Local-Momentum Autoregression and the Modeling of the Interest Rate Term Structure

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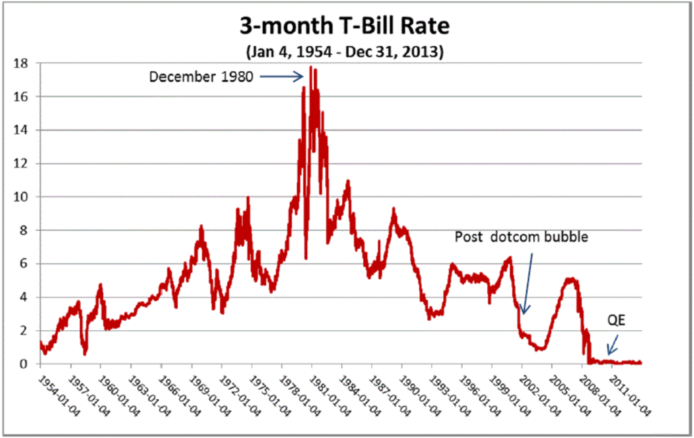
Overview

- This paper presents a new model for interest rates, to capture:
  - Very long-term mean reversion
  - Shorter-run autocorrelation
  - Very short run momentum

US 3-month T-bill rate, Jan 1954 – Dec 2013

Clearly persistent, possibly mean-reverting, periods of momentum



The "LM-CTAR" model

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- The LM-CTAR model additionally allows for momentum:

$$\begin{aligned} \Delta X_t &= \kappa_x \left(\mu_t - X_{t-1}\right) + \omega \left(\bar{X}_{(t-1)|n} - X_{t-1}\right) + \sigma_x \varepsilon_t \\ \Delta \mu_t &= \kappa_\mu \left(\bar{\mu} - \mu_{t-1}\right) + \sigma_\mu \epsilon_t \\ \bar{X}_{(t-1)|n} &\equiv \sum_{i=1}^{n} b_i X_{t-i} \end{aligned}$$

Related work

- A time series "mean-reverting" to a time-varying central point:
  - Balduzzi, Das and Foresi (1998): CTAR model for bond yields
  - Engle and Lee (1999): GARCH model with time-varying mean
  - Barndorff-Nielsen and Shephard (2002), and others: two factor models for stochastic volatility
- An autoregression with time-varying persistence:
  - Aït-Sahalia (1996) and Ang and Bekaert (2002): AR is close to random walk near "middle" of distribution, mean-reverting for extreme values
  - Diebold and Inoue (2001): regime switching AR processes can appear to have long memory
  - "Threshold AR" and "Smooth transition AR" models: see Granger and Teräsvirta (1993) for a survey

Some comments on the paper

- I like the paper, and it was interesting to think about how a time series model can try to capture the features of US interest rates

- I have a few questions and comments for the author

The "local momentum" component

- The new term here is the LM term: $\bar{X}_{(t-1)|n} \equiv \sum_{i=1}^{n} b_i X_{t-i}$
- A few questions about $\bar{X}_{(t-1)|n}$:

1. How do we optimally choose n? Author sets $n = 7$, which seems reasonable, but so would many other choices.
2. Intuition on how n will change with the sampling frequency? Would optimal n for daily data be 5 times larger than that for weekly data?
3. What is gained by the lag coefficients $\{b_i\}_{i=1}^{n}$? The author imposes these to be $1/n$ in estimation, which seems reasonable. Does he envisage a scenario when this would be relaxed?
4. If $\{b_i\}_{i=1}^{n}$ are not fixed, then they are unidentified when $\omega = 0$, making estimation and inference a bit trickier.

Realiability of standard inference methods

- The models considered here may be stationary, but they are **very** close to being non-stationary
  - AR(1) coefficient in Table 2 is 0.9974
  - LM-AR $\rho(B)$ coefficient is 0.9957
  - Although LM-CTAR $\rho(B)$ coefficient for is only 0.8259 (why?)
- Tese models may be inside the stationary region of the parameter space, but they are so close to the boundary that standard inference methods may be unrealiable
- Perhaps run some simulations to see whether standard methods work, using the parameter estimates reported in the paper

Moving from one interest rate to the term structure

- This paper considers both a scalar time series process, and its generalization to the entire term structure (which is nice)
- What is the motivation for the additional AR(1) process that appears in this generalization (eq 14)?

$$r_t = \underbrace{X_t}_{LM-CTAR} + \underbrace{\nu_t}_{AR(1)} + \underbrace{\epsilon_t}_{AR(0)}$$

- The LM-CTAR model already contains (i) an AR(1) for the central tendency factor, μ_t (ii) an AR(1) for the interest rate (iii) an MA(7) for the local momentum effect
- The combined model for the term structure rates is thus quite complicated. Are all these AR/MA parameters are well identified?

Simple models w/ structural breaks vs. complicated models

- One approach: keep generalizing a time series model until it fits the data over a (long) sample period
  - Out-of-sample performance? Over-fitting?
  - Stability of model parameters over a 50+ year period?
  - Identification of parameters in the various components of the model?
- Alternative: consider simpler time series models, estimated over shorter samples
  - Formal tests for structural breaks (eg, Bai, 1995–now)
  - Estimate using rolling window (eg, Fan, Farmen and Gijbels, 1998)
  - Long-run predictions are harder, term structure extension may be hard
- It would be interesting to see comparisons of the LM-CTAR model not only with models it nests (ie, with constant parameters) but some other alternatives as well.