \alpha_0 + \alpha_1 |Disagree_{it}| + \Gamma' Controls_{it} + \epsilon_{it}.$$

$Controls$ represents a vector of control variables. $|Disagree_{it}|$ represents the absolute value of the corresponding disagreement measure. $Size$ is the natural logarithm of one plus the market value of equity six months prior to the end date of quarter t. MTB is the market-to-book ratio six months prior to the end date of quarter t. $S\&P$ is an indicator variable that equals one if a firm belongs to the S&P 500 index six months prior to the end date of quarter t. $Dispersion$ is the standard deviation of first forecasts issued by analysts during the six-month window prior to the end date of quarter t, divided by the stock price one month prior to the first forecast of the six-month window. $Inst$ is the percentage of institutional ownership six months prior to the end date of quarter t. $log(Price)$ is the natural logarithm of one plus the stock price six months prior to the end date of quarter t. The standard error is clustered at the firm level

4.4 Summary

In summary, the results from the regression analyses are consistent with our measures reflecting disagreement. As predicted, the disagreement measures are positively associated with trading volume and negatively associated with bid-ask spread. The measures' validity is also robust to adjustments for the bias and walk-down patterns in analyst earnings forecasts. We caution that such adjustments are likely unwarranted. Prior evidence indicates that investors do not fully adjust for forecast biases (So 2013; Veenman and Verwijmeren 2018; Lundholm and Rogo 2020), in which case those biases themselves would be a source of disagreement among investors. Furthermore, the results also suggest that our measures reflect a distinct source of belief dispersion in that it is empirically distinct from a list of factors associated with information asymmetry or overall uncertainty.

5 Disagreement and expected returns

We apply our measure of disagreement to assess whether there is a relation between disagreement and expected returns, which has been theoretically attributed to higher order beliefs (Bloomfield and Fischer 2011; Banerjee 2011) as well as constraints on short selling (Miller 1977; Hong and Stein 2007). Each of these theories suggests different mechanisms, which are not mutually exclusive, that could drive an association between disagreement and expected returns.

Under the higher order beliefs theory, disagreement implies that investors believe that others' beliefs evolve with error. Those perceived errors, in turn, influence investors' perceptions of future price uncertainty. If that uncertainty is priced, then the disagreement influences expected returns.

To illustrate the intuition underlying the higher order beliefs mechanism, we develop a simple model of trade, in the Appendix, that relies upon the information and belief structure employed previously. That is, two analysts disagree about the noise in the information each impounds into his or her individual forecasts. This information and belief structure is integrated into a two-period overlapping generations model of trade in which there is a continuum of investors who engage in two rounds of trade: one before the forecasts are released and one after their release. The terminal value is realized after the second round of trade, after which claims are paid. Finally, all of the investors have preferences characterized by a negative exponential utility function with risk aversion parameter normalized to 1; all investors can invest in a risk free asset with gross return normalized to 1; the supply of the risky asset per investor each period is normalized to 1; and half of the investors agree with one analyst, while the other half agree with the other.

Within the context of the model, we show that, absent disagreement, the expected return is independent of the noise in the signals. The quality of the information has no impact on the expected returns over the duration of the two periods, because, ultimately, the risk that must be borne by investors is just a function of the fundamental risk of the asset, $2s$.[10] Once disagreement is introduced, however, the model predicts that expected returns will increase with the extent of that disagreement. The reason

[10] The observation that information revelation does not influence the discount when there is no disagreement aligns with the result of Christensen et al. (2010). Note that, when the gross rate of return on the risk free asset exceeds 1, the timing of when the fundamental uncertainty is resolved can matter (Bloomfield and Fischer 2011).

is, as highlighted previously, when there is more disagreement regarding the noise in the signals received, investors believe that the second round price is excessively volatile, given the information made available to the market. As a consequence, they discount the asset's price more severely, which results in the asset offering a higher expected return.[11]

The second theory that predicts a relation between disagreement and returns stems from Miller (1977), who employs a single period model to show how short-selling constraints cause an equilibrium equity price to be influenced more by the higher valuations of optimists than the lower valuations of pessimists. As a consequence, holding average beliefs constant, greater disagreement implies a higher price. The observation obtained in the single period model extends to a model with multiple periods and, as argued by Hong and Stein (2007), has further implications for other observable statistics, such as trading volume and returns. For our purposes, the reasoning implies that increases (decreases) in the extent of disagreement over a period should be associated with higher (lower) returns for that period.

While both theories predict that disagreement can be positively associated with returns, tests of the predictions conceptually require somewhat different measures. The first requires a disagreement measure that reflects the average disagreement for each trading period within the return window of interest. Our measure, which reflects the average disagreement over a period, likely satisfies this requirement. The second requires a measure that reflects the change in disagreement between two points in time, that is, the first day of the return window and the last day of the return window. Our measure is somewhat less suited for this purpose, because it does not try to measure disagreement at specific points in time. To the extent that our disagreement measure for a quarter or its change over that quarter is associated with the change in disagreement regarding equity value between the beginning and end of a quarter, it will be useful for testing the predicted implication of the theory.

5.1 The association between disagreement and returns

We initially do not attempt to explicitly parse the two theories and instead assess whether there is a positive relation between returns and our disagreement measure after controlling for other predictors of returns. If such a relation exists, it could be attributable to disagreement influencing returns through higher order beliefs or, if the measure (i.e., the level) reflects the change in disagreement during a period, short-selling constraints being binding for some subset of firm-quarters.

To construct a sample to test for a relation between the disagreement measure and contemporaneous returns, we match monthly stock returns to firms' fiscal quarters,

[11] While our illustration predicts a positive relation between the extent of disagreement and contemporaneous returns, antecedent research involving higher order beliefs (Bloomfield and Fischer 2011; Banerjee 2011) illustrates how disagreement can also lower expected returns. Within the context of our simple model, we can obtain an analogous result if we assume the disagreement pertains solely to the covariance of the signals with the terminal value as opposed to the variance of those signals. The intuition is analogous to that in our illustrative model, in the sense that the disagreement causes investors to perceive that the second period price will vary less than the information warrants.

defined as the period within three months prior to the fiscal-quarter end date. We drop stocks trading at lower than five dollars per share following Banerjee (2011), require CRSP share codes of 10 or 11 (i.e., common shares), and remove instances of overlapping return windows (i.e., cases of fiscal-year end-date changes, resulting in the previous fiscal quarter end date being less than three months before the current fiscal-quarter end date). Our inferences regarding the effects of disagreement, however, are insensitive to these requirements. We express returns in percentage points.

The regression model applied to the sample is:

$$R_{im} = \beta_0 + \beta_1 |Disagree_{im}| + \Gamma' X_{im} + \eta_{im}, \tag{7}$$

where R_{im} is the monthly return of firm i in month m, $|Disagree_{im}|$ is firm i's disagreement measure in month m, and X_{im} is a vector of controls for other firm characteristics. The regressions control for several well-known cross-sectional firm characteristics associated with expected returns, including log of firm market capitalization ($Size$), log of book-to-market ratio ($log(BTM)$), return momentum ($Momentum$), lagged monthly return, return volatility ($Volatility$), share turnover ($Volume$), and the number of analysts ($Coverage$) (Lee and So 2017). We also control for analyst forecast dispersion ($Dispersion$) in light of its relation with overall uncertainty, which can affect expected returns (e.g., Zhang 2006). We measure return volatility, share turnover, the number of analysts, and forecast dispersion during month m so that we can capture their effects on contemporaneous returns. For return momentum, we use the cumulative stock return of the 12-month window ending in month $m - 1$. We measure market capitalization one month prior to the return date and book value of equity six months prior to the return date. All explanatory variables are standardized to have mean of zero and standard deviation of one, to facilitate comparison of their coefficient magnitudes. We report standard errors clustered by month to account for unobserved cross-sectional correlation in stock returns. Statistical inferences regarding the effects of disagreement are robust to using Fama-Macbeth standard errors (untabulated).

Table 5 finds consistent evidence that our disagreement measures relate positively to contemporaneous stock returns. The results are robust to including controls for other cross-sectional firm characteristics (columns (2) and (4)). Using coefficient estimates in column (2), a one standard deviation increase in disagreement is associated with a 0.41 percentage point higher stock returns. The economic magnitude of disagreement represents about 48% of the effect of market capitalization, 26% of the effect of return volatility, and 1.65 times the effect of forecast dispersion.

Regarding the coefficients on the control variables, firms with larger market capitalization, higher return volatility, and higher forecast dispersion have lower returns. A negative relation between return volatility and average returns might seem puzzling but has been documented by many studies (e.g., Ang et al. 2006; Ang et al. 2009). The negative coefficient for forecast dispersion is consistent with Banerjee's (2011) finding that portfolios with higher contemporaneous forecast dispersion have lower returns. We find that greater book-to-market ratio and return momentum are associated with greater returns, consistent with prior studies (e.g., Johnson et al. 2020), although the coefficients on these variables are not statistically significant at

Table 5 Disagreement and contemporaneous returns

Variables	(1) Return	(2) Return	(3) Return	(4) Return
$\lvert Disagree^c \rvert$	0.391***	0.414***		
	[0.069]	[0.053]		
$\lvert Disagree^d \rvert$			0.892***	0.934***
			[0.148]	[0.115]
$Dispersion$		−0.251***		−0.251***
		[0.081]		[0.081]
$Size$		−0.869***		−0.878***
		[0.233]		[0.233]
$log(BTM)$		0.148		0.146
		[0.171]		[0.171]
$Momentum$		0.158		0.153
		[0.168]		[0.169]
$Volatility$		−1.591**		−1.589**
		[0.670]		[0.671]
$Volume$		0.135		0.137
		[0.173]		[0.173]
R_{m-1}		0.127		0.128
		[0.285]		[0.285]
$Coverage$		0.356***		0.379***
		[0.117]		[0.117]
Observations	345,849	312,079	345,849	312,079
R−squared	0.001	0.017	0.001	0.017

This table examines the relation between disagreement and contemporaneous returns. The regression model is:

$$R_{im} = \beta_0 + \beta_1 |Disagree_{im}| + \Gamma' X_{im} + \eta_{im}.$$

R_{im} is the stock return of firm i in month m, measured in percentage points. $|Disagree_{im}|$ represents the absolute value of the corresponding disagreement measure for firm i in month m. X_{im} represents a vector of cross-sectional firm characteristics that explain expected returns. $Dispersion_{im}$ is the standard deviation of the first forecasts issued by analysts in month m for the quarter end earnings, divided by the stock price at the end of month $m-1$. $Size_{im}$ is the natural logarithm of market value at the end of month $m-1$. $log(BTM_{im})$ is the natural logarithm of book-to-market ratio, where the book value of equity is measured at least six months prior to month m. $Momentum_{im}$ is the cumulative stock return of the past 12 months. $Volatility_{im}$ is the standard deviation of daily stock returns in month m. $Volume_{im}$ is the mean of trading volume divided by shares outstanding in month m. $Coverage_{im}$ is the number of analysts covering a firm in month m. All explanatory variables except for $|Disagree^d|$ are standardized to have mean zero and variance one. The standard error is clustered by month

the 10% level. The coefficients on the lagged monthly return and share turnover are also insignificant. Analyst coverage is positively associated with stock returns.

To provide more evidence, we examine the association between returns and disagreement at the portfolio level. For each month, we sort firms into five portfolios based on the quintiles of $|Disagree^c|$ of the month and compute equal- and value-weighted returns for each portfolio. We also sort firms into two portfolios based on the level of $|Disagree^d|$ (recall that the measure ends up taking only 0 and 1) and compute equal- and value-weighted returns for the two portfolios. For each portfolio time series, we regress the monthly portfolio returns on the contemporaneous Fama-French three factors (market, size, and book-to-market factors) and the momentum factor. The regression model is:

$$R_{pm} - Rf_m = \alpha_p + \beta_{1p} MktRf_m + \beta_{2p} SMB_m + \beta_{3p} HML_m + \beta_{4p} UMD_m + \eta_{pm}, \quad (8)$$

where R_{pm} is the raw return of portfolio p in month m, and Rf_m, $MktRf_m$, SMB_m, HML_m, UMD_m are, respectively, the risk free rate, the Fama-French three factors (market, size, and book-to-market factors), and the momentum factor in month m. Robust standard errors are reported. We are interested in the difference in the portfolio α between the high and low disagreement portfolios (e.g., $\alpha_5 - \alpha_1$). If disagreement increases expected returns, the difference in the portfolio α should be positive.

Table 6 reports our findings using Eq. 8. Panels A and B report the results for the equal- and value-weighted portfolio return regressions based on portfolios sorted on $|Disagree^c|$ quintiles, and Panels C and D report the results based on portfolios sorted on $|Disagree^d|$. The rows present coefficient estimates using Eq. 8, with the highest disagreement portfolio reported in the top row, followed by portfolios based on lower levels of disagreement. For example, the first row of Panel A reports the coefficient estimates for the equal-weighted portfolio based on the highest $|Disagree^c|$ quintile. The coefficient differences between the highest and lowest disagreement portfolios are reported in the last row of each panel.

We find that, across all panels, greater disagreement is associated with a larger α (i.e., larger intercept). For example, using estimates from Panel A, moving from the lowest disagreement quintile to the highest, the portfolio α increases from -0.71% to 0.31%. The increase is both statistically and economically significant. Inferences from other panels resemble Panel A. In additional untabulated analyses, we continue to find a positive relation between firm- and portfolio-level returns and the three alternative disagreement measures defined in Section 4.3.1. Overall, we document a positive association between disagreement and contemporaneous expected returns at the firm and portfolio level.

5.2 Short-selling constraints and the association between disagreement and returns

To provide some insight into whether the documented association between disagreement and returns is attributable to just one or both theories, we explicitly consider how the cost of short selling moderates the association, where the cost is measured by equity loan fees from Markit following Beneish et al. (2015) and Zhou and Zhou (2020). If the association is driven primarily by a relation between disagreement and perceived uncertainty, there should be no moderating effect attributable to the cost of short selling. If the association is primarily driven by the cost of short selling, we

Table 6 Returns and disagreement at the portfolio level

	MKTRF	SMB	HML	UMD	Intercept
Panel A: Equal-weighted portfolio based on $\lvert Disagree^c \rvert$ quintiles					
$Q5(High)$	1.036***	0.665***	−0.061	−0.170***	0.313***
	[0.022]	[0.041]	[0.060]	[0.052]	[0.086]
$Q4$	1.062***	0.665***	0.002	−0.127***	0.182***
	[0.018]	[0.032]	[0.037]	[0.030]	[0.062]
$Q3$	1.075***	0.660***	0.024	−0.097***	0.050
	[0.021]	[0.031]	[0.032]	[0.021]	[0.063]
$Q2$	1.106***	0.714***	0.114***	−0.039**	−0.407***
	[0.026]	[0.026]	[0.030]	[0.015]	[0.061]
$Q1(Low)$	1.095***	0.856***	0.157***	−0.089**	−0.714***
	[0.034]	[0.038]	[0.052]	[0.037]	[0.091]
High−Low	−0.059	−0.190***	−0.218***	−0.081	1.027***
Standard error	0.040	0.056	0.080	0.064	0.125
Panel B: Value-weighted portfolio based on $\lvert Disagree^c \rvert$ quintiles					
$Q5(High)$	0.978***	−0.065	−0.125**	−0.065	0.177**
	[0.022]	[0.040]	[0.058]	[0.054]	[0.083]
$Q4$	1.021***	−0.034	−0.019	−0.024	0.175**
	[0.024]	[0.035]	[0.048]	[0.041]	[0.080]
$Q3$	0.981***	−0.006	−0.055	−0.040	−0.081
	[0.021]	[0.034]	[0.043]	[0.029]	[0.070]
$Q2$	1.058***	0.047	−0.005	0.051	−0.351***
	[0.029]	[0.038]	[0.040]	[0.033]	[0.086]
$Q1(Low)$	1.082***	0.213***	0.022	−0.038	−0.572***
	[0.042]	[0.050]	[0.065]	[0.034]	[0.110]
High−Low	−0.104**	−0.279***	−0.147*	−0.027	0.749***
Standard error	0.047	0.064	0.087	0.063	0.138
Panel C: Equal-weighted portfolio based on $\lvert Disagree^d \rvert$					
$1(High)$	1.056***	0.673***	−0.008	−0.124***	0.150***
	[0.016]	[0.028]	[0.035]	[0.031]	[0.055]
$0(Low)$	1.089***	0.807***	0.129***	−0.069***	−0.600***
	[0.026]	[0.027]	[0.034]	[0.021]	[0.068]
High−Low	−0.033	−0.134***	−0.138***	−0.054	0.751***
Standard error	0.031	0.039	0.049	0.037	0.087

Table 6 (continued)

	MKTRF	SMB	HML	UMD	Intercept
Panel D: Value-weighted portfolio based on $\lvert Disagree^d \rvert$					
$1(High)$	0.994***	−0.056***	−0.056**	−0.015	0.081**
	[0.010]	[0.014]	[0.022]	[0.020]	[0.035]
$0(Low)$	1.055***	0.151***	0.053	0.009	−0.486***
	[0.027]	[0.036]	[0.040]	[0.024]	[0.078]
High−Low	−0.061**	−0.207***	−0.110**	−0.024	0.567***
Standard error	0.029	0.039	0.045	0.031	0.086

This table examines the association between returns and disagreement at the portfolio level. For each month, we sort firms into five portfolios based on the quintiles of $\lvert Disagree^c \rvert$ of the month and compute equal- and value-weighted returns for each portfolio. We also sort firms into two portfolios based on the level of $\lvert Disagree^d \rvert$ and compute equal- and value-weighted returns for the two portfolios. For each portfolio time series, we regress the monthly portfolio returns on the contemporaneous Fama-French three factors (market, size, and book-to-market) and the momentum factor. The regression model is:

$$R_{pm} - Rf_m = \alpha_p + \beta_{1p} MktRf_m + \beta_{2p} SMB_m + \beta_{3p} HML_m + \beta_{4p} UMD_m + \eta_{pm},$$

where R_{pm} is the raw return of portfolio p in month m, and Rf_m, $MktRf_m$, SMB_m, HML_m, and UMD_m are, respectively, the risk free rate, the Fama-French three factors (market, size, and book-to-market factors), and the momentum factor in month m. Panels A and B report the results for the equal- and value-weighted portfolio regressions based on $\lvert Disagree^c \rvert$ quintiles; Panels C and D report results based on $\lvert Disagree^d \rvert$. The coefficient differences between the highest and lowest disagreement portfolios are reported in the last row of each panel. All returns are expressed in percentages. Robust standard errors are reported

expect the association to be almost nonexistent for those firms for which the cost of short selling is extremely low. If both mechanisms operate, we expect a significant association between disagreement and expected returns, even for those equities with extremely low costs of short selling, and the association will be larger for those equities with high costs of short selling.

As discussed previously, testing the theory involving short-selling constraints requires a measure that reflects the change in disagreement between the beginning and end of the period. Our disagreement measure may not proxy for that change particularly well. Hence we consider two proxies for that change using our measure: the measure itself and the change in the measure (i.e., disagreement in quarter q relative to quarter $q - 1$). To conduct our tests, we run two separate regressions of monthly stock returns on the disagreement measure as well as the change in the disagreement measure and consider how the association is moderated by the cost of short selling. We consider the moderating influence of the cost of short selling by creating five quintiles of equity loan fees, with the lowest quintile being the equities with the lowest fees. The empirical difference in the short-selling costs between the top and bottom quintiles is large. The average (median) short-selling cost of the top quintile is 270 (68) basis points, compared to 36 (37) basis points for the bottom quintile (untabulated).

When we use the disagreement measure itself (i.e., its level), the regression model is:

$$R_{im} = \beta_0 + \beta_1 |Disagree_{im}| + \sum_{j=2}^{5} \beta_j |Disagree_{im}| * Short_{im}^{j^{th}\ quintile} + \sum_{j=2}^{5} \beta_{4+j} Short_{im}^{j^{th}\ quintile} + \Gamma' X_{im} + \eta_{im}, \tag{9}$$

where $Short^{j^{th}\ quintile}$ is an indicator variable that equals one if equity loan fees belong to the j^{th} quintile and zero otherwise, and X represents the same set of cross-sectional firm characteristics included in Eq. 7. When we use the change in disagreement instead, we replace $|Disagree_{im}|$ with $\Delta|Disagree_{im}|$, which represents the change in disagreement in month m, relative to the most recent quarter. All continuous variables in the regression are standardized to have mean of zero and standard deviation of one to facilitate comparisons across coefficient estimates. The standard errors are clustered at the calendar month level to account for unobserved cross-sectional correlation in the error term.

Table 7 presents the results. Columns (1) and (2) report results for the level of disagreement, $|Disagree|$, and columns (3) and (4) for the change in disagreement, $\Delta|Disagree|$. Columns (1) and (3) use the continuous measure, and columns (2) and (4) the discrete measure. Like Miller (1977) and Hong and Stein (2007), we find a more pronounced positive association between disagreement (both levels and changes) and returns when short-selling constraints belong to the top quintile. The coefficient estimates are statistically significant at the 10% level or better.

The findings of a more pronounced positive association between disagreement (both levels and changes) and returns in Table 7 raise the possibility that the findings in Table 5 might entirely reflect the implications of the forces identified by Miller (1977) and Hong and Stein (2007). We offer two pieces of evidence against this interpretation. First, as columns (1) and (2) of Table 7 demonstrate, the level of disagreement is positively associated with returns, even when short-selling constraints are very low (i.e., they belong to the bottom quintile). For these stocks, short selling is relatively cheap. Therefore it is unlikely that constraints in short selling suppress the opinions of pessimists, which, in turn, results in a positive association between disagreement and returns. Second, as short-selling constraints increase but remain below the top quintile, there is not a statistically significant increase in the relation between disagreement and returns. If the forces identified by Miller (1977) and Hong and Stein (2007) were to dominate, we would expect a monotonic increase in the relation, because pessimistic views become more likely to be suppressed by increasingly binding short-selling constraints.

For robustness purposes, we also run the regressions underlying Table 7 using the three alternative measures of disagreement defined in Section 4.3.1. The untabulated results document that the inferences drawn from Table 7 are unaffected for the most part when using these alternative measures. We continue to find a more pronounced positive and statistically significant (at the 10% level or better) association between disagreement (both levels and changes) and returns when short-selling constraints

Table 7 Disagreement, short-selling constraint, and returns

Variables	(1) Return	(2)	Variables	(3) Return	(4)
Disagreement measure	Continuous	Discrete	Disagreement measure	Continuous	Discrete
$\|Disagree\|$	0.298***	0.727***	$\Delta\|Disagree\|$	0.251***	0.452***
	[0.078]	[0.192]		[0.079]	[0.130]
$\|Disagree\| * Short^{2^{nd} quintile}$	0.009	-0.030	$\Delta\|Disagree\| * Short^{2^{nd} quintile}$	-0.049	-0.046
	[0.105]	[0.253]		[0.098]	[0.173]
$\|Disagree\| * Short^{3^{rd} quintile}$	0.066	0.069	$\Delta\|Disagree\| * Short^{3^{rd} quintile}$	-0.061	-0.161
	[0.105]	[0.222]		[0.090]	[0.145]
$\|Disagree\| * Short^{4^{th} quintile}$	0.143	0.331	$\Delta\|Disagree\| * Short^{4^{th} quintile}$	0.038	-0.009
	[0.117]	[0.263]		[0.092]	[0.155]
$\|Disagree\| * Short^{5^{th} quintile}$	0.319***	0.500**	$\Delta\|Disagree\| * Short^{5^{th} quintile}$	0.207**	0.269*
	[0.117]	[0.229]		[0.092]	[0.144]
Short quintile indicators	Yes	Yes	Short quintile indicators	Yes	Yes
Controls	Yes	Yes	Controls	Yes	Yes
Observations	297,602	297,602	Observations	288,586	288,586
R-squared	0.017	0.017	R-squared	0.017	0.017

This table examines the relation between disagreement, short-selling constraint, and contemporaneous returns. The regression model in columns (1) and (2) is:

$$R_{im} = \beta_0 + \beta_1 |Disagree_{im}| + \sum_{j=2}^{5} \beta_j |Disagree_{im}| * Short_{im}^{j^{th}\ quintile} + \sum_{j=2}^{5} \beta_{4+j} Short_{im}^{j^{th}\ quintile} + \Gamma' X_{im} + \eta_{im}.$$

R_{im} is the stock return of firm i in month m, measured in percentage points. $|Disagree_{im}|$ represents the absolute value of the corresponding disagreement measure of firm i in month m. $Short^{j^{th}\ quintile}$ is an indicator variable for the j^{th} quintile of short-selling constraint. We follow Beneish et al. (2015) and Zhou and Zhou (2020) and measure short-selling constraint using equity loan fees from Markit. X_{im} represents a vector of cross-sectional firm characteristics that explain expected returns, discussed in Table 5. The regression model in columns (3) and (4) is:

$$R_{im} = \beta_0 + \beta_1 \Delta|Disagree_{im}| + \sum_{j=2}^{5} \beta_j \Delta|Disagree_{im}| * Short_{im}^{j^{th}\ quintile} + \sum_{j=2}^{5} \beta_{4+j} Short_{im}^{j^{th}\ quintile} + \Gamma' X_{im} + \eta_{im}.$$

$\Delta|Disagree_{im}|$ is the change in unsigned disagreement (i.e., the absolute value) relative to the most recent quarter. Columns (1) and (3) report results using the continuous disagreement measures, namely $|Disagree^c|$ and $\Delta|Disagree^c|$, and columns (2) and (4) report results using the the discrete disagreement measures, namely $|Disagree^d|$ and $\Delta|Disagree^d|$. All continuous explanatory variables are standardized to have mean of zero and standard deviation of one. The standard error is clustered by month

belong to the top quintile. Disagreement (both levels and changes) remains positively associated with returns in the bottom quintile of short-selling constraints, but the association is significant at the 10% level or better only for $|Disagree^{control}|$ and $|Disagree|^{res}$ in case of levels and only for $\Delta|Disagree|^{res}$ in case of changes.

In summary, the results in Tables 5, 6, and 7 are consistent with disagreement influencing expected returns through two channels: (1) higher order beliefs and (2) short-selling constraints. Consistent with the first channel, we find a significant association between disagreement and returns for firms with a very low cost of short selling, and, consistent with the second channel, we find that the association is larger for those firms that are most costly to short.

6 Conclusion

In this study, we have constructed a measure of disagreement (i.e., divergence of opinions) using analyst earnings forecasts. Our measure is theoretically motivated with the observation that, when analysts agree, the law of iterated expectations applies and a regression of an analyst's forecast on the previous forecast issued by another analyst should produce a slope coefficient of one. This logic is then extended to motivate using the slope coefficient's deviation from one as a measure of disagreement.

After motivating the approach for measuring disagreement, we apply it using analyst forecasts of quarterly earnings and validate the resulting estimates by showing that they are positively associated with trading volume and negatively associated with bid-ask spread, even after controlling for other pertinent variables. The measure estimates suggest that disagreement is pervasive among analysts. For example, our estimates suggest the presence of statistically significant disagreement at the 1% level in a two-tailed test in 67% of firm quarters. In addition, 95% of our estimates are consistent with analysts perceiving overreaction by others, which suggests that the observed disagreement is largely attributable to perceptions of overreaction as opposed to underreaction.

Having validated the disagreement measure, we conclude by applying it to test for an association between disagreement and expected returns, which is predicted by models involving concerns about higher order beliefs (Banerjee 2011; Bloomfield and Fischer 2011) and models involving short-selling constraints (Miller 1977; Hong and Stein 2007). Using our measure, we provide evidence consistent with the mechanisms underlying both theories being at play.

From a broader perspective, our study offers an approach for measuring disagreement, and our findings suggest that disagreement may be an economically meaningful construct when considered separate from information asymmetry. We acknowledge, however, that the approach's applications have limitations. In particular, our measurement approach requires sequential forecasts of some future event and is infeasible absent those forecasts. It is less useful if the theory being tested requires a disagreement measure for a specific point in time, as opposed to one requiring an average measure for a period. The measure reflects disagreement about the particular event being forecast and, as a consequence, might not capture all dimensions of disagreement. Nonetheless, we believe our measurement approach can be used in many settings and hope that others find the manner in which we have motivated our approach to be useful in constructing alternative measures when our particular approach is not applicable or feasible.

Appendix: Illustrative higher order beliefs model

To illustrate how disagreement influences returns through higher order beliefs, we offer a simple two-period overlapping generations (OLG) model of trade. In the model there is an initial round of trade, after which two analysts, A and B, forecast in sequence. After these forecasts, a second round of trade occurs. The asset's terminal value is then realized and claims are paid. There is a continuum of investors in the market in each period; each has negative exponential utility with a risk aversion parameter normalized to 1 (i.e., the utility of wealth w is $-exp[-w]$); the supply of the asset is normalized to one share per investor; and, in addition to the risky asset, each investor can invest in a risk free asset with a gross return normalized to one.

The terminal value of the asset is the realization of earnings $\tilde{e} = \mu+\tilde{a}+\tilde{b}$, where μ is the prior mean and $\tilde{a}$ and $\tilde{b}$ are independent mean-zero normally distributed random variables with variance s. Analyst A privately observes the realization of $\tilde{y}_A = \tilde{a} + \tilde{\alpha}$ prior to forecasting, and B privately observes A's forecast and the realization of $\tilde{y}_B = \tilde{b} + \tilde{\beta}$ prior to forecasting. Both analysts believe $\tilde{\alpha}$ and $\tilde{\beta}$ are independent mean-zero normally distributed random variables, but they disagree about their variances. Analyst A believes the variances of $\tilde{\alpha}$ and $\tilde{\beta}$ are σ and $\sigma + \delta$ respectively, where $\sigma > 0$ and $\delta > -\sigma$, and B believes the converse. Hence, if $\delta = 0$ the two analysts agree, and if $\delta > (<)\ 0$ each believes the other is overreacting (underreacting) to private information. Finally, we assume that half the investors agree with A and the other half agree with B.

Using backward induction, we first solve for the price at the second stage of trading. The demand of investor type $I \in A, B$ is

$$D_{I2} = \frac{E_I[\tilde{e}|f_A, f_B] - P_2}{Var_I[\tilde{e}|f_A, f_B]}.$$

The conditional expectations (after plugging in the forecasts) are $E_A[\tilde{e}|f_A, f_B] = \mu + \frac{s}{s+\sigma} y_A + \frac{s}{s+\sigma+\delta} y_B$ and $E_B[\tilde{e}|f_A, f_B] = \mu + \frac{s}{s+\sigma+\delta} y_A + \frac{s}{s+\sigma} y_B$, while the conditional variance is commonly $Var_I[\tilde{e}|f_A, f_B] = \frac{s\sigma}{s+\sigma} + \frac{s(\sigma+\delta)}{s+\sigma+\delta}$. Market clearing requires that $\frac{1}{2} D_{A2} + \frac{1}{2} D_{B2} = 1$. Hence the equilibrium price is

$$P_2 = \mu + \frac{1}{2}\left(\frac{s}{s+\sigma} + \frac{s}{s+\sigma+\delta}\right)(y_A + y_B) - \left(\frac{s\sigma}{s+\sigma} + \frac{s(\sigma+\delta)}{s+\sigma+\delta}\right).$$

Given P_2, the demand of investor type $I \in A, B$ in the first stage of trading is

$$D_{I1} = \frac{E_I[\tilde{P}_2] - P_1}{Var_I[\tilde{P}_2]}.$$

The common expectation is $E[P_2] = \mu - \left(\frac{s\sigma}{s+\sigma} + \frac{s(\sigma+\delta)}{s+\sigma+\delta}\right)$, and the common variance is $Var[P_2] = \frac{1}{4}\left(\frac{s}{s+\sigma} + \frac{s}{s+\sigma+\delta}\right)^2 (2s + 2\sigma + \delta)$. Market clearing requires that

$\frac{1}{2}D_{A1} + \frac{1}{2}D_{B1} = 1$. Hence the equilibrium price is

$$\begin{aligned} P_1 &= \mu - \left(\frac{s\sigma}{s+\sigma} + \frac{s(\sigma+\delta)}{s+\sigma+\delta}\right) - \frac{1}{4}\left(\frac{s}{s+\sigma} + \frac{s}{s+\sigma+\delta}\right)^2 (2s+2\sigma+\delta) \\ &= \mu - \left(\frac{s\sigma}{s+\sigma} + \frac{s(\sigma+\delta)}{s+\sigma+\delta}\right) - \frac{s^2(2s+2\sigma+\delta)^3}{4(s+\sigma)^2(s+\sigma+\delta)^2}. \end{aligned}$$

Hence price equals the common expectation of the terminal value, μ, less a discount for risk, $\left(\frac{s\sigma}{s+\sigma} + \frac{s(\sigma+\delta)}{s+\sigma+\delta}\right) + \left(\frac{s^2(2s+2\sigma+\delta)^3}{4(s+\sigma)^2(s+\sigma+\delta)^2}\right)$. When there is no disagreement, $\delta = 0$, that discount collapses to $2s$, which is just the agreed upon fundamental uncertainty. Most notably, the amount of fundamental uncertainty resolved via the signals, which is determined by σ, does not influence the discount, because the fundamental uncertainty must be borne regardless of when it is resolved.

If we differentiate this price with respect to δ, we obtain

$$\begin{aligned} \frac{\partial P_1}{\partial \delta} &= -\frac{s^2}{(s+\sigma+\delta)^2} + \frac{s^2(s+\sigma-\delta)(2s+2\sigma+\delta)^2}{4(s+\sigma)^2(s+\sigma+\delta)^3} \\ &= -\frac{\delta s^2\left(\frac{7}{4}(s+\sigma)^2 + \left(\delta + \frac{3}{2}(s+\sigma)\right)^2\right)}{4(s+\sigma)^2(s+\sigma+\delta)^3}, \end{aligned}$$

which has the opposite sign as that of δ. Hence, when there is disagreement, $\delta \neq 0$, the discount is increasing in the extent of that disagreement.

If we assume that $\mu > \left(\frac{s\sigma}{s+\sigma} + \frac{s(\sigma+\delta)}{s+\sigma+\delta}\right) + \left(\frac{s^2(2s+2\sigma+\delta)^3}{4(s+\sigma)^2(s+\sigma+\delta)^2}\right) > 0$ so that P_1 is positive, which is empirically reasonable, there is a positive relation between the magnitude of disagreement and expected returns. The expected return between the initial round of trade and the realization of earnings, $E\left[\frac{\tilde{e}-P_1}{P_1}\right] = \frac{\mu}{P_1} - 1$, is increasing in the magnitude of δ (i.e., the absolute value of δ) because P_1 is decreasing in the magnitude of δ.

To provide some intuition for why the discount for risk is increasing in disagreement regarding the noise in the signals, as represented by $|\delta|$, we show how increases in disagreement cause investors to perceive that the second round price is excessively volatile, relative to what it should be if all investors agreed. In particular, if all investors agreed with type $I \in \{A, B\}$ investors, the second period price would be

$$\begin{aligned} P_2^{ND} &= \frac{s}{s+\sigma}y_I + \frac{s}{s+\sigma+\delta}y_J - Var\left[\tilde{e}|y_A, y_B\right] \\ &= \frac{s}{s+\sigma}y_I + \frac{s}{s+\sigma+\delta}y_J - \left(\frac{s\sigma}{s+\sigma} + \frac{s\,(\sigma+\delta)}{s+\sigma+\delta}\right) \end{aligned}$$

and the first period price would be

$$\begin{aligned} P_1^{ND} &= -Var[\tilde{P}_2^{ND}] - Var[\tilde{e}|y_A, y_B] \\ &= -Var[\frac{s}{s+\sigma}\tilde{y}_I + \frac{s}{s+\sigma+\delta}\tilde{y}_J] - Var[\tilde{e}|y_A, y_B] \\ &= \left(\frac{s^2}{s+\sigma} + \frac{s^2}{s+\sigma+\delta}\right) - \left(\frac{s\sigma}{s+\sigma} + \frac{s\,(\sigma+\delta)}{s+\sigma+\delta}\right) \\ &= -2s. \end{aligned}$$

The fact that P_1^{ND} is only a function of the prior uncertainty regarding $\tilde{e}$, $2s$, is not surprising and is consistent with what we obtain when we assume no disagreement with $\delta = 0$. With disagreement among the investors, each investor believes $Var[\tilde{P}_2]$ differs from $Var[\tilde{P}_2^{ND}]$, which gives rise to a different first period price discount. In particular, the first period price is

$$P_1 = -Var[\tilde{P}_2] - Var[\tilde{e}|y_A, y_B] = -Var[\tilde{P}_2] + Var[\tilde{P}_2^{ND}] - 2s$$

where $Var\left[\tilde{P}_2\right]$ is the common variance for second period price. Furthermore,

$$Var\left[\tilde{P}_2\right] - Var[\tilde{P}_2^{ND}] = \frac{\delta^2 s^2 (2s + 2\sigma + \delta)}{4\,(s+\sigma)^2\,(s+\sigma+\delta)^2}.$$

Hence, as long as there is disagreement, $|\delta| > 0$, all investors believe that the second period price, P_2, is excessively volatile, which manifests in a larger discount for risk. Furthermore, differentiating that excess price volatility with respect to δ yields

$$\frac{\partial\left(Var\left[\tilde{P}_2\right] - Var[\tilde{P}_2^{ND}]\right)}{\partial\delta} = \frac{\delta s^2\left(\frac{7}{4}(s+\sigma)^2 + \left(\delta + \frac{3}{2}(s+\sigma)\right)^2\right)}{4(s+\sigma)^2(s+\sigma+\delta)^3},$$

which implies that the perceived excess price volatility is decreasing in δ for $\delta < 0$ and increasing in δ for $\delta > 0$. Consequently, the extent of the perceived excess price volatility and the discount are increasing in the extent of disagreement, $|\delta|$.

References

Abarbanell, J.S., W.N. Lanen, and R.E. Verrecchia. 1995. Analysts' forecasts as proxies for investor beliefs in empirical research. *Journal of Accounting and Economics* 20(1): 31–60.

Ang, A., R.J. Hodrick, Y. Xing, and X. Zhang. 2006. The cross-section of volatility and expected returns. *The Journal of Finance* 61(1): 259–299.

Ang, A., R.J. Hodrick, Y. Xing, and X. Zhang. 2009. High idiosyncratic volatility and low returns: International and further US evidence. *Journal of Financial Economics* 91(1): 1–23.

Avramov, D., T. Chordia, G. Jostova, and A. Philipov. 2009. Dispersion in analysts' earnings forecasts and credit rating. *Journal of Financial Economics* 91(1): 83–101.

Balakrishnan, K., M.B. Billings, B. Kelly, and A. Ljungqvist. 2014. Shaping liquidity: on the causal effects of voluntary disclosure. *The Journal of Finance* 69(5): 2237–2278.

Bamber, L.S., O.E. Barron, and T.L. Stober. 1997. Trading volume and different aspects of disagreement coincident with earnings announcements. *The Accounting Review* 72(4): 575–597.

Banerjee, S. 2011. Learning from prices and the dispersion in beliefs. *The Review of Financial Studies* 24(9): 3025–3068.

Banerjee, S., R. Kaniel, and I. Kremer. 2009. Price drift as an outcome of differences in higher-order beliefs. *The Review of Financial Studies* 22(9): 3707–3734.

Banerjee, S., and I. Kremer. 2010. Disagreement and learning: Dynamic patterns of trade. *The Journal of Finance* 65(4): 1269–1302.

Barber, B.M., and T. Odean. 2001. Boys will be boys: Gender, overconfidence, and common stock investment. *The Quarterly Journal of Economics* 116(1): 261–292.

Barron, O.E., O. Kim, S.C. Lim, and D.E. Stevens. 1998. Using analysts' forecasts to measure properties of analysts' information environment. *The Accounting Review* 73(4): 421–433.

Beneish, M.D., C.M. Lee, and D.C. Nichols. 2015. In short supply: Short-sellers and stock returns. *Journal of Accounting and Economics* 60(2-3): 33–57.

Bernhardt, D., M. Campello, and E. Kutsoati. 2006. Who herds?. *Journal of Financial Economics* 80(3): 657–675.

Bloomfield, R., and P.E. Fischer. 2011. Disagreement and the cost of capital. *Journal of Accounting Research* 49(1): 41–68.

Bradshaw, M.T., L.F. Lee, and K. Peterson. 2016. The interactive role of difficulty and incentives in explaining the annual earnings forecast walkdown. *The Accounting Review* 91(4): 995–1021.

Bradshaw, M.T., and R.G. Sloan. 2002. GAAP Versus the street: an empirical assessment of two alternative definitions of earnings. *Journal of Accounting Research* 40(1): 41–66.

Brown, L.D., A.C. Call, M.B. Clement, and N.Y. Sharp. 2015. Inside the "black box" of sell-side financial analysts. *Journal of Accounting Research* 53(1): 1–47.

Cao, H.H., and H. Ou-Yang. 2008. Differences of opinion of public information and speculative trading in stocks and options. *The Review of Financial Studies* 22(1): 299–335.

Chae, J. 2005. Trading volume, information asymmetry, and timing information. *The Journal of Finance* 60(1): 413–442.

Christensen, P.O., L.E. de la Rosa, and G.A. Feltham. 2010. Information and the cost of capital: an ex ante perspective. *The Accounting Review* 85(3): 817–848.

Clement, M., R. Frankel, and J. Miller. 2003. Confirming management earnings forecasts, earnings uncertainty, and stock returns. *Journal of Accounting Research* 41(4): 653–679.

Daniel, K., and D. Hirshleifer. 2015. Overconfident investors, predictable returns, and excessive trading. *Journal of Economic Perspectives* 29(4): 61–88.

Daniel, K., D. Hirshleifer, and A. Subrahmanyam. 1998. Investor psychology and security market under-and overreactions. *The Journal of Finance* 53(6): 1839–1885.

Diether, K.B., C.J. Malloy, and A. Scherbina. 2002. Differences of opinion and the cross section of stock returns. *The Journal of Finance* 57(5): 2113–2141.

Dugar, A., and S. Nathan. 1995. The effect of investment banking relationships on financial analysts' earnings forecasts and investment recommendations. *Contemporary Accounting Research* 12(1): 131–160.

Garfinkel, J.A. 2009. Measuring investors' opinion divergence. *Journal of Accounting Research* 47(5): 1317–1348.

Garfinkel, J.A., and J. Sokobin. 2006. Volume, opinion divergence, and returns: A study of post–earnings announcement drift. *Journal of Accounting Research* 44(1): 85–112.

Gervais, S., and T. Odean. 2001. Learning to be overconfident. *The Review of Financial Studies* 14(1): 1–27.

Goetzmann, W.N., and M. Massa. 2005. Dispersion of opinion and stock returns. *Journal of Financial Markets* 8(3): 324–349.

Grinblatt, M., and M. Keloharju. 2009. Sensation seeking, overconfidence, and trading activity. *The Journal of Finance* 64(2): 549–578.

Harris, M., and A. Raviv. 1993. Differences of opinion make a horse race. *The Review of Financial Studies* 6(3): 473–506.

Harrison, J.M., and D.M. Kreps. 1978. Speculative investor behavior in a stock market with heterogeneous expectations. *The Quarterly Journal of Economics* 92(2): 323–336.

Heflin, F.L., K.W. Shaw, and J.J. Wild. 2005. Disclosure policy and market liquidity: Impact of depth quotes and order Sizes. *Contemporary Accounting Research* 22(4): 829–865.

Hong, H., and J.C. Stein. 2007. Disagreement and the stock market. *Journal of Economic Perspectives* 21(2): 109–128.

Johnson, T.C. 2004. Forecast dispersion and the cross section of expected returns. *The Journal of Finance* 59(5): 1957–1978.

Johnson, T.L., J. Kim, and E.C. So. 2020. Expectations management and stock returns. *The Review of Financial Studies* 33(10): 4580–4626.

Kandel, E., and N.D. Pearson. 1995. Differential interpretation of public signals and trade in speculative markets. *Journal of Political Economy* 103(4): 831–872.

Kondor, P. 2012. The more we know about the fundamental, the less we agree on the price. *The Review of Economic Studies* 79(3): 1175–1207.

Lee, C.M., and E.C. So. 2017. Uncovering expected returns: Information in analyst coverage proxies. *Journal of Financial Economics* 124(2): 331–348.

Lehmer, T., B. Lourie, and D.M. Shanthikumar. 2020. Broker trading volume: A conflict of interest? Review of Accounting Studies, forthcoming.

Leuz, C., and R.E. Verrecchia. 2000. The economic consequences of increased disclosure. *Journal of Accounting Research* 38(Supplement): 91–124.

Liu, X.G., and R. Natarajan. 2012. The effect of financial analysts' strategic behavior on analysts' forecast dispersion. *The Accounting Review* 87(6): 2123–2149.

Lundholm, R., and R. Rogo. 2020. Do excessively volatile forecasts impact investors? Review of Accounting Studies 25: 636–671.

Lundholm, R.J., and R. Rogo. 2016. Do analyst forecasts vary too much? *Journal of Financial Reporting* 1(1): 101–123.

Miller, E.M. 1977. Risk, uncertainty, and divergence of opinion. *The Journal of Finance* 32(4): 1151–1168.

Morgan, J., and P.C. Stocken. 2003. An analysis of stock recommendations. *The RAND Journal of Economics* 34(1): 183–203.

Odean, T. 1999. Do investors trade too much? *American Economic Review* 89(5): 1279–1298.

Richardson, S., S.H. Teoh, and P.D. Wysocki. 2004. The walk-down to beatable analyst forecasts: the role of equity issuance and insider trading incentives. *Contemporary Accounting Research* 21(4): 885–924.

Sadka, R., and A. Scherbina. 2007. Analyst disagreement, mispricing, and liquidity. *The Journal of Finance* 62(5): 2367–2403.

Scheinkman, J.A., and W. Xiong. 2003. Overconfidence and speculative bubbles. *Journal of Political Economy* 111(6): 1183–1220.

So, E.C. 2013. A new approach to predicting analyst forecast errors: Do investors overweight analyst forecasts? *Journal of Financial Economics* 108(3): 615–640.

Veenman, D., and P. Verwijmeren. 2018. Do investors fully unravel persistent pessimism in analysts' earnings forecasts? *The Accounting Review* 93(3): 349–377.

Wang, H., J. Yan, and J. Yu. 2017. Reference-dependent preferences and the risk-return trade-off. *Journal of Financial Economics* 123(2): 395–414.

Zhang, X.F. 2006. Information uncertainty and stock returns. *The Journal of Finance* 61(1): 105–137.

Zhou, F.S., and Y. Zhou. 2020. The dog that did not bark: Limited price efficiency and strategic nondisclosure. *Journal of Accounting Research* 58(1): 155–197.