

**NYU Stern School of Business**  
**Department of Information, Operations & Management Sciences**  
**IOMS Colloquium Series**

**DATE:** Friday, April 1, 2016

**TIME:** 12:30 PM – 1:45PM (Lunch served at 12:15 pm)

**PLACE:** KMC 4-60

**ABSTRACTS:**

**Feature-based Dynamic Pricing**

*Ilan Lobel (NYU Stern)*

We consider the problem faced by a firm who receives highly differentiated products in an online fashion and needs to price them in order to sell them to its customer base. Products are described by vectors of features and the market value of each product is linear in the values of the features. The firm does not initially know the values of the different features, but it can learn the values of the features based on whether products were sold at the posted prices in the past. This model is motivated by a question in online advertising, where impressions arrive over time and can be described by vectors of features. We first consider a multi-dimensional version of binary search, and show that it has exponential worst-case regret. We then show that a modification of the prior algorithm, where uncertainty sets are replaced by their Lowner-John ellipsoids has a worst-case regret that is quadratic in the dimensionality of the feature space and logarithmic in the time horizon. Joint work with Maxime Cohen and Renato Paes Leme.

**Reflected Brownian Motion in a Wedge**

*Peter Lakner (NYU Stern)*

Reflected Brownian motion (RBM) in a wedge introduced by Varadhan and Williams in a seminal paper in 1984, is a 2-dimensional stochastic process whose state space is a wedge  $S$  with angle  $\zeta$  at the vertex. In the interior of the wedge the process behaves as a 2-dimensional Brownian motion, whereas upon hitting the boundary of  $S$  it is reflected back into the interior at an angle that depends upon which edge of the boundary has been hit. The angles of reflection off the edges are denoted by  $\theta_1$  and  $\theta_2$ . These are measured with respect to the inward facing normal vector off each edge, with angles directed toward the vertex assumed to be positive. In a series of papers Williams showed that the parameter  $\alpha = (\theta_1 + \theta_2)/\zeta$  plays a crucial role in the study of this process. In particular, it is known that for  $1 \leq \alpha < 2$  the RBM is not a semimartingale, i.e., can not be decomposed as a sum of a local martingale and a process with sample paths of bounded variation. In the present paper we show that for  $1 \leq \alpha < 2$  the RBM is a Dirichlet process, i.e., it has a decomposition  $Z = X + Y$ , where  $X$  is a two-dimensional Brownian motion, and  $Y$  has zero quadratic variation. Furthermore, we show that for  $\alpha < p$  the strong  $p$ -variation of the sample paths of  $Y$  is finite, and for  $0 < p \leq \alpha$  the same  $p$ -

variation is infinity. We also show that on excursion intervals of  $Z$  away from the origin  $(Z, Y)$  satisfies the Skorohod problem for  $X$ . However, on the entire time horizon  $(Z, Y)$  does not satisfy the Skorohod problem for  $X$ , since the paths of  $Y$  do not have bounded variation. In a series of papers Kang and Ramanan introduced the concept of Extended Skorohod Problem (ESP), which does not require bounded variation for the sample paths of  $Y$ . They have shown that  $(Z, Y)$  satisfies the ESP for  $X$  if the vectors representing the angles of the reflection off the edges point exactly in the opposite direction, i.e.,  $\alpha = 1$ . We show that  $(Z, Y)$  satisfies the ESP for  $X$  for all values  $1 \leq \alpha < 2$ . Joint work with Josh Reed.