Word of Mouth, Noise-driven Volatility, and Public Disclosure[1]

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Abstract

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This paper shows that word-of-mouth communications among investors impose an endogenous cost of amplifying supply shocks and increasing price volatility even when the communications are assumed to be unbiased and truthful. We also examine how a firm adjusts its disclosure precision in response to these private communications, and derive a necessary and sufficient condition under which the two information channels are complements or substitutes. Our analyses generate predictions about how market depth, volatility, and firms' disclosure qualities would change as technological innovations, such as social media, facilitate investors' interpersonal communications.

Keywords: Word of Mouth; Disclosure; Price volatility; Complementarity.

JEL Classifications: D82; G14; M41.

1 Introduction

Word-of-mouth communications are an important way investors learn and transmit information and have been increasingly relevant in light of technological innovations.[1] To the extent that investors learn both from firms' public disclosures and from private word of mouth, we aim to address two questions. First, how do investors' private information communications affect firms' information environment, and is the effect different from that resulting from firms' public disclosures? Second, how would a firm adjust its public disclosures in response to more active private communications among investors? In particular, will a more intensive exchange of private information among investors reduce the need for the firm's public disclosure (i.e., substitutive), or actually incentivize the firm to provide more precise public disclosure (i.e., complementary)?

Practitioners divide over the implications of word-of-mouth communications. Some claim that investors' private communications will crowd out firms' public disclosures. The thinking is, by facilitating private information discovery, word of mouth improves investors' private information and, therefore, lowers their reliance on firms' public disclosures according to Bayes' rule. Opponents point out that, instead of making investors better informed, word-of-mouth communications often introduce misleading rumors that can cause mis-pricing if investors have bounded rationality. They argue that firms should disclosure more to mitigate the damage caused by rumors disseminated via word of mouth. Our analyses cast doubt on the reasoning of both sides of the arguments. On the one hand, we show that even if word of mouth improves

[1]Shiller (2015) argues that innovations such as the telephone, e-mail, chat rooms, and social media facilitate interpersonal word-of-mouth communications and, hence, the spread of information among individuals.

investors' private information and lowers their reliance on the firm's public disclosures, that does not translate into a lower provision of public disclosures. On the other hand, while we show that word of mouth can indeed drive the price (ex post) away from its fundamental value and, hence, induce more public disclosure, our results arise *without* assuming any biased rumors or bounded rationalities. In other words, we show that a "dark side" of investors word of mouth arises endogenously even if one makes the most benevolent assumption that such private communications are unbiased, truthful, and strictly increase investors' private information.

Our model consists of an equilibrium asset market, a continuum of risk-averse investors, and a manager who operates the firm and chooses the precision of the public disclosure. The risk-averse manager (he) chooses an unobservable effort and an observable disclosure precision, and then sells his shares in a competitive market similar to Hellwig (1980) and Diamond and Verrecchia (1981). In addition to the public disclosure and an endowed private signal, each investor (she) engages in private word-of-mouth communications prior to trading. We use the technology from the information percolation literature (see Duffie and Manso, 2007; Duffie et al., 2009) to model private word-of-mouth communications among a continuum of investors. In particular, each investor meets others at a sequence of Poisson arrival time with a mean arrival rate that is common across all investors. Upon meeting, the two investors exchange information with each other. More active word-of-mouth communications are captured by a higher arrival rate in the Poisson process. Given the normally distributed signal structure, word of mouth increases the precision of investors' private information by increasing the number of signals they learn over time.

As both public disclosure and private word-of-mouth communications provide information to the investors, we first analyze the similarities and differences between the two information channels in terms of their impact on price volatility. We show that, from an ex ante perspective, price volatility comes from two sources: (1) *fundamental-driven volatility*, attributed to the uncertainty of the underlying firm value, and (2) *noise-driven volatility*, caused by the noisy supply that is independent of the firm's value.[2] Our results show that both private word of mouth and public disclosure increase fundamental-driven volatility by increasing the covariance between price and firm value. The similarity between the two information channels in increasing the fundamental-driven volatility is consistent with the well-known result that giving investors more information reduces the ex post uncertainty but increases the uncertainty ex ante (that is, before the information is revealed).[3]

Interestingly, disclosure and word of mouth can have an *opposite* effect on noise-driven volatility. In particular, we show that while public disclosure unambiguously mitigates the impact of the noisy supply on price volatility, word of mouth often amplifies such noise realizations and, therefore, drives the price further away from its fundamental. The key to understanding the result is investors' attempt to learn others' private information from the market price. As word-of-mouth communications make investors' private information more precise, equilibrium price aggregates the information dispersed among the investors more effectively. Anticipating a more informative price, each investor will optimally place a higher weight on the observed market price in forming her belief about the firm's value. Ironically, when investors jointly put more "trust" in the market price in making inferences, noises contained

[2]Denote by v (and p) the firm's value (and price). We decompose $\text{var}(p)$ into $\text{cov}(p, v)$ and $\text{cov}(p, p - v)$ and prove in Proposition 3 that $\text{cov}(p, p - v)$ is tied to the noisy supply.

[3]For example, Hirshleifer (1971) states "the anticipation of public information becoming available in advance of trading adds a significant distributive risk to the underlying technological risk."

in the market price are amplified endogenously, as investors cannot differentiate the supply shock from the underlying value. In contrast, as public disclosure becomes more precise, investors correctly attribute a more informative price to the public signal and, therefore, do not "overly" read into the price to infer others' information. Without affecting investors' reliance on price, public disclosure mitigates the impact of the supply shock by directly lowering each investor's residual uncertainty and, therefore, making the aggregate demand more responsive to investors' private information.

We further endogenize the manager's disclosure choice and examine how the optimal disclosure quality will change if private word of mouth becomes more active. We derive the necessary and sufficient condition under which the two are complements. The driving force behind the complementarity between the two information channels is their opposite effects on *noise-driven volatility.* As word-of-mouth communications amplify the noise-driven volatility, they indirectly increase the manager's marginal benefit of improving public disclosure because disclosure unambiguously lowers this type of volatility. We show that the public disclosure and private word of mouth are complements if and only if the variance of the noise supply is high. Intuitively, the cost of word-of-mouth communications in endogenously amplifying the supply shock is particularly severe when this shock is volatile to begin with. In this case, the call for a more precise public disclosure to lower the otherwise exacerbated noise-driven volatility outweighs the intrinsic substitutability between the two channels in increasing the fundamental-driven volatility.

Our analytical results allow us to conduct comparative statics to predict which type of firms is more likely to increase or decrease its public disclosure quality in response to more active word-of-mouth communications. The result shows that, all else equal, a firm is more likely to increase its public disclosure quality following more active word of mouth if (1) investors are more risk averse or (2) their private information endowment is less precise. We also show that, after endogenizing the firm's public disclosure, more active word of mouth unambiguously lowers the market depth. These empirical predictions are relevant in light of the recent discussions on the consequence of the development of social media that facilitates interpersonal communications (e.g., Bartov et al. (2015); Blankespoor et al. (2014); Jung et al. (2017)).

This paper is related to the literature on the relation between public and private information. Several papers show that releasing public information can crowd out private information acquisition by reducing the rents received by informed investors (e.g., Fischer and Stocken, 2010; Gao and Liang, 2013; Han and Yang, 2013). Amador and Weill (2010) show a different crowding-out mechanism: more precise public information obscures the aggregation of agents' private information by making individuals' actions less sensitive to their private signals. Chen et al. (2014) show that, when investors have short horizons and are asymmetrically informed, public information can increase or decrease price informativeness by indirectly affecting the degree to which price reveals investors' private information. While the quality of public disclosures is generally taken as given in prior studies (see Goldstein and Yang (2017) for a review), we examine how the firm's disclosure policy responds as investors' private information becomes more precise due to private communications.[4] Our results show that the two information channels can be complements if we take into account the information aggregation role of price.

Prior studies have shown informational complementarity in various settings.

[4] Voluntary disclosure literature (e.g., Dye (1985); Verrecchia (1983)) instead focuses on the manager's ex-post information withholding decision. Stocken (2013) provides a nice review of the subject.

Boot and Thakor (2001) show public disclosure can strengthen investors' incentives to acquire private information, if the two are assumed to be complementary in understanding the fundamentals. Arya et al. (2017) demonstrate natural synergies between accounting reports and stock prices in directing firm strategies. Diamond and Verrecchia (1991) consider a setting in which only some investors have private information, and the firm increases its public disclosure to lower information asymmetry and hence its cost of capital. Goldstein and Yang (2015) show that investors' information acquisition can be complements if their information is about different pieces of the fundamental value. Hellwig and Veldkamp (2009) show that if agents' actions are assumed to be strategic complements, then their information acquisitions are also strategic complements. We identify a new complementarity mechanism that does not require investors' incentives to coordinate, higher-order beliefs, short horizon, or a division between informed and uninformed investors.

Our paper is also related to the long standing wisdom that releasing public information, although lowers uncertainty ex post, increases ex ante uncertainty and price volatility (e.g., Dutta and Nezlobin, 2017; Hirshleifer, 1971). The asset price in these studies is typically determined by a single/representative investor's expected value of an asset using Bayes' rule. In contrast, we derive the market price from a market-clearing condition among a continuum of investors, and decompose the price volatility into fundamental-driven and noise-driven. We show that the Bayes' pricing rule in single-investor models mechanically assumes away noise-driven volatility, which is central to our paper. Moving away from single-investor settings, we show that public disclosures can instead *reduce* ex ante price volatility thanks to their impact on the (often overlooked) noise-driven volatility.

The paper proceeds as follows. Section 2 describes the model. Section 3 takes the disclosure precision as exogenous and analyze the similarity and difference between public disclosure and private word of mouth. Section 4 endogenizes the disclosure precision and derives the necessary and sufficient condition under which disclosure and word of mouth are complementary or substitutive. Section 5 discusses empirical predictions, and Section 6 concludes.

2 Setup

The model consists of a risk-averse manager who operates a firm and a continuum of risk-averse investors. At the beginning of the game, the manager chooses an unobservable effort $a \geq 0$ at a personal cost $C(a) = \frac{1}{2}a^2$. The manager's effort a increases the firm's value v in the following stochastic manner:

$$v = a + \phi, \tag{1}$$

where ϕ is normally distributed, $\phi \sim N(0, \sigma_\phi^2)$, and its precision is $\tau_\phi = 1/\sigma_\phi^2$.

Given a realization of the firm value v, the firm is traded in a competitive market, and the market-clearing price p is determined. The manager owns an exogenous amount of shares, which we normalize to one for clear notation.[5] In choosing his unobservable effort a at $t = 0$, the manager maximizes his expected CARA utility as follows:

$$U^M = \mathbb{E}\left[-\exp\left(-\rho\left(p - C(a)\right)\right)\right], \tag{2}$$

[5] A literal interpretation is that the manager/entrepreneur initially owns 100 percent of the firm and later sells the firm at $t = 2$. However, this normalization (hence the entrepreneur-IPO interpretation) is not important: we can assume that the manager owns $\alpha < 1$ fraction of the firm and verify that our results carry over qualitatively.

where ρ is the manager's constant absolute risk aversion, p is the equilibrium price at which he sells his shares, and $C(a)$ is his cost of effort.

The price p is determined in a competitive market similar to Hellwig (1980) and Diamond and Verrecchia (1981). There is a continuum of investors $i \in [0,1]$ and a risk-free asset that serves as the numeraire. To prevent fully revealing prices, we assume the supply of the firm's shares ε (i.e., the risky asset) to be random: $\varepsilon \sim N(0, \sigma_\varepsilon^2)$. Each investor is endowed with w_0 units of the risk-free asset and has the same exponential utility function:

$$U_i = -\exp(-W_i/r), \tag{3}$$

where W_i is i's ending wealth and r is the common risk tolerance.

Prior to the trading stage, the firm publicly discloses a signal x informative about the firm's value:

$$x = v + \zeta, \tag{4}$$

with $\zeta \sim N(0, \sigma_x^2)$. The precision of the public disclosure, $\tau_x = 1/\sigma_x^2$, is publicly chosen by the manager at $t = 0$. The disclosure choice τ_x, as argued in Diamond and Verrecchia (1991), can be interpreted as the choice of an accounting technique or a committed policy of making earnings guidance or other forecasts.[6]

In addition to the the firm's public disclosure, each investor $i \in [0,1]$ receives a private signal y_i about v at the beginning of the game, and

$$y_i = v + \eta_i, \tag{5}$$

[6]This assumption is standard in the literature. See, for example, Admati and Pfleiderer (2000); Fishman and Hagerty (1989); Kanodia and Lee (1998); Kurlat and Veldkamp (2015).

where $\eta_i \sim N(0, \sigma_\eta^2)$ is independent across all investors and the exogenous precision $\tau_\eta = 1/\sigma_\eta^2$ is the same across all investors.

We allow investors to communicate with others (e.g., family members or friends) via word of mouth, and we use the technology developed by Duffie and Manso (2007) and Duffie et al. (2009) to model such private communications. In particular, each investor meets other investors at a sequence of Poisson arrival time with a mean arrival rate λ that is common across all investors. When two investors meet, they exchange their initial signal and other signals that they received in previously meetings (if any). The Poisson arrival rate $\lambda \geq 0$ is exogenous and a higher λ corresponds to more active word-of-mouth communications ($\lambda = 0$ means that no one shares information with others). The technology studied in the literature assumes truthful communications and, therefore, is silent about the rumor-dissemination aspect of word-of-mouth communications. Instead, we show that word-of-mouth communications impose an endogenous cost of amplifying supply shocks and increasing price volatility (and, hence, call for better public disclosure) even if the communications are assumed to be truthful and unbiased. Figure 1 summarizes the sequence of the game.

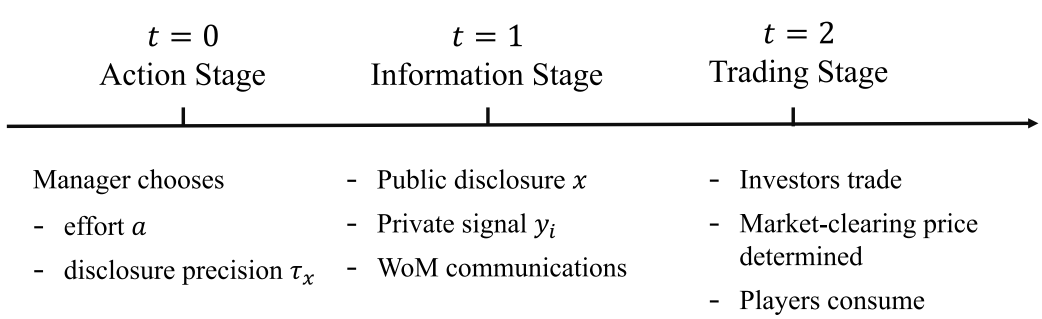

Figure 1: Time line

3 Analysis with Exogenous Disclosure Precision

In this section, we take the precision of the public disclosure as given. Our focus is to demonstrate how public disclosure and private word of mouth can have qualitatively different effects on the firm's price volatility, even though word of mouth is modeled in a way that increases investors' posterior precisions as the public disclosure does. The analysis not only adds to our understanding of the two competing information channels but also builds a foundation for the next section in which we endogenize the disclosure precision.

3.1 Equilibrium

Our model is built on Hellwig (1980) and incorporates a public disclosure, an unobservable effort, and private word-of-mouth communications. We start by describing how to keep track of and incorporate investors' heterogenous beliefs caused by word-of-mouth communications. Given the joint-normal information structure, it is sufficient for the purpose of updating investors' beliefs about v that each investor i tells the counterpart, at each meeting, her current conditional mean $\tilde{\mu}_i$ the total number of signals N_i from which $\tilde{\mu}_i$ is derived. The number N_i is initially one (as each individual is endowed with one signal), and is then incremented at each meeting by the number of signals N_j gathered by her counterpart j prior to the meeting. Denote by μ_T the cross-sectional distribution of N_i at the trading state; Andrei and Cujean (2017) derive in their Proposition 1 that the cross-sectional average of the number of signals at the time of trading T is a constant:

$$\bar{\mathbf{N}} = \sum_{n=1,2,3...} n\,\mu_T(n) = 1 + e^{(T-1)\lambda}\,(e^{\lambda} - 1). \tag{6}$$

The equilibrium is solved in three steps. We first reason from investors' perspective and solve for the linear pricing function that clears the market, while taking investors' *conjecture* $\hat{a}$ about the manager's effort as given. In particular, we guess and verify the following linear pricing function in equilibrium:

$$p(\hat{a}) = \hat{\alpha}_0 + \hat{\alpha}_v v + \hat{\alpha}_x \zeta - \hat{\alpha}_\varepsilon \varepsilon, \tag{7}$$

where the coefficients depend on the conjectured effort $\hat{a}$ but not the actual a that is unobservable by assumption. As we show in the proof, the constant term $\hat{\alpha}_0$ is the only coefficient that depends on the conjectured effort $\hat{a}$. This is not surprising because, from investors' perspective, different values of $\hat{a}$ correspond to different prior means of the firm's value.

In the second step, we reason from the manager's perspective. The manager, taking the market conjecture $\hat{a}$ and the pricing function (7) as given, chooses a to maximize his payoff (2). Given the CARA-normal setup, this is equivalent to maximizing the following certainty equivalent:

$$\max_a \mathbb{E}[p|a, \hat{a}, \tau_x] - C(a) - \frac{\rho}{2}\mathrm{var}(p|a, \hat{a}, \tau_x),$$

where $\mathbb{E}[p|a, \hat{a}]$ and $\mathrm{var}(p|a, \hat{a})$ are derived from (7). The first-order condition yields the manager's best response

$$a^*(\hat{a}, \tau_x) = \hat{\alpha}_v. \tag{8}$$

In the third step, we impose rational expectations to determine the equilibrium. That is, the conjectured effort equals the actual one in equilibrium (i.e., $a^* = \hat{a}$) and, therefore, the conjectured linear pricing function coincides with the actual market-clearing price. We summarize the equilibrium in Proposition 1 and defer the details to the Appendix.[7]

Proposition 1 (Equilibrium with Exogenous Precision) *Fixing the public precision τ_x, there exists a unique linear pricing function as follows:*

$$p = \alpha_0 + \alpha_v v + \alpha_x \zeta - \alpha_\varepsilon \varepsilon, \tag{9}$$

where $\alpha_0 = \frac{\tau_\phi}{\tau_\phi + L} a^*, \alpha_v = \frac{L}{\tau_\phi + L}, \alpha_x = \frac{\tau_x}{\tau_\phi + L}, \alpha_\varepsilon = \frac{\bar{\mathbf{N}} \tau_\eta r \tau_\varepsilon + \frac{1}{r}}{\tau_\phi + L}$, $L = \tau_x + \bar{\mathbf{N}} \tau_\eta + \left(\bar{\mathbf{N}} \tau_\eta r\right)^2 \tau_\varepsilon$, *and* $\bar{\mathbf{N}}$ *is the cross-sectional average number of signals defined in* (6). *In equilibrium, the manager's unobservable effort is*

$$a^* = \alpha_v = \operatorname{cov}(p, v)\, \tau_\phi. \tag{10}$$

Proof. All proofs are in the Appendix. ■

The pricing function (9) suggests that, all else equal, the market-clearing price p will be higher if the firm's fundamental v is higher, the asset supply ε is lower, or the common noise ζ contained in the public disclosure x is higher. In contrast, the

[7] We show in the proof that the coefficient $\hat{\alpha}_v$ in (7) is independent of the investor's conjecture $\hat{a}$ and only depends on the primitives in the model that are commonly known. Therefore, equation (8) suggests that the manager has a dominant strategy a^* in the sense that it is independent of the market's belief $\hat{a}$. Such a dominate-strategy-like response rules out potential multiple equilibria. We thank Phillip Stocken for helping us with the uniqueness argument.

idiosyncratic noises η_i contained in investors' private signals y_i do not affect the price because they are aggregated away by the law of large numbers. It is also easy to verify that the market-clearing price p is efficient in that $\mathbf{E}[p] = v$.

To understand the manager's equilibrium effort $a^* = \alpha_v$, note that while the market-clearing price (9) satisfies $\mathbf{E}[p] = v$ in equilibrium, it is formed in a process only partially responsive to its fundamental value v (and hence effort a) in the sense that $\frac{d}{dv}\mathbf{E}[p] = \alpha_v < 1$. This partial responsiveness arises because investors always attach some weight to the conjectured effort $\hat{a}$ that the manager takes as given and cannot affect (see also Holmström and Tirole, 1993; Edmans and Manso, 2011 for similar argument). The manager's moral hazard problem arises because the rate at which his effort increases the market price $\frac{d}{da}\mathbf{E}[p] = \alpha_v$ is strictly lower than the rate at which it increases the firm's value $\frac{d}{da}v = 1$. Thus, the coefficient $\alpha_v = \frac{d}{da}\mathbf{E}[p]$ measures the manager's perceived marginal benefit of exerting effort. Information quality affects the manager's effort choice by changing his perceived marginal benefit α_v.

Proposition 1 shows that, while investors are heterogeneously informed in terms of their signal precisions, the equilibrium price depends only on the average precision of their private signals $\bar{\mathbf{N}}\tau_\eta$. This has been shown in prior literature (e.g., Lambert et al., 2011) to be a standard feature of the classical noisy rational expectation models.[8] The average-precision-only feature makes it clear that our complementarity result between public and private information channels is different from earlier studies that rely on a differentiation between better and less informed investors (e.g., Diamond and Verrecchia, 1991). Our model suggests that word-of-mouth communications have a cost in amplifying noises even if they increase all investors' information equally.

[8]Because of the average-precision-only feature, single-period competitive market microstructure models like ours are unable to study questions unique to heterogenous precisions caused by word-of-mouth communications. We discuss this limitation and future research in Conclusion.

3.2 Public Disclosure versus Private Word of Mouth

In this subsection, we compare public disclosure and private word-of-mouth communications, focusing on their impact on price volatility. To analyze the effect of information on ex ante price volatility, we first decompose var(p) into two parts:

$$\text{var}(p) = \underbrace{\text{cov}(p, v)}_{\text{Fundamental-driven volatility}} + \underbrace{\text{cov}(p, p - v)}_{\text{Noise-driven volatility}} . \tag{11}$$

The *fundamental-driven* volatility $\text{cov}(p, v)$ measures how the price p covaries with firm value v, while the *noise-driven* volatility $\text{cov}(p, p - v)$ measures how price covaries systematically with the noise that causes the price to deviate from its fundamental value. As will become clear in Proposition 3, we use the term "noise-driven" because $\text{cov}(p, p - v)$ is caused by the noisy supply ε that moves the price but is independent of firm value.

Decomposing the price volatility as in equation (11) provides a novel perspective to analyze the similarities and differences between public disclosure and private word of mouth. As we summarize below, the two information channels have a similar effect on the fundamental-driven component and qualitatively different effects on the noise-driven component. This difference is a key step towards unveiling the complementarity between public and private information.

Proposition 2 (Similarity and Difference) *While both public disclosure and private word of mouth increase the fundamental-driven volatility cov*(p, v)*, they have different effects on the noise-drive volatility cov*$(p, p - v)$*. In particular,*

(i). More precise public disclosure lowers the noise-driven volatility; while

(ii). More active private word of mouth amplifies the noise-driven volatility when the variance of the noisy supply satisfies $\sigma_\varepsilon^2 \in (\underline{\sigma}_\varepsilon^2, \bar{\sigma_\varepsilon}^2)$.

We specify the exogenous boundaries $\underline{\sigma}_\varepsilon$ and $\bar{\sigma_\varepsilon}$ in the Appendix. Part (i) of Proposition 2 is not surprising. It echoes the wisdom that giving investor(s) more information prior to trading reduces uncertainty ex post but increases uncertainty ex ante (e.g., Dutta and Nezlobin (2017); Hirshleifer (1971); Veldkamp (2011)).

Part (ii) of Proposition 2 is the new insight, and suggests that different information channels affect investors' inferences from prices in different ways. As private word of mouth becomes more active (i.e., a higher λ), investors' private information becomes more precise on average, and hence they trade more intensely. As a result, investors' private information is better aggregated by market price, making the price more informative. Anticipating that the price is more informative about others' private signals, investors optimally rely more heavily on it in forming their beliefs. Ironically, when investors place more *trust* in the market price in making inferences, noises contained in the market price (i.e., the noisy supply) are also amplified endogenously, for investors cannot differentiate whether a price change is due to supply noise or the underlying value.

Why would the same argument not apply to more precise public disclosures, even though they also make the price more informative? The answer is that public disclosure is observed by *everyone*. When making inferences from a (more informative) price, investors correctly attribute the higher informativeness to public disclosure being more precise; therefore, they do *not* "overly" read into the price to infer others' private information.

The difference between public disclosure and private word of mouth is best illustrated by analyzing the Bayesian updating of an "average/representative investor" R – a theoretical construct whose private information is as precise as the cross-sectional average precision among all the investors $\bar{\mathbf{N}}\tau_\eta$. Denote by $\mathcal{F}_R = \{p, x, y_R, \hat{a}\}$ the information set of the representative investor R, where p is the market price, x is the firm's public disclosure, y_R is the private signal with a precision of $\bar{\mathbf{N}}\tau_\eta$, and $\hat{a}$ is the market's conjecture on the manager's effort. Bayes' rule implies that R's posterior mean is a precision-weighted average of the elements in her information set as follows (we derive the weights w_0^R, w_p^R, w_x^R, and w_y^R in the Appendix):[9]

$$\mathbb{E}\left(v|\mathcal{F}_R\right) = w_0^R \hat{a} + w_p^R p + w_x^R x + w_y^R y_R. \tag{12}$$

Moreover, the coefficient w_p^R satisfies

$$\frac{\partial}{\partial \lambda} w_p^R > 0, \quad \frac{\partial}{\partial \tau_x} w_p^R = 0. \tag{13}$$

Equation (13) formalizes our claim that private word of mouth induces investors to place more trust in the market price in forming their beliefs, while public disclosure has no such effect.

The notations introduced above also help us analyze noise-driven volatility $\mathrm{cov}(p, p-v)$ – the novel component of the analysis. The closed-form expression of $\mathrm{cov}(p, p-v)$ is complicated because it is a non-linear function of the pricing coefficients in (9), which, in turn, depend on the primitives of the model in a complex manner. Nonetheless, we show in Proposition 3 that the noise-driven volatility can be expressed

[9] This "average" investor R is a theoretical construct helpful in illustrating intuition. Note that our definition does *not* require R to hold the aggregate beliefs of the continuum of investors, which would require taking average across all investors' private signals and, therefore, perfectly reveals the firm's value v by the laws of large numbers.

in a surprisingly simply way that has a sharp economic interpretation.

Proposition 3 (Noise-driven volatility) *The noise-driven volatility is*

$$cov(p, p-v) = \sigma_\varepsilon^2 \times \frac{var^2(v|\mathcal{F}_R)}{r^2\left(1-w_p^R\right)}, \tag{14}$$

where σ_ε^2 is the variance of the noisy supply and w_p^R is the extent to which an investor with average precision relies on the market price in forming her belief in (12)*. The weight w_p^R increases with the intensiveness of private word of mouth but is unaffected by the quality of the public disclosure.*

We can use Proposition 3 to understand the condition under which private word of mouth amplifies noises, that is, $\sigma_\varepsilon^2 \in (\underline{\sigma}_\varepsilon^2, \bar{\sigma_\varepsilon}^2)$ in Proposition 2. Inspection of equation (14) reveals the two countervailing effects private word of mouth has on the noise-driven volatility. On the one hand, more active private communications induce investors to put more weight w_p^R on price in forming their Bayesian beliefs – this is the inference effect argued previously that tends to amplify the supply noise contained in the price. On the other hand, private communications also lower investors' residual uncertainty $\text{var}(v|\mathcal{F}_R)$, which tends to lower $\text{cov}(p, p-v)$. The thinking behind this second effect is that risk-averse investors trade more intensely when residual uncertainty is low, and more intensive trading collectively makes the price more responsive to investors' private information and, hence, less sensitive to the noisy supply ε. Proposition 2 shows that investors' inference effect (the denominator in equation (14)) is the dominant effect as long as the supply noise is *not* too extreme. The intuition can be illustrated by analyzing the limiting case of $\sigma_\varepsilon^2 \to 0$ and $\sigma_\varepsilon^2 \to \infty$: the market price p will be either completely uninformative (for $\sigma_\varepsilon \to \infty$) or perfectly informative (for $\sigma_\varepsilon \to 0$). In both cases, however, the marginal effect of word of mouth λ on investors' inference effect, measured by the magnitude of $\frac{\partial}{\partial\lambda} w_p^R$, diminishes to zero.[10] The condition $\sigma_\varepsilon^2 \in (\underline{\sigma_\varepsilon}^2, \bar{\sigma_\varepsilon}^2)$ then follows immediately from a simple continuity argument. Proposition 3 also explains why public disclosures (i.e., higher τ_x) always reduce noise-driven volatility: a higher τ_x lowers the residual uncertainty $\text{var}(v|\mathcal{F}_R)$ *without* affecting the investors' reliance w_p^R on price (recall equation (13)).

Figure 2 illustrates the similarities and difference between public disclosures and private word of mouth via a numerical example. While both information channels make the market-clearing price more informative (i.e., a higher $\text{var}^{-1}(v|p)$), only more active word of mouth (but not more precise disclosure) triggers investors to rely more on price in forming their beliefs, measured by a higher w_P^R. The increased reliance on price in turn amplifies the noise-driven volatility $\text{cov}(p, p - v)$.

3.3 Discussion of Noise-driven Volatility

Noise-driven volatility characterized in Proposition 3 is a novel part of the paper and central throughout our analyses. It is helpful to put this new concept in the context of prior literature that studies how public disclosure affects ex-ante price volatility. In those studies, price is often determined by a single/representative investor's expected value of the firm (e.g., Dutta and Nezlobin, 2017). We show in Corollary 1 that the Bayes' pricing rule used in those single-investor studies mechanically assumes away the noise-driven volatility $\text{cov}(p, p - v)$.

Corollary 1 *The noise-driven volatility* $\text{cov}(p, p - v) \equiv 0$ *if the price is determined by a single investor's expectation* $\mathbb{E}(v|\mathcal{F})$ *given her information* $\mathcal{F}$.

[10]That is, $\lim_{\sigma_\varepsilon\to\infty} \frac{\partial}{\partial\lambda} w_p^R = \lim_{\sigma_\varepsilon\to 0} \frac{\partial}{\partial\lambda} w_p^R = 0$.

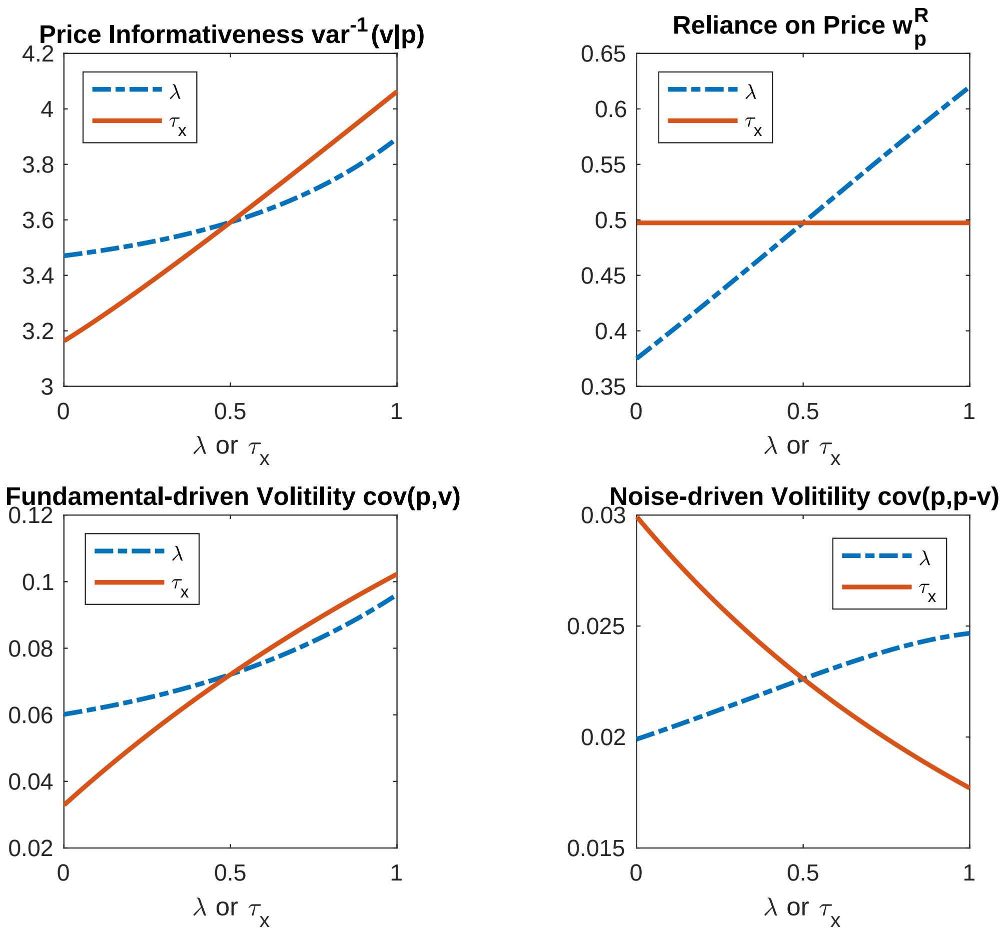

Figure 2: Similarities and differences between disclosure τ_x and word of mouth λ $(\tau_\eta = 0.1, \tau_\varepsilon = 6, \tau_\phi = 3, r = 1)$

In each figure, the solid line is plotted by fixing $\lambda = 0.5$ and varying τ_x. The dotted line is plotted by fixing $\tau_x = 0.5$ and varying λ. p is the market-clearing price, v is the realized firm value, and w_p^R is the extent to which an investor with average precision relies on the market price in forming her belief in (12).

The result can be illustrated by drawing an analogy to linear regression: mathematically, the price $p = \mathbb{E}[v|\mathcal{F}]$ in the benchmark is the projection of firm value v onto the investor's information set $\mathcal{F}$, and hence $(p - v)$ is the unexplained "regression error." The result $\text{cov}(p, p - v) = 0$ then follows, as the regression error $p - v$ is orthogonal to the information set $\mathcal{F}$, on which is the projection p.

In contrast, noise-driven volatility arises when we allow price to aggregate multiple investors' private information. Intuitively, $cov(p, p - v)$ arises because the noisy supply prevents investors from making perfect inferences about others' information from the price. For instance, investors cannot tell whether a price increase is driven by (1) other investors receiving favorable private signals and hence increasing their demand or (2) a decrease in asset supply caused by a negative supply shock. As a result, a higher price caused by a negative supply shock ε will be partially interpreted by everyone as others receiving good signals, which in turn fuels up investors' estimates of firm value.

Analyzing noise-driven volatility can also overturn some insights derived from single-investor models. For instance, a well-known result from single-investor models is that public disclosures, although lower residual uncertainty ex post, increase ex ante uncertainty and hence price volatility (e.g., Dutta and Nezlobin, 2017; Hirshleifer, 1971). Our next result shows that, if we move away from single-investor settings, public disclosures can instead *reduce* price volatility thanks to their impact on the noise-driven component that was often overlooked in single-investor models.

Corollary 2 *Unlike in single-investor models assuming $p = \mathbb{E}[v|\mathcal{F}]$, more precise public disclosures in our model can lower ex ante price volatility* $\text{var}(p)$ *because of the noise-driven volatility. In particular,* $\frac{d}{d\tau_x}\text{var}(p) < 0$ *if and only if* $\tau_x < \bar{\tau}_x$.

We characterize the exogenous threshold $\bar{\tau}_x$ in the appendix. Corollary 2 addresses a natural question of Proposition 2 in which we show that more precise disclosure has two countervailing effects on price volatility: it increases fundamental-driven volatility $\mathrm{cov}(p, v)$ but lowers noise-driven volatility $\mathrm{cov}(p, p - v)$. The corollary shows that the well-known effect of public disclosures on increasing fundamental-driven volatility can be dominated by their effect on lowering the (often overlooked) noise-driven volatility. As we will show next, noise-driven volatility is also key to deriving complementarity in our model where public and private information channels would have been perfectly substitutive had we assumed a single-investor setup.

In summary, deriving the noise-driven volatility $\mathrm{cov}(p, p - v)$ is important in our model for several reasons. First, Proposition 2 shows that public and private information have qualitative different (similar) effects on price volatility when we take into account (rule out) noise-driven volatility. Second, Corollary 2 shows that public information can reduce (unambiguously increase) price volatility when we consider (rule out) noise-driven volatility. Third, and more to our main message, we show in the next section that analyzing the inferences associated to the noise-driven volatility gives rises to an endogenous complementarity between public and private information.

4 Endogenous Disclosure and Complementarity

In the analysis thus far, we have taken the precision of public disclosure as exogenously given. We now endogenize the precision and show that public disclosure and word-of-mouth communications can be complements once we account for the information aggregation role of the market price. To solve for the optimal disclosure precision, the manager takes the subgame equilibrium shown in Proposition 1 as given and chooses an optimal τ_x^* to maximize his expected payoff (2) at $t = 0$. Given the exponential-normal setup, the problem is equivalent to maximizing his certainty equivalent:

$$\tau_x^* = \arg\max_{\tau_x} \mathbb{E}\left(p(\tau_x)\right) - C\left(a^*(\tau_x)\right) - \frac{\rho}{2}\text{var}\left(p(\tau_x)\right), \tag{15}$$

where $p(\tau_x)$ and $a^*(\tau_x)$ are characterized as in Proposition 1. The corresponding first-order condition for τ_x is

$$\frac{d\left[\mathbb{E}[p|a^*(\tau_x)] - C(a^*(\tau_x)) - \text{cov}(p, p-v)\right]}{d\tau_x}|_{\tau_x=\tau_x^*} = \frac{\rho}{2} \times \frac{d\,\text{cov}(p,v)}{d\tau_x}|_{\tau_x=\tau_x^*}. \tag{16}$$

The left-hand side of equation (16) is the marginal benefit of more precise public disclosures, both in terms of motivating a higher managerial effort (hence the expected price $\mathbb{E}[p|a^*(\tau_x)]$) and in terms of lowering the *noise-driven* price volatility $\text{cov}(p, p - v)$. The right-hand side is the marginal cost of public disclosure in increasing the *fundamental-driven* price volatility. We explained in Proposition 2 the thinking behind the marginal cost $\frac{d\,\text{cov}(p,v)}{d\tau_x} > 0$ and marginal benefit $\frac{d\,\text{cov}(p,p-v)}{d\tau_x} < 0$. To see the benefit of precise public disclosures in motivating managerial effort a^*, recall from Proposition 1 that the equilibrium price p is only partially responsive to the manager's effort a because the investor always attaches some weight to the conjectured effort $\hat{a}$. Providing the investor with more precise signals makes the price more responsive to the firm's *true* value – captured by a higher $\text{cov}(p, v)$ – rather than to the *conjectured* value. In other words, an increase in τ_x ties the manager's wealth more closely to his unobservable action and hence increases the manager's perceived marginal benefit of effort.

Solving the first-order condition (16) yields a unique optimal precision τ_x^*. We summarize the equilibrium in Proposition 4.

Proposition 4 (Equilibrium) *When the precision of public disclosure is endogenous,* *the game has a unique linear equilibrium in which the disclosure precision as follows:*

$$\tau_x^* = \max\{0,\, 2\frac{1}{r^2\tau_\varepsilon} + \frac{2}{\rho}\tau_\phi^2 - \tau_\phi + \bar{\mathbf{N}}\tau_\eta - \left(\bar{\mathbf{N}}\tau_\eta r\right)^2 \tau_\varepsilon\}, \tag{17}$$

with $\tau_x^* > 0$ *if and only if* $\sigma_\varepsilon^2 > \frac{r^2\left[\sqrt{\left(\frac{2}{\rho}\tau_\phi^2 - \tau_\phi + \bar{\mathbf{N}}\tau_\eta\right)^2 + 8\left(\bar{\mathbf{N}}\tau_\eta\right)^2} - \left(\frac{2}{\rho}\tau_\phi^2 - \tau_\phi + \bar{\mathbf{N}}\tau_\eta\right)\right]}{4}$*. Substituting the value of* τ_x^* *into Proposition 1 fully characterizes the equilibrium.*

How would the manager adjust the public disclosure precision τ_x^* as more active word-of-mouth communications improve investors' private information? We are interested in showing when and why word-of-mouth communications can motivate the manager to provide more precise public disclosure (i.e., complementarity).

In order to highlight the necessity of investors' *inferences* from prices in generating the complementarity, we analyze a *single-investor* benchmark (hence the inferences from prices are absent). We use the following result to show that more precise private information would unambiguously crowd out the provision of public disclosure if the investor does not learn from prices.

Lemma 1 (Substitutability in a single investor benchmark) *Suppose the price* $p = \mathbb{E}(v|x, y)$ *is set by a single investor given the public disclosure* x *and her private signal* y*. The unique equilibrium is such that* $a^* = \frac{\tau_x^* + \tau_\eta}{\tau_\phi + \tau_x^* + \tau_\eta}$ *and the disclosure precision* $\tau_x^* = \max\{0, \frac{(2\tau_\phi - \rho)\tau_\phi}{\rho} - \tau_\eta\}$*. Importantly, more precise private information crowds out public disclosure; that is, for any* $\tau_x^* > 0$*,*

$$\frac{d\tau_x^*}{d\tau_\eta} < 0. \tag{18}$$

The simple Bayesian pricing rule $p = \mathbb{E}(v|x, y)$ is used only to be consistent with prior studies that assume a single investor economy, and the substitutive result $\frac{d\tau_x^*}{d\tau_\eta} < 0$ is robust to alternative pricing rules.[11] To understand the substitutive result, note that public and private information (τ_x and τ_η) are perfectly substitutive in motivating managerial effort, $a^* = \text{cov}(p, v)\,\tau_\phi$. The two types of information are again perfectly substitutive in increasing price volatility $\text{var}(p)$, which, as shown earlier, degenerates into $\text{cov}(p, v)$ under the Bayesian pricing rule.[12] In the exposition of the first-order condition (16), a more precise private signal not only lowers the marginal benefit of public disclosure in providing managerial incentives but also increases the marginal cost of disclosure in further increasing the fundamental-driven price volatility.

One may expect a similar relation between disclosure and private word of mouth, for the latter also improves investors' private information in our model. However, our next result shows that the casual thinking is incorrect: public disclosures and private word-of-mouth communications can be endogenous complements once we model the information aggregation role of the market price.

Proposition 5 (Complementarity) *Public disclosures and private word-of-mouth communications are endogenous complements if and only if the variance of the noisy supply is large. That is, for any* $\tau_x^* > 0$,

$$\frac{d}{d\lambda}\tau_x^* > 0 \;\; \textit{if and only if} \;\; \sigma_\varepsilon^2 > 2\bar{\mathbf{N}}\tau_\eta r^2. \tag{19}$$

What drives the complementarity is the *opposite* effects disclosure and word of

[11] We can introduce a noisy supply ε and derive the price from the market-clearing condition by a *single* investor with CARA utility as in our main model. The market-clearing price would be $p = E[v|x, y] - \frac{\text{var}(v|x,y)}{r}\,\varepsilon$, where ε is the supply shock. In contract to Proposition 2, however, τ_x and τ_η would have the *same* effect on noise-driven volatility $\text{cov}(p, p - v)$ even though $\text{cov}(p, p - v) \neq 0$ in this case. Therefore, one can verify the substitutive result $\frac{d\tau_x^*}{d\tau_\eta} < 0$.

[12] Inspecting Corollary 1 and equation (11) together verifies the claim.

mouth have on the noise-driven volatility $\mathrm{cov}(p, p - v)$. Recall from Proposition 2 that private word of mouth can amplify noise-driven volatility, which, in turn, increases the marginal benefit of improving public disclosures in mitigating such volatility. It is the increased marginal benefit of disclosure in the intensity of word of mouth that leads to the complementary result. Recall from equation (14) that the noise-driven volatility $\mathrm{cov}(p, p - v)$ can be traced to variance of the noisy supply, σ_ε^2. Proposition 5 shows that, in equilibrium (i.e., with the optimal τ_x^*), the cost of private communications in amplifying the supply shock is particularly severe when the shock is volatile, i.e., $\sigma_\varepsilon^2 > 2\bar{\mathbf{N}}\tau_\eta r^2$. In this case, the call for a more precise public disclosure to lower the (otherwise exacerbated) noise-driven volatility outweighs the intrinsic substitutability between τ_x and λ in providing managerial incentives and in increasing the fundamental-driven volatility, $\mathrm{cov}(p, v)$.

The manager's risk aversion plays an important role in our analysis (in particular, his disutility from facing a more volatile price). In a way, our emphasis on the manager's utility is in line with (Beyer et al., 2010, p.305) who write: "[i]t is management and not the "firm" that makes disclosure decisions. As a result, the costs and benefits of disclosure that explain disclosure decisions reflect management's utility and disutility from making a disclosure." Beyer et al. (2010) reviewed that "[m]ost models assume that the managers attempt to maximize share price." In this model, we complement these studies by also considering the manager's disutility associated with price volatility.[13] While we acknowledge that some managers may even prefer a more volatile price, the risk-aversion assumption and, in our opinion, the incentives to avoid volatilities apply to at least some managers.

[13]For example, Bagnoli and Watts (2017) study a voluntary disclosure model in a risk-neutral setup. They show that negative pressures (exogenous event that results in the market reducing its expectation of the firm value) can force the firm to disclose information that the firm withheld initially.

5 Empirical Implications

Our model is relevant to the increasing use of social media in the capital markets. The New York Stock Exchange (NYSE) recently noted that "social media has become a crucial source of information for the financial services community."[14] A natural question is how firms' public disclosure quality would react in response to a competing information source. Recall that in Proposition 5 we identify the necessary and sufficient condition $\sigma_\varepsilon^2 > 2\bar{\mathbf{N}}\tau_\eta r^2$ under which active word of mouth and public disclosure are complements.[15] Our next result conducts comparative statics to the critical threshold $\Sigma \doteq 2\bar{\mathbf{N}}\tau_\eta r^2$ above which word of mouth leads to more precise public disclosures. The result is helpful in predicting cross-sectionally which types of firms are more likely to improve (or lower) their public disclosure quality when technological innovations such as social media facilitate private information sharing. The idea is that while econometricians do not observe the exact value of σ_ε^2 that the manager knows, they know that $\sigma_\varepsilon^2 > \Sigma$ is more likely to satisfy if Σ becomes smaller.

Proposition 6 (Comparative statics about disclosure) *From the econometrician's perspective, a firm is more likely to increase its public disclosure in response to an increase in private word-of-mouth communications if:*

(i) Investors are more risk averse: $\frac{d}{d\,r^{-1}}\Sigma < 0$*, or*

(ii) Investors' initial private signal endowment is noisier: $\frac{d}{d\sigma_\eta}\Sigma < 0$.

[14] "NYSE Technologies and SMA to Distribute Social Media Analysis Data via SFTI" on NYSE Technologies (https://nysetechnologies.nyx.com).

[15] Upon discussing empirical predictions, we confine attention to the more interesting case in which the disclosure precision $\tau_x^* > 0$. Proposition 4 provides the necessary and sufficient condition.

Our next prediction speaks to the effect of word-of-mouth communications on market depth, after taking into account the endogenous precision of public disclosure characterized in Proposition 4. We use the inverse of coefficient α_ε in the pricing function (9) to measure the market depth (as in, say, Vives, 2010; Han and Yang, 2013). The idea behind the measure, as argued in Vives (2010), is that a change of noise trading by one unit moves prices by α_ε; a market is deep if a noise trader shock is absorbed without moving prices much, which happens when α_ε is low.

Proposition 7 (Lowering market depth) *More active word-of-mouth communications reduce the market depth; that is, $\frac{d}{d\lambda}\alpha_\varepsilon^{-1} < 0$ for $\forall \tau_x^* > 0$.*

This result is in contrast to the non-monotonic relationship shown in prior literature. For example, Grossman and Stiglitz (1980) explicitly state in their Conjecture 7 that market depth α_ϵ^{-1} is non-monotonic in the quality of private information (the fraction of informed investors in their case). The reason we derive an unambiguous negative correlation is that our model introduces a firm's public disclosure and allows the manager to choose the disclosure quality τ_x^* (see Proposition 4). Allowing the manager to adjust the firm's disclosure quality is descriptive to us. When testing Proposition 7, however, cautions should be given if a firm is in a specific industry or a sensitive period in which both mandatory and voluntary disclosures are highly regulated to the extent that its manager has little or no discretion over its disclosure quality. In this case, the disclosure quality τ_x should be treated as given exogenously, and more active word of mouth λ would have a non-monotonic effect on market depth as seen in prior literature.[16]

[16] Market depth is not the focus of the model and, hence, we view Proposition 7 as a side result. Kim and Verrecchia (1994) show how disclosure can reduce liquidity by affecting information asymmetry. More recently, Caskey et al. (2015) study the effect of information dissemination in networks on bid-ask spread in a sequential trade model á la Glosten and Milgrom (1985).

6 Conclusion

This paper shows that word-of-mouth communications can endogenously amplify noises contained in the asset supply and increase price volatility even if the communications are assumed to be truthful, and to increase investors' posterior precisions. Contrary to the casual intuition that more precise private information would compete with and crowd out firms' public disclosures, we show that firms often commit to more precise public disclosure when investors' private information becomes more precise as a result of their word-of-mouth communications. This complementarity arises because the two information channels have qualitatively different effects on the firm's price volatility. Public disclosures unambiguously mitigate the impact of the noisy supply on price volatility, whereas word-of-mouth communications often amplify such noise, making price more volatile ex ante. When the asset supply is volatile, public disclosures will be particularly valuable in lowering the noise-driven volatility that would otherwise be exacerbated by investors' private word-of-mouth communications.

Our results suggest that, all else equal, a firm is more likely to increase the quality of its public disclosure if investors are more risk averse or if their private information endowment is less precise. We also show that, after endogenizing the firm's public disclosure, more active word of mouth unambiguously lowers the market depth. These empirical predictions are relevant in light of the recent debate on the consequence of technological innovations, such as social media, that facilitate investors' interpersonal communications.

Our model makes a few simplifying assumptions: one-shot game, single-firm economy, and no earnings management. We make these assumptions to maintain tractability, for our model already features a continuum of investors, risk-aversion, and a continuous support for state, message, and action spaces. While these assumptions are standard in many disclosure models, they are nonetheless limiting. We view the restriction to a one-shot game the most limiting because a static game is silent about the dynamic aspect of word-of-mouth communications. We are unaware of models that provide a tractable way to study the interactions between firm's disclosures and (a continuum of) investors' private communications in a dynamic market setting, and it seems to be an interesting avenue for future research.

A Appendix: Proofs

Proof of Proposition 1: We organize the proof in two steps. We first reason from the investors' perspective: we take the investors' conjecture $\hat{a}$ about the manager's effort as given, and solve for the linear pricing function that clears the market (given the conjecture $\hat{a}$). We then reason from the manager's perspective in the second step: we take the linear pricing function derived in the first step as given and solve for the manager's optimal effort choice a^*. The rationality condition ensures that $\hat{a} = a^*$ in equilibrium.

Step 1: We guess and verify the following linear pricing equilibrium:

$$p = \hat{\alpha}_0 + \hat{\alpha}_v v + \hat{\alpha}_x \zeta - \hat{\alpha}_\varepsilon \varepsilon, \tag{A.1}$$

where the coefficients can depend on the investors' conjecture $\hat{a}$ (among other primitives of the model) but not on the manager's actual effort a, which is unobservable by its nature.

Consider the demand of the risky asset from any investor i who observes (i) the public signal x, (ii) the market price p, and (iii) N_i independent private signals $\{y_k = v + \eta_k\}_{k=1}^{N_i}$ via word-of-mouth communications prior to trading. Two observations of investor i's information set are helpful. First, the N_i signals are informationally equivalent to observing a signal $y_i = v + \varepsilon_i$, where $\varepsilon_i = \frac{\sum_{k=1}^{N_i} \eta_k}{N_i}$ and $\varepsilon_i \sim N\left(0, \frac{1}{N_i \tau_\eta}\right)$ follow because $\{\eta_k\}_{k=1}^{N_i}$ are independently distributed. Second, the market price p is informationally equivalent to $q \doteq \frac{p - \hat{\alpha}_x x - \hat{\alpha}_0}{\hat{\alpha}_v - \hat{\alpha}_x} = \frac{p - \hat{\alpha}_x (v + \zeta) - \hat{\alpha}_0}{\hat{\alpha}_v - \hat{\alpha}_x} = v - \frac{\hat{\alpha}_\varepsilon}{\hat{\alpha}_v - \hat{\alpha}_x}\varepsilon$. Note that q is easier to work with because its mean is v. We can express the investor i's information set as $\mathcal{F}_i = \{y_i, x, q, \hat{a}\}$, where $\hat{a}$ is the investors' conjecture of the manager's effort.

The joint normality implies that

$$\operatorname{var}(v|\mathcal{F}_i) = \frac{1}{\left(\frac{\hat{\alpha}_v-\hat{\alpha}_x}{\hat{\alpha}_\varepsilon}\right)^2\tau_\varepsilon+\tau_x+\tau_\phi+N_i\tau_\eta}, \tag{A.2}$$

$$\begin{aligned}\mathbb{E}(v|\mathcal{F}_i) &= \frac{\left(\frac{\hat{\alpha}_v-\hat{\alpha}_x}{\hat{\alpha}_\varepsilon}\right)^2\tau_\varepsilon q+\tau_x x+\tau_\phi\hat{a}+N_i\tau_\eta y_i}{\left(\frac{\hat{\alpha}_v-\hat{\alpha}_x}{\hat{\alpha}_\varepsilon}\right)^2\tau_\varepsilon+\tau_x+\tau_\phi+N_i\tau_\eta} \\ &= \frac{\left(\frac{\hat{\alpha}_v-\hat{\alpha}_x}{\hat{\alpha}_\varepsilon}\right)^2\tau_\varepsilon\frac{p-\hat{\alpha}_x x-\hat{\alpha}_0}{\hat{\alpha}_v-\hat{\alpha}_x}+\tau_x x+\tau_\phi\hat{a}+N_i\tau_\eta y_i}{\left(\frac{\hat{\alpha}_v-\hat{\alpha}_x}{\hat{\alpha}_\varepsilon}\right)^2\tau_\varepsilon+\tau_x+\tau_\phi+N_i\tau_\eta}.\end{aligned} \tag{A.3}$$

Therefore, investor i's demand for the risky-asset is

$$\begin{aligned}D_i &= \frac{r(\mathbb{E}(v|\mathcal{F}_i)-p)}{\operatorname{var}(v|\mathcal{F}_i)} \\ &= r\Bigg[\left(\frac{\hat{\alpha}_v-\hat{\alpha}_x}{\hat{\alpha}_\varepsilon}\right)^2\tau_\varepsilon\frac{p-\hat{\alpha}_x x-\hat{\alpha}_0}{\hat{\alpha}_v-\hat{\alpha}_x}+\tau_x x+\tau_\phi\hat{a}+N_i\tau_\eta y_i \\ &\quad -p\left(\left(\frac{\hat{\alpha}_v-\hat{\alpha}_x}{\hat{\alpha}_\varepsilon}\right)^2\tau_\varepsilon+\tau_x+\tau_\phi+N_i\tau_\eta\right)\Bigg].\end{aligned} \tag{A.4}$$

Integrating D_i over the continuum of investors and making use of the market-clearing condition $\int_i D_i di = \varepsilon$, we can show the following:

$$\begin{aligned}& r\Bigg[\left(\frac{\hat{\alpha}_v-\hat{\alpha}_x}{\hat{\alpha}_\varepsilon}\right)^2\tau_\varepsilon\frac{p-\hat{\alpha}_x x-\hat{\alpha}_0}{\hat{\alpha}_v-\hat{\alpha}_x}+\tau_x x+\tau_\phi\hat{a}+\int_i N_i\tau_\eta y_i di \\ & -p\left(\left(\frac{\hat{\alpha}_v-\hat{\alpha}_x}{\hat{\alpha}_\varepsilon}\right)^2\tau_\varepsilon+\tau_x+\tau_\phi+\int_i N_i di\tau_\eta\right)\Bigg] = \varepsilon,\end{aligned}$$

$$\Leftrightarrow \quad r\Bigg[\left(\frac{\hat{\alpha}_v-\hat{\alpha}_x}{\hat{\alpha}_\varepsilon}\right)^2\tau_\varepsilon\frac{p-\hat{\alpha}_x\left(v+\zeta\right)-\hat{\alpha}_0}{\hat{\alpha}_v-\hat{\alpha}_x}+\tau_x\left(v+\zeta\right)+\tau_\phi\hat{a}+\bar{\mathbf{N}}\tau_\eta v$$
$$-p\left(\left(\frac{\hat{\alpha}_v-\hat{\alpha}_x}{\hat{\alpha}_\varepsilon}\right)^2\tau_\varepsilon+\tau_x+\tau_\phi+\bar{\mathbf{N}}\tau_\eta\right)\Bigg]=\varepsilon,$$

from which we know the market-clearing price is

$$p=\frac{-\left(\frac{\hat{\alpha}_v-\hat{\alpha}_x}{\hat{\alpha}_\varepsilon}\right)^2\tau_\varepsilon\frac{\hat{\alpha}_x\zeta+\hat{\alpha}_0}{\hat{\alpha}_v-\hat{\alpha}_x}+\tau_x\zeta+\tau_\phi\hat{a}+\left(-\left(\frac{\hat{\alpha}_v-\hat{\alpha}_x}{\hat{\alpha}_\varepsilon}\right)^2\tau_\varepsilon\frac{\hat{\alpha}_x}{\hat{\alpha}_v-\hat{\alpha}_x}+\tau_x+\bar{\mathbf{N}}\tau_\eta\right)v-\frac{\varepsilon}{r}}{\left(\frac{\hat{\alpha}_v-\hat{\alpha}_x}{\hat{\alpha}_\varepsilon}\right)^2\tau_\varepsilon+\tau_x+\tau_\phi+\bar{\mathbf{N}}\tau_\eta-\left(\frac{\hat{\alpha}_v-\hat{\alpha}_x}{\hat{\alpha}_\varepsilon}\right)^2\tau_\varepsilon\frac{1}{\hat{\alpha}_v-\hat{\alpha}_x}}. \tag{A.5}$$

To determine the coefficients, we impose the rational condition that the conjectured pricing function (A.1) coincides with the true market-clearing price (A.5) in equilibrium. That is, the coefficients satisfy:

$$\hat{\alpha}_0=\frac{-\left(\frac{\hat{\alpha}_v-\hat{\alpha}_x}{\hat{\alpha}_\varepsilon}\right)^2\tau_\varepsilon\frac{\hat{\alpha}_0}{\hat{\alpha}_v-\hat{\alpha}_x}+\tau_\phi\hat{a}}{\left(\frac{\hat{\alpha}_v-\hat{\alpha}_x}{\hat{\alpha}_\varepsilon}\right)^2\tau_\varepsilon+\tau_x+\tau_\phi+\bar{\mathbf{N}}\tau_\eta-\left(\frac{\hat{\alpha}_v-\hat{\alpha}_x}{\hat{\alpha}_\varepsilon}\right)^2\tau_\varepsilon\frac{1}{\hat{\alpha}_v-\hat{\alpha}_x}}, \tag{A.6}$$

$$\hat{\alpha}_v=\frac{-\left(\frac{\hat{\alpha}_v-\hat{\alpha}_x}{\hat{\alpha}_\varepsilon}\right)^2\tau_\varepsilon\frac{\hat{\alpha}_x}{\hat{\alpha}_v-\hat{\alpha}_x}+\tau_x+\bar{\mathbf{N}}\tau_\eta}{\left(\frac{\hat{\alpha}_v-\hat{\alpha}_x}{\hat{\alpha}_\varepsilon}\right)^2\tau_\varepsilon+\tau_x+\tau_\phi+\bar{\mathbf{N}}\tau_\eta-\left(\frac{\hat{\alpha}_v-\hat{\alpha}_x}{\hat{\alpha}_\varepsilon}\right)^2\tau_\varepsilon\frac{1}{\hat{\alpha}_v-\hat{\alpha}_x}}, \tag{A.7}$$

$$\hat{\alpha}_x=\frac{-\left(\frac{\hat{\alpha}_v-\hat{\alpha}_x}{\hat{\alpha}_\varepsilon}\right)^2\tau_\varepsilon\frac{\hat{\alpha}_x}{\hat{\alpha}_v-\hat{\alpha}_x}+\tau_x}{\left(\frac{\hat{\alpha}_v-\hat{\alpha}_x}{\hat{\alpha}_\varepsilon}\right)^2\tau_\varepsilon+\tau_x+\tau_\phi+\bar{\mathbf{N}}\tau_\eta-\left(\frac{\hat{\alpha}_v-\hat{\alpha}_x}{\hat{\alpha}_\varepsilon}\right)^2\tau_\varepsilon\frac{1}{\hat{\alpha}_v-\hat{\alpha}_x}}, \tag{A.8}$$

$$\hat{\alpha}_\varepsilon=\frac{\frac{1}{r}}{\left(\frac{\hat{\alpha}_v-\hat{\alpha}_x}{\hat{\alpha}_\varepsilon}\right)^2\tau_\varepsilon+\tau_x+\tau_\phi+\bar{\mathbf{N}}\tau_\eta-\left(\frac{\hat{\alpha}_v-\hat{\alpha}_x}{\hat{\alpha}_\varepsilon}\right)^2\tau_\varepsilon\frac{1}{\hat{\alpha}_v-\hat{\alpha}_x}}. \tag{A.9}$$

The system of linear equations shown above determines the pricing coefficients:

$$\hat{\alpha}_0=\frac{\tau_\phi}{\tau_\phi+L}\times\hat{a},\ \hat{\alpha}_v=\frac{L}{\tau_\phi+L},\ \hat{\alpha}_x=\frac{\tau_x}{\tau_\phi+L},\ \hat{\alpha}_\varepsilon=\frac{\bar{\mathbf{N}}\tau_\eta r\tau_\varepsilon+\frac{1}{r}}{\tau_\phi+L}, \tag{A.10}$$

where $\hat{a}$ is the investor's conjecture about the manager's unobservable effort a and we define the term L as follows to economize notations:

$$L = \tau_x + \bar{\mathbf{N}}\tau_\eta + \left(\bar{\mathbf{N}}\tau_\eta r\right)^2 \tau_\varepsilon. \tag{A.11}$$

Step 2: We next solve for the manager's equilibrium effort choice. In particular, the manager takes the linear pricing function charactered above as given and chooses a to maximize his certainty equivalent:

$$\max_a \mathbb{E}[p|a, \hat{a}] - C(a) - \frac{\rho}{2}\text{var}(p|a, \hat{a}), \tag{A.12}$$

where $\mathbb{E}[p|a, \hat{a}] = \hat{\alpha}_0 + \hat{\alpha}_v \times a$ and $\text{var}(p|a, \hat{a}) = \frac{L}{\tau_\phi\left(L+\tau_\phi\right)} + \frac{1}{\left(L+\tau_\phi\right)^2}\left(\frac{1}{r^2\tau_\varepsilon} + \bar{\mathbf{N}}\tau_\eta\right)$ follow from the linear pricing function characterized in Step 1. Inspecting the first-order condition yields the manager's best response as follows:

$$a^*(\tau_x) = \hat{\alpha}_v = \frac{L}{\tau_\phi + L}. \tag{A.13}$$

Since the manager's best response is independent of the investors' conjecture $\hat{a}$, $\hat{a}$ must equal $a^*(\tau_x) = \frac{L}{\tau_\phi + L}$ to be correct in equilibrium. Finally, replacing the conjectured effort $\hat{a}$ in (A.10) with the equilibrium effort a^* yields the equilibrium linear pricing coefficients $(\alpha_0, \alpha_v, \alpha_x, \alpha_\varepsilon)$ shown in the proposition. ∎

Proof of Proposition 2: Given the equilibrium pricing function derived in

Proposition 1, it is straightforward to verify that

$$\operatorname{cov}(p,v)=\frac{L}{\tau_\phi+L}\frac{1}{\tau_\phi}, \tag{A.14}$$

$$\operatorname{cov}\left(p,p-v\right)=\frac{1}{\left(L+\tau_\phi\right)^2}\left(\frac{1}{r^2\tau_\varepsilon}+\bar{\mathbf{N}}\tau_\eta\right). \tag{A.15}$$

One can show the following for the fundamental-driven volatility $\operatorname{cov}(p,v)$:

$$\begin{aligned}\frac{\partial\operatorname{cov}\left(p,v\right)}{\partial\tau_x}&=\frac{\partial\operatorname{cov}\left(p,v\right)}{\partial L}\frac{\partial L}{\partial\tau_x}\\&=\frac{1}{\tau_\phi\left(L+\tau_\phi\right)^2}\cdot 1>0,\end{aligned} \tag{A.16}$$

and

$$\begin{aligned}\frac{\partial\operatorname{cov}\left(p,v\right)}{\partial\lambda}&=\frac{\partial\operatorname{cov}\left(p,v\right)}{\partial L}\frac{\partial L}{\partial\bar{\mathbf{N}}}\frac{\partial\bar{\mathbf{N}}}{\partial\lambda}\\&=\frac{1}{\tau_\phi\left(L+\tau_\phi\right)^2}\cdot\left(\tau_\eta+2\bar{\mathbf{N}}\left(\tau_\eta r\right)^2\tau_\varepsilon\right)\cdot\frac{\partial\bar{\mathbf{N}}}{\partial\lambda}>0,\end{aligned} \tag{A.17}$$

where the last inequality follows from $\frac{\partial\bar{\mathbf{N}}}{\partial\lambda}>0$. The results verify the claim that $\operatorname{cov}(p,v)$ increases in both disclosure τ_x and word of mouth λ.

Regarding the noise-driven volatility $\operatorname{cov}(p,p-v)$, we obtain

$$\begin{aligned}\frac{\partial\operatorname{cov}\left(p,p-v\right)}{\partial\tau_x}&=\frac{\partial\operatorname{cov}\left(p,p-v\right)}{\partial L}\frac{\partial L}{\partial\tau_x}\\&=-2\frac{\left(\frac{1}{r^2\tau_\varepsilon}+\bar{\mathbf{N}}\tau_\eta\right)}{\left(L+\tau_\phi\right)^3}\cdot 1<0,\end{aligned} \tag{A.18}$$

which suggests more precise public disclosure lowers the noise-driven volatility. On the other hand, one can show

$$
\begin{aligned}
&\frac{\partial \operatorname{cov}(p, p-v)}{\partial \lambda} \\
=&\frac{\partial \operatorname{cov}(p, p-v)}{\partial L} \frac{\partial L}{\partial \bar{\mathbf{N}}} \frac{\partial \bar{\mathbf{N}}}{\partial \lambda}+\frac{\tau_\eta}{\left(L+\tau_\phi\right)^2} \frac{\partial \bar{\mathbf{N}}}{\partial \lambda} \\
=&-2 \frac{\left(\frac{1}{r^2 \tau_\varepsilon}+\bar{\mathbf{N}} \tau_\eta\right)}{\left(L+\tau_\phi\right)^3} \cdot\left(\tau_\eta+2 \bar{\mathbf{N}}\left(\tau_\eta r\right)^2 \tau_\varepsilon\right) \cdot \frac{\partial \bar{\mathbf{N}}}{\partial \lambda}+\frac{\tau_\eta}{\left(L+\tau_\phi\right)^2} \cdot \frac{\partial \bar{\mathbf{N}}}{\partial \lambda} \\
=&\frac{\tau_\eta\left(\tau_\phi+\tau_x-5 \bar{\mathbf{N}} \tau_\eta-\frac{2}{r^2 \tau_\varepsilon}-3\left(\bar{\mathbf{N}} \tau_\eta r\right)^2 \tau_\varepsilon\right)}{\left(L+\tau_\phi\right)^3} \cdot \frac{\partial \bar{\mathbf{N}}}{\partial \lambda},
\end{aligned} \tag{A.19}
$$

which is positive if and only if $\tau_\varepsilon \in (\underline{\tau}_\varepsilon, \bar{\tau}_\varepsilon)$, where

$$
\underline{\tau}_\varepsilon \doteq \frac{\tau_\phi+\tau_x-5 \bar{\mathbf{N}} \tau_\eta-\sqrt{\left(\tau_\phi+\tau_x-5 \bar{\mathbf{N}} \tau_\eta\right)^2-24\left(\bar{\mathbf{N}} \tau_\eta r\right)^2}}{6\left(\bar{\mathbf{N}} \tau_\eta r\right)^2}, \tag{A.20}
$$

$$
\bar{\tau}_\varepsilon \doteq \frac{\tau_\phi+\tau_x-5 \bar{\mathbf{N}} \tau_\eta+\sqrt{\left(\tau_\phi+\tau_x-5 \bar{\mathbf{N}} \tau_\eta\right)^2-24\left(\bar{\mathbf{N}} \tau_\eta r\right)^2}}{6\left(\bar{\mathbf{N}} \tau_\eta r\right)^2}. \tag{A.21}
$$

The sets $(\underline{\tau}_\varepsilon, \bar{\tau}_\varepsilon)$ is not empty if and only if $\tau_\phi + \tau_x > \left(5 + 2\sqrt{6}\right) \bar{\mathbf{N}} \tau_\eta$. Collecting the conditions proves the result (let $\underline{\sigma}_\varepsilon^2 = 1/\bar{\tau}_\varepsilon$ and $\bar{\sigma}_\varepsilon^2 = 1/\underline{\tau}_\varepsilon$.) ■

Proof of Proposition 3: Denote by $\mathcal{F}_R = \{y_R, x, p, \hat{a}\}$ the representative investor R's information set. As shown in the proof of Lemma 1 below, $\mathcal{F}_R$ is informationally equivalent to $\{y_R, x, q, \hat{a}\}$, where $q = \frac{p - \hat{\alpha}_x x - \alpha_0}{a_v - a_x}$ is informationally equivalent to p and handy for Bayesian updating. R's conditional variance is

$$\begin{aligned}\operatorname{var}(v|\mathcal{F}_R) &= \frac{1}{\left(\frac{\alpha_v-\alpha_x}{\alpha_\varepsilon}\right)^2\tau_\varepsilon+\tau_x+\tau_\phi+\bar{\mathbf{N}}\tau_\eta} \\ &= \frac{1}{\tau_\phi+L},\end{aligned} \tag{A.22}$$

where $L, \bar{\mathbf{N}}$, and the coefficients $(\alpha_0, \alpha_v, \alpha_x, \alpha_\varepsilon)$ are derived in Proposition 1.

Investor R's conditional mean is:

$$\begin{aligned}\mathbb{E}(v|\mathcal{F}_R) &= \frac{\left(\frac{\alpha_v-\alpha_x}{\alpha_\varepsilon}\right)^2\tau_\varepsilon q+\tau_x x+\tau_\phi\hat{a}+\bar{\mathbf{N}}\tau_\eta y_R}{var^{-1}(v|\mathcal{F}_R)} \\ &= \frac{\left(\frac{\alpha_v-\alpha_x}{\alpha_\varepsilon}\right)^2\tau_\varepsilon\frac{p-\alpha_x x-\alpha_0}{\alpha_v-\alpha_x}+\tau_x x+\tau_\phi\hat{a}+\bar{\mathbf{N}}\tau_\eta y_R}{\tau_\phi+L} \\ &= \frac{\left(\bar{\mathbf{N}}\tau_\eta r\right)^2\tau_\varepsilon\frac{p-\frac{\tau_x}{\tau_\phi+L}x-\frac{\tau_\phi}{\tau_\phi+L}\hat{a}}{\frac{\bar{\mathbf{N}}\tau_\eta+\left(\bar{\mathbf{N}}\tau_\eta r\right)^2\tau_\varepsilon}{\tau_\phi+L}}+\tau_x x+\tau_\phi\hat{a}+\bar{\mathbf{N}}\tau_\eta y_R}{\tau_\phi+L} \\ &= \frac{\left(\bar{\mathbf{N}}\tau_\eta r\right)^2\tau_\varepsilon}{\bar{\mathbf{N}}\tau_\eta+\left(\bar{\mathbf{N}}\tau_\eta r\right)^2\tau_\varepsilon}\left(p-\frac{\tau_x}{\tau_\phi+L}x-\frac{\tau_\phi}{\tau_\phi+L}\hat{a}\right) \\ &\quad +\frac{\tau_x}{\tau_\phi+L}x+\frac{\tau_\phi}{\tau_\phi+L}\hat{a}+\frac{\bar{\mathbf{N}}\tau_\eta}{\tau_\phi+L}y_R.\end{aligned} \tag{A.23}$$

We can represent $\mathbb{E}(v|\mathcal{F}_R) = w_0^R\hat{a}+w_p^R p+w_x^R x+w_y^R y_R$, where

$$w_p^R = \frac{\left(\bar{\mathbf{N}}\tau_\eta r\right)^2\tau_\varepsilon}{\bar{\mathbf{N}}\tau_\eta+\left(\bar{\mathbf{N}}\tau_\eta r\right)^2\tau_\varepsilon}, \tag{A.24}$$

$$w_0^R = \left(1-w_p^R\right)\frac{\alpha_0}{a^*}, \tag{A.25}$$

$$w_x^R = \left(1-w_p^R\right)\alpha_x, \tag{A.26}$$

$$w_y^R = \frac{\bar{\mathbf{N}}\tau_\eta}{\tau_\phi+L} = \left(1-w_p^R\right)\left(\alpha_v-\alpha_x\right). \tag{A.27}$$

To show that $\mathrm{cov}(p, p-v) = (1-w_p^R)\alpha_\varepsilon^2\sigma_\varepsilon^2 = \sigma_\varepsilon^2 \times \frac{\mathrm{var}^2(v|\mathcal{F}_R)}{r^2\left(1-w_p^R\right)}$, we first obtain

$$\begin{aligned}
\mathrm{cov}\left(p, p-v\right) &= \mathbb{E}\left[p\left(p-v\right)\right] - \mathbb{E}\left(p\right)\mathbb{E}\left(p-v\right) \\
&= \mathbb{E}\left[\mathbb{E}\left[p\left(p-v\right)|\mathcal{F}_R\right]\right] - \mathbb{E}\left(p\right)\cdot 0 \\
&= \mathbb{E}\left[\mathbb{E}\left[p|\mathcal{F}_R\right]\mathbb{E}\left[p-v|\mathcal{F}_R\right]\right] \\
&= \mathbb{E}\left[p\left(p-\mathbb{E}\left(v|\mathcal{F}_R\right)\right)\right], \qquad \text{(A.28)}
\end{aligned}$$

where the second from the last equality makes use of the fact that p is part of the information set $\mathcal{F}_R$ and, hence, can be treated as a constant given $\mathcal{F}_R$. Substituting $\mathbb{E}\left(v|\mathcal{F}_R\right)$ as we derived earlier, we obtain

$$\begin{aligned}
\mathrm{cov}\left(p, p-v\right) &= \mathbb{E}\left[p\left(1-w_p^R\right)\left(p-\alpha_x x - \left(\alpha_v-\alpha_x\right)y_R - \alpha_0\right)\right] \\
&= \left(1-w_p^R\right)\mathbb{E}\left[p\left(\alpha_\varepsilon\varepsilon + \left(\alpha_v-\alpha_x\right)\left(y_R-v\right)\right)\right] \\
&= \left(1-w_p^R\right)\mathbb{E}\left[p\cdot\alpha_\varepsilon\varepsilon\right] + \left(1-w_p^R\right)\left(\alpha_v-\alpha_x\right)\mathbb{E}\left[p\cdot\left(y_R-v\right)\right] \\
&= \left(1-w_p^R\right)\alpha_\varepsilon^2\sigma_\varepsilon^2, \qquad \text{(A.29)}
\end{aligned}$$

where we use the fact that $\mathbb{E}\left[p\left(y_R-v\right)\right]=0$ in the last step. Moreover, one can show

$$1-w_p^R = \frac{\bar{\mathbf{N}}\tau_\eta}{\bar{\mathbf{N}}\tau_\eta + \left(\bar{\mathbf{N}}\tau_\eta r\right)^2\tau_\varepsilon} = \frac{1}{1+r^2\bar{\mathbf{N}}\tau_\eta\tau_\varepsilon}, \qquad \text{(A.30)}$$

$$\alpha_\varepsilon = \frac{\frac{1}{r}+r\bar{\mathbf{N}}\tau_\eta\tau_\varepsilon}{\tau_\phi+L} = \frac{1}{r}\frac{\mathrm{var}\left(v|\mathcal{F}_R\right)}{1-w_p^R}. \qquad \text{(A.31)}$$

Hence, we can derive

$$\begin{aligned}
\left(1-w_p^R\right)\alpha_\varepsilon^2\sigma_\varepsilon^2 &= \left(1-w_p^R\right)\left(\frac{1}{r}\frac{\operatorname{var}\left(v|\mathcal{F}_R\right)}{1-w_p^R}\right)^2\frac{1}{\tau_\varepsilon} \\
&= \frac{1}{r^2\tau_\varepsilon}\times\frac{\operatorname{var}^2(v|\mathcal{F}_R)}{\left(1-w_p^R\right)}. \qquad (A.32)
\end{aligned}$$

Substituting $1/\tau_\varepsilon = \sigma_\varepsilon^2$ proves equation (14) in Proposition 3. ■

Proof of Corollary 1: The result follows the nature of mathematical projection as explained in the text. ■

Proof of Corollary 2: Recall from (11) that $\operatorname{var}(p) = \operatorname{cov}(p,v) + \operatorname{cov}(p,p-v)$, we know

$$\begin{aligned}
\frac{\partial\operatorname{var}\left(p\right)}{\partial\tau_x} &= \frac{\partial\operatorname{cov}\left(p,v\right)}{\partial\tau_x} + \frac{\partial\operatorname{cov}\left(p,p-v\right)}{\partial\tau_x} \\
&= \frac{1}{\tau_\phi\left(L+\tau_\phi\right)^2} - 2\frac{\left(\frac{1}{r^2\tau_\varepsilon}+\bar{\mathbf{N}}\tau_\eta\right)}{\left(L+\tau_\phi\right)^3} \\
&= \frac{L+\tau_\phi-2\tau_\phi\left(\frac{1}{r^2\tau_\varepsilon}+\bar{\mathbf{N}}\tau_\eta\right)}{\tau_\phi\left(L+\tau_\phi\right)^3}, \qquad (A.33)
\end{aligned}$$

which is negative if and only if $\tau_x < \bar{\tau}_x \doteq 2\tau_\phi\left(\frac{1}{r^2\tau_\varepsilon}+\bar{\mathbf{N}}\tau_\eta\right) - \tau_\phi - \bar{\mathbf{N}}\tau_\eta - \left(\bar{\mathbf{N}}\tau_\eta r\right)^2\tau_\varepsilon$. ■

Proof of Proposition 4 : Substituting $a^*\left(\tau_x\right) = \frac{L}{L+\tau_\phi}$ from Proposition 1, we can express the manager's objective function U^M as

$$
\begin{aligned}
U^M &= \mathbb{E}[p] - \frac{\rho}{2}\operatorname{var}(p) - C(a) \\
&= \frac{L}{L+\tau_\phi} - \frac{\rho}{2}\left(\frac{L}{\tau_\phi\left(L+\tau_\phi\right)} + \frac{1}{\left(L+\tau_\phi\right)^2}\left(\frac{1}{r^2\tau_\varepsilon} + \bar{\mathbf{N}}\tau_\eta\right)\right) - \frac{1}{2}\left(\frac{L}{L+\tau_\phi}\right)^2 \\
&= \frac{1}{2} - \frac{\tau_\phi^2}{2\left(L+\tau_\phi\right)^2} - \frac{\rho}{2}\left(\frac{L}{\tau_\phi\left(L+\tau_\phi\right)} + \frac{1}{\left(L+\tau_\phi\right)^2}\left(\frac{1}{r^2\tau_\varepsilon} + \bar{\mathbf{N}}\tau_\eta\right)\right).
\end{aligned} \tag{A.34}
$$

The first-order condition is

$$
\begin{aligned}
\frac{\partial U^M}{\partial \tau_x} &= \frac{\tau_\phi^2}{\left(L+\tau_\phi\right)^3} - \frac{\rho}{2}\frac{1}{\left(L+\tau_\phi\right)^2} + \frac{\rho\left(\frac{1}{r^2\tau_\varepsilon} + \bar{\mathbf{N}}\tau_\eta\right)}{\left(L+\tau_\phi\right)^3} \\
&= \frac{\tau_\phi^2 - \frac{\rho}{2}\left(L+\tau_\phi\right) + \rho\left(\frac{1}{r^2\tau_\varepsilon} + \bar{\mathbf{N}}\tau_\eta\right)}{\left(L+\tau_\phi\right)^3} = 0.
\end{aligned} \tag{A.35}
$$

Plugging $L = \tau_x + \bar{\mathbf{N}}\tau_\eta + \left(\bar{\mathbf{N}}\tau_\eta r\right)^2\tau_\varepsilon$ into (A.35), we solve for the optimal precision:

$$
\tau_x^* = 2\frac{1}{r^2\tau_\varepsilon} + \frac{2}{\rho}\tau_\phi^2 - \tau_\phi + \bar{\mathbf{N}}\tau_\eta - \left(\bar{\mathbf{N}}\tau_\eta r\right)^2\tau_\varepsilon, \tag{A.36}
$$

with $\tau_x^* > 0$ if and only if $\sigma_\varepsilon^2 = \frac{1}{\tau_\varepsilon} > \frac{r^2\left[\sqrt{\left(\frac{2}{\rho}\tau_\phi^2 - \tau_\phi + \bar{\mathbf{N}}\tau_\eta\right)^2 + 8\left(\bar{\mathbf{N}}\tau_\eta\right)^2} - \left(\frac{2}{\rho}\tau_\phi^2 - \tau_\phi + \bar{\mathbf{N}}\tau_\eta\right)\right]}{4}$. ■

Proof of Lemma 1: Denote by $\hat{a}$ the investor's conjecture about the manager's effort choice a. Given the joint-normal information structure, simple Bayes rule implies that we can rewrite the price $p = \mathbb{E}[v|\hat{a}, x, y]$ as follows:

$$
\begin{aligned}
p &= \frac{\tau_\phi}{\tau_\phi + \tau_x + \tau_\eta}\mathbb{E}[v|\hat{a}] + \frac{\tau_x}{\tau_\phi + \tau_x + \tau_\eta}x + \frac{\tau_\eta}{\tau_\phi + \tau_x + \tau_\eta}y \\
&= \frac{\tau_\phi\hat{a} + \tau_x x + \tau_\eta y}{\tau_\phi + \tau_x + \tau_\eta}.
\end{aligned} \tag{A.37}
$$

Given a public precision τ_x and the conjectured $\hat{a}$, the manager chooses a to maximize his certainty equivalent:

$$\max_a \mathbb{E}[p|a, \hat{a}] - C(a) - \frac{\rho}{2}\text{var}(p|a, \hat{a}), \tag{A.38}$$

where we can use (A.37) to show

$$\mathbb{E}[p|a, \hat{a}] = \frac{\tau_\phi}{\tau_\phi + \tau_x + \tau_\eta}\hat{a} + \frac{\tau_x + \tau_\eta}{\tau_\phi + \tau_x + \tau_\eta}a, \tag{A.39}$$

and

$$\begin{aligned}\text{var}(p|a, \hat{a}) &= \left(\frac{\tau_x + \tau_\eta}{\tau_\phi + \tau_x + \tau_\eta}\right)^2 \frac{1}{\tau_\phi} + \left(\frac{\tau_x}{\tau_\phi + \tau_x + \tau_\eta}\right)^2 \frac{1}{\tau_x} + \left(\frac{\tau_\eta}{\tau_\phi + \tau_x + \tau_\eta}\right)^2 \frac{1}{\tau_\eta} \\ &= \frac{\tau_x + \tau_\eta}{\tau_\phi + \tau_x + \tau_\eta}\frac{1}{\tau_\phi}. \end{aligned} \tag{A.40}$$

The optimal effort a^* satisfies the first-order condition $\frac{d\mathbb{E}[p|a,\hat{a}]}{da}|_{a=a^*} = \frac{dC(a)}{da}|_{a=a^*}$, from which we know

$$a^*(\tau_x) = \frac{\tau_x + \tau_\eta}{\tau_\phi + \tau_x + \tau_\eta}. \tag{A.41}$$

Note that investors' conjecture $\hat{a}$ must equal a^* in equilibrium, for they know from (A.41) that a^* is the manager's dominant strategy for a given τ_x.

The manager takes $a^*(\tau_x)$ as given and chooses the precision of public disclosure τ_x to maximize his certainty equivalent:

$$\max_{\tau_x} \mathbb{E}[p|a^*(\tau_x)] - C(a^*(\tau_x)) - \frac{\rho}{2}\text{var}(p|a^*(\tau_x)). \tag{A.42}$$

The corresponding first-order condition for τ_x is (assuming an interior solution, i.e., $\tau_x^* > 0$):

$$\frac{d}{d\tau_x}\left(\mathbb{E}[p|a^*(\tau_x)] - C(a^*(\tau_x))\right)|_{\tau_x=\tau_x^*} = \frac{\rho}{2} \times \frac{d}{d\tau_x}\text{var}(p)|_{\tau_x=\tau_x^*}. \tag{A.43}$$

Substituting (A.39), (A.40), and (A.41), one can rewrite (A.43) as

$$\frac{d}{d\tau_x}\left(\frac{\tau_x+\tau_\eta}{\tau_\phi+\tau_x+\tau_\eta} - \frac{1}{2}\left(\frac{\tau_x+\tau_\eta}{\tau_\phi+\tau_x+\tau_\eta}\right)^2\right)|_{\tau_x=\tau_x^*} = \frac{\rho}{2}\cdot\frac{d}{d\tau_x}\left(\frac{\tau_x+\tau_\eta}{\tau_\phi+\tau_x+\tau_\eta}\frac{1}{\tau_\phi}\right)|_{\tau_x=\tau_x^*},$$

and solve for τ_x^* as

$$\tau_x^* = \frac{(2\tau_\phi-\rho)\tau_\phi}{\rho} - \tau_\eta. \tag{A.44}$$

Equation (A.44) verifies the necessary and sufficient condition $\rho < \frac{2\tau_\phi^2}{\tau_\phi+\tau_\eta}$ for an interior solution, i.e., $\tau_x^* > 0$. The claim $\frac{d\tau_x^*}{d\tau_\eta} < 0, \forall \tau_x^* > 0$ also follows easily. ■

Proof of Proposition 5: Based on the result in Proposition 4, when $\tau_x^* > 0$, it is easy to show

$$\frac{\partial\tau_x^*}{\partial\bar{\mathbf{N}}} = \tau_\eta - 2\bar{\mathbf{N}}\left(\tau_\eta r\right)^2\tau_\varepsilon, \tag{A.45}$$

which is positive if and only if $\sigma_\varepsilon^2 \doteq \frac{1}{\tau_\varepsilon} > 2\bar{\mathbf{N}}\tau_\eta r^2$. ■

Proof of Proposition 6: It is easy to verify that $\Sigma \doteq 2\bar{\mathbf{N}}\tau_\eta r^2$ in Proposition 5 increases in r and τ_η. ■

Proof of Proposition 7: Substituting τ_x^* characterized in Proposition 4 into the linear pricing function, one can rewrite the coefficient α_ε as

$$\alpha_\varepsilon = \frac{\left(\frac{1}{r} + r\bar{\mathbf{N}}\tau_\eta\tau_\varepsilon\right)}{\tau_\phi + L} = \frac{\frac{1}{r} + r\bar{\mathbf{N}}\tau_\eta\tau_\varepsilon}{2\left(\frac{1}{r^2\tau_\varepsilon} + \bar{\mathbf{N}}\tau_\eta\right) + \frac{2}{\rho}\tau_\phi^2}. \tag{A.46}$$

Straightforward calculation shows

$$\begin{aligned}\frac{d}{d\lambda}\alpha_\varepsilon^{-1} &= \frac{d\frac{2\left(\frac{1}{r^2\tau_\varepsilon}+\bar{\mathbf{N}}\tau_\eta\right)+\frac{2}{\rho}}{\frac{1}{r}+r\bar{\mathbf{N}}\tau_\eta\tau_\varepsilon}}{d\bar{\mathbf{N}}}\frac{d\bar{\mathbf{N}}}{d\lambda} \\ &= -\frac{\frac{2}{\rho}r\tau_\eta\tau_\varepsilon}{\left(\frac{1}{r} + r\bar{\mathbf{N}}\tau_\eta\tau_\varepsilon\right)^2}\frac{d\bar{\mathbf{N}}}{d\lambda} < 0,\end{aligned} \tag{A.47}$$

which proves the proposition. ■

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