Timely Loan Loss Provisioning: A Double-Edged Sword?*

Preliminary and Incomplete

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Abstract

We investigate the impact of loan loss provisioning on the prudential regulation of banks. We study two provisioning models: an incurred loss model and an expected loss model. Relative to the incurred loss model, an expected loss model improves efficiency as it allows the bank regulator to intervene in the bank's operations in a timely manner to curb inefficient ex-post asset-substitution. However, taking real effects into account, our analysis uncovers a potential cost of the expected loss model. We show that when the bank is highly leveraged, it responds to timely regulatory intervention under the expected loss model by originating riskier loans so that timely intervention induces timelier risk-taking. By appropriately tailoring the bank's capital requirements to the expected loss model, the regulator may improve the efficiency of the expected loss model.

Keywords: Capital Requirements; Loan Loss Provisioning; Real Effects.

JEL codes: G21, G28, M41, M48.

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1 Introduction

The recent adoption of the Current Expected Credit Loss model (CECL) for the recognition and measurement of credit losses for loans and debt securities is arguably one of the most consequential accounting changes to impact banks and financial institutions.[1] Under the new standard, banks are required to replace the "incurred loss" model with an "expected loss" model. A key difference between the two accounting models is that under the incurred loss model, banks "delay recognition of credit losses until they have been incurred," whereas under the expected loss model, banks must recognize "the full amount of credit losses that are expected."[2] While the Financial Accounting Standards Board (FASB) argue that the new standard results in "more timely and relevant information," others have argued that it "could actually produce negative economic consequences."[3] Most notably, banks are concerned that the forecasts of future credit losses are often unreliable and false loss recognition may lower bank capital ratios and reduce lending to the economy, thus making credit supplies procyclical and worsening economic downturns. To shed some light on this important debate, we develop an economic model to study the trade–offs in moving from an "incurred loss" model to an "expected loss" model.

In our model, a representative bank is subject to shareholder–debtholder conflicts. The bank's shareholders have incentives to take excessive risk by either: 1) increasing the *ex ante* risk of the bank's loan portfolio via exerting less effort to screen borrowers, and/or 2) engaging in *ex post* asset substitution to replace low-risk loans with high-risk ones. To discipline excessive risk-taking, a regulator imposes minimum capital requirements. Importantly, the level of capital depend on the information on loan losses reported by the bank's accounting system. Under an incurred loss

[1] See public round table meeting on credit losses, January 28, 2019, available at https://www.youtube.com/watch?v=FH7xWZsCX9s&t=2866s

[2] See FASB, Accounting Standards Update No. 2016-13 Financial Instruments-Credit Losses (Topic 326).

[3] See the comment letter by Tom Quaadman, U.S. Chamber of Commerce, available at https://www.centerforcapitalmarkets.com/wp-content/uploads/2019/01/Chamber-CECL-Letter.pdf?#.

model, the bank does not report a loan loss until it is realized, whereas under the expected loss model, the bank must additionally provide an early but imprecise report of the expected loan loss. Recognizing a large loan loss erodes the bank's capital taking the bank close to, or even below, its regulatory capital requirement, thus triggering regulatory intervention. The regulator can then choose whether to liquidate or restructure the bank's loan portfolio before it pays off.

We first study a setting in which the bank's *ex ante* choice of loan risk is kept fixed. We show that timely recognition of loan losses under the expected loss model is always beneficial. Such a benefit arises because the regulator may utilize the timely information to curb the bank's *ex post* asset-substitution. Whenever the bank recognizes a large loan loss in the interim, the regulator rationally anticipates that the bank has a strong incentive to asset-substitute and thus liquidates the bank early. More importantly, we show that the benefit of timely loss recognition is strictly positive, despite the possibility of false alarm costs caused by the imprecise information inherent in an expected loss model. The reason is that a rational and benevolent regulator fully internalizes the false-alarm costs and chooses an action that results in the highest surplus in expectation. In other words, our analysis shows that the usual false alarm arguments are not sufficient to overturn the benefits of timeliness offered by expected loss models.

However, once we consider the real effects of timely loss recognition on banks' risk choices, our model shows that early intervention is a double-edged sword. While timely intervention always curbs ex post asset substitution, it may induce the bank to originate safer or riskier loans. In particular, when the bank's leverage is high, it invests in riskier loans under the expected loss model. This results in a surplus loss that outweighs the benefit of timely loss recognition, thereby making the expected loss model inferior to the incurred loss model.

To understand the risk-disciplining effect of early intervention, note that under the incurred loss model, given the bank's anticipation of future asset substitution, its incentives to screen borrowers *ex ante* decrease. In contrast, under the expected loss model, since the bank is restrained from switching to riskier loans later, its incentives to originate safer loans increase.

The risk-aggravating effect of early intervention is more subtle. Note that under the incurred loss model, the bank prefers to defer taking excessive amounts of risk until it receives more precise information on the performance of its loan portfolio. However, under the expected loss model, the option value of waiting is constrained by regulatory intervention. Anticipating this, the bank shifts the timing of its risk-taking earlier by originating riskier loans under the expected loss model.

In equilibrium, the bank trades off these two real effects in determining how much risk to take *ex ante*. We show that, when the bank is highly leveraged, the risk-aggravating effect of early intervention dominates its risk-disciplining effect, inducing the bank to originate riskier loans. Intuitively, a highly-leveraged bank has a strong incentive to asset-substitute. When that possibility is constrained by early intervention, the bank responds by building up more risk in its loan portfolio *ex ante* so that timely intervention triggers even timelier risk-taking by the bank. We find that, when we take into account the real effects of accounting measurements, the expected loss becomes inferior when banks are heavily leveraged.

An implication of our model is that the regulator should take into account such real effects when designing the capital requirement policy that, in turn, governs the bank's leverage. Stated differently, changing the accounting method for loan loss provisioning requires the bank regulator to simultaneously adjust the capital adequacy ratios. We show that by appropriately tailoring the bank's capital requirements to information about loan losses, the regulator can improve the efficiency of the expected loss model.

2 Related Literature

To be added later.

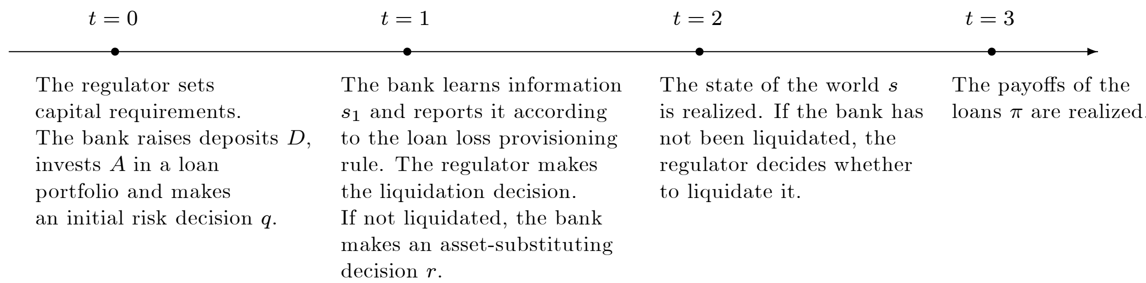

Figure 1: Timeline of the baseline model

3 The Model

We examine an environment with four dates that includes a representative bank and a bank regulator. Figure 1 summarizes the sequence of events.

At $t = 0$, the bank is endowed with an amount of exogenous equity E and chooses the size $A \in \left[0, \bar{A}\right]$ and risk $q \in [0, 1]$ of investments in a loan portfolio. $\bar{A}$ is the maximum size of the loan portfolio and is chosen to be sufficiently large. For a size of $A > E$, the bank borrows $D = A - E$ from depositors. We assume that deposits are fully insured and we normalize the risk-free deposit rate to zero.

We model the return on the loan portfolio as follows. The outcome of each loan is binary: either the loan succeeds or it defaults. The bank chooses a costly effort q to screen risky borrowers. Absent any screening effort ($q = 0$), the bank always generates a "high-risk loan" that returns β with probability $\tau \in (0, 1)$ and 0 with probability $1 - \tau$. To improve the performance of the loan portfolio, the bank exerts effort q at a cost of $C(q)$ where the cost function $C(.)$ satisfies the standard properties: $C(0) = 0$, $C(1) = \infty$, $C'(0) = 0$, $C'(1) = \infty$, and $C'' > 0$. Conditional on a choice of $q > 0$, the bank receives the high-risk loan with probability $1 - q$ and the low-risk loan with probability q. The low-risk loan returns α with probability s and 0 with probability $1 - s$. The random variable s represents the (inverse) default risk of the low-risk loan and has a distribution $H(.)$ and a density $h(.)$ with full support on $[\tau, 1]$ so that the low-risk loan always has a lower default risk than the high-risk loan. This assumption guarantees that the bank's effort q

reduces the default risk of the loan portfolio by decreasing the likelihood of high-risk loans in its loan portfolio. To reflect the risk-return trade-off, we assume that $\beta > \alpha > 1$ such that the bank demands a higher interest rate on the high-risk loan.

In originating its loan portfolio, the bank is subject to a minimum capital requirement, $\gamma \in (0,1)$, imposed by the regulator:

$$\frac{E}{A} \geq \gamma. \tag{1}$$

In the expression of the capital ratio in (1), the numerator is the accounting value of the bank's equity and the denominator is the accounting value of the bank's assets. The asset side of the bank's balance sheet contains the loan portfolio and the liability side of the balance sheet contains the deposits. Under the "effective interest rate" method of the FASB rules, the initial values of both the loans and the deposits are recognized at the initial contractual principal, A and D, respectively (Ryan, 2007). Therefore, the value of total assets is A and the value of total liabilities is D. By the accounting identity, the equity value is $A - D = E$.[4]

At $t = 1$, the bank observes some early information s_1 regarding the default risk of its loan portfolio s. The random variable $\widetilde{s}_1$ has a distribution $G(.)$ and a density $g(.)$ with full support on $[\tau, 1]$. We interpret s_1 as new information about a non-incurred loan value change that arrives at the intermediate date, and reflects the change in expectation of future loan losses. The arrival of such information affects the bank's assessment of the default likelihood and future loan losses. Formally, we model the bank's updated assessment of loan performance using the posterior distribution

[4] We consider the case that the regulator uses a simple leverage ratio to regulate capital. This is consistent with the newly proposed Basel III framework. In particular, Section V of Basel Committee (2010) provides a discussion of the use of leverage ratios in Basel III, that "the Committee agreed to introduce a simple, transparent, non-risk based leverage ratio that is calibrated to act as a credible supplementary measure to the risk based capital requirements." Under the previous Basel II regulatory framework, bank regulators may use some risk-weighted measure of assets to regulate capital. By Basel II, assets are partitioned into different groups based on their risk and these different groups are assigned different weights. However, in the actual implementations of Basel II, the exact partitions of assets are very coarse. For example, the risk weight for most commercial loans and investment exposures held by banks is 1. If we assign a weight of 1 to the loan portfolio, the resulting risk-weighted capital ratio is identical to the simple leverage ratio we have in the model.

of s given s_1, denoted by $F(s|s_1)$ with a density $f(s|s_1)$. An important goal of our paper is to determine whether the bank's accounting system should report the early information in its financial statements, i.e., by taking a provision for the expected loan losses. We study two accounting systems. We refer to the accounting system in which the bank does not report s_1 and hence doesn't recognize loan losses early as an "incurred loss model (IL)" while the accounting system in which the bank reports s_1 and therefore recognizes loan losses early as "the expected loss model (EL)." Our model therefore provides a parsimonious and tractable way to capture a key difference between an incurred loss model and an expected loss model. The expected loss model requires a more timely recognition of non-incurred loan losses whereas the incurred loss model does not recognize loan losses until they are incurred.

The timing of loan loss recognition matters in our model because reporting a large loan loss, i.e., a low realization of s_1 in the intermediate date may trigger intervention by the regulator.[5] Recognizing a large loan loss leads to an impairment of the bank's capital and puts the bank on the verge of violating the capital requirement that, in turn, triggers regulatory intervention. For simplicity, we consider one form of intervention: the regulator liquidates the bank by either selling or restructuring the bank's loans.[6]. Given the information s_1, the regulator optimally liquidates the bank's loans if the total expected payoffs from liquidation exceed the total payoffs from continuation. We assume that the regulator learns the type of the loan at the liquidation stage. If a low-risk loan is liquidated, we assume that the liquidation payoff is $L_\alpha < \alpha$ whereas if a high-risk loan is liquidated, we assume that the liquidation payoff is $L_\beta < \beta$. Furthermore, liquidating risky subprime loans entails losses in expectation and regulators often recover very little residual value.

[5] In our main analyses, we ignore the uninteresting case in which the regulator intervenes at $t = 1$ even if the bank does not report any loan losses (as in the incurred loss model). In practice, regulatory intervention needs to be triggered by the release of verifiable adverse information such that a large reported loan loss. We have also considered an extension in which the regulator may intervene even in the absent of bank's report. We find that our main result is robust under this alternative assumption.

[6] Note that, in practice, the regulator could also recapitalize the banks by forcing them to issue equity.

We therefore assume that:

Assumption 1: $\tau\beta > L_\beta$.

In case of continuation at $t = 1$, based on its updated assessment of the loan performance $F(s|s_1)$, the bank may engage in asset substitution $r \in \{0, 1\}$ .The variable $r = 0$ implies no asset substitution so that the bank does not change its original loan portfolio whereas the variable $r = 1$ implies that the bank changes its original loan portfolio by substituting the low-risk loans in the loan portfolio with high-risk loans. That is, conditional on $r = 1$, the bank always receives the high-risk loan.

At $t = 2$, the default risk s is realized and observed by the bank. Since the default risk is realized, the bank reports s, regardless of whether the bank follows the incurred loss model or the expected loss model. Upon receiving the new report of loan losses s, the regulator decides whether to liquidate the bank, conditional on allowing the bank to continue at $t = 1$.

At $t = 3$, the terminal payoffs of the loans are realized. The payoff π of the bank's loan portfolio is as follows. If the loan is low-risk and liquidated, $\pi = L_\alpha$ but if the loan is low-risk and continued, $\pi = \alpha$ with probability s and $\pi = 0$ with probability $1 - s$. If the loan is high-risk and is liquidated, $\pi = L_\beta$ whereas in case of continuation, $\pi = \beta$ with probability τ and $\pi = 0$ with probability $1 - \tau$. The regulator compensates depositors if the bank fails, i.e., $\pi < D$, with a lump sum payment which we assume is financed via a frictionless *ex ante* tax.

Finally, to create a demand for prudential regulation, we impose the following two assumptions throughout our entire analysis of the model:

Assumption 2: A high-risk loan is value-destroying whereas a low-risk loan is value-creating. This assumption ensures that the regulator has an incentive to discipline the bank from investing in high-risk loans. It turns out that a sufficient condition for Assumption 2 is

$$L_\alpha \geq 1 > \tau\beta. \tag{2}$$

The first part of the inequality ensures that the low-risk loan generates at least zero net present value (NPV) even in case of liquidation whereas the second part ensures that the high-risk loan always generates negative NPV.

Assumption 3: The bank always prefers to invest in the high-risk loan if it lends the maximal extent, i.e., $A = \bar{A}$.

This assumption rules out the degenerate case in which the bank can achieve the first-best by choosing the highest leverage and lending the maximal extent, thus making the capital requirement constraint undesirable. A sufficient condition for this assumption is

$$\tau(\beta - 1) > \left(\int_{\tau}^{\frac{L_\alpha}{\alpha}} f(s|s_1)\, ds\right)(L_\alpha - 1) + \left(\int_{\frac{L_\alpha}{\alpha}}^{1} sf(s|s_1)\, ds\right)(\alpha - 1), \tag{3}$$

for any $s_1 \in [\tau, 1]$.

4 Analysis

4.1 Exogenous loan risk

As a benchmark, we first solve a variant of our model treating the bank's *ex ante* risk q as exogenous.

4.1.1 Incurred loss model with exogenous risk

We analyze the incurred loss model in which the bank does not report s_1 so that the regulator can only intervene at $t = 2$ after the default risk s is realized. We solve the model using backward induction. At $t = 2$, given default risk s, the regulator decides whether to liquidate the bank by comparing the total expected payoff from liquidation with the total payoffs from continuation. If the bank's loan turns out to be high-risk, it generates L_β upon liquidation and $\tau\beta > L_\beta$ upon continuation. Therefore, the regulator never liquidates the high-risk loan. On the other hand, if the loan turns out to be low-risk, it generates L_α upon liquidation and $s\alpha$ upon continuation. Therefore, the regulator liquidates the low-risk loan if and only if

$$s < \frac{L_\alpha}{\alpha}. \tag{4}$$

Next, we solve for the bank's asset-substituting decision r at $t = 1$ conditional on the early information s_1. Denote the bank's shareholders (hereafter the bank's) expected payoff by $U(q, r|s_1)$, which depends on its *ex ante* risk choice q and interim asset-substituting decision r. If the bank chooses not to engage in asset substitution so that $r = 0$, its payoff is:

$$\begin{aligned} U(q,0|s_1) &= (1-q)\tau(A\beta-(A-E)) \qquad (5)\\ &+q\left(\left[\int_{\frac{L_\alpha}{\alpha}}^{1} sf\left(s|s_1\right)ds\right]\left(A\alpha-(A-E)\right)+\Pr\left[s<\frac{L_\alpha}{\alpha}|s_1\right]\left(AL_\alpha-(A-E)\right)\right)\left(\right.\end{aligned}$$

Conditional on obtaining a high-risk loan with probability $1 - q$, the bank receives $A\beta - (A - E)$ after repaying depositors with probability τ, and receives 0 with probability $1 - \tau$. Conditional on obtaining a low-risk loan with probability q, the bank's payoff depends on the regulator's liquidation decision at $t = 2$. If $s \geq L_\alpha/\alpha$ so that the regulator doesn't liquidate the bank, the bank receives a net payoff of $A\alpha - (A - E)$ after repaying depositors with probability s, and receives 0 with probability $1 - s$. But if $s < L_\alpha/\alpha$, the regulator liquidates the bank so that the bank receives the certain liquidation payoff $AL_\alpha - (A - E)$ after repaying depositors. Note that $AL_\alpha - (A - E) > 0$ because $L_\alpha \geq 1$.

If the bank engages in asset substitution so that $r = 1$, its payoff is

$$U\left(q, 1|s_1\right) = \tau(A\beta - (A - E)), \tag{6}$$

because the bank always obtains the high-risk loan and is not liquidated. The bank chooses asset substitution if and only if

$$U(q, 1|s_1) > U(q, 0|s_1) \tag{7}$$

which reduces into

$$\tau(A\beta - (A - E)) > \left(\left[\int_{\frac{L_\alpha}{\alpha}}^{1} s f\left(s|s_1\right) ds \right] \left(A\alpha - (A - E)\right) + \Pr\left[s < \frac{L_\alpha}{\alpha} | s_1 \right] \left(AL_\alpha - (A - E)\right) \right). \tag{8}$$

The left hand side of (8) represents the bank's payoff from investing in a high-risk loan and does not depend on s_1. As s_1 increases, the posterior distribution of s shifts to the right so that the right hand side of (8) which represents the bank's payoff from investing in a low-risk loan increases in s_1. As a result, there exists a unique cutoff $\bar{s}_1(A)$ such that the bank engages in asset substitution if and only if its early information regarding the low-risk loan's performance is below the cutoff, i.e., $s_1 < \bar{s}_1(A)$. We formally state the bank's equilibrium asset-substituting decision in the following lemma.

Proposition 1 *Conditional on the early information s_1, there exists a unique threshold $\bar{s}_1(A) \in [\tau, 1]$ such that the bank makes the asset-substituting decision ($r = 1$) if and only if $s_1 < \bar{s}_1(A)$, where $\bar{s}_1(A)$ is given by*

$$\tau = \left[\int_{\frac{L_\alpha}{\alpha}}^{1} s f\left(s|\bar{s}_1(A)\right) ds \right] \frac{A\alpha - (A - E)}{A\beta - (A - E)} + \Pr\left[s < \frac{L_\alpha}{\alpha} |\bar{s}_1(A) \right] \frac{AL_\alpha - (A - E)}{A\beta - (A - E)}. \tag{9}$$

The threshold $\bar{s}_1(A)$ increases in the size of the bank A, decreases in the face value of the low-risk loan α and increases in the face value of the high-risk loan β.

Proposition 1 is intuitive. It states that when the bank expects the performance of its loan portfolio to deteriorate, its incentives to engage in asset substitution increase. Furthermore, such incentives become sharper when either the relative payoff of the high-risk loan to the low-risk loan β/α increases and/or the bank's leverage A increases. The latter result suggests a beneficial role for capital requirements in curbing asset substitution: a higher capital ratio γ (equivalently, a lower A) induces the bank to set a lower leverage, thus weakening its asset-substituting incentive in the interim. To highlight this role, the following corollary characterizes the effect of asset size A on the asset-substitution threshold.

Corollary 1 *There exists two cutoffs on the bank's size $A_{\max}$ and $A_{\min}$ such that:*

$$\begin{cases} \bar{s}_1(A) = \tau & \text{if } A \in [E, A_{\min}]; \\ \bar{s}_1(A) \in (\tau, 1) \text{ and } \frac{\partial \bar{s}_1(A)}{\partial A} > 0 & \text{if } A \in (A_{\min}, A_{\max}); \\ \bar{s}_1(A) = 1 & \text{if } A \in [A_{\max}, \bar{A}]. \end{cases}$$

The cutoffs $A_{\max}$ and $A_{\min}$ increase in the face value of the low-risk loan α and decrease in the face value of the high-risk loan β.

Corollary 1 is a direct consequence of Proposition 1. It shows that when the leverage of the bank is extremely low (i.e., the bank faces a tight capital requirement), the bank never engages in asset substitution, whereas when the leverage is extremely high, the bank always chooses to asset-substitute. For intermediate values of leverage, the bank's assessment of its loan performance matters and the bank engages in asset substitution if and only if such assessment deteriorates.

Finally, we solve for the bank's choice of asset size A at $t = 0$ given its equilibrium asset substitution strategy at date 1. The bank chooses A to maximize

$$U(q) = \int_{\tau}^{\bar{s}_1(A)} U(q, 1|s_1)\, g(s_1)\, ds_1 + \int_{\bar{s}_1(A)}^{1} U(q, 0|s_1)\, g(s_1)\, ds_1 - C(q), \tag{10}$$

subject to the capital requirement, $\frac{E}{A} \leq \gamma$. The capital requirement essentially sets an upper-bound on the bank's asset size, i.e., $A \leq \frac{E}{\gamma}$. Differentiating $U(q)$ with respect to A yields:

$$\begin{aligned}
& \int_{\bar{s}_1(A)}^{1} (1-q)\tau(\beta-1) + q\left(\left[\int_{\frac{L_\alpha}{\alpha}}^{1} sf(s|s_1)\, ds\right](\alpha-1) + \Pr\left[s < \frac{L_\alpha}{\alpha}|s_1\right](L_\alpha - 1)\right) g(s_1)\, ds_1 \\
& + \int_{\tau}^{\bar{s}_1(A)} \tau(\beta-1) g(s_1)\, ds_1 \\
> \;& 0.
\end{aligned} \tag{11}$$

Therefore, the bank always chooses to invest up to the maximum allowed under the capital requirement, i.e., $A^* = \frac{E}{\gamma}$. It turns out that throughout our analysis, the bank always sets $A^* = \frac{E}{\gamma}$, making the bank's asset size choice A isomorphic to the minimum capital ratio set by the regulator γ. For brevity, we therefore omit discussions of asset size choice in future analysis.

4.1.2 Expected loss model with exogenous risk

We now analyze the expected loss model in which the bank reports s_1 so the regulator can either intervene early at $t = 1$ based on s_1 or late at $t = 2$ based on s. We again solve the model using backward induction. Note that the regulator's liquidation decision at $t = 2$ is the same as that in the incurred loss model because under both models, the regulator receives the full information s about the default risk. Facing the same liquidation policy at $t = 2$, the bank's asset-substituting decision also stays the same across both models. Nonetheless, there exists a key difference between the two loan loss models: under the expected loss model, the regulator may choose to intervene early at $t = 1$ upon receiving the bank's updated loan loss report s_1.

More specifically, conditional on a high-risk loan, the regulator never liquidates, as explained previously. Conditional on a low-risk loan, two key factors come into play when the regulator decides whether to liquidate: 1) the information s_1 and 2) the bank's future asset-substituting decision r. While the regulator receives a constant payoff L_α from liquidation, her expected payoff from continuation depends on both s_1 and r. If $s_1 \geq \bar{s}_1(A)$, the regulator rationally anticipates that the bank will keep the low-risk loan whose expected surplus is

$$\left[\int_{\frac{L_\alpha}{\alpha}}^{1} sf\left(s|s_1\right) ds\right] \alpha + \Pr\left[s < \frac{L_\alpha}{\alpha}|s_1\right] L_\alpha > L_\alpha. \tag{12}$$

That is, absent the asset-substitution problem, the regulator should never liquidate the bank early based on the timely but imperfect information s_1. This is because, the regulator can always postpone the decision to a later date (i.e., $t = 2$) when better information arrives (i.e., the default risk s is fully realized).

However, given the bank's asset substitution incentives, early intervention becomes desirable to curb such behavior. To see this, note that when the loan performance deteriorates, i.e., $s_1 < \bar{s}_1(A)$, the regulator anticipates that the bank will switch to the high-risk loan, which generates an expected surplus of $\tau\beta$. Since $L_\alpha \geq 1 > \tau\beta$ (Assumption 2), by curbing asset substitution, early intervention generates expected surplus gains. This represents the social benefit of timely intervention. Note, however, that the regulator faces a non trivial trade-off in intervening early in the bank's operations: early intervention is imperfect as it relies on imprecise information s_1 about future default risk s resulting in false alarm costs. For instance, the regulator may liquidate a bank that is financially sound ($s > L_\alpha/\alpha$) but nevertheless receives some bad interim information ($s_1 < \bar{s}_1(A)$). A rational and benevolent regulator will trade off these false-alarm costs against the benefits of intervention and choose an action that results in the highest social surplus *in expectation.* We formally state the regulator's liquidation decision at $t = 1$ in the following proposition.

Proposition 2 *Under the expected loss model, conditional on the early information s_1, the regulator liquidates the bank at $t = 1$ if and only if the bank has a low-risk loan and $s_1 < \bar{s}_1(A)$.*

4.1.3 Surplus comparison with exogenous risk

With the equilibrium characterized, for a given leverage A, we compare the surplus between under the incurred loss model and the expected loss model, holding the bank's *ex ante* risk choice q fixed. The surplus under incurred loss is

$$W_{IL}(A) = (1-q)A\tau\beta + q\Big(A\int_{\tau}^{\bar{s}_1(A)} \tau\beta f(s_1)ds_1 \\ + A\int_{\bar{s}_1(A)}^{1}\Big(\Big[\int_{\frac{L_\alpha}{\alpha}}^{1} sf\left(s|s_1\right)ds\Big]\alpha + \Pr\Big[s < \frac{L_\alpha}{\alpha}|s_1\Big]L_\alpha\Big)f(s_1)ds_1\Big) - A. \quad (13)$$

With probability $1-q$, the bank gets a high-risk loan and the expected surplus is $\tau\beta$, whereas with probability q, the bank gets a low-risk loan. The bank keeps the low-risk loan if $s_1 \geq \bar{s}_1(A)$ and switches to a high-risk one if $s_1 < \bar{s}_1(A)$.

Under the expected loss model, the surplus is

$$W_{EL}(A) = (1-q)A\tau\beta + q\Big(A\int_{\tau}^{\bar{s}_1(A)} L_\alpha f(s_1)ds_1 \\ + A\int_{\bar{s}_1(A)}^{1}\Big(\Big[\int_{\frac{L_\alpha}{\alpha}}^{1} sf\left(s|s_1\right)ds\Big]\alpha + \Pr\Big[s < \frac{L_\alpha}{\alpha}|s_1\Big]L_\alpha\Big)f(s_1)ds_1\Big) - A. \quad (14)$$

The only difference from the incurred loss model is that, if the bank gets a low-risk loan which occurs with probability q and $s_1 < \bar{s}_1(A)$, the bank is liquidated under the expected loss model whereas the bank is allowed to continue and asset-substitute under the incurred loss model. Recognizing this difference, we obtain:[7]

$$W_{EL}(A) - W_{IL}(A) = qA \left[\int_{\tau}^{\bar{s}_1(A)} (L_\alpha - \tau\beta)\, f(s_1) ds_1 \right] \geq 0. \tag{15}$$

The expected loss model always dominates the incurred loss model. Apparently, since the expected loss model dominates for all ranges of asset sizes A, it also dominates when the regulator sets the capital requirement policy optimally. We formally state this result in the following proposition.

Proposition 3 *Fixing the bank's ex ante risk q of the loan portfolio, the expected loss model always dominates the incurred loss model.*

Proposition 3 implies that the additional information revealed by the expected loan loss model generates an expected benefit because it allows the regulator to intervene in a more timely manner in banks' operations to curb inefficient asset substitution. Interestingly, such benefit cannot be overturned by the false-alarm cost stemming from using the imperfect information of the expected loss model. The reason is that, the regulator rationally takes into account the (im)precision of the early information and therefore internalizes such false alarm costs in determining whether to intervene. The preceding result therefore supports claims made by proponents of expected loss models who have argued that by providing more timely information about the performance of banks' loans, expected loss models would curb banks' excessive risk taking behavior. Proposition 3 confirms those views *as long as the ex ante riskiness of the banks' loan portfolios are kept fixed.* However, in the presence of real effects, banks may respond to changes in the regulator's intervention strategy that, in turn, depend on the loan loss provisioning model. We next investigate the impact of the loan loss provisioning models on banks' *ex ante* risk portfolios.

[7] The inequality is strict if $\bar{s}_1(A) > \tau$ (i.e., $A \in [E, A_{\min}]$ from Corollary 1).

4.2 Endogenous loan risk

We now analyze the full model in which the bank can choose the riskiness q of the loans at the origination stage.

4.2.1 Incurred loss model with endogenous risk

We start with the incurred loss model. For a given bank size A, the bank chooses risk q that solves

$$q_{IL}^{*} \in \arg\max_{q\in[0,1]} U(q) = \underbrace{\int_{\tau}^{\bar{s}_1(A)} U(q,1|s_1)\, g(s_1)\, ds_1}_{\text{expected payoff given asset substitution}} + \underbrace{\int_{\bar{s}_1(A)}^{1} U(q,0|s_1)\, g(s_1)\, ds_1}_{\text{expected payoff given no asset substitution}} - C(q)\,. \tag{16}$$

Note that if $s_1 < \bar{s}_1(A)$, the bank's payoff $U(q,1|s_1)$ is independent of its initial risk choice because the bank will engage in asset substitution and change the loan portfolio into a high-risk one. Thus, the *ex ante* risk choice only matters when the bank does not asset substitute. The higher the likelihood of interim asset substitution, the lower the bank's incentives to engage in costly *ex ante* screening, i.e., q_{IL}^{*} decreases in $\bar{s}_1(A)$. As the bank's leverage becomes very large, i.e., $A \in [A_{\max}, \bar{A}]$ so that $\bar{s}_1(A) = 1$, the bank always engages in interim asset substitution making the *ex ante* risk decision moot. In this case, since screening is costly, the bank chooses not to screen the borrowers *ex ante* so that $q_{IL}^{*} = 0$.

The first-order condition of the preceding equation with respect to q yields:

$$\int_{\bar{s}_1(A)}^{1} \frac{\partial U(q,0|s_1)}{\partial q} g(s_1)\, ds_1 = C'(q_{IL}^{*})\,. \tag{17}$$

The right hand side of equation (17) captures the marginal cost of screening borrowers whereas the left hand side captures the marginal benefit of screening stemming from reducing the future default risk. To see the latter effect, note that from equation (5),

$$\frac{\partial U\left(q,0|s_1\right)}{\partial q} = \left[\int_{\frac{L_\alpha}{\alpha}}^{1} sf\left(s|s_1\right)ds\right]\Big(A\alpha-(A-E)\Big)+\Pr\left[s<\frac{L_\alpha}{\alpha}|s_1\right]\Big(AL_\alpha-(A-E)\Big)\Big)-\tau(A\beta-(A-E)), \tag{18}$$

which is the incremental payoff from investing in the low-risk loan vs. the high-risk loan. It is positive because, by the incentive-compatibility constraint (8), the bank strictly prefers the low-risk loan for $s_1 \geq \bar{s}_1$.

We formally state the bank's *ex ante* risk choice in the following proposition.

Proposition 4 *Under the incurred loss model, the bank makes the risk choice q^*_{IL} such that, for $A \in [E, A_{\max})$, $q^*_{IL} \in (0,1)$ where q^*_{IL} solves equation (17), whereas for $A \in [A_{\max}, \bar{A}]$, $q^*_{IL} = 0$. The risk choice q^*_{IL} decreases in the size of the bank A, increases in the face value of the low-risk loan α and decreases in the face value of the high-risk loan β.*

4.2.2 Expected loss model with endogenous risk

The bank chooses risk q that solves

$$q^*_{EL} \in \arg\max_{q\in[0,1]} U\left(q\right) = \int_{\tau}^{\bar{s}_1(A)} U\left(q,1|s_1\right)g\left(s_1\right)ds_1 + \int_{\bar{s}_1(A)}^{1} U\left(q,0|s_1\right)g\left(s_1\right)ds_1 - C\left(q\right). \tag{19}$$

Note that when $s_1 < \bar{s}_1\left(A\right)$, the bank's expected payoff under the expected loss model differs from that under the incurred loss model. Under the expected loss model, if the regulator expects the performance of the bank's portfolio to deteriorate, the regulator intervenes and liquidates the bank with low-risk loan in order to prevent the bank from engaging in asset substitution. Using this equilibrium property, we obtain:

$$U\left(q,1|s_1\right) = q(AL_\alpha-(A-E)) + (1-q)\,\tau\left(A\beta-(A-E)\right). \tag{20}$$

Taking the first-order condition yields:

$$\int_{\tau}^{\bar{s}_1(A)} \left[(AL_\alpha - (A-E)) - \tau (A\beta - (A-E)) \right] g(s_1)\, ds_1 + \int_{\bar{s}_1(A)}^{1} \frac{\partial U(q, 0|s_1)}{\partial q} g(s_1)\, ds_1 = C'(q^*_{EL}). \tag{21}$$

To compare the risk choices across the loan loss models, we plug the first-order condition (17) on q^*_{IL} into the first order condition (21) to obtain:

$$\int_{\tau}^{\bar{s}_1(A)} \left[(AL_\alpha - (A-E)) - \tau (A\beta - (A-E)) \right] g(s_1) ds_1 = C'(q^*_{EL}) - C'(q^*_{IL}). \tag{22}$$

Since $C'' > 0$, $q^*_{EL} \geq q^*_{IL}$ if and only if

$$\underbrace{AL_\alpha - (A-E)}_{\text{risk disciplining effect}} - \underbrace{\tau (A\beta - (A-E))}_{\text{risk aggravating effect}} \geq 0. \tag{23}$$

The left hand side of (23) captures two supplementary forces at play in the region of $s_1 \in [\tau, \bar{s}_1(A)]$ under the expected loss model that are not present in the incurred loss model. The first term is positive and captures the risk-disciplining effect of early intervention. By reducing asset substitution, early intervention increases the bank's incentives to engage in costly screening as explained above. This is precisely the argument made by proponents of expected loss models: by reducing *ex post* asset substitution, regulatory intervention disciplines *ex ante* risk taking. However, when real effects are taken into account, our model shows that early intervention is a double-edged sword: while timely intervention curbs asset substitution, it could lead to timelier risk-taking. This is captured by the second term of (23). To understand this risk-aggravating effect, note that under the incurred loss model, the bank prefers to defer taking excessive amounts of risk until it receives more precise information on the performance of its loan portfolio. However, under the expected loss model, the option value of waiting for more precise information diminishes due to regulatory intervention that preempts asset substitution. Anticipating this, the bank shifts the timing of its risk-taking earlier by originating riskier loans *ex ante* under the expected loss model. In equilibrium, the bank trades off these two effects in determining how much risk to take *ex ante*. We next show that whether the risk-aggravating effect exceeds the risk-disciplining effect depends on the bank's leverage. It is straightforward to verify that (23) holds if and only if

$$A \leq A_r \equiv \frac{1-\tau}{\tau\beta + (1-\tau) - L_\alpha} E.[8] \tag{24}$$

If the bank has a low leverage ($A < A_r$) so that the risk-aggravating effect is not too severe, the bank chooses a lower risk under the expected loss model, i.e., $q^*_{EL} > q^*_{IL}$. Otherwise, if the bank has a high leverage ($A > A_r$) so that the risk-aggravating effect dominates, the bank chooses a higher risk under the expected loss model, i.e., $q^*_{EL} < q^*_{IL}$. Intuitively, a highly-leveraged bank has a strong incentive to asset-substitute. When that possibility is eliminated by regulatory intervention, the bank responds by building up more risk in its loan portfolio *ex ante.*

Finally, when A is sufficiently large and close to $\bar{A}$, the first-order condition (21) becomes:

$$\int_\tau^1 \left[\bar{A}\left(L_\alpha - \tau\beta\right) - (1-\tau)\left(\bar{A} - E\right)\right] g(s_1) ds_1 < 0. \tag{25}$$

Note that we used again the property that $\bar{s}_1\left(\bar{A}\right) = 1$. An application of the intermediate value theorem thus suggests that, there exists some cutoff $A_e \in (E, A_{\max})$ such that for $A \geq A_e$, we have $q^*_{EL} = 0$. The cutoff A_e is defined by

$$\int_\tau^{\bar{s}_1(A_e)} \left[A_e\left(L_\alpha - \tau\beta\right) - (1-\tau)\left(A_e - E\right)\right] g\left(s_1\right) ds_1 + \int_{\bar{s}_1(A_e)}^1 \frac{\partial U\left(q, 0|s_1\right)}{\partial q} g\left(s_1\right) ds_1 = 0. \tag{26}$$

[8] Note that using Assumption 3, we can show that $\tau\beta + (1-\tau) - L_\alpha > 0$ so that A_r is strictly positive.

In other words, whenever when $A \geq A_{\max}$, the region in which $q^*_{EL} = 0$ is larger than that of $q^*_{IL} = 0$ We formally state these results in the following proposition.

Proposition 5 *Under the expected loss model, the bank makes the risk choice q^*_{EL} such that, if $A \in [E, A_e)$, $q^*_{EL} \in (0,1)$ and solves equation (21). Otherwise, for $A \in [A_e, \bar{A}]$, $q^*_{EL} = 0$. In addition, $q^*_{EL} < q^*_{IL}$ if and only if $A > A_r$. For $A > A_r$, the risk choice q^*_{EL} decreases in the size of the bank A, increases in the face value of the low-risk loan α and decreases in the face value of the high-risk loan β.*

Proposition 5 implies that when real effects are taken into account, there are potential costs of adopting the expected loss model. While timely regulatory intervention provides discipline in terms of curbing *ex post* inefficient asset substitution, banks may respond to such intervention by originating riskier loan portfolios that potentially reduce surplus. We next investigate whether the costs triggered by such real effects may outweigh the benefits of timely intervention under an expected loss model.

4.2.3 Surplus comparison with endogenous risk

To compare the results with endogenous risk choice with those with exogenous risk choice (Proposition 3), we first compare the surplus between the two models for *all ranges* of asset sizes A.

Proposition 6 *With the risk choice endogenized, there exist two thresholds A_r, A_e where $A_r \leq A_e$, such that:*

1. *if $A \leq A_r$, the expected loss model dominates the incurred loss model;*

2. *if $A \geq A_e$, the incurred loss model dominates the expected loss model.*

Proposition 6 shows that, when we take into account the real effects of accounting measurements, the more timely information under the expected loss model is no longer always socially desirable.

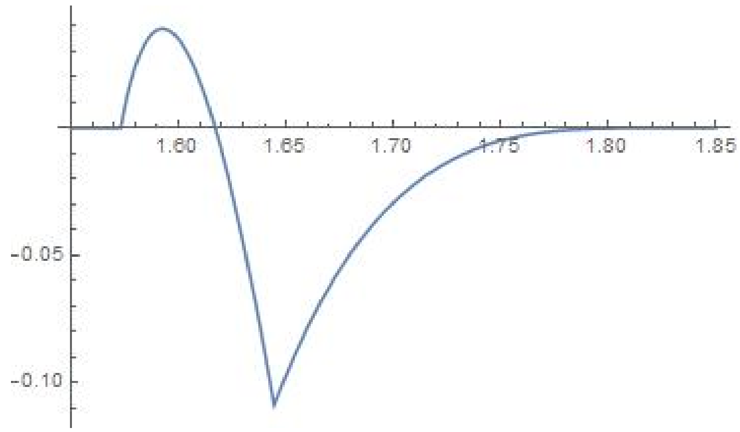

Figure 2: Plot of welfare difference between the expected loss model and the incurred loss model as a function of asset size

While the expected loss model remains superior when banks' leverages are low, it becomes inferior when banks are heavily leveraged. To see the latter, consider the case that $A \geq A_e$ so that $q^*_{EL} = 0 < q^*_{IL}$. When the bank is highly leveraged, under the expected loss model, the bank always chooses the high-risk loan whereas under the incurred loss model, the bank chooses the low-risk loan with some probability, thus resulting in a higher surplus.

The fact that the expected loss model improves the surplus if the leverage is sufficiently low and reduces the surplus if the leverage is sufficiently high suggests there exists a unique cutoff such that the expected loss model reduces the surplus when the leverage is above the cutoff. Although the complexity of our model prevents us from obtaining an analytical proof, numerical analyses suggest that such a conjecture is indeed true, as illustrated in Figure 2 which shows that $W_{EL} < W_{IL}$ if and only if the leverage A is large.

An implication of Proposition 6 is that the regulator should take into account such real effects when designing the capital requirement policy that, in turn, governs the bank's leverage. Stated differently, changing the accounting method for loan loss provisioning requires the bank regulator to simultaneously adjust the capital adequacy ratios. To shed some light on how the regulator should make such adjustments, we next solve for the optimal capital ratios under the two loan loss models.

Recall that the bank always chooses to invest up to the maximum allowed under the capital requirement, i.e., $A^* = \frac{E}{\gamma}$. Thus choosing the capital ratio γ is isomorphic to choosing the bank size A. For expositional purposes, we describe the regulator's choice of γ as if she were choosing A. We first reproduce the surplus under the incurred loss model, equation (13), here:

$$
\begin{aligned}
W_{IL}(A) = (1 - q_{IL}^*)A\tau\beta + q_{IL}^*\Bigg(A \int_{\tau}^{\bar{s}_1(A)} \tau\beta g(s_1) ds_1 \\
+ A \int_{\bar{s}_1(A)}^{1} \left(\left[\int_{\frac{L_\alpha}{\alpha}}^{1} sf\left(s|s_1\right) ds \right] \alpha + \Pr\left[s < \frac{L_\alpha}{\alpha} | s_1 \right] L_\alpha \right) g(s_1) ds_1 \Bigg) - A. \quad (27)
\end{aligned}
$$

Taking the first-order condition yields

$$
\begin{aligned}
\frac{\partial W_{IL}}{\partial A} &= NPV_{IL}\left(\bar{s}_1(A)\right) + q_{IL}^* A \frac{\partial \bar{s}_1}{\partial A} \left(\tau\beta - \left[\int_{\frac{L_\alpha}{\alpha}}^{1} sf\left(s|\bar{s}_1(A)\right) ds \right] \alpha - \Pr\left[s < \frac{L_\alpha}{\alpha} | \bar{s}_1(A) \right] L_\alpha \right) g(\bar{s}_1(A)) \\
&\quad + \frac{\partial q_{IL}^*}{\partial A} \left(A \int_{\bar{s}_1}^{1} \left(\left[\int_{\frac{L_\alpha}{\alpha}}^{1} sf\left(s|s_1\right) ds \right] \alpha + \Pr\left[s < \frac{L_\alpha}{\alpha} | s_1 \right] L_\alpha - \tau\beta \right) g(s_1) ds_1 \right). \quad (28)
\end{aligned}
$$

The first term in $\frac{\partial W_{IL}}{\partial A}$,

$$
\begin{aligned}
NPV_{IL}\left(\bar{s}_1(A)\right) &= (1 - q_{IL}^*)\tau\beta + q_{IL}^* \Bigg(\int_{\tau}^{\bar{s}_1} \tau\beta g(s_1) ds_1 \\
&\quad + \int_{\bar{s}_1}^{1} \left(\left[\int_{\frac{L_\alpha}{\alpha}}^{1} sf\left(s|s_1\right) ds \right] \alpha + \Pr\left[s < \frac{L_\alpha}{\alpha} | s_1 \right] L_\alpha \right) g(s_1) ds_1 \Bigg) - 1, \quad (29)
\end{aligned}
$$

measures the per-unit NPV from the bank's loan portfolio and represents the potential social benefit of increasing the bank's size. It is straightforward to verify the NPV is positive if and only if the asset size is sufficiently small. The reason is that, if the bank is highly leveraged, it will convert its entire loan portfolio into high-risk in the interim, which generates a negative NPV in expectation by Assumption 2.

The other two terms in $\frac{\partial W_{IL}}{\partial A}$ are both negative and represent the social costs of increasing the bank's size. In particular, the second term captures the effect of increasing the asset size in motivating the bank to asset substitute *ex post* whereas the third term captures the effect of increasing the asset size in discouraging the bank from exerting screening effort *ex ante*.[9]

The regulator sets the optimal capital requirement ratio by trading off the above benefit and the costs. We denote the optimal bank size under the incurred loss model by A_{IL}^* which solves the first-order condition (28). The optimal bank size under the expected loss model can be similarly derived. We denote it by A_{EL}^*.[10] Because A_{IL}^* and A_{EL}^* are defined by implicit solutions to differential equations, in general, we are unable to compare them. However, we next derive a sufficient condition under which $A_{EL}^* > A_{IL}^*$, i.e., the regulator should lower the capital ratio in response to the adoption of the expected loss model.

Proposition 7 *If the surplus functions are sufficiently concave, then the regulator sets tighter capital requirements under the incurred loss model than under the expected loss model, i.e.,* $A_{IL}^* < A_{EL}^*$ *so that the expected loss model dominates the incurred loss model, i.e.,* $W_{EL}(A_{EL}^*) > W_{IL}(A_{IL}^*)$.

Proposition 7 states that when the marginal cost of increasing bank size is sufficiently high compared to the marginal benefit, the regulator may be able to relax the capital requirements when banks use the expected loss model for loan loss provisioning. More importantly, provided that the capital requirements are optimally set, the expected loss model generates higher surplus than the incurred loss model. While Proposition 6 states that, fixing the capital requirement policy, the additional information released under expected loss may impair surplus, Proposition 7 shows that such adverse effect can be overturned if the regulator can appropriately fine-tune the capital

[9] Mathematically, the second term in $\frac{\partial W_{IL}}{\partial A}$ is negative because 1) from Proposition 1, $\frac{\partial \bar{s}_1}{\partial A} > 0$ and 2) $\left[\int_{\frac{L_\alpha}{\alpha}}^1 sf\left(s|\bar{s}_1(A)\right)ds\right]\alpha + \Pr\left[s < \frac{L_\alpha}{\alpha}|\bar{s}_1(A)\right]L_\alpha > L_\alpha > \tau\beta$ (Assumption 2). The third term is negative because, 1) from Proposition 4, $\frac{\partial q_{IL}^*}{\partial A} < 0$ and 2) $\left[\int_{\frac{L_\alpha}{\alpha}}^1 sf\left(s|s_1\right)ds\right]\alpha + \Pr\left[s < \frac{L_\alpha}{\alpha}|s_1\right]L_\alpha > L_\alpha > \tau\beta$.

[10] The first-order condition that defines A_{EL}^* is complicated and given in the proof of Proposition 7.

ratio in response to the loan loss model. Under the optimal capital requirements, the expected loss model supplies the regulator with valuable early warning signals on banks' loan performance, thus facilitating regulatory intervention and improving the social surplus. Indeed, given the better information under expected loss, the regulator can actually relax the capital constraint since the regulation has become more effective. These results echo the recent call for better coordination between accounting and bank regulation.

5 Conclusion

To be added later.

Appendix: proofs

Proof. of Proposition 1: The existence of the cutoff $\bar{s}_1$ has been proved in the main text. We now derive some comparative statics on $\bar{s}_1 \in (\tau, 1)$, which is defined such that

$$\tau = \left[\int_{\frac{L_\alpha}{\alpha}}^{1} sf\left(s|\bar{s}_1\right) ds\right] \frac{A\alpha - (A-E)}{A\beta - (A-E)} + \Pr\left[s < \frac{L_\alpha}{\alpha}|\bar{s}_1\right] \frac{AL_\alpha - (A-E)}{A\beta - (A-E)}. \tag{30}$$

which is equivalent to

$$\tau\left(A\beta - (A-E)\right) = \left[\int_{\frac{L_\alpha}{\alpha}}^{1} sf\left(s|\bar{s}_1\right) ds\right] \left(A\alpha - (A-E)\right) + \Pr\left[s < \frac{L_\alpha}{\alpha}|\bar{s}_1\right] \left(AL_\alpha - (A-E)\right). \tag{31}$$

Taking the derivative of (31) with respect to A, we get

$$\begin{aligned}\tau(\beta-1) = &\left[\frac{\partial \bar{s}_1}{\partial A}\int_{\frac{L_\alpha}{\alpha}}^{1} \frac{\partial sf\left(s|\bar{s}_1\right)}{\partial s_1} ds\right] \left(A\alpha - (A-E)\right) + \left[\frac{\partial \bar{s}_1}{\partial A}\int_{\tau}^{\frac{L_\alpha}{\alpha}} \frac{\partial f\left(s|\bar{s}_1\right)}{\partial s_1} ds\right] \left(AL_\alpha - (A-E)\right)\\ &+ \left[\int_{\frac{L_\alpha}{\alpha}}^{1} sf\left(s|\bar{s}_1\right) ds\right] (\alpha - 1) + \Pr\left[s < \frac{L_\alpha}{\alpha}|\bar{s}_1\right] (L_\alpha - 1),\end{aligned} \tag{32}$$

which is equivalent to

$$\frac{\partial \bar{s}_1}{\partial A} = \frac{\tau(\beta-1) - \left[\int_{\frac{L_\alpha}{\alpha}}^{1} sf\left(s|\bar{s}_1\right) ds\right](\alpha-1) - \Pr\left[s < \frac{L_\alpha}{\alpha}|\bar{s}_1\right](L_\alpha - 1)}{\left[\int_{\frac{L_\alpha}{\alpha}}^{1} \frac{\partial sf(s|\bar{s}_1)}{\partial s_1} ds\right]\left(A\alpha - (A-E)\right) + \left[\int_{\tau}^{\frac{L_\alpha}{\alpha}} \frac{\partial f(s|\bar{s}_1)}{\partial s_1} ds\right]\left(AL_\alpha - (A-E)\right)}. \tag{33}$$

Hence, we have $\frac{\partial \bar{s}_1}{\partial A} \geq 0$.

Taking the derivative of (31) with respect to β, we get

$$\tau A = \left[\frac{\partial \bar{s}_1}{\partial \beta}\int_{\frac{L_\alpha}{\alpha}}^{1} \frac{\partial sf\left(s|\bar{s}_1\right)}{\partial s_1} ds\right] \left(A\alpha - (A-E)\right) + \left[\frac{\partial \bar{s}_1}{\partial \beta}\int_{\tau}^{\frac{L_\alpha}{\alpha}} \frac{\partial f\left(s|\bar{s}_1\right)}{\partial s_1} ds\right] \left(AL_\alpha - (A-E)\right), \tag{34}$$

which is equivalent to

$$\frac{\partial \bar{s}_1}{\partial \beta} = \frac{\tau A}{\left[\int_{\frac{L_\alpha}{\alpha}}^{1} \frac{\partial s f(s|\bar{s}_1)}{\partial s_1} ds\right] \left(A\alpha - (A - E)\right) + \left[\int_{\tau}^{\frac{L_\alpha}{\alpha}} \frac{\partial f(s|\bar{s}_1)}{\partial s_1} ds\right] \left(AL_\alpha - (A - E)\right)}. \tag{35}$$

Hence, we have $\frac{\partial \bar{s}_1}{\partial \beta} \geq 0$.

Finally, taking the derivative of (31) with respect to α, we get

$$\begin{aligned} 0 = \left[\frac{\partial \bar{s}_1}{\partial \alpha} \int_{\frac{L_\alpha}{\alpha}}^{1} \frac{\partial s f(s|\bar{s}_1)}{\partial s_1} ds + \frac{L_\alpha^2}{\alpha^3} f\left(\frac{L_\alpha}{\alpha}|\bar{s}_1\right)\right] \left(A\alpha - (A - E)\right) + \left[\int_{\frac{L_\alpha}{\alpha}}^{1} s f(s|\bar{s}_1)\, ds\right] A \\ + \left[\frac{\partial \bar{s}_1}{\partial \alpha} \int_{\tau}^{\frac{L_\alpha}{\alpha}} \frac{\partial f(s|\bar{s}_1)}{\partial s_1} ds - \frac{L_\alpha}{\alpha^2} f\left(\frac{L_\alpha}{\alpha}|\bar{s}_1\right)\right] \left(AL_\alpha - (A - E)\right), \end{aligned} \tag{36}$$

which is equivalent to

$$\frac{\partial \bar{s}_1}{\partial \alpha} = \frac{-\frac{L_\alpha^2}{\alpha^3} f\left(\frac{L_\alpha}{\alpha}|\bar{s}_1\right)\left(A\alpha - (A - E)\right) - \left[\int_{\frac{L_\alpha}{\alpha}}^{1} s f(s|\bar{s}_1)\, ds\right] A + \frac{L_\alpha}{\alpha^2} f\left(\frac{L_\alpha}{\alpha}|\bar{s}_1\right)\left(AL_\alpha - (A - E)\right)}{\left[\int_{\frac{L_\alpha}{\alpha}}^{1} \frac{\partial s f(s|\bar{s}_1)}{\partial s_1} ds\right] \left(A\alpha - (A - E)\right) + \left[\int_{\tau}^{\frac{L_\alpha}{\alpha}} \frac{\partial f(s|\bar{s}_1)}{\partial s_1} ds\right] \left(AL_\alpha - (A - E)\right)}. \tag{37}$$

Hence, we have $\frac{\partial \bar{s}_1}{\partial \alpha} \leq 0$. ■

Proof. of Corollary 1: At one extreme, $A = E$ (the lower bound for A), condition (8) becomes

$$\tau\beta > \left[\int_{\frac{L_\alpha}{\alpha}}^{1} s f(s|s_1)\, ds\right] \alpha + \Pr\left[s < \frac{L_\alpha}{\alpha}|s_1\right] L_\alpha. \tag{38}$$

Inequality (38) is never satisfied by Assumption 1:

$$\tau\beta < 1 \leq \left[\int_{\frac{L_\alpha}{\alpha}}^{1} s f(s|s_1 = \tau)\, ds\right] \alpha + \Pr\left[s < \frac{L_\alpha}{\alpha}|s_1 = \tau\right] L_\alpha. \tag{39}$$

That is, at $A = E$, the bank always makes the no asset-substituting decision such that $\bar{s}_1 = \tau$. Intuitively, the bank fully internalizes the surplus consequences of the risk decision and makes the efficient no asset-substituting decision.

At the other extreme of $A = \bar{A}$, by Assumption 3, the bank always makes the asset-substituting decision such that $\bar{s}_1 = 1$. Notice that from the bank's incentive-compatibility constraint (8), this requires that

$$\begin{aligned}&\tau(\bar{A}\beta - (\bar{A} - E)) \\ &\qquad > \left[\int_{\frac{L_\alpha}{\alpha}}^{1} sf\left(s|s_1\right)ds\right]\left(\bar{A}\alpha - (\bar{A} - E)\right) + \Pr\left[s < \frac{L_\alpha}{\alpha}|s_1\right]\left(\bar{A}L_\alpha - (\bar{A} - E)\right). \quad (40)\end{aligned}$$

Since $\left[\int_{\frac{L_\alpha}{\alpha}}^{1} sf\left(s|\bar{s}_1\right)ds\right]$ is increasing in $\bar{s}_1$, the right-hand side of (9) is increasing in $\bar{s}_1$, and by the intermediate value theorem, there exists some $A_{\min} > E$ such that $\bar{s}_1\left(A_{\min}\right) = \tau$ and some $A_{\max} < \bar{A}$ such that $\bar{s}_1\left(A_{\max}\right) = 1$. For $A \in [A_{\min}, A_{\max}]$, $\bar{s}_1\left(A\right) \in [\tau, 1]$; for $A < A_{\min}$, $\bar{s}_1\left(A\right) = \tau$; for $A > A_{\max}$, $\bar{s}_1\left(A\right) = 1$. The variable $A_{\max}$ is defined such that

$$\tau = \left[\int_{\frac{L_\alpha}{\alpha}}^{1} sf\left(s|s_1 = 1\right)ds\right]\frac{A_{\max}\alpha - (A_{\max} - E)}{A_{\max}\beta - (A_{\max} - E)} + \Pr\left[s < \frac{L_\alpha}{\alpha}|s_1 = 1\right]\frac{A_{\max}L_\alpha - (A_{\max} - E)}{A_{\max}\beta - (A_{\max} - E)}, \quad (41)$$

which is equivalent to

$$\tau(A_{\max}\beta - (A_{\max} - E)) = \left[\int_{\frac{L_\alpha}{\alpha}}^{1} sf\left(s|s_1 = 1\right)ds\right]\left(A_{\max}\alpha - (A_{\max} - E)\right) + \Pr\left[s < \frac{L_\alpha}{\alpha}|s_1 = 1\right]\left(A_{\max}L_\alpha - (A_{\max} - E)\right). \quad (42)$$

Taking the derivative of (42) with respect to β, we get

$$\tau(\beta - 1)\frac{\partial A_{\max}}{\partial \beta} + \tau A_{\max} = \frac{\partial A_{\max}}{\partial \beta}\left(\int_{\frac{L_\alpha}{\alpha}}^{1} sf\left(s|s_1 = 1\right)ds(\alpha - 1) + \Pr\left[s < \frac{L_\alpha}{\alpha}|s_1 = 1\right]\left(L_\alpha - 1\right)\right), \quad (43)$$

which is equivalent to

$$\frac{\partial A_{\max}}{\partial\beta} = \frac{\tau A_{\max}}{\int_{\frac{L_\alpha}{\alpha}}^{1} sf\left(s|s_1=1\right)ds(\alpha-1) + \Pr\left[s < \frac{L_\alpha}{\alpha}|s_1=1\right]\left(L_\alpha - 1\right) - \tau(\beta-1)} < 0. \tag{44}$$

Taking the derivative of (42) with respect to α, we get

$$\frac{\partial A_{\max}}{\partial\alpha} = \frac{-\int_{\frac{L_\alpha}{\alpha}}^{1} sf(s|s=1)dsA_{\max} - \frac{L_\alpha^2}{\alpha^3}f\left(\frac{L_\alpha}{\alpha}|s_1=1\right)\left(A_{\max}\alpha - (A_{\max}-E)\right)}{\int_{\frac{L_\alpha}{\alpha}}^{1} sf\left(s|s_1=1\right)ds(\alpha-1) + \Pr\left[s < \frac{L_\alpha}{\alpha}|s_1=1\right]\left(L_\alpha - 1\right) - \tau(\beta-1)}$$
$$+ \frac{\frac{L_\alpha}{\alpha^2}f\left(\frac{L_\alpha}{\alpha}|s_1=1\right)\left(A_{\max}L_\alpha - (A_{\max}-E)\right)}{\int_{\frac{L_\alpha}{\alpha}}^{1} sf\left(s|s_1=1\right)ds(\alpha-1) + \Pr\left[s < \frac{L_\alpha}{\alpha}|s_1=1\right]\left(L_\alpha - 1\right) - \tau(\beta-1)} > 0. \tag{45}$$

The variable $A_{\min}$ is defined such that

$$\tau = \left[\int_{\frac{L_\alpha}{\alpha}}^{1} sf\left(s|s_1=0\right)ds\right]\left(\frac{A_{\min}\alpha - (A_{\min}-E)}{A_{\min}\beta - (A_{\min}-E)}\right) + \Pr\left[s < \frac{L_\alpha}{\alpha}|s_1=0\right]\left(\frac{A_{\min}L_\alpha - (A_{\min}-E)}{A_{\min}\beta - (A_{\min}-E)}\right), \tag{46}$$

Hence, similar computations yield to

$$\frac{\partial A_{\min}}{\partial\beta} < 0 \text{ and } \frac{\partial A_{\min}}{\partial\alpha} > 0. \tag{47}$$

■

Proof. of Proposition 2: See the main text. ■

Proof. of Proposition 3: See the main text. ■

Proof. of Proposition 4: We know that, for $A \in [E, A_{\max})$, q_{IL}^* is such that

$$\int_{\bar{s}_1(A)}^{1} \left[\int_{\frac{L_\alpha}{\alpha}}^{1} sf\left(s|s_1\right)ds\right]\left(\left(A\alpha-(A-E)\right)+\Pr\left[s<\frac{L_\alpha}{\alpha}|s_1\right]\left(AL_\alpha-(A-E)\right)\right)g(s_1)ds_1$$
$$-\int_{\bar{s}_1(A)}^{1} \tau\left(A\beta-(A-E)\right)f(s_1)ds_1 = C'\left(q_{IL}^*\right). \quad (48)$$

Using the implicit function theorem,

$$\begin{aligned}
& C''(q_{IL}^*)\frac{\partial q_{IL}^*}{\partial A} \\
= & \int_{\bar{s}_1(A)}^{1} \left[\int_{\frac{L_\alpha}{\alpha}}^{1} sf\left(s|s_1\right)ds\right]\left((\alpha-1)+\Pr\left[s<\frac{L_\alpha}{\alpha}|s_1\right](L_\alpha-1)-\tau\left(\beta-1\right)\right)g(s_1)ds_1 \\
& +\frac{\partial \bar{s}_1}{\partial A}\left(\left[\int_{\frac{L_\alpha}{\alpha}}^{1} sf\left(s|\bar{s}_1(A)\right)ds\right]\left(\left(A\alpha-(A-E)\right)+\Pr\left[s<\frac{L_\alpha}{\alpha}|\bar{s}_1(A)\right]\left(AL_\alpha-(A-E)\right)\right.\right. \\
- & \left.\left.\tau\left(A\beta-(A-E)\right)\right)\right)g(\bar{s}_1(A)) \\
= & \int_{\bar{s}_1(A)}^{1} \left(\left[\int_{\frac{L_\alpha}{\alpha}}^{1} sf\left(s|s_1\right)ds\right]\left((\alpha-1)+\Pr\left[s<\frac{L_\alpha}{\alpha}|s_1\right]\left(L_\alpha-1\right)-\tau\left(\beta-1\right)\right)\right)g(s_1)ds_1.
\end{aligned}$$

The second step uses

$$\left[\int_{\frac{L_\alpha}{\alpha}}^{1} sf\left(s|\bar{s}_1(A)\right)ds\right]\left(\left(A\alpha-(A-E)\right)+\Pr\left[s<\frac{L_\alpha}{\alpha}|\bar{s}_1(A)\right]\left(AL_\alpha-(A-E)\right)\right)=\tau\left(A\beta-(A-E)\right).$$

Note that assumption (3) implies

$$\left[\int_{\frac{L_\alpha}{\alpha}}^{1} sf\left(s|s_1\right)ds\right]\left((\alpha-1)+\Pr\left[s<\frac{L_\alpha}{\alpha}|s_1\right]\left(L_\alpha-1\right)\right)<\tau\left(\beta-1\right),$$

which implies that

$$\frac{\partial q_{IL}^*}{\partial A}<0.$$

Taking the derivative of (48) with respect to α, we get

$$\int_{\bar{s}_1(A)}^{1} \frac{\partial}{\partial \alpha} \left[\int_{\frac{L_\alpha}{\alpha}}^{1} sf\left(s|s_1\right) ds\right] \left(A\alpha - (A-E)\right) + \Pr\left[s < \frac{L_\alpha}{\alpha}|s_1\right] \left(AL_\alpha - (A-E)\right) g(s_1)ds_1$$
$$+ \frac{\partial \bar{s}_1}{\partial \alpha}\Bigg(- \left[\int_{\frac{L_\alpha}{\alpha}}^{1} sf\left(s|s_1\right) ds\right] \left(A\alpha - (A-E)\right) - \Pr\left[s < \frac{L_\alpha}{\alpha}|s_1\right] \left(AL_\alpha - (A-E)\right)$$
$$+ \tau\left(A\beta - (A-E)\right)\Bigg) g(\bar{s}_1) = C''\left(q^*_{IL}\right) \frac{\partial q^*_{IL}}{\partial \alpha}. \quad (49)$$

Hence, we get $\frac{\partial q^*_{IL}}{\partial \alpha} > 0$.

Lastly, taking the derivative of (48) with respect to β, we get

$$- \int_{\bar{s}_1(A)}^{1} \tau A g(s_1) ds_1$$
$$+ \frac{\partial \bar{s}_1}{\partial \beta}\Bigg(- \left[\int_{\frac{L_\alpha}{\alpha}}^{1} sf\left(s|s_1\right) ds\right] \left(A\alpha - (A-E)\right) - \Pr\left[s < \frac{L_\alpha}{\alpha}|s_1\right] \left(AL_\alpha - (A-E)\right)$$
$$+ \tau\left(A\beta - (A-E)\right)\Bigg) g(\bar{s}_1) = C''\left(q^*_{IL}\right) \frac{\partial q^*_{IL}}{\partial \beta}. \quad (50)$$

Hence, we get $\frac{\partial q^*_{IL}}{\partial \beta} < 0$. ■

Proof. of Proposition 5: We know that

$$C'(q^*_{IL}) + \int_{\tau}^{\bar{s}_1(A)} [A(L_\alpha - \tau\beta) - (1-\tau)(A-E)] g(s_1) ds_1 = C'(q^*_{EL}). \quad (51)$$

Taking the derivative of (51) with respect to A, we get

$$C''(q^*_{EL}) \frac{\partial q^*_{EL}}{\partial A} = C''(q^*_{IL}) \frac{\partial q^*_{IL}}{\partial A} + \int_{\tau}^{\bar{s}_1(A)} [L_\alpha - 1 - \tau(\beta - 1)] g(s_1) ds_1$$
$$+ \frac{\partial \bar{s}_1}{\partial A}\left(A(L_\alpha - \tau\beta) - (1-\tau)(A-E)\right) g(\bar{s}_1(A)). \quad (52)$$

The second term on the right-hand side is negative whereas the third term on the right-hand side is negative if and only $A > A_r$. Therefore, we have $\frac{\partial q_{EL}^*}{\partial A} < 0$ for $A > A_r$.

Taking the derivative of (51) with respect to α, we get

$$C''(q_{EL}^*)\frac{\partial q_{EL}^*}{\partial \alpha} = C''(q_{IL}^*)\frac{\partial q_{IL}^*}{\partial \alpha} + \frac{\partial \bar{s}_1}{\partial \alpha}\left(A(L_\alpha - \tau\beta) - (1-\tau)(A-E)\right) g(\bar{s}_1(A)). \tag{53}$$

The second-term on the right-hand side is positive if and only if $A > A_r$. Therefore, we have $\frac{\partial q_{EL}^*}{\partial \alpha} > 0$ for $A > A_r$.

Lastly, taking the derivative of (51) with respect to β, we get

$$C''(q_{EL}^*)\frac{\partial q_{EL}^*}{\partial \beta} = C''(q_{IL}^*)\frac{\partial q_{IL}^*}{\partial \beta} + \frac{\partial \bar{s}_1}{\partial \beta}\left(A(L_\alpha - \tau\beta) - (1-\tau)(A-E)\right) g(\bar{s}_1(A)) - \int_\tau^{\bar{s}_1} \tau A g(s_1) ds_1. \tag{54}$$

Therefore, we get $\frac{\partial q_{EL}^*}{\partial \beta} < 0$ for $A > A_r$. ■

Proof. of Proposition 6: The surplus is given by

$$W_{IL}(A) = (1 - q_{IL}^*)\, A\tau\beta + q_{IL}^*\left(A\int_\tau^{\bar{s}_1(A)} \tau\beta g(s_1) ds_1 + A\int_{\bar{s}_1(A)}^1 \left(\left(\int_{\frac{L_\alpha}{\alpha}}^1 sf(s|s_1)\, ds\right)\alpha + \Pr\left[s < \frac{L_\alpha}{\alpha}|s_1\right] L_\alpha\right) g(s_1) ds_1\right) - A,$$

and

$$W_{EL}(A) = (1 - q_{EL}^*)\, A\tau\beta + q_{EL}^*\left(A\int_\tau^{\bar{s}_1(A)} L_\alpha g(s_1) ds_1 + A\int_{\bar{s}_1(A)}^1 \left(\left(\int_{\frac{L_\alpha}{\alpha}}^1 sf(s|s_1)\, ds\right)\alpha + \Pr\left[s < \frac{L_\alpha}{\alpha}|s_1\right] L_\alpha\right) g(s_1) ds_1\right) - A.$$

First, consider the case that $A \in [A_{\max}, \bar{A}]$. We have $q^*_{IL} = q^*_{EL} = 0$, which implies that $W_{IL}(A) = W_{EL}(A) = A\tau\beta - A$.

Second, if $A \in [A_e, A_{\max})$, we have $q^*_{EL} = 0$ and $q^*_{IL} \in (0,1)$, which implies that

$$W_{EL}\left(A\right) = A\tau\beta - A < 0, \tag{55}$$

and

$$\begin{aligned} W_{IL}(A) &= A\tau\beta - A \\ &\quad + q^*_{IL} \left(A \int_{\bar{s}_1}^{1} \left(\left[\int_{\frac{L_\alpha}{\alpha}}^{1} sf\left(s|s_1\right) ds \right] \alpha + \Pr\left[s < \frac{L_\alpha}{\alpha} |s_1 \right] L_\alpha - \tau\beta \right) g(s_1) ds_1 \right). \end{aligned}$$

That is, $W_{IL}(A) > W_{EL}(A)$. Then, if $A \in (A_{\min}, A_r]$, we have $q^*_{EL} \geq q^*_{IL} > 0$. Thus

$$\begin{aligned} & W_{EL}(A) - W_{IL}(A) \\ = \; & q^*_{EL} \left(A \int_{\tau}^{\bar{s}_1} L_\alpha g(s_1) ds_1 + A \int_{\bar{s}_1}^{1} \left(\left[\int_{\frac{L_\alpha}{\alpha}}^{1} sf\left(s|s_1\right) ds \right] \alpha + \Pr\left[s < \frac{L_\alpha}{\alpha} |s_1 \right] L_\alpha \right) g(s_1) ds_1 - A\tau\beta \right) \\ & - q^*_{IL} \left(A \int_{\tau}^{\bar{s}_1} \tau\beta g(s_1) ds_1 + A \int_{\bar{s}_1}^{1} \left(\left[\int_{\frac{L_\alpha}{\alpha}}^{1} sf\left(s|s_1\right) ds \right] \alpha + \Pr\left[s < \frac{L_\alpha}{\alpha} |s_1 \right] L_\alpha \right) g(s_1) ds_1 - A\tau\beta \right) \\ \geq \; & q^*_{IL} \left(A \int_{\tau}^{\bar{s}_1} L_\alpha g(s_1) ds_1 + A \int_{\bar{s}_1}^{1} \left(\left[\int_{\frac{L_\alpha}{\alpha}}^{1} sf\left(s|s_1\right) ds \right] \alpha + \Pr\left[s < \frac{L_\alpha}{\alpha} |s_1 \right] L_\alpha \right) g(s_1) ds_1 - A\tau\beta \right) \\ & - q^*_{IL} \left(A \int_{\tau}^{\bar{s}_1} \tau\beta g(s_1) ds_1 + A \int_{\bar{s}_1}^{1} \left(\left[\int_{\frac{L_\alpha}{\alpha}}^{1} sf\left(s|s_1\right) ds \right] \alpha + \Pr\left[s < \frac{L_\alpha}{\alpha} |s_1 \right] L_\alpha \right) g(s_1) ds_1 - A\tau\beta \right) \\ = \; & q^*_{IL} A \int_{\tau}^{\bar{s}_1} \left(L_\alpha - \tau\beta\right) g(s_1) ds_1 > 0. \end{aligned} \tag{56}$$

Lastly, if $A \in [E, A_{\min}]$, we have $q^*_{EL} = q^*_{IL}$ and $\bar{s}_1 = \tau$, which implies that

$$\begin{aligned} W_{IL}(A) = W_{EL}(A) = {} & \left(1 - q^*_{IL}\right) A\tau\beta \\ & + q^*_{IL} A \int_{\tau}^{1} \left(\left[\int_{\frac{L_\alpha}{\alpha}}^{1} s f\left(s|s_1\right) ds \right] \alpha + \Pr\left[s < \frac{L_\alpha}{\alpha} | s_1 \right] L_\alpha \right) g(s_1) ds_1 - A. \quad (57) \end{aligned}$$

Proof. of Proposition 7: The first-order condition on A^*_{EL} is given by:

$$\begin{aligned} \frac{\partial W_{EL}}{\partial A} = {} & NPV_{EL}\left(\bar{s}_1(A)\right) + q^*_{EL} A \frac{\partial \bar{s}_1}{\partial A} \left(L_\alpha - \left[\int_{\frac{L_\alpha}{\alpha}}^{1} s f\left(s|\bar{s}_1(A)\right) ds \right] \alpha - \Pr\left[s < \frac{L_\alpha}{\alpha} | \bar{s}_1(A) \right] L_\alpha \right) g(\bar{s}_1(A)) \\ & + A \frac{\partial q^*_{EL}}{\partial A} \left(\int_{\tau}^{\bar{s}_1} L_\alpha g(s_1) ds_1 + \int_{\bar{s}_1}^{1} \left(\left[\int_{\frac{L_\alpha}{\alpha}}^{1} s f\left(s|s_1\right) ds \right] \alpha + \Pr\left[s < \frac{L_\alpha}{\alpha} | s_1 \right] L_\alpha \right) g(s_1) ds_1 - \tau\beta \right), \end{aligned}$$

where

$$\begin{aligned} NPV_{EL}\left(\bar{s}_1(A)\right) = {} & (1 - q^*_{EL})\tau\beta + q^*_{EL} \left(\int_{\tau}^{\bar{s}_1(A)} L_\alpha g(s_1) ds_1 \right. \\ & \left. + \int_{\bar{s}_1(A)}^{1} \left(\left[\int_{\frac{L_\alpha}{\alpha}}^{1} s f\left(s|s_1\right) ds \right] \alpha + \Pr\left[s < \frac{L_\alpha}{\alpha} | s_1 \right] L_\alpha \right) g(s_1) ds_1 \right) - 1. \end{aligned}$$

Taking the difference between $\frac{\partial W_{EL}}{\partial A}$ and $\frac{\partial W_{IL}}{\partial A}$ gives that:

$$
\begin{aligned}
& \frac{\partial \left(W_{EL} - W_{IL}\right)}{\partial A} \qquad (58)\\
= \; & \frac{\partial q_{EL}^*}{\partial A} \left(A \int_{\tau}^{\bar{s}_1} L_\alpha g(s_1) ds_1 + A \int_{\bar{s}_1}^{1} \left(\left[\int_{\frac{L_\alpha}{\alpha}}^{1} s f\left(s|s_1\right) ds \right] \alpha + \Pr\left[s < \frac{L_\alpha}{\alpha} | s_1 \right] L_\alpha \right) g(s_1) ds_1 - A\tau\beta \right) \\
& - \frac{\partial q_{IL}^*}{\partial A} \left(A \int_{\tau}^{\bar{s}_1} \tau\beta g(s_1) ds_1 + A \int_{\bar{s}_1}^{1} \left(\left[\int_{\frac{L_\alpha}{\alpha}}^{1} s f\left(s|s_1\right) ds \right] \alpha + \Pr\left[s < \frac{L_\alpha}{\alpha} | s_1 \right] L_\alpha \right) g(s_1) ds_1 - A\tau\beta \right) \\
& + q_{EL}^* \left(\int_{\tau}^{\bar{s}_1} L_\alpha g(s_1) ds_1 + \int_{\bar{s}_1}^{1} \left(\left[\int_{\frac{L_\alpha}{\alpha}}^{1} s f\left(s|s_1\right) ds \right] \alpha + \Pr\left[s < \frac{L_\alpha}{\alpha} | s_1 \right] L_\alpha \right) g(s_1) ds_1 - \tau\beta \right) \\
& - q_{IL}^* \left(\int_{\tau}^{\bar{s}_1} \tau\beta g(s_1) ds_1 + \int_{\bar{s}_1}^{1} \left(\left[\int_{\frac{L_\alpha}{\alpha}}^{1} s f\left(s|s_1\right) ds \right] \alpha + \Pr\left[s < \frac{L_\alpha}{\alpha} | s_1 \right] L_\alpha \right) g(s_1) ds_1 - \tau\beta \right) \\
& + q_{EL}^* \frac{\partial \bar{s}_1}{\partial A} A g(\bar{s}_1) \left(L_\alpha - \left(\left[\int_{\frac{L_\alpha}{\alpha}}^{1} s f\left(s|\bar{s}_1\right) ds \right] \alpha + \Pr\left[s < \frac{L_\alpha}{\alpha} | \bar{s}_1 \right] L_\alpha \right) \right) \\
& - q_{IL}^* \frac{\partial \bar{s}_1}{\partial A} A g(\bar{s}_1) \left(\tau\beta - \left(\left[\int_{\frac{L_\alpha}{\alpha}}^{1} s f\left(s|\bar{s}_1\right) ds \right] \alpha + \Pr\left[s < \frac{L_\alpha}{\alpha} | \bar{s}_1 \right] L_\alpha \right) \right)
\end{aligned}
$$

We know that for $A < A_r$, we have $q_{EL}^* > q_{IL}^*$. Hence, for $A < A_r$, we have

$$
\begin{aligned}
& q_{EL}^* \left(\int_{\tau}^{\bar{s}_1} L_\alpha g(s_1) ds_1 + \int_{\bar{s}_1}^{1} \left(\left[\int_{\frac{L_\alpha}{\alpha}}^{1} s f\left(s|s_1\right) ds \right] \alpha + \Pr\left[s < \frac{L_\alpha}{\alpha} | s_1 \right] L_\alpha \right) g(s_1) ds_1 - \tau\beta \right) \\
& - q_{IL}^* \left(\int_{\tau}^{\bar{s}_1} \tau\beta g(s_1) ds_1 + \int_{\bar{s}_1}^{1} \left(\left[\int_{\frac{L_\alpha}{\alpha}}^{1} s f\left(s|s_1\right) ds \right] \alpha + \Pr\left[s < \frac{L_\alpha}{\alpha} | s_1 \right] L_\alpha \right) g(s_1) ds_1 - \tau\beta \right) > 0.
\end{aligned}
$$

Further, we know that

$$
C'(q_{IL}^*) + \int_{\tau}^{\bar{s}_1(A)} [A(L_\alpha - \tau\beta) - (1-\tau)(A-E)] g(s_1) ds_1 = C'(q_{EL}^*), \qquad (59)
$$

which implies that for A close to $A_{\min}$, we have $\bar{s}_1(A)$ close to τ and q_{EL}^* close to q_{IL}^*. Hence, for

A close to $A_{\min}$, we have

$$
\begin{aligned}
& q_{EL}^{*}\frac{\partial \bar{s}_1}{\partial A}Ag(\bar{s}_1)\left(L_\alpha-\left(\left[\int_{\frac{L_\alpha}{\alpha}}^{1} sf\left(s|\bar{s}_1\right)ds\right]\alpha+\Pr\left[s<\frac{L_\alpha}{\alpha}|\bar{s}_1\right]L_\alpha\right)\right) \\
& -q_{IL}^{*}\frac{\partial \bar{s}_1}{\partial A}Ag(\bar{s}_1)\left(\tau\beta-\left(\left[\int_{\frac{L_\alpha}{\alpha}}^{1} sf\left(s|\bar{s}_1\right)ds\right]\alpha+\Pr\left[s<\frac{L_\alpha}{\alpha}|\bar{s}_1\right]L_\alpha\right)\right) \\
= \; & \left(q_{EL}^{*}-q_{IL}^{*}\right)\frac{\partial \bar{s}_1}{\partial A}Ag(\bar{s}_1)\left(L_\alpha-\left(\left[\int_{\frac{L_\alpha}{\alpha}}^{1} sf\left(s|\bar{s}_1\right)ds\right]\alpha+\Pr\left[s<\frac{L_\alpha}{\alpha}|\bar{s}_1\right]L_\alpha\right)\right) \\
& +q_{IL}^{*}\frac{\partial \bar{s}_1}{\partial A}Ag(\bar{s}_1)\left(L_\alpha-\tau\beta\right).
\end{aligned}
$$

The second term is positive while the first term is negative. The second term dominates the first term if $q_{EL}^{*}-q_{IL}^{*}$ is sufficiently small, which holds when A close to $A_{\min}$.

Lastly, we know that

$$
\begin{aligned}
C''(q_{EL}^{*})\frac{\partial q_{EL}^{*}}{\partial A} = C''(q_{IL}^{*})\frac{\partial q_{IL}^{*}}{\partial A} &+ \int_{\tau}^{\bar{s}_1(A)}\left[L_\alpha-1-\tau(\beta-1)\right]g(s_1)ds_1 \\
&+\frac{\partial \bar{s}_1}{\partial A}\left(A(L_\alpha-\tau\beta)-(1-\tau)(A-E)\right)g(\bar{s}_1(A)), \quad (60)
\end{aligned}
$$

which implies that for A close to $A_{\min}$, we have $\frac{\partial q_{EL}^{*}}{\partial A}>\frac{\partial q_{IL}^{*}}{\partial A}$. Hence, for A close to $A_{\min}$, we have

$$
\begin{aligned}
& \frac{\partial q_{EL}^{*}}{\partial A}\left(A\int_{\tau}^{\bar{s}_1}L_\alpha g(s_1)ds_1+A\int_{\bar{s}_1}^{1}\left(\left[\int_{\frac{L_\alpha}{\alpha}}^{1} sf\left(s|s_1\right)ds\right]\alpha+\Pr\left[s<\frac{L_\alpha}{\alpha}|s_1\right]L_\alpha\right)g(s_1)ds_1-A\tau\beta\right) \\
& -\frac{\partial q_{IL}^{*}}{\partial A}\left(A\int_{\tau}^{\bar{s}_1}\tau\beta g(s_1)ds_1+A\int_{\bar{s}_1}^{1}\left(\left[\int_{\frac{L_\alpha}{\alpha}}^{1} sf\left(s|s_1\right)ds\right]\alpha+\Pr\left[s<\frac{L_\alpha}{\alpha}|s_1\right]L_\alpha\right)g(s_1)ds_1-A\tau\beta\right) \\
> \; & 0.
\end{aligned}
$$

Therefore, for all A close to $A_{\min}$, we get

$$
\frac{\partial\left(W_{EL}-W_{IL}\right)}{\partial A}>0, \quad (61)
$$

i.e., there exists a cutoff $A_+ \in (A_{\min}, A_e)$ such that, for all $A \in [E, A_+)$, we have $\frac{\partial(W_{EL}-W_{IL})}{\partial A} > 0$.

Finally, suppose that $A^*_{IL} \in [E, A_+)$ and $\frac{\partial^2 W_{EL}}{\partial A^2} < 0$. Plugging $A = A^*_{IL}$ into $\frac{\partial W_{EL}}{\partial A}$ gives that

$$\frac{\partial W_{EL}}{\partial A}|_{A=A^*_{IL}} > \frac{\partial W_{IL}}{\partial A}|_{A=A^*_{IL}} = 0. \tag{62}$$

The inequality uses $\frac{\partial(W_{EL}-W_{IL})}{\partial A} > 0$ for $A < A_+$. By the concavity of W_{EL}, $\frac{\partial W_{EL}}{\partial A}|_{A=A^*_{IL}} > 0 = \frac{\partial W_{EL}}{\partial A}|_{A=A^*_{EL}}$ implies that $A^*_{IL} < A^*_{EL}$. Since $W_{EL}(A^*_{EL}) > W_{EL}(A^*_{IL})$, Proposition 6 implies that $W_{EL}(A^*_{EL}) > W_{EL}(A^*_{IL}) > W_{IL}(A^*_{IL})$. ■