Manipulation, Panic Runs, and the Short Selling Ban[*]

Pingyang Gao

Booth School of Business

The University of Chicago

pingyang.gao@chicagobooth.edu

Xu Jiang

Fuqua School of Business

Duke University

xu.jiang@duke.edu

Jinzhi Lu

College of Business

City University of Hong Kong

jinzhilu@cityu.edu.hk

Abstract

This paper identifies conditions under which a short-selling ban improves the ex-ante firm value. Short selling improves price discovery and enables stakeholders to make better investment decisions. However, manipulative short selling can arise as a self-fulfilling equilibrium, resulting in inefficient investment decisions. The adverse effect is amplified by the firm's vulnerability to panic runs. Overall, short selling reduces the ex-ante firm value if both manipulative short selling is strong and the firm is very vulnerable to runs. The results contribute to our understanding of the function of short selling in the capital markets and to the controversy around the regulations against short selling.

1 Introduction

This paper presents a model to evaluate the efficiency consequences of banning short sales. Ever since the first regulation against short selling was enacted by Amsterdam exchange in 1610, such regulations have been controversial (e.g., Bris et al. (2007)). A salient feature of the restrictions on short selling is that they are often imposed on financial stocks and during bad times. Historically, short-sale ban was imposed after crashes, e.g. the Dutch stock market crash in the seventeenth century and the South Sea Bubble burst in the eighteenth century. More recently, in September 2008 the SEC banned short-sales of shares of 799 companies for two weeks and the U.K. and Japan declared a ban on short selling for "as long as it takes" to stabilize the markets. Similarly, in August 2011, France, Spain, Italy, and Belgium imposed temporary bans on short selling for some financial stocks during the European sovereign debt crisis.

Proponents for short selling make a straightforward argument. Like other selling and buying, short selling allows investors to express their negative opinions through trading and improves the informativeness of stock prices. The price discovery, in turn, leads to better decisions and more efficient allocation of capital in the economy.

In contrast, opponents of short selling have argued that short sellers may manipulate the market through "bear raids." Speculators with initial short positions may employ various tactics, including spreading rumors, to drive down the share prices in order to close their initial short positions at a lower cost. Goldstein and Guembel (2008) (hereafter referred to as GG) presents a model in which manipulative short selling, whereby an uninformed speculator shorts a firm's stock and earns a profit, can arise as a self-fulfilling equilibrium. Moreover, GG show that such a self-fulfilling equilibrium does not hold where the uninformed speculator chooses to buy.

In this paper, we combine both arguments to evaluate the short-selling (SS) ban on the ex-ante efficiency. We augment a coordination game with the MSS (hereafter referred to as MSS) from GG. There is a speculator who receives information about the state with a known probability. After privately learning about whether he has received an informative

signal or not, the speculator trades in a market in the style of Kyle (1985) in which the stock price endogenously reflects some of the speculator's information. A continuum of investors then observe the stock price and make their decisions that collectively affect the firm's cash flow. Conditional on the stock price, the coordination subgame yields a unique equilibrium using the global games methodology. We then solve for the speculator's trading strategy, characterize the entire equilibrium, and compare the equilibrium outcomes under two regimes of allowing and banning short selling.

Our main result is that the SS ban improves the ex-ante efficiency if the firm's vulnerability to runs is sufficiently high and the speculator's information quality is not sufficiently high. Financial firms are more vulnerable to runs, whereas crises are often associated with high degree of uncertainty in the market.

To see why the high vulnerability to run is necessary, it is useful to consider a benchmark where the investment decision is made by a representative investor, as studied in GG. In this case, the SS ban always reduces the ex-ante efficiency despite the equilibrium existence of manipulative short selling. The intuition for this somewhat surprising result is compelling. The investor can always ignore the stock price in making the decision and thus cannot be worse off with the stock price. The implication is that a short selling ban that cannot discriminate between informed and uninformed short selling always reduces the ex-ante efficiency. The existence of rational bear raids is not sufficient to justify the ban on short selling and is merely a secondary consequence of informative prices.

The medium level of the speculator's information quality is also necessary. To see this, consider two extremes. At one extreme, if the market is populated mainly by informed speculators, then MSS arises but very infrequently. The informational benefit from the frequent informed short selling dominates the cost of the infrequent MSS. As a result, the SS ban strictly reduces the amount of information conveyed through prices and thus efficiency. At the other extreme, if the market is mainly populated by uninformed speculators, then based on the intuition in GG, manipulative short selling does not arise in equilibrium because price is very uninformative. As a result, the SS ban removes the informed short selling and thus reduces the efficiency only when the probability that the speculator is present is intermediate,

or, equivalently, the speculator's information quality is at a medium level.

The intuition behind the main result is as follows. In a coordination game, the investors' investment decisions are driven not only by information about the fundamental but also by their concern about others' actions. Since investors use the stock price to update their beliefs about both the state and others' decisions, the investors' responsiveness to the stock price results from both motivations. The former improves while the latter reduces the efficiency of the investment decisions from the society's perspective. We show that the SS ban essentially reduces the stock price informativeness and makes investors less sensitive to the stock price. Such a reduction in the sensitivity to the stock price reduces the use of information in the investment decision but also mitigates the coordination failure. When the speculator's information is not sufficiently good in expectation (as the probability of the speculator being present is not sufficiently high) and when the coordination friction is sufficiently high, the informational loss is dominated by the improvement of coordination, and, as a result, the efficiency is improved.

Our result provides a possible justification for the SS ban on the financial firms' stocks during the financial crisis. It is well-known that the significant mismatch of assets and liabilities for financial institutions results in them being vulnerable to panic-based runs that are caused by coordination failure, which has been both theoretically micro-founded (e.g. Diamond and Dybvig (1983), Goldstein and Pauzner (2005), Morris and Shin (2000)) and empirically documented (e.g. Chen et al. (2010), Gorton and Metrick (2012)). It is also well-known that uncertainty is the highest during crisis periods and after major shocks (e.g. Bloom (2009), Bloom et al. (2018)). We show that the interaction between MSS and panic-based run generates an adverse effect on firms' investment decisions so severe that a ban on SS is justified, in particular when there is a lot of uncertainty and when firms are more vulnerable to panic-based runs, i.e. financial firms during crisis periods.

Our paper is related to the literature on short sales. Short sales are a basic component in modern finance theories of asset pricing and portfolio choice. Most theoretical studies thus have viewed short sales as an institutional constraint and focused on identifying its consequences (e.g., Miller (1977), Diamond and Verrecchia (1987), Duffie et al. (2002), Abreu

and Brunnermeier (2003), Scheinkman and Xiong (2003)). In most of these studies, banning short sales has an adverse effect on efficiency.

A few papers have studied the ex-post consequences of short selling in the presence of rigid frictions. The study that is most closely related to ours is GG. We build on the manipulative short selling equilibrium in GG and extend GG to model a coordination decision-making game. This extension generates our main result that the SS ban can improve efficiency, while in GG the SS ban cannot improve efficiency despite removing the manipulative short selling. Brunnermeier and Oehmke (2013) show that short selling forces firms with market-based leverage requirement to liquidate the illiquid assets. In their model there is no informational feedback from the stock price to real decisions. The effect of stock price on the liquidation decision is assumed. Liu (2014) also studies a coordination game with short selling. In his model, investors in the coordination game receive private information and observes the stock price as public information. Short selling is assumed to add noise into the stock price and makes the public information noisier. In contrast, short selling in our model allows speculator's private information to be endogenously impounded into the stock price and makes the stock price more informative. Liu (2016) studies the interaction between an investors' coordination game with interbank market trading and focuses on the feedback loop between interbank market rate and the coordination game. Interbank market serves as a provision of liquidity and banks do not learn any information from the interbank market. In contrast, we focus on the interaction between managers learning from price and the coordination game.

Our study also makes a methodological contribution to the coordination game with market manipulation. We use the global games methodology to obtain the unique equilibrium of the coordination game to conduct welfare analysis. However, the market manipulation component from GG employs two rounds of trading and is too complicated to be combined with the coordination game. We use a one-round trading setting to simplify the market manipulation component and integrate it with the coordination game. This formulation of market manipulation may be used in other settings.

Finally, our paper is related to the literature on the welfare effects of public information in coordination games (Angeletos and Pavan (2007)). Morris and Shin (2002) is probably

the first to show that in settings with coordination motives, more precise public information may decrease welfare when private information is sufficiently precise. In our setting there is no private information per se so even with coordination, more public information should increase welfare. The detrimental effect of more public information comes from the interaction of coordination with feedback effect.

The rest of the paper is organized as follows. Section 2 introduces the model set up. Section 3 highlights the multiple equilibria problem of the model. In section 4, we pin-down the unique equilibrium using the global games technique. In section 5, the main results, specifically the result that short selling could be detrimental to bank value, are presented. Section 6 concludes.

2 Model setup

Our model augments a coordination game with the manipulative short selling from GG. We start with the coordination game. Consider a risk-neutral economy with no discounting and four dates $(t = 0, 1, 2, 3)$, one firm with an underlying project, and a continuum of investors. The underlying state θ is either good or bad with equal probability, *i.e.*, $\theta \in \{H, L\}$ with $\Pr(\theta = H) = \frac{1}{2}$.

At $t = 2$, after observing a signal to be described below, each investor makes a binary investment decision, *i.e.*, $a_i \in \{0, 1\}$. $a_i = 1$ means that agent i invests. If the investor does not invest, she receives a payoff normalized to 0. If she invests, then her payoff u is jointly determined by the state θ and the aggregate non-investing population $l \equiv \int_{i \in [0,1]} (1 - a_i) di$:

$$u = \theta - H\delta l.$$

The parameter $\delta \geq 0$ captures the degree of strategic complementarity among investors' investment decisions and the multiplier H of the second term is a scaling factor. δ is often referred to as the project's vulnerability to runs. The project's aggregate value is

$$v = (1 - l)(\theta - H\delta l).$$

So far we have a standard coordination game. The only information source for investors is the stock price endogenously determined in a Kyle setting. Specifically, there are three types of traders. The first is a speculator who learns perfectly about θ with probability $\alpha \in (0, 1]$ and nothing with the complementary probability, that is, the speculator observes a signal $s \in \{H, L, \emptyset\}$. We call a speculator with $s = H(L)$ as a positively (negatively) informed speculator and a speculator with no information ($s = \emptyset$) as an uninformed speculator. The speculator chooses an order $d(s) \in \{-1, 0, 1\}$. The second group of traders are liquidity traders who trade for reasons orthogonal to state θ. Their aggregate order is denoted as $\widetilde{n}$, which is normally distributed with mean zero and variance σ_n^2. Finally, the third group of traders is the market maker who observes the total order flow $q = d + \widetilde{n}$ and sets price equal to the expected firm value, taking into account that the investors may learn from the stock price:

$$P = E[v|q].$$

The stock price and the order flow have identical information content. For simplicity, we assume that investors observe the order flow (instead of the stock price).

In sum, the timeline of the events is as follows.

At $t = 0$, the speculator's information endowment s is realized.

At $t = 1$, the trading occurs. Both the stock price and the order flow are observed.

At $t = 2$, investors observe the order flow q and make decisions.

At $t = 3$, the firm's terminal cash flows are realized.

As a benchmark, GG's setting is special case of our model with $\delta = 0$.[1]

We make a few assumptions before proceeding.

---

[1] Another difference between our setting and that of GG is the assumption that noise trading is normally distributed rather than discretely distributed. As will be discussed below, this assumption allows us to solve the unique equilibrium with only one round of trading instead of two rounds of trading as in GG, resulting in tractable analysis of welfare when we introduce the coordination friction into the feedback model.

$$A1 \quad : \quad H > 0 > L$$

$$A2 \quad : \quad \frac{H+L}{H} = 1 + \frac{L}{H} \equiv 1 - \gamma > \delta$$

$$A3 \quad : \quad \gamma > \overline{\gamma}$$

$A1$ states that it is socially optimal to invest in the good state and not to invest in the bad state. $A2$ guarantees that in the absence of any information, the default choice is to invest. It enhances the assumption in GG that $H + L > 0$ to accommodate the coordination game represented by δ. Finally, $A3$ requires that the feedback effect is sufficiently strong so that the investors' decisions can be influenced by the stock price even in the absence of the coordination failure (see Proposition 3 of GG). To see this, note that $\gamma > \overline{\gamma}$ is equivalent to $1 - \overline{\gamma} > \frac{H+L}{H}$, i.e. $\frac{H+L}{H}$ cannot be too high. $\overline{\gamma}$ is defined by equation (21) in the Appendix.

As is typical in the feedback literature, γ is a measure of the strength of the informational feedback effect. A higher γ indicates that the investment decision is more sensitive to information.

In addition, we introduce two tier-breakers. First, if the negatively (positively) informed speculator is indifferent among $d \in \{-1, 0, 1\}$, he always chooses $d = -1$ ($d = 1$). This rules out a degenerate equilibrium where no trading takes place. Second, if the uninformed speculator is indifferent among $d \in \{-1, 0, 1\}$, he chooses not to trade. This assumption biases against finding equilibria where the uninformed speculator trades.

The efficiency is defined as the expected firm value that aggregates the payoffs to all investors:

$$V \equiv E[(1 - l)(\theta - H\delta l)]. \tag{1}$$

A perfect Bayesian equilibrium (PBE) of our model consists of the speculator's trading strategy $d(s)$, each investor's withdrawal strategy $a_i(q)$ and beliefs about the fundamental θ such that (1) both the speculator and the investors maximize their respective objective functions, given their beliefs and the strategies of others and (2) each investor uses Bayes' Rule, if possible, to update beliefs about θ.

3 The equilibrium

3.1 The first-best benchmark

Before proceeding, we first solve for the first-best benchmark. Since there are both coordination friction and informational friction in our setting, the first-best benchmark consists of a single agent (i.e. no coordination friction) who knows θ (i.e. no informational friction) and chooses investment at date 1 to maximize V as in equation (1). Equivalently, she chooses $l \in [0, 1]$ to maximize

$$E[(1 - l)(\theta - H\delta l)|l, \theta] = (1 - l)(\theta - H\delta l),$$

with the optimal solution in Lemma 1.

Lemma 1
$$l^{FB} = \begin{cases} 0 \; \textit{if } \theta = H \\ 1 \; \textit{if } \theta = L \end{cases},$$

resulting in $V^{FB} = \frac{1}{2}H$.

Lemma 1 is intuitive. When the investor knows about the state, the assumption that $H > 0 > L$ ensures that she will continue investing when $\theta = H$ and withdraw all of her investment when $\theta = L$. The first-best firm value therefore is the probability of $\theta = H$ times the firm value when $\theta = H$, i.e. $\frac{1}{2}H$.

We now solve for the equilibrium in the general case using the backward induction.

3.2 The coordination subgame

We first solve for the subgame after the investors have observed the order flow q. We conjecture and later verify that q satisfies maximum likelihood ratio property (MLRP), that is, a higher q indicates that $\theta = H$ is more likely. Intuitively, positively informed speculator always chooses $d = 1$ and negatively informed speculator always chooses $d = -1$. Given the conjectures, regardless of the uninformed speculator's choice of d, a higher order flow indicates that it

is more likely that the order flow comes from positively informed speculator and therefore a higher probability of $\theta = H$.

Now consider the strategy of investor i when observing the order flow q. If she chooses to withdraw, then she gets 0 for sure. If she chooses to continue, then her expected payoff differential is

$$\Delta(q) = E[\theta|q] - \delta H E[l|q].$$

As is standard in coordination game, multiple equilibria arise if q is common knowledge, making it difficult to conduct comparative statics. We apply the global games methodology to obtain the unique equilibrium. Specifically, we assume that each investor receives a noisy signal q_i and focus on the equilibrium when the noise converges to 0:

$$q_i = q + \varepsilon_i,$$

where $\varepsilon_i \sim N(0, \sigma_\varepsilon^2)$ reflects the idiosyncratic noise in each investor's observation of the order flow q. Equivalently, ε_i can be interpreted as the individual specific difference of investors' interpretation of the information context of the order flow q. Both interpretations generate the same results. As is standard in the global games literature (e.g. Morris and Shin (2000), Goldstein and Pauzner (2005), Bouvard et al. (2015) and Gao and Jiang (2018)), we will focus on the limiting case as σ_ε approaches 0. This results in a unique equilibrium of the coordination game.

Lemma 2 *The investors play a common threshold strategy. The common threshold q^* is determined by the following equation.*

$$E[\theta|q^*] - \frac{H\delta}{2} = 0. \tag{2}$$

Lemma 2 characterizes the equilibrium common threshold in an intuitive manner. An investor uses her signal q_i to forecast both the state θ and other investors' actions that collectively determine l. The first use of information is summarized in the component $E[\theta|q^*]$, the marginal investor's expectation of the fundamental. Since θ is binary, the expectation is

fully characterized by the conditional probability $\beta(q_j^*) \equiv \Pr(\theta = H | q_j^*)$. A higher β means that the marginal investor has to be more optimistic about the state to invest.

Investors also use q_i to forecast other investors' signals and actions. At $q_i = q^*$, she conjectures that exactly half of the other investors will get a signal higher than q^* and stay whereas the other half will get a signal lower than q^* and withdraw. Therefore, she expects that half of the investors will stay: $E[l|q^*] = \frac{1}{2}$.

Collecting these two expectations and imposing the equilibrium condition that the marginal investor has to be indifferent between continuing and running at the threshold q^*, i.e. $\Delta(q^*) = 0$, we obtain equation (2) that uniquely determines the common threshold q^*.

Using Bayes' rule to express $E[\theta|q^*]$ in terms of $\beta(q^*)$ in equation (2) results in

$$\beta(q^*) = \frac{\frac{\delta}{2} + \gamma}{1 + \gamma}. \tag{3}$$

Note that $\beta(q^*)$ is an equilibrium variable and depends on the trading strategy of the speculator, to which we turn now.

3.3 The trading decision and the equilibrium

Anticipating the unique equilibrium for the subgame of coordination, the speculator chooses his trading strategy, which then determines the price and thus investors' investment strategies. The trading strategy and the investors' investment decisions are then jointly determined by solving a fixed point problem.

The following Lemma echoes the result from GG that the manipulation is only one-sided. It is never optimal for the speculator to buy when he is uninformed.

Lemma 3 $d(\emptyset) = 1$ *is always a dominated strategy given that* $d^*(H) = 1$ *and* $d^*(L) = -1$ *or* 0, *for any conjecture of the market maker's strategy of the uninformed speculator.*

When short selling is banned, it is straightforward to show that $d^*(L) = 0$. Lemma 3 then implies that $d^*(\emptyset) = 0$, which is the next Proposition. Note that we add subscript to q^* as q^* clearly depends on whether short sales are banned or not, with q_B^* denoting the regime where SS are banned and q_A^* denoting the regime where SS are allowed.

Proposition 1 *Suppose short sales are banned. Then the positively informed speculator buys while other speculators do not trade, i.e. $d^*(H) = 1$ and $d^*(L) = d^*(\emptyset) = 0$. The investment threshold is q_B^*, defined in equation (10) in the appendix. $q_B^* < \frac{1}{2}$ and is increasing in α and δ.*

Proposition 1 is intuitive. The positively informed speculator finds it optimal to buy since his information will only be partially reflected in prices, resulting in a lower expected buying price and a positive expected profit. The negatively informed speculator wants to short sell but is not able to because of the ban. He clearly finds it not optimal to buy and therefore chooses not to trade. Similarly, the uninformed speculator finds it not optimal to buy from Lemma 3 and therefore chooses not to trade.

We now consider the case when short selling is allowed.

Proposition 2 *Suppose short sales are allowed, then*

1. the informed speculator trades in the direction of his information: i.e. $d^*(H) = 1$ and $d^*(L) = -1$;

2. $d(\emptyset) = -1$ is the unique equilibrium if and only if $\alpha > \underline{\alpha}(\delta)$ where $\underline{\alpha}(\delta)$ is defined in equation (13).

3. the investment threshold is q_A^* when $d(\emptyset) = -1$ and q_{NT}^* when $d(\emptyset) = 0$, defined in equations (15) and (16) in the appendix, respectively. Both q_A^* and q_{NT}^* is negative and increasing in α and δ.

When short selling is allowed, the possibility of manipulative short selling complicates the derivation of the equilibrium. SS lowers the order flow and induces more investors not to invest. This has two effects on the speculator's profit, which we now elaborate.

First, in the absence of the feedback effect, the market maker interprets the lower order flow as a bad signal about the state and lowers the price accordingly. Since the market maker cannot distinguish whether the SS originates from the negatively informed speculator or the uninformed speculator, the price is set as a weighted average conditional on the speculator being negatively informed or uninformed. Thus, SS is profitable for the informed speculator
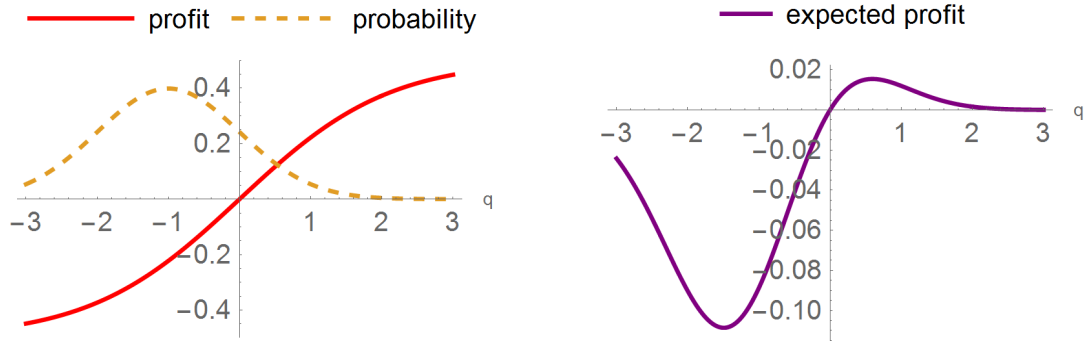
11

Figure 1: Expected profit of the uninformed speculator when choosing $d = -1$

but not profitable for the uninformed speculator. Specifically, in the absence of the feedback effect, the expected profit from short selling of the uninformed speculator, $\Pi(\alpha)$, can be written as

$$\Pi(\alpha) \propto \int_0^{+\infty} (\beta(q) - \frac{1}{2}) dF(q) + \int_{-\infty}^0 (\beta(q) - \frac{1}{2}) dF(q),$$

where F is the cumulative distribution function of q. Since there is no feedback, the investment will always be carried out as the project is ex ante positive NPV. When $q > 0$, the order flow is so high that the market maker believes that $\beta(q) > \frac{1}{2}$ and the uninformed speculator will be making money whereas the opposite occurs when $q < 0$. As shown in Figure 1, in the absence of feedback effect, short selling shifts the distribution of the order flow (i.e. $F(q)$) to the left, increasing the loss region and shrinking the profit region. Therefore in the absence of the feedback effect, the uninformed speculator will generate a negative expected profit from short selling and short selling is therefore not optimal for him.

Second, the lower price also affects investment decisions through the feedback effect. Agents use the stock price to make inferences about the state and about other investors' decisions. A lower price is a bad signal about the state and thus discourages investors from investing. The reduction in the investment indeed reduces the firm's terminal cash flow and

12

creates a self-fulfilling equilibrium. Both the informed and uninformed speculator can profit from this informational feedback effect. Specifically, in the presence of the feedback effect, the project will not be carried out when $q < q^*$ as $\beta(q)$ becomes so small that every investor will withdraw, implying that the price will be zero and thus the uninformed speculator makes zero profit rather than suffering a loss when $q < q^*$ in the absence of feedback, i.e.

$$\Pi(\alpha) \propto \int_0^{+\infty} (\beta(q) - \frac{1}{2}) dF(q) + \int_{q^*}^0 (\beta(q) - \frac{1}{2}) dF(q).$$

As shown in Figure 2, feedback effect, by terminating investment when q is sufficiently small, reduces the loss region and thus increses the expected payoff from short selling for speculators. Figure 3 shows that this effect is stronger when α becomes larger. Intuitively, higher α increases the informativeness of negative order flow and reduces the loss region even more. Therefore, while the negatively informed speculator always short-sells, the uninformed speculator short sells if and only if the profit from the informational feedback effect compensates for the loss from his informational disadvantage, a condition satisfied when the fraction of informed speculator is sufficiently large.

We thus know that for uninformed SS to be an equilibrium, it has to be that $\Pi(\rho, \alpha) \geq 0$ where $\rho = \Pr(d(\emptyset) = -1)$. Since $\Pi(\alpha, \rho)$ is increasing in α, when α is sufficiently large (i.e. $\alpha > \underline{\alpha}(\delta)$), $\Pi(\alpha, \rho) > 0 \,\forall \rho$, making $d(\emptyset) = -1$ a dominant strategy. When α is sufficiently small (i.e. $\alpha < \alpha_1(\delta)$), $\Pi(\alpha, \rho) < 0 \,\forall \rho$, making $d(\emptyset) = 0$ a dominant strategy. When α is in the middle, i.e. when $\alpha \in (\alpha_1(\delta), \underline{\alpha}(\delta))$, neither $d(\emptyset) = -1$ nor $d(\emptyset) = 0$ is dominant, resulting in at least three equilibria: two pure strategie equlibria where $\rho = 1$ and $\rho = 0$ and a mixed strategy equilibrium where $\rho_{-1} \in (0, 1)$.[2]

From now on, we focus on the equilibrium that $d(\emptyset) = -1$ if and only if $\alpha > \underline{\alpha}(\delta)$. This equilibrium also benefits SS the most as it removes all MSS whenever MSS is not the unique equilibrium. Setting $\delta = 0$, Proposition 2 replicates the main result in GG that manipulative short selling arises when the probability that the speculator is informed is sufficiently high. We extend this result to our setting of a coordination game in which $\delta > 0$.

---

[2]We say at least three equilibria as it is possible that ρ_{-1} may not be unique.
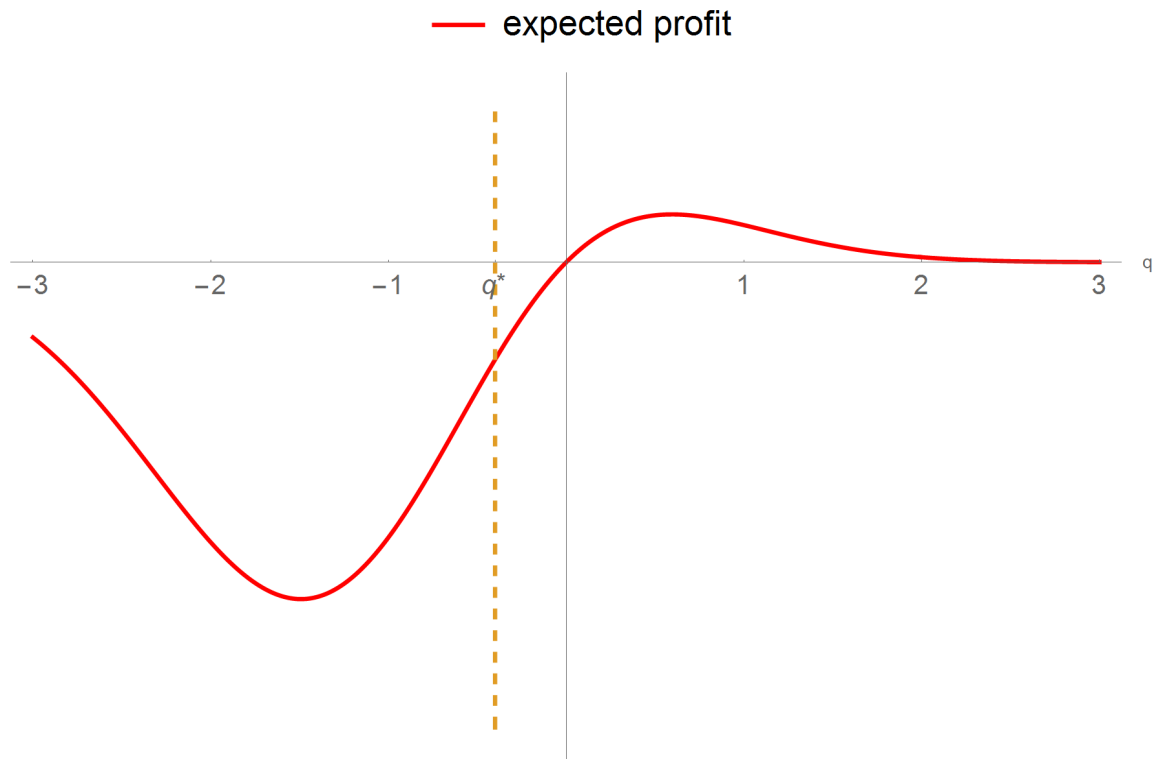
Figure 2: Expected profit of the uninformed speculator in the presence of feedback

4 The analysis

Having characterized the equilibrium when short-sale is allowed and when it is banned, we are ready to present our main result about the efficiency of banning short selling.

We have defined the efficiency as the ex-ante expected firm value V in equation (1). After some algebra, for a given regime $j \in \{A, B\}$, we can write it generally as

$$V = V^{FB} - \frac{1}{2}H(\varepsilon^H + \gamma \varepsilon^L).$$

Note that $V^{FB} = \frac{1}{2}$ is the efficiency in the first-best case as stated in Lemma 1. Relative to the first-best, the efficiency is ultimately reduced by two types of errors in the investment decisions, under-investment in the good state and over-investment in the bad state, with the respective probabilities denoted as ε^H and ε^L. Underinvestment reduces 1 unit of efficiency by foregoing the payoff H in the good state, while overinvestment generates a loss of $L = -\gamma H$ when the state is bad.
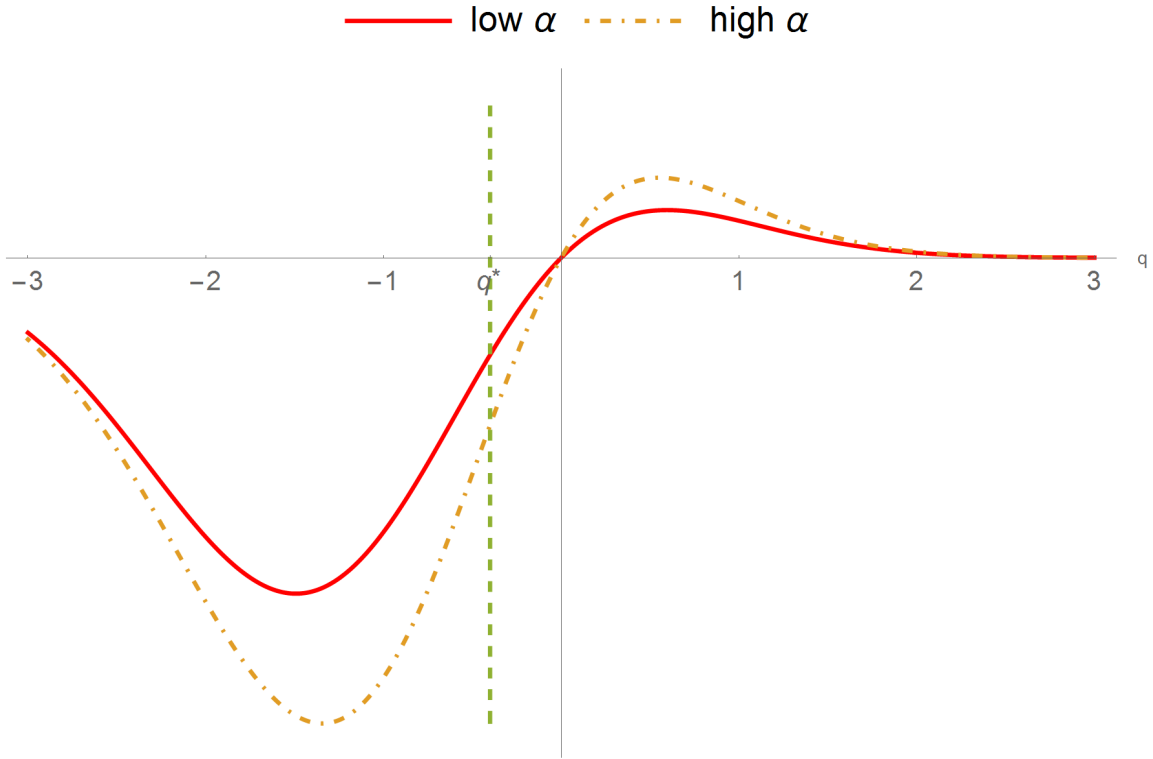
Figure 3: Expected profit of the uninformed speculator with different α

The ban affects the speculator's trading strategy, the information content of the order flow, the investors' investment decisions and ultimately the efficiency. We analyze each effect in turn.

First, the ban affects the speculator's trading strategy and investors' investment decisions. It does not affect the buy order from the positively informed speculator but replaces the sell order from both the negatively informed speculator and (possibly) the uninformed speculator with no trading. Accordingly, the ban does not affect the order flow distribution when the speculator is positively informed but shifts the distribution to the right by one in other cases. Rationally anticipating the consequences of the SS ban on the information content of the stock price, the investors adjust their investment decisions accordingly. The order flow threshold, if increasing, will increase smaller than 1 as the investors are not sure whether the banned SS comes from the informed or the uninformed speculator.

Lemma 4 also shows that, counterintuitively, banning SS may not result in an increase in the order flow threshold, i.e. q_B^* is larger if and only if α is sufficiently large. To understand

this, note that banning SS has two effects: first, as discussed above, it pushes the distribution of the order flow of the negatively (and possibly uninformed) speculator to the right (the "rightward shifting effect", therefore increasing q_B^*; second, such pushing of the distribution to the right also decreases the relative informativeness of order flow (the "informativeness decrease effect" in infering the speculator's information since the distributions of the order flow of positively and negatively informed speculators are now closer. Such decrease of the informativeness makes order flow a more noisy signal and decreases q_B^*. The reason is that the default action in the absence of any information is to stay, resulting in more noisy order flow making each agent more likely to stay. When α is sufficiently large, the order flow is sufficiently informative that the second effect is dominated by the first effect, resulting in a larger q_B^*.[3]

Lemma 4 *The SS ban changes the investment threshold as follows: $q_B^* < q_A^* + 1$ and $q_B^* < q_{NT}^* + 1$. We also have $q_B^* > q_A^*$ and $q_B^* > q_{NT}^*$ if and only if $\alpha > \alpha_2$, where α_2 is defined in equation (12) in the appendix.*

From Proposition 2 we know that MSS is the unique equilibrium when $\alpha \geq \underline{\alpha}$. From Lemma 4 we know that $q_A^* < q_B^* < q_A^* + 1$ and $q_{NT}^* < q_B^* < q_{NT}^* + 1$ when $\alpha > \alpha_2$. We are not able to sign the difference of α_2 and $\underline{\alpha}$, however. The next Proposition shows that regardless of whether α_2 is bigger than $\underline{\alpha}$, banning SS improves efficiency if and only if α is not much bigger than $\underline{\alpha}$, which exists only when δ is not too small. The intuition, however, is slightly different when α_2 is bigger than $\underline{\alpha}$ versus when α_2 is smaller than $\underline{\alpha}$. We first present the intuition when α_2 is smaller than $\underline{\alpha}$, followed by the case when α_2 is larger than $\underline{\alpha}$.

When $\alpha_2 < \underline{\alpha}$, given the order flow distribution and the investment threshold, the invest- ment errors in various scenarios are summarized in Table 1 when $\alpha > \underline{\alpha} > \alpha_2$. In state θ for a given regime $j \in \{A, B\}$, the expected investment error ε_j^θ can be decomposed by speculator

---

[3]A possible reason for q_B^* to be smaller is that we take the speculator's order flow as exogenous. In principle, the speculator would trade more aggressively when σ_n^2 is larger (Kyle (1985)), which may result in q_B^* always larger. We leave this interesting question for future research.

Table 1: Investment errors in various scenarios when $\alpha > \underline{\alpha}$

j	A	B	$B - A$	$B - A$	Prob*cost
ε_{Ij}^{H}	$\Phi(\frac{q_A^*-1}{\sigma_n})$	$\Phi(\frac{q_B^*-1}{\sigma_n})$	$\Phi(\frac{q_B^*-1}{\sigma_n}) - \Phi(\frac{q_A^*-1}{\sigma_n})$	$+$	α
ε_{Ij}^{L}	$1 - \Phi(\frac{q_A^*+1}{\sigma_n})$	$1 - \Phi(\frac{q_B^*}{\sigma_n})$	$\Phi(\frac{q_A^*+1}{\sigma_n}) - \Phi(\frac{q_B^*}{\sigma_n})$	$+$	αr
ε_{Uj}^{H}	$\Phi(\frac{q_A^*+1}{\sigma_n})$	$\Phi(\frac{q_B^*}{\sigma_n})$	$\Phi(\frac{q_B^*}{\sigma_n}) - \Phi(\frac{q_A^*+1}{\sigma_n})$	$-$	$(1-\alpha)$
ε_{Uj}^{L}	$1 - \Phi(\frac{q_A^*+1}{\sigma_n})$	$1 - \Phi(\frac{q_B^*}{\sigma_n})$	$\Phi(\frac{q_A^*+1}{\sigma_n}) - \Phi(\frac{q_B^*}{\sigma_n})$	$+$	$(1-\alpha) r$

type, whether the speculator is informed (I) or uninformed (U):

$$\varepsilon_j^\theta = \alpha \varepsilon_{Ij}^\theta + (1 - \alpha) \varepsilon_{Uj}^\theta.$$

Consider first the informed speculator who buys in the good state and shorts in the bad state. In the good state, the order flow has a mean of 1 and variance of σ_n^2 because the positively informed speculator buys. Thus, the probability of underinvestment is $\varepsilon_{Ij}^{H} = \Phi(\frac{q_j^*-1}{\sigma_n})$ for $j \in \{A, B\}$. This explains the second row in both tables. Similarly, in the bad state, the order flow has a mean of -1 if SS is allowed and 0 if SS is banned. The probability of overinvestment is thus $\varepsilon_{Ij}^{L} = 1 - \Phi(\frac{q_j^* - d_{Ij}^*(L)}{\sigma_n})$, where $d_{Ij}^*(L)$ is the negatively informed speculator's trading strategy in regime $j \in \{A, B\}$. This explains the third row in both tables.

Therefore, when the speculator is informed, the effect of the SS ban on the quality of the investment decision is captured by

$$\Delta \varepsilon_I^\theta = \varepsilon_{IB}^\theta - \varepsilon_{IA}^\theta.$$

Now consider the uninformed speculator who shorts when SS is allowed and does not trade when SS is banned. The order flow has a mean of $d_{Uj}^*(\theta)$ in regime j and state θ, and the investment errors can be expressed as in the fourth and fifth row of both tables. Therefore, when the speculator is uninformed, the effect of the SS ban on the quality of the investment decision is captured by

$$\Delta \varepsilon_U^\theta = \varepsilon_{UB}^\theta - \varepsilon_{UA}^\theta.$$

We can characterize the ban's effect on investment efficiencies as follows.

Lemma 5 *When $\alpha > \underline{\alpha} > \alpha_2$, the SS ban affects the accuracy of the investment decisions as follows.*

1. *when the speculator is informed, the ban reduces the investment accuracy, that is, $\Delta\varepsilon_I^\theta > 0$ for any θ;*

2. *when the speculator is not informed, the ban increases investment accuracy in the good state and reduces investment accuracy in the bad state. That is, $\Delta\varepsilon_U^H < 0$ and $\Delta\varepsilon_U^L > 0$.*

3. *$\Delta\varepsilon_I^L = \Delta\varepsilon_U^L = -\Delta\varepsilon_U^H = \Delta\varepsilon_0$.*

Lemma 5 is intuitive. First, the ban suppresses the information from the negatively informed speculator and increases the overinvestment in the bad state, despite the rational adjustment by investors. The ban removes the sell order from the negatively informed speculator and thus degrades the informational value of the order flow. Hence, $\Delta\varepsilon_I^L > 0$. Second, the ban also increases the investment error when the speculator is positively informed, that is, $\Delta\varepsilon_I^H > 0$. Even though the ban does not affect the equilibrium strategy of the positively informed speculator (i.e. to buy), it changes the equilibrium strategy of other types of speculators whose order flow cannot be distinguished from the positively informed speculator. In particular, the suppression of the sell orders dilutes the information content of the positively informed speculator's buy order, which adversely affects the investors' use of information in the investment decisions. Collectively, these two channels explain Part 1 of Lemma 5 and capture the conventional wisdom that the SS ban reduces the efficiency by degrading the information value of the stock price.

When the speculator engages in manipulative short selling, that is, shorting when he is uninformed, how the SS ban affects the investment accuracy depends on the state. In the good state, the ban reduces underinvestment induced by such MSS, that is, $\Delta\varepsilon_U^H < 0$. In the bad state, the ban results in more overinvestment as there should be no investment in the bad state, that is, $\Delta\varepsilon_U^L > 0$. The reason is that in the absence of information and manipulative trading, the default action is to invest. By suppressing such information (that the speculator

is uninformed), the ban leads to less investment, resulting in less errors when the state is good and more errors when the state is bad. This explains Part 2 of Lemma 5.

Finally, Part 3 of Lemma 5 shows an articulate relationship among the investment errors. First, the uninformed speculator's trading strategies affect investment errors in a symmetric manner. When the ban reduces the underinvestment in the good state, it increases the overinvestment in the bad state by the same amount, that is, $\Delta \varepsilon_U^H = -\Delta \varepsilon_U^L$. Second, in the bad state, both the uninformed and informed speculators use the same trading strategy across the two regimes. Thus, the ban has the same effect on the investment accuracy, that is, $\Delta \varepsilon_I^L = \Delta \varepsilon_U^L$.

In sum, the ban reduces efficiency when the speculator is informed, regardless of the uninformed speculator's trading strategy. The ban improves (decreases) efficiency when the uninformed speculator engages in manipulative short selling and decreases (improves) efficiency when the uninformed speculator does not trade and the state is good (bad). Collecting the investment error terms and weighting them by their probabilities and associated consequences, we can write out the efficiency difference across the two regimes, resulting in

$$
\begin{aligned}
\Delta V & \equiv & 2(V_{Ban} - V_{Allowed}) \\
& = & H\{-\alpha \Delta \varepsilon_I^H - [\alpha \gamma - (1-\alpha)(1-\gamma)]\Delta \varepsilon_0\}.
\end{aligned} \tag{4}
$$

As will be shown in the proof of Proposition 3, ΔV is decreasing in α when $\alpha > \underline{\alpha}$. Intuitively, the discussion above suggests that banning informed SS always increases decision errors but banning MSS may decrease decision errors. Thus, when the probability of informed trading is higher, the efficience loss from banning informed SS (i.e. $\alpha \Delta \varepsilon_I^H$) more than outweighs the potentially efficiency loss from banning MSS (i.e. $[\alpha \gamma - (1-\alpha)(1-\gamma)]\Delta \varepsilon_0$). Therefore, ΔV can only be positive if α is not too big. As we will discuss in more detail below, if banning MSS still increases decision errors, which occurs when δ is too small, then ΔV can never be positive. This completes the discussion for the intuition when $\alpha_2 < \underline{\alpha}$.

When $\alpha_2 \geq \underline{\alpha}$, Lemma 5 will still apply and the intuition will be the same when $\alpha > \alpha_2$,

i.e. $\Delta V > 0$ when α is not too bigger than α_2. When $\underline{\alpha} < \alpha \leq \alpha_2$, however, since $q_B^* \leq q_A^*$, we have $\Delta \varepsilon_I^H < 0$, i.e. even banning informed SS will increase the efficiency when $\theta = H$. Intuitively, $q_B^* \leq q_A^*$ implies that the order flow is so noisy that banning SS results in an even lower threshold and thus less run. Since less run is good when $\theta = H$, $\Delta \varepsilon_I^H < 0$. As a result, when ΔV is positive when banning informed SS is unambiguously bad, then ΔV must be even more positive when banning informed SS is sometimes good, i.e. $\Delta V > 0$ when $\underline{\alpha} < \alpha \leq \alpha_2$. This completes our discussion.

We now summarize our main result, based on the discussion above, in the next Proposition.

Proposition 3 *Banning SS improves the efficiency if and only if $\alpha \in (\underline{\alpha}(\delta), \alpha^*(\delta))$, where $\alpha^*(\delta)$ is defined in the appendix. This set is empty at $\delta = 0$ and nonempty when δ is sufficiently large.*

We first illustrate Proposition 3 with two special cases, one in the absence of coordination friction $\delta = 0$ and the other with the extreme coordination friction $\delta \to 1 - \gamma$. The proofs of two Corollaries are omitted as they are contained in the proof of Proposition 3.

Corollary 1 *Consider the special case in which $\delta \to 0$. If SS is allowed, then MSS arises if and only if $\alpha > \lim_{\delta \to 0^-} \underline{\alpha}(\delta) > 1 - \gamma$.*

Corollary 1 focuses on the case with $\delta = 0$, resulting in a single-person decision making setting with no coordination friction. In this case, the argument from GG shows that the ban reduces the efficiency. Since the single investor can always ignore a signal, she cannot be worse off from having additional information. Since the ban reduces the stock price informativeness, it reduces the efficiency.

Alternatively, we can also use the expression of ΔV to analyze the ban's consequences. In particular, there is an endogenous connection between δ and the possible value of α for MSS to be optimal, which is best illustrated by focusing on the extreme values of δ. Note that the first term of ΔV is clearly negative when $\alpha > \max(\alpha_2, \underline{\alpha})$, as discussed before. Thus, the sign of ΔV crucially depends on the sign of the second term.

When $\delta \to 0$, for MSS to be optimal (i.e. $q_A^* > -\infty$), it is necessary that $\alpha > 1 - \gamma$. Otherwise, if $\alpha \leq 1 - \gamma$, then the stock price cannot be informative enough to change the investor's decision. In other words, short sales can come from both the negatively informed speculator and the uninformed speculator. The cost of suppressing the former dominates the benefit of suppressing the latter because α has to be sufficiently large. Note that $\alpha > 1 - \gamma$ results in $\alpha\gamma - (1 - \alpha)(1 - \gamma)$ being positive, i.e. conditional on the state being bad, the beneficial effect from preventing MSS of the uninformed speculator is dominated by the cost of preventing informed SS. This explains why the set is empty when $\delta = 0$.

To understand the result in Proposition 3 of why the set is nonempty when δ is sufficiently large, it is useful to prove the following property about $\underline{\alpha}(\delta)$ and $\alpha_2(\delta)$.

Lemma 6 *Both $\underline{\alpha}(\delta)$ and $\alpha_2(\delta)$ are decreasing in δ.*

Lemma 6 is intuitive. As the coordination concern becomes more severe, investors are more pessimistic and they are only willing to stay if the probability of the good state is sufficiently high. Anticipating this fragility, manipulative short selling is more likely. Similarly, when investors are more pessimistic, it is more likely the SS ban increases the threshold as the threshold will be quite high when there is no SS ban, which is as if that the order flow being very informative, resulting in the rightward shifting effect dominating the informativeness effect. In fact, as δ approaches $1 - \gamma$, an uninformed investor short sells even as the fraction of informed speculators approaches 0, i.e. $\underline{\alpha}(\delta) \to 0$ (and $\alpha_2(\delta) \to 0$), as illustrated in the following Corollary.

Corollary 2 *Consider the special case in which $\delta \to 1-\gamma$. $\lim_{\delta \to 1-\gamma} \underline{\alpha}(\delta) = \lim_{\delta \to 1-\gamma} \alpha_2(\delta) = 0$. The investment threshold is $\lim_{\delta \to 1-\gamma} q_A^* = 0$, $\lim_{\delta \to 1-\gamma} q_B^* = \frac{1}{2}$. The errors $\lim_{\delta \to 1-\gamma} \Delta\varepsilon_I^H = \lim_{\delta \to 1-\gamma} \Delta\varepsilon_0 > 0$.*

When $\alpha \to 0$, clearly $\alpha\gamma - (1 - \alpha)(1 - \gamma) < 0$, i.e. conditional on the state being bad, the beneficial effect from preventing MSS of the uninformed speculator is dominated by the cost of preventing informed SS since the likelihood of informed SS becomes sufficiently small. The second term of ΔV therefore becomes positive. If the first term is positive, then ΔV is unambiguously positive. Even if the first term is negative, the magnitude decreases when α

becomes smaller as the benefit from informed SS decreases regardless of whether the state is good or bad when the probability of being informed decreases. In fact, when $\delta \to 1 - \gamma$, we can calculate that

$$\Delta V \to 2H(\frac{1-\gamma}{2} - \alpha)[\Phi(\frac{1}{\sigma_n}) - \Phi(\frac{1}{2\sigma_n})]$$

Note that $\Phi(\frac{1}{\sigma_n}) - \Phi(\frac{1}{2\sigma_n}) > 0$, that is, the ban reduces the sensitivity of investors' decisions to the order flow. In addition, $\frac{1-\gamma}{2}$ and α represent the respective effects of coordination and information on investors' investment decisions. When $\frac{1-\gamma}{2} > \alpha$, then the coordination effect dominates the information effect. In this case, the ban, by mitigating the investors' response to the order flow, improves the efficiency. Otherwise, the ban reduces the efficiency. By continuity, we can prove the more general result that the SS ban improves the efficiency if and only if α is not sufficiently high and δ is sufficiently high, i.e. the set is non-empty when δ is sufficiently large.

As a summary, in the absence of coordination friction (i.e. when $\delta = 0$), informed SS improves efficiency and MSS by uninformed speculators, by terminating good projects, reduces efficiency. However, MSS is secondary as it is sustained only by informed SS. In other words, MSS is only optimal for uninformed speculators if the price is sufficiently informative, i.e. informed SS is sufficiently strong. This results in the benefit of informed SS always dominating the ineffiency of MSS and thus a decrease of firm value from informed SS, as documented in GG. We introduce another source of MSS based on panic runs from coordination failure (i.e. when $\delta > 0$). The introduction of panic runs reduces investment (i.e. $\frac{\partial q_A^*}{\partial \delta} > 0$) even in the absence of informed SS, thus making MSS more prevalent (i.e. $\frac{\partial \alpha(\delta)}{\partial \delta} < 0$) and not necessarily secondary. In fact, when δ is large and when α is small, our channel of MSS sustained by panic runs is strong and the channel from GG is mild, resulting in SS reducing efficiency overall and a SS ban increasing efficiency.

To better understand Proposition 3, we illustrate using a numerical example. For ease of notation we write ΔV as $\Delta V(\alpha, \delta)$. We assume that $H = 1$, $\sigma_n = \frac{1}{2}$, $L = -0.5$. Note that in this case $1 - \gamma = 0.5$. When $\delta = 0$, we can numerically calculate that $\underline{\alpha}(0) \simeq 0.91 > \alpha_2(0) = 0.51$. In addition, $q_{SA}^*(\underline{\alpha}(0), 0) \simeq -0.2 \in (-\frac{1}{2}, 0)$ and $\Delta V(\underline{\alpha}(0), 0) \simeq -0.0875 < 0$,

i.e. banning SS decreases firm value. When we increase δ and keep $\alpha = \underline{\alpha}(0)$, SS is clearly still optimal for the uninformed speculator. When $\delta = 0.48$, $\Delta V(\underline{\alpha}(0), 0.45) \simeq -0.0882 < \Delta V(\underline{\alpha}(0), 0)$, i.e. banning SS is even worse. However, when $\delta = 0.48$, $\underline{\alpha}(0.48)$ decreases to around $0.03 > \alpha_2(0.48) \approx 0.0268$, which enlarges the range for SS to be optimal for the uninformed speculator. For example, when $\alpha = 0.2 > \underline{\alpha}(0.48)$, $\Delta V(0.2, 0.48) = 0.0145 > 0$. In addition, note that α cannot be too large. For example, when $\alpha = 0.4$, $\Delta V(0.4, 0.48) = -0.0168 < 0$, i.e. banning SS is bad when $\alpha = 0.4$.

5 Empirical and policy implications

Our results provide several empirical and policy implications.

First, as suggested in GG, in the presence of MSS, the first-best policy would always be a discriminative ban on uninformed speculators. This policy eliminates MSS while preserving the information conveyed through trading of informed speculators. However, such a policy is impractical due to the difficulty in judging whether the speculator is informed or not. In the absence of such discriminative ban, our result provides a rationale for indiscriminately banning all SS, as the interaction of MSS with feedback effect and strategic complementarities reduces firm value by forgoing productive investments in good states. Such forgone productive investments may dominate the information through trading of informed speculators.

Second, our results suggest when SS ban is more likely to be efficient. Specifically, SS ban is more likely to be efficient for 1) stocks of firms that exhibit strong complementarities and thus are more vulnerable to runs (e.g. financial institutions); 2) for period when speculators do not have very precise information (e.g. during uncertain economic periods) and; 3) for situations where investors are sensitive to information (e.g. when feedback effect is strong). Our results thus are consistent with the anecdotal evidence of SS ban for financial stocks during crisis periods. Our results also predict that such SS ban is more effective when firms have more to learn from the stock market, as empirically proxied in Chen et al. (2007).

6   Conclusions

We propose a model to provide justification for SS ban. The benefit of SS ban stems from the elimination of MSS where uninformed speculators short-sells, which can be sustained by both panic runs by investors due to strategic complementarity and the possibility of market learning from informed speculators (i.e. the feedback effect). We show that when strategic complementarities are sufficiently strong and market uncertainty is sufficiently high, MSS, by inducing firms to abandon positive NPV projects, can destroy value so much that the benefit of banning MSS outweighs the cost of not being able to learn from informed speculators. To the extent that financial institutions exhibit strong degree of complementarities and crisis periods exhibit high market uncertainty, our results are consistent with anecdotal evidence that SS bans are typically imposed on financial stocks during crisis periods.

7   Appendix: Proofs

7.1   Proof of Lemma 1

Proof. The derivative with respect to l results in

$$\frac{\partial[(1-l)(\theta - H\delta l)]}{\partial l} = -\theta + H\delta(2l - 1)$$

Thus, when $\theta = H$, $\frac{\partial[(1-l)(\theta - H\delta l)]}{\partial l} < 0$, resulting in $l^{FB} = 0$. When $\theta = L$, $\frac{\partial[(1-l)(\theta - H\delta l)]}{\partial l}$ is either always positive or initially negative but eventually positive. In the former case, $l^{FB} = 1$. In the latter case, the optimal solution is corner and one has to compare $V(l = 0)$ and $V(l = 1)$. Note that $V(l = 0) = L < V(l = 1) = 0$. Therefore $l^{FB} = 1$ when $\theta = L$. ∎

7.2   Proof of Lemma 2:

Proof. The proof of the Lemma is organized in three steps. We first prove that the investors play a common threshold strategy. It is proved in two steps. In the first step, we show that all investors use the same strategy. In the second step, we show that the equilibrium strategy must be a single threshold. The proof of the second step assumes that it is optimal for the

positively informed speculator to choose $d = 1$ and the negatively informed speculator to choose $d = -1$. This will be proved at the very end after step 2. In the third step we derive the common threshold strategy.

Step 1: All investors use the same strategy

Suppose that investor i chooses to withdraw if and only if $q_i \in S_i$, and investor j chooses to run if and only if $q_j \in S_j$, where S_i and S_j are subsets of the real line. Suppose that $S_i \neq S_j$. This implies that at least one of the sets of S_i or S_j must be non-empty. Without loss of generality suppose that S_i is not empty. This implies that there exists a q_0 such that $q_0 \in S_i$ but $q_0 \notin S_j$. This implies that upon observing $q_i = q_0$, investor i stays but investor j withdraws, i.e. $\Delta(q_i = q_0) > 0 \geq \Delta(q_j = q_0)$, which is a contradiction as $\Delta(q_i = q_0) = \Delta(q_j = q_0)$ as $\Delta(q)$ only depends on q for any fixed speculators' strategies. Therefore all investors use the same strategy.

Step 2: The equilibrium strategy must a threshold strategy

Upon observing q_i, investor i's expected payoff of staying relative to withdrawing is

$$
\begin{aligned}
&\Delta(q_i) \\
=\ & \Pr(\theta = H|q_i)(H - \delta H E[l|q_i]) + \Pr(\theta = L|q_i)(L - \delta H E[l|q_i]) \\
=\ & L + \Pr(\theta = H|q_i)(H - L) - \delta H E[l|q_i] \\
=\ & L + \beta(q_i)(H - L) - \delta H E[l|q_i]
\end{aligned}
$$

Assume now that positively informed speculator will choose $d = +1$ and negatively informed speculator will choose $d = -1$. Denote the uninformed speculator's strategy by choosing $d = 1$ with probability ρ_1, choosing $d = -1$ with probability ρ_{-1} and choosing $d = 0$ with probability $1 - \rho_1 - \rho_{-1}$. Note that it is impossible for the uninformed speculator to mix between $d = -1$ and $d = 1$ as he needs to be indifferent between choosing $d = -1$ and $d = 1$, which is only possible if $E[P|\emptyset] = E[V|\emptyset]$. This implies that the uninformed speculator is indifferent between any trading strategies. In this case the uninformed speculator will not trade, according to the tiebreaker. Therefore we only need to consider either $\rho_1 = 0$ or $\rho_{-1} = 0$. We prove the case when $\rho_1 = 0$ as the proof when $\rho_{-1} = 0$ is essentially the same.

Denote the density of q conditional on θ as g. Then following Bayes' Rule,

$$
\begin{aligned}
&\beta(q_i) \\
&= \frac{g(q_i|\theta = H, \rho_{-1}, \alpha)}{g(q_i|\theta = H, \rho_{-1}, \alpha) + g(q_i|\theta = L, \rho_{-1}, \alpha)} \\
&= \frac{\frac{g(q_i|\theta=H,\rho_{-1},\alpha)}{g(q_i|\theta=L,\rho_{-1},\alpha)}}{\frac{g(q_i|\theta=H,\rho_{-1},\alpha)}{g(q_i|\theta=L,\rho_{-1},\alpha)} + 1}
\end{aligned}
\tag{5}
$$

Note that

$$
\begin{aligned}
&\frac{g(q_i|\theta = H)}{g(q_i|\theta = L)} \\
&= \frac{\alpha\phi(\frac{q_i-1}{\sqrt{\sigma_n^2+\sigma_\varepsilon^2}}) + (1-\alpha)[(1-\rho_{-1})\phi(\frac{q_i}{\sqrt{\sigma_n^2+\sigma_\varepsilon^2}}) + \rho_{-1}\phi(\frac{q_i+1}{\sqrt{\sigma_n^2+\sigma_\varepsilon^2}})]}{\alpha\phi(\frac{q_i+1}{\sqrt{\sigma_n^2+\sigma_\varepsilon^2}}) + (1-\alpha)[(1-\rho_{-1})\phi(\frac{q_i}{\sqrt{\sigma_n^2+\sigma_\varepsilon^2}}) + \rho_{-1}\phi(\frac{q_i+1}{\sqrt{\sigma_n^2+\sigma_\varepsilon^2}})]} \\
&= \frac{\alpha\frac{\phi(\frac{q_i-1}{\sqrt{\sigma_n^2+\sigma_\varepsilon^2}})}{\phi(\frac{q_i+1}{\sqrt{\sigma_n^2+\sigma_\varepsilon^2}})} + (1-\alpha)[(1-\rho_{-1})\frac{\phi(\frac{q_i}{\sqrt{\sigma_n^2+\sigma_\varepsilon^2}})}{\phi(\frac{q_i+1}{\sqrt{\sigma_n^2+\sigma_\varepsilon^2}})} + \rho_{-1}]}{\alpha + (1-\alpha)[(1-\rho_{-1})\frac{\phi(\frac{q_i}{\sqrt{\sigma_n^2+\sigma_\varepsilon^2}})}{\phi(\frac{q_i+1}{\sqrt{\sigma_n^2+\sigma_\varepsilon^2}})} + \rho_{-1}]} \\
&= \frac{\alpha e^{\frac{2q_i}{(\sigma_n^2+\sigma_\varepsilon^2)}} + (1-\alpha)[(1-\rho_{-1})e^{\frac{2q_i+1}{2(\sigma_n^2+\sigma_\varepsilon^2)}} + \rho_{-1}]}{\alpha + (1-\alpha)[(1-\rho_{-1})e^{\frac{2q_i+1}{2(\sigma_n^2+\sigma_\varepsilon^2)}} + \rho_{-1}]}
\end{aligned}
$$

Thus, $\lim_{q_i \to -\infty} \beta(q_i) = \frac{(1-\alpha)\rho_{-1}}{\alpha+2(1-\alpha)\rho_{-1}} \leq \frac{1-\alpha}{2-\alpha}$ and $\lim_{q_i \to +\infty} \beta(q_i) = 1$. Therefore by continuity, upper dominance region exists and there exists finite $\bar{q}$ such that $\Delta(q_i) > 0$ if $q > \bar{q}$. If $\frac{(1-\alpha)\rho_{-1}}{\alpha+2(1-\alpha)\rho_{-1}}H + \frac{\alpha+(1-\alpha)\rho_{-1}}{\alpha+2(1-\alpha)\rho_{-1}}L < 0$, then the lower dominance region exists and by continuity, there exists finite $\underline{q}$ such that $\Delta(q_i) < 0$ if $q < \underline{q}$. If $\frac{(1-\alpha)\rho_{-1}}{\alpha+2(1-\alpha)\rho_{-1}}H + \frac{\alpha+(1-\alpha)\rho_{-1}}{\alpha+2(1-\alpha)\rho_{-1}}L \geq 0$, then the lower dominance region does not exist.

Similarly, when $\rho_{-1} = 0$, we can show that the lower dominance region always exists and thus $\underline{q}$ exists but the upper dominance region may not exist.

As a summary, in equilibrium we either have 1) $\Delta(q_i) < 0$ if $q < \underline{q}$ and $\Delta(q_i) > 0$ if $q > \bar{q}$ or 2) $\Delta(q_i) < 0$ if $q < \underline{q}$ or 3) $\Delta(q_i) > 0$ if $q > \bar{q}$ for some finite $\underline{q}$ and $\bar{q}$. We now show that under either of the three scenarios, common threshold strategy is the equilibrium.

For case 1) and 2), denote $q_B = \sup\{q_i : \Delta(q_i) < 0\}$, i.e. the highest signal below which a

investor prefers to withdraw. Note that it is possible for $q_B = +\infty$, which then implies that $\Delta(q_B) \leq 0$. If all investors use threshold strategy then investor i will withdraw when $q_i < q_B$, for any i. Suppose in equilibrium they do not use threshold strategy, then for investor i, there exists signals smaller than q_B such that an investor observing q_i will stay. Denote q_A to be the largest of them, i.e. $q_A = \sup\{q_i < q_B : \Delta(q_i) \geq 0\}$. $\underline{q} < q_A < q_B$ and thus is finite. This implies that investors in the range $[q_A, q_B]$ and $(-\infty, \underline{q})$ will withdraw for sure, while investors in the range of $[\underline{q}, q_A)$ may choose to stay or withdraw. Denote the strategies of the investors in the range of $[\underline{q}, q_A)$ by $n(q_i) \in [0, 1]$. Since investors are indifferent upon observing q_A, we have

$$\Delta(q_A) = 0 \geq \Delta(q_B)$$

On the other hand, note that

$$
\begin{aligned}
&E[l|q_A] \\
&= \Phi(\frac{\underline{q} - q_A}{\sqrt{2}\sigma_\varepsilon}) + \int_{\underline{q}}^{q_A} n(q_j)\frac{1}{\sqrt{2}\sigma_\varepsilon}\phi(\frac{q_j - q_A}{\sqrt{2}\sigma_\varepsilon})dq_j + \Phi(\frac{q_B - q_A}{\sqrt{2}\sigma_\varepsilon}) - \frac{1}{2}
\end{aligned}
$$

and

$$
\begin{aligned}
&E[l|q_B] \\
&= \Phi(\frac{\underline{q} - q_B}{\sqrt{2}\sigma_\varepsilon}) + \int_{\underline{q}}^{q_A} n(q_j)\frac{1}{\sqrt{2}\sigma_\varepsilon}\phi(\frac{q_j - q_B}{\sqrt{2}\sigma_\varepsilon})dq_j + \frac{1}{2} - \Phi(\frac{q_A - q_B}{\sqrt{2}\sigma_\varepsilon}) \\
&= \Phi(\frac{\underline{q} - q_B}{\sqrt{2}\sigma_\varepsilon}) + \int_{\underline{q}}^{q_A} n(q_j)\frac{1}{\sqrt{2}\sigma_\varepsilon}\phi(\frac{q_j - q_B}{\sqrt{2}\sigma_\varepsilon})dq_j + \Phi(\frac{q_B - q_A}{\sqrt{2}\sigma_\varepsilon}) - \frac{1}{2}
\end{aligned}
$$

where we used $\Phi(-x) = 1 - \Phi(x)$ to arrive at the last equality.

Thus

$$
\begin{aligned}
&E[l|q_B] - E[l|q_A] \\
&= \Phi(\frac{\underline{q} - q_B}{\sqrt{2}\sigma_\varepsilon}) - \Phi(\frac{\underline{q} - q_A}{\sqrt{2}\sigma_\varepsilon}) + \int_{\underline{q}}^{q_A} n(q_j)\frac{1}{\sqrt{2}\sigma_\varepsilon}[\phi(\frac{q_j - q_B}{\sqrt{2}\sigma_\varepsilon}) - \phi(\frac{q_j - q_A}{\sqrt{2}\sigma_\varepsilon})]dq_j
\end{aligned}
$$

Since $q_B > q_A$, $\phi(\frac{q_j - q_B}{\sqrt{2}\sigma_\varepsilon}) - \phi(\frac{q_j - q_A}{\sqrt{2}\sigma_\varepsilon}) < 0$ for any $q_j \in [\underline{q}, q_A]$. Therefore

$$E[l|q_B] - E[l|q_A]$$
$$\leq \quad \Phi(\frac{\underline{q} - q_B}{\sqrt{2}\sigma_\varepsilon}) - \Phi(\frac{\underline{q} - q_A}{\sqrt{2}\sigma_\varepsilon}) < 0$$

In addition, $\beta(q_i)$ is increasing in q_i, which results in $\beta(q_B) > \beta(q_A)$. Correspondingly, $\Delta(q_B) > \Delta(q_A)$ and thus, the contradiction. Therefore all investors use the same threshold strategy in equilibrium.

For case 1) and 3), denote $q_C = \inf\{q_i : \Delta(q_i) > 0\}$, i.e. the lowest signal above which a investor will stay. Note that it is possible for $q_C = -\infty$, which then implies that $\Delta(q_C) \geq 0$. If all investors use threshold strategy then investor i will stay when $q_i > q_C$, for any i. Suppose in equilibrium they do not use threshold strategy, then for investor i, there exists signals larger than q_C such that a investor observing q_i will withdraw. Denote q_D to be the smallest of them, i.e. $q_D = \sup\{q_i > q_C : \Delta(q_i) \leq 0\}$. $q_C < q_D < \bar{q}$ and thus is finite. This implies that investors in the range $[q_C, q_D]$ and $(\bar{q}, +\infty)$ will stay for sure, investors in the range of $(-\infty, q_C)$ will withdraw for sure, while investors in the range of $[q_D, \bar{q})$ may choose to stay or withdraw and denote their strategies by $n(q_i) \in [0, 1]$. Since investors are indifferent upon observing q_D, we have

$$\Delta(q_D) = 0 \leq \Delta(q_C)$$

Using similar techniques as above we can show that this inequality cannot hold. Therefore all investors use the same threshold strategy in equilibrium.

To complete step 2, we now prove that the positively informed speculator will buy and the negatively informed speculator will sell. We will prove the optimal strategy for the positively informed speculator as the proof for the optimal strategy of the negatively informed speculators essentially the same.

Consider the strategy of the positively informed speculator. Since the speculator knows that $\theta = H$, he knows that the equity value is $(1-l)(H-\delta Hl)$. On the other hand, conditional

on total order flow q, stock price

$$
\begin{aligned}
P(q) &= \Pr(\theta = H|q)[1 - l(q)][H - \delta Hl(q)] + \Pr(\theta = L|q)[1 - l(q)][L - \delta Hl(q)] \\
&= [1 - l(q)][\Pr(\theta = H|q)(H - \delta Hl(q)) + \Pr(\theta = L|q)(L - \delta Hl(q))]
\end{aligned}
$$

Therefore, when the speculator takes action d, the speculator's profit will be

$$
\begin{aligned}
\Pi(d) &= d(E[V|d] - E[P|d]) \\
&= d(E[(1 - l)(H - \delta Hl)|d] - E[(1 - l)\{\Pr(\theta = H|q)(H - \delta Hl) + \Pr(\theta = L|q)(L - \delta Hl)\}|d]) \\
&= dE[(1 - l)\Pr(\theta = L|q)(H - L)|d]
\end{aligned}
$$

Since $E[(1 - l)\Pr(\theta = L|q)(1 - L)|d] \geq 0$, $\Pi(1) \geq \Pi(0) \geq \Pi(-1)$. The tiebreaker assumption then implies that $d = 1$ for the positively informed speculator.

Step 3: Derive the common threshold

Given that each investor uses a common threshold strategy, we now solve for the common threshold, denoted as q^*. Threshold strategy implies that an investor will be indifferent when observing q^*, i.e. q^* satisfies the indifferent condition.

$$
\begin{aligned}
&\Delta(q^*) \\
&= \beta(q^*)(H - \delta HE[l|q^*]) + [1 - \beta(q^*)](L - \delta HE[l|q^*]) \\
&= \beta(q^*)(H - L) - \delta HE[l|q^*] + L \\
&= 0
\end{aligned}
$$

When $q = q^*$,

$$
\begin{aligned}
&E[l|q^*] \\
&= \Pr(q_j \leq q^*|q_i = q^*) \\
&= \frac{1}{2}
\end{aligned}
$$

, resulting in

$$\beta(q^*) = \frac{\frac{\delta}{2} + \gamma}{1 + \gamma} \tag{6}$$

As will be shown in Lemma 3, $\beta(q^*)$ is strictly increasing in q^*. Therefore, equation (6) has at most one solution. The expression of $\beta(q^*)$, however, depends on q^* and therefore the uninformed speculators' strategies (as the informed speculator always buy when observing $\theta = H$ and sell when observing $\theta = L$). ■

7.3 Proof of Lemma 3:

Proof. We prove that $d(\emptyset) = 1$ is dominated when SS is allowed as the proof for when SS is banned is essentially the same.

As shown in the proof of Lemma 2, the uninformed speculator will not mix between $d = 1$ and $d = -1$. Therefore the market maker will conjecture that either the uninformed speculator mixes between $d = -1$ and $d = 0$ or $d = 1$ and $d = 0$. We prove the case for that the conjecture being that the uninformed speculator mixes between $d = -1$ and $d = 0$ as the proof for the other case is essentially the same. Suppose the market maker conjectures that the uninformed speculator chooses $d = -1$ with probability $\widehat{\rho}_{-1}$ and chooses $d = 0$ with probability $1 - \widehat{\rho}_{-1}$, we can calculate from equation (5) that

$$
\begin{aligned}
&\beta(q, \widehat{\rho}_{-1}, \alpha) \\
=~& \Pr(\theta = H | q, \widehat{\rho}_{-1}, \alpha) \\
=~& \frac{g(q|\theta = H, \widehat{\rho}_{-1}, \alpha)}{g(q|\theta = H, \widehat{\rho}_{-1}, \alpha) + g(q|\theta = L, \widehat{\rho}_{-1}, \alpha)} \\
=~& \frac{\frac{g(q|\theta=H,\widehat{\rho}_{-1},\alpha)}{g(q|\theta=L,\widehat{\rho}_{-1},\alpha)}}{\frac{g(q|\theta=H,\widehat{\rho}_{-1},\alpha)}{g(q|\theta=L,\widehat{\rho}_{-1},\alpha)} + 1} \\
=~& \frac{\frac{\alpha\phi(\frac{q-1}{\sqrt{\sigma_n^2+\sigma_\varepsilon^2}})+(1-\alpha)[(1-\widehat{\rho}_{-1})\phi(\frac{q}{\sqrt{\sigma_n^2+\sigma_\varepsilon^2}})+\widehat{\rho}_{-1}\phi(\frac{q+1}{\sqrt{\sigma_n^2+\sigma_\varepsilon^2}})]}{\alpha\phi(\frac{q+1}{\sqrt{\sigma_n^2+\sigma_\varepsilon^2}})+(1-\alpha)[(1-\widehat{\rho}_{-1})\phi(\frac{q}{\sqrt{\sigma_n^2+\sigma_\varepsilon^2}})+\widehat{\rho}_{-1}\phi(\frac{q+1}{\sqrt{\sigma_n^2+\sigma_\varepsilon^2}})]}}{\frac{\alpha\phi(\frac{q-1}{\sqrt{\sigma_n^2+\sigma_\varepsilon^2}})+(1-\alpha)[(1-\widehat{\rho}_{-1})\phi(\frac{q}{\sqrt{\sigma_n^2+\sigma_\varepsilon^2}})+\widehat{\rho}_{-1}\phi(\frac{q+1}{\sqrt{\sigma_n^2+\sigma_\varepsilon^2}})]}{\alpha\phi(\frac{q+1}{\sqrt{\sigma_n^2+\sigma_\varepsilon^2}})+(1-\alpha)[(1-\widehat{\rho}_{-1})\phi(\frac{q}{\sqrt{\sigma_n^2+\sigma_\varepsilon^2}})+\widehat{\rho}_{-1}\phi(\frac{q+1}{\sqrt{\sigma_n^2+\sigma_\varepsilon^2}})]} + 1}
\end{aligned} \tag{7}
$$

To prove that $d(\emptyset) = 1$ is dominated for any conjecture $\widehat{\rho}_{-1}$, we show that the trading

profit from choosing $d(\emptyset) = 1$ given any conjecture $\widehat{\rho}_{-1}$ is negative. More generally, for any pure strategy $d(\emptyset)$, the trading profit is

$$\Pi(d(\emptyset), \widehat{\rho}_{-1}, \alpha)$$
$$\equiv d(\emptyset)\{E[V|d(\emptyset)] - E[P(d(\emptyset), \widehat{\rho}_{-1}, \alpha)|d(\emptyset)]\}$$

Note that when $\sigma_\varepsilon \to 0$,

$$l(q)$$
$$= \Pr(\Phi(\frac{q^*(\widehat{\rho}_{-1}, \alpha) - q}{\sigma_\varepsilon}))$$
$$\to \begin{cases} 1 \text{ when } q < q^*(\widehat{\rho}_{-1}, \alpha) \\ 0 \text{ when } q > q^*(\widehat{\rho}_{-1}, \alpha) \end{cases}$$

Therefore

$$E[V|d(\emptyset)]$$
$$= \frac{1}{2}H[1 - \Phi(\frac{q^*(\widehat{\rho}_{-1}, \alpha) - d(\emptyset)}{\sigma_n})] + \frac{1}{2}L[1 - \Phi(\frac{q^*(\widehat{\rho}_{-1}, \alpha) - d(\emptyset)}{\sigma_n})]$$
$$= \frac{1}{2}(H + L)[1 - H\Phi(\frac{q^*(\widehat{\rho}_{-1}, \alpha) - d(\emptyset)}{\sigma_n})]$$

Similarly, when $\sigma_\varepsilon \to 0$, the market maker sets

$$P(q, \widehat{\rho}_{-1}) = \begin{cases} \beta(q, \widehat{\rho}_{-1}, \alpha)H + [1 - \beta(q, \widehat{\rho}_{-1}, \alpha)]L \text{ if } q > q^*(\widehat{\rho}_{-1}, \alpha) \\ 0 \text{ if } q \le q^*(\widehat{\rho}_{-1}, \alpha) \end{cases}$$

Therefore

$$\Pi(d(\emptyset), \widehat{\rho}_{-1}, \alpha)$$
$$= d(\emptyset)E[V|d(\emptyset)] - E[P(d(\emptyset), \widehat{\rho}_{-1}, \alpha)|d(\emptyset)]$$
$$= d(\emptyset)\int_{q^*(\widehat{\rho}_{-1}, \alpha)}^{+\infty} [\frac{1}{2} - \beta(q, \widehat{\rho}_{-1}, \alpha)](H - L)\frac{1}{\sigma_n}\phi(\frac{q - d(\emptyset)}{\sigma_n})dq \qquad (8)$$

Insert $d(\emptyset) = 1$ and equation (7) into equation (8) and let $\sigma_\varepsilon \to 0$ results in

$$\Pi(1, \widehat{\rho}_{-1}, \alpha)$$
$$= \frac{H-L}{2} \int_{q^*(\widehat{\rho}_{-1}, \alpha)}^{+\infty} \frac{\phi(\frac{q-1}{\sigma_n})\alpha\frac{1}{\sigma_n}[\phi(\frac{q+1}{\sigma_n}) - \phi(\frac{q-1}{\sigma_n})]}{\alpha\phi(\frac{q-1}{\sigma_n}) + \alpha\phi(\frac{q+1}{\sigma_n}) + 2(1-\alpha)[(1-\widehat{\rho}_{-1})\phi(\frac{q}{\sigma_n}) + \widehat{\rho}_{-1}\phi(\frac{q+1}{\sigma_n})]} dq$$
$$\equiv \frac{H-L}{2} \int_{q^*(\widehat{\rho}_{-1}, \alpha)}^{+\infty} h(\alpha, q, \widehat{\rho}_{-1})[\phi(\frac{q+1}{\sigma_n}) - \phi(\frac{q-1}{\sigma_n})] dq$$

where

$$h(\alpha, q, \widehat{\rho}_{-1})$$
$$\equiv \frac{\alpha\phi(\frac{q-1}{\sigma_n})}{\alpha\phi(\frac{q-1}{\sigma_n}) + \alpha\phi(\frac{q+1}{\sigma_n}) + 2(1-\alpha)[(1-\widehat{\rho}_{-1})\phi(\frac{q}{\sigma_n}) + \widehat{\rho}_{-1}\phi(\frac{q+1}{\sigma_n})]}$$
$$= \frac{\alpha\frac{\phi(\frac{q-1}{\sigma_n})}{\phi(\frac{q+1}{\sigma_n})}}{\alpha\frac{\phi(\frac{q-1}{\sigma_n})}{\phi(\frac{q+1}{\sigma_n})} + \alpha + 2(1-\alpha)[(1-\widehat{\rho}_{-1})\frac{\phi(\frac{q}{\sigma_n})}{\phi(\frac{q+1}{\sigma_n})} + \widehat{\rho}_{-1}]}$$
$$= \frac{e^{\frac{2q}{\sigma_n^2}}}{\alpha e^{\frac{2q}{\sigma_n^2}} + \alpha + 2(1-\alpha)[(1-\widehat{\rho}_{-1})e^{\frac{2q+1}{2\sigma_n^2}} + \widehat{\rho}_{-1}]}$$

Note that the sign of $\Pi(1, \widehat{\rho}_{-1}, \alpha)$ is the same as the sign of $\int_{q^*(\widehat{\rho}_{-1}, \alpha)}^{+\infty} h(\alpha, q, \widehat{\rho}_{-1})\frac{1}{\sigma_n}[\phi(\frac{q+1}{\sigma_n}) - \phi(\frac{q-1}{\sigma_n})]dq$.

Using integration by parts,

$$\int_{q^*(\widehat{\rho}_{-1}, \alpha)}^{+\infty} h(\alpha, q, \widehat{\rho}_{-1})\frac{1}{\sigma_n}[\phi(\frac{q+1}{\sigma_n}) - \phi(\frac{q-1}{\sigma_n})]dq$$
$$= h(\alpha, q, \widehat{\rho}_{-1})[\Phi(\frac{q+1}{\sigma_n}) - \Phi(\frac{q-1}{\sigma_n})]|_{q^*(\widehat{\rho}_{-1}, \alpha)}^{+\infty}$$
$$\quad - \int_{q^*(\widehat{\rho}_{-1}, \alpha)}^{+\infty} \frac{\partial h(\alpha, q, \widehat{\rho}_{-1})}{\partial q}[\Phi(\frac{q+1}{\sigma_n}) - \Phi(\frac{q-1}{\sigma_n})]dq$$
$$= -h(\alpha, q^*(\widehat{\rho}_{-1}, \alpha), \widehat{\rho}_{-1})[\Phi(\frac{q^*(\widehat{\rho}_{-1}, \alpha)+1}{\sigma_n}) - \Phi(\frac{q^*(\widehat{\rho}_{-1}, \alpha)-1}{\sigma_n})]$$
$$\quad - \int_{q^*(\widehat{\rho}_{-1}, \alpha)}^{+\infty} \frac{\partial h(\alpha, q, \widehat{\rho}_{-1})}{\partial q}[\Phi(\frac{q+1}{\sigma_n}) - \Phi(\frac{q-1}{\sigma_n})]dq$$

The first term is negative as $\Phi(\frac{q^*(\widehat{\rho}_{-1}, \alpha)+1}{\sigma_n}) - \Phi(\frac{q^*(\widehat{\rho}_{-1}, \alpha)-1}{\sigma_n}) > 0$ and $h(\alpha, q^*(\widehat{\rho}_{-1}, \alpha), \widehat{\rho}_{-1}) > 0$.

The second term is also negative as $\Phi(\frac{q+1}{\sigma_n}) - \Phi(\frac{q-1}{\sigma_n}) > 0$ and

$$
\begin{aligned}
& \frac{\partial h(\alpha, q, \widehat{\rho}_{-1})}{\partial q} \\
= \ & \frac{2e^{\frac{2q}{\sigma_n^2}}[e^{\frac{2q+1}{2\sigma_n^2}}(1-\alpha)(1-\widehat{\rho}_{-1}) + 2\widehat{\rho}_{-1}(1-\alpha) + \alpha]}{\{\alpha e^{\frac{2q}{\sigma_n^2}} + \alpha + 2(1-\alpha)[(1-\widehat{\rho}_{-1})e^{\frac{2q+1}{2\sigma_n^2}} + \widehat{\rho}_{-1}]\}^2} \\
> \ & 0
\end{aligned}
$$

Therefore $\Pi(1, \widehat{\rho}_{-1}, \alpha) < 0$ for any $\widehat{\rho}_{-1}$, making $d(\emptyset) = 1$ a dominated strategy. ∎

7.4   Proof of Proposition 1:

Proof. The optimal trading strategies of the speculators follow from Lemma 2 and Lemma 3. We now derive the investment threshold, denoted as q_B^*.

Insert $\widehat{\rho}_{-1} = 0$ into equation (7) and rearranging terms results in

$$
\begin{aligned}
& \beta(q, 0, \alpha) \\
= \ & \frac{\phi(\frac{q-1}{\sqrt{\sigma_n^2 + \sigma_\varepsilon^2}})}{\alpha\phi(\frac{q}{\sqrt{\sigma_n^2 + \sigma_\varepsilon^2}}) + (2-\alpha)\phi(\frac{q-1}{\sqrt{\sigma_n^2 + \sigma_\varepsilon^2}})}
\end{aligned}
$$

Insert the expression of $\beta(q, 0, \alpha)$ into equation (6) and rearranging terms results in

$$
\alpha \frac{\phi(\frac{q_B^*-1}{\sqrt{\sigma_n^2 + \sigma_\varepsilon^2}})}{\phi(\frac{q_B^*}{\sqrt{\sigma_n^2 + \sigma_\varepsilon^2}})} + 1 - \alpha = \frac{\frac{\delta}{2} + \gamma}{1 - \frac{\delta}{2}} \tag{9}
$$

Note that the left hand side of equation (9) is increasing in q_B^*. When $q_B^* \to +\infty$, the left hand side becomes $+\infty$, which is clearly larger than the right hand side. When $q_B^* \to -\infty$, the left hand side becomes $1-\alpha$. Thus, equation (9) has a solution if and only if $1-\alpha < \frac{\frac{\delta}{2} + \gamma}{1 - \frac{\delta}{2}}$, or $\alpha > 1 - \frac{\frac{\delta}{2} + \gamma}{1 - \frac{\delta}{2}}$. Solving equation (9) results in

$$
q_B^* = (\sigma_n^2 + \sigma_\varepsilon^2)\ln\frac{\alpha - (1 - \frac{\frac{\delta}{2} + \gamma}{1 - \frac{\delta}{2}})}{\alpha} + \frac{1}{2} \tag{10}
$$

When $\alpha \leq 1 - \frac{\frac{\delta}{2}+\gamma}{1-\frac{\delta}{2}}$, then $q_B^* = -\infty$. Since $\ln \frac{\alpha - (1-\frac{\frac{\delta}{2}+\gamma}{1-\frac{\delta}{2}})}{\alpha} < 0$, $q_B^* < \frac{1}{2}$.

Finally, q_B^* is strictly increasing in α and δ as

$$\frac{\partial q_B^*}{\partial \alpha} = (\sigma_n^2 + \sigma_\varepsilon^2)\frac{1 - \frac{\frac{\delta}{2}+\gamma}{1-\frac{\delta}{2}}}{\alpha[\alpha - (1 - \frac{\frac{\delta}{2}+\gamma}{1-\frac{\delta}{2}})]} > 0$$

and

$$\frac{\partial q_B^*}{\partial \delta} = (\sigma_n^2 + \sigma_\varepsilon^2)\frac{\alpha}{\alpha - (1 - \frac{\frac{\delta}{2}+\gamma}{1-\frac{\delta}{2}})}\frac{2(H-L)}{(\delta - 2L)^2} > 0$$

■

7.5 Proof of Proposition 2:

Proof. The proposition is proved in four steps. From Lemma 3 we know that $d(\emptyset) = 1$ is never optimal, which leaves the possibility of either $d(\emptyset) = -1$ being optimal, $d(\emptyset) = 0$ being optimal or a mixed strategy between $d(\emptyset) = -1$ and $d(\emptyset) = 0$ being optimal. In step 1, we derive the conditions where $d(\emptyset) = 0$ is the dominant strategy and where $d(\emptyset) = -1$ is the dominant strategy, respectively. In step 2, we further reduce those conditions into restrictions on exogenous parameters. Finally in step 3 we derive the investment threshold when $d(\emptyset) = -1$ is optimal which we denote as q_A^* and the investment threshold when $d(\emptyset) = 0$ is optimal which we denote as q_{NT}^*. We then derive some comparative statics of q_A^* and q_{NT}^*, which will be used in subsequent analysis.

Step 1: $d(\emptyset) = 0$ is a dominant strategy when $\Pi(-1, 1, \alpha) < 0$ and $d(\emptyset) = -1$ is a dominant strategy when $\Pi(-1, 0, \alpha) > 0$.

Recall that $d(\emptyset) = -1$ is dominant if $\Pi(-1, \widehat{\rho}_{-1}, \alpha) > 0$ for any $\widehat{\rho}_{-1} \in [0, 1]$, which translates into

$$\int_{q^*(\widehat{\rho}_{-1}, \alpha)}^{+\infty} [\beta(q, \widehat{\rho}_{-1}, \alpha) - \frac{1}{2}](H - L)\frac{1}{\sigma_n}\phi(\frac{q+1}{\sigma_n})dq > 0$$

, or, equivalently,

$$\int_{q^*(\widehat{\rho}_{-1},\alpha)}^{+\infty} [\beta(q,\widehat{\rho}_{-1},\alpha) - \frac{1}{2}]\frac{1}{\sigma_n}\phi(\frac{q+1}{\sigma_n})dq$$

$$\equiv \int_{q^*(\widehat{\rho}_{-1},\alpha)}^{+\infty} M(\alpha,q,\widehat{\rho}_{-1},-1)dq > 0$$

where

$$M(\alpha,q,\widehat{\rho}_{-1},d) \equiv [\beta(q,\widehat{\rho}_{-1},\alpha) - \frac{1}{2}]\frac{1}{\sigma_n}\phi(\frac{q-d}{\sigma_n})$$

From Lemma **??** we know that $\int_{q^*(\widehat{\rho}_{-1},\alpha)}^{+\infty} M(\alpha,q,\widehat{\rho}_{-1},-1)$ is increasing in $\widehat{\rho}_{-1}$. Therefore for $d(\emptyset) = -1$ to be a dominant strategy. Similarly for $d(\emptyset) = 0$ to be a dominant strategy, we need $\Pi(-1,\widehat{\rho}_{-1},\alpha) < 0$ for any $\widehat{\rho}_{-1} \in [0,1]$, which is true if $\Pi(-1,1,\alpha) = \int_{q^*(1,\alpha)}^{+\infty} M(\alpha,q,1,-1)dq < 0$ from Lemma **??** and Step 1 is proved.

Step 2: $\int_{q^*(1,\alpha)}^{+\infty} M(\alpha,q,1,-1)dq < 0$ **if and only if** $\alpha < \alpha_1$ **and** $\int_{q^*(0,\alpha)}^{+\infty} M(\alpha,q,0,-1)dq > 0$ **if and only if** $\alpha > \alpha_0$.

We prove the first statement as the proof for the second statement is essentially the same.

Writing out the expression of $\int_{q^*(1,\alpha)}^{+\infty} M(\alpha,q,1,-1)dq$ and rearranging terms results in

$$\int_{q^*(1,\alpha)}^{+\infty} M(\alpha,q,1,-1)dq$$

$$= \int_{q^*(1,\alpha)}^{+\infty} [\frac{\alpha\phi(\frac{q-1}{\sigma_n}) + (1-\alpha)\phi(\frac{q+1}{\sigma_n})}{\alpha\phi(\frac{q-1}{\sigma_n}) + (2-\alpha)\phi(\frac{q+1}{\sigma_n})} - \frac{1}{2}](H-L)\frac{1}{\sigma_n}\phi(\frac{q+1}{\sigma_n})dq$$

$$= (H-L)\int_{q^*(1,\alpha)}^{+\infty} \frac{\alpha(e^{\frac{2q}{\sigma_n^2}} - 1))}{2[\alpha e^{\frac{2q}{\sigma_n^2}} + (2-\alpha)]}\frac{1}{\sigma_n}\phi(\frac{q+1}{\sigma_n})dq$$

$$\equiv \frac{(H-L)}{2}\int_{q^*(1,\alpha)}^{+\infty} f(\alpha,q,\sigma_n)dq$$

From Lemma 7, $\int_{q^*(1,\alpha)}^{+\infty} M(\alpha,q,1,-1)dq < 0$ if and only if $\alpha < \alpha_1$ where $\alpha_1 \in (1 - \frac{\frac{\delta}{2}+\gamma}{1-\frac{\delta}{2}}, 1)$ is the unique solution that $\int_{q^*(1,\alpha_1)}^{+\infty} f(\alpha_1,q,\sigma_n)dq = 0$, or, equivalently,

$$\int_{q^*(1,\alpha_1)}^{+\infty} \frac{\alpha_1}{2 + \alpha_1(e^{\frac{2q}{\sigma_n^2}} - 1)}\frac{1}{\sigma_n}[\phi(\frac{q+1}{\sigma_n}) - \phi(\frac{q-1}{\sigma_n})]dq = 0 \tag{11}$$

35

We can use essentially the same proof as Lemma 7 to show that $\int_{q^*(0,\underline{\alpha})}^{+\infty} M(\alpha, q, 0, -1)dq > 0$ if and only if $\alpha > \alpha_0$ where $\alpha_0 \in (1 - \frac{\frac{\delta}{2} + \gamma}{1 - \frac{\delta}{2}}, 1)$ is the unique solution that $\int_{q^*(0,\underline{\alpha})}^{+\infty} M(\alpha, q, 0, -1)dq = 0$, or, equivalently,

$$\int_{q^*(0,\underline{\alpha})}^{+\infty} \frac{\alpha(e^{\frac{2q}{\sigma_n^2}} - 1)}{2 + \alpha(e^{\frac{2q}{\sigma_n^2}} - 1) + 2(1-\alpha)(e^{\frac{2q+1}{2\sigma_n^2}} - 1)} \frac{1}{\sigma_n}[\phi(\frac{q+1}{\sigma_n}) - \phi(\frac{q-1}{\sigma_n})]dq = 0 \qquad (12)$$

Step 2 is therefore proved.

Define

$$\underline{\alpha} = \max(\alpha_1, \alpha_0) \qquad (13)$$

Note we are not able to compare whether α_0 or α_1 is larger. However, when $\alpha \geq \alpha_1$, $\Pi(-1, 1, \alpha) \geq 0$, implying that $d(\emptyset) = -1$ is a Nash equilibrium. Similarly, when $\alpha \leq \alpha_0$, $\Pi(-1, 0, \alpha) \leq 0$, implying that $d(\emptyset) = 0$ is a Nash equilibrium. Therefore when $\alpha_1 < \alpha_0$, both $d(\emptyset) = -1$ and $d(\emptyset) = 0$ and at least one mixed strategy would be equilibria. When $\alpha_1 \geq \alpha_0$, there is no pure strategy equilibrium but at least one mixed stragegy equilibrium. Regardless of the cases, $d(\emptyset) = -1$ is the unique equilibrium if and only if $\alpha > \underline{\alpha}$.

Step 3: Derive the investment thresholds when $\rho_{-1} = 1$ and when $\rho_{-1} = 0$

From equation (7) we have

$$\beta(q, \rho_{-1}, \alpha)$$
$$= \frac{\frac{\alpha\phi(\frac{q-1}{\sqrt{\sigma_n^2 + \sigma_\varepsilon^2}}) + (1-\alpha)[(1-\rho_{-1})\phi(\frac{q}{\sqrt{\sigma_n^2 + \sigma_\varepsilon^2}}) + \rho_{-1}\phi(\frac{q+1}{\sqrt{\sigma_n^2 + \sigma_\varepsilon^2}})]}{\alpha\phi(\frac{q+1}{\sqrt{\sigma_n^2 + \sigma_\varepsilon^2}}) + (1-\alpha)[(1-\rho_{-1})\phi(\frac{q}{\sqrt{\sigma_n^2 + \sigma_\varepsilon^2}}) + \rho_{-1}\phi(\frac{q+1}{\sqrt{\sigma_n^2 + \sigma_\varepsilon^2}})]}}{\frac{\alpha\phi(\frac{q-1}{\sqrt{\sigma_n^2 + \sigma_\varepsilon^2}}) + (1-\alpha)[(1-\rho_{-1})\phi(\frac{q}{\sqrt{\sigma_n^2 + \sigma_\varepsilon^2}}) + \rho_{-1}\phi(\frac{q+1}{\sqrt{\sigma_n^2 + \sigma_\varepsilon^2}})]}{\alpha\phi(\frac{q+1}{\sqrt{\sigma_n^2 + \sigma_\varepsilon^2}}) + (1-\alpha)[(1-\rho_{-1})\phi(\frac{q}{\sqrt{\sigma_n^2 + \sigma_\varepsilon^2}}) + \rho_{-1}\phi(\frac{q+1}{\sqrt{\sigma_n^2 + \sigma_\varepsilon^2}})]} + 1}$$

When $\rho_{-1} = 1$, denote the investment threshold as q_A^*. Insert the expression of $\beta(q_A^*, \rho_{-1}, \alpha)$ into equation (6) and rearranging terms result in

$$\alpha\frac{\phi(\frac{q_A^* - 1}{\sqrt{\sigma_n^2 + \sigma_\varepsilon^2}})}{\phi(\frac{q_A^* + 1}{\sqrt{\sigma_n^2 + \sigma_\varepsilon^2}})} + 1 - \alpha = \frac{\frac{\delta}{2} + \gamma}{1 - \frac{\delta}{2}} \qquad (14)$$

First note that equation (14) has a solution $\alpha \in (0, 1)$ only if $1 - \alpha < \frac{\frac{\delta}{2} + \gamma}{1 - \frac{\delta}{2}}$, or, equivalently, $\alpha > 1 - \frac{\frac{\delta}{2} + \gamma}{1 - \frac{\delta}{2}}$. When $\alpha \leq 1 - \frac{\frac{\delta}{2} + \gamma}{1 - \frac{\delta}{2}}$, then

$$\alpha \frac{\phi(\frac{q_A^* - 1}{\sqrt{\sigma_n^2 + \sigma_\varepsilon^2}})}{\phi(\frac{q_A^* + 1}{\sqrt{\sigma_n^2 + \sigma_\varepsilon^2}})} + 1 - \alpha \geq \frac{\frac{\delta}{2} + \gamma}{1 - \frac{\delta}{2}}$$

, implying that it is always optimal for the investors to stay, or, equivalently, $q_A^* = -\infty$.

When $\alpha > 1 - \frac{\frac{\delta}{2} + \gamma}{1 - \frac{\delta}{2}}$, one can also solve for a close-form solution of q_A^* from equation (14) to be

$$
\begin{aligned}
q_A^* &= \frac{\sigma_n^2 + \sigma_\varepsilon^2}{2} \{\ln[\alpha - (1 - \frac{\frac{\delta}{2} + \gamma}{1 - \frac{\delta}{2}})] - \ln \alpha\} \\
&= \frac{q_B^* - \frac{1}{2}}{2} \quad\quad\quad\quad\quad\quad\quad\quad\quad (15)
\end{aligned}
$$

where the last equality comes from comparing q_A^* and q_B^* from equation (10). Again since $\ln[\alpha - (1 - \frac{\frac{\delta}{2} + \gamma}{1 - \frac{\delta}{2}})] - \ln \alpha < 0$, $q_A^* < 0$.

Since $q_A^* = \frac{q_B^* - \frac{1}{2}}{2}$,

$$\frac{\partial q_A^*}{\partial \alpha} = \frac{1}{2} \frac{\partial q_B^*}{\partial \alpha} > 0$$

and

$$\frac{\partial q_A^*}{\partial \delta} = \frac{1}{2} \frac{\partial q_B^*}{\partial \delta} > 0$$

where the comparative statics for q_B^* come from Proposition 1.

When $\rho_{-1} = 0$, we denote the threshold as q_{NT}^* and we can similarly calculate that

$$\frac{\alpha \frac{\phi(\frac{q_{NT}^* - 1}{\sqrt{\sigma_n^2 + \sigma_\varepsilon^2}})}{\phi(\frac{q_{NT}^*}{\sqrt{\sigma_n^2 + \sigma_\varepsilon^2}})} + 1 - \alpha}{\alpha \frac{\phi(\frac{q_{NT}^* + 1}{\sqrt{\sigma_n^2 + \sigma_\varepsilon^2}})}{\phi(\frac{q_{NT}^*}{\sqrt{\sigma_n^2 + \sigma_\varepsilon^2}})} + 1 - \alpha} = \frac{\frac{\delta}{2} + \gamma}{1 - \frac{\delta}{2}} \quad\quad (16)$$

The solution of equation (16) always exists and is unique as the left hand side of equation (16) is decreasing in q_{NT}^*, which we show below. When $q_{NT}^* \to -\infty$, the left hand side

approaches $+\infty$ which is clearly larger than the right hand side. In addition, when $q_{NT}^* \to$

$+\infty$, the right hand side approaches 0 which is clearly smaller.

Since $\frac{\frac{\delta}{2}+\gamma}{1-\frac{\delta}{2}} < 1$, $\alpha\frac{\phi(\frac{q_{NT}^*-1}{\sqrt{\sigma_n^2+\sigma_\varepsilon^2}})}{\phi(\frac{q_{NT}^*}{\sqrt{\sigma_n^2+\sigma_\varepsilon^2}})} + 1 - \alpha < \alpha\frac{\phi(\frac{q_{NT}^*+1}{\sqrt{\sigma_n^2+\sigma_\varepsilon^2}})}{\phi(\frac{q_{NT}^*}{\sqrt{\sigma_n^2+\sigma_\varepsilon^2}})} + 1 - \alpha$, resulting in $q_{NT}^* < 0$.

We now show that q_{NT}^* is increasing in α and δ. Recall from equation (16) that when

$\sigma_\varepsilon \to 0$ and after rearranging terms

$$R(q_{NT}^*, \alpha) \equiv \frac{\alpha e^{-\frac{2q_{NT}^*+1}{2\sigma_n^2}} + 1 - \alpha}{\alpha e^{\frac{2q_{NT}^*-1}{2\sigma_n^2}} + 1 - \alpha} = \frac{1-\frac{\delta}{2}}{\frac{\delta}{2}+\gamma} \tag{17}$$

Differentiate equation (17) with respect to δ results in

$$\frac{\partial R}{\partial q_{NT}^*}\frac{dq_{NT}^*}{d\delta} = \frac{\partial(\frac{1-\frac{\delta}{2}}{\frac{\delta}{2}+\gamma})}{\partial\delta} = -\frac{2(1+\gamma)}{(\delta+2\gamma)^2} < 0$$

Therefore $\frac{dq_{NT}^*}{d\delta} > 0$ as

$$\frac{\partial R}{\partial q_{NT}^*} = -\frac{\alpha e^{-\frac{q_{NT}^*}{\sigma_n^2}}[(1-\alpha)e^{\frac{1}{2\sigma_n^2}} + 2\alpha e^{\frac{q_{NT}^*}{\sigma_n^2}} + (1-\alpha)e^{\frac{4q_{NT}^*+1}{2\sigma_n^2}}]}{\sigma_n^2[(1-\alpha)e^{\frac{1}{2\sigma_n^2}} + \alpha e^{\frac{q_{NT}^*}{\sigma_n^2}}]^2} < 0$$

Differentiate equation (17) with respect to α results in

$$\frac{\partial R}{\partial\alpha} + \frac{\partial R}{\partial q_{NT}^*}\frac{dq_{NT}^*}{d\alpha} = 0$$

Therefore

$$\frac{dq_{NT}^*}{d\alpha} = -\frac{\frac{\partial R}{\partial\alpha}}{\frac{\partial R}{\partial q_{NT}^*}} > 0$$

as

$$\frac{\partial R}{\partial\alpha} = \frac{2e^{\frac{1}{2\sigma_n^2}}(e^{-\frac{q_{NT}^*}{\sigma_n^2}} - e^{\frac{q_{NT}^*}{\sigma_n^2}})}{[(1-\alpha)e^{\frac{1}{2\sigma_n^2}} + \alpha e^{\frac{q_{NT}^*}{\sigma_n^2}}]^2} > 0$$

due to $q_{NT}^* < 0$. ∎

Lemma 7

$$\int_{q^*(1,\alpha)}^{+\infty} f(\alpha, q, \sigma_n)dq \equiv \int_{q^*(1,\alpha)}^{+\infty} \frac{\alpha(e^{\frac{2q}{\sigma_n^2}} - 1))}{2[\alpha e^{\frac{2q}{\sigma_n^2}} + (2 - \alpha)]} \frac{1}{\sigma_n} \phi(\frac{q+1}{\sigma_n})dq$$

is increasing in α *and* $\int_{q^*(1,\alpha)}^{+\infty} f(\alpha, q, \sigma_n)dq > 0$ *when* $\alpha \to 1$ *and* $\int_{q^*(1,\alpha)}^{+\infty} f(\alpha, q, \sigma_n)dq < 0$
when $\alpha \le 1 - \frac{\frac{\delta}{2}+\gamma}{1-\frac{\delta}{2}}$. *Therefore there exists a unique* $\alpha_1 \in (1 - \frac{\frac{\delta}{2}+\gamma}{1-\frac{\delta}{2}}, 1)$ *such that* $\int_{q^*(1,\alpha_1)}^{+\infty} f(\alpha_1, q, \sigma_n)dq = 0$.

7.6 Proof of Lemma 7:

Proof. When $\delta < 1 - \gamma$, and $\alpha \le 1 - \frac{\frac{\delta}{2}+\gamma}{1-\frac{\delta}{2}}$, $q^*(1, \alpha) = -\infty$, as shown in step 4 of the proof of Proposition 2.

Note that we can write $\int_{-\infty}^{+\infty} f(\alpha, q, \sigma_n)dq$ as

$$\int_{-\infty}^{+\infty} f(\alpha, q, \sigma_n)dq$$
$$= \int_{-\infty}^{0} f(\alpha, q, \sigma_n)dq + \int_{0}^{+\infty} f(\alpha, q, \sigma_n)dq$$
$$= \int_{0}^{+\infty} f(\alpha, -q, \sigma_n)dq + \int_{0}^{+\infty} f(\alpha, q, \sigma_n)dq$$
$$= \int_{0}^{+\infty} [f(\alpha, q, \sigma_n) + f(\alpha, -q, \sigma_n)]dq$$

where we used change of variables to arrive at the second inequality. Note that $f(\alpha, q, \sigma_n) > 0 > f(\alpha, -q, \sigma_n)$ as $f(\alpha, q, \sigma_n) > 0$ if and only if $q > 0$. Note that

$$\frac{f(\alpha, q, \sigma_n)}{-f(\alpha, -q, \sigma_n)} = \frac{2 + \alpha(e^{-\frac{2q}{\sigma_n^2}} - 1)}{2 + \alpha(e^{\frac{2q}{\sigma_n^2}} - 1)} < 1$$

as

$$e^{-\frac{2q}{\sigma_n^2}} < e^{\frac{2q}{\sigma_n^2}}$$

Therefore $f(\alpha, q, \sigma_n) + f(\alpha, -q, \sigma_n) < 0$ and therefore $\int_{-\infty}^{+\infty} f(\alpha, q, \sigma_n)dq < 0$.

When $\alpha > 1 - \frac{\frac{\delta}{2}+\gamma}{1-\frac{\delta}{2}}$, $q^*(1, \alpha) < 0$.

39

Through changing variables, for any $N > 0$, we have

$$\int_{q^*(1,\alpha)}^0 f(\alpha, q, \sigma_n) dq$$

$$= \frac{1}{N} \int_{Nq^*(1,\alpha)}^0 f(\alpha, \frac{q}{N}, \sigma_n) dq$$

$$= \frac{1}{N} \int_0^{-Nq^*(1,\alpha)} f(\alpha, -\frac{q}{N}, \sigma_n) dq$$

Therefore

$$\int_{q^*(1,\alpha)}^{+\infty} f(\alpha, q, \sigma_n) dq$$

$$= \frac{1}{N} \int_0^{-Nq^*(1,\alpha)} f(\alpha, -\frac{q}{N}, \sigma_n) dq + \int_0^{+\infty} f(\alpha, q, \sigma_n) dq$$

$$= \int_0^{-Nq^*(1,\alpha)} [\frac{1}{N} f(\alpha, -\frac{q}{N}, \sigma_n) + f(\alpha, q, \sigma_n)] dq + \int_{-Nq^*(1,\alpha)}^{+\infty} f(\alpha, q, \sigma_n) dq$$

In addition, $q^*(1,\alpha)$ is increasing in α as

$$\frac{\partial q^*(1,\alpha)}{\partial \alpha} = \frac{\sigma_n^2 + \sigma_\varepsilon^2}{2} \frac{1 - \frac{\frac{\delta}{2} + \gamma}{1 - \frac{\delta}{2}}}{\alpha[\alpha - (1 - \frac{\frac{\delta}{2} + \gamma}{1 - \frac{\delta}{2}})]} > 0$$

Therefore, the derivative of $\int_{q^*(1,\alpha)}^{+\infty} f(\alpha, q, \sigma_n) dq$ with respect to α is

$$\frac{\partial}{\partial \alpha} \int_{q^*(1,\alpha)}^{+\infty} f(\alpha, q, \sigma_n) dq$$

$$= [\frac{1}{N} f(\alpha, q^*(1,\alpha), \sigma_n) + f(\alpha, -Nq^*(1,\alpha), \sigma_n)] N \frac{\partial(-q^*(1,\alpha))}{\partial \alpha}$$

$$+ \int_0^{-Nq^*(1,\alpha)} \frac{\partial}{\partial \alpha} [\frac{1}{N} f(\alpha, -\frac{q}{N}, \sigma_n) + f(\alpha, q, \sigma_n)] dq$$

$$- f(\alpha, -Nq^*(1,\alpha), \sigma_n) N \frac{\partial(-q^*(1,\alpha))}{\partial \alpha} + \int_{-Nq^*(1,\alpha)}^{+\infty} \frac{\partial}{\partial \alpha} f(\alpha, q, \sigma_n) dq$$

$$= f(\alpha, q^*(1,\alpha), \sigma_n) \frac{\partial(-q^*(1,\alpha))}{\partial \alpha}$$

$$+ \int_0^{-Nq^*(1,\alpha)} \frac{\partial}{\partial \alpha} [\frac{1}{N} f(\alpha, -\frac{q}{N}, \sigma_n) + f(\alpha, q, \sigma_n)] dq$$

$$+ \int_{-Nq^*(1,\alpha)}^{+\infty} \frac{\partial}{\partial \alpha} f(\alpha, q, \sigma_n) dq$$

40

Now take the limit of $\frac{\partial}{\partial\alpha}\int_{q^*(1,\alpha)}^{+\infty}f(\alpha,q,\sigma_n)dq$ when $N\to+\infty$. The first term is positive as $f(\alpha,q^*(1,\alpha),\sigma_n)<0$ and $\frac{\partial(-q^*(1,\alpha))}{\partial\alpha}=-\frac{\partial q^*(1,\alpha)}{\partial\alpha}<0$. The second term is positive as

$$
\lim_{N\to+\infty}\frac{\partial}{\partial\alpha}[\frac{1}{N}f(\alpha,-\frac{q}{N},\sigma_n)+f(\alpha,q,\sigma_n)]
$$

$$
=\quad\frac{\partial}{\partial\alpha}f(\alpha,q,\sigma_n)
$$

$$
\to\quad\frac{\sqrt{\frac{2}{\pi}}e^{-\frac{(q+1)^2}{2\sigma_n^2}}\left(e^{\frac{2q}{\sigma_n^2}}-1\right)}{\sigma_n\left(\alpha(e^{\frac{2q}{\sigma_n^2}}-1)+2\right)^2}>0\text{ when }q>0
$$

The third term converges to zero as $-Nq^*(1,\alpha)\to+\infty$. Therefore $\frac{\partial}{\partial\alpha}\int_{q^*(1,\alpha)}^{+\infty}f(\alpha,q,\sigma_n)dq>0$ when $N\to+\infty$, implying that $\frac{\partial}{\partial\alpha}\int_{q^*(1,\alpha)}^{+\infty}f(\alpha,q,\sigma_n)dq>0$ as we can choose a number N_0 sufficiently large so that we can change variables and express $\int_{q^*(1,\alpha)}^{+\infty}f(\alpha,q,\sigma_n)dq$ as

$$
\int_{q^*(1,\alpha)}^{+\infty}f(\alpha,q,\sigma_n)dq
$$

$$
=\quad\int_0^{-N_0q^*(1,\alpha)}[\frac{1}{N_0}f(\alpha,-\frac{q}{N_0},\sigma_n)+f(\alpha,q,\sigma_n)]dq+\int_{-N_0q^*(1,\alpha)}^{+\infty}f(\alpha,q,\sigma_n)dq
$$

Thus $\int_{q^*(1,\alpha)}^{+\infty}f(\alpha,q,\sigma_n)dq$ is increasing in α. When $\alpha\to1-\frac{\frac{\delta}{2}+\gamma}{1-\frac{\delta}{2}}$, $q^*(1,\alpha)\to-\infty$ and $\int_{q^*(1,\alpha)}^{+\infty}f(\alpha,q,\sigma_n)dq<0$, as we already show. When $\alpha\to1$, $q^*(1,\alpha)\to\frac{\sigma_n^2}{2}\ln\frac{\frac{\delta}{2}+\gamma}{1-\frac{\delta}{2}}$. Note that when $\delta<1-\gamma$, $\frac{\frac{\delta}{2}+\gamma}{1-\frac{\delta}{2}}<1$. In addition, $\frac{\frac{\delta}{2}+\gamma}{1-\frac{\delta}{2}}$ increases in δ, resulting in $q^*(1,\alpha)$ increasing in δ. Since when fixing α, $\int_{q^*(1,\alpha)}^{+\infty}f(\alpha,q,\sigma_n)dq$ increases in $q^*(1,\alpha)$ when $q^*(1,\alpha)<0$, $\int_{q^*(1,\alpha)}^{+\infty}f(\alpha,q,\sigma_n)dq$ increases in δ. When $\delta\to1-\gamma$, $\frac{\frac{\delta}{2}+\gamma}{1-\frac{\delta}{2}}\to1$ and $q^*(1,\alpha)\to0$ and $\int_{q^*(1,\alpha)}^{+\infty}f(\alpha,q,\sigma_n)dq>0$. When $\delta\to0$, $\frac{\frac{\delta}{2}+\gamma}{1-\frac{\delta}{2}}\to\gamma$, implying that $q^*(1,\alpha)$ is increasing in γ. When $\gamma\to0$, then $q^*(1,\alpha)\to-\infty$ and $\int_{q^*(1,\alpha)}^{+\infty}f(\alpha,q,\sigma_n)dq<0$. When $\gamma\to1$ then $q^*(1,\alpha)\to0$ and $\int_{q^*(1,\alpha)}^{+\infty}f(\alpha,q,\sigma_n)dq>0$. Therefore there exists a unique $\alpha_1\in(1-\frac{\frac{\delta}{2}+\gamma}{1-\frac{\delta}{2}},1)$ which is also a function of γ and δ such that short-selling is optimal form the uninformed speculator if $\alpha>\alpha_1$ and $\gamma>\gamma_1$ where γ_1 is the unique solution to

$$
\int_{\frac{\sigma_n^2}{2}\ln\gamma_1}^{+\infty}f(1,q,\sigma_n)dq=0 \tag{18}
$$

41

and α_1 is the unique solution to

$$\int_{\frac{\sigma_n^2}{2}\{\ln[\alpha_1-(1-\frac{\frac{\delta}{2}+\gamma}{1-\frac{\delta}{2}}))-\ln\alpha_1]\}}^{+\infty} f(\alpha_1,q,\sigma_n)dq = 0$$

Since $\gamma_1 \leq \gamma$, the Lemma is proved. ■

7.7 Proof of Lemma 4:

Proof. Recall from equation (15) that

$$q_B^* = 2q_A^* + \frac{1}{2}$$

Therefore $q_A^* < q_B^*$ if and only if

$$q_A^* > -\frac{1}{2}$$

Recall that when $\sigma_\varepsilon \to 0$,

$$q_A^* = \frac{\sigma_n^2}{2} \ln[1 - \frac{\frac{1-\gamma-\delta}{1-\frac{\delta}{2}}}{\alpha}]$$

is increasing in α. Thus, $-\frac{1}{2} < q_A^* < 0$ if and only if

$$\alpha > \alpha_2 \equiv \frac{1 - \gamma - \delta}{(1 - \frac{\delta}{2})(1 - e^{-\frac{1}{\sigma_n^2}})} > 1 - \frac{\frac{\delta}{2} + \gamma}{1 - \frac{\delta}{2}} \tag{19}$$

Note that equation (19) will only be satisfied if

$$\frac{1 - \gamma - \delta}{(1 - \frac{\delta}{2})(1 - e^{-\frac{1}{\sigma_n^2}})} < 1 \tag{20}$$

. Note that $\frac{1-\gamma-\delta}{(1-\frac{\delta}{2})(1-e^{-\frac{1}{\sigma_n^2}})}$ is decreasing in δ so equation (20) will be satisfied if

$$1 - \gamma < 1 - e^{-\frac{1}{\sigma_n^2}}$$

, or, equivalently,

$$\gamma > e^{-\frac{1}{\sigma_n^2}}$$

42

Now define

$$\overline{\gamma} = \max(\gamma_1, e^{-\frac{1}{\sigma_n^2}}) \tag{21}$$

Thus, when $\gamma > \overline{\gamma}$ and $\alpha > \alpha_2$, $q_A^* > -\frac{1}{2}$.

Note that In addition, since $q_A^* > -\frac{1}{2}$, we also have $q_B^* < q_A^* + 1$, resulting in $q_A^* < q_B^* < q_A^* + 1$ when $\gamma > \gamma_2$.

We then have that when $\gamma > \gamma_1$ and $\alpha > \alpha_2$, $q_A^* < q_B^* < q_A^* + 1$ and Proposition 2 holds.

Finally, note that $q_B^* = 2q_A^* + \frac{1}{2}$ and $-\frac{1}{2} < q_A^* < 0$ implies that $-\frac{1}{2} < q_B^* < \frac{1}{2}$. Note that this also implies that

$$\phi(\frac{q_B^*}{\sigma_n}) > \phi(\frac{q_A^* + 1}{\sigma_n})$$

as

$$|q_B^*| < \frac{1}{2} < q_A^* + 1$$

, which will be used in the proof of Proposition 3.

We now show that $q_{NT}^* < q_B^* < q_{NT}^* + 1$.

First we show that $-\frac{1}{2} < q_{NT}^* < 0$ when $-\frac{1}{2} < q_A^* < 0$.

We already know from Proposition 2 that $q_{NT}^* < 0$. Suppose that $q_{NT}^* \leq -\frac{1}{2}$. Since $q_{NT}^* = q^*(0, \alpha)$ and $q_A^* = q^*(1, \alpha)$ and we know from the proof of Lemma ?? that $q^*(\rho, \alpha)$ is non-increasing in ρ when $q^*(\rho, \alpha) \leq -\frac{1}{2}$. This implies that $q_A^* \leq q_{NT}^* \leq -\frac{1}{2}$, which is a contradiction. Therefore $-\frac{1}{2} < q_{NT}^* < 0$. This results in $q_{NT}^* + 1 > q_B^*$ as $q_B^* < \frac{1}{2}$ from Proposition 1.

Next, Combining equation (10) and equation (16), rearranging terms and take the limit of $\sigma_\varepsilon \to 0$ results in

$$\alpha \frac{\phi(\frac{q_B^* - 1}{\sigma_n})}{\phi(\frac{q_B^*}{\sigma_n})} + 1 - \alpha = \frac{\alpha \frac{\phi(\frac{q_{NT}^* - 1}{\sigma_n})}{\phi(\frac{q_{NT}^*}{\sigma_n})} + 1 - \alpha}{\alpha \frac{\phi(\frac{q_{NT}^* + 1}{\sigma_n})}{\phi(\frac{q_{NT}^*}{\sigma_n})} + 1 - \alpha}$$

When $-\frac{1}{2} < q_{NT}^* < 0$, $\alpha \frac{\phi(\frac{q_{NT}^* + 1}{\sigma_n})}{\phi(\frac{q_{NT}^*}{\sigma_n})} + 1 - \alpha < \alpha + 1 - \alpha = 1$, resulting in $\frac{\phi(\frac{q_B^* - 1}{\sigma_n})}{\phi(\frac{q_B^*}{\sigma_n})} > \frac{\phi(\frac{q_{NT}^* - 1}{\sigma_n})}{\phi(\frac{q_{NT}^*}{\sigma_n})}$ and thus $q_B^* > q_{NT}^*$. Therefore $q_{NT}^* < q_B^* < q_{NT}^* + 1$. ∎

7.8   Proof of Lemma 5:

Proof. Note that when $\alpha > \underline{\alpha} > \alpha_2$,

$$\Delta \varepsilon_I^H$$
$$= \varepsilon_{IB}^H - \varepsilon_{IA}^H$$
$$= \Phi(\frac{q_B^* - 1}{\sigma_n}) - \Phi(\frac{q_A^* - 1}{\sigma_n}) > 0$$

and

$$\Delta \varepsilon_I^L$$
$$= \varepsilon_{IB}^L - \varepsilon_{IA}^L$$
$$= \Phi(\frac{q_A^* + 1}{\sigma_n}) - \Phi(\frac{q_B^*}{\sigma_n}) > 0$$

as $q_A^* < q_B^* < q_A^* + 1$.

Part 1 is thus proved.

In addition,

$$\Delta \varepsilon_U^H$$
$$= \varepsilon_{UB}^H - \varepsilon_{UA}^H$$
$$= \Phi(\frac{q_B^*}{\sigma_n}) - \Phi(\frac{q_A^* + 1}{\sigma_n}) < 0$$

as $q_B^* < q_A^* + 1$.

Finally,

$$\Delta \varepsilon_U^L$$
$$= \varepsilon_{UB}^L - \varepsilon_{UA}^L$$
$$= -[\Phi(\frac{q_B^*}{\sigma_n}) - \Phi(\frac{q_A^* + 1}{\sigma_n})]$$
$$= \Delta \varepsilon_I^L = -\Delta \varepsilon_U^H > 0$$

Thus, part 2 and 3 are proved. ∎

7.9   Proof of Proposition 3:

Proof. We first prove the proposition when $\alpha_2 \leq \underline{\alpha}$, resulting in $q_A^* > -\frac{1}{2}$ when $\alpha > \underline{\alpha}$. It is proved in four steps. In the final step we prove the proposition when $\alpha_2 > \underline{\alpha}$.

Step 1: Prove that SS is uniquely optimal for the uninformed speculator if $\alpha > \underline{\alpha}(\delta)$.

This follows from Proposition 2.

Step 2: Prove that $\frac{\partial \Delta V}{\partial \delta} < 0$ and $\frac{\partial \Delta V}{\partial \alpha} < 0$.

Recall from equation (4) that

$$\Delta V$$
$$= -H\{\alpha \Delta \varepsilon_I^H + [\alpha\gamma - (1-\alpha)(1-\gamma)]\Delta \varepsilon_0\}$$
$$= -H\{\alpha \Delta \varepsilon_I^H + [\alpha - (1-\gamma)]\Delta \varepsilon_0\}.$$

Therefore

$$\frac{\partial \Delta V}{\partial \delta}$$
$$\propto -\alpha \frac{\partial \Delta \varepsilon_I^H}{\partial \delta} + [\alpha - (1-\gamma)]\frac{\partial \Delta \varepsilon_0}{\partial \delta}$$
$$= -\frac{1}{\sigma_n}\frac{\partial q_A^*}{\partial \delta}\{\alpha[2\phi(\frac{q_B^*-1}{\sigma_n}) - \phi(\frac{q_A^*-1}{\sigma_n})] + [\alpha - (1-\gamma)][\phi(\frac{q_A^*+1}{\sigma_n}) - 2\phi(\frac{q_B^*}{\sigma_n})]\}$$

Since $\frac{\partial q_A^*}{\partial \delta} > 0$ from Proposition 2,

$$sgn(\frac{\partial \Delta V}{\partial \delta})$$
$$= sgn\{-\alpha[2\phi(\frac{q_B^*-1}{\sigma_n}) - \phi(\frac{q_A^*-1}{\sigma_n})] - [\alpha - (1-\gamma)][\phi(\frac{q_A^*+1}{\sigma_n}) - 2\phi(\frac{q_B^*}{\sigma_n})]\}$$

Note that

$$-\alpha[2\phi(\frac{q_B^* - 1}{\sigma_n}) - \phi(\frac{q_A^* - 1}{\sigma_n})] - [\alpha - (1-\gamma)][\phi(\frac{q_A^* + 1}{\sigma_n}) - 2\phi(\frac{q_B^*}{\sigma_n})]$$

$$= -2\alpha\phi(\frac{q_B^* - 1}{\sigma_n}) + 2[\alpha - (1-\gamma)]\phi(\frac{q_B^*}{\sigma_n}) + \alpha\phi(\frac{q_A^* - 1}{\sigma_n}) - [\alpha - (1-\gamma)]\phi(\frac{q_A^* + 1}{\sigma_n}) \quad (22)$$

Rearranging terms in equations (9) and (14) result in

$$\frac{\phi(\frac{q_A^* - 1}{\sigma_n})}{\phi(\frac{q_A^* + 1}{\sigma_n})} = \frac{1}{\alpha}[\frac{\frac{\delta}{2} + \gamma}{1 - \frac{\delta}{2}} - (1-\alpha)] \quad (23)$$

and

$$\frac{\phi(\frac{q_B^* - 1}{\sigma_n})}{\phi(\frac{q_B^*}{\sigma_n})} = \frac{1}{\alpha}[\frac{\frac{\delta}{2} + \gamma}{1 - \frac{\delta}{2}} - (1-\alpha)] \quad (24)$$

Inserting equations (23) and (24) into equation (22) and rearranging terms result in

$$-2\alpha\phi(\frac{q_B^* - 1}{\sigma_n}) + 2[\alpha - (1-\gamma)]\phi(\frac{q_B^*}{\sigma_n}) + \alpha\phi(\frac{q_A^* - 1}{\sigma_n}) - [\alpha - (1-\gamma)]\phi(\frac{q_A^* + 1}{\sigma_n})$$

$$= 2\phi(\frac{q_B^*}{\sigma_n})\{-\alpha\frac{1}{\alpha}[\frac{\frac{\delta}{2} + \gamma}{1 - \frac{\delta}{2}} - (1-\alpha)] + \alpha - (1-\gamma)\} + \phi(\frac{q_A^* + 1}{\sigma_n})\{\alpha\frac{1}{\alpha}[\frac{\frac{\delta}{2} + \gamma}{1 - \frac{\delta}{2}} - (1-\alpha)] - [\alpha - (1-\gamma)]\}$$

$$= 2\phi(\frac{q_B^*}{\sigma_n})(\gamma - \frac{\frac{\delta}{2} + \gamma}{1 - \frac{\delta}{2}}) - \phi(\frac{q_A^* + 1}{\sigma_n})(\gamma - \frac{\frac{\delta}{2} + \gamma}{1 - \frac{\delta}{2}})$$

$$= [2\phi(\frac{q_B^*}{\sigma_n}) - \phi(\frac{q_A^* + 1}{\sigma_n})](\gamma - \frac{\frac{\delta}{2} + \gamma}{1 - \frac{\delta}{2}}) < 0$$

as $2\phi(\frac{q_B^*}{\sigma_n}) - \phi(\frac{q_A^* + 1}{\sigma_n}) > \phi(\frac{q_B^*}{\sigma_n}) - \phi(\frac{q_A^* + 1}{\sigma_n}) > 0$ from Lemma 4 and $\gamma - \frac{\frac{\delta}{2} + \gamma}{1 - \frac{\delta}{2}} < 0$. Therefore $\frac{\partial \Delta V}{\partial \delta} < 0$.

Taking the derivative of ΔV with respect to α results in

$$\frac{\partial \Delta V}{\partial \alpha} \propto -\alpha\frac{\partial \Delta\varepsilon_I^H}{\partial \alpha} + [\alpha - (1-\gamma)]\frac{\partial \Delta\varepsilon_0}{\partial \alpha} - \Delta\varepsilon_I^H - \Delta\varepsilon_0$$

$$= -\alpha\frac{\partial \Delta\varepsilon_I^H}{\partial \alpha} + [\alpha - (1-\gamma)]\frac{\partial \Delta\varepsilon_0}{\partial \alpha} - \Delta\varepsilon_I^H - \Delta\varepsilon_0$$

$$= -\frac{1}{\sigma_n}\frac{\partial q_A^*}{\partial \alpha}\{\alpha[2\phi(\frac{q_B^* - 1}{\sigma_n}) - \phi(\frac{q_A^* - 1}{\sigma_n})] + [\alpha - (1-\gamma)][\phi(\frac{q_A^* + 1}{\sigma_n}) - 2\phi(\frac{q_B^*}{\sigma_n})]\} - \Delta\varepsilon_I^H - \Delta\varepsilon_0$$

Again, since $\frac{\partial q_A^*}{\partial \alpha} > 0$ from Proposition 2,

$$sgn(-\frac{1}{\sigma_n}\frac{\partial q_A^*}{\partial \alpha}\{\alpha[2\phi(\frac{q_B^*-1}{\sigma_n}) - \phi(\frac{q_A^*-1}{\sigma_n})] + [\alpha - (1-\gamma)][\phi(\frac{q_A^*+1}{\sigma_n}) - 2\phi(\frac{q_B^*}{\sigma_n})]\})$$

$$= sgn\{-\alpha[2\phi(\frac{q_B^*-1}{\sigma_n}) - \phi(\frac{q_A^*-1}{\sigma_n})] - [\alpha - (1-\gamma)][\phi(\frac{q_A^*+1}{\sigma_n}) - 2\phi(\frac{q_B^*}{\sigma_n})]\} \qquad (25)$$

Again inserting equations (23) and (24) into equation (25) and rearranging terms result in

$$-\alpha[2\phi(\frac{q_B^*-1}{\sigma_n}) - \phi(\frac{q_A^*-1}{\sigma_n})] - [\alpha - (1-\gamma)][\phi(\frac{q_A^*+1}{\sigma_n}) - 2\phi(\frac{q_B^*}{\sigma_n})]$$

$$= -2\alpha\phi(\frac{q_B^*-1}{\sigma_n}) + 2[\alpha - (1-\gamma)]\phi(\frac{q_B^*}{\sigma_n}) + \alpha\phi(\frac{q_A^*-1}{\sigma_n}) - [\alpha - (1-\gamma)]\phi(\frac{q_A^*+1}{\sigma_n})$$

$$= 2\phi(\frac{q_B^*}{\sigma_n})\{-\alpha\frac{1}{\alpha}[\frac{\frac{\delta}{2}+\gamma}{1-\frac{\delta}{2}} - (1-\alpha)] + \alpha - (1-\gamma)\} + \phi(\frac{q_A^*+1}{\sigma_n})\{\alpha\frac{1}{\alpha}[\frac{\frac{\delta}{2}+\gamma}{1-\frac{\delta}{2}} - (1-\alpha)] - [\alpha - (1-\gamma)]\}$$

$$= 2\phi(\frac{q_B^*}{\sigma_n})(\gamma - \frac{\frac{\delta}{2}+\gamma}{1-\frac{\delta}{2}}) - \phi(\frac{q_A^*+1}{\sigma_n})(\gamma - \frac{\frac{\delta}{2}+\gamma}{1-\frac{\delta}{2}})$$

$$= [2\phi(\frac{q_B^*}{\sigma_n}) - \phi(\frac{q_A^*+1}{\sigma_n})](\gamma - \frac{\frac{\delta}{2}+\gamma}{1-\frac{\delta}{2}})$$

$$< 0$$

where we used equation (22) to arrive at the first equality.

Therefore $\frac{\partial \Delta V}{\partial \alpha} < 0$ as the rest two terms of $\frac{\partial \Delta V}{\partial \alpha}$ are clearly negative.

Step 3: Prove that $\Delta V(\delta = 0, \alpha = \underline{\alpha}(0) + \varepsilon) < 0$, $\Delta V(\delta = 1 - \gamma, \alpha = \underline{\alpha}(1-\gamma) + \varepsilon) > 0$ and $\Delta V(\delta = 1 - \gamma, \alpha = 1) < 0$ for ε sufficiently small.

Recall from equation (4) that

$$\Delta V$$

$$= H\{-\alpha\Delta\varepsilon_I^H - [\alpha\gamma - (1-\alpha)(1-\gamma)]\Delta\varepsilon_0\}$$

$$= H\{-\alpha\Delta\varepsilon_I^H - [\alpha - (1-\gamma)]\Delta\varepsilon_0\}.$$

Recall from the proof of Proposition 2 that $\underline{\alpha}(\delta) > 1 - \frac{\frac{\delta}{2}+\gamma}{1-\frac{\delta}{2}}$. Therefore $\underline{\alpha}(0) > 1-\gamma$. This implies that both $-\alpha\Delta\varepsilon_I^H$ and $[\alpha - (1-\gamma)]\Delta\varepsilon_0$ is negative. Therefore $\Delta V(\delta = 0, \alpha) < 0$.

When $\delta \to 1 - \gamma$, $q_A^* \to 0$, $\underline{\alpha} \to 0$ and $q_B^* \to \frac{1}{2}$. In addition,

$$\Delta \varepsilon_I^H \to \Phi(-\frac{1}{2\sigma_n}) - \Phi(-\frac{1}{\sigma_n})$$

and

$$\Delta \varepsilon_0 \to \Phi(\frac{1}{\sigma_n}) - \Phi(\frac{1}{2\sigma_n})$$

Note that

$$\Phi(-\frac{1}{2\sigma_n}) - \Phi(-\frac{1}{\sigma_n})$$
$$= [1 - \Phi(\frac{1}{2\sigma_n})] - [1 - \Phi(\frac{1}{\sigma_n})]$$
$$= \Phi(\frac{1}{\sigma_n}) - \Phi(\frac{1}{2\sigma_n})$$

where we use $\Phi(-x) = 1 - \Phi(x)$ to arrive at the second equality. Thus

$$\Delta V \to -\alpha H[\Phi(-\frac{1}{2\sigma_n}) - \Phi(-\frac{1}{\sigma_n})] - [\alpha - (1-\gamma)][\Phi(\frac{1}{\sigma_n}) - \Phi(\frac{1}{2\sigma_n})]$$
$$= (1 - 2\alpha - \gamma)H[\Phi(\frac{1}{\sigma_n}) - \Phi(\frac{1}{2\sigma_n})],$$

which is positive if and only if $\alpha < \frac{1}{2}(1-\gamma)$. Therefore, $\Delta V(\delta = 1-\gamma, \alpha = \underline{\alpha}(1-\gamma)+\varepsilon) > 0$ when $\varepsilon < \frac{1}{2}(1 - \gamma)$.

Finally,

$$\Delta V(\delta = 1 - \gamma, \alpha = 1) = H[(-1 - \gamma)[\Phi(\frac{1}{\sigma_n}) - \Phi(\frac{1}{2\sigma_n})] < 0$$

Thus, step 3 is proved.

Step 4: Define $\alpha^*(\delta)$ and show that $\Delta V > 0$ if $\alpha \in (\underline{\alpha}(\delta), \alpha^*(\delta))$

Define $\alpha^{**}(\delta)$ to be the unique solution, if exists, of

$$\Delta V(\delta, \alpha^{**}(\delta)) = 0 \tag{26}$$

The uniqueness comes from 1) there is no solution when $\alpha \leq \underline{\alpha}(\delta)$ from Lemma **??** and 2) when $\alpha > \underline{\alpha}(\delta)$, ΔV is decreasing in α.

48

Thus when the unique solution $\alpha^{**}(\delta)$ exists, which must be that $\alpha^{**}(\delta) > \underline{\alpha}(\delta)$.

Now define

$$\alpha^*(\delta) = \begin{cases} \alpha^{**}(\delta) \text{ if the unique } \alpha^{**}(\delta) \text{ exists} \\ \underline{\alpha}(\delta) \text{ if } \alpha^*(\delta) \text{ does not exist} \end{cases}$$

Since ΔV is decreasing in α when $\alpha > \underline{\alpha}(\delta)$, $\Delta V > 0$, implying banning SS increases firm value as it removes MSS if $\alpha \in (\underline{\alpha}(\delta), \alpha^*(\delta))$. This set may be empty but clearly is not empty when δ is sufficiently close to $1 - \gamma$, as $\Delta V(\delta = 1 - \gamma, \alpha = \underline{\alpha}(\delta) + \varepsilon) > 0$ shown in the third step and by continuity.

Step 5: Show that $\Delta V > 0$ if $\alpha \in (\underline{\alpha}(\delta), \alpha^*(\delta))$ when $\underline{\alpha}(\delta) > \alpha_2(\delta)$.

Note that since $q_A^* > -\frac{1}{2}$ when $\alpha > \alpha_2(\delta)$. Thus, following steps 1 to 4 we can establish that $\Delta V > 0$ if $\alpha \in (\alpha_2(\delta), \alpha^*(\delta))$ where $\alpha^*(\delta)$ is the unique solution of equation (26). Recall that

$$\Delta V$$
$$= H\{-\alpha\Delta\varepsilon_I^H - [\alpha\gamma - (1-\alpha)(1-\gamma)]\Delta\varepsilon_0\}$$
$$= H\{-\alpha\Delta\varepsilon_I^H - [\alpha - (1-\gamma)]\Delta\varepsilon_0\}$$

Thus, $\alpha^*(\delta) < 1 - \gamma$, resulting in $\alpha - (1 - \gamma) < 0$ when $\alpha \in (\underline{\alpha}(\delta), \alpha_2(\delta)]$. Since $\Delta\varepsilon_0 > 0$ and $\Delta\varepsilon_I^H \leq 0$ when $\alpha \in (\underline{\alpha}(\delta), \alpha_2(\delta)]$ as $q_B^* \leq q_A^*$, we have $\Delta V > 0$ when $\alpha \in (\underline{\alpha}(\delta), \alpha_2(\delta)]$. The proof of step 5 is thus complete. ∎

7.10 Proof of Lemma 6:

Proof. Since $\alpha_2(\delta) = \frac{1-\gamma-\delta}{(1-\frac{\delta}{2})(1-e^{-\frac{1}{\sigma_n^2}})}$, it is straightforward to show that $\alpha_2(\delta)$ is decreasing in δ as the derivative is $-\frac{2}{(2-\delta)^2(1-e^{-\frac{1}{\sigma_n^2}})} < 0$.

To prove that $\underline{\alpha}(\delta)$ is decreasing in δ, it is equivalent to show that both α_2 as defined in equation (12) and α_2 as defined in equation (11) is decreasing in δ. We prove the case for α_2

as the case for α_1 is similar. Recall that α_2 is defined as

$$\int_{q_{NT}^*(\alpha_2,\delta)}^{+\infty} \frac{\alpha_2(e^{\frac{2q}{\sigma_n^2}}-1)}{2+\alpha_2(e^{\frac{2q}{\sigma_n^2}}-1)+2(1-\alpha_2)(e^{\frac{2q+1}{2\sigma_n^2}}-1)} \frac{1}{\sigma_n}[\phi(\frac{q+1}{\sigma_n})-\phi(\frac{q-1}{\sigma_n})]dq$$

$$\equiv \int_{q_{NT}^*(\alpha_2,\delta)}^{+\infty} L(\alpha_2,q)dq$$

$$= 0$$

Take derivative with respect to δ results in

$$\int_{q_{NT}^*(\alpha_2(\delta),\delta)}^{+\infty} \frac{\partial}{\partial \alpha_2}L(\alpha_2,q)dq\frac{\partial \alpha_2(\delta)}{\partial \delta}+L(\alpha_2,q_{NT}^*(\alpha_2(\delta),\delta))dq[\frac{\partial q_{NT}^*(\alpha_2,\delta)}{\partial \alpha_2}\frac{\partial \alpha_2(\delta)}{\partial \delta}+\frac{\partial q_{NT}^*(\alpha_2,\delta)}{\partial \delta}]=0$$

Rearranging terms result in

$$\frac{\partial \alpha_2(\delta)}{\partial \delta}=-\frac{L(\alpha_2,q_{NT}^*(\alpha_2(\delta),\delta))\frac{\partial q_{NT}^*(\alpha_2,\delta)}{\partial \delta}}{\int_{q_{NT}^*(\alpha_2(\delta),\delta)}^{+\infty}\frac{\partial}{\partial \alpha_2}L(\alpha_2,q,\sigma_n)dq+L(\alpha_2,q_{NT}^*(\alpha_2(\delta),\delta))\frac{\partial q_{NT}^*(\alpha_2,\delta)}{\partial \alpha_2}}$$

The numerator is positive as $\frac{\partial q_{NT}^*(\alpha_2(\delta),\delta)}{\partial \delta}>0$ from Proposition 2.

The denominator is also positive as it has been shown in the proof of Proposition 2 that $\int_{q_{NT}^*(\alpha_2(\delta),\delta)}^{+\infty}\frac{\partial}{\partial \alpha_2}L(\alpha_2,q,\sigma_n)dq>0$ and $\frac{\partial q_{NT}^*(\alpha_2,\delta)}{\partial \underline{\alpha}}>0$ from Proposition 2. ∎

References

Abreu, D., and M.K. Brunnermeier, 2003, Bubbles and crashes, *Econometrica* 71, 173–204.

Angeletos, G., and A. Pavan, 2007, Efficient use of information and social value of information, *Econometrica* 75, 1103–1142.

Bloom, Nicholas, 2009, The impact of uncertainty shocks, *econometrica* 77, 623–685.

Bloom, Nicholas, Max Floetotto, Nir Jaimovich, Itay Saporta-Eksten, and Stephen J Terry, 2018, Really uncertain business cycles, *Econometrica* 86, 1031–1065.

Bouvard, M., P. Chaigneau, and A. De Motta, 2015, Transparency in the financial system: Rollover risk and crises, *The Journal of Finance* 70, 1805–1837.

Bris, A., W.N. Goetzmann, and N. Zhu, 2007, Efficiency and the bear: Short sales and markets around the world, *The Journal of Finance* 62, 1029–1079.

Brunnermeier, M.K., and M. Oehmke, 2013, Predatory short selling, *Review of Finance* 18, 2153–2195.

Chen, Q., I. Goldstein, and W. Jiang, 2007, Price informativeness and investment sensitivity to stock price, *Review of Financial Studies* 20, 619–650.

Chen, Q., I. Goldstein, and W. Jiang, 2010, Payoff complementarities and financial fragility: Evidence from mutual fund outflows, *Journal of Financial Economics* 97, 239–262.

Diamond, D.W., and P.H. Dybvig, 1983, Bank runs, deposit insurance, and liquidity, *The Journal of Political Economy* 91, 401–419.

Diamond, D.W., and R.E. Verrecchia, 1987, Constraints on short-selling and asset price adjustment to private information, *Journal of Financial Economics* 18, 277–311.

Duffie, D., N. Garleanu, and L.H. Pedersen, 2002, Securities lending, shorting, and pricing, *Journal of Financial Economics* 66, 307–339.

Gao, P., and X. Jiang, 2018, Reporting choices in the shadow of bank runs, *Journal of Accounting and Economics* 65, 85–108.

Goldstein, I., and A. Guembel, 2008, Manipulation and the allocational role of prices, *The Review of Economic Studies* 75, 133–164.

Goldstein, I., and A. Pauzner, 2005, Demand–deposit contracts and the probability of bank runs, *Journal of Finance* 60, 1293–1327.

Gorton, G., and A. Metrick, 2012, Securitized banking and the run on repo, *Journal of Financial Economics* 104, 425–451.

Kyle, A.S., 1985, Continuous auctions and insider trading, *Econometrica* 53, 1315–1335.

Liu, X., 2014, Short-selling attacks and creditor runs, *Management Science* 61, 814–830.

Liu, X., 2016, Interbank market freezes and creditor runs, *The Review of Financial Studies* 29, 1860–1910.

Miller, E.M., 1977, Risk, uncertainty, and divergence of opinion, *The Journal of finance* 32, 1151–1168.

Morris, S., and H.S. Shin, 2000, Rethinking multiple equilibria in macroeconomic modelling, *NBER Macroeconomics Annual* 15, 139–182.

Morris, S., and H.S. Shin, 2002, Social value of public information, *The American Economic Review* 92, 1521–1534.

Scheinkman, J.A., and W. Xiong, 2003, Overconfidence and speculative bubbles, *Journal of political Economy* 111, 1183–1220.