From Population Growth to Firm Demographics:
Implications for Concentration, Entrepreneurship and the Labor Share[*]

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Abstract

The US economy has undergone a number of puzzling changes in recent decades. Large firms now account for a greater share of economic activity, new firms are being created at a slower rate, and workers are getting paid a smaller share of GDP. This paper shows that changes in population growth provide a unified quantitative explanation for these long-term changes. The mechanism goes through firm entry rates. A decrease in population growth lowers firm entry rates, shifting the firm-age distribution towards older firms. Heterogeneity across firm age groups combined with an aging firm distribution replicates the observed trends. Micro data show that an aging firm distribution fully explains i) the concentration of employment in large firms, ii) and trends in average firm size and exit rates, key determinants of the firm entry rate. Building on empirical work that documents a negative relationship between firm size and labor share, we show that firm aging induced by population growth increases the market share of larger firms, leading to a decline in the aggregate labor share.

J.E.L. Codes: J11, E13, E20, L16, L26
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1 Introduction

Three long-term changes in the US economy have attracted a great deal of attention. First, economic activity is being concentrated in larger firms. For example, the fraction of workers employed by large firms increased by 6 percentage points since 1978. Second, the entrepreneurship rate — the ratio of new firms to total firms — has nearly halved since the 1970s. Third, the share of GDP going to labor, once thought to be stable, has declined since 1975. What explains these changes?

Our analysis begins by highlighting the importance of changing firm demographics—an aging firm distribution combined with heterogeneity by firm age—in driving these aggregate trends. We document that the increase in employment concentration is entirely driven by changing firm demographics. There has been no change in employment concentration within firm-age categories. Nevertheless, aggregate concentration has increased because an aging firm distribution shifts weights towards older firms, which have higher employment concentration. We document that changing firm demographics can also account for changes in two related variables: average firm size and the aggregate firm exit rate. Conditional on age, these variables have changed little over time. Because older firms are larger and exit at lower rates, an aging firm distribution leads to an increase in average firm size and a decline in the aggregate exit rate.

The decline in the entrepreneurship rate can be analyzed through the lens of a simple accounting identity. The firm entry rate equals the aggregate exit rate minus the growth in average firm size plus labor force growth,[1]

$$
\underbrace{\lambda}_{\text{Entry Rate}} = \underbrace{\xi}_{\text{Exit Rate}} - \underbrace{\widehat{e}}_{\text{AFS Growth}} + \underbrace{\widehat{N}}_{\text{LF Growth}} .
\tag{1}
$$

The exit rate and average firm size are constant in stationary equilibria of standard firm dynamics models. Therefore, changes in labor force growth are a natural candidate to explain changes in the firm entry rate. Holding the exit rate and average firm size constant, can a change in labor force growth explain the observed drop in firm entry rates? No. US labor force growth has declined, but not by enough. Figure 1 shows US civilian labor force growth rates by decade. Since the 1970s, labor force growth has declined by 2pp, which is only one-third of the 6pp decline in the entry rate. The remaining two-thirds is attributed to changes in the exit rate and changes in the growth rate of average firm size.

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[1]This identity comes from the definition of average firm size, $e \equiv N/M$, where N is the number of workers and M is the number of firms. It follows that the growth rate in the number of firms equals the growth rate in the number of workers minus the growth rate of average firm size, $\widehat{M} = \widehat{N} - \widehat{e}$. The growth in the number of firms also depends on firm entry and exit, $\widehat{M} = \lambda - \xi$. Combining these two equations leads to identity (1). We measure $\widehat{N}$ using labor force growth. Other measures of $\widehat{N}$ are discussed in Section 5.2.
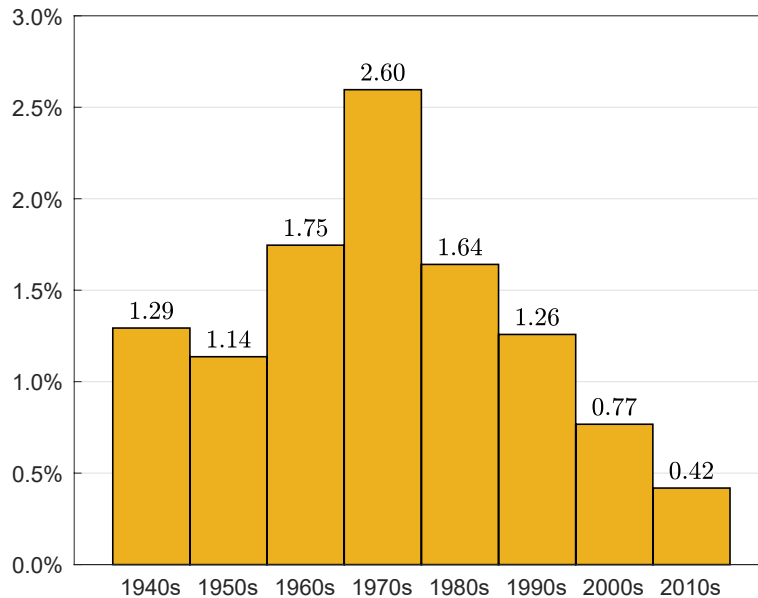
Figure 1: Civilian Labor Force Growth Rate

We show that changes in labor force growth lead to changes in both the aggregate exit rate and average firm size. Consider an increase in labor force growth. The increase in labor supply must be met by a corresponding increase in labor demand. Incumbent firms are limited by scale, so they cannot absorb the entire increase in labor supply. The residual labor demand must therefore be absorbed by new firms. The increase in firm entry shifts the firm-age distribution towards younger firms, which have higher exit rates and lower size.

To be consistent with the data, the changes in labor force growth should change aggregate variables while maintaining stability of these variables by firm age. While this property holds along a balanced growth path, it is not clear that it carries over to transitions. This distinction is of interest because the growth in average firm size in the data is non-zero, indicating that the US economy is going through a transition. The theoretical challenge is to show that an evolving firm-age distribution along the transition path is consistent with stability of firm-level variables by age. We derive sufficient conditions for the existence of such an equilibrium in a general framework that incorporates standard models of perfect and imperfect competition.

The transitional dynamics of firm entry depend on the entire history of past entry. Firm entry fills the gap between labor supply and incumbent labor demand. Therefore, entry depends on total labor demand by incumbents in each age group, which in turn is determined by past entry, survival probabilities and average size. This leads to the *dynamic entry equation*, which relates current entry to the distributed lag of past entries. The dynamic nature of entry implies that changes in current entry affect future entry, through

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the firm-age distribution.

How much of the secular changes experienced by the US economy can be explained by changes in labor force growth? We can answer this question by feeding the labor force series into the dynamic entry equation. This exercise requires knowledge of (i) the initial firm-age distribution and (ii) average size and survival probabilities for all age groups. Data on these variables is limited. We use two alternative approaches to impute the initial distribution and firm-age variables from the data. The first approach does not impose a functional form on these variables. The second approach specifies a stochastic process for employment and derives an initial distribution consistent with a balanced growth path. Both approaches yield similar results.

We find the decline in labor force growth can explain the majority of the observed decline in firm entry rates from 1978 to 2014. In addition, changes in labor force growth explain well two episodes in the data: the pre-1978 increase in the entry rate, and the large fluctuations in the entry rate around World War II. As in the data, the post-1978 decline in labor force growth generates a 2pp decline in the exit rate, a 6pp increase in employment concentration, and an aging of the firm distribution. Declining labor force growth also generates an increase in average firm size, as in the data, but slightly overshoots. This occurs because average firm size for older ages declined slightly after the year 2000 in the data.

We evaluate the quantitative role of history dependence, transitional dynamics and the feedback effect of firm demographics in generating the aggregate patterns by repeating the quantitative exercise in three counterfactual economies. To evaluate the role of history dependence, we assume that labor force growth was constant before 1978. This eliminates one-third of the decline in the entry rates. We shut down transitional dynamics by comparing the entry rates along balanced growth paths with the 1978 and 2014 labor force growth rates. This eliminates half of the decline in the entry rate. We shut down the feedback effect from firm demographics by assuming exit rates and average firm size are equal across age groups. This eliminates two-thirds of the decline in entry rates. The counterfactual exercises show that history dependence, transitional dynamics and firm demographics all play quantitatively important roles in shaping the aggregate trends.

We use labor force projections to predict the future path of entry rates. Despite a projected decline in labor force growth, we find that future entry rates bounce back by 1pp. This can be understood using identity (1). The exit rate is projected to increase because the glut of firms born in the years of high labor force growth will have mostly died off and will have been replaced by younger firms, which have higher exit rates on average. The projected growth in average firm size hits zero because the economy is expected to converge to a balanced growth path. Both these forces more than counteract the effect of the future

decline in labor force growth.

We next turn to the labor share. A recent set of papers document two facts about labor shares: (i) firm-level labor shares are negatively related to firm size and (ii) almost all of the decline in the aggregate labor share is due to reallocation from high to low labor share units, rather than a decline within labor share units.[2] Because older firms tend to be larger, an aging of the firm distribution corresponds to reallocation of value added towards larger firms. It follows that a decline in labor force growth lowers the aggregate labor share, by shifting the firm-age distribution towards older firms. This is consistent with the data: the decline in the corporate labor share in Karabarbounis and Neiman (2014) is coincident with the decline in labor force growth from 1980 onwards. Recent work by Koh, Santaeulalia-Llopis and Zheng (2018) extends the measurement of the corporate labor share back to 1947, and finds a hump-shaped pattern. This is the pattern predicted by labor force growth, which also follows a hump shape.

We close by showing that changes in population growth are the primary driver of changes in labor force growth. We decompose labor force growth into three components: birth rates 16 years prior, the growth in participation rates, and a residual term that captures rates of migration, death and institutionalization. Birth rates account for the bulk of the changes in labor force growth.

Related Literature. Our paper builds on a wealth of recent empirical evidence from seemingly disconnected strands of the literature. One strand of the literature has documented changes in entry rates and the age distribution of firms. Reedy and Strom (2012) documents declining firm entry, while Pugsley and Şahin (2018), Decker, Haltiwanger, Jarmin and Miranda (2014), Hathaway and Litan (2014a), Gourio, Messer and Siemer (2015) and Davis and Haltiwanger (2014) document the pervasiveness of this decline across geographic areas and industries. Decker, Haltiwanger, Jarmin and Miranda (2014), Hathaway and Litan (2014b) and Pugsley and Şahin (2018) document the aging of the firm distribution and link it to declining firm entry. A different strand of the literature has documented trends in the aggregate labor share and the rise in concentration. Karabarbounis and Neiman (2014) find that the decline in the labor share is primarily a within-industry rather than a cross-industry phenomenon. Grullon, Larkin and Michaely (2017) document increased concentration across most U.S. industries, whereas Barkai (2017) and Autor, Dorn, Katz, Patterson and Van Reenen (2017) both document a positive correlation between industry concentration and the decline in the labor share. Our paper incorporates all of these em-

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[2]Hartman-Glaser, Lustig and Xiaolan (2019) documents this pattern by showing that the capital share has been increasing for the largest public firms in the US. Autor, Dorn, Katz, Patterson and Van Reenen (2017) document the same pattern using US Census Data. Kehrig and Vincent (2018) document the reallocation for manufacturing establishments.

pirical findings into one unified explanation.

We are not the first paper to propose the decline in labor force growth as an explanation for the decline in firm entry rates. Using lagged fertility rates as an instrument, Karahan, Pugsley and Şahin (2018) find that the entry rate is highly elastic to changes in labor supply across US states.[3] The authors then explore the role of labor force growth in the steady state of a Hopenhayn (1992a)-style model. There are two main differences between our papers. First, we aim to explain a broader set of facts, such as the increase in concentration and the decline of the labor share. Second, our study focuses on transitional dynamics, allowing us to uncover how the history of past entry matters for current entry and firm demographics.

To the best of our knowledge, ours is the first paper that jointly explains the evolution of entrepreneurship, concentration, and the labor share. Alternative explanations have been proposed for a subset of these trends.[4] One related, but distinct, explanation is that of the aging of the workforce (Liang, Wang and Lazear, 2018; Kopecky, 2017; Engbom, 2017).[5] We note that a decline in labor force growth is a different phenomenon than an aging workforce. Another explanation that has gained considerable attention is that of the rise in market power, as measured by increasing markups (Loecker and Eeckhout, 2017). Our framework shows that it is possible to generate an increase in concentration without decreasing competition.[6]

The rest of the paper is organized as follows. Section 2 presents the data. Section 3 presents the theoretical results. Section 4 presents the quantitative findings. Section 5 discusses drivers of labor force growth and alternative measures of labor supply. Section 6 concludes.

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[3]Hathaway and Litan (2014c) also note a correlation between declining firm entry rates and population growth across geographic regions. Other explanations for the decline in entrepreneurship include the decline in corporate taxes (Neira and Singhania, 2017), the decline in interest rates, (Liu, Mian and Sufi, 2018; Chatterjee and Eyigungor, 2018), and skill-biased technical change (Salgado, 2018; Jiang and Sohail, 2017).

[4]Explanations specific to the labor share decline include an increase in firm-level volatility (Hartman-Glaser, Lustig and Xiaolan, 2019), the treatment of intangible capital (Koh, Santaeulalia-Llopis and Zheng, 2018), the decline in the relative price of capital (Karabarbounis and Neiman, 2014), capital accumulation (Piketty and Zucman, 2014), import competition and globalization (Elsby, Hobijn and Sahin, 2013), and corporate taxes (Kaymak and Schott, 2018).

[5]More broadly, population aging has been linked to slower growth in advanced economies; see Cooley and Henriksen (2018).

[6]Rossi-Hansberg, Sarte and Trachter (2018) also show that increasing concentration at the aggregate level need not be generated by a decline in competition. They present evidence that the positive trend observed in national product-market concentration becomes a negative trend when focusing on measures of local concentration.

2 Data

We obtain data on firms from the Business Dynamics Statistics (BDS) produced by the US Census Bureau. The BDS dataset covers the 1977 to 2014 period. It has near universal coverage of private sector firms with paid employees.

We start by looking at the time series evolution of concentration, average firm size and the aggregate exit rate in US data; see top panel of Figure 2. We measure concentration as the share of employment by firms with 250+ employees. Figure 2 shows that concentration in the US has increased from about 51% to 57%.[7] Average firm size in the US has increased steadily from about 20 employees to about 24 employees. The aggregate exit rate has declined steadily from about 9.5 percentage points to about 7.5 percentage points. The bottom panel of Figure 2 shows the time series of concentration, average firm size and exit rates broken down by firm age. None of the aggregate changes have occurred within firm-age bins. For example, a typical five year old firm has the same size in 1980 and 2014, with no discernible trend. The same pattern holds for concentration and exit rates: conditional on age, concentration and exit rates do not exhibit a trend over the 1977-2014 time period. It follows that the aggregate trends in concentration, average firm size and exit rates are not being driven by changes in the corresponding variables within firm-age categories.

The bottom panel of Figure 2 also shows patterns for each variable by firm age. Concentration and average firm size increase with age. Firm exit rates decrease with age. These patterns suggest that changes in the age composition of firms drive the aggregate trend in each variable. In order to investigate this formally we run the following regression,

$$y_{ajt} = \beta_0 + \beta_y\, year + \sum_a \beta_a\, age + \sum_j \beta_j\, sector + \sum_a \sum_j \beta_{aj}(age \times sector) + \varepsilon_{ajt},$$

where y_{ajt} equals the share of employment by firms with 250+ employees, log average firm size or firm exit rates. We start with a specification that features *year* with an intercept term. The coefficient on *year* captures the aggregate trend in dependent variable. We then add age controls and see how the year coefficient changes. For the average firm size and firm exit rate regressions, we add further controls for sector and age-sector interaction effects in successive specifications.[8]

The regression results confirm that changes in the age composition drive the aggregate

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[7]The increase in concentration is robust to the firm size cutoff. For size cutoffs of 5, 10, 20, 50, 100, 250, 500, 1000, 2500, 5000, and 10,000 employees, the share of employment increased by 1.6, 3.1, 4.3, 5.4, 6.0, 5.7, 5.1, 4.6, 3.9, 3.1, and 2.4 percentage points, respectively.

[8]To protect the identity of firms, the Business Dynamics Statistics do not report data on share of employment by firm size, age and sector. Therefore, we cannot include controls for sector and age-sector interactions in the concentration regression.
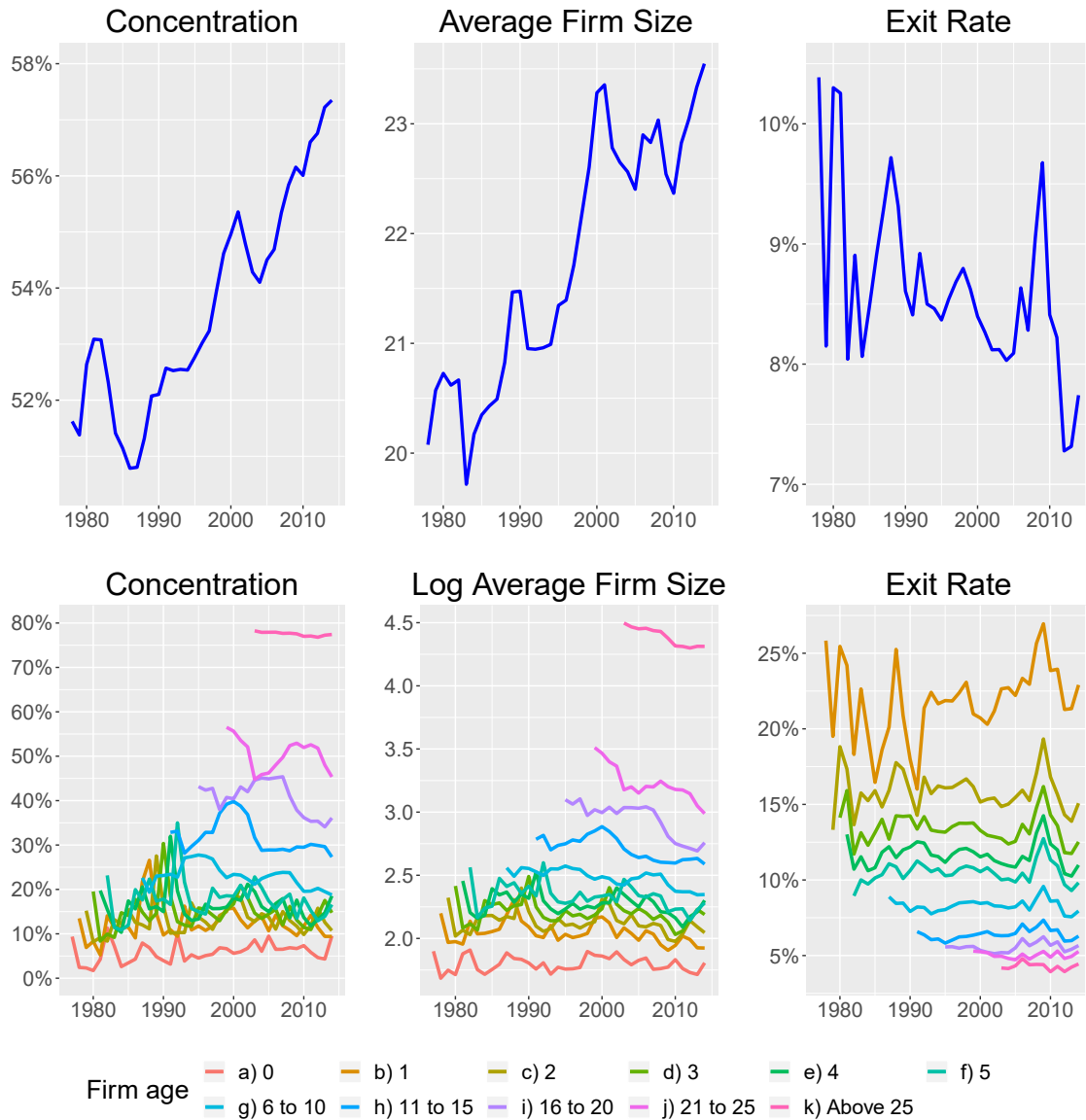
Figure 2

Notes. Concentration is the share of employment in firms with 250+ employees. Concentration within an age category is share of employment in firms with 250+ employees within the age category divided by total employment in the age category. The *Above 25* age category includes firms labeled 26+ and Left Censored firms in the Business Dynamics Statistics. Average firm size is number of workers per firm.

trends.[9] Without controls, the average trend across age groups and sectors in each variable is statistically significant and non-zero. Once we control for age, the trend disappears or reverses sign. The inclusion of controls for sector and age-sector interactions has no further effect on the trend. The coefficients on the age controls exhibit the same patterns as Figure 2: they increase with age for average firm size and concentration, and decrease with age

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[9]The regression tables are presented in Tables B-1 to B-3 in Appendix B.

for exit rates.

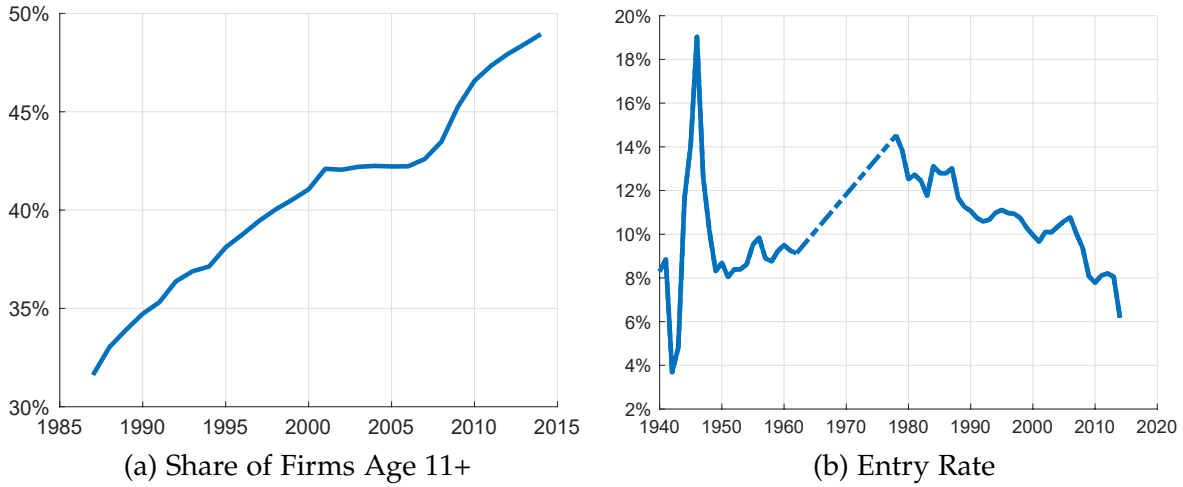(a) Share of Firms Age 11+

(b) Entry Rate

Figure 3

Sources. (a) BDS. (b) Entry rate 1940-1962: Survey of Current Business. Entry rate 1978-2014: BDS.
Notes. The entry rate from 1963 to 1977 is linearly interpolated.

Figure 3a presents direct evidence that US firms are aging. The figure shows that the share of firms aged 11+ has risen steadily from 32 percent in 1986 to 48 percent in 2014. Figure 3b shows the contemporaneous decline in entry rates. The entry rate series can be extended back to 1940. Two episodes stand out in the early period of the entry rate series. First, the entry rate displayed large fluctuations around World War II. Second, the entry rate displayed an apparent increase before 1978.[10]

3 Theory

Firms have a common discount factor β. There is a fixed endowment of a labor N_t, which is inelastically supplied and also the numeraire. Firms are confronted with an aggregate state Z and an idiosyncratic state s. The aggregate state Z is determined as part of the equilibrium. The idiosyncratic state s follows a Markov process with conditional distribution $F(s_{t+1}|s_t)$, which we assume is continuous and nondecreasing. Let $R(s, n, Z)$ denote the revenue function, where n is productive labor demand. Firms have a fixed cost of operation $c_f(s)$ denominated in units of labor, which can include an entrepreneur and other labor overhead. Firm revenue net of operating cost is $R(s, n, Z) - c_f(s)$. Assume that net revenue is increasing in all arguments, concave in n, and supermodular in (s, n) and (Z, n),

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[10]The entry rate from 1940 to 1962 comes from the now discontinued Survey of Current Business. The entry rate from 1963 to 1977 is linearly interpolated. The apparent increase in that period is consistent with the increase in the entry rate for *establishments* documented by Karahan, Pugsley and Şahin (2018).

i.e. $R_{sn} > 0$ and $R_{Zn} > 0$ if differentiable. Let $\pi(s, Z)$ and $n(s, Z)$ denote the profit and employment functions, respectively. These functions are increasing in s and Z.

The value of a firm is given by the Bellman equation:

$$v(s, \mathbf{Z}_t) = \max\left\{0, \pi(s, Z_t) + \beta Ev(s', \mathbf{Z}_{t+1}|s)\right\}$$

when confronted with a deterministic path of $\mathbf{Z}_t = \{Z_\tau\}_{\tau \geq t}$. The value of exit is normalized to zero, while the right hand side under the maximization is the continuation value for the firm. It is easy to show that this value is increasing in s and Z_t when nonzero. Let

$$s_t^* = \inf\left\{s|\pi(s, Z_t) + \beta Ev(s', \mathbf{Z}_{t+1}|s) > 0\right\}. \tag{2}$$

A firm is shut down iff $s \leq s_t^*$.

The technology for entry of a new firm is as follows. Upon paying a cost of entry of c_e units of labor, entrants draw their initial productivity from the distribution G. The productivity draws are independent across entrants and time. Prior to entry, the expected value of an entrant net of the entry cost is

$$v^e(\mathbf{Z}_t) = \int v(s, \mathbf{Z}_t) dG(s) - c_e. \tag{3}$$

Let μ_t denote the measure of firms operating at time t, where for a fixed set A of firm types, $\mu_t(A)$ measures the magnitude of firms that at time t have $s_{it} \in A$. Given an initial measure μ_0, the exit thresholds s_t^* together with mass of entrants m_t determine uniquely the sequence of measures $\{\mu_t\}$ recursively as follows. For any set of productivities A, define

$$\mu_{t+1}(A) = m_{t+1}\left(\int_{s\in A, s\geq s_{t+1}^*} dG(s)\right) + \int\int_{s\in A, s\geq s_{t+1}^*} dF(s|x) d\mu_t(x) \tag{4}$$

The first term in the right hand side corresponds to entrants, excluding those that exit immediately, while the second term corresponds to incumbents after the realization of new productivities, excluding those that exit.

3.1 Examples

Our formulation is general and can encompass models of perfect and imperfect competition.

Perfect Competition. Firms produce a homogeneous good with a decreasing returns to scale production technology $q(s, n)$, where s can be interpreted as a productivity shock.

The model is the standard entry and exit model considered in the literature based on Hopenhayn (1992b). In this example, the revenue function $R(s, n, Z)$ is simply equal to $Zq(s, n)$, where Z is the equilibrium price p of the good. The production function $q(s, n)$ allows firm level labor shares to be decreasing in firm size, which depends on s.

Monopolistic Competition with Constant Elasticity. Each firm i produces a differentiated good with linear production function, $q(s) = sn(s)$. The representative consumer has preferences over intermediate goods given by the aggregator

$$U = \left(\int c(s)^\eta \, d\mu(s) \right)^{1/\eta}$$

with $0 < \eta < 1$, where $c(s)$ denotes consumption of a good of type s by the consumer. Total population is N and the measure of firms is μ, so output per firm is $q(s) = Nc(s)$. First order conditions for the choice of $c(s)$ are given by

$$U^{1-\eta} c(s)^{\eta-1} = \theta p(s),$$

where θ is the multiplier of the budget constraint of the consumer.[11] Letting $Z = NU^{1-\eta}\theta^{-1}$, the revenue function is $R(s, n, Z) = Z(sn)^\eta$, which satisfies all our assumptions above.

3.2 Equilibrium

Let $M_t = \int d\mu_t(s)$ denote the total mass of firms. Labor market clearing requires that:

$$\int n(s, Z_t) \, d\mu_t(s) + \int c_f(s) d\mu_t(s) + m_t c_e = N_t. \tag{5}$$

The first term is productive labor demand, the second term overhead and the third labor utilized for entry, e.g. entrepreneurs in startups. The right hand side represents total labor inelastically supplied.

An equilibrium for a given sequence $\{N_t\}$ and given initial measure μ_0 is given by shutdown thresholds $\{s_t^*\}$, mass of entrants $\{m_t\}$, measures of firms $\{\mu_t\}$ and aggregate states $\mathbf{Z}_t = \{Z_t\}$ such that:

1. Exit: Shutdown thresholds are given by equation (2);

2. Entry: No rents for entrants, $v^e(\mathbf{Z}_t) \leq 0$ and $v^e(\mathbf{Z}_t) m_t = 0$;

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[11]An alternative equivalent formulation is that U represents a final good produced by perfectly competitive firms with the production function aggregator given above. In that case, θ^{-1} is the price of the final good.

3. Resource constraint (5) holds.

4. Law of motion: The sequence μ_t is generated recursively by equation (4) given the initial measure μ_0.

We focus on equilibria with strictly positive entry, which is the relevant case in reality. The existence and uniqueness of such an equilibrium can be proved along the lines of Hopenhayn (1992b). Along the lines of Hopenhayn (1992a), it can be shown that a stationary equilibrium exists and is unique when labor N_t grows at a constant rate. Here we generalize this result to the case where labor is growing at non-constant rates. In particular, we provide conditions for the existence and uniqueness of a *constant aggregate state equilibrium*, $Z_t = Z^*$ for all t. Under the above assumptions, it can be shown that Z^* is unique and corresponds to the aggregate state in the stationary equilibrium in Hopenhayn (1992a). In what follows, we develop the existence argument.

For existence, we need to show that the equilibrium conditions hold in every period. Let Z^* be such that the entry condition holds, $v^e(Z^*) = 0$. Let $s_t^* = s^*$ be the corresponding shutdown threshold, so that the exit condition holds. Given μ_0, we construct the sequence μ_t recursively such that the law of motion holds.

It remains to verify that the resource constraint holds. Let S_a denote the probability that an entrant survives at least a periods, i.e. that the state $s_{i\tau} \geq s^*$ for ages τ from 0 to a. Let $\tilde{\mu}_a$ denote the cross-sectional probability distribution of productivities for firms in the cohort of age a. These can be obtained recursively as follows:

1. Let $S_0 = (1 - G(s^*))$. Let $\tilde{\mu}_0(ds) = G(ds)/S_0$ denote the distribution of entrant productivity draws conditional on $s \geq s^*$.

2. Let $S_a = S_{a-1} \int P(s_a \geq s^*|s_{a-1}) d\tilde{\mu}_{a-1}(s_{a-1})$, where the term under the integral is the probability that a firm in cohort $a - 1$ is not shutdown in the next period, and let

$$\tilde{\mu}_a(ds) = \frac{\int P(ds_a|s_{a-1}) d\tilde{\mu}_{a-1}(s_{a-1})}{S_a/S_{a-1}}.$$

Let $\tilde{e}_a = \int(n(s, Z^*) + c_f(s))d\tilde{\mu}_a$ denote the average employment of a firm in the age n cohort. Let E_{ta} denote total employment by that cohort at time t. In addition to average employment $\tilde{e}_a$, total employment E_{ta} depends on the original mass of entrants in that cohort and the survival rate,

$$E_{ta} = m_{t-a}S_a\tilde{e}_a.$$

Total employment by incumbents (i.e. excluding new entrants) at time t is the sum of employment by cohorts with age greater than one, $E_t^I = \sum_1^t E_{ta}$. On adding E_t^I and total

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employment by entrants $m_t (S_0 \tilde{e}_0 + c_e)$, we recover the resource constraint:

$$N_t = m_t (S_0 \tilde{e}_0 + c_e) + E_t^I. \tag{6}$$

Given that Z^* is constant, S_a and $\tilde{e}_a$ are known at time t. Because m_{t-a}, and therefore E_t^I, are also known at time t, the only unknown in the above equation is m_t. It follows that equation (6) implicitly determines m_t such that the resource constraint holds. If m_t is strictly positive, all equilibrium conditions hold and the existence argument is complete. This occurs provided that $E_t^I < N_t$ in every period t. The following proposition provides sufficient conditions for strictly positive entry.

Proposition 1 (Constant Aggregate State Equilibrium). *Suppose that N_t is a nondecreasing sequence and $S_a \tilde{e}_a$ is non-increasing. Then the aggregate state and the exit threshold are constant in the unique equilibrium, $Z_t = Z^*$ and $s_t^* = s^*$.*

The intuition is as follows. Because N_t is a nondecreasing sequence, a sufficient condition for $E_{t+1}^I < N_{t+1}$, which guarantees strictly positive entry in period $t + 1$, is that $E_{t+1}^I < N_t$. Note that

$$N_t = m_t S_0 \tilde{e}_0 + m_{t-1} S_1 \tilde{e}_1 + ... + m_0 S_t \tilde{e}_t + m_t c_e$$
$$E_{t+1}^I = m_t S_1 \tilde{e}_1 + m_{t-1} S_2 \tilde{e}_2 + ... + m_0 S_{t+1} \tilde{e}_{t+1}.$$

Therefore E_{t+1}^I is the inner product of the same vector of the mass of entrants as N_t, with a forward shift in the corresponding terms $S_a \tilde{e}_a$ and without the entry cost term $m_t c_e$. A sufficient condition for $N_t - E_{t+1}^I > 0$ every period is that $S_a \tilde{e}_a$ decreases with a. For a given cohort, this condition is equivalent to saying that the total employment of the cohort is decreasing in age. In the data, survival rates are decreasing in a but average size of a cohort, when properly calibrated, is increasing. Therefore the sufficient condition holds when shutdown rates are sufficiently high to offset the growth in average size. In the model, this property is easy to verify given the stochastic process for the idiosyncratic shocks s_{it} and the shutdown threshold s^*.[12] Note that the conditions in Proposition 1 are sufficient, but not necessary, for a constant aggregate state equilibrium. Now we discuss various properties of the equilibrium.

Corollary 1 (Time Invariance). *Exit rates by age, average firm size by age, and size distributions by age are time invariant in a constant aggregate state equilibrium.*

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[12]Models that assume permanent productivity shocks and exogenous exit trivially satisfy this condition. The same holds true for the models where productivity shocks are redrawn with some probability from the same distribution as entrants, e.g. Mortensen and Pissarides (1994).

This Corollary follows because a constant aggregate state implies that firm exit decisions and optimal scale of operation do not change over time. The law of large numbers applies to a cohort at each age, and therefore the firm demographic variables, S_a and $\tilde{e}_a$, and the size distribution by age are time invariant. It follows that the constant aggregate state equilibrium qualitatively generates the constancy by age of exit rates, average firm size and employment concentration as observed in the data. As a consequence, the Corollary implies that changes in aggregate variables, which weighted averages involving firm demographic variables S_a and $\tilde{e}_a$, will be entirely due to changes in weights.

Because employment by incumbents E_t^I depends on S_a and $\tilde{e}_a$, the mass of entrants in equation (6) depends on firm demographics. With strictly positive entry, we can solve for m_t to obtain the following result.

Corollary 2 (Dynamic Entry Equation). *The mass of entrants in equilibrium is given by*

$$m_t = \frac{N_t - \sum_{a=1}^{\infty} m_{t-a} S_a \tilde{e}_a}{S_0 \tilde{e}_0 + c_e}. \tag{7}$$

Because S_a and $\tilde{e}_a$ are time invariant in equilibrium, the dynamic entry equation implies that the mass of entrants m_t is linear in N_t. It follows that, in equilibrium, changes in N_t are accommodated along the extensive margin by changes in entry. The dynamic entry equation also shows that that entry in the constant aggregate state equilibrium is history dependent: current entry m_t depends on past entry m_{t-a}. Given N_t, higher entry in the past lowers current entry by increasing the mass of incumbent firms m_{t-a}.

3.3 The Turnover of Firms

In this section we examine the determinants of aggregate rates of entry and exit. In particular, we highlight the role of firm demographics, i.e. the age distribution of firms, in determining aggregate entry and exit rates. We show that changes in firm demographics have important feedback effects on the entry rate along transitions, i.e. when population N_t is growing at non-constant rates.

The mass of aggregate exit at time t, denoted X_t, is the sum of exit masses of different age cohorts. Exit of firms of age a equals the difference in survival rates $S_{a-1} - S_a$ multiplied by the size of the cohort at entry, m_{t-a}. We follow here the convention that the age at which a firm is shut down corresponds to the age at which the firm was last productive. The model allows for entrants to exit immediately without producing, so the mass of immediate exits $m_t(1 - S_0)$ are excluded from aggregate exit. It follows that the mass of aggregate exit

is given by

$$X_t = \sum_{a=1}^{t} m_{t-a} \left(S_{a-1} - S_a \right).$$

The number of firms at $t - 1$ is given by

$$M_{t-1} = \sum_{a=1}^{t} m_{t-a} S_{a-1}.$$

Let $\omega_{ta} \equiv m_{t-a} S_a / M_t$ denote the share of firms of age a in the total mass of firms at time t. The hazard rate of exit for a firm of age $a - 1$ is $(S_{a-1} - S_a)/S_{a-1}$. The aggregate exit rate $\xi_t \equiv X_t / M_{t-1}$ can be expressed as the weighted average of hazard rates of exit of different cohorts

$$\xi_t = \sum_{a=1}^{t} \omega_{t-1,a-1} \left(\frac{S_{a-1} - S_a}{S_{a-1}} \right). \tag{8}$$

The hazard rates of exit are fixed by the Time Invariance Corollary. Therefore the aggregate exit rate is only a function of the age distribution of firms, which in turn is determined by past entry. This formula highlights the role of firm demographics in determining the aggregate exit rate. Because the hazard rates are different across firm ages, a change in the age distribution of firms affects the aggregate exit rate. The exception, of course, is when hazard rates are the same for all cohorts. In that case, firm demographics plays no role.

Now consider entry rates. Following the convention about exit, we define $m_t S_0$ as the measure of entry.[13] Let $e_t = N_t / M_t$ denote average employment. The rate of growth in the number of firms is

$$\frac{M_t}{M_{t-1}} = \frac{N_t}{N_{t-1}} \frac{e_{t-1}}{e_t}. \tag{9}$$

Letting $\overline{S}_t$ denote the average survival rate from $t - 1$ to t. The mass of firms M_t can be decomposed into the mass of surviving incumbents plus the mass of entrants,

$$M_t = \overline{S}_t M_{t-1} + m_t S_0.$$

Solving for M_t in (9) and substituting in the above equation gives the following expression for the entry rate

$$\lambda_t \equiv \frac{m_t S_0}{M_{t-1}} = \frac{N_t}{N_{t-1}} \frac{e_{t-1}}{e_t} - \overline{S}_t, \tag{10}$$

which is the discrete-time version of identity (1).

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[13]If we had we assumed that all entrants must produce for at least one period, then $S_0 = 1$ and m_t would be measured entry.

Long-Run vs. Adjustment Path.   Suppose we are on a balanced growth path and population grows at a constant rate g. Average employment e_t is constant along this path. The cohort entry weights m_{t-a} decay as a function of age at the rate $1+g$, so $m_{t-a} = (1+g)^{-a}m_t$. The aggregate exit rate along the balanced growth path, denoted ξ_t^B, follows from (8)

$$\xi_t^B = \frac{\sum_{a=1}^{\infty}(1+g)^{-a}\left(S_{a-1} - S_a\right)}{\sum_{a=0}^{\infty}(1+g)^{-a}S_a}, \tag{11}$$

which is independent of t. Because e_t is constant and $\overline{S}_t = 1 - \xi_t$, the entry rate in (10) along the balanced growth path, denoted λ_t^B, is also independent of t.[14] We have

$$\lambda_t^B = g + \xi_t^B.$$

The entry rate equals the sum of the population growth rate and the exit rate. The intuition is simple. Entrants must replace the exiting firms. In addition, because average employment is constant, the total mass of firms needs to grow at the rate of population growth, g, to clear the labor market. Therefore, the entry rate must be enough to also create this extra employment.

More generally, when labor grows at non-constant rates and we are in a constant aggregate state equilibrium, changes in firm demographics will have feedback effects on entry. Aggregate exit rates depend on the age distribution of firms and thus the history of past entry. Because conditional exit rates are decreasing in age, a larger share of young firms will be associated with a higher aggregate exit rate and consequently higher entry. In addition, changes in average employment will further impact the entry rate. The initial rise in entry rates will increase the share of younger firms which tend to be smaller. This should lower average firm size and, from equation (10), further increase the rate of entry. Thus a rise in population growth will lead to increased entry rates over and above those needed to accommodate the increase in the labor supply. This multiplier effect will operate similarly in the opposite direction when population growth decreases.

Theorem 1 (Feedback Effect of Firm Demographics)**.** *Assume hazard rates of exit $(S_{a-1} - S_a)/S_{a-1}$ are decreasing in age, and average firm size $\tilde{e}_a$ is increasing in age. An increase (decrease) in the rate of population growth will result in an increase (decrease) of entry rates over and beyond the rate of increase (decrease) in population.*

This theorem highlights the importance firm demographic variables in determining en-

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[14]The same holds in a model where productivity shocks are fully persistent or randomly redrawn from the same distribution as the one faced by entrants (as in Mortensen and Pissarides (1994)), average firm size is constant so the above formula applies. In particular this means that the rate of entry is independent of history and only depends on current population growth. If exit rates are not age dependent, the same will also be true for exit.

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try along transitions. Suppose there are no firm demographics, i.e. S_a and $\tilde{e}_a$ do not change with firm age. In that special case, changes in the age distribution of firms does not affect the aggregate exit rate and average employment. Therefore, the feedback effect is absent and changes in the entry rate along a transition are only due to changes in population growth.

4 Quantitative Analysis

The theory section provided conditions under which changes in labor force growth generate constancy of firm demographics variables by age, while allowing for aggregate variables to evolve due to changes in the age-distribution. In this section, we explore the quantitative implications of the theory. We address the following questions. Can changes in labor force growth, combined with feedback from firm demographics, quantitatively generate the secular changes experienced by the US economy? What is the role of the feedback mechanism? What is the role of history dependence, and therefore the importance of the Baby Boom? How do we expect entry rates and firm demographics to evolve from here on?

These quantitative questions can be answered using the dynamic entry equation (7). This equation determines the evolution of the firm-age distribution given an exogenous labor force series, an initial age distribution, and firm demographics variables for all ages. There is reliable data on labor force growth. However, there is limited data on the initial age distribution and firm demographics variables. Specifically, the Census does not assign an age to firms born before 1977. This implies that (i) the 1977 firm-age distribution is unknown, and that (ii) firm demographics variables for firms born before 1977 are also unknown. Given that labor force data go back to 1940, we can shift the search for the initial distribution back to 1940. Doing so allows us to obtain the 1977 distribution that is consistent with historical labor force growth. In what follows, we present two alternative approaches of imputing the 1940 age distribution and firm demographic variables for older firms.

First approach. This approach obtains the firm demographic variables for older ages using a non-parametric extrapolation. We partition ages 26-101 into two-year bins, which gives us 38 moments each for exit rates, average size and concentration by age. The moments are chosen to match the time series of the corresponding variables for *Left Censored* firms — the group of firms born before 1977. The time series for each variable ranges from 1977 to 2014, and therefore contains 38 observations. Because the minimum age of the Left Censored group increases every year, these data reveal new information about older ages

with each passing year. To determine the age distribution in 1940, we partition the distribution into 38 bins and pick the weights to match the employment weights of Left Censored firms from 1977 to 2014. The dynamic entry equation requires two more parameters, S_0 and c_e. The ratio of these parameters determines the volatility of the entry rate in the dynamic entry equation. We normalize c_e to unity and set S_0 to match the volatility of the entry rate. Our extrapolation strategy matches well the time series of exit rates, average firm size, concentration and employment shares of Left Censored Firms; see Appendix C.1.

Second approach. This approach relies on the insight that specifying a stochastic process for employment is sufficient to obtain both the firm demographic variables and the 1940 age distribution. The employment process consists of the distribution of entrant employment, the evolution of employment over time and an exit rule. Therefore, the employment process implies values for the firm demographic variables for all ages. Assuming the US economy was in a balanced growth path, the 1940 age distribution corresponds to the stationary distribution of the employment process. Our calibrated process matches 5-year growth and exit rates, along with average size and concentration of entrants. Because 1978 is the first year in our data, we calibrate the employment process to match average firm size in 1978. The last parameter required for the dynamic entry equation is the entry cost c_e, which we calibrate to match the entry rate in 1978. Any model consistent with this employment process will generate the same firm demographic variables by age; Appendix C.2 provides an example. Table 1 compares the extrapolated values from the second approach to the data.

Findings. Because both approaches yield similar results, we only present the results from the second approach here, which we refer to as the *model*. The results from the frist approach are presented in Appendix C.1.

We begin by presenting the findings for the entry rate in Figure 4. We highlight three distinct episodes that the model matches well. First, the model generates the steady decline in the entry rate observed between 1978 and 2014. The entry rate in the data declined from 14.5 to 6.2 percent, whereas the entry rate in the model declined from 14.4 to 8.1 percent. Second, the model generates the apparent increase in the entry rate before 1978. This increase is driven by the steady increase in labor force growth during the same time period. The third episode is related to World War II. The years around the war exhibited large fluctuations in the entry rate. The labor force growth series also exhibits large fluctuations around the same time, corresponding to large numbers of civilians leaving the labor force to join the war effort and then returning after the war. Through the lens of our model, these large labor force growth fluctuations translate into similarly large fluctuations in the entry

Table 1: Firm Demographic Variables by Age

Age	Exit rate		Average firm size		Concentration	
	Data(%)	Model(%)	Data	Model	Data(%)	Model(%)
0	–	–	6.05	5.35	5.90	5.87
1	21.85	22.24	7.73	6.01	12.29	7.53
2	15.86	15.67	8.46	6.71	13.29	9.07
3	13.43	12.67	9.14	7.47	14.83	10.68
4	11.68	10.90	9.77	8.34	16.45	12.44
5	10.48	9.70	10.36	9.29	17.84	14.43
6-10	8.32	7.85	11.98	12.66	23.00	22.38
11-15	6.40	6.21	15.08	20.52	31.85	37.62
16-20	5.56	5.44	18.81	30.46	40.68	50.85
21-25	4.99	5.01	24.03	41.43	50.47	60.25
Above 25	4.29	4.45	81.59	72.70	78.91	73.90

Notes. Concentration is the share of employment in firms with 250+ employees within the age category divided by total employment in the age category.

rate. The ability of the calibrated model to match both the long term trends and short term fluctuations suggests that changes in labor force growth play a central role in the evolution of the entry rate.

Figure 5 shows how the aggregate exit rate, average firm size and concentration evolve in the model and the data. The model does an excellent job of matching the decline in the aggregate exit rate since 1978. Exit rate declines from 10.5 to 8.4 percent in the model whereas it drops from 10.4 to 7.7 percent in the data. Average firm size increases in both the model and the data. The model, however, overshoots the magnitude of the increase. This occurs because average firm size by age in the model is constant over time, whereas the average size of firms aged 11+ in the data declined after 2000; see Figure 2. The model also does an excellent job of matching the increase in concentration observed in the data. Starting in 1978, concentration in the model increases from 51.0 to 59.2 percent versus 51.6 to 57.4 percent in the data.

Given that the model does a good job of matching the firm demographics variables and the aggregate time series, it must be the case that model matches matches firm aging well. Since 1987, the share of 11+ firms in the data increased by 17 percentage points compared to 14 percentage points in the model. The increase in the employment share of older firms is also captured by the model. Since 1987, the employment share of firms age 11+ increased by 14 percentage points in the data. The employment share in the model increases by 11 percentage points.
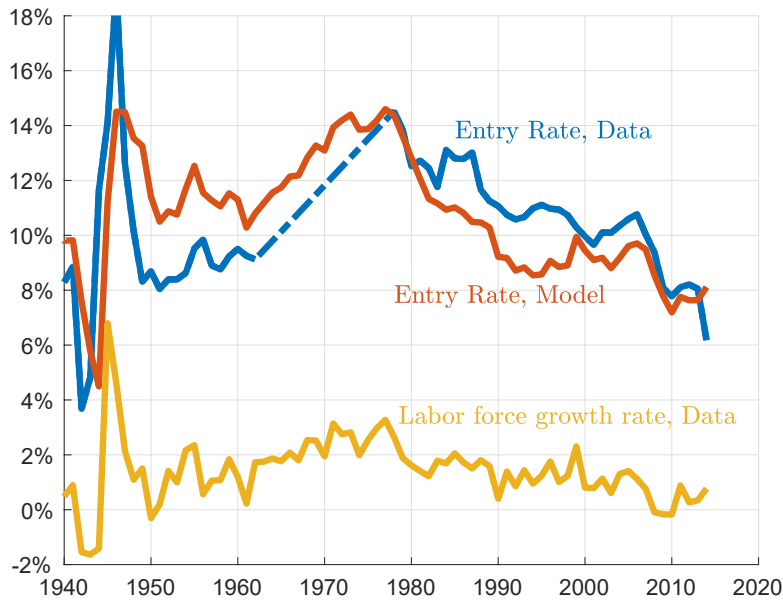
Figure 4

Notes. The entry rate from 1963 to 1977 is linearly interpolated.
Sources. Entry rate 1940-1962: Survey of Current Business. Entry rate 1978-2014: Business Dynamics Statistics.
Labor force growth 1940-1946: Lebergott (1964). Labor force growth 1947-2014: Current Population Survey.
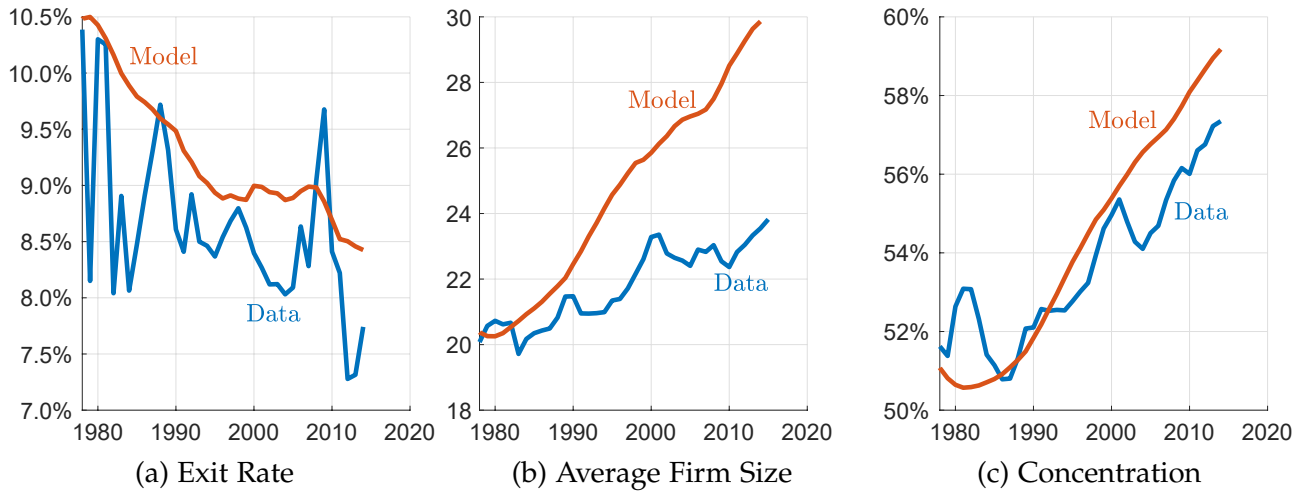

(a) Exit Rate   (b) Average Firm Size   (c) Concentration

Figure 5

Notes. Concentration is the share of employment in firms with 250+ employees.

4.1 Decomposition and Counterfactuals

In this section we decompose the change in entry rates into the effect coming from changes in labor force growth, changes in exit rate, changes in average firm size growth, and a residual corresponding to changes in entry labor. The three left columns of Table 2 present the decomposition for the Benchmark economy. The entry rate declines by 6.26 percentage points between 1978 and 2014. Of this decline, 1.88 percentage points (or 30%) correspond

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to a decline in labor force growth, 2.05 percentage points (or 33%) correspond to a decline in exit rate, and 1.96 percentage points (or 31%) correspond to an *increase* in the growth rate of average firm size. The growth rate of average firm size increases because the time series of average firm size is U-shaped and reaches its minimum in 1980. The remaining 0.37 percentage points (or 6%) residual is due to the change in labor allocated to the creation of entrants.

Table 2: Counterfactuals

	Benchmark			No Rise			No Transition			No Firm Demographics		
	1978	2014	$|\Delta|$	1978	2014	$|\Delta|$	1978	2014	$|\Delta|$	1978	2014	$|\Delta|$
LF Growth, $\hat{N}$	2.65	0.77	1.88	2.65	0.77	1.88	2.65	0.77	1.88	2.65	0.77	1.88
Exit Rate, ξ	10.48	8.43	2.05	10.11	8.53	1.58	10.11	9.14	0.97	11.65	11.65	0
AFS Growth, $\hat{e}$	−1.19	0.77	1.96	0	0.74	0.74	0	0	0	0	0	0
Residual	0.06	−0.31	0.37	0	−0.28	0.28	0	0	0	0.08	−0.26	0.34
Entry Rate, λ	14.38	8.12	**6.26**	12.76	8.28	**4.48**	12.76	9.91	**2.85**	14.38	12.16	**2.22**

Notes. All values are in percentage points. With entry costs denominated in units of labor, identity (1) holds approximately, $\lambda \approx \hat{N} + \xi - \hat{e}$. The residual corresponds to the growth rate of labor allocated towards the creation of entrants.

In order to better understand the quantitative importance of history dependence and firm demographics, we run three counterfactual exercises. The first counterfactual assumes that labor force growth was constant before 1978. We refer to this as the *No Rise* counterfactual, because it shuts down the pre-1978 rise in labor force growth. The second counterfactual shuts down the transition and instead compares two balanced growth paths. We refer to this as the *No Transition* counterfactual. The last counterfactual shuts down the feedback effect from firm demographics by assuming that there are no differences in exit rates and average firm size across age groups. We label this the *No Firm Demographics* counterfactual.

The middle-left columns of Table 2 report the decomposition for the No Rise counterfactual. This economy starts off in a balanced growth path in 1978 corresponding to the 1978 labor force growth rate of 2.65 percent. All the other parameter values are identical to their values in the benchmark economy. The entry rate declines by 4.48 percentage points, or roughly two-thirds of the decline in the benchmark. The entry rate in the counterfactual is at a lower level in 1978, but catches up with the benchmark by 2014. As Table 2 shows, the No Rise counterfactual has a lower entry rate in 1978 for two reasons. First, the growth in average firm size in that economy is zero because average firm size does not change on a balanced growth path, while the benchmark exhibits negative average firm size growth. Second, the 1978 exit rate in the counterfactual is smaller than the benchmark. This is because the pre-1978 rise in labor force growth in the benchmark shifts the age distribution

towards younger firms, which exit at higher rates.

The middle-right columns of Table 2 report the decomposition for the No Transition counterfactual. As in the No Rise counterfactual, the economy is on a balanced growth path (BGP) in 1978. We compare the 1978 BGP to an economy on a balanced growth path with the 2014 labor force growth. The entry rate in the 2014 BGP is 2.85 percentage points lower than in the 1978 BGP, which is roughly half of the 6.26 percentage point decline in the benchmark. This occurs for two primary reasons. First, because average firm size is constant along a balanced growth path, comparing balanced growth paths misses the contribution of $\hat{e}$. Second, firms are younger in the 2014 BGP relative to the benchmark in 2014. Therefore, the exit rate in the 2014 BGP is higher than the benchmark in 2014.

The last three columns of Table 2 report the decomposition for the No Firm Demographics counterfactual. In this economy, firms do not grow or shrink and therefore do not exhibit differences in average firm size or exit rates across age groups.[15] The entry rate declines by 2.22 percentage points, or roughly one third of the decline in the benchmark. Without firm demographics, the feedback effects are missing: there is no change in aggregate exit rates or average firm size over time. Therefore the decline in entry rates corresponds simply to the decline in the labor force growth rate, plus a small contribution of changes in entry labor from the transition.

The main message from these counterfactual exercises is that history dependence, transitional dynamics and firm demographics are all quantitatively important to study the decline in entry rates. We next explore what projections of labor force growth imply for future entry rates.

4.2 Entry Rate Projections

The benchmark calibration can be used to project firm entry rates going forward. The Bureau of Labor Statistics (BLS) publishes projections of labor force growth up until the year 2060. We feed the BLS projections into the benchmark model and compute firm entry rates. Figure 6 presents our findings.

The BLS projects that the labor force will slowly converge to a growth rate of about 0.25 percent by the year 2060. Through the lens of our model, these projections imply that the entry rate will rise from the 8.1 percent in 2014 to 9.1 percent in 2060. The reason for the rebound is twofold. First the exit rate increases from 8.4 percent to 8.8 percent. This is

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[15]We shut down firm demographics by setting the transition matrix to be the identity matrix, $F = \mathbf{I}$, implying that the productivity of a firm equals its productivity drawn at birth. Firms do not grow or shrink in this economy. Without firm growth there is no endogenous exit and introducing exogenous exit rate is necessary to recover stationarity. We set the exogenous exit rate at 11.65 percent so as to match the benchmark entry rate in 1978.
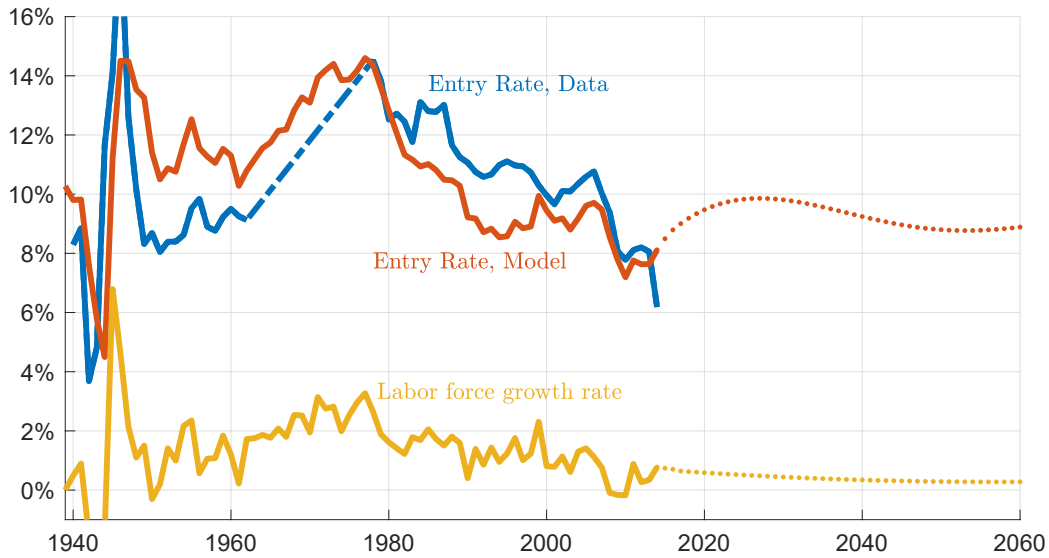
Figure 6: Projections

because firms are older along the transition than in the 2060 balanced growth path, and older firms exit at lower rates. Second, average firm size stops growing by 2060 adding an extra 0.7 percentage points to the entry rate. Together these two forces more than offset the lower labor force growth rate in 2060.

The projections also show that the convergence to the new balanced growth path is non-monotonic. The entry rate rises above and then declines to its stationary level. This cycle in entry rates is due to the dynamic nature of entry. As past entrants age, they grow at slower rates and cannot absorb as much of the growth in labor supply. This creates room for new firms, raising the entry rate. As these new firms age and grow, they absorb a larger fraction of the growth in labor supply, lowering the entry rate and generating firm aging. This cycle repeats until convergence.

4.3   The Aggregate Labor Share

In recent work, Hartman-Glaser, Lustig and Xiaolan (2019), Kehrig and Vincent (2018) and Autor, Dorn, Katz, Patterson and Van Reenen (2017) document a negative relationship between firm size and labor share. These studies find that almost all of the decline of the aggregate labor share is due to reallocation of value-added from high to low labor share units, rather than a decline in labor share within units. It follows that the decline in the aggregate labor share is primarily due to changes in weights corresponding to the size distribution of firms. Firm aging provides a mechanism that results in such a change in the size distribution. In this section, we explore quantitatively what firm aging, driven by labor force growth, implies for the aggregate labor share. To conduct this exercise, we

need to generate a negative relationship between firm size and labor share. This negative relationship can be generated in various ways without affecting the results. For example, the negative relationship could arise because larger firms produce using technologies that are less labor intensive as in Guimaraes and Gil (2019). We use the mechanism proposed by Autor, Dorn, Katz, Patterson and Van Reenen (2017), in which labor shares decline with firm size because of overhead labor.

A firm's labor share can be broken down into the share of value added paid to production workers and to overhead labor. In equilibrium, the share paid to production workers is equal to α for all firms. Therefore, all differences in firm-level labor shares are due to the share paid to overhead labor. We have

$$\text{Labor share} = \alpha + \frac{wc_f(s)}{py(s)} = \alpha \left(1 + \frac{c_f(s)}{n(s)} \right) \tag{12}$$

If all firms have the same overhead, $c_f(s) = c_{fa}$, then firm-level labor shares are decreasing in firm size. In our calibration we pick a functional form that allows $c_f(s)$ to vary with firm size, $c_f(s) = c_{fa} + c_{fb}s^{\frac{1}{1-\alpha}}$. This captures the intuitive idea that larger firms require greater labor overhead. The slope of the overhead function c_{fb} is calibrated to match the standard deviation of log-labor productivity reported in Bartelsman, Haltiwanger and Scarpetta (2013). In spite of requiring higher levels of overhead labor, larger firms in the calibrated model have lower labor shares because the ratio $c_f(s)/n(s)$ declines with firm size. Firm aging reallocates market shares towards older firms, which are larger and have lower labor shares.[16] As a result, the aggregate labor share declines. Figure 7 plots the cumulative change in the aggregate labor share in the model and the data. Quantitatively, firm aging generates a significant drop in the aggregate labor share.

We compare the model generated decline to two measures of labor share in the data. First, we take the corporate labor share from 1975-2010 as measured by Karabarbounis and Neiman (2014). Second, we consider an alternative measure of the corporate labor share proposed by Koh, Santaeulalia-Llopis and Zheng (2018).[17] The model generates a decline comparable to both the series. The Koh, Santaeulalia-Llopis and Zheng (2018) series exhibit

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[16]This mechanism is consistent with labor share dynamics in the data: Kehrig and Vincent (2018) document that reallocation occurs towards units that lower their labor share, as opposed to those that have a low level of the labor share.

[17]This measure of the aggregate labor share is different because it accounts for changes in the way the Bureau of Economic Analysis treats intellectual property products. Prior to 1999, intellectual property was treated as a business or consumption expenditure. However, over time the BEA has started treating intellectual property as capital, affecting the measurement of the labor share. While we rely on overhead labor to generate the negative correlation between firm size and labor share, the negative relationship could just as well arise because larger firms make greater investments in intellectual property products, and therefore have lower measured labor shares.
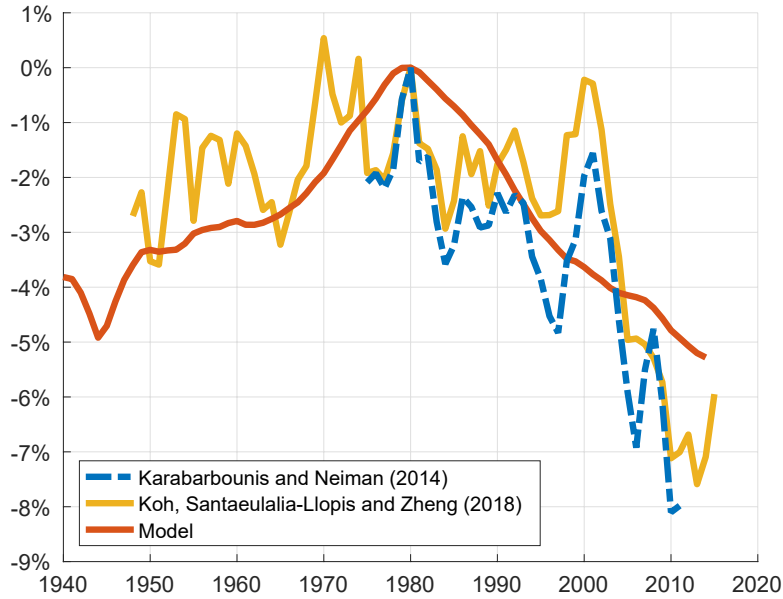
Figure 7: Corporate Labor Share (Normalized to 1980)

an increase in the labor share from 1947 to 1980, generating a hump-shaped aggregate labor share overall. The model matches this hump-shaped pattern well. The intuition is simple. From 1940-1980, the aggregate labor share increases with the entry rate because entrants are small in size, and therefore have higher labor shares. From 1978 onwards, as firms age and grow in size the share of firms with low labor shares increases, leading to a decline in the aggregate labor share.

5   Labor Force Growth: Drivers and Measures

5.1   Sources of Labor Force Growth

What are the main drivers of labor force growth? To answer this question, we decompose labor force growth into each of its components. We start from the BLS' definition of labor force,

$$\text{LF}_t = \text{CNP16}_t \times \text{PR}_t$$

where LF_t is the civilian labor force at time t, CNP16_t is the civilian noninstitutional population age 16 and over at time t, and PR_t is the participation rate at time t. It follows that labor force growth rate is the sum of the growth rate of each component,

$$\text{LF Growth Rate}_t = \text{CNP16 Growth Rate}_t + \text{PR Growth Rate}_t.$$

We can further decompose CNP16 growth rate at time t into the birth rate at time $t - 16$
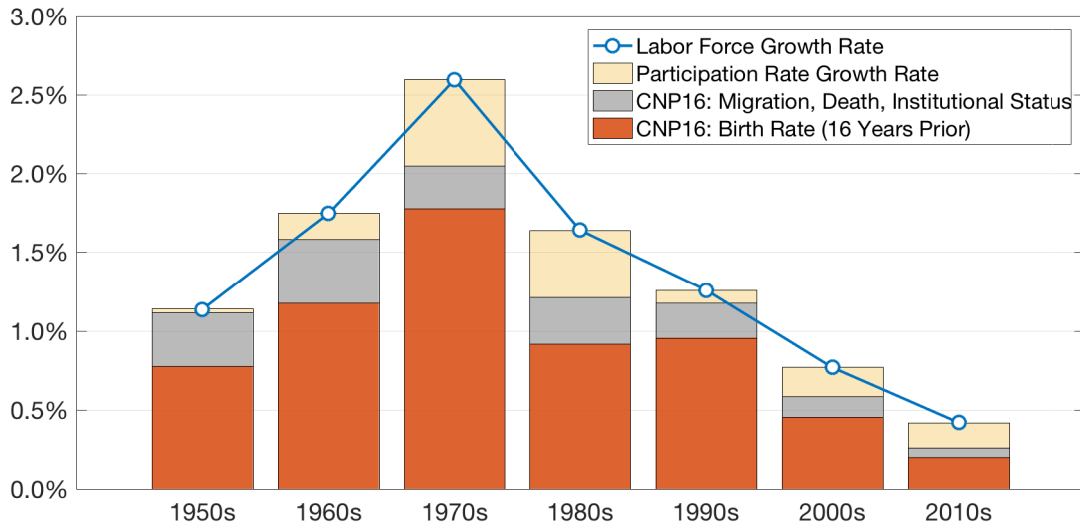
Figure 8: Decomposition of Labor Force Growth

and a residual term Other$_t$

$$\text{CNP16 Growth Rate}_t = \text{Birth Rate}_{t-16} + \text{Other}_t,$$

where the Other$_t$ term includes death rates, net migration rates, and rates of entry and exit into institutional status. Figure 8 plots the labor force growth rate by decade, dividing the bars into the percentage contribution of growth in participation rates, birth rates, and other. In spite of the increase in female labor force participation starting in the 1950s, the contribution of the growth in participation rate term is small. This occurs because male labor force participation declined over the same time period, counteracting the increase in female labor force participation. On average, birth rates 16 years prior account for 64 percent of the changes in labor force growth across decades. Therefore, we conclude that birth rates account for the bulk of the changes in labor force growth.[18]

5.2 Measures of Labor Supply

One potential source of concern when using the civilian labor force as a measure of labor supply is that it includes the unemployed population, those employed by government, and the self-employed. Figure 9 shows that the pattern for total employment growth (excludes the unemployed) and for private sector employment growth (excludes the self-employed

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[18]The actual contribution of the birth rate to labor force growth is likely higher than 64 percent because the birth rate also has an effect on participation rates. For example, an important fraction of the decline of participation rates since the year 2000 is due to the baby boomer generation reaching the age of 55 and over, whose age group has low participation rates.

and those working for government) follow a similar rise and fall pattern as labor force growth.
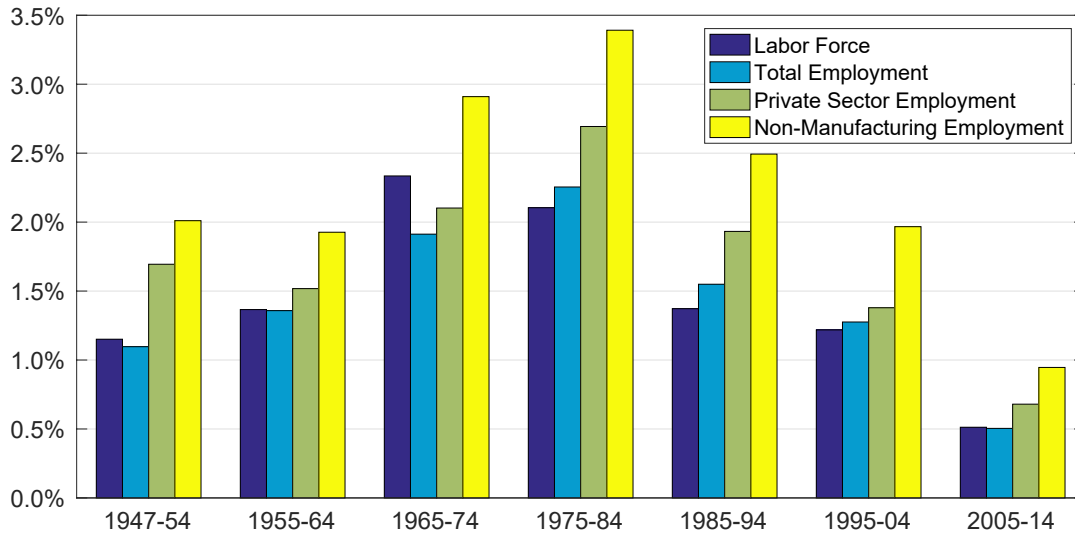

Figure 9: Growth Rates

Sources. Current Population Survey and Current Employment Statistics.
Notes. Data starts in 1947. Decade cutoffs are chosen so that full business cycles fall within the decade bin, effectively capturing the trend component in growth rates.

The manufacturing sector is another potential source of concern, as it has experienced overall negative employment growth since the 1980s (Fort, Pierce and Schott, 2018). This raises the possibility that an exodus of workers from manufacturing into non-manufacturing reverses the trend of declining employment growth in non-manufacturing sectors. Figure 9 shows that this is not the case. Non-manufacturing employment growth also follows a similar rise and fall pattern as labor force growth.

The decline of manufacturing employment does not have a large effect on nonmanufacturing employment growth partly because the flow of workers out of manufacturing is small compared to the flow of workers entering the labor force. From 1977 to 2014, manufacturing employment shrank by 6 million workers while the labor force grew by 57 million workers.

6   Conclusion

Recent decades have witnessed a decline in firm entry and exit rates, and an increase in employment concentration and average firm size. None of these changes have occurred within firm-age bins. The interplay of population growth and firm demographics can generate both the stability within firm-age bins and the aggregate behavior of these variables. History dependence, transitional dynamics and feedback effects from firm demographics

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are all quantitatively important in generating the aggregate trends. Given a negative relationship between firm size and labor shares, firm aging induced by population growth also replicates the hump-shaped pattern of the aggregate labor share.

This paper explored one set of aggregate trends. The US economy has undergone other important changes during recent decades. For example, it seems that US productivity growth has slowed down and that markups charged by US firms have increased. To the extent that these trends are related, it is likely that demographic forces play an important role. We leave these questions to future work.

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Appendix A Data Appendix

Civilian Labor Force Growth Rate 1940-2014.   Civilian labor force data comes from the Bureau of Labor Statistics (BLS) Current Population Survey for the years 1947 to 2014, and from Lebergott (1964) from 1940 to 1946. The civilian labor force definition in BLS includes population 16 years of age and over while the definition in Lebergott includes population 14 years of age and over. We can use Lebergott's series from 1947 to 1960 to compare the difference in growth rates using either definition. Figure A-1 shows that the labor force growth rates of ages 14+ and 16+ are nearly identical.
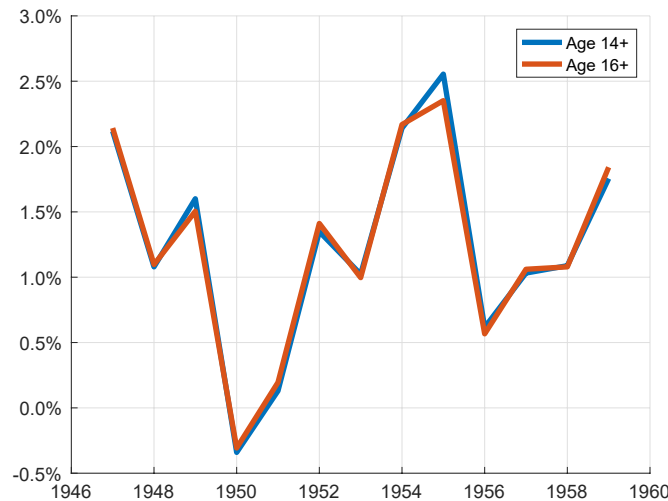

Figure A-1: US Civilian Labor Force Growth Rate

Firm Data 1978-2014.   Firm data comes from the U.S. Census Bureau's Business Dynamics Statistics (BDS). The BDS dataset has near universal coverage of private sector firms with paid employees. BDS data starts in 1977, but common practice suggests dropping 1977 and 1978 due to suspected measurement error (e.g. Moscarini and Postel-Vinay, 2012). We drop entry rates for 1977, but keep 1978, as calibrating to 1978 or 1979 does not affect our quantitative results (the model matches the entry rate in both 1978 and 1979 almost exactly).

Firm Entry Rates 1940-1962.   The firm entry rate is obtained from the now-discontinued U.S. Department of Commerce's Survey of Current Business. This dataset includes all nonfarm businesses, including firms with zero employees. The entry rate is defined as 'New Businesses' divided by 'Operating Businesses'. The 1963 edition was the last one to report a 'Business Population and Turnover' section. From 1963, the Survey of Current Business reported 'Business Incorporations' instead, which only include stock corporations.

Birth Rates. The 1930 to 2000 birth rate series is from the CDC National Center for Health Statistics.

Employment 1947-2014. Total employment corresponds to the civilian employment in the BLS Current Population Survey. Private sector employment and manufacturing employment are from the BLS Current Employment Statistics (Establishment Survey). Non-manufacturing employment is private sector employment minus manufacturing employment.

Labor Force Projections. Projections of labor force growth are from the BLS; see Toossi (2016).

Appendix B   Firm Age Regressions

Table B-1: Regression of concentration (employment share of firms sized 250+) on year

Variable	Specification	
	(1)	(2)
Year	0.003***	-0.000
	(0.001)	(0.000)
AGE:		
Age 0		0.065***
		(0.007)
Age 1		0.129***
		(0.007)
Age 2		0.139***
		(0.007)
Age 3		0.154***
		(0.007)
Age 4		0.171***
		(0.007)
Age 5		0.185***
		(0.008)
Age 6 to 10		0.237***
		(0.008)
Age 11 to 15		0.326***
		(0.009)
Age 16 to 20		0.415***
		(0.010)
Age 21 to 25		0.514***
		(0.011)
Age Above 25		0.785***
		(0.012)
R^2	0.026	0.984
Observations	351	313

*** $p < 0.01$; ** $p < 0.05$; * $p < 0.1$

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Table B-2: Regression of log average firm size on year

Variable	Specification			
	(1)	(2)	(3)	(4)
Year	0.009***	-0.005***	-0.005***	-0.005***
	(0.001)	(0.001)	(0.000)	(0.000)
AGE:				
Age 0		1.845***	1.413***	1.467***
		(0.024)	(0.016)	(0.025)
Age 1		2.091***	1.658***	1.725***
		(0.025)	(0.016)	(0.025)
Age 2		2.181***	1.748***	1.815***
		(0.025)	(0.017)	(0.025)
Age 3		2.257***	1.824***	1.877***
		(0.026)	(0.017)	(0.026)
Age 4		2.329***	1.896***	1.950***
		(0.026)	(0.017)	(0.026)
Age 5		2.388***	1.955***	2.012***
		(0.026)	(0.017)	(0.027)
Age 6 to 10		2.536***	2.103***	2.169***
		(0.029)	(0.018)	(0.029)
Age 11 to 15		2.758***	2.325***	2.333***
		(0.031)	(0.019)	(0.031)
Age 16 to 20		2.987***	2.555***	2.482***
		(0.034)	(0.021)	(0.034)
Age 21 to 25		3.261***	2.828***	2.589***
		(0.038)	(0.022)	(0.038)
Age Above 25		4.385***	3.952***	3.176***
		(0.042)	(0.024)	(0.044)
SECTOR CONTROLS	No	No	Yes	Yes
SECTOR×AGE CONTROLS	No	No	No	Yes
Observations	3,159	2,817	2,817	2,817
R^2	0.014	0.977	0.994	0.996

*** $p < 0.01$; ** $p < 0.05$; * $p < 0.1$

Table B-3: Regression of exit rate on year

Variable	Specification			
	(1)	(2)	(3)	(4)
Year	-0.146***	-0.014**	-0.014***	-0.014***
	(0.011)	(0.006)	(0.005)	(0.004)
AGE:				
Age 1		21.967***	19.677***	19.304***
		(0.178)	(0.187)	(0.334)
Age 2		16.227***	13.938***	12.786***
		(0.182)	(0.189)	(0.339)
Age 3		13.759***	11.469***	10.850***
		(0.185)	(0.191)	(0.344)
Age 4		12.116***	9.826***	9.467***
		(0.189)	(0.193)	(0.350)
Age 5		10.842***	8.553***	8.420***
		(0.192)	(0.195)	(0.355)
Age 6 to 10		8.743***	6.454***	6.791***
		(0.211)	(0.208)	(0.386)
Age 11 to 15		6.815***	4.525***	5.263***
		(0.229)	(0.219)	(0.417)
Age 16 to 20		6.010***	3.721***	4.692***
		(0.249)	(0.233)	(0.457)
Age 21 to 25		5.532***	3.243***	4.536***
		(0.274)	(0.251)	(0.509)
Age Above 25		4.697***	2.408***	4.597***
		(0.307)	(0.275)	(0.585)
SECTOR CONTROLS	No	No	Yes	Yes
SECTOR×AGE CONTROLS	No	No	No	Yes
Observations	2,817	2,475	2,475	2,475
R^2	0.061	0.962	0.975	0.978

*** $p < 0.01$; ** $p < 0.05$; * $p < 0.1$

Appendix C Quantitative Appendix

C.1 First Approach

This approach performs a non-parametric extrapolation to get firm demographic variables for older firms. By the Time Invariance Corollary, we can infer exit rates, average firm size and concentration by age from sample averages of the observed levels. This strategy applies directly for ages 0 to 5. For ages 6-25, the BDS provides this data in five-year bins (e.g. 6-10). We interpolate the firm demographic variables for these intermediate ages. Ages 26-101 are divided into 38 bins, with the last bin corresponding to ages greater than or equal to 101. The value of average firm size and concentration in these bins is set to match the 38 years of the corresponding time series of the Left Censored group. We extrapolate firm exit rates by age linearly.

Figures C-2 presents the extrapolated values of average size, exit rates and concentration by age. Figure C-2d shows that the resulting 1940 distribution is similar to the age-distribution along the balanced growth path, which is what we use in the second approach. Figure C-3 shows the match of our extrapolation strategy the time series of exit rates, average firm size, concentration and employment shares of Left Censored firms. Figure C-4 shows the implied evolution of entry rates, average firm size, exit rates and concentration that arise by feeding the time series of labor force through the dynamic entry equation.

C.2 Second Approach

This approach calibrates an employment process consistent with either with a model of perfect competition or with a model of monopolistic competition with constant elasticity. These models are isomorphic to the interpretation of the curvature parameter in the revenue function; see Hopenhayn (2011).

The model period is set to one year. The time discount factor β is set to 0.96. The steady-state labor force growth rate g is set to the standard value of one percent. The production function of a firm is $q(n, s) = sn^{\alpha}$ with the curvature parameter α set to its standard value of 0.64. In the perfect competition model this parameter captures the managerial span of control. In the monopolistic competition model this parameter maps to η, which corresponds to an elasticity of substitution of $1/(1 - \eta)$.

Firm productivity follows an AR(1) process,

$$\log(s_{t+1}) = \mu_s + \rho \log(s_t) + \varepsilon_{t+1}; \qquad \varepsilon_{t+1} \sim \mathcal{N}(0, \sigma_\varepsilon^2) \tag{A-1}$$
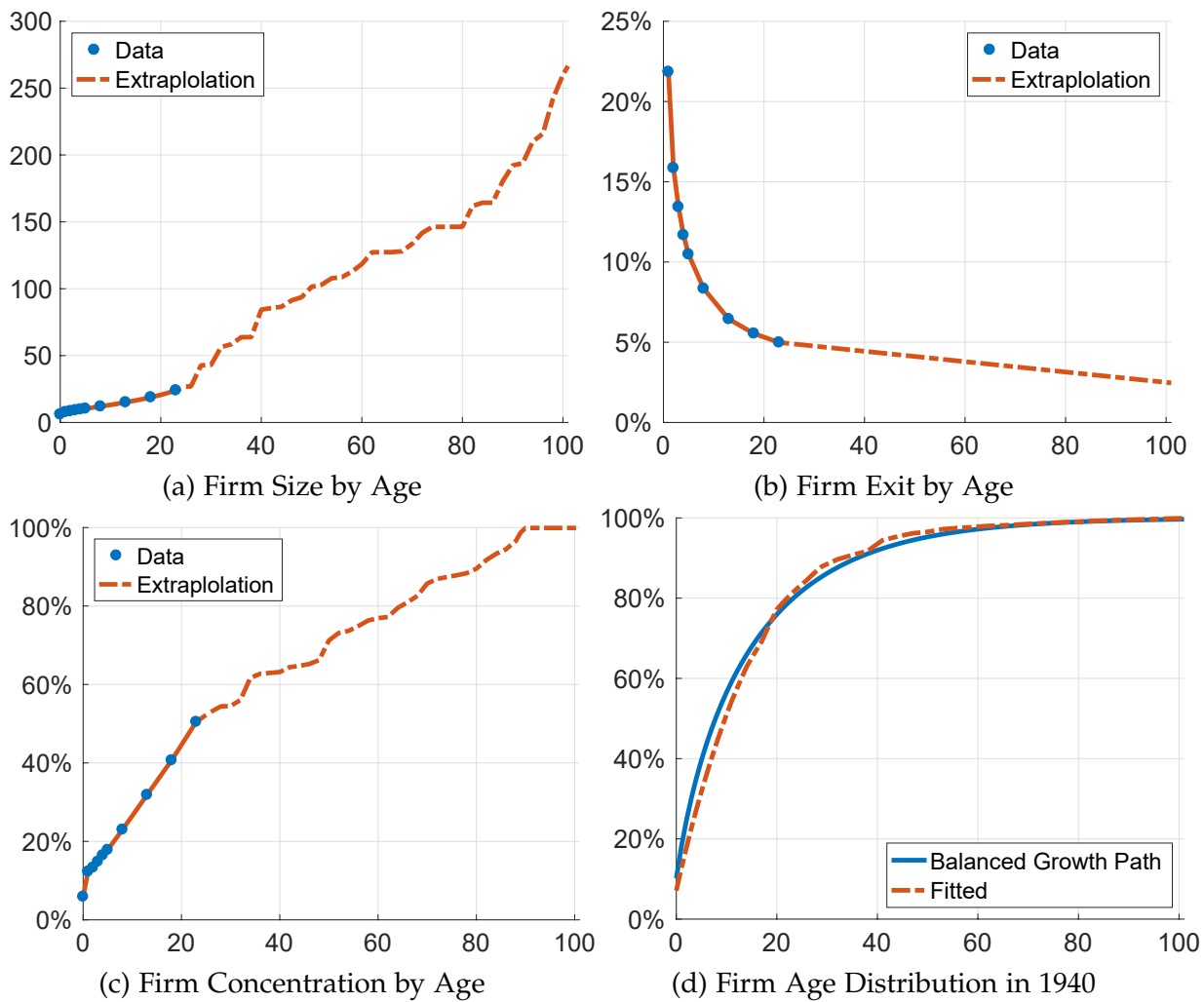
Figure C-2

Notes. Dots are the sample average for the age group. Value of age groups with multiple ages were assigned to the intermediate age (e.g. the mean of the 6 to 10 age group was assigned to age 8). Values in between dots are interpolated. Dashed lines are extrapolations set to match moments of the left-censored cohort in Figure C-3. Figure C-2d compares the initial distribution used in the first approach exercise (fitted) with a balanced growth path distribution (with labor force growth of 1 percent).

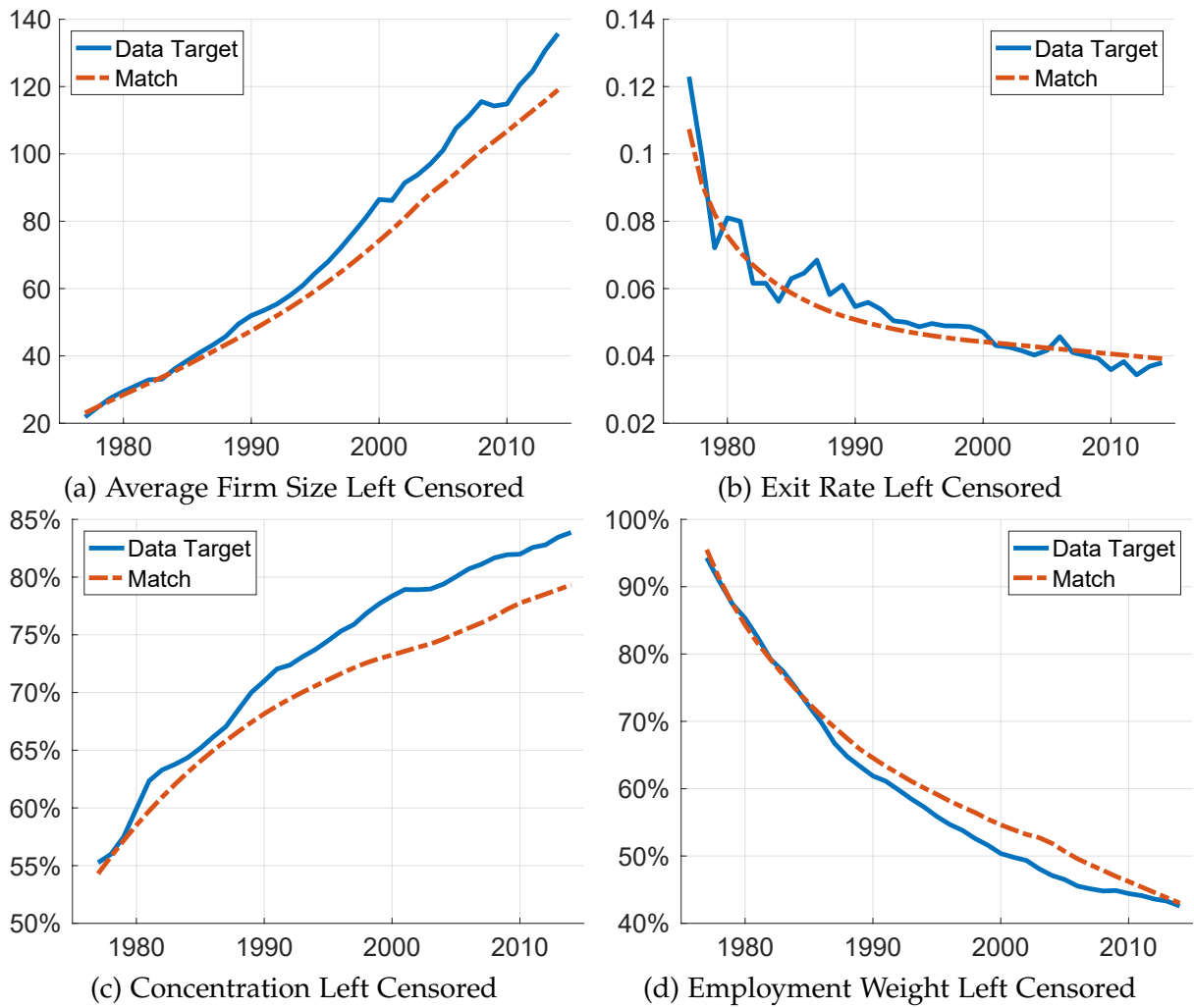(a) Average Firm Size Left Censored   (b) Exit Rate Left Censored

(c) Concentration Left Censored   (d) Employment Weight Left Censored

Figure C-3: Moments of the Left Censored cohort

Notes. Left censored firms are those born before 1977.

(a) Entry Rate   (b) Average Firm Size

(c) Exit Rate   (d) Concentration
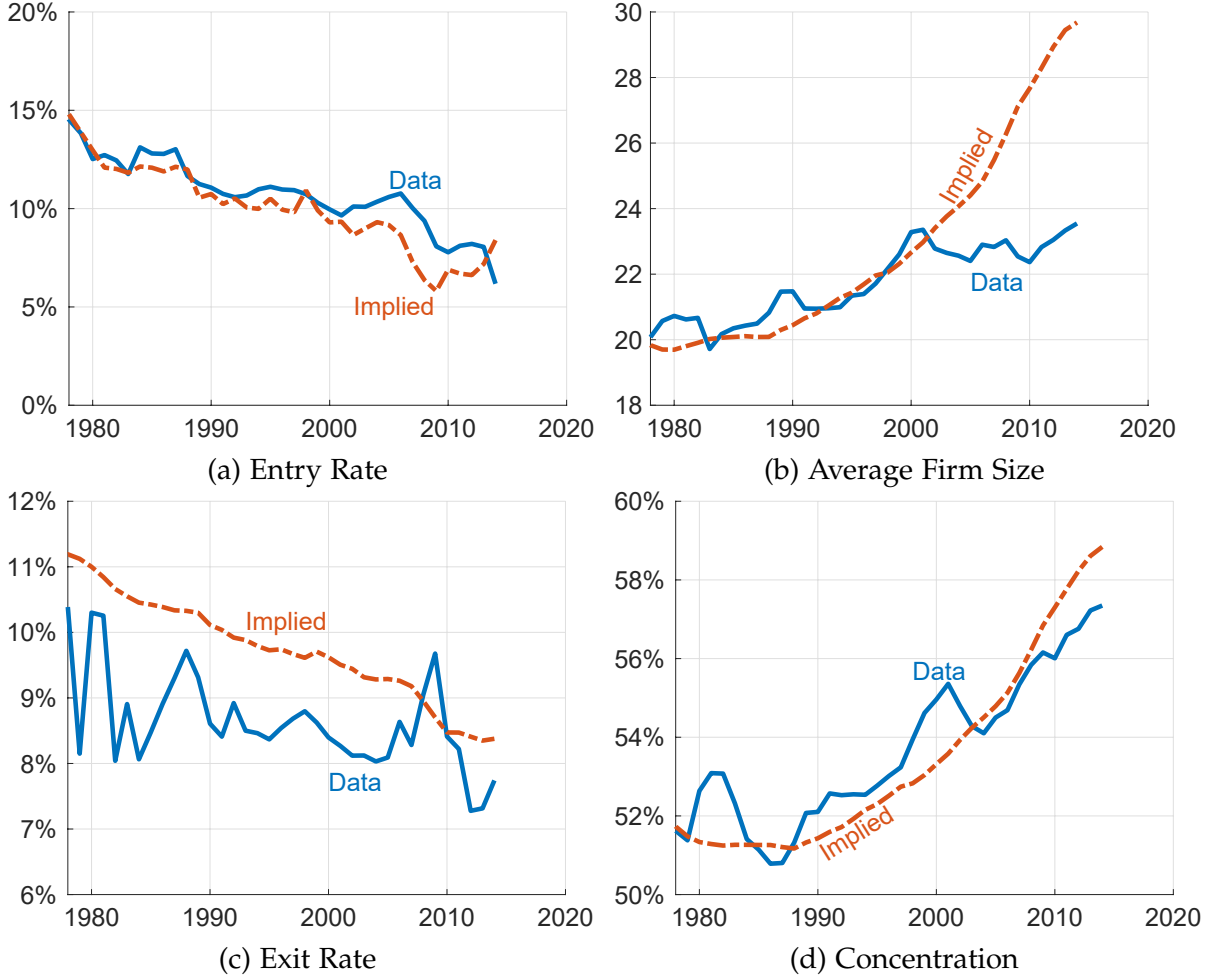
Figure C-4

with ρ as the persistence, μ_s as the drift and σ_ε^2 as the variance of shocks. The distribution of entrant productivities G is lognormal with mean μ_G and variance σ_G^2. We allow overhead labor to increase monotonically with firm productivity, $c_f(s) = c_{fa} + c_{fb}s^{\frac{1}{1-\alpha}}$, to capture the intuitive idea that overhead labor increases with the number of production workers in the firm.

As shown in Hopenhayn and Rogerson (1993), this setting implies that productive employment follows an AR(1) process,

$$\log(n_{t+1}) = \frac{1-\rho}{1-\alpha}\left(\frac{\mu_s}{1-\rho} + \log\alpha\right) + \rho\log(n_t) + \left(\frac{1}{1-\alpha}\right)\varepsilon_{t+1} \qquad (\text{A-2})$$

where the aggregate state Z^* is normalized to one. Doing so gets around the identification problem that arises because Z^* and the idiosyncratic shock enter the firm's revenue function multiplicatively. The total employment process at the firm level is a composite process that depends on the productive employment process, the constant exit threshold s^* and the

process followed by overhead labor $c_f(s)$.

In total, we have 8 parameters c_{fa}, c_{fb}, μ_s, ρ, σ_ε^2, μ_G, σ_G^2 and c_e that need to be calibrated. We jointly calibrate these parameters to match $Z^* = 1$, 5-year conditional growth rates, 5-year unconditional exit rates, average entrant size, average concentration of entrants, average firm size in 1978, entry rate in 1978 and the average dispersion of log labor productivity for the year 1993 to 2001.

Some justification for the choice of moments is in order. From the dynamic entry equation, matching the average entrant size in 1978 is necessary to match the entry rate in 1978, so we target this moment. Average entrant size in the model is constant over time. It is determined primarily by μ_G, the mean of the entrant productivity distribution G. The variance σ_G^2 determines the thickness of the right tail of G, and therefore targets the concentration of entrants. The variance of the productivity process σ_ε^2 affects the weight on productivity gridpoints at which firms exit, so it primarily targets the 5-year exit rate. The operating cost intercept c_{fa} plays an important role in determining average firm size. The persistence parameter ρ determines how quickly firms grow, so we use it to target the 5-year growth rate of firms. The operating cost slope c_{fb} plays an important role in matching labor productivity dispersion, so we use it to target the average dispersion deviation of log labor productivity reported in Bartelsman, Haltiwanger and Scarpetta (2013). Table C-4 summarizes the calibration targets in the model and shows that the parsimonious model does a good job of matching the targets.

Table C-4

		Assigned	
	Value	Definition	Basis
β	0.96	Discount factor	Standard
α	2/3	Curvature	Standard
g	0.01	Labor force growth rate (SS)	Standard

		Jointly Calibrated				
	Parameters			Moments		
	Value	Definition			Data	Model
c_e	3e−7	Entry cost	Entry rate 1978		14.75%	14.33%
c_{fa}	3.760	Operating cost intercept	Avg. firm size 1978		20.08	20.08
c_{fb}	0.002	Operating cost slope	SD log-LP 1993-01		0.58	0.60
μ_G	−11.189	Mean of G	Avg. entrant size 1978		5.40	5.36
σ_G^2	3.966	Variance of G	Avg. conc. of entrants		5.90%	5.87%
μ_s	−0.025	Drift in AR(1)	$Z^* = 1$		—	—
ρ	0.973	Persistence of AR(1)	5-year growth rate		70.49%	73.82%
σ_ε^2	0.073	Variance of AR(1) shocks	5-year exit rate		48.42 %	45.83%