Robots, Trade, and Luddism: A Sufficient Statistic Approach to Optimal Technology Regulation*

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Abstract

Technological change, from the advent of robots to expanded trade opportunities, tends to create winners and losers. When are such changes welcome? How should government policy respond? We provide a general theory of optimal technology regulation in a second best world, with rich heterogeneity across households, linear taxes on the subset of firms affected by technological change, and a nonlinear tax on labor income. Our first result shows that, despite limited tax instruments, technological progress is always welcome and valued in the same way as in a first best world. Our second set of results consists of three optimal tax formulas, with minimal structural assumptions, involving sufficient statistics that can be implemented using evidence on the distributional impact of new technologies, such as robots and trade. Our final result is a comparative static exercise illustrating that while distributional concerns create a rationale for non-zero taxes on robots and trade, the magnitude of these taxes may decrease as the process of automation and globalization deepens and inequality increases.

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1 Introduction

Robots and artificial intelligence technologies are on the rise. So are imports from China and other developing countries. These changes create opportunities for some workers, destroy opportunities for others, and generate significant distributional consequences, as documented in the recent empirical work of Autor, Dorn and Hanson (2013) and Acemoglu and Restrepo (2017b) for the United States.

When should technological change be welcome? Should any policy response be in place? And if so, how should we manage new technologies? Should we become more luddites as machines become more efficient or more protectionist as trade opportunities expand? The goal of this paper is to provide a general second-best framework to help address these and other related questions.

Answers to these questions necessarily depend on the range of available policy instruments. At one extreme, if lump-sum transfers are available, as in the Second Welfare Theorem, or if linear taxes are available on all goods and factors, as in Diamond and Mirrlees (1971a,b) and Dixit and Norman (1980), then distribution can be done without distorting production. In such cases, production efficiency implies the optimality of zero taxes on robots and free trade. At another extreme, in the absence of any policy instrument, whenever technological progress creates at least one loser, a welfare criterion must be consulted and the status quo may be preferred.

Here, we focus on a more realistic intermediate situation where tax instruments are available, but are more limited than those ensuring production efficiency. We consider two sets of technologies, which we refer to as *old* and *new*. For instance, firms using the new technology may be producers of robots or traders that export some goods in exchange for others. Since we are interested in the optimal regulation of the new technology, we do not impose any restriction on the linear taxes faced by firms using that technology, e.g. taxes on robots or trade. In contrast, we restrict the set of taxes that can be imposed on households' labor supply to be a function of their income, but not on their type, as in Mirrlees (1971). In the economic environment that we consider, the after-tax wage structure can be influenced by tax policy, but not completely controlled, which creates a meaningful trade-off between redistribution and efficiency.

Our first result focuses on the welfare impact of new technologies under the assumption that constrained, but optimal policies are in place. We offer a novel envelope result that generalizes the evaluation of productivity shocks in first-best environments, as in Solow (1957) and Hulten (1978), to distorted economies. Because of restrictions on the set of available tax instruments, marginal rates of substitution may not be equalized across agents and marginal rates of transformation may not be equalized between new and old technology firms. Yet, "Immiserizing Growth," as in Bhagwati (1958), never arises. Provided that governments are free to tax new technology firms, the welfare impact of technological change can be measured in the exact same way as in first best environments, despite not being first best. Terms-of-trade shocks, for instance, are beneficial if and only if they raise the value of the trade balance at current quantities.

A direct implication of our envelope result is that even if new technologies tend to have a disproportionate effect on the wages of less skilled workers, and one cares about redistribution, this does not create any new rationale for taxes and subsidies on innovation, that is, no reason to distort technology adoption by firms.

Our second set of results characterizes the structure of optimal taxes on new technology firms. In a two-type environment, Naito (1999) has proven that governments seeking to redistribute income from high- to low-skill workers may have incentives to depart from production efficiency. Doing so manipulates relative wages, which cannot be taxed directly, and relaxes incentive compatibility constraints. Our general analysis goes beyond this qualitative insight by allowing for rich heterogeneity across households and deriving optimal tax formulas expressed in terms of sufficient statistics that are, at least in principle, empirically measurable.

Specifically, we provide three novel optimal tax formulas. Each formula provides different insights and involves its own set of sufficient statistics, but they all give a central role to the impact on the wage distribution. Although the response of wages to robots or trade is of obvious empirical interest for descriptive reasons, our formulas show how it also provides a sufficient statistic for optimal policy design. Given knowledge of this statistic, the specific structure of the economy leading to a change in wages can be left in the background. For example, it is not necessary to take a stand on how robots and workers may be combined to perform different tasks, or on how production processes may get fragmented across countries. While these features may be critical in shaping the impact of new technologies on wages, all that is needed according to our formulas is knowledge of this impact, not how it comes about.

We illustrate the usefulness of our approach by exploring the magnitude of optimal taxes on robots and trade. We focus on the third of our formulas, which can be implemented without taking a stand on preferences for redistribution, since it does not involve social welfare weights. Using the reduced-form evidence of Acemoglu and Restrepo (2017b) on the impact of robots in the United States, we find efficient taxes on robots ranging from 1% to 3.7%. In contrast, the evidence of Chetverikov, Larsen and Palmer (2016) on the impact of Chinese imports on U.S. inequality points towards much smaller efficient tariffs, between 0.03% to 0.11%. While the estimated impact of robots and Chinese imports on wages is of similar magnitude, robots account for a much smaller share of the U.S. economy. According to our third formula, this calls for a much larger tax on robots than trade.

Our final result is a comparative static exercise that asks: as progress in Artificial Intelligence and other areas makes for cheaper and better robots, should we tax them more? Or in a trade context, does hyper-globalization call for hyper-protection? We do so by constructing a simple economy in which the government has Rawlsian preferences and cheaper machines, either robots or imported machines from China, increase in inequality. In spite of both the government's extreme preference for redistribution and the negative impact of technological progress on inequality, we show that improvements in new technologies are associated with lower taxes on firms using those technologies. Thus, as the process of automation and globalization deepens, more inequality may best be met with lower Luddism and less protectionism.

Related Literature

Our paper makes three distinct contributions to the existing literature. The first one is a new perspective on the welfare impact of technological progress in the presence of distortions. In a first best world, the impact of small productivity shocks can be evaluated, absent any restriction on preferences and technology, using a simple envelope argument as in Solow (1957) and Hulten (1978). With distortions, evaluating the welfare impact of productivity shocks, in general, requires additional information about whether such shocks aggravate or alleviate underlying distortions. In an environment with markups, for instance, this boils down to whether employment is reallocated towards goods with higher or lower markups, as in Baeqee and Farhi (2017). If the aggravation of distortions is large enough, technological progress may even lower welfare, as discussed by Bhagwati (1971). Here, we follow a different approach. Our analysis builds on the idea that while economies may be distorted and tax instruments may be limited, the government may still have access to policy instruments to regulate the new technology. If so, we show that the envelope results of Solow (1957) and Hulten (1978) still hold, with direct implications for the taxation of innovation and the valuation of new technologies.

Our second contribution is a general characterization of the structure of optimal taxes in environments with restricted factor income taxation. In so doing, we fill a gap between the general analysis of Diamond and Mirrlees (1971a,b) and Dixit and Norman (1980), which assumes that linear taxes on all factors are available, and specific examples, typically with two goods and two factors, in which only income taxation is available, as in the original work of Naito (1999), and subsequent work by Guesnerie (1998), Spector (2001), Naito (2006), and Jacobs (2015).[1] On the broad spectrum of restrictions on available policy instruments, one can also view our analysis as an intermediate step between the work of Diamond and Mirrlees (1971a,b) and Dixit and Norman (1980) and the trade policy literature where it is common to assume that the only instruments available for redistribution are trade taxes. In fact, one of our three formulas is a strict generalization of the formulas reviewed by Helpman (1997), including Grossman and Helpman's (1994) tariff formula.

Our third contribution is to offer a more specific application of our general formulas to the taxation of robots and trade. In recent work, Guerreiro, Rebelo and Teles (2017) and Thuemmel (2018) have studied a model with both heterogeneous workers as well as robots. Assuming factor-specific taxes are unavailable, they find a non-zero tax on robots to be generally optimal, in line with Naito (1999). Although we share the same rationale for finding nonzero taxes on robots, based on Naito (1999), our main goal is not to sign the tax on robots, nor to explore a particular production structure, but instead to offer tax formulas highlighting key sufficient statistics needed to determine the level of taxes, with fewer structural assumptions. In this way, our formulas provide a foundation for empirical work as well as the basis for novel comparative static results. In another recent contribution, Hosseini and Shourideh (2018) analyze a multi-country Ricardian model of trade with input-output linkages and imperfect mobility of workers across sectors. Although sector-specific taxes on labor are not explicitly allowed, these missing taxes can be perfectly mimicked by the available tax instruments. By implication, their economy provides an alternative implementation but fits Diamond and Mirrlees (1971b,a) and Dixit and Norman (1980, 1986), where households face a complete set of linear taxes, including sector-specific taxes on labor. Production efficiency and free trade then follow, just as they did in Diamond and Mirrlees (1971a,b).[2]

[1] In the first three papers, like in Dixit and Norman (1980), the new technology is international trade. In another related trade application, Feenstra and Lewis (1994) study an environment where governments cannot subject different worker types to different taxes, but can offer subsidies to workers moving from one industry to another in response to trade. They provide conditions under which such a trade adjustment assistance program are sufficient to guarantee Pareto gains from trade, as in Dixit and Norman (1980).

[2] A separate line of work, e.g. Itskhoki (2008), Antras, de Gortari and Itskhoki (2017) and Tsyvinski and Werquin (2018), studies technological changes such as trade or robots, without considering taxes on these new technologies, but instead focusing on how the income tax schedule may respond to these changes.

2 Environment

We consider an economy with an arbitrary number of goods and a continuum of heterogeneous households supplying labor. Households have the same preferences, but differ in their skills. We allow this heterogeneity to be multi-dimensional, unlike the classical one-dimensional Mirrleesian model. For instance, a household may be more productive at some tasks, but less productive at others, as in a Roy model. Households sell their labor in competitive labor markets and pay nonlinear taxes on their earnings to the government. Production is carried out by competitive firms. The government may linearly tax transactions between firms and households as well as the transactions that take place between firms, inducing production inefficiency. This is the focus of our analysis.

2.1 Preferences

All households have identical and weakly separable preferences between goods and labor. The utility of household θ is given by

$$U(\theta) = u(C(\theta), n(\theta)),$$
$$C(\theta) = v(c(\theta)),$$

where $C(\theta)$ is the sub-utility that household θ derives from consuming goods, $n(\theta)$ is her labor supply, $c(\theta) \equiv \{c_i(\theta)\}$ is her vector of good consumption, and u and v are quasi-concave and strictly increasing utility functions.

2.2 Technology

Households are distinguished by their skill $\theta \in \Theta \subseteq \mathbb{R}^n$ with distribution F. Each skill type θ provides a distinct labor input for use in production. We assume that, for at least one of the elements of $\theta = (\theta_1, \theta_2, \ldots, \theta_n)$, higher values are associated with higher productivity (thus, commanding higher wages).

We divide technologies into two types, which we refer to as *old* and *new*, each associated with a distinct production set. In our applications, the old technology is how most production takes place, while the new technology captures either trade with the rest of the world or the production of machines, like robots. The dichotomy between old and new technologies is what allows us to consider the taxation of transactions between firms and the resulting aggregate production inefficiency. Without such taxation we could consolidate technology into a single aggregate production set.

Old Technology. Let $y \equiv \{y_i\}$ denote the vector of total net output by old technology firms and let $n \equiv \{n(\theta)\}$ denote the schedule of their total labor demand. Positive values for y_i represent output, while negative y_i represent inputs. The production set associated with the old technology corresponds to all production plans (y, n) such that

$$G(y, n) \leq 0,$$

where G is some convex and homogeneous function of (y, n). Homogeneity of G implies constant returns to scale.

Except for constant returns to scale, we impose no restriction on the old technology. This allows for arbitrary production networks and global supply chains. For instance, old technology firms may be able to produce a final good by executing a continuum of tasks, with each task chosen to be performed by either workers or robots, as in Acemoglu and Restrepo (2017a), or by domestic or foreign workers, as in Grossman and Rossi-Hansberg (2008). In such environments, the production possibility frontier G can be derived from a subproblem that solves the optimal assignment of workers and other inputs to tasks. The commodity vector y then consists of the final good produced and the intermediate goods demanded, i.e., the robots or foreign labor services supplied by new technology firms, but omits tasks as they become subsumed in the definition of G. Appendix B.1 provides the formal mapping between production functions that explicitly model tasks and our general production possibility frontier G.

New Technology. Let $y^* \equiv \{y_i^*\}$ denote the vector of total net output by new technology firms. The production set associated with the new technology corresponds to all production plans y^* such that

$$G^*(y^*; \phi) \leq 0,$$

where G^* is some convex and homogeneous function of y^* and $\phi > 0$ is a productivity parameter. We assume that G^* is decreasing in ϕ so that an increase in ϕ corresponds to an improvement in the new technology.

Unlike the old technology, the new technology does not employ labor directly. This assumption fits well our applications to robots and trade. In the first case, new technology firms may be robot-producers that transform a composite of all other goods in the economy, call it gross output, into robots. This is the standard way to model capital accumulation in a neoclassical growth model.[3] In the second case, new technology firms may

[3]Due to constant returns to scale, profits are zero for the owners of new technology firms. In our framework, one can capture windfall gains of initial owners by introducing endowments of robots. This

be traders who export and import goods,

$$G^*(y^*;\phi) = \bar{p}(\phi) \cdot y^*,$$

where $\bar{p}(\phi) \equiv \{\bar{p}_i(\phi)\}$ denotes the vector of world prices, and $\cdot$ denotes the inner product of two vectors. An increase in ϕ corresponds to a positive terms-of-trade shock that may be due to a change in transportation costs or productivity in the rest of the world.

Abstracting from labor in the new technology is convenient, as it implies that wages are determined by the old technology. New technology has an effect on wages, through its effect on the structure of production within the old technology, but not directly through employment. For a fixed value of ϕ, this restriction is without loss of generality since the new technology sector can always be defined as the last stage of production where taxation is imposed, as described in Appendix B.2. For comparative static exercises, the omission of labor from G^* implicitly restricts attention to changes in ϕ that are labor-neutral in the sense that that they do not induce changes in wages for given prices faced by the old technology firms.

Resource Constraint. For all goods, the demand by households is equal to the supply by old and new technology firms,

$$\int c(\theta) dF(\theta) = y + y^*.$$

2.3 Prices and Taxes

Factors. Let $w \equiv \{w(\theta)\}$ denote the schedule of wages faced by firms. Because of income taxation, a household with ability θ and labor earnings $w(\theta)n(\theta)$ retains

$$w(\theta)n(\theta) - T(w(\theta)n(\theta)),$$

where $T(w(\theta)n(\theta))$ denotes its total tax payment. Crucially, the income tax schedule T is the same for all households. This rules out agent-specific lump-sum transfers, in contrast to the Second Welfare Theorem, as well as factor-specific linear taxes, in contrast to the analysis of Diamond and Mirrlees (1971a,b) and Dixit and Norman (1980).

leads to issues similar to those found in the literature on capital taxation, with the optimum possibly calling for expropriatory levels of taxation of such initial wealth.

Goods. Let $p^* \equiv \{p_i^*\}$ denote the vector of good prices faced by new technology firms. Because of ad-valorem taxes $t^* \equiv \{t_i^*\}$, these prices may differ from the vector of good prices $p \equiv \{p_i\}$ faced by old technology firms and households,

$$p_i = (1 + t_i^*)p_i^*, \text{ for all } i.$$

Production inefficiency arises if $t^* \neq 0$ because of the wedge created between the two technologies. In the robot context the tax in question might be a tax on robots produced by the new technology and employed in the old technology. In a trade context, an import tariff or an export subsidy on good i corresponds to $t_i^* > 0$, whereas an import subsidy or an export tax corresponds to $t_i^* < 0$. Since demand and supply only depend on relative prices, we can normalize prices and taxes such that $p_1 = p_1^* = 1$ and $t_1^* = 0$. We maintain this normalization throughout.[4]

3 Equilibrium, Social Welfare, and Government Problem

We now define the equilibrium for this economy, introduce our general social welfare criterion, and describe the government problem.

3.1 Equilibrium

An equilibrium consists of an allocation, $c \equiv \{c(\theta)\}$, $n \equiv \{n(\theta)\}$, $C \equiv \{C(\theta)\}$, $y \equiv \{y_i\}$, and $y^* \equiv \{y_i^*\}$, prices and wages, $p \equiv \{p_i\}$, $p^* \equiv \{p_i^*\}$, and $w \equiv \{w(\theta)\}$, as well as an income tax schedule, T, and taxes on new technology firms, $t^* \equiv \{t_i^*\}$, such that: (i) households maximize their utility, (ii) firms maximize profits, (iii) markets clear, and (iv) prices satisfy the non-arbitrage condition, as described in Appendix C.

The equilibrium determination of wages is central to our analysis. As shown in Appendix C, profit maximization by old technology firms implies a wage schedule

$$w(p, n; \theta)$$

that depends on prices p and labor n. By affecting the labor demand of old technology firms, changes in p affect wages. Given the limited ability of the government to tax different factors differently, this creates a pecuniary motive for taxing goods. This is the key

[4] Section 5.3 discusses the generalization of our results to environments where ad-valorem taxes on old technology firms, $t \equiv \{t_i\}$, are also available. In such case, the government may also create a wedge between the prices $p \equiv \{p_i\}$ faced by old technology firms and those faced by consumers, $q \equiv \{q_i\}$.

mechanism at play in our optimal tax formulas.

3.2 Social Welfare

We consider a general social welfare criterion that depends on the distribution of individual well-beings, not the particular well-being of certain agents. Any consumption and labor supply schedule $(c, n) \equiv \{c(\theta), n(\theta)\}$ is associated with a utility schedule $U \equiv \{U(\theta)\}$. This, in turn, induces a cumulative distribution over utilities, summarized by the utility levels $\bar{U} \equiv \{\bar{U}(z)\}$ associated with each quantile $z \in [0, 1]$. The social welfare objective is assumed to be a strictly increasing function W of this induced distribution,

$$W(\bar{U}).$$

When θ is one dimensional and higher-θ households achieve higher utility, as in the standard Mirrleesian setup, this nests the special case of a weighted utilitarian objective. When θ is multidimensional, our assumption about W only restricts Pareto weights to be the same for all households θ earning the same wage, since they obtain the same utility.

3.3 Government Problem

The government problem is to select a competitive equilibrium with taxes that maximizes social welfare. A compact statement of the government problem is as follows:

$$\max_{(c,n,y,y^*,p,p^*,w,T,t^*,\bar{U})\in\Omega} W(\bar{U})$$

subject to

$$G^*(y^*; \phi) \leq 0,$$

where the feasible set Ω imposes all equilibrium conditions except the resource constraint associated with the new technology, $G^*(y^*; \phi) \leq 0$, as also described in Appendix C.

4 The Valuation of Technological Change

We first study the welfare impact of new technologies under the assumption that constrained, but optimal policies are in place. In the presence of distortions, it is well-known that technological progress may lower welfare. This is what Edgeworth (1884) and Bhagwati (1958) refer to as "Economic Damnification" and "Immesirizing Growth". Since governments may want to depart from production efficiency in order to achieve their distributional objectives in the present environment, one might expect the previous phenomenon to arise here as well. We now demonstrate that this is not the case.

4.1 An Envelope Result

Let $V(\phi)$ denote the value function associated with the government problem. As shown in Appendix D, it is also equal to the value function of the relaxed problem,

$$V(\phi) = \max_{(c,N,n,y,y^*,p,p^*,w,T,t^*,\bar{U}) \in \Omega_R} W(\bar{U})$$

subject to

$$G^*(y^*;\phi) \leq 0,$$

with $\Omega_R \subset \Omega$ requiring utility maximization by households, profit maximization by old technology firms, and market clearing, but not profit maximization by new technology firms. Crucially, this implies that Ω_R is independent of ϕ.

Now consider a small productivity shock from ϕ to $\phi + d\phi$. The Envelope Theorem immediately implies

$$\frac{dV}{d\phi} = -\gamma \frac{\partial G^*}{\partial \phi}, \tag{1}$$

with $\gamma > 0$ the Lagrange multiplier associated with the constraint $G^*(y^*;\phi) \leq 0$.[5]

This envelope condition can be thought of as a generalized version of Hulten's (1978) Theorem. In spite of the economy not being first-best, the welfare impact of technological progress can be measured in the exact same way as in first-best environments. This leads to our first proposition.

Proposition 1. *Technological change increases social welfare, $dV/d\phi > 0$, if and only if it expands the production possibility of new technology firms, that is, if and only if $\partial G^*/\partial \phi < 0$.*

The critical assumption here is not that there are no distortions or, equivalently, that our planner has enough tax instruments to target any underlying distortion. In fact, the planning problem described above would also hold in an environment with externalities and other market imperfections, such as price and wage rigidities leading to labor market distortions. Proposition 1 instead follows from the assumption that, in spite of distortions

[5]In general, the Lagrange multiplier γ could be equal to zero if preferences were satiated or if the government was unable, or unwilling due to distributional concerns, to transfer extra income to households. The assumption that u, v, and W are strictly increasing rules out the first issue. Appendix E.3 shows that the availability of non-linear income taxation rules out the second issue as well.

and restrictions on the set of available instruments, our planner still has a tax instruments t^* to control the new technology firms, which is where technological change occurs.

In the case of international trade, for which $G^*(y^*;\phi) = \sum_i \bar{p}_i(\phi) y_i^*$, equation (1) implies that a country gains from a terms-of-trade shock if and only if it raises the value of its net exports, evaluated at the initial quantities,

$$\sum_i \bar{p}_i'(\phi)(-y_i^*) > 0.$$

Intuitively, whenever world prices $\bar{p}(\phi)$ fluctuate, the government has the option to maintain domestic prices p unchanged by adjusting the (specific) tax on each good i by $-\bar{p}_i'(\phi)$. The sole impact of this adjustment is to change tax revenues by $-\sum_i \bar{p}_i'(\phi) y_i^*$. Although this may not be the optimal response, the possibility of such a response is sufficient to evaluate the welfare impact of a terms-of-trade shock at the margin.[6]

4.2 Regulation of Innovation (Not!)

Our envelope result has direct implications for the regulation of innovation. To see this, suppose that there exists a set of feasible new technologies, Φ, that can be restricted by the government. The profit maximization problem of new technology firms is given by

$$y^*, \phi \in \max_{\tilde{y}^*, \tilde{\phi} \in \bar{\Phi}} \{ p^* \cdot \tilde{y}^* | G^*(\tilde{y}^*; \tilde{\phi}) \leq 0 \},$$

where $\bar{\Phi} \subset \Phi$ is the set of technologies allowed by the government. The government's problem, in turn, takes the general form,

$$\max_{(c,n,y,y^*,p,p^*,w,T,t^*,\bar{U}) \in \Omega, \phi \in \Phi} W(c,n)$$

subject to

$$G^*(y^*;\phi) \leq 0.$$

By equation (1), the optimal technology ϕ^* simply satisfies

$$\frac{\partial G^*(y^*;\phi^*)}{\partial \phi} = 0.$$

[6] The previous observation does not depend on whether the country is a small open economy or not. The same result holds when international prices depend on y^*.

Provided that taxes of new technology firms, t^*, have been set such that they find it optimal to produce y^*, conditional on ϕ^*, they will also find it optimal to choose ϕ^* , if allowed to do so. It follows that the government does not need to affect the direction of innovation, in spite of its potential distributional implications. More generally, there may be externalities across firms that directly call for subsidizing, or taxing, innovation. In such environments, the implication of our envelope result is that distributional concerns do not add a new motive for taxing, or subsidizing, innovation.

5 Optimal Technology Regulation

Our second set of results characterizes the structure of taxes on new technology firms. Specifically, we provide optimal tax formulas expressed in terms of sufficient statistics and using minimal structural assumptions.

5.1 The Trade-off Between Efficiency and Redistribution

Our tax formulas are derived by starting from an initial equilibrium with taxes (t^*, T) and engineering marginal changes δt^* in the taxes on the new technology firms, potentially accompanied by changes in the nonlinear tax schedule δT, such that all the equilibrium conditions are met except $G^*(y^*;\phi) \leq 0$. These marginal tax changes, in turn, induce equilibrium marginal adjustments in prices δp, wages δw, quantities δy^*, labor δn, and, ultimately, social welfare. Our three formulas differ in whether the nonlinear tax is adjusted and how it is adjusted.

We start with an intermediate result that encompasses all cases, providing a condition that the marginal tax changes δt^* and δT as well as the marginal adjustments δp, δw, δy^* and δn must satisfy so that welfare is not improved by the variation. For any household at the quantile $z \in [0, 1]$ of both the utility and wage distribution, let $\bar{w}(z)$, $\bar{n}(z)$, $\bar{x}(z)$, and $\bar{c}(z)$ denote the common wage, labor supply, earnings, and consumption vector, respectively, and let $\tau(z) \equiv T'(\bar{x}(z))$ denote the marginal income tax rate. Using the previous notation, our first optimal tax result can be stated as follows.

Lemma 1. *Optimal taxes satisfy*

$$(p^* - p) \cdot \delta y^* - \int \tau(z)\bar{w}(z)\delta\bar{n}(z)\, dz$$
$$= \int [\bar{\lambda}(z) - 1][(1 - \tau(z))\bar{n}(z)\, \delta\bar{w}(z) - \bar{c}(z) \cdot \delta p - \delta T(z)]\, dz, \quad (2)$$

where $\bar{\lambda}(z)$ measures the social marginal benefit of allocating income to households at quantile z, as described in Appendix E.1.

Lemma 1 captures the trade-off between efficiency and redistribution. It states that for a marginal change in taxes not to improve welfare, its marginal costs in terms of efficiency should be equal to its marginal benefit in terms of redistribution. The formal proof is based on a standard variational argument similar to those used, for instance, in Saez (2001) and Tsyvinski and Werquin (2018) to characterize properties of the income tax schedule. The novelty here is to use Lemma 1 to study the structure of optimal commodity taxes, thereby opening up the door for generalizations and empirical applications of Naito's (1999) original insights.

Efficiency considerations are reflected in two fiscal externalities on the left-hand side of equation (2). The first term represents the change in revenues from the linear tax t^*, also equal to the marginal increase in the deadweight burden or "Harberger triangle"; the second term captures the change in revenue from the non-linear income tax schedule. These changes in revenue are not internalized by private agents and thus represent a change in efficiency.

Distributional considerations are represented by the right-hand side. It evaluates the change in utility in monetary terms directly perceived by a household, weighted by $\bar{\lambda}(z) - 1$. By an envelope argument, the marginal change in real income for z is given by $(1 - \tau(z))\bar{n}(z)\,\delta\bar{w}(z) - \bar{c}(z) \cdot \delta p - \delta T(z)$. The first two terms, $(1 - \tau(z))\bar{n}(z)\,\delta\bar{w}(z) - \delta T(z)$, capture the change in income, due to both the change in before-tax income as well as the change in the tax schedule. The term $-\bar{c}(z) \cdot \delta p$ adjusts this change in income by a household-specific measure of inflation.

It is also important to note that all variables and adjustments in equation (2) are expressed in terms of the percentile z, not the underlying skills θ. This implies that one can collapse heterogeneity and proceed *as if* there were of a single dimension of heterogeneity. From a theoretical standpoint, this feature derives both from our assumption that the government's social welfare function only depends on the distribution of utility levels $\{\bar{U}(z)\}$ and from the fact that changes in the wages $\{\bar{w}(z)\}$ must be equal to the average changes in the wage of the households originally at a quantile z (even though they may move to different quantiles after the tax change). The formal argument, which is related to Reynolds Transport Theorem, can be found given in Appendix E.1.

For empirical purposes, the reduction to a single dimension of heterogeneity is crucial. It implies that researchers can focus measurement on the wage distribution, as is often done in practice. Despite the movements of individual workers across the wage distribution, panel data is not required, a repeated cross-section of wages is sufficient for the purposes of Lemma 1.

5.2 Three Tax Formulas

We now explore three feasible tax variations, each leading to a novel optimal tax formula. All variations lead to changes in y^* in any desired direction, but differ with respect to the nonlinear labor income tax schedule. In particular, we consider variations with

i. no change in income taxes, $\delta T = 0$;

ii. no change in (the distribution of) labor supply, $\delta \bar{n} = 0$;

iii. no change in (the distribution of) utility, $\delta \bar{U} = 0$.

To obtain a formula for the tax on good $i \neq 1$ in each of these cases, we focus on a marginal changes with $\delta y_i^*, \delta y_1^* \neq 0$ and $\delta y_j^* = 0$ for $j \neq i, 1$. For notational convenience, all formulas are expressed in terms of the price gap $p_i - p_i^*$ between old and new technology firms.

No change in income taxes. Our first variation is the simplest. It leaves the labor income tax schedule unchanged. Hence, the last term in equation (2) vanishes.

Formula 1 ($\delta T = 0$)**.** *Optimal taxes satisfy*

$$p_i - p_i^* \\ = \int \left[(1-\bar{\lambda}(z)) \Big((1-\tau(z)) \bar{n}(z) \frac{\delta \bar{w}(z)}{\delta y_i^*}|_{\delta T=0} - \bar{c}(z) \frac{\delta p}{\delta y_i^*}|_{\delta T=0} \Big) - \tau(z) \bar{w}(z) \frac{\delta \bar{n}(z)}{\delta y_i^*}|_{\delta T=0} \right] dz.$$

While our first formula follows mechanically from Lemma 1, itoffers a strict generalization of, as well as a new perspective on, the tax formulas found in the political economy of trade literature. The models discussed in Helpman's (1997) review, for instance, focuses the special case with quasi-linear preferences, with inelastic labor supply, without labor income taxation, and with sector-specific factors of production. Under quasi-linear preferences, our formula becomes

$$p_i - p_i^* = \int \left[(1-\bar{\lambda}(z)) ((1-\tau(z)) \bar{n}(z) \frac{\delta \bar{w}(z)}{\delta y_i^*}|_{\delta T=0}) - \tau(z) \bar{n}(z) \varepsilon_w(z) \frac{\delta \bar{w}(z)}{\delta y_i^*}|_{\delta T=0} \right] dz,$$

where $\varepsilon_w(z)$ denotes the elasticity of labor supply with respect to the wage at quantile z, i.e. the percentage change in labor supply caused by a one-percent change in the wage of any household with earnings initially in that quantile. Under the other three restrictions, with each z now corresponding to the index of a specific factor, so that $\frac{\delta \bar{w}(z)}{\delta y_i^*}|_{\delta T=0} = \frac{d\bar{w}(z)}{dy_i^*} = 0$ for $z \neq i$, one obtains

$$p_i - p_i^* = (\bar{\lambda}(i) - 1)\left(-\frac{d(\bar{n}(i)\bar{w}(i))}{dy_i^*}\right).$$

This expression coincides with the tariff formula derived by Helpman (1997) for various political-economy models, including the lobbying model of Grossman and Helpman (1994), where tariffs are expressed as a function of the the ratio of domestic output to imports, the elasticity of import demand, and whether sectors are politically organized or not.

Our alternative way of expressing optimal tariffs succinctly captures the essence of trade protection as redistribution. It states that protection in a given sector i, as measured by the gap between the domestic and foreign prices, should be equal to the Pareto weight that the government puts on workers from sector i (relative to other sectors) times the marginal impact of a decrease in imports on the wage bill of these workers. Different political-economy models—direct democracy (Mayer, 1984), political support function (Hillman, 1982), tariff formation function (Findlay and Wellisz, 1982), electoral competition (Magee et al., 1989), and influence-driven contributions (Grossman and Helpman, 1994)—simply correspond to different Pareto weights, $\bar{\lambda}(i)$.

No change in (the distribution of) labor supply. Our second variation engineers a change in the income tax schedule that keeps the distribution of labor supply unchanged. Because there is a continuum of ways to do this we also maintain resource feasibility. In the canonical Mirrleesian case, where θ is one dimensional, this variation ensures that labor supply is unchanged for each household. When θ is multidimensional, there are not enough instruments to ensure this, but still enough instruments to keep the distribution unchanged.

Formula 2 ($\delta \bar{n} = 0$)**.** *Optimal taxes satisfy*

$$p_i - p_i^* = \int \bar{\psi}(z)(1-\tau(z))\bar{w}(z)\bar{n}(z)\,\frac{\delta \omega(z)}{\delta y_i^*}|_{\delta \bar{n}=0}\,dz,$$

where $\omega(z) = \bar{w}'(z)/\bar{w}(z)$ and the coefficient $\bar{\psi}(z)$ depends only on household preferences and social weights, not technology, as described in Appendix E.2.

Formula 2 only features one distributional term on the right-hand side. Because the distribution of labor is unchanged, the labor fiscal externality is not present. Although the expression involves fewer terms than before, who wins and who loses is no longer mechanical, as it now incorporates the adjustment in the income tax schedule required to keep the distribution of labor supply unchanged.

One interesting difference between Formula 1 and Formula 2 is the complete absence of the adjustment in real income due to price changes. Intuitively, when the income tax schedule is modified to keep labor unchanged, it must offset these price changes. After all, the optimal labor supply decisions depend on the tradeoff between n and the attainable aggregate consumption C. Note also that labor supply elasticities do not enter Formula 2; again, this is due to the fact that the distribution of labor is kept unchanged.

Compared to Formula 1, only changes in relative wages, as captured by $\delta\omega$ appear, weighted by distributional concerns summarized in $\bar{\psi}(z)$. Note that it is the change in the growth rate $\omega(z) = w'(z)/w(z)$ rather than the level $w(z)$ immediately implies that a zero tax is optimal if the proportional impact on wages is uniform across z. This reflects the fact that only relative wages matter for incentives. To see this more formally, consider the incentive compatibility constraints that hold at any equilibrium

$$u(\bar{C}(z), \bar{n}(z)) \geq u(\bar{C}(z'), \bar{n}(z')\tfrac{\bar{w}(z')}{\bar{w}(z)}),$$

where $\bar{C}(z)$ is the indirect utility from consumption, given income $\bar{w}(z)\bar{n}(z) - T(\bar{w}(z)\bar{n}(z))$ and prices p. In Formula 2, $\omega(z)$ is the local counterpart to $\frac{\bar{w}(z')}{\bar{w}(z)}$. It captures the fact that a household of type z that earns the same amount as one of type z' must work $\hat{n}(z, z')$ where $\bar{w}(z)\hat{n}(z, z') = \bar{w}(z')\bar{n}(z')$. Hence, changes in relative wages may tighten or loosen incentive compatibility constraints, not changes in the overall wage level.

Formula 2 takes a particularly simple form in the special case where households' preferences over aggregate consumption and labor are quasi-linear and the government is Rawlsian. As described in Appendix E.2, quasi-linearity implies $\bar{\psi}(z) = \bar{\Lambda}(z) - z$ with $\bar{\Lambda}(z) \equiv (\int_0^z \bar{\lambda}(\tilde{z})d\tilde{z})/(\int \bar{\lambda}(\tilde{z})d\tilde{z})$. In the Rawlsian benchmark, Formula 2 therefore further simplifies into

$$p_i - p_i^* = \int (1-z)(1-\tau(z))\bar{w}(z)\bar{n}(z)\, \frac{\delta\omega(z)}{\delta y_i^*}|_{\delta\bar{n}=0}\, dz.$$

We will come back to this special case in Section 6.

No change in (the distribution of) utility. Our third variation engineers a change in the income tax schedule to keep the distribution of utility unchanged.

Formula 3 ($\delta \bar{U} = 0$)**.** *Optimal taxes satisfy*

$$p_i - p_i^* = \int \tau(z)\bar{w}(z)\bar{n}(z)\,\frac{\varepsilon(z)}{\varepsilon(z)+1}\,\frac{1}{\omega(z)}\frac{\delta\omega(z)}{\delta y_i^*}|_{\delta\bar{U}=0}\,dz,$$

where $\varepsilon(z)$ is the consumption-compensated elasticity of labor supply.

Formula 3 presents two attractive features. First, it depends on a labor supply elasticity $\varepsilon(z)$ that is free of general equilibrium considerations, whatever preferences may be. In the case of Formula 1, the same feature was only true under quasi-linear preferences. As described in Appendix E.2, $\varepsilon(z)$ corresponds to the percentage change in labor supply $\bar{n}(z)$ at a given quantile z caused by a one-percent change in the marginal income tax rate at that same quantile, holding the indirect utility from consumption, $\bar{C}(z)$, fixed. When preferences over consumption and leisure are additively separable, $\varepsilon(z)$ reduces to the uncompensated labor supply elasticity.

Second, unlike Formula 1 and 2, our third formula does not require any information on the welfare weights $\bar{\lambda}(z)$. This is because, by construction, the distribution of utility is unaffected by our variation; thus, social welfare is unaffected. As a result, distributional considerations vanish and only efficiency considerations remain, captured by the labor fiscal externality. Remarkably, tedious calculations relegated in Appendix E.2 show that the fiscal externality takes an extremely simple form, with the change in labor given by

$$\delta\bar{n}(z) = \frac{\varepsilon(z)}{\varepsilon(z)+1}\,\frac{1}{\omega(z)}\delta\omega(z).$$

For purposes of implementation, the fact that welfare weights do not enter Formula 3 must be welcomed. It opens up the possibility of a purely empirical evaluation, without any subjective choice over the social welfare objective. Our formula effectively rules out a first-order dominant improvement in the distribution of utilities. Our general strategy is reminiscent of the one used by Dixit and Norman (1986) to show the existence of Pareto gains from trade by constructing commodity taxes such that all households are kept at the same utility level under free trade, while simultaneously increasing the fiscal revenues of the government. Here, we show that unless Formula 3 holds, one can also construct changes in taxes that increase fiscal revenues, while holding utility fixed at all quantiles of the wage distribution.

The existence of a nonlinear income tax schedule T plays a crucial role, as evidenced by the presence of the marginal tax rates $\tau(z)$ in the formula. Everything else being equal, higher marginal taxes $\tau(z)$ potentiate fiscal externalities and demand larger t_i^*. To take an extreme case, if marginal taxes were zero, $\tau(z)$, then the formula immediately implies $t_i^* =$

0. This should come as no surprise: the First Welfare Theorem holds in our environment, so the absence of taxation leads to a Pareto optimum that cannot be improved upon.

5.3 Discussion

Limited Factor Taxation and Wage Manipulation. Despite their differences, all three formulas give center stage to the change in the wage schedule, as either captured by the change in the wage level w, as in Formula 1, or wage growth ω, as in Formulas 2 and 3. Our formulas make clear that changes in the wage schedule, which may be of empirical interest for descriptive reasons, is actually a sufficient statistic for optimal policy design. Given knowledge of this statistic, the underlying structure of the economy leading to the change in wages can be left in the background.

Unlike in Diamond and Mirrlees (1971a,b) and Dixit and Norman (1980), the government here cannot achieve its distributional objectives by taxing workers of different types at different rates. To achieve the same objectives, it is now forced to manipulate wages before taxes, which it can do by influencing, through taxes t^*, the prices p that firms face, and, in turn, their demand for workers of different types, as in Naito (1999).

Taxes on Old and New Technology Firms. Our three formulas have assumed that only taxes on new technology firms were available. What if taxes on old technology firms were available as well? In a trade context, this would mean the possibility of imposing production taxes rather than import tariffs or export taxes. We now describe how our results extend to environments with ad-valorem taxes on old technology firms $t \equiv \{t_i\}$.

If taxes on both old and new technology firms are available, the government can create wedges between the prices faced by old technology firms, $p \equiv \{p_i\}$, the prices faced by new technology firms, $p^* \equiv \{p_i^*\}$, and the prices faced by consumers, $q \equiv \{q_i\}$, with $q_i = (1 + t_i)p_i$ and $q_i = (1 + t_i^*)p_i$. In such environments, the trade-off between efficiency and redistribution described in Lemma 1 generalizes to

$$(p^* - q) \cdot \delta c - (p^* - p) \cdot \delta y - \int \tau(z)\bar{w}(z)\delta\bar{n}(z)\, dz \\ = \int [\bar{\lambda}(z) - 1][(1 - \tau(z))\bar{n}(z)\, \delta\bar{w}(z) - \bar{c}(z) \cdot \delta q - \delta T(z)]\, dz. \quad (3)$$

Compared to equation (2), the first term on the left-hand side is now split into two, reflecting the fact the fiscal externality associated with changes in consumption and output are now different. In addition, the price deflator on the right-hand side is now given $\bar{c}(z) \cdot \delta q$, reflecting the fact that consumer prices are now given by q rather than p.

Using the standard Atkinson and Stiglitz's (1976) logic, one can show that for a feasible variation not to improve welfare, q and p^* should be equalized. That is, there should be no taxes on new technology firms. One can view the fact that the government would only tax old technology firms as an expression of the Targeting Principle. Since wages depend on the labor demand of those firms, redistribution through wage manipulation is best achieved by taxing them, without introducing any additional consumption distortion.[7]

Given the equality between q and p^*, one can use the same steps as in Section 5.2 to go from equation (3) to each of our formulas. The only difference between our old formulas and the new ones is that the differentials on the right-hand side should now be taken with respect to δy rather than δy^*, reflecting again the fact that only the output of old technology firms is being distorted, not the consumption of households.[8]

Full versus Partial Optimality. All our formulas must coincide and hold at an optimum, for any given welfare function. Away from an optimum, each formula instead highlights an alternative way to increase welfare through a different marginal change in taxes. In this way, our formulas do not require the income tax schedule T to be optimized conditional on the taxes on new technology firms t^*.

Absent full optimality, however, a given formula may fail to detect welfare-improving changes in t^*. To see this more clearly, consider again the extreme case where $\tau(z) = 0$. In this situation, Formula 3 leads to $t_i^* = 0$. But suppose that the government has Rawlsian preferences, so that the absence of labor income taxation is undesirable. In this case, the government enjoys many ways of improving welfare. It could change the nonlinear tax schedule only, or it could also change the linear tax t^*, along the lines of Formula 1, or change both using Formula 2. In this case, these two formulas may detect an improvement, even if Formula 3 does not.

Although there is no reason for our three formulas to coincide in general, the fact that they do coincide at an optimum can be used to provide an alternative perspective on the absence of social weights in Formula 3. At an optimum, this absence can be thought of as reflecting a *revealed preference* on the part of the government. Intuitively, the underlying

[7] Mayer and Riezman (1987) establish a similar result in a trade context with inelastic factor supply and no income taxation. If both producer and consumer taxes are available, they show that only the former should be used. This result, however, requires preferences to be homothetic, as discussed in Mayer and Riezman (1989). Our result does not require this restriction. This reflects the fact that we have access to non-linear income taxation and that preferences are weakly separable. Hence, absent the wage manipulation motive, there is no rationale for commodity taxation, as in Atkinson and Stiglitz (1976).

[8] The counterpart of Formula 1 provides a strict generalization—to an environment with endogenous labor supply, nonlinear income taxation, as well as general preferences and technology—of the optimal production taxes in Dixit (1996).

social preferences for the utility of different groups of households, as measured by $\bar{\lambda}(z)$, get revealed by the marginal tax rate $\tau(z)$ after controlling for the distortionary cost of redistribution, as measured by the labor supply elasticity $\varepsilon(z)$.[9]

A Pigouvian Interpretation. Interestingly, all three formulas provide a direct expression for the tax rate. This differs from the optimal linear tax literature (e.g. Diamond and Mirrlees, 1971b), which usually derives a system of simultaneous equations, with the entire set of tax rates on the left hand side. We could have stated our formulas in such forms by focusing on price variations such that $\delta p_i \neq 0$ and $\delta p_j = 0$ for $j \neq i$. In vector and matrix notation, Formula 3, for instance, would then become

$$[D_p y^*]_{\delta \bar{U}=0}(p^* - p) = \int \tau(z)\bar{w}(z)\bar{n}(z)\,\frac{\varepsilon(z)}{\varepsilon(z)+1}\,\frac{1}{\omega(z)}[\nabla_p \omega(z)]_{\delta \bar{U}=0}\,dz$$

with $[D_p y^*]_{\delta \bar{U}=0} \equiv \{(\delta y_j^*/\delta p_i)|_{\delta \bar{U}=0}\}$ and $[\nabla_p \omega(z)]_{\delta \bar{U}=0} \equiv \{(\delta \omega(z)/\delta p_i)|_{\delta \bar{U}=0}\}$Our formulas, which focus on quantity variations, are more akin to the Pigouvian tax literature, which provides an explicit expression for the tax on each good in terms of its externality. Indeed, we favor a Pigouvian interpretation of our three formulas, as correcting for distributional externalities: if an extra unit of y_i^* is increased, then this has an impact on the wage schedule that, in turn, affects distribution and social welfare; the tax asks agents to pay for these marginal effects.

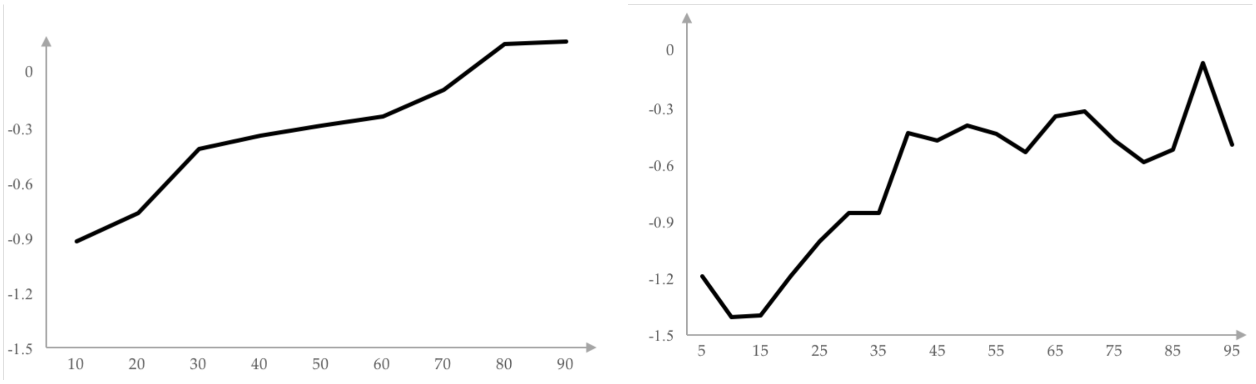
5.4 Quantitative Examples

We now illustrate through two examples, robots and trade, how our theoretical results can be combined with existing reduced-form evidence to provide estimates of optimal taxes. We restrict attention to Formula 3, which allows us to dispense with any assumption on welfare weights. According to this formula, the ad-valorem tax t_m^* on either robots or imports must satisfy

$$t_m^* = \int \tau(z)\frac{\bar{w}(z)\bar{n}(z)}{p_m^* y_m^*}\,\frac{\varepsilon(z)}{\varepsilon(z)+1}\,\frac{\delta \ln \omega(z)}{\delta \ln y_m^*}|_{\delta \bar{U}=0}\,dz. \tag{4}$$

Robot Example. To measure the efficient tax on robots in the United States using equation (4), the key input is the elasticity of relative wages with respect to the number of robots, $\frac{\delta \ln \omega(z)}{\delta \ln y_m^*}|_{\delta \bar{U}=0}$. To recover that elasticity, the ideal experiment would engineer a

[9]This is the idea behind Werning's (2007) test of whether an income tax schedule is Pareto optimal. Namely, it is if the inferred Pareto weights are all positive.



(a) Robots (Acemoglu and Restrepo, 2017b) **(b)** Chinese Imports (Chetverikov, Larsen and Palmer, 2016)

Figure 1: Semi-Elasticity of wages, $\frac{d \ln w(z)}{d y_m} \times 100$, across quantiles of US wage distribution.

marginal change in taxes that increase the number of robots in the entire United States by one unit, while holding utility fixed, and record the differential changes in wages between consecutive quantiles of the income distribution. As a first proxy for such an ideal experiment, we propose to use the empirical estimates from Acemoglu and Restrepo (2017b). Using a difference-in-difference strategy, the previous authors have estimated the effect of industrial robots, defined as "an automatically controlled, reprogrammable, and multipurpose [machine]" on different quantiles of the wage distribution between 1990 and 2007 across US commuting zones. We interpret their estimates as the semi-elasticity of wages with respect to robots, $\eta_{AR}(z) \simeq \frac{\delta \ln w(z)}{\delta y_m^*}|_{\delta \bar{U}=0}$, where y_m^* is expressed as number of robots per thousand workers.[10] These estimates are reported in Figure 1a.

Under the previous interpretation, the elasticity that we are interested in is given by

$$\frac{\delta \ln \omega(z)}{\delta \ln y_m^*}|_{\delta \bar{U}=0} = \frac{y_m^*}{\Delta \ln \bar{w}(z)} \times \Delta \eta_{AR}(z),$$

where $\Delta \ln \bar{w}(z)$ and $\Delta \eta_{AR}(z)$ denote changes between consecutive deciles of the wage distribution. In the United States in 2007, the number of robots per thousand workers is slightly greater than one, $y_m^* \simeq 1.2$. This leads to an average elasticity $\frac{\delta \ln \omega(z)}{\delta \ln y_m^*}$ across deciles around 0.5.

Given estimates of the previous elasticities, the only additional information required

[10] As discussed in Appendix E.4, a sufficient condition for this approximation to be valid is that the indirect effects of exogenous changes in the price of robots, p_m, on relative wages, $\omega(z)$, and the demand for robots, y_m^*, caused by the endogenous response of the households' labor supply is negligible relative to the direct effects of changes in the price of robots on those same variables. We view this approximation as a useful starting point.

to evaluate the optimal tax on robots given by equation (4) is: (i) total spending on robots, $p_m^* y_m^*$, which we obtain from Graetz and Michaels (2018); (ii) labor earnings, $\bar{w}(z)\bar{n}(z)$, which we compute from the World Wealth and Income Database; (iii) marginal income tax rates, $\tau(z)$, which we compute from Guner, Kaygusuz and Ventura (2014); and (iv) labor supply elasticities, $\epsilon(z)$, which we obtain from Chetty (2012).

For a labor supply elasticity of $\epsilon = 0.1$, in the low range of the micro-estimates reviewed in Chetty (2012), equation (4) leads to an optimal tax on robots equal to $t_m^* = 1\%$. For higher labor supply elasticities of $\epsilon = 0.3$ or $\epsilon = 0.5$, the previous tax goes up to $t_m^* = 2.7\%$ and $t_m^* = 3.7\%$, respectively.

Trade Example. Our second example focuses on Chinese imports. Again, we propose to use estimates obtained from a difference-in-difference strategy as a proxy for the elasticity that we are interested in, $\frac{\delta \ln \omega(z)}{\delta \ln y_m^*}|_{\delta \bar{U}=0}$. Using the same empirical strategy as in Autor, Dorn and Hanson (2013), Chetverikov, Larsen and Palmer (2016) have estimated the effect on log wages of a \$1,000 increase in Chinese imports per worker at different percentiles of the wage distribution, as described in Figure 1b. Following the same approach as in the case of robots, we can transform the previous semi-elasticities into elasticities using $y_m^* \simeq 2.2$ as the value of Chinese imports, in thousands of US dollars, per worker for the United States in 2007.

Interestingly, the average value of the relative wage elasticity, $\frac{d \ln \omega}{d \ln y_m^*}$, is of the same order of magnitude as the one implied by the estimates of Acemoglu and Restrepo (2017b), around 0.5. Compared to the robot example, however, the ratio of total labor earnings to total Chinese imports in 2007 is only 26.4, an order of magnitude smaller than the ratio to total spending on robots, around 245.[11] This leads to an optimal tax on Chinese imports that is much smaller than the tax on robots: $t_m^* = 0.03\%$ for $\epsilon = 0.1$, 0.08% for $\epsilon = 0.3$, and 0.11% for $\epsilon = 0.5$.

6 Comparative Statics

Our final result focuses on comparative static issues. By way of a simple example, we propose to tackle the following question: If improvements in new technologies raise inequality, should taxes be raised further as well?

[11] Graetz and Michaels (2018) estimate the share of robots in total capital services to be around 0.64%.

6.1 A Simple Economy

Consider a special case of the environment presented in Section 3. There is one final good, indexed by f, and one intermediate good, indexed by m, which could be either robots that are produced domestically or machines that are imported from abroad. Households have one-dimensional skills θ, uniformly distributed over $[0,1]$, and quasi-linear preferences,

$$U(\theta) = C(\theta) - \frac{(n(\theta))^{1+1/\varepsilon}}{1+1/\varepsilon}, \tag{5}$$

with $C(\theta)$ the consumption of the unique final good, which we use as our numeraire, $p_f = p_f^* = 1$, and ε the constant labor supply elasticity.[12]

Old technology firms produce the final good, $y_f \geq 0$, using workers, $n \equiv \{n(\theta)\}$, and machines, $y_m \leq 0$, as an input. Their production set is given by

$$G(y_f, y_m, n) = y_f - \max_{\{\tilde{y}_m(\theta)\}} \{ \int g(\tilde{y}_m(\theta), n(\theta); \theta) dF(\theta) | \int \tilde{y}_m(\theta) dF(\theta) \leq -y_m \}, \tag{6}$$

with $g(\tilde{y}_m(\theta), n(\theta); \theta)$ a Cobb-Douglas production function,

$$g(\tilde{y}_m(\theta), n(\theta); \theta) = \exp(\alpha(\theta)) \cdot (\frac{\tilde{y}_m(\theta)}{\beta(\theta)})^{\beta(\theta)} (\frac{n(\theta)}{1-\beta(\theta)})^{1-\beta(\theta)}, \tag{7}$$

where $\tilde{y}_m(\theta)$ represents the number of machines combined with workers of type θ to produce the final good, $\alpha(\theta) \equiv \frac{\alpha \ln(1-\theta)}{\beta \ln(1-\theta) - 1}$, and $\beta(\theta) \equiv \frac{\beta \ln(1-\theta)}{\beta \ln(1-\theta)-1}$, with $\alpha, \beta > 0$. New technology firms produce machines, $y_m^* \geq 0$, using the final good, $y_f^* \leq 0$,

$$G^*(y_f^*, y_m^*; \phi) = \phi y_f^* + y_m^*, \tag{8}$$

where ϕ measures the productivity of machine producers.

Let p_m and p_m^* denote the price of robots faced by old and new technology firms. Profit maximization by new technology firms implies

$$p_m^* = 1/\phi,$$

whereas profit maximization by old technology firms implies

$$w(p_m; \theta) = (1-\theta)^{-1/\gamma(p_m)},$$

[12] In the one-dimensional case, the assumption that θ is uniformly distributed is only assumes that households can be indexed by their quantiles in the wage distribution.

with $\gamma(p_m) \equiv 1/(\alpha - \beta \ln p_m)$. Under the restriction that $\gamma(p_m) > 0$, which we maintain throughout, wages are increasing in θ and Pareto distributed with shape parameter equal to $\gamma(p_m)$ and lower bound equal to 1. By construction, more skilled workers tend to use machines relatively more, since $\beta(\theta)$ is increasing in θ. So an increase in the price of machines tends to lower their wages relatively more, which decreases inequality,

$$\frac{d \ln \omega(\theta)}{d \ln p_m} = -\frac{d \ln \gamma(p_m)}{d \ln p_m} = -\beta\gamma(p_m) < 0.$$

Here, because of additive separability in production, machines directly affect inequality by affecting the relative marginal products of workers of different skills, but not indirectly through further changes in their relative labor supply, as in Stiglitz (1982).

6.2 Technological Progress, Greater Inequality, and Lower Taxes

In the spirit of keeping this example as simple as possible, we propose to study how t_m^* varies with ϕ in the case of a government with extreme preferences for redistribution. Namely, we assume that the government has Rawlsian preferences, which corresponds to the weighted utilitarian objective of Section 3.2, with the distribution of Pareto weights Λ being a Dirac at $\theta = 0$.

For comparative static purposes, a limitation of our three formulas is that they involve the entire schedule of marginal income tax rates. These are themselves endogenous objects that will respond to changes in ϕ. In Appendix F, we demonstrate how to solve for $\tau(\theta)$ and obtain the following formula for the optimal Rawlsian tax,

$$\frac{t_m^*}{1+t_m^*} = \frac{\frac{\epsilon}{\epsilon+1}\frac{d \ln \omega}{d \ln y_m^*}\tau^*}{1 - \frac{\epsilon}{\epsilon+1}\frac{d \ln \omega}{d \ln y_m^*}\tau^*}\frac{1-s_m}{s_m}, \tag{9}$$

where the elasticity of relative wages, $\frac{d \ln \omega}{d \ln y_m^*} \equiv -\beta\gamma(p_m)\frac{\partial \ln p_m}{\partial \ln |y_m(p_m,n)|}$, is now constant across agents; $\tau^* \equiv \frac{\epsilon+1}{\epsilon+1+\epsilon\gamma(p_m)}$ corresponds to the optimal marginal tax rate that would be imposed in the absence of a tax on machines, as in Diamond (1998), Saez (2001), and Scheuer and Werning (2017); and $s_m \equiv \frac{p_m y_m^*}{\int x(\theta) dF(\theta) + p_m y_m^*}$ measures the share of machines in gross output. After expressing the three previous statistics, $\frac{d \ln \omega}{d \ln y_m^*}$, τ^*, and s_m, as functions of t_m^* and ϕ, we can apply the Implicit Function Theorem to determine the monotonicity of the optimal Rawlsian tax, as we do in Appendix F. This leads to the following proposition.

Proposition 2. *In a simple economy where equations (5)-(8) hold, the optimal Rawlsian tax t_m^* is decreasing with the productivity ϕ of new technology firms.*

By construction, more machines always increase inequality in this simple economy, $\frac{d \ln \omega}{d \ln y_m^*} > 0$. So one should always tax new technology firms. For comparative static purposes, however, the relevant question is whether this effect gets exacerbated as the new technology improves. Here, one can check that $\frac{\partial}{\partial \phi} \frac{d \ln \omega}{d \ln y_m^*} < 0$ both because relative wages are becoming less responsive to the price of machines, $\frac{\partial}{\partial \phi} \left| \frac{d \ln \omega}{d \ln y_m^*} \right| < 0$, and because the demand for machines is becoming more elastic, $\frac{\partial}{\partial \phi} \left| \frac{\partial \ln p_m}{\partial \ln |y_m(p_m, n)|} \right| < 0$, due to the increase in the labor supply of high-skilled workers whose demand for machines is more elastic. One can also check that these two effects dominate the increase in the marginal tax rate, $\frac{\partial \tau^*}{\partial \phi} > 0$, in response to greater inequality. For a given share of machines s_m, this implies that the total fiscal externality associated with new machines decreases. Since the share of machines increase with improvements in the new technology, $\frac{\partial s_m}{\partial \phi} > 0$, the fiscal externality per machine a fortiori decreases and so does the tax on machines.

As this simple example illustrates, cheaper robots may lead to a higher share of robots in the economy, more inequality, but a *lower* optimal tax on robots. Likewise, more imports and more inequality, in spite of the government having extreme distributional concerns and imports causing inequality, may be optimally met with less trade protection. This does not occur because redistribution is becoming more costly as the economy gets more open.[13] Here, the elasticity of labor supply is fixed and the marginal tax rate τ^* increases with ϕ. This also does not occur because redistribution through income taxation is becoming more attractive. Everything else being equal, an increase in τ^* raises the tax on imports. Rather the decrease in the tax on imports captures a standard Pigouvian intuition: as ϕ increases, the total fiscal externality associated with imports increases, but the marginal impact does not, leading to a lower value for the optimal tax.

7 Conclusion

Our paper has focused on two broad questions. First, how should we value technological progress? Second, how should government policy respond to technological change?

Our answer to the first question is that in second best environments—where income taxation is available, but taxes on specific factors are not—technological progress can still be valued using a simple envelope argument. The critical assumption behind this result is not the absence of any distortion, like in a first best environment, but rather that the government has enough instruments to control the behavior of new technology firms, which

[13] This is the point emphasized by Itskhoki (2008) and Antras, de Gortari and Itskhoki (2017) in an economy where entrepreneurs can decide whether to export or not. This makes labor supply decisions more elastic in an open economy, which may reduce redistribution at the optimum.

are the only ones directly subject to technological change. This offers a new perspective on technological change that stands in sharp contrast with the existing results of a large literature concerned with distortions and welfare. It implies, in particular, that innovation should not be distorted because of distributional concerns.

From a trade perspective, our envelope result further suggests that provided that governments have access to a full set of trade policy instruments, the general law of comparative advantage (Deardorff, 1980) may remain valid in economies subject to imperfect competition, political-economy forces, and various distortions; that the only rationale for trade agreements in all these economies may remain to correct terms-of-trade externalities (Bagwell and Staiger, 1999, 2001, 2012a,b); and that in spite of distributional concerns and various labor market imperfections, the welfare gains from trade may still be given by the integral below the import demand curve (Costinot and Rodríguez-Clare, 2018).

Our answer to the second question is that in such second best environments, there is a case for taxing new technology firms, with each of our three formulas offering a precise answer to what optimal taxes on new technology firms should be as a function of a few sufficient statistics. Although our three formulas differ in important ways, they all give center stage to changes in the wage schedule. This reflects a general Pigouvian motive for optimal technology regulation to correct distributional externalities. When one extra unit is produced using the new technology, either in the form of a robot or imports from abroad, this has an impact on the wage schedule that, in turn, affects distribution and social welfare; the optimal tax asks agents to pay for these marginal effects.

Perhaps surprisingly, we have also provided an example showing that more robots or more trade may go hand in hand with more inequality and lower taxes, despite robots or trade being responsible for the rise in inequality, and governments having extreme preferences for redistribution. Although there is always a distributional externality to be corrected, the marginal impact of either robots or trade is what matters for the magnitude of the tax, and that marginal impact is always falling in this example. In short, hyper automation and hyper globalization do not necessarily call for hyper taxation.

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A Notation

Consider a function $h : \mathbb{R}^n \rightarrow \mathbb{R}^m$ such that

$$h(x_1, ..., x_n) = (h_1(x_1, ..., x_n), ..., h_m(x_1, ..., x_n)).$$

Throughout our appendix, we use the following notation

$$h_{x_i}(x_1, ..., x_n) \equiv \frac{\partial h(x_1, ..., x_n)}{\partial x_i} \text{ for all } i = 1, ..., n,$$
$$h_{j,x_i}(x_1, ..., x_n) \equiv \frac{\partial h_j(x_1, ..., x_n)}{\partial x_i} \text{ for all } i = 1, ..., n \text{ and } j = 1, ..., m,$$

Whenever there is no risk of confusion, we also drop arguments from functions so that, for instance, h implicitly stands for $h(x_1, ..., x_n)$.

B Section 2

B.1 Tasks in Old Technology

Our environment nests economies in which a final good is produced using a continuum of tasks. To see this formally, consider an economy that produces a final good f using a continuum of tasks indexed by j,

$$y_f = g_f(\{y(j)\}),$$

with $y(j) \geq 0$ the output of task j and g_f a constant returns to scale production function. Each task, in turn, is produced using domestic workers and robots, as in Acemoglu and Restrepo (2017a), or domestic and foreign workers, as in Grossman and Rossi-Hansberg (2008),

$$y(j) = g_j(y_m(j), \{n(\theta, j)\}), \text{ for all } j,$$

with $y_m(j) \geq 0$ the number of robots or foreign workers used to perform task j, $n(\theta, j) \geq 0$ the number of domestic workers, and g_j a constant return to scale production function. The production possibility frontier is then given by

$$G(y_f, y_m, n) = \max_{\{y(j), y_m(j), n(\theta,j)\}} g_f(\{y(j)\})$$

subject to

$$
\begin{aligned}
y(j) &\leq g_j(y_m(j), \{n(\theta, j)\}), \text{ for all } j, \\
n(\theta) &\geq \int n(\theta, j) dj, \text{ for all } \theta, \\
-y_m &\geq \int y_m(j) dj,
\end{aligned}
$$

with $y_f \geq 0$ the output of the final good and $y_m \leq 0$ the total amount of robots or foreign workers demanded by old technology firms.

B.2 Labor in New Technology

In Section 2, we have argued that it is without loss of generality to assume the new technology G^* does not employing labor. Here, we provide the formal argument.

Suppose that the production sets associated with the old and new technology are such that

$$
\begin{aligned}
\hat{G}(\hat{y}, \hat{n}) &\leq 0, \\
\hat{G}^*(\hat{y}^*, \hat{n}^*) &\leq 0.
\end{aligned}
$$

First, define

$$
y = \begin{pmatrix} \hat{y} \\ \hat{y}^* \end{pmatrix}
$$

Next, define G so that the set of (y, n) satisfying $G(y, n) \leq 0$ coincides with the set of (y, n) for which there exists $\hat{n}$ and $\hat{n}^*$ such that

$$
\begin{aligned}
\hat{G}(\hat{y}, \hat{n}) &\leq 0, \\
\hat{G}^*(\hat{y}^*, \hat{n}^*) &\leq 0, \\
\hat{n} + \hat{n}^* &= n.
\end{aligned}
$$

Last, define G^* such that $G^*(y^*) \leq 0$ is satisfied if and only if

$$
y^* = \begin{pmatrix} \hat{y} \\ \hat{y}^* \end{pmatrix}
$$

is such that $\hat{y} \leq -\hat{y}^*$.

C Section 3

C.1 Competitive Equilibrium with Taxes

We provide a formal definition of a competitive equilibrium with taxes.

Demand. Households maximize utility taking prices p and the income tax schedule T as given. Since preferences are weakly separable, the demand of any household θ is given by the two-step problem

$$c(\theta) \in \text{argmax}_{\tilde{c}(\theta)}\{v(\tilde{c}(\theta))|p \cdot \tilde{c}(\theta) \leq w(\theta)n(\theta) - T(w(\theta)n(\theta))\}, \tag{C.1}$$

$$n(\theta) \in \text{argmax}_{\tilde{n}(\theta)}\{u(C(p, w(\theta)\tilde{n}(\theta) - T(w(\theta)\tilde{n}(\theta))), \tilde{n}(\theta))\}, \tag{C.2}$$

with $C(p, R(\theta)) \equiv \max_{\tilde{c}(\theta)}\{v(\tilde{c}(\theta))|p \cdot \tilde{c}(\theta) \leq R(\theta)\}$ the indirect utility function of an household with after-tax earnings $R(\theta) \equiv w(\theta)n(\theta) - T(w(\theta)n(\theta))$.

Supply. Firms maximize profits taking prices p and p^* as given,

$$y, n \in \text{argmax}_{\tilde{y},\tilde{n}}\{p \cdot \tilde{y} - \int w(\theta)\tilde{n}(\theta)dF(\theta) \mid G(\tilde{y}, \tilde{n}) \leq 0\}, \tag{C.3}$$

$$y^* \in \text{argmax}_{\tilde{y}^*}\{p^* \cdot \tilde{y}^* | \; G^*(\tilde{y}^*; \phi) \leq 0\}. \tag{C.4}$$

Market Clearing. Demand equals supply for all goods,

$$\int c_i(\theta)dF(\theta) = y_i + y_i^* \qquad \text{for all } i. \tag{C.5}$$

Linear Taxation. Prices satisfy

$$p_i = (1 + t_i^*)p_i^* \qquad \text{for all } i. \tag{C.6}$$

Equilibrium. A competitive equilibrium with taxes (T, t^*) is an allocation $c \equiv \{c(\theta)\}$, $n \equiv \{n(\theta)\}$, $y \equiv \{y_i\}$, $y^* \equiv \{y_i^*\}$, prices and wages $p \equiv \{p_i\}$, $p^* \equiv \{p_i^*\}$, and $w \equiv \{w(\theta)\}$, such that:

i. households maximize their utility, condition (C.1) and (C.2);

ii. firms maximize their profits, conditions (C.3) and (C.4);

iii. good markets clear, condition (C.5);

iv. prices satisfy the non-arbitrage condition (C.6).

C.2 Wage Schedule

Let $y(p,n)$ denote the supply of old technology firms, that is the solution to

$$\max_{\tilde{y}}\{p \cdot \tilde{y} \mid G(\tilde{y},n) \leq 0\}.$$

The first-order conditions of (C.3) imply a wage schedule $w(p,n) \equiv \{w(p,n;\theta)\}$ such that

$$w(\theta) = -G_{n(\theta)}(y(p,n),n)/G_{y_1}(y(p,n),n) \text{ for all } \theta.$$

C.3 Government Problem

For any vector of consumption and labor supply, $c \equiv \{c(\theta)\}$ and $n \equiv \{n(\theta)\}$, the utility of quantile $z \in [0,1]$ is given by

$$\int_{u(v(c(\theta)),n(\theta)) \leq \bar{U}(z)} dF(\theta) = z. \tag{C.7}$$

For any y^*, construct p^* such that

$$p_i^* = G_{y_i^*}^*(y^*;\phi)/G_{y_1^*}^*(y^*;\phi) \text{ for all } i. \tag{C.8}$$

Define the feasible set

$$\Omega = \{(c,n,y,y^*,p,p^*,w,T,t^*,\bar{U}) \text{ such that equations (C.1)-(C.3) and (C.5)-(C.8) hold}\}.$$

The problem of selecting a competitive equilibrium with taxes that maximizes social welfare can then be expressed as

$$\max_{(c,n,y,y^*,p,p^*,w,T,t^*,\bar{U}) \in \Omega} W(\bar{U})$$

subject to

$$G^*(y^*;\phi) \leq 0.$$

D Section 4

Define the feasible set

$$\Omega_R = \{(c,N,n,y,y^*,p,p^*,w,T,t^*,\bar{U}) \text{ such that equations (C.1)-(C.3), (C.5), and (C.7) hold}\} \subset \Omega.$$

Consider the value function $V_R(\phi)$ associated with the relaxed version of the government problem

$$V_R(\phi) = \max_{(c,N,n,y,y^*,p,p^*,w,T,t^*,\bar{U}) \in \Omega_R} W(\bar{U})$$

subject to

$$G^*(y^*;\phi) \leq 0.$$

Since $\Omega \subset \Omega_R$, we must have

$$V_R(\phi) \geq V(\phi).$$

Conversely, take a solution $(c, N, n, y, y^*, p, p^*, w, T, t^*, \bar{U})$ of the relaxed problem. Construct $\tilde{p}^*$ and $\tilde{t}^*$ such that

$$\tilde{p}_i^* = G^*_{y_i^*}(y^*)/G^*_{y_1^*}(y^*) \text{ for all } i,$$
$$\tilde{t}_i^* = p_i/p_i^* - 1 \text{ for all } i.$$

Since $(c, N, n, y, y^*, p, \tilde{p}^*, w, T, \tilde{t}^*, \bar{U}) \in \Omega$, we must have

$$V_R(\phi) \leq V(\phi).$$

This establishes that $V(\phi) = V_R(\phi)$, as argued in Section 4.

E Section 5

E.1 Lemma 1

Preliminaries. The next lemma will be used to reduce the dimensionality from θ to the percentile z in our optimality conditions.

Lemma 2. *Suppose $\theta \in \Theta \subseteq \mathbb{R}^n$, $\epsilon \in \mathbb{R}$ and the function $w(\theta, \epsilon)$ is a.e. differentiable and increasing in θ_n, define*

$$Z(W, \epsilon) = \int_{w(\theta,\epsilon) \leq W} dF(\theta)$$

and let $\bar{w}(z, \epsilon)$ denote the inverse of Z with respect to W. Then

$$\bar{w}_\epsilon(z, \epsilon) = \mathbb{E}[w_\epsilon(\theta, \epsilon) \mid w(\theta, \epsilon) = \bar{w}(z, \epsilon)].$$

Proof. Denote the inverse of $W = w(\theta, \epsilon)$ for θ_n as $\hat{\theta}(\theta_{-n}, W, \epsilon)$. Then

$$Z(W, \epsilon) = \int F(\hat{\theta}(\theta_{-n}, W, \epsilon) \mid \theta_{-n}) \, dF(\theta_{-n}). \tag{E.1}$$

Since $w(\theta_{-n}, \hat{\theta}(\theta_{-n}, W, \epsilon), \epsilon) = W$ it follows that

$$\hat{\theta}_W(\theta_{-n}, W, \epsilon) = \frac{1}{w_{\theta_n}(\theta_{-n}, \hat{\theta}(\theta_{-n}, W, \epsilon), \epsilon)}$$

$$\hat{\theta}_\epsilon(\theta_{-n}, W, 0) = -\frac{w_\epsilon(\theta_{-n}, \hat{\theta}(\theta_{-n}, W, \epsilon), \epsilon)}{w_{\theta_n}(\theta_{-n}, \hat{\theta}(\theta_{-n}, W, \epsilon), \epsilon)}$$

Since $\bar{w}(Z(W,\epsilon),\epsilon) = W$ it follows that

$$\bar{w}_\epsilon(z,\epsilon) = -\bar{w}_z(z,\epsilon) Z_\epsilon(\bar{w}(z,\epsilon),\epsilon) = -\frac{1}{Z_W(\bar{w}(z,\epsilon),\epsilon)} Z_\epsilon(\bar{w}(z,\epsilon),\epsilon)$$

Using these expressions and differentiating (E.1) gives

$$\bar{w}_\epsilon(z,\epsilon) = \frac{1}{\int \frac{f(\hat{\theta}(\theta_{-n},\bar{w}(z,\epsilon),\epsilon)|\theta_{-n})}{w_{\theta_n}(\theta_{-n},\hat{\theta}(\theta_{-n},\bar{w}(z,\epsilon),\epsilon),\epsilon)} dF(\theta_{-n})} \times \int \frac{f(\hat{\theta}(\theta_{-n},\bar{w}(z,\epsilon),\epsilon) \mid \theta_{-n})}{w_{\theta_n}(\theta_{-n},\hat{\theta}(\theta_{-n},\bar{w}(z,\epsilon),\epsilon),\epsilon)} w_\epsilon(\theta_{-n},\hat{\theta}(\theta_{-n},\bar{w}(z,\epsilon),\epsilon),\epsilon)\, dF(\theta_{-n}).$$

To see that this establishes the desired equality, define

$$G(W \mid \theta_{-n}) = F(\hat{\theta}(\theta_{-n}, W, \epsilon) \mid \theta_{-n})$$

the c.d.f. for $W = w(\theta,\epsilon)$ conditional on θ_{-n}; differentiating, one sees that

$$g(W \mid \theta_{-n}) = \frac{f(\hat{\theta}(\theta_{-n}, W, \epsilon) \mid \theta_{-n})}{w_{\theta_n}(\theta_{-n}, \hat{\theta}(\theta_{-n}, W, \epsilon), \epsilon)}$$

represents the associated conditional density. Noting that

$$g(\theta_{-n} \mid W) = \frac{g(W \mid \theta_{-n}) dF(\theta_{-n})}{\int g(W \mid \theta_{-n}) dF(\theta_{-n})}$$

the result then follows: $\bar{w}_\epsilon(z,\epsilon) = \mathbb{E}[w_\epsilon(\theta,\epsilon) \mid w(\theta,\epsilon) = \bar{w}(z,\epsilon)]$. □

The following simple corollary will be employed below, for any differentiable function $N(w,\epsilon)$ define $n(\theta,\epsilon) = N(w(\theta,\epsilon),\epsilon)$ and $\bar{n}(z,\epsilon) = N(\bar{w}(z,\epsilon),\epsilon)$ then

$$\bar{n}_\epsilon(z,\epsilon) = \mathbb{E}[n_\epsilon(\theta,\epsilon) \mid w(\theta,\epsilon) = \bar{w}(z,\epsilon)].$$

Proof of Lemma 1. We shall engineer variations that ensure all the equilibrium conditions are met except $G^*(y^*;\phi) \leq 0$. We must evaluate how the variation affects welfare $W(\bar{U})$ and $G^*(y^*;\phi)$. To do so, define the Lagrangian

$$\mathcal{L} = W(\bar{U}) - \gamma G^*(y^*;\phi)$$

with Lagrange multiplier $\gamma > 0$.[14] At an optimum the variation in $\mathcal{L}$ must be set to zero.

[14]The formal argument establishing that γ cannot be equal to zero is given in Appendix E.3.

The new tax schedule is given by

$$t_i^*(\epsilon) = t_i^* + \epsilon \hat{t}_i^* \text{ for all } i,$$
$$T(x,\epsilon) = T(x) + \epsilon \hat{T}(x) \text{ for all } x \geq 0,$$

for some arbitrary vector new technology taxes, $\hat{t}^*$, income tax schedule, $\hat{T}$, and $\epsilon \in \mathbb{R}$. We let $\{c(\theta,\epsilon)\}$, $\{n(\theta,\epsilon)\}$, $y(\epsilon) \equiv \{y_i(\epsilon)\}$, and $y^*(\epsilon) \equiv \{y_i^*(\epsilon)\}$ denote the associated equilibrium allocation and we let $p(\epsilon) \equiv \{p_i(\epsilon)\}$, $p^*(\epsilon) \equiv \{p_i^*(\epsilon)\}$ and $\{w(\theta,\epsilon)\}$ denote the associated prices and wages. They are given by conditions (C.1)-(C.3) and (C.5)-(C.8). The only equilibrium condition that is not imposed is $G^*(y^*(\epsilon);\phi) = 0$.

Let $W(\{\bar{U}(z,\epsilon)\})$ denote the social welfare associated with a given value of ϵ. Since households have identical preferences, all households of a given quantile z of the utility distribution have wage $\bar{w}(z,\epsilon)$ and achieve utility

$$\bar{U}(z,\epsilon) = \max_{\tilde{n}(\theta)} u(C(p(\epsilon), R(\bar{w}(z,\epsilon)\tilde{n}(\theta),\epsilon)), \tilde{n}(\theta)),$$

with labor supply $\bar{n}(z,\epsilon)$, consumption, $\bar{c}(z,\epsilon)$, and earnings, $\bar{x}(z,\epsilon) \equiv \bar{w}(z,\epsilon)\bar{n}(z,\epsilon)$. Let $\bar{U}_\epsilon(z,\epsilon) \equiv \partial\bar{U}(z,\epsilon)/\partial\epsilon$ denote the marginal change in utility at quantile z. The Envelope Theorem implies

$$\bar{U}_\epsilon(z,\epsilon) = \bar{u}_C(z,\epsilon)\, \bar{C}_R(z,\epsilon)\, [\bar{R}_x(z,\epsilon)\bar{n}(z,\epsilon)\bar{w}_\epsilon(z,\epsilon) + \bar{R}_\epsilon(z,\epsilon) - \bar{c}(z,\epsilon) \cdot p_\epsilon],$$

with

$$\begin{aligned}
\bar{u}_C(z,\epsilon) &\equiv u_C(C,n)|_{C=C(p(\epsilon),\bar{R}(z,\epsilon)),n=\bar{n}(z,\epsilon)}, \\
\bar{C}_R(z,\epsilon) &\equiv C_R(p,R)|_{p=p(\epsilon),R=\bar{R}(z,\epsilon)}, \\
\bar{R}(z,\epsilon) &\equiv R(\bar{x}(z,\epsilon),\epsilon), \\
\bar{R}_x(z,\epsilon) &\equiv R_x(x,\epsilon)|_{x=\bar{x}(z,\epsilon),\epsilon}, \\
\bar{R}_\epsilon(z,\epsilon) &\equiv R_\epsilon(x,\epsilon)|_{x=\bar{x}(z,\epsilon),\epsilon}.
\end{aligned}$$

This further implies

$$\frac{dW(\{\bar{U}(z,\epsilon)\})}{d\epsilon} = \int \frac{\partial W}{\partial \bar{U}(z)} \bar{u}_C(z,\epsilon)\, \bar{C}_R(z,\epsilon)\, [\bar{R}_x(z,\epsilon)\bar{n}(z,\epsilon)\bar{w}_\epsilon(z,\epsilon) - \bar{c}(z,\epsilon) \cdot p_\epsilon + \bar{R}_\epsilon(z,\epsilon)]\, dz. \quad \text{(E.2)}$$

Now consider the change in the cost of resources used by the new technology,

$$\frac{dG^*(y^*(\epsilon);\phi)}{d\epsilon} = \sum_i G^*_{y_i^*} y^*_{i,\epsilon} + \int G^*_{n^*(\theta)} n^*_\epsilon(\theta,\epsilon) dF(\theta).$$

Using equation (C.8), this can be rearranged as

$$\frac{dG^*(y^*(\epsilon);\phi)}{d\epsilon} = G^*_{y_1}\{(p^* - p)\cdot y^*_\epsilon + p\cdot \int c_\epsilon(\theta,\epsilon)dF(\theta) - p\cdot y_\epsilon\}. \tag{E.3}$$

From the first-order conditions associated with (C.3), we know that

$$p\cdot\frac{\partial y(p,n)}{\partial p_i} = 0 \text{ for all } i,$$
$$p\cdot\frac{\partial y(p,n)}{\partial n(\theta)} = w(\theta) \text{ for all } \theta.$$

It follows that

$$p\cdot y_\epsilon = p\cdot[\sum_i \frac{\partial y(p,n)}{\partial p_i}p_{i,\epsilon} + \int \frac{\partial y(p,n)}{\partial n(\theta)}n_\epsilon(\theta,\epsilon)dF(\theta)] = \int w(\theta)n_\epsilon(\theta,\epsilon)dF(\theta), \tag{E.4}$$

From the budget constraint associated with (C.1), we also know that

$$\frac{d}{d\epsilon}[p(\epsilon)\cdot\int c(\theta,\epsilon)dF(\theta)] = \frac{dR(w(\theta,\epsilon)n(\theta,\epsilon),\epsilon)}{d\epsilon}.$$

It follows that

$$\begin{aligned} p\cdot\int c_\epsilon(\theta,\epsilon)dF(\theta) = &-\int p_\epsilon\cdot c(\theta,\epsilon)dF(\theta) + \int [R_\epsilon(w(\theta,\epsilon)n(\theta,\epsilon),\epsilon) \\ &+ R_x(w(\theta,\epsilon)n(\theta,\epsilon),\epsilon)\big(w_\epsilon(\theta,\epsilon)n(\theta,\epsilon) + w(\theta,\epsilon)n_\epsilon(\theta,\epsilon)\big)\big]dF(\theta). \end{aligned} \tag{E.5}$$

Combining (E.3)-(E.5) and using $w(\theta,\epsilon)n(\theta,\epsilon) = x(\theta,\epsilon)$, we get

$$\begin{aligned} \frac{dG^*(y^*(\epsilon);\phi)}{d\epsilon} &= G^*_{y_1}\{(p^* - p)\cdot y^*_\epsilon - \int p_\epsilon\cdot c(\theta,\epsilon)dF(\theta) \\ &\int [R_\epsilon(x(\theta,\epsilon),\epsilon) + R_x(x(\theta,\epsilon),\epsilon)w_\epsilon(\theta,\epsilon)n(\theta,\epsilon) - (1 - R_x(x(\theta,\epsilon),\epsilon))w(\theta,\epsilon)n_\epsilon(\theta,\epsilon))]dF(\theta)\}. \end{aligned}$$

Applying the corollary of Lemma 2, we have

$$\bar{n}_\epsilon(z,\epsilon) = \int n_\epsilon(\theta,\epsilon)f(\theta|w(\theta,\epsilon) = \bar{w}(z,\epsilon))d\theta.$$

Thus we can rearrange the previous expression as

$$\begin{aligned} \frac{dG^*(y^*(\epsilon);\phi)}{d\epsilon} = G^*_{y_1}\{(p^* - p)\cdot y^*_\epsilon &+ \int [\bar{R}_\epsilon(z,\epsilon) + \bar{R}_x(z,\epsilon)\bar{n}(z,\epsilon)\bar{w}_\epsilon(z,\epsilon) - \bar{c}(z,\epsilon)\cdot p_\epsilon]dz \\ &- \int (1 - \bar{R}_x(z,\epsilon))\bar{w}(z,\epsilon)\bar{n}_\epsilon(z,\epsilon)]dz\}. \end{aligned} \tag{E.6}$$

A necessary condition for a feasible variation not to improve welfare is that

$$\frac{dW(\{\bar{U}(z,\epsilon)\})}{d\epsilon}\bigg|_{\epsilon=0} - \gamma\,\frac{dG^*(y^*(\epsilon);\phi)}{d\epsilon}\bigg|_{\epsilon=0} = 0.$$

Combining equations (E.2) and (E.6), this implies

$$\int [\bar{\lambda}(z)-1][(1-\tau(z))\bar{n}(z)\bar{w}_\epsilon(z) - \bar{c}(z)\cdot p_\epsilon - \bar{T}_\epsilon(z)]dz$$
$$= (p^*-p)\cdot y^*_\epsilon - \int \tau(z)\bar{w}(z)\bar{n}_\epsilon(z)dz,$$

with

$$\bar{\lambda}(z) \equiv \frac{(\partial W/\partial \bar{U}(z))|_{\epsilon=0}\;\bar{u}_C(z,0)\,\bar{C}_R(z,0))}{\gamma G^*_{y_1}(y^*(0))},$$
$$\tau(z) \equiv 1-\bar{R}_x(z,0),$$
$$\bar{T}_\epsilon(z) \equiv -\bar{R}_\epsilon(z,0),$$

as well as the obvious short hand notation, $\bar{n}(z)\equiv\bar{n}(z,0)$ etc.

E.2 Proposition 1

Preliminaries. We introduce the following definitions,

$$\tilde{C}(x,\epsilon) \equiv C(R(x,\epsilon),p(\epsilon)),$$
$$MRS(C,n) \equiv -\frac{u_n(C,n)}{u_C(C,n)}.$$

For the household at the quantile z of the utility distribution, we then have

$$\bar{U}(z,\epsilon) = u(\tilde{C}(\bar{x}(z,\epsilon),\epsilon),\bar{n}(z,\epsilon))], \tag{E.7}$$
$$MRS(\tilde{C}(\bar{x}(z,\epsilon),\epsilon),\bar{n}(z,\epsilon)) = \bar{w}(z,\epsilon)\tilde{C}_x(\bar{x}(z,\epsilon),\epsilon), \tag{E.8}$$
$$\bar{x}(z,\epsilon) = \bar{w}(z,\epsilon)\bar{n}(z,\epsilon), \tag{E.9}$$

where the second equation corresponds to the first-order condition associated with C.2.

Differentiating equations (E.7)-(E.9) with respect to ϵ implies

$$\bar{U}_\epsilon = \bar{u}_C(\tilde{C}_x\bar{n}\bar{w}_\epsilon + \tilde{C}_\epsilon), \tag{E.10}$$
$$\bar{n}_\epsilon = \frac{(\tilde{C}_{xx}\bar{w}\bar{n}+\tilde{C}_x)\bar{w}_\epsilon + \tilde{C}_{x\epsilon}\bar{w} - MRS_C(\tilde{C}_x\bar{n}\bar{w}_\epsilon+\tilde{C}_\epsilon)}{(MRS_C\tilde{C}_x\bar{w} + MRS_n - \tilde{C}_{xx}\bar{w}^2)} \tag{E.11}$$

Below we use equations (E.10) and (E.11) to construct variations such that $\bar{n}_\epsilon = 0$ and $\bar{U}_\epsilon = 0$.

Variation $\delta T = 0$. Consider a first variation such that

$$t_i^*(\epsilon) = t_i^* + \epsilon \hat{t}_i^* \text{ for all } i,$$
$$T(x,\epsilon) = T(x) \text{ for all } x \geq 0,$$

From Lemma 1, if $T_\epsilon = 0$, then

$$(p^* - p) \cdot y_\epsilon^* - \int \tau(z)\bar{w}(z)\bar{n}_\epsilon(z)\, dz = \int (\bar{\lambda}(z) - 1)((1 - \tau(z))\bar{n}(z)\, \bar{w}_\epsilon(z) - \bar{c}(z) \cdot p_\epsilon)\, dz. \quad \text{(E.12)}$$

Pick $\hat{t}^* \equiv \{\hat{t}_i^*\}$ such that $y_{1,\epsilon}^*, y_{i,\epsilon}^* \neq 0$ and $y_{j,\epsilon}^* = 0$ for all $j \neq 1, i$. This implies

$$p_i - p_i^*$$
$$= \int \left[(1 - \bar{\lambda}(z))\left((1 - \tau(z))\bar{n}(z) \frac{\delta \bar{w}(z)}{\delta y_i^*}|_{\delta T = 0} - \bar{c}(z) \frac{\delta p}{\delta y_i^*}|_{\delta T = 0}\right) - \tau(z)\bar{w}(z) \frac{\delta \bar{n}(z)}{\delta y_i^*}|_{\delta T = 0} \right] dz.$$

Variation $\delta n = 0$. Consider a variation

$$t_i^*(\epsilon) = t_i^* + \epsilon \hat{t}_i^* \text{ for all } i,$$
$$T(x,\epsilon) = T(x) + \epsilon \hat{T}(x) \text{ for all } x \geq 0,$$

such that $\bar{n}_\epsilon = 0$ and $(dG^*(y^*(\epsilon))/d\epsilon)_{\epsilon=0} = 0$. From Lemma 1, we know that if $\bar{n}_\epsilon(z) = 0$ for all z, then

$$(p^* - p) \cdot y_\epsilon^* = \int [\bar{\lambda}(z) - 1][\bar{R}_x(z)\bar{n}(z)\bar{w}_\epsilon(z) - \bar{c}(z) \cdot p_\epsilon + \bar{R}_\epsilon(z)]dz.$$

By definition of $\tilde{C}(x,\epsilon) \equiv C(R(x,\epsilon), p(\epsilon))$, we must also have

$$\tilde{C}_x(\bar{x}(z), 0) = \bar{C}_R(z)\bar{R}_x(z), \quad \text{(E.13)}$$
$$\tilde{C}_\epsilon(\bar{x}(z), 0) = \bar{C}_R(z)[\bar{R}_\epsilon - \bar{c}(z) \cdot p_\epsilon]. \quad \text{(E.14)}$$

Thus, we can rearrange the previous expression as

$$(p^* - p) \cdot y_\epsilon^* = \int [\tilde{C}_x(\bar{x}(z), 0)\bar{n}(z)\bar{w}_\epsilon(z) + \tilde{C}_\epsilon(\bar{x}(z), 0)] \frac{[\bar{\lambda}(z) - 1]}{\bar{C}_R(z)} dz. \quad \text{(E.15)}$$

Let us now compute $\tilde{C}_\epsilon$. By equation (E.11), for $\bar{n}_\epsilon = 0$, $\tilde{C}_\epsilon$ must solve

$$\tilde{C}_{x\epsilon}(\bar{x}(z), 0) - \rho(z)\tilde{C}_\epsilon(\bar{x}(z), 0) = g(z),$$

with

$$\rho(z) \equiv \frac{MRS_C(\tilde{C}(\bar{x}(z),0),\bar{n}(z))}{\bar{w}(z)} \tag{E.16}$$

$$g(z) \equiv -\frac{\bar{w}_\epsilon(z)}{\bar{w}(z)}\tilde{C}_x(\bar{x}(z),0)[1+\frac{\tilde{C}_{xx}(\bar{x}(z),0)\bar{w}(z)\bar{n}(z)}{\tilde{C}_x(\bar{x}(z),0)}]+\rho(z)\tilde{C}_x(\bar{x}(z),0)\bar{n}(z)\bar{w}_\epsilon(z). \tag{E.17}$$

The general solution to this ODE is

$$\tilde{C}_\epsilon(\bar{x}(z),0)=\int_0^z \exp[\int_{\tilde{z}}^z \rho(v)d\bar{x}(v)]g(\tilde{z})dx(\tilde{z})+\exp[\int_0^z \rho(v)d\bar{x}(v)]\tilde{C}_\epsilon(\bar{x}(0),0). \tag{E.18}$$

The initial condition $\tilde{C}_\epsilon(\bar{x}(0),0)$, in turn, is given by the requirement that $(dG^*(y^*(\epsilon))/d\epsilon)_{\epsilon=0}=0$. Using equations (E.6), (E.13), and (E.14), we can rearrange this requirement as

$$(p^*-p)\cdot y_\epsilon^*+\int[\tilde{C}_x(\bar{x}(z),0)\bar{n}(z)\bar{w}_\epsilon(z)+\tilde{C}_\epsilon(\bar{x}(z),0)]\frac{1}{\bar{C}_R(z)}dz=0.$$

Combining this expression with equation (E.18) leads to

$$\tilde{C}_\epsilon(\bar{x}(0),0)=\frac{(p-p^*)\cdot y_\epsilon^*-\int[\tilde{C}_x(\bar{x}(z),0)\bar{n}(z)\bar{w}_\epsilon(z)+\int_0^z \exp[\int_{\tilde{z}}^z \rho(v)d\bar{x}(v)]g(\tilde{z})dx(\tilde{z})]\frac{1}{\bar{C}_R(z)}dz}{\int \exp[\int_0^z \rho(v)d\bar{x}(v)]\frac{1}{\bar{C}_R(z)}dz}. \tag{E.19}$$

Next, substituting for $\tilde{C}_\epsilon$ in (E.15) using (E.18) and (E.19), we obtain

$$(p^*-p)\cdot y_\epsilon^*=\int[\tilde{C}_x(\bar{x}(z),0)\bar{n}(z)\bar{w}_\epsilon(z)\exp[-\int_0^z \rho(v)d\bar{x}(v)]+\int_0^z \exp[-\int_0^{\tilde{z}} \rho(v)d\bar{x}(v)]g(\tilde{z})dx(\tilde{z})]d\Upsilon(z),$$

with

$$d\Upsilon(z)\equiv\left[\frac{\int \exp[\int_0^{\tilde{z}}\rho(v)d\bar{x}(v)]\frac{1}{\bar{C}_R(\tilde{z})}d\tilde{z}}{\int \exp[\int_0^{\tilde{z}}\rho(v)d\bar{x}(v)]\frac{\bar{\lambda}(\tilde{z})}{\bar{C}_R(\tilde{z})}d\tilde{z}}\bar{\lambda}(z)-1\right]\frac{\exp[\int_0^z \rho(v)d\bar{x}(v)]}{\bar{C}_R(z)}dz.$$

Integrating by parts, we get

$$(p^*-p)\cdot y_\epsilon^*=-\int \xi(z)\{\tilde{C}_{xx}\bar{x}_z(z)\bar{w}_\epsilon\bar{n}+\tilde{C}_x\bar{w}_{\epsilon z}\bar{n}+\tilde{C}_x\bar{w}_\epsilon\bar{n}_z-\rho(z)\bar{x}_z(z)\tilde{C}_x\bar{w}_\epsilon\bar{n}+g(z)x_z\}dz.$$

with $\xi(z)\equiv\exp[-\int_0^z\rho(v)d\bar{x}(v)]\int_0^z d\Upsilon(\tilde{z})$. Substituting for $g(z)$ using (E.17) implies

$$(p^*-p)\cdot y_\epsilon^*=-\int \xi(z)\tilde{C}_x\{-\frac{\bar{w}_\epsilon}{\bar{w}}\bar{x}_z(z)+\bar{w}_{\epsilon z}\bar{n}+\bar{w}_\epsilon\bar{n}_z\}dz.$$

Using the fact that $\bar{x}_z(z)=\bar{w}_z\bar{n}+\bar{w}\bar{n}_z$, one can further check that

$$-\frac{\bar{w}_\epsilon}{\bar{w}}\bar{x}_z(z)+\bar{w}_{\epsilon z}\bar{n}+\bar{w}_\epsilon\bar{n}_z=(\bar{w}\bar{n})\omega_\epsilon,$$

with $\omega \equiv \frac{d \ln \bar{w}}{dz}$. This leads to

$$(p^* - p) \cdot y^*_\epsilon = -\int \bar{\psi}(z)(1-\tau(z))\bar{w}(z)\bar{n}(z)\omega_\epsilon(z)dz$$

with

$$\bar{\psi}(z) \equiv \int_0^z \left\{ \left[\frac{\int \exp[\int_0^{z'} \rho(v)d\bar{x}(v)] \frac{\bar{\lambda}(\tilde{z})}{\bar{C}_R(z')} dz'}{\int \exp[\int_0^{z'} \rho(v)d\bar{x}(v)] \frac{\bar{\lambda}(z')}{\bar{C}_R(z')} dz'} - 1 \right] \frac{\bar{C}_R(z)}{\bar{C}_R(\tilde{z})} \exp[\int_z^{\tilde{z}} \rho(v)d\bar{x}(v)] \right\} d\tilde{z}.$$

Pick $\hat{T}$ and $\hat{t}^* \equiv \{\hat{t}^*_i\}$ such that, in addition to $\bar{n}_\epsilon = 0$, $\tilde{C}_\epsilon(\bar{x}(0), 0)$ satisfies (E.19), and $y^*_{1,\epsilon}, y^*_{i,\epsilon} \neq 0$ and $y^*_{j,\epsilon} = 0$ for all $j \neq 1, i$. Then

$$p_i - p^*_i = \int \bar{\psi}(z)(1-\tau(z))\bar{w}(z)\bar{n}(z)\frac{\delta\omega}{\delta y^*_i}|_{\delta n=0}\, dz.$$

Variation $\delta U = 0$. Consider a variation

$$t^*_i(\epsilon) = t^*_i + \epsilon\hat{t}^*_i \text{ for all } i,$$
$$T(x,\epsilon) = T(x) + \epsilon\hat{T}(x) \text{ for all } x \geq 0,$$

such that $\bar{U}_\epsilon = 0$. From Lemma 1, we know that

$$(p^* - p) \cdot y^*_\epsilon - \int \tau(z)\bar{w}(z)\bar{n}_\epsilon(z)\, dz$$
$$= \int (\bar{\lambda}(z) - 1)[(1-\tau(z))\bar{n}(z)\,\bar{w}_\epsilon(z) - \bar{c}(z)\cdot p_\epsilon - T_\epsilon(z)]\, dz.$$

In the proof of Lemma 1, we have already established that

$$\bar{U}_\epsilon(z) = \bar{u}_C(z)\,\bar{C}_R(z)[(1-\tau(z))\bar{n}(z)\,\bar{w}_\epsilon(z) - \bar{c}(z)\cdot p_\epsilon - T_\epsilon(z)].$$

It follows that if $\bar{U}_\epsilon = 0$, then

$$(p^* - p) \cdot y^*_\epsilon = \int \tau(z)\bar{w}(z)\bar{n}_\epsilon(z)\, dz. \tag{E.20}$$

Let us now compute $\bar{n}_\epsilon(z)$. By equation (E.10), $\bar{U}_\epsilon = 0$ implies

$$\tilde{C}_x(\bar{x}(z), 0)\bar{n}(z)\bar{w}_\epsilon(z) + \tilde{C}_\epsilon(\bar{x}(z), 0) = 0. \tag{E.21}$$

Differentiating with respect to z implies

$$\left(\tilde{C}_{\epsilon x} + \tilde{C}_{xx}\bar{w}_\epsilon\bar{n}\right)(\bar{n}_z\bar{w} + \bar{n}\bar{w}_z) + \tilde{C}_x\bar{w}_{\epsilon z}\bar{n} + \tilde{C}_x\bar{w}_\epsilon\bar{n}_z = 0. \tag{E.22}$$

Differentiating (E.8) with respect to z further implies

$$\bar{n}_z = \frac{\tilde{C}_{xx}\varpi\bar{n} + \tilde{C}_x - MRS_C\tilde{C}_x\bar{n}}{MRS_C\tilde{C}_x\varpi + MRS_n - \tilde{C}_{xx}\varpi^2}\varpi_z.$$

Substituting for $\bar{n}_z$ in equation (E.22), we get

$$\begin{aligned}&\left(\tilde{C}_{\epsilon x} + \tilde{C}_{xx}\varpi_\epsilon\bar{n}\right)\left(\tilde{C}_x\varpi + MRS_n\bar{n}\right)\varpi_z \\ &\quad + \left(MRS_C\tilde{C}_x\varpi + MRS_n - \tilde{C}_{xx}\varpi^2\right)\tilde{C}_x\varpi_{\epsilon z}\bar{n} + \tilde{C}_x\varpi_\epsilon\left(\tilde{C}_{xx}\varpi\bar{n} + \tilde{C}_x - MRS_C\tilde{C}_x\bar{n}\right)\varpi_z = 0.\end{aligned}$$

Let $\varepsilon(z) \equiv \frac{MRS(\tilde{C}(\bar{x}(z),0),\bar{n}(z))}{MRS_n(\tilde{C}(\bar{x}(z),0),\bar{n}(z))\bar{n}(z)}$ denote the consumption-compensated elasticity of labor supply. Using equation (E.8), the previous expression can be rearranged as

$$\begin{aligned}\tilde{C}_{\epsilon x} = -\tilde{C}_{xx}\varpi_\epsilon\bar{n} - \frac{\varepsilon\left(MRS_C\tilde{C}_x\varpi\bar{n} + MRS_n\bar{n} - \tilde{C}_{xx}\varpi^2\bar{n}\right)}{1+\varepsilon}\frac{\varpi_{\epsilon z}}{\varpi_z\varpi} \\ + \frac{\varepsilon\left(MRS_C\tilde{C}_x\varpi\bar{n} - \tilde{C}_x\varpi - \tilde{C}_{xx}\varpi^2\bar{n}\right)}{1+\varepsilon}\frac{\varpi_\epsilon}{\varpi^2}.\end{aligned}$$

Together with equation (E.21), equation (E.11) further implies

$$\bar{n}_\epsilon = \frac{(\tilde{C}_{xx}\varpi\bar{n} + \tilde{C}_x)\varpi_\epsilon + \tilde{C}_{x\epsilon}\varpi}{(MRS_C\tilde{C}_x\varpi + MRS_n - \tilde{C}_{xx}\varpi^2)}.$$

Substituting for $\tilde{C}_{\epsilon x}$ leads to

$$\bar{n}_\epsilon = \frac{\tilde{C}_x\varpi_\epsilon - \frac{\varepsilon}{1+\varepsilon}\left(\left(MRS_C\tilde{C}_x\varpi + MRS_n - \tilde{C}_{xx}\varpi^2\right)\bar{n}\frac{\varpi_{\epsilon z}}{\varpi_z} - \left(MRS_C\tilde{C}_x\bar{n}\varpi - \tilde{C}_x\varpi - \tilde{C}_{xx}\varpi^2\bar{n}\right)\frac{\varpi_\epsilon}{\varpi}\right)}{(MRS_C\tilde{C}_x\varpi + MRS_n - \tilde{C}_{xx}\varpi^2)}.$$

Differentiating $\omega \equiv \frac{d\ln\varpi}{dz}$ with respect to ϵ also implies

$$\frac{\omega_\epsilon}{\omega} = \frac{1}{\frac{\varpi_z}{\varpi}}\frac{\partial}{\partial\epsilon}\left(\frac{\varpi_z}{\varpi}\right) = \frac{1}{\frac{\varpi_z}{\varpi}}\left(\frac{\varpi_{z\epsilon}}{\varpi} - \frac{\varpi_z\varpi_\epsilon}{\varpi^2}\right) = \frac{\varpi_{z\epsilon}}{\varpi_z} - \frac{\varpi_\epsilon}{\varpi}.$$

Combining the two previous expressions we obtain

$$\bar{n}_\epsilon = \frac{\frac{\tilde{C}_x\varpi_\epsilon}{1+\varepsilon} - \frac{\varepsilon}{1+\varepsilon}\bar{n}MRS_n\frac{\varpi_{\epsilon z}}{\varpi_z} - \frac{\varepsilon}{1+\varepsilon}\bar{n}\frac{\omega_\epsilon}{\omega}\left(MRS_C\tilde{C}_x\varpi - \tilde{C}_{xx}\varpi^2\right)}{(MRS_C\tilde{C}_x\varpi + MRS_n - \tilde{C}_{xx}\varpi^2)}.$$

By equation (E.8), we know that

$$\frac{\tilde{C}_x\varpi_\epsilon}{1+\varepsilon} = \frac{MRS}{1+\varepsilon}\frac{\varpi_\epsilon}{\varpi} = \frac{\varepsilon}{1+\varepsilon}\bar{n}MRS_n\frac{\varpi_\epsilon}{\varpi}.$$

It follows that

$$\bar{n}_\epsilon = -\frac{\varepsilon}{1+\varepsilon}\bar{n}\frac{\omega_\epsilon}{\omega}.$$

Substituting for $\bar{n}_\epsilon$ into (E.20), we obtain

$$(p^* - p)\cdot y^*_\epsilon = -\int \tau(z)\bar{w}(z)\bar{n}(z)\frac{\varepsilon(z)}{1+\varepsilon(z)}\frac{\omega_\epsilon(z)}{\omega(z)}\,dz.$$

Pick $\hat{T}$ and $\hat{t}^* \equiv \{\hat{t}^*_i\}$ such that, in addition to $\bar{U}_\epsilon = 0$, $y^*_{1,\epsilon}, y^*_{i,\epsilon} \neq 0$ and $y^*_{j,\epsilon} = 0$ for all $j \neq 1, i$. Then

$$p_i - p^*_i = \int \tau(z)\bar{w}(z)\bar{n}(z)\,\frac{\varepsilon(z)}{\varepsilon(z)+1}\frac{1}{\omega(z)}\frac{\delta\omega(z)}{\delta y^*_i}|_{\delta U=0}\,dz.$$

E.3 Lagrange Multiplier is Strictly Positive

In Footnote 5, we have argued that γ must be strictly positive. We have later used this result in the proof of Lemma 1. We now provide the formal argument showing that γ cannot be equal to zero at an optimum in our environment.

Start with $\gamma \geq 0$. Following the same steps as in the proof of Lemma 1, one can show that for any variation that satisfies all the equilibrium conditions except $G^*(y^*;\phi) \leq 0$, we must have

$$\int [\tilde{\lambda}(z) - \gamma][(1-\tau(z))\bar{n}(z)\bar{w}_\epsilon(z) - \bar{c}(z)\cdot p_\epsilon - \bar{T}_\epsilon(z)]dz$$
$$= \gamma[(p^* - p)\cdot y^*_\epsilon - \int \tau(z)\bar{w}(z)\bar{n}_\epsilon(z)dz],$$

with

$$\tilde{\lambda}(z) \equiv \frac{(\partial W/\partial \bar{U}(z))|_{\epsilon=0}\;\bar{u}_C(z,0)\;\bar{C}_R(z,0))}{G^*_{y_1}(y^*(0))}.$$

Now construct a variation in the taxes on old technology firms and the income tax schedule,

$$t^*_i(\epsilon) = t^*_i + \epsilon\hat{t}^*_i \text{ for all } i,$$
$$T(x,\epsilon) = T(x) + \epsilon\hat{T}(x) \text{ for all } x \geq 0,$$

such that $\bar{n}_\epsilon = 0$, $p_\epsilon = 0$, and, in turn, $\bar{w}_\epsilon = 0$. For any such variation, the same steps as in the proof our second formula leads to the counterpart of equation (E.15),

$$\gamma[(p^* - p)\cdot y^*_\epsilon + \int \frac{\tilde{C}_\epsilon(\bar{x}(z),0)}{\bar{C}_R(z)}dz] = \int \frac{\tilde{C}_\epsilon(\bar{x}(z),0)\tilde{\lambda}(z)}{\bar{C}_R(z)}dz, \tag{E.23}$$

with $\tilde{C}(x,\epsilon) \equiv C(R(x,\epsilon), p(\epsilon))$ the indirect utility of households with earnings x.

By equation (E.11), for $\bar{n}_\epsilon = 0$ and $\bar{w}_\epsilon = 0$, the change in indirect utility $\tilde{C}_\epsilon$ must solve

$$\tilde{C}_{x\epsilon}(\bar{x}(z),0) - \rho(z)\tilde{C}_\epsilon(\bar{x}(z),0) = 0,$$

with

$$\rho(z) \equiv \frac{MRS_C(\tilde{C}(\bar{x}(z),0),\bar{n}(z))}{\bar{w}(z)}.$$

The solution to this ODE is

$$\tilde{C}_\epsilon(\bar{x}(z),0) = \exp[\int_0^z \rho(v)d\bar{x}(v)]\tilde{C}_\epsilon(\bar{x}(0),0). \tag{E.24}$$

Now, pick $\hat{T}(0)$ such that $\tilde{C}_\epsilon(\bar{x}(0),0) > 0$. By equation (E.24), we therefore have $\tilde{C}_\epsilon(\bar{x}(z),0) > 0$ for all z. For such a variation, the right-hand side of equation (E.23) is strictly positive. So the left-hand side must be strictly positive as well. This establishes that γ cannot be zero.

E.4 Quantitative Examples

Consider the robot example. We think of the mapping between theory and data as follows. In theory, we need $\delta \ln \omega(z)|_{\delta \bar{U}=0}$ and $\delta y_m^*|_{\delta \bar{U}=0}$ to implement Formula 3. In the data, we observe $\delta \ln \omega(z)|_{data}$ and $\delta y_m^*|_{data}$. In a competitive equilibrium, we know that $\omega(z)$ and y_m^* depend both on the price of robots p_m and the vector of labor supply, n. Totally differentiating, we therefore have

$$\delta \ln \omega(z)|_{\delta \bar{U}=0} = \frac{\partial \ln \omega(z)}{\partial \ln p_m} \delta \ln p_m|_{\delta \bar{U}=0} + \int (\frac{\partial \ln \omega(z)}{\partial \ln n(\theta)} \delta \ln n(\theta)|_{\delta \bar{U}=0}) d\theta,$$
$$\delta y_m^*|_{\delta \bar{U}=0} = \frac{\partial y_m^*}{\partial \ln p_m} \delta \ln p_m|_{\delta \bar{U}=0} + \int (\frac{\partial y_m^*}{\partial \ln n(\theta)} \delta \ln n(\theta)|_{\delta \bar{U}=0}) d\theta,$$

whereas

$$\delta \ln \omega(z)|_{data} = \frac{\partial \ln \omega(z)}{\partial \ln p_m} \delta \ln p_m|_{data} + \int (\frac{\partial \ln \omega(z)}{\partial \ln n(\theta)} \delta \ln n(\theta)|_{data}) d\theta,$$
$$\delta y_m^*|_{data} = \frac{\partial y_m^*}{\partial \ln p_m} \delta \ln p_m|_{data} + \int (\frac{\partial y_m^*}{\partial \ln n(\theta)} \delta \ln n(\theta)|_{data}) d\theta.$$

For our approximation to be valid, $\frac{\delta \ln \omega(z)|_{\delta \bar{U}=0}}{\delta y_m^*|_{\delta \bar{U}=0}} \simeq \frac{\delta \ln \omega(z)|_{data}}{\delta y_m^*|_{data}}$, a sufficient condition is therefore that the indirect labor supply effects caused by changes in the price of robots, both along our variation and in the data) are small relative to the direct price effects:

$$\frac{\partial \ln \omega(z)}{\partial \ln p_m} \delta \ln p_m|_{\delta \bar{U}=0} \gg \int (\frac{\partial \ln \omega(z)}{\partial \ln n(\theta)} \delta \ln n(\theta)|_{\delta \bar{U}=0}) d\theta,$$
$$\frac{\partial y_m^*}{\partial \ln p_m} \delta \ln p_m|_{\delta \bar{U}=0} \gg \int (\frac{\partial y_m^*}{\partial \ln n(\theta)} \delta \ln n(\theta)|_{\delta \bar{U}=0}) d\theta,$$
$$\frac{\partial \ln \omega(z)}{\partial \ln p_m} \delta \ln p_m|_{data} \gg \int (\frac{\partial \ln \omega(z)}{\partial \ln n(\theta)} \delta \ln n(\theta)|_{data}) d\theta,$$
$$\frac{\partial y_m^*}{\partial \ln p_m} \delta \ln p_m|_{data} \gg \int (\frac{\partial y_m^*}{\partial \ln n(\theta)} \delta \ln n(\theta)|_{data}) d\theta.$$

The trade example can be dealt with in a similar manner.

F Section 6

F.1 Preliminaries

Government Problem. Suppose, as in Section 6, that skill heterogeneity is one-dimensional, $\theta \in [0,1]$, so that we can write social welfare as a function of the utility of each skill type, $W(\{U(\theta)\})$. Using the revelation principle, we can express the government problem as

$$\max_{U,n,p} W(\{U(\theta)\})$$

subject to

$$\theta \in \operatorname{argmax}_{\tilde{\theta}} u \left(C(n(\tilde{\theta}), U(\tilde{\theta})), n(\tilde{\theta}) \frac{w(p,n;\tilde{\theta})}{w(p,n;\tilde{\theta})} \right),$$
$$G^*(c(p,n,U) - y(p,n); \phi) \leq 0,$$

where $C(n(\theta), U(\theta))$ is the aggregate consumption required to achieve utility $U(\theta)$ given labor supply $n(\theta)$, that is the solution to $u(C, n(\theta)) = U(\theta)$, and $c(p,n,U) = \int c(p, C(n(\theta), U(\theta))) dF(\theta)$ is the total demand for goods conditional on prices, p, labor supply, $n \equiv \{n(\theta)\}$, and utility levels, $U \equiv \{U(\theta)\}$, with $c(p,C)$ the solution to $\min_{\tilde{c}(\theta)} \{p \cdot \tilde{c} | v(\tilde{c}) \geq C\}$.

The envelope condition associated with the Incentive Compatibility constraint gives

$$U'(\theta) = -u_n(C(n(\theta), U(\theta)), n(\theta)) n(\theta) \omega(p,n;\theta)$$

with $\omega(p,n;\theta) \equiv \frac{w_\theta(p,n;\theta)}{w(p,n;\theta)}$. For piecewise differentiable allocations, the envelope condition and monotonicity of the mapping from wages, $w(p,n;\theta)$, to before-tax earnings, $w(p,n;\theta)n(\theta)$ is equivalent to incentive compatibility. We will focus on cases where $w(p,n;\theta)$ is increasing in θ, which for a given allocation can be interpreted as a normalization or ordering of θ. Under the previous conditions, we can rearrange our planning problem as

$$\max_{U,n,p} W(\{U(\theta)\})$$

subject to

$$U'(\theta) = -u_n(C(n(\theta), U(\theta)), n(\theta))n(\theta)\omega(p, n; \theta),$$

$$G^*(c(p, n, U) - y(p, n); \phi) \leq 0.$$

Under the functional-form assumptions of Section 6 this simplifies further into

$$\max_{U,n,p_m} \int U(\theta) d\Lambda(\theta) \tag{F.1a}$$

subject to

$$U'(\theta) = h'(n(\theta))n(\theta)\omega(p_m; \theta), \tag{F.1b}$$

$$\phi[\int (U(\theta) + h(n(\theta)))dF(\theta) - y_f(p_m, n)] - y_m(p_m, n) \leq 0, \tag{F.1c}$$

with $h(n(\theta)) \equiv \frac{(n(\theta))^{1+1/\varepsilon}}{1+1/\varepsilon}$ and $\Lambda(\theta) = 1$ for all θ, that is full weight at $\theta = 0$.

Lagrangian. The Lagrangian associated with the planner's problem (F.1) is given by

$$\mathcal{L} = \int U(\theta) d\Lambda(\theta) + \int \mu(\theta) \left(U'(\theta) - h'(n(\theta))n(\theta)\omega(p_m; \theta)\right) d\theta \\ - \gamma[\phi[\int (U(\theta) + h(n(\theta)))dF(\theta) - y_f(p_m, n)] - y_m(p_m, n)].$$

Integrating by parts, we get

$$\mathcal{L} = \int U(\theta) d\Lambda(\theta) - \int \mu'(\theta) U(\theta) d\theta + U(1)\mu(1) - U(0)\mu(0) \\ - \int \mu(\theta) h'(n(\theta)) n(\theta) \omega(p_m; \theta) d\theta \\ - \gamma[\phi[\int (U(\theta) + h(n(\theta)))dF(\theta) - y_f(p_m, n)] - y_m(p_m, n)].$$

Since $U(1)$ and $U(0)$ are free we must have

$$\mu(0) = \mu(1) = 0.$$

First-order Conditions: $U(\theta)$. The first-order condition with respect to $U(\theta)$ leads to

$$\lambda(\theta) - \mu'(\theta) - \gamma\phi f(\theta) = 0.$$

Since $\mu(0) = 0$, integrating between 0 and θ, we get

$$\mu(\theta) = \Lambda(\theta) - \gamma\phi F(\theta).$$

Since $\mu(1) = 0$, we must also have

$$1 - \gamma\phi = 0.$$

Combining the two previous observations, we obtain

$$\frac{\mu(\theta)}{\gamma\phi} = \Lambda(\theta) - F(\theta). \tag{F.2}$$

First-order Conditions: $n(\theta)$. The first-order condition with respect to $n(\theta)$ is given by

$$\begin{aligned}\gamma\phi[y_{f,n(\theta)}(p_m, n) - h'(n(\theta)) + \frac{1}{\phi} y_{m,n(\theta)}(p_m, n\}]f(\theta) \\ = \mu(\theta)[h''(n(\theta))n(\theta) + h'(n(\theta))]\omega(p_m;\theta),\end{aligned} \tag{F.3}$$

with $y_{f,n(\theta)} \equiv \partial y_f / \partial n(\theta)$ and $y_{m,n(\theta)} \equiv \partial y_m / \partial n(\theta)$. The first-order conditions of old technology firms, new technology firms, and households imply

$$\begin{aligned} y_{f,n(\theta)}(p_m, n) + p_m y_{m,n(\theta)}(p_m, n) &= w(\theta), \\ p_m^* &= 1/\phi, \\ h'(n(\theta)) &= w(\theta)(1 - \tau(\theta)). \end{aligned}$$

Thus, we can rearrange equation (F.3) into

$$\gamma\phi[w(\theta)\tau(\theta) + (p_m^* - p_m)y_{m,n(\theta)}]f(\theta) = \mu(\theta)h'(n(\theta))[\frac{\varepsilon(\theta)+1}{\varepsilon(\theta)}]\omega(p_m;\theta), \tag{F.4}$$

with $\varepsilon(\theta) \equiv d\ln(n(\theta))/d\ln h'(n(\theta)))$.

First-order Conditions: p_m. The first-order condition with respect to p_m is given by

$$\gamma\phi(p_m^* - p_m)y_{m,p_m} = \int \mu(\theta)h'(n(\theta))n(\theta)\omega_{p_m}(p_m;\theta)d\theta, \tag{F.5}$$

with $y_{m,p_m} \equiv \partial y_m / \partial p_m$ and $\omega_{p_m} \equiv \partial \omega / \partial p_m$.

Optimal Tax on Machines Using equation (F.4) to substitute for $\mu(\theta)h'(n(\theta))/\gamma$ in equation (F.5) and noting that $\partial \ln y_m(p_m, n(\theta); \theta)/\partial \ln n(\theta) = 1$,[15] we obtain

$$p_m - p_m^* = \frac{\int \frac{\varepsilon(\theta)}{\varepsilon(\theta)+1} \cdot \frac{d\ln \omega(\theta)}{d\ln |y_m|} \cdot \tau(\theta) \cdot x(\theta) dF(\theta)}{|y_m|[1 - \int \frac{\varepsilon(\theta)}{\varepsilon(\theta)+1} \cdot \frac{d\ln \omega(\theta)}{d\ln |y_m|} \cdot \frac{y_m(\theta)}{|y_m|} dF(\theta)]},$$

with $\frac{d\ln \omega(\theta)}{d\ln |y_m|} = \frac{\partial \ln p_m}{\partial \ln |y_m(p_m,n)|} \frac{d\ln \omega(p_m;\theta)}{d\ln p_m}$. Using $y_m^* = |y_m|$ and $t_m^* = p_m/p_m^* - 1$, this implies

$$t_m^* = \frac{\int \frac{\varepsilon(\theta)}{\varepsilon(\theta)+1} \frac{d\ln \omega(\theta)}{d\ln y_m^*} \tau(\theta) x(\theta) dF(\theta)}{p_m^* y_m^* [1 - \int \frac{\varepsilon(\theta)}{\varepsilon(\theta)+1} \frac{d\ln \omega(\theta)}{d\ln y_m^*} \frac{y_m(\theta)}{y_m^*} dF(\theta)]}. \tag{F.6}$$

Optimal Income Tax. Equations (F.2) and (F.4) imply

$$[w(\theta)\tau(\theta) + (p_m^* - p_m)y_{m,n(\theta)}] = \frac{[\Lambda(\theta) - F(\theta)]}{f(\theta)} h'(n(\theta))[\frac{\varepsilon(\theta)+1}{\varepsilon(\theta)}]\omega(p_m;\theta).$$

Using again the fact that $\partial \ln y_m(p_m, n(\theta); \theta)/\partial \ln n(\theta) = 1$ and $h'(n(\theta)) = w(\theta)(1 - \tau(\theta))$, from the first-order condition of the household's utility maximization problem, this leads to

$$\tau(\theta) = \tau^*(\theta) - \frac{p_m - p_m^*}{p_m} \frac{p_m y_m(\theta)}{x(\theta)} (1 - \tau^*(\theta)), \tag{F.7}$$

with

$$\tau^*(\theta) \equiv \frac{1}{1 + \frac{\varepsilon(\theta)}{\varepsilon(\theta)+1} \cdot \frac{f(\theta)}{(\Lambda(\theta) - F(\theta))\omega(p_m;\theta)}}.$$

Equation (9). Combining equations (F.6) and (F.7), we obtain

$$\frac{t_m^*}{1 + t_m^*} = \frac{\int \frac{\varepsilon(\theta)}{\varepsilon(\theta)+1} \cdot \frac{d\ln \omega(\theta)}{d\ln y_m^*} \cdot \tau^*(\theta) \cdot x(\theta) dF(\theta)}{p_m y_m^* [1 - \int \frac{\varepsilon(\theta)}{\varepsilon(\theta)+1} \cdot \frac{d\ln \omega(\theta)}{d\ln y_m^*} \cdot \tau^*(\theta) \cdot \frac{y_m(\theta)}{y_m^*} dF(\theta)]} \tag{F.8}$$

In the parametric example of Section (6), we have assumed

$$\varepsilon(\theta) = \varepsilon \text{ for all } \theta, \tag{F.9}$$

$$\Lambda(\theta) = 1 \text{ for all } \theta, \tag{F.10}$$

$$f(\theta) = 1 \text{ for all } \theta, \tag{F.11}$$

$$F(\theta) = \theta \text{ for all } \theta. \tag{F.12}$$

[15] Recall that $y_m(p_m, n(\theta); \theta)$ is implicitly defined as the solution to $p_m = \partial g(y_m(\theta), n(\theta); \theta)/\partial y_m(\theta)$. Since $g(\cdot, \cdot; \theta)$ is homogeneous of degree one, this is equivalent to $p_m = \partial g(y_m(\theta)/n(\theta), 1; \theta)/\partial y_m(\theta)$. Differentiating, we therefore get $\partial \ln y_m(p_m, n(\theta); \theta)/\partial \ln n(\theta) = 1$.

We therefore immediately get

$$\tau^*(\theta, p_m) = \frac{1}{1 + \frac{\varepsilon}{\varepsilon+1} \cdot \frac{1}{(1-\theta)\omega(p_m;\theta)}}. \tag{F.13}$$

In Section 6, we have also established that

$$w(p_m;\theta) = (1-\theta)^{-1/\gamma(p_m)},$$

which implies

$$\omega(p_m;\theta) = \frac{1}{\gamma(p_m)} \cdot \frac{1}{1-\theta}.$$

Substituting into equation (F.13), we therefore get

$$\tau^*(\theta) = \frac{1}{1 + \frac{\varepsilon}{\varepsilon+1}\gamma(p_m)} \equiv \tau^*. \tag{F.14}$$

In Section (6), we have also established that

$$\frac{d\ln\omega(p_m;\theta)}{d\ln p_m} = -\beta\gamma(p_m),$$

which implies

$$\frac{d\ln\omega(\theta)}{d\ln|y_m|} = -\beta\gamma(p_m)\frac{d\ln p_m}{d\ln|y_m(p_m,n)|} = \frac{d\ln\omega}{d\ln y_m^*}. \tag{F.15}$$

Combining equations (F.8), (F.9), (F.14) and (F.15), we obtain

$$\frac{t_m^*}{1+t_m^*} = \frac{\frac{\varepsilon}{\varepsilon+1}\frac{d\ln\omega}{d\ln y_m^*}\tau^*}{1 - \frac{\varepsilon}{\varepsilon+1}\frac{d\ln\omega}{d\ln y_m^*}\tau^*(\theta,p_m)} \frac{\int x(\theta)dF(\theta)}{p_m y_m^*}, \tag{F.16}$$

Letting $s_m \equiv \frac{p_m y_m^*}{\int x(\theta)dF(\theta) + p_m y_m^*}$, this leads to equation (9).

F.2 Proposition 2

From equation (9), we know that

$$\frac{t_m^*}{1+t_m^*} = \frac{\frac{\varepsilon}{\varepsilon+1}\frac{d\ln\omega}{d\ln y_m^*}\tau^*}{1 - \frac{\varepsilon}{\varepsilon+1}\frac{d\ln\omega}{d\ln y_m^*}\tau^*} \frac{1-s_m}{s_m},$$

with

$$
\begin{aligned}
\frac{d\ln\omega}{d\ln|y_m|} &= -\beta\gamma(p_m)\frac{d\ln p_m}{d\ln|y_m(p_m,n)|},\\
\tau^* &= \frac{1}{1+\frac{\varepsilon}{\varepsilon+1}\gamma(p_m)},\\
s_m &= \frac{p_m|y_m(p_m,n)|}{\int x(\theta)dF(\theta)+p_m|y_m(p_m,n)|},
\end{aligned}
$$

This expression can be rearranged as

$$
\frac{t_m^*}{1+t_m^*} = \frac{\Phi}{\rho-\Phi}\frac{1-s_m}{s_m} \tag{F.17}
$$

with

$$
\begin{aligned}
\Phi &= -\frac{\varepsilon\beta\gamma(p_m)}{(\varepsilon+1)+\varepsilon\gamma(p_m)},\\
\rho &= \frac{\partial\ln|y_m(p_m,n)|}{\partial\ln p_m}.
\end{aligned} \tag{F.18}
$$

We first demonstrate that Φ, s_m, and ρ can be expressed as functions of t_m^* and ϕ.

Using the fact that $p_m = (1+t_m^*)/\phi$, we can immediately rearrange equation (F.18) as

$$
\Phi = -\frac{\varepsilon\beta\gamma((1+t_m^*)/\phi)}{(\varepsilon+1)+\varepsilon\gamma((1+t_m^*)/\phi)} \equiv \Phi(t_m^*,\phi). \tag{F.19}
$$

To express s_m and ρ as a function of t_m^* and ϕ, we further need to solve for the optimal labor supply of each agent, $n(\theta)$, which itself depends on the marginal income tax rates, $\tau(\theta)$. Together with equations (F.14), equation (F.7) implies

$$
\tau(\theta) = \frac{\varepsilon+1-\frac{t_m^*}{1+t_m^*}\frac{p_m y_m(\theta)}{x(\theta)}\varepsilon\gamma(p_m)}{\varepsilon+1+\epsilon\gamma(p_m)}.
$$

From the first-order condition of the old technology firms, we know that

$$
\frac{p_m y_m(\theta)}{x(\theta)} = -\beta\ln(1-\theta), \tag{F.20}
$$

which leads to

$$
\tau(\theta) = \frac{\varepsilon+1+\frac{t_m^*}{1+t_m^*}\beta\varepsilon\gamma(p_m)\ln(1-\theta)}{\varepsilon+1+\varepsilon\gamma(p_m)}. \tag{F.21}
$$

The optimal labor supply is given by the agent's first-order condition

$$
n(\theta) = ((1-\tau(\theta))w(\theta))^{\varepsilon}. \tag{F.22}
$$

Combining equations (F.21) and (F.22) with the fact that $w(p_m;\theta) = (1-\theta)^{-1/\gamma(p_m)}$, we get

$$n(\theta) = (\frac{\varepsilon\gamma(p_m)}{\varepsilon+1+\varepsilon\gamma(p_m)})^{\varepsilon}(1-\frac{t_m^*}{1+t_m^*}\beta\ln(1-\theta))^{\varepsilon}(1-\theta)^{-\varepsilon/\gamma(p_m)},$$

and in turn,

$$\int w(\theta)n(\theta)d\theta = (\frac{\varepsilon\gamma(p_m)}{\varepsilon+1+\varepsilon\gamma(p_m)})^{\varepsilon}\int(1-\frac{t_m^*}{1+t_m^*}\beta\ln(1-\theta))^{\varepsilon}(1-\theta)^{-\frac{1+\varepsilon}{\gamma(p_m)}}d\theta$$

Using equation (F.20), we further get

$$p_m y_m(p_m, n(\theta);\theta) = -\beta\ln(1-\theta)(\frac{\varepsilon\gamma(p_m)}{\varepsilon+1+\varepsilon\gamma(p_m)})^{\varepsilon}(1-\frac{t_m^*}{1+t_m^*}\beta\ln(1-\theta))^{\varepsilon}(1-\theta)^{-\frac{1+\varepsilon}{\gamma(p_m)}}, \quad \text{(F.23)}$$

and in turn,

$$p_m|y(p_m,n)| = -\beta(\frac{\varepsilon\gamma(p_m)}{\varepsilon+1+\varepsilon\gamma(p_m)})^{\varepsilon}\int\ln(1-\theta)(1-\frac{t_m^*}{1+t_m^*}\beta\ln(1-\theta))^{\epsilon}(1-\theta)^{-\frac{1+\varepsilon}{\gamma(p_m)}}d\theta. \quad \text{(F.24)}$$

The aggregate share of robots is therefore given by

$$s_m = \frac{\int\beta\ln(1-\theta)(1-\frac{t_m^*}{1+t_m^*}\beta\ln(1-\theta))^{\varepsilon}(1-\theta)^{-\frac{1+\varepsilon}{\gamma((1+t_m^*)/\phi)}}d\theta}{\int(\beta\ln(1-\theta)-1)(1-\frac{t_m^*}{1+t_m^*}\beta\ln(1-\theta))^{\varepsilon}(1-\theta)^{-\frac{1+\varepsilon}{\gamma((1+t_m^*)/\phi)}}d\theta} \equiv s_m(t_m^*,\phi), \quad \text{(F.25)}$$

where we have again used $p_m = (1+t_m^*)/\phi$. The elasticity ρ can be computed in a similar manner. From equation (F.20) and the fact that $w(p_m;\theta) = (1-\theta)^{-1/\gamma(p_m)}$, we get

$$p_m y(p_m, n(\theta);\theta) = -\beta\ln(1-\theta)n(\theta)(1-\theta)^{-1/\gamma(p_m)}.$$

Using the previous expression with the definition of $\rho \equiv \frac{\partial\ln|y_m(p_m,n)|}{\partial\ln p_m}$, we get

$$\rho = \int\frac{y(p_m,n(\theta);\theta)}{|y_m(p_m,n)|}\frac{d\ln w(p_m;\theta)}{d\ln p_m}d\theta - 1.$$

Combining the previous expressions with equations (F.23), (F.24), and using the fact that $\frac{d\ln w(p_m;\theta)}{d\ln p_m} = \beta\ln(1-\theta)$ and $p_m = (1+t_m^*)/\phi$, we get

$$\rho = \frac{\int(\beta\ln(1-\theta)-1)\ln(1-\theta)(1-\frac{t_m^*}{1+t_m^*}\beta\ln(1-\theta))^{\varepsilon}(1-\theta)^{-\frac{1+\varepsilon}{\gamma((1+t_m^*)/\phi)}}d\theta}{\int\ln(1-\theta)(1-\frac{t_m^*}{1+t_m^*}\beta\ln(1-\theta))^{\varepsilon}(1-\theta)^{-\frac{1+\varepsilon}{\gamma((1+t_m^*)/\phi)}}d\theta} \equiv \rho(t_m^*,\phi). \quad \text{(F.26)}$$

At this point, we have established that the three statistics in equation (F.17) can be expressed as $\Phi(t_m^*, \phi)$, $\rho(t_m^*, \phi)$, and $s_m(t_m^*, \phi)$. We can therefore rearrange equation (F.17) as

$$H(t_m^*, \Phi(t_m^*, \phi), \rho(t_m^*, \phi), s_m(t_m^*, \phi)) = 0,$$

with

$$H(t_m^*, \Phi, \rho, s_r) \equiv \frac{\Phi}{\rho - \Phi} \cdot \frac{1 - s_m}{s_m} - \frac{t_m^*}{1 + t_m^*}.$$

By the Implicit Function Theorem, we have

$$\frac{dt_m^*}{d\phi} = -\frac{dH/d\phi}{dH/dt_m^*}. \tag{F.27}$$

Since the tax on robots is chosen to maximize welfare, the second derivative of the government's value function, expressed as a function of t_m^* only, must be negative. Noting that H corresponds to its first derivative—which is equal to zero at the optimal tax—we therefore obtain

$$dH/dt_m^* < 0. \tag{F.28}$$

Since $\gamma(\cdot)$ is a strictly increasing function, equation (F.19) implies

$$\frac{\partial \Phi(t_m^*, \phi)}{\partial \phi} > 0. \tag{F.29}$$

To establish the monotonicity of s_m and ρ with respect to ϕ, it is convenient to introduce the following function:

$$d(t_m^*, \phi, \zeta; \theta) = (1 - \beta \frac{t_m^*}{1 + t_m^*} \ln(1 - \theta))^{\epsilon} (1 - \theta)^{-\frac{1+\epsilon}{\gamma((1+t_m^*)/\phi)}} (\ln(1 - \theta))^{-\zeta}.$$

By construction, d is log-supermodular in (ϕ, ζ, θ). Since log-supermodularity is preserved by integration, the following function,

$$D(\phi, \zeta) = \int d(t_m^*, \phi, \zeta; \theta) d\theta,$$

is also log-supermodular. It follows that

$$\frac{D(\phi, \zeta = 0)}{D(\phi, \zeta = -1)} = \frac{\int (1 - \beta \frac{t_m^*}{1+t_m^*} \ln(1 - \theta))^{\varepsilon} (1 - \theta)^{-\frac{1+\varepsilon}{\gamma((1+t_m^*)/\phi)}} d\theta}{\int (\ln(1 - \theta))(1 - \beta \frac{t_r^*}{1+t_r^*} \ln(1 - \theta))^{\varepsilon} (1 - \theta)^{-\frac{1+\varepsilon}{\gamma((1+t_m^*)/\phi)}} d\theta} \text{ is increasing in } \phi,$$

$$\frac{D(\phi, \zeta = -2)}{D(\phi, \zeta = -1)} = \frac{\int (\ln(1 - \theta))^2 (1 - \beta \frac{t_m^*}{1+t_m^*} \ln(1 - \theta))^{\varepsilon} (1 - \theta)^{-\frac{1+\varepsilon}{\gamma((1+t_m^*)/\phi)}} d\theta}{\int (\ln(1 - \theta))(1 - \beta \frac{t_r^*}{1+t_m^*} \ln(1 - \theta))^{\varepsilon} (1 - \theta)^{-\frac{1+\varepsilon}{\gamma((1+t_m^*)/\phi)}} d\theta} \text{ is decreasing in } \phi.$$

Noting that

$$s_m = \frac{1}{1 - \frac{1}{\beta}\frac{D(\phi,\zeta=0)}{D(\phi,\zeta=-1)}},$$
$$\rho = \beta\frac{D(\phi,\zeta=-2)}{D(\phi,\zeta=-1)} - 1,$$

we obtain that

$$\frac{\partial s_m(t_m^*,\phi)}{\partial \phi} > 0, \tag{F.30}$$
$$\frac{\partial \rho(t_m^*,\phi)}{\partial \phi} < 0. \tag{F.31}$$

Since $\frac{\partial H}{\partial \Phi} < 0$, $\frac{\partial H}{\partial s_m} < 0$, and $\frac{\partial H}{\partial \rho} > 0$, inequalities (F.29)-(F.31) imply

$$\frac{dH}{d\phi} = \frac{\partial H}{\partial \Phi}\frac{\partial \Phi}{\partial \phi} + \frac{\partial H}{\partial s_m}\frac{\partial s_m}{\partial \phi} + \frac{\partial H}{\partial \rho}\frac{\partial \rho}{\partial \phi} < 0.$$

Combining this observation with equation (F.27) and (F.28), we conclude that $dt_m^*/d\phi > 0$.