Accounting Manipulation, Peer Pressure, and Internal Control[*]

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Abstract

We study firms' investment in internal control to reduce accounting manipulation. We first show the peer pressure for manipulation: one manager manipulates more if he suspects that reports of peer firms are more likely to be manipulated. As a result, one firm's investment in internal control has a positive externality on peer firms. It reduces its own manager's manipulation, which, in turn, mitigates the manipulation pressure on managers in peer firms. Firms do not internalize this positive externality and thus underinvest in their internal control over financial reporting. The problem of underinvestment provides one justification for regulatory intervention in firms' internal control choices.

1 Introduction

The wave of accounting frauds and restatements in the early 2000s (e.g., Enron, WorldCom) exposed the staggering failure of internal control over financial reporting in many firms (GAO (2002)). Until the early 2000s, a firm's internal control decisions had long been deemed its private domain and outside the purview of the securities regulations that had traditionally focused on disclosure of such decisions (Ribstein (2002), Coates (2007)). However, the prevalence and magnitude of the failures eroded the support for such practices and eventually led to passage of the Sarbanes–Oxley Act of 2002 (SOX) in the U.S., and similar legislature in other countries. In addition to enhanced disclosure requirements, SOX also mandated substantive measures to deter and detect accounting fraud.[1] Their mandatory nature has made these measures controversial (e.g., Romano (2005), Hart (2009)). Even for those who felt that something needed to be done about the firms' internal control over financial reporting, it may not be clear why it should be done through regulations. Is there a case for regulation that intervenes in firms' internal control decisions? Do firms have the right incentives to choose optimal levels of internal control to assure the veracity of their financial statements? In fact, Romano (2005), in an influential critique of SOX, argues that, "The central policy recommendation of this Article is that the corporate governance provisions of SOX should be stripped of their mandatory force and rendered optional for registrants."

We construct a model to study firms' investment in internal control over financial reporting. In the model, firms can invest in costly internal control to detect and deter managers' accounting manipulation. We show that such an investment by one firm has a positive externality on its peers. At the core of the channel for this externality is the peer pressure for accounting manipulation among firms: one manager's incentive to manipulate increases in his expectation that peer firms' reports are manipulated. As a result, a firm's investment in internal control reduces its own manager's manipulation, which, in turn, mitigates the pressure for manipulation on managers in peer firms. Since the firm does not internalize this externality, it underinvests in internal control over financial reporting. Regulatory intervention can

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[1] These mandates range from independent audit committees, auditor partner rotations, prohibition of non-audit service provided by auditors, and executives' certification and auditors' attestation to an internal control system.

improve the value of all firms by mandating a floor of internal control investment.

In our model, there are two firms with correlated fundamentals, indexed by A and B. Each manager's payoff is a weighted average of the current stock price and the fundamental value of *his own firm*. Investors rely on accounting reports to set stock prices. Accounting manipulation boosts accounting reports and allows a bad manager who successfully manipulates to be pooled with the truly good ones. Investors rationally conjecture this pooling result and discount the pool accordingly to break even. In the equilibrium, the bad manager who successfully manipulates obtains an inflated stock price at the expense of the truly good manager. Accounting manipulation is detrimental to firm value and firms do have private incentives to invest in their internal control over financial reporting.

In such a setting, peer pressure for accounting manipulation arises. By peer pressure we mean that one manager manipulates more if he expects that the other firm's report is more likely to be manipulated in equilibrium. In other words, the two managers' manipulation decisions are strategic complements in the sense of Bulow, Geanakoplos, and Klemperer (1985). The mechanism works as follows. Consider manager A's manipulation decision. Rational investors utilize reports from both firms in setting the stock price of firm A due to their correlated fundamentals. Investors compare report A with report B to distinguish between the truly good firm A and the bad firm that successfully manipulates. Manipulation of report B reduces its informativeness and makes it less useful for investors of firm A to cull out the bad one with successful manipulation. Anticipating that his fraudulent report is less likely to be confronted by report B, manager A expects a higher benefit from manipulation and thus increases his manipulation. The manipulation of report B creates "pressure" on manager A to manipulate because the opportunity cost for manager A not to manipulate is higher.

To further see this intuition, consider a special case in which two firms' fundamentals are perfectly correlated and manager B does not manipulate. As a result, manager A will also not manipulate because any successful manipulation will be confronted by the report from manager B. If manager B is expected to slightly manipulate, manager A now anticipates that his fraudulent report is sometimes camouflaged and thus has an incentive to manipulate.

Peer pressure for manipulation creates a positive externality of the firm's costly invest-ment in internal control. Firm B's investment in internal control reduces its own manager's manipulation. The reduction of manipulation in firm B mitigates the pressure for manipula-tion on manager A, resulting in lower manipulation by manger A. However, firm B does not internalize this externality and underinvests in the internal control. This underinvestment in internal control by individual firms suggests a rationale for regulatory intervention that imposes some floor of internal control over financial reporting.

We define peer pressure formally as the strategic complementarity among managers' manipulation decisions. Strategic complementarity is related, but different from spillover. Spillover refers to the effect that manager A manipulates less when firm B is present. Strate-gic complementarity refers to the effect that manager A manipulates less when manager B is expected to manipulate less in equilibrium. In Section 4.1, we delineate the relation be-tween the two notions. The spillover effect occurs if and only if two firms' fundamentals are correlated. In contrast, strategic complementarity requires the additional condition that manipulation reduces the report's informativeness. In other words, if manipulation does not affect the report's informativeness, then spillover exists but not strategic complementarity. Our main result on the positive externality of internal control is driven by strategic comple-mentarity, not by spillover.

The peer pressure for manipulation is often alleged in practice. One of the best known and most extreme examples is the telecommunications industry around the turn of the mil-lennium (see Bagnoli and Watts (2010) footnote 1 for detailed references to such allegations). When WorldCom turned to aggressive and, ultimately, illegal reporting practices to boost its performance, peers firms were under enormous pressure to perform. Horowitz (2003) claims: "Once WorldCom started committing accounting fraud to prop up their numbers, all of the other telecoms had to either (a) commit accounting fraud to keep pace with World-Com's blistering growth rate, or (b) be viewed as losers with severe consequences." Qwest and Global Crossing ended up with accounting frauds while AT&T and Sprint took a series of actions that aimed to shore up their short-term performance at the expense of long-term viability. While these companies had plenty of their own problems, the relentless capital

market pressure undoubtedly made matters worse (see Sadka (2006)).

The peer pressure mechanism also generates new empirical predictions. The central prediction is that peer firms' manipulation decisions are correlated, even after controlling for their own fundamentals and characteristics. An exogenous increase in one firm's manipulation incentive also elevates the manipulation incentives in peer firms. For example, if one firm's manager is given stronger incentive pay, the model predicts that not only will the manager engage in manipulation, but also that managers from peer firms will behave in a similar manner. In another example, one bank's loan loss provisioning might be increasing in the peer average even after controlling for the bank and its peers' fundamentals. Some recent papers have examined how one firm's fraudulent accounting affects investment decisions in peer firms (e.g., Gleason, Jenkins, and Johnson (2008), Beatty, Liao, and Yu (2013)). Our model suggests an additional effect that one firm's accounting manipulation and internal control imposes on its peer firms.

1.1 Literature review and contributions

We make three contributions. First, we provide a rational explanation of the peer pressure for manipulation arising from the capital market. Peer pressure for manipulation has been studied in other contexts. Both Bagnoli and Watts (2010) and Einhorn, Langberg, and Versano (2016) study the interactions of firms' reporting choices and product market competition in a Cournot oligopoly setting. One result in Bagnoli and Watts (2010) (Result 5) shows numerically that two firms' misreporting decisions are strategic substitutes when the firms' misreporting cost difference is sufficiently large. Einhorn, Langberg, and Versano (2016) present an interesting "cross-firm earnings management" mechanism: firm A attempts to influence its investors' belief by changing its production decision, which alters firm B's manipulation that, in turn, affects investors' use of report B in assessing firm A. They show that such cross-firm earnings management could serve as a commitment device for the oligopoly to reduce production and improve profitability.

The peer pressure for manipulation, in the form of collusion of actions, can also be induced by contractual payoff links among agents within the same firm (e.g., Bagnoli and Watts

(2001), Baldenius and Glover (2010), Cheng (2011), Glover (2012), Glover and Xue (2014), and Friedman (2014)). Carlin, Davies, and Iannaccone (2012) show that firms' disclosure decisions interact when an exogenous tournament structure of payoffs exists to firms. There are also behavioral explanations for peer pressure for manipulation in that one manager's unethical behavior diminishes the moral sanction for others to engage in the same behavior (e.g., Mittendorf (2006), Mittendorf (2008), Fischer and Huddart (2008)). This explanation is often labeled as reporting culture, code of ethics, or social norms.

Our paper complements these explanations from a capital market pressure perspective in that managers intervene in the reporting process to influence capital markets' inferences regarding their firms. Capital market pressure is often viewed as a major motivation for accounting manipulation (Graham, Harvey, and Rajgopal (2005)). We assume neither the contractual links among managers nor the complementarity of manipulation costs.

Petacchi and Nagar (2014) study a model of "enforcement thinning" in which manipulation decisions among firms are also strategic complements. A regulator's budget of enforcement against frauds is fixed. As the number of firms engaging in accounting manipulation increases, the probability that each firm will be subject to the regulator's investigation becomes smaller and thus more firms engage in manipulation.

Our second contribution is to provide one rationale for regulating firms' internal control over financial reporting. As Leuz and Wysocki (2007) have pointed out, understanding the positive externalities of regulations is crucial for their justification in the first place. Our model suggests that the proposal in Romano (2005)–that the internal control mandates in SOX should be made optional–is flawed. Competition among firms (or state laws) does not lead to socially optimal investment in internal control.

This relates our paper to the literature on the externalities of disclosure and corporate governance. Beyer, Cohen, Lys, and Walther (2010) provide an excellent summary of various potential rationales for disclosure regulation. Dye (1990), Admati and Pfleiderer (2000) and Lambert, Leuz, and Verrecchia (2007) study how truthful disclosure by one firm can affect the decisions of investors of other firms. Dye and Sridhar (1995) study a model of voluntary disclosure with verifiable messages in which firms' information receipt is uncertain

but correlated. They show that one firm's disclosure threshold depends on the number of peer firms as well as the nature of the private information. Kartik, Lee, and Suen (2014) also study a voluntary disclosure model with verifiable messages in which two agents forecast the same fundamental. They show that the agents' disclosure strategies are strategic complements when concealing information is costly, but strategic substitutes when disclosing information is costly. They also examine a model of costly signaling similar to that in Kartik (2009) and show that the sender's incentive to misreport decreases as public information quality improves.

There have also been a series of recent papers that study the externality of managerial compensation and corporate governance in monitoring managerial consumption of private benefit. The key channel is through an imperfection in the labor market for managers (e.g., Acharya and Volpin (2010), Dicks (2012), Thanassoulis (2012)).

Our model is concerned with accounting manipulation because of our focus on the externality of internal control. Disclosure alone in our model does not solve the underinvestment problem. In our model, although the firms' internal control decisions are perfectly observed by investors and peer managers, the underinvestment problem persists.

Finally, the manipulation component of our model belongs to the class of multi-firm signal-jamming models (e.g., Stein (1989)).[2] Heinle and Verrecchia (2016) also examine a multi-firms setting. The two models differs in at least two respects. First, Heinle and Verrecchia (2016) focus on the spillover effect while our model focuses on the strategic complementarity. As we have previously discussed in this introduction, our result on the positive externality is driven by strategic complementarity, not by spillover. To see the difference, consider an exogenous increase in manager B's capital market concern. In our model, such an shock would lead to an increase in manager A's manipulation, but would not affect manager A's manipulation in Heinle and Verrecchia (2016). Second, a critical feature in Heinle and Verrecchia (2016) is that firms ex-ante benefit from manipulation (see pages 5-6 in Heinle and Verrecchia (2016)).

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[2]Signal-jamming models have been widely used to study economic consequences of earnings management. We refer readers to some recent surveys, including Leuz and Wysocki (2007), Ronen and Yaari (2008), Beyer, Cohen, Lys, and Walther (2010), Ewert and Wagenhofer (2012), and Stocken (2013), due to the size of the literature (as partially evidenced by the number of surveys).

As a result, the issue of installing internal control to curtail manipulation in their framework degenerates.

A key driver behind strategic complementarity is our modeling feature that manipulation reduces the report's informativeness. This feature is most transparent in a binary structure in our baseline model (see also e.g., Chen, Hemmer, and Zhang (2007), Strobl (2013), Bertomeu (2013)). It may not always arise in continuous structures (e.g., Fischer and Verrecchia (2000), Dye and Sridhar (2004), Guttman, Kadan, and Kandel (2006), Beyer and Guttman (2012), and Heinle and Verrecchia (2016)).

More broadly, the manipulation component of our model is also related to the multi-agent career-concern literature (e.g., Meyer and Vickers (1997), Holmstrom (1999), Dewatripont, Jewitt, and Tirole (1999)). The career concern models often focus on endogenous contracting with agents in addition to implicit career concerns and intertemporal dynamics. Moreover, the actions in these models are productive efforts. Nonetheless, the manipulation component of our model shares a key feature with career concern models in that agents take costly actions in an attempt to influence outsiders' rational inference. A conceptual difference between our model and multi-agent career concern models such as Meyer and Vickers (1997) is that we focus on strategic complementarity while Meyer and Vickers (1997) exploit spillover. The central question in Meyer and Vickers (1997) is how an agent responds to the presence of other agents. There is also informational spillover: one agent's outcome is useful in evaluating other agents because their environments are correlated. However, strategic complementarity does not arise in Meyer and Vickers (1997) because efforts do not affect the outcome's informativeness.

The rest of the paper proceeds as follows. Section 2 describes the model. Section 3 solves the equilibria and examines the strategic relation among firms' manipulation and internal control investment. Section 4 discusses some extensions and Section 5 concludes.

2 The Model

The economy consists of two firms, indexed by A and B. There are four dates, $t = 0, 1, 2,$ and 3. All parties are risk neutral and the risk-free rate is normalized to 1.

Each firm has a project that pays out *gross* cash flow s_i, $i \in \{A, B\}$, at $t = 3$. s_i is either high ($s_i = 1$) or low ($s_i = 0$). The prior probability that $s_i = 1$ is θ_i. The firm's *net* cash flow at $t = 3$, denoted as V_i, differs from the gross cash flow s_i for two reasons explained below. We refer to the net cash flow V_i as the firm's long-term value and the gross cash flow s_i as the firm's fundamental or type.

The payoff function of manager i is

$$U_i = \delta_i P_i + (1 - \delta_i) V_i, \ i \in \{A, B\}. \tag{1}$$

The manager's interests are not fully aligned with the firm's long-term value V_i. Instead, the manager cares about both the long-term firm value at $t = 3$ and the short-term stock price P_i at $t = 2$. $\delta_i \in (0, 1)$ measures the manager's relative focus on the two.

Managers' concern for short-term stock price performance is empirically descriptive. For example, Stein (1988) argues that takeovers would force managers to tender their shares at market prices even if they would rather hold the stocks for a longer term. In another example, Narayanan (1985) and Rajan (1994) contend that managers' reputation concerns could lead them to focus on the short-term stock prices at the expense of the firms' long-term value. Alternatively, managers' stock-based compensation or equity funding for new projects could also induce them to focus on firms' short-term stock price performance.

The stock price P_i at $t = 2$ is influenced by both firms' accounting reports. Each firm's financial reporting process is as follows. At $t = 1$, each manager privately observes the fundamental s_i. After observing his type s_i, each manager issues an accounting report, $r_i \in \{0, 1\}$. The good manager always reports truthfully in equilibrium, i.e., $r_i(s_i = 1) = 1$. The bad manager with $s_i = 0$ may manipulate the report. The probability that the bad manger

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issues a good report, i.e., $r_i(s_i = 0) = 1$, is

$$\mu_i \equiv \Pr(r_i = 1 | s_i = 0, m_i, q_i) = m_i(1 - q_i).$$

μ_i is the probability that the bad firm successfully issues a fraudulent report. This probability is determined jointly by the manager's manipulation decision, m_i, and the firm's internal control choice, q_i. $m_i \in [0, 1]$ is the bad manager's efforts to overstate the report. To economize on notations, we often use m_i to denote the bad manager's manipulation $m_i(s_i = 0)$ and omit the argument $s_i = 0$ whenever no confusion arises. Manipulation effort m_i is the manager's choice at $t = 1$ after he has observed s_i. m_i reduces the firm's long-term value by $C_i(m_i)$. $C_i(m_i)$ also has the standard properties of a cost function (similar to the Inada conditions): $C_i(0) = 0$, $C_i'(0) = 0$, $C_i'(m_i) > 0$ for $m_i > 0$, $C_i'(1) = \infty$ and $C_i'' > c$. c is a constant sufficiently large to guarantee that the manager's equilibrium manipulation choice is unique.

$q_i \in [0, 1]$ denotes the quality of the firm's internal control over financial reporting. It is interpreted as the probability that the manager's overstatement is detected and prevented by the internal control system. q_i is the firm's choice at $t = 0$ and reduces the firm's cash flow by $K_i(q_i)$. $K_i(q_i)$ has the standard properties of a cost function as well: $K_i(0) = 0$, $K_i'(0) = 0$, $K_i'(q_i) > 0$ for $q_i > 0$, $K_i'(1) = \infty$, and $K_i'' > k$. k is a constant sufficiently large to guarantee that the firms' equilibrium internal control choice is unique. Unlike m_i, the firm's choice of q_i is publicly observable.

Overall, the bad manager can take actions to inflate the report, but his attempts are checked by the internal control system. We model the cost of manipulation as a reduction in the firm's long-term value. Both accrual manipulation and real earnings management are costly to the firm (e.g., Kedia and Philippon (2009)). When the manager engages in accrual manipulation, the cost includes not only the direct cost of searching for opportunities, but also the indirect cost of the managers' distraction and the associated actions to cover up the manipulation. Real earnings management directly distorts the firm's decisions and decreases the firm's cash flow. Our results are also robust to the alternative interpretation that C_i is the

manager's private cost, such as the psychic suffering, the potential reputation loss, and the possible legal consequences (e.g., Karpoff, Lee, and Martin (2008)), if we modify the model with an additional assumption that, ex ante, at $t = 0$ the firm would internalize part of the manager's ex-post manipulation cost through some sort of individual participation constraint for the manager. Such an extension is provided in Section 4.[3]

The firm's net cash flow (i.e., the long-term firm value) can now be written as

$$V_i = s_i - C_i(m_i(s_i)) - K_i(q_i).$$

V_i is lower than the gross cash flow s_i by two terms, the cost of manipulation C_i, and the cost of internal control K_i.

Finally, the only connection between the two firms is the correlation between their gross cash flows or types, s_i.[4] The correlation coefficient ρ can be either positive or negative. For simplicity we assume away the trivial case of $\rho = 0$. In addition, $\rho \in [\underline{\rho}, 1]$ is bounded from below by $\underline{\rho} \equiv \max\{-\sqrt{\frac{\theta_A}{1-\theta_A}\frac{\theta_B}{1-\theta_B}}, -\sqrt{\frac{1-\theta_A}{\theta_A}\frac{1-\theta_B}{\theta_B}}\}$, instead of -1. $\underline{\rho} < 0$, and approaches -1 (i.e., two Bernoulli variables are perfectly negatively correlated) only if their marginal probabilities satisfy $\theta_A = 1 - \theta_B$.

In sum, the timeline of the model is summarized as follows.

1. $t = 0$, firm i publicly chooses its internal control quality q_i;

2. $t = 1$, manager i privately chooses manipulation m_i after privately observing s_i;

3. $t = 2$, investors set stock price P_i after observing both report r_A and r_B;

4. $t = 3$, cash flows are realized and paid out.

The equilibrium solution concept is Perfect Bayesian Equilibrium (PBE). A PBE is characterized by the set of decisions and prices, $\{q_A^*, q_B^*, m_A^*(s_A), m_B^*(s_B), P_A^*(r_A, r_B), P_B^*(r_A, r_B)\}$, such that

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[3]We thank an anonymous referee for pointing this out.

[4]There is empirical literature documenting the intra-industry information transfer, e.g., Foster (1981), Han, Wild, and Ramesh (1989), and Thomas and Zhang (2008).

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1. $q_i^* = \arg\max_{q_i} E_0[V_i]$ maximizes the long-term firm value expected at $t = 0$;

2. $m_i^*(s_i) = \arg\max_{m_i(s_i)} E_1[U_i|s_i]$ maximizes the manager's payoff expected at $t = 1$ after observing s_i;

3. $P_i^*(r_A, r_B) = E_2[V_i|r_A, r_B]$ is set to be equal to investors' expectation of the long-term firm value, conditional on both firms' reports (r_A, r_B);

4. The players have rational expectations in each stage. In particular, both the manager's and investors' beliefs about the other manager's manipulation are consistent with Bayesian rules, if possible.

3   The Analysis

In this section we analyze the model in sequence. We first examine how one manager's manipulation decision is influenced by his expectation of the manipulation of his peer firm's report and then study the design of internal control.

3.1   Equilibrium manipulation decisions

3.1.1   Equilibrium manipulation with only one firm

To highlight the driving forces behind the peer pressure for manipulation, we start with a benchmark with only firm A (or, equivalently, the two firms' fundamentals are not correlated). We solve the benchmark case of a single firm by backward induction. Investors at $t = 2$ set stock price $P_A^*(r_A)$ to be equal to their expectation of the firm value V_A upon observing report r_A. Since they do not observe the manager's actual choice of manipulation m_A, investors conjecture that the manager has chosen m_A^* in equilibrium if his type is bad, and 0 otherwise. Recall that the probability of the bad type succeeding in manipulation is $\mu_A = m_A(1 - q_A)$. Since q_A is observable to investors at $t = 2$, investors thus conjecture $\mu_A^* = m_A^*(1 - q_A)$. Expecting this data-generating process, investors use the Bayesian rule to update their belief about the firm type. Define $\theta_A(r_A) \equiv \Pr(s_A = 1|r_A)$ as investors' posterior belief about the firm being a good type conditional on report r_A.

We first have $\theta_A(0) = 0$. Since the good firm always issues the favorable report $r_A = 1$ and only the bad firm may issue the unfavorable report $r_A = 0$, investors learn for certain that the firm issuing $r_A = 0$ is a bad type.

Upon observing the favorable report $r_A = 1$, investors are uncertain about the firm type. The favorable report $r_A = 1$ can be issued by either the truly good firm ($s_A = 1$) or the bad firm with successful manipulation ($s_A = 0$). The population of the former is θ_A and of the latter is $(1 - \theta_A)\mu_A^*$. Using this knowledge, investors update their belief about the firm type as follows:

$$\theta_A(1) \equiv \Pr(s_A = 1 | r_A = 1) = \frac{\theta_A}{\theta_A + (1 - \theta_A)\mu_A^*}. \tag{2}$$

Investors become more optimistic upon observing the favorable report $r_A = 1$ because the probability of issuing the favorable report is higher for the good firm than for the bad firm, i.e., $1 \geq \mu_A^*$. However, investors do discount the favorable report to reflect the possibility that it is manipulated. If the bad firm cannot manipulate ($i, e., \mu_A^* = 0$), then $\theta_A(1) = 1$. Investors take the favorable report at face value and do not discount it at all. If the bad firm always succeeds in manipulation (i.e., $\mu_A^* = 1$), then $\theta_A(1) = \theta_A$. Investors completely discard the favorable report. If the probability of manipulation is in between, $\theta_A(1) \in (\theta_A, 1)$.

The stock price $P_A^*(r_A)$ is then set to be equal to investors' expectation of the firm value V_i :

$$P_A^*(r_A) = \theta_A(r_A) + (1 - \theta_A(r_A))(0 - C_A^*) - K_A(q_A). \tag{3}$$

The good firm generates a gross cash flow of 1. The bad firm generates a gross cash flow of 0 and incurs the manipulation cost of $C_A^* = C_A(m_A^*)$. Both types pay the internal control cost $K_A(q_A)$.[5] It is obvious that $P_A^*(1) - P_A^*(0) = \theta_A(1)(1 + C_A^*) > 0$. Investors pay a higher price for the favorable report, despite the manipulation. As a result, the manager who cares about the short-term stock price prefers report $r_A = 1$ to $r_A = 0$.

Anticipating the investors' pricing response to report $P_A^*(r_A)$, the bad manager chooses m_A to maximize his expected utility $E_1[U_A(m_A)|s_A = 0]$ defined in Equation 1. We denote

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[5] We normalize the gross cash flow of the bad type to be 0. As a result, we have negative net cash flow for the bad firm. This can be easily fixed by introducing a positive baseline gross cash flow that is large enough to cover the cost of manipulation and internal control. Such a setting complicates the notations but would affect none of our formal results.

the manager's best response to the investors' conjecture m_A^* as $\tilde{m}_A^*(m_A^*)$ or simply $\tilde{m}_A^*$. Its first order condition is

$$H^A(m_A)|_{m_A=\tilde{m}_A^*(m_A^*)} \equiv \delta_A \frac{\partial \mu_A}{\partial m_A}\theta_A(1)(1 + C_A^*) - (1 - \delta_A) C_A'(m_A) = 0. \qquad (4)$$

Equation 4 describes the trade-off of the manipulation decision. The first term is the marginal benefit of manipulation, which increases the firm's chance of issuing the favorable report $r_A = 1$, at a marginal rate of $\frac{\partial \mu_A}{\partial m_A} = 1 - q_A$. The favorable report, in turn, increases its stock price by $P_A^*(1) - P_A^*(0) = \theta_A(1)(1 + C_A^*)$. The second term is the marginal cost. Manipulation reduces the firm's future cash flow by $C_A(m_A)$. Since the manager has a stake of $1-\delta_A$ in the long-term firm value, he bears part of the manipulation cost as well. The manager thus chooses the optimal manipulation level such that the marginal benefit is equal to the marginal cost. We use $H^A(m_A)$ defined in Equation 4 to denote the difference of the marginal benefit and marginal cost for an arbitrary manipulation m_A. Therefore, $\tilde{m}_A(m_A^*)$, defined by $H^A(\tilde{m}_A(m_A^*)) = 0$, characterizes the manager's best response to investors' conjecture m_A^*. In equilibrium, the investors' conjecture is consistent with the manager's optimal choice, that is, $\tilde{m}_A(m_A^*) = m_A^*$. This rational expectations requirement implies that the optimal choice m_A^* is defined by $H^A(m_A^*) = 0$. This manipulation game in general could have multiple equilibria, that is, the Equation $H^A(m_A^*) = 0$ can have multiple solutions.[6] Since the multiplicity is not our focus, we obtain the unique equilibrium by assuming that both the manipulation cost function and, later, the internal control cost functions are sufficiently convex (see Cooper and John (1988)). With the unique equilibrium determined, we conduct comparative statics to understand the determinants of the manager's optimal manipulation choice.

Lemma 1 *When there is only firm A, there exists a unique equilibrium $\{m_A^*(s_A), P_A^*(r_A)\}$ characterized collectively by Equation 4, $m_A^*(s_A = 1) = 0$, and Equation 3. $m_A^*(s_A = 0)$ is increasing in θ_A and δ_A, and decreasing in q_A.*

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[6]Notice that multiple equilibria may arise even with a single firm because the game is essentially a two-person game between investors and the manager. Each equilibria is characterized by imposing investors' rational expectation on the manager's best response to investors' conjecture, i.e., $\tilde{m}_A(m_A^*) = m_A^*$. The resulting equation is $H(m_A^*) = 0$ which may have multiple roots, say m_{Aj}^*, $j \in N \geq 2$. Then if investors believe that the manager has chosen any one of the m_{Aj}^*, it is indeed optimal for the manager to choose m_{Aj}^*. Thus there are N Nash equilibria.

These properties of the equilibrium manipulation decisions are standard. First, m_A^* is increasing in the investors' prior belief about the firm type (θ_A) before observing report r_A. When θ_A is higher, investors expect that report $r_A = 1$ is more likely to come from the good firm and thus attach a higher valuation to it. The bad manager takes advantage of this optimism by increasing manipulation. Second, the manager manipulates more if he has a stronger capital market concern. Third, internal control reduces manipulation. All else being equal, an improvement in internal control quality detects manipulation more often and reduces the information asymmetry between the manager and investors. As a result, the bad manager's incentive to manipulate diminishes.

Our single-firm model is a variant of the signal-jamming model (e.g., Stein (1989)) with an important difference. It has the defining feature that even though the manager attempts to influence investors' belief through unobservable and costly manipulation, investors with rational expectations are not systematically misled. *On average* they see through the manipulation and break even. As such, the manipulation eventually hurts the firm value through the distorted decisions.

Our model differs from Stein (1989) in that information asymmetry between investors and the manager persists in equilibrium. The manager knows his type while investors observe only report $r_A = 1$ that is a noisy signal of the manager's type s_A. This information asymmetry is consequential for investors' pricing. Since investors make the pricing decision only conditional on report r_A, the same stock price $P_A^*(1)$ is paid to both the good firm ($s_A = 1$) and the bad firm ($s_A = 0$) with successful manipulation. Investors rationally anticipate this information asymmetry and price protect themselves by discounting both types of firms. However, such non-discriminatory discounting implies that the stock price $P_A^*(1)$ is too low from the perspective of the good manager, but too high from the perspective of the bad manager, who has successfully manipulated. Even though manipulation does not systematically mislead investors, it does reduce the report's informativeness to investors.

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3.1.2 Equilibrium manipulation with two firms

We now extend the model to include two firms to study the strategic complementarity. When the two firms' fundamentals are correlated, investors can also use the peer firm's report to improve their pricing decisions. This informational spillover affects managers' manipulation, as studied in other multi-firm models (e.g., Heinle and Verrecchia (2016)). However, we will focus on the strategic complementarity or peer pressure for manipulation: Does manager A's incentive to manipulate increase in his expectation that manager B has successfully manipulated report r_B?

We redefine the notations to accommodate the addition of firm B. Investors now use both report r_A and r_B to update their beliefs and set the stock price. Denote $\theta_A(r_A, r_B) \equiv \Pr(s_A = 1 | r_A, r_B)$ as the investors' posterior about firm A being a good type after observing reports r_A and r_B. We also use ϕ to denote a null signal. For example, $\theta_A(r_A, \phi)$ is the investors' posterior after observing r_A but before observing r_B. Thus, $\theta_A(r_A, \phi) = \theta_A(r_A)$, which is defined in the single-firm case. Similarly, we denote $P_A^*(r_A, r_B)$ as the stock price conditional on r_A and r_B. In addition, although both investors and manager A observe firm B's internal control q_B and report r_B, neither observes manager B's actual choice of manipulation m_B. Thus, both investors and manager A have to conjecture manager B's equilibrium manipulation choices. Rational expectations require that in equilibrium the conjectures by both investors and manager A are the same as manager B's equilibrium choice m_B^*. Moreover, since q_B is observable, investors and manager A conjecture that the probability that manager B successfully issues a fraudulent report is $\mu_B^* = m_B^*(1 - q_B)$.

Investors use report r_A and r_B to update their belief about s_A. Note that r_A and r_B are independent conditional on s_A. Thus, investors' belief can be updated in two steps. First, investors use r_A to update their prior from θ_A to the posterior $\theta_A(r_A, \phi)$. Second, treating $\theta_A(r_A, \phi)$ as a new prior for s_A, investors then use report r_B to update their belief, just as in a single-firm case. The conditional-independence assumption allows us to convert the two-firm

case into two iterations of the single-firm case.[7]

The equilibrium stock price $P_A^*(r_A, r_B)$ is now set to be equal to investors' expectations of the firm value V_i conditional on r_A and r_B :

$$P_A^*(r_A, r_B) = \theta_A(r_A, r_B) + (1 - \theta_A(r_A, r_B))(0 - C_A^*) - K_A(q_A). \tag{5}$$

As in the single-firm case (Eqn 3), the unfavorable report $r_A = 0$ reveals $s_A = 0$ perfectly. As a result, the additional report r_B does not affect investors' belief, i.e., $\theta_A(0, \phi) = \theta_A(0, 1) = \theta_A(0, 0)$. Thus we focus on the favorable report $r_A = 1$.

Conditional on $r_A = 1$, report r_B will also affect investors' belief about s_A, and the effect can go in either direction. Consider first the case of positive correlation, $\rho > 0$. If firm B issues report $r_B = 1$, it improves investors' belief that $s_B = 1$. Because s_B is positively correlated with s_A, investors also believe that $s_A = 1$ is more likely. Hence, $\theta_A(1, 1) - \theta_A(1, \phi) > 0$. Report $r_B = 1$ induces investors to discount $r_A = 1$ less and enhances the credibility of $r_A = 1$. In this sense, report $r_B = 1$ provides camouflage for manager A's fraudulent report. On the other hand, if firm B issues an unfavorable report $r_B = 0$, then investors revise their beliefs about both firms' types and we have $\theta_A(1, 0) - \theta_A(1, \phi) < 0$. Report $r_B = 0$ reduces the credibility of report $r_A = 1$. In this sense, report $r_B = 0$ confronts the fraudulent report, $r_A = 1$. Thus, manager A prefers $r_B = 1$ to $r_B = 0$.

Consider the other case of $\rho < 0$. In this situation, if firm B issues a favorable report $r_B = 1$, investors are more pessimistic about firm A's type, i.e., $\theta_A(1, 1) - \theta_A(1, \phi) < 0$. Otherwise, if firm B issues an unfavorable report $r_B = 0$, investors revise their belief about $s_A = 1$ upward and pay firm A a higher price. From the perspective of the bad manager A,

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[7]The claim is proved as follows. We can rewrite the conditional probability

$$\Pr(s_A | r_A, r_B) = \frac{\Pr(s_A | r_B) \Pr(r_A | s_A, r_B)}{\sum_{s_A} \Pr(s_A | r_B) \Pr(r_A | s_A, r_B)} = \frac{\Pr(s_A | r_B) \Pr(r_A | s_A)}{\sum_{s_A} \Pr(s_A | r_B) \Pr(r_A | s_A)}.$$

The first step uses the conditional-probability function definition and the Bayes rule, while the second step utilizies the conditional independence result that $\Pr(r_A | s_A, r_B) = \Pr(r_A | s_A)$. The rewriting makes clear that adding report r_B to investors' information set is equivalent to replacing investors' prior of $\Pr(s_A)$ with a new one, $\Pr(s_A | r_B)$. Similarly, we can change the order of r_A and r_B :

$$\Pr(s_A | r_A, r_B) = \frac{\Pr(s_A | r_A) \Pr(r_B | s_A, r_A)}{\sum_{s_A} \Pr(s_A | r_A) \Pr(r_B | s_A, r_A)} = \frac{\Pr(s_A | r_A) \Pr(r_B | s_A)}{\sum_{s_A} \Pr(s_A | r_A) \Pr(r_B | s_A)}.$$

report $r_B = 1$ confronts his fraudulent report $r_A = 1$, while report $r_B = 0$ provides cover. Therefore, manager A prefers $r_B = 0$ to $r_B = 1$.

The magnitude by which investors revise their belief upon observing r_B depends on the report B's informativeness about s_B and the correlation ρ between s_A and s_B. As we discussed in the single-firm case, the report B's informativeness about s_B is decreasing in μ_B^*. Investors attach less credibility to $r_B = 1$, as they suspect that it is more likely from a bad manager. We summarize these discussions in the next lemma.

Lemma 2 *When there are two firms,*

    1. if $\rho > 0$, $\theta_A(1,1) - \theta_A(1,\phi)$ and $\theta_A(1,1) - \theta_A(1,0)$ are positive and decreasing in μ_B^*;

    2. if $\rho < 0$, $\theta_A(1,1) - \theta_A(1,\phi)$ and $\theta_A(1,1) - \theta_A(1,0)$ are negative and increasing in μ_B^*.

With investors' equilibrium response to the reports, we can verify that, as in the single-firm case, manager A has incentive to manipulate because $P_A^*(1, r_B) - P_A^*(0, r_B) = \theta_A(1, r_B)(1 + C_A^*) > 0$ for any r_B.

Anticipating the investors' pricing response, manager A chooses manipulation at $t = 1$. At the time that manager A chooses his manipulation, he does not observe report r_B or manager B's actual choice of manipulation m_B. Instead, he conjectures that manager B will choose manipulation m_B^* and thus succeed in issuing a fraudulent report with probability $\mu_B^* = m_B^*(1 - q_B)$. His best response to μ_B^*, denoted as $\tilde{m}_A^*(\mu_B^*)$, is determined by the following first order condition:

$$H^A(m_A; m_B^*)|_{m_A = \tilde{m}_A^*} \equiv \delta_A \frac{\partial \mu_A}{\partial m_A} E_{r_B}[\theta_A(1, r_B)|s_A = 0](1 + C_A^*) - (1 - \delta_A) C_A'(m_A) = 0. \quad (6)$$

Equation 6 is similar to Equation 4 in the single-firm case except that $\theta_A(1)$ is replaced by $E_{r_B}[\theta_A(1, r_B)|s_A = 0]$ (and we have also imposed investors' rational expectations about m_A^*). Manager A bases his manipulation on his expectation about investors' belief about his firm averaged over report r_B. We call $E_{r_B}[\theta_A(1, r_B)|s_A = 0]$ the manager's expected payoff (from successful manipulation) for ease of notation. Since μ_B^* is not affected by manager A's actual choice of m_A, we can treat μ_B^* as a parameter in Equation 6 and examine the

manager's best (manipulation) response to parameter μ_B^*. Since μ_B^* and m_B^* differ only by an observable constant, we use $\tilde{m}_A^*(.)$ as the manager's best response to both m_B^* and μ_B^* to save on notations.

Proposition 1 *There exists a unique best response of manager A's manipulation upon $s_A = 0$ to the probability that manager B issues a fraudulent report, $\tilde{m}_A^*(\mu_B^*)$, which is characterized by Equation 6. For any interior μ_B^*, $\tilde{m}_A^*(\mu_B^*)$ is increasing in μ_B^*. In particular, $\tilde{m}_A^*(\mu_B^*)$ is increasing in m_B^* and decreasing in q_B.*

Proposition 1 states that manager A's incentive to manipulate is increasing in his *belief* that manager B will succeed in manipulating report r_B. The belief arises from both his conjecture of manager B's manipulation m_B^* and his observation of firm B's internal control choice q_B. This resembles some aspects of the peer pressure for manipulation we have previously discussed in the introduction. When manager A expects manager B to manipulate more, the opportunity cost for manager A to refrain from manipulating is higher.

We discuss the intuition behind Proposition 1. To fix the idea, suppose the new manager of firm B has a higher capital market concern, δ_B. Both investors and manager A observe this change and conjecture that the equilibrium m_B^* and μ_B^* will be higher (according to Lemma 1). We need to explain why manager A responds to this expected increase in μ_B^* by increasing his own manipulation.

An extreme case can illustrate our intuition. Consider a special case $\rho = 1$ so that s_A and s_B are perfectly correlated. Suppose we start with $\delta_B = 0$ (and $\delta_A > 0$) so that $m_B^* = 0$ and $\mu_B^* = 0$. Since manager B has no capital market concern and never manipulates, investors do not discount report $r_B = 1$. Even though the bad manager A has a capital market concern, he will not manipulate either. He fears that report r_B will be equal to 0 with a probability 1 and investors' belief will be equal to $\theta_A(1, 0) = 0$, regardless of his report r_A. The perfect correlation between s_A and s_B means that he privately knows that $s_B = 0$ for certain and thus $r_B = 1$ with probability 0. Suppose manager B's capital market concern δ_B becomes positive so that $m_B^* > 0$ and $\mu_B^* > 0$. The bad manager A will now engage in a positive amount of manipulation. He anticipates that investors will discount $r_B = 1$ by

slightly narrowing the price difference $\theta_A(1,1) - \theta_A(1,0)$. However, he also expects that there is a positive probability of receiving $r_B = 1$, which results in a positive expected payoff of manipulation. Thus, as μ_B^* moves away from 0, manager A starts to manipulate as well.

We now explain the intuition for a general ρ. Inspecting Equation 6 reveals that manager A's manipulation decision is driven by his belief about investors' belief about his firm type that is conditional on $r_A = 1$ averaged across report r_B, $E_{r_B}[\theta_A(1, r_B)|s_A = 0]$. We start with investors' belief about firm A that is conditional on $r_A = 1$ averaged across report r_B, i.e., $E_{r_B}[\theta_A(1, r_B)]$. Investors' rationality implies that the arrival of report r_B does not affect their beliefs about firm A on average. The law of iterated expectations implies that $E_{r_B}[\theta_A(1, r_B)] = \theta_A(1)$, which is independent of the distribution of report r_B. Even though μ_B^* affects the distribution of r_B and the realizations of r_B affect investors' ex-post beliefs about firm A, investors rationally understand the impact of μ_B^* and adjust their beliefs accordingly. We discuss this adjustment process in detail as it is useful for subsequent results about manager A's belief about investors' beliefs. We write out investors' average expectation of $s_A = 1$ as

$$E_{r_B}[\theta_A(1, r_B)] = \theta_A(1,0) + \Pr(r_B = 1|r_A = 1)[\theta_A(1,1) - \theta_A(1,0)]. \tag{7}$$

Since $\theta_A(1,0)$ is not a function of μ_B^*, μ_B^* affects the investors' expectation through two channels. First, as μ_B^* increases, investors expect to receive $r_B = 1$ more often. That is, $\Pr(r_B = 1|r_A = 1)$ is increasing in μ_B^*. We call this the probability effect of μ_B^*. Second, rationally anticipating the inflation of the distribution of report r_B, investors discount report $r_B = 1$ accordingly. That is, $\theta_A(1,1) - \theta_A(1,0)$, the belief difference investors attach to report $r_B = 1$ relative to $r_B = 0$, moves towards 0 as μ_B^* increases, as can be seen from Lemma 2. We term this effect the discounting effect of μ_B^*. Investors' rationality means that they discount report r_B to fully offset their expectations of inflation in the distribution of report r_B. In other words, the probability effect and the discounting effect cancel out each other perfectly, thus μ_B^* does not affect the investors' belief about s_A averaged over r_B, $E_{r_B}[\theta_A(1, r_B)]$.

We now turn to the effect of μ_B^* on manager A's belief about investors' beliefs about firm

A averaged over report r_B, $E_{r_B}[\theta_A(1, r_B)|s_A = 0]$. We can write out this belief in a similar way to Equation 7:

$$E_{r_B}[\theta_A(1, r_B)|s_A = 0] = \theta_A(1, 0) + \Pr(r_B = 1|s_A = 0)[\theta_A(1, 1) - \theta_A(1, 0)]. \qquad (8)$$

The comparison reveals that manager A's ex-ante belief about investors' belief differs from investors' ex-ante belief. Conditional on r_B, manager A's belief about investors' belief is the same as investors' belief. However, manager A and investors have different expectations about the distribution of report r_B. Investors rely on $r_A = 1$ to forecast the distribution of r_B while manager A has more precise information $s_A = 0$. In particular, an increase in μ_B^* makes $r_B = 1$ more frequent if firm B is a bad type. Investors observing $r_A = 1$ believe that firm B is a bad type with probability $\Pr(s_B = 0|r_A = 1)$. In contrast, the bad manager with private information $s_A = 0$ knows that firm B is the bad type with probability $\Pr(s_B = 0|s_A = 0)$. A simple calculation reveals that

$$\Pr(s_B = 0|s_A = 0) - \Pr(s_B = 0|r_A = 1) \propto \rho.$$

"$\propto$" reads as "has the same sign as." Manager A believes that firm B is more (less) likely to be the bad type than investors believe if $\rho > 0$ ($\rho < 0$). From the perspective of the bad manager A, as μ_B^* increases, investors either underestimate the probability effect when $\rho > 0$, or overestimate it when $\rho < 0$. Manager A benefits from both errors. To see this, consider the case of $\rho > 0$ first. Investors observing $r_A = 1$ underestimate the probability that manager B is a bad type and thus the probability that $r_B = 1$ results from manipulation. As a result, they do not discount $r_B = 1$ sufficiently in evaluating firm A. This insufficient discounting of $r_B = 1$ increases manager A's payoff from delivering $r_A = 1$ and thus induces manager A to manipulate more. Similarly, consider the case of $\rho < 0$. With $\rho < 0$, the bad manager knows that firm B is less likely to be a bad type than investors believe, i.e., $\Pr(s_B = 0|s_A = 0) < \Pr(s_B = 0|r_A = 1)$ for $\rho < 0$. As μ_B^* increases, from the bad manager's perspective, investors overestimate the probability that firm B is a bad type and thus the probability that $r_B = 1$ results from successful manipulation. As a result, they discount

$r_B = 1$ excessively in evaluating firm A. This excessive discounting of $r_B = 1$ increases manager A's payoff from issuing $r_A = 1$ and thus induces him to further manipulate.

In sum, as μ_B^* increases, report r_B becomes less informative about firm A and investors become more uncertain about s_A. As a result, manager A manipulate more, which explains the strategic complementarity.

Kartik, Lee, and Suen (2014) have proven the general statistical result that, loosely speaking, a more informative experiment will on average bring the posteriors of two agents with different priors closer to their respective priors. The authors call this the "information-validates-the-prior (IVP) theorem." The IVP theorem provides another way to view the intuition of Proposition 1. As we have discussed at the end of the previous section, information asymmetry occurs in equilibrium between manager A and the investors in our model. The bad manager A's belief about s_A is lower than that of the investors. Firm B's report can be viewed as an informative (though endogenous) experiment about s_A. As manager B increases manipulation in equilibrium, the informativeness of the experiment becomes lower. According to the IVP theorem, the disagreement or information asymmetry between manager A and his investors increases. As a result, manager A manipulates more. Therefore, Proposition 1 provides another application of the IVP theorem.

To fully pin down the effect of the internal control on manipulation, we have to endogenize firm B's manipulation decision as well. $H^A(\tilde{m}_A^*; m_B^*) = 0$ defined by Equation 6 characterizes manager A's best response to manager B's equilibrium choice. Using the same procedure, we can solve manager B's best response to manager A's equilibrium choice, $\tilde{m}_B^*(m_A^*)$. It is characterized by the following Equation:

$$H^B(m_B; m_A^*)|_{m_B = \tilde{m}_B^*} \equiv \delta_B (1 - q_B) E_{r_A}[\theta_B(1, r_A)|s_B = 0](1 + C_B^*) - (1 - \delta_B) C_B'(m_B) = 0. \tag{9}$$

Equation 6 and Equation 9 jointly determine the managers' optimal choices of manipulation (m_A^*, m_B^*) through $m_A^* = \tilde{m}_B^*(m_A^*)$ and $m_B^* = \tilde{m}_A^*(m_B^*)$. Again the equilibrium is unique under our assumption that the manipulation cost functions are sufficiently convex. After we solve for the unique equilibrium, the optimal manipulation decisions (m_A^*, m_B^*) can be

expressed as functions of the firms' choices of internal control (q_A, q_B) and we can examine the equilibrium effect of internal control on manipulation.

Proposition 2 *The unique pair of managers' best responses to firms' internal control decisions, $\{m_A^*(q_A, q_B), m_B^*(q_B, q_A)\}$, are collectively characterized by Equation 6 and Equation 9. In addition, $\frac{\partial m_A^*(q_A, q_B)}{\partial q_A} < 0$ and $\frac{\partial m_B^*(q_B, q_A)}{\partial q_A} < 0$.*

Proposition 2 confirms that a firm's internal control has an externality on its peer firm. An improvement in one firm's internal control quality reduces not only its own manager's manipulation (i.e., $\frac{\partial m_A^*(q_A, q_B)}{\partial q_A} < 0$), but also the peer manager's manipulation (i.e., $\frac{\partial m_B^*(q_B, q_A)}{\partial q_A} < 0$) via mitigation of the peer pressure for manipulation. Lemma 1 shows that a firm's internal control directly deters its own manager's manipulation and reduces μ_A^* (the probability that report A is manipulated). Proposition 1 suggests that, through symmetry between firm A and B, a lower μ_A^* results in a lower m_B^*. In other words, firm A's internal control indirectly mitigates the peer pressure for manipulation on manager B, and reduces manager B's manipulation. Of course, the reduction of manipulation by manager B alleviates the pressure on manager A as well, setting into motion a loop of feedbacks. Through this loop, the effect of a firm's internal control on its own manager's manipulation is amplified.

3.2 Equilibrium internal control decisions

In the previous section, we characterized the managers' manipulation decisions at $t = 1$, taking the firms' internal control choices q_A and q_B at $t = 0$ as given. In this section we endogenize the firms' internal control decisions. We show that even though firms have a private incentive to invest in internal control, they underinvest in internal control due to the externality described in Proposition 2.

After understanding managers' manipulation decisions, we fold back to $t = 0$ and consider firm A's private incentive to invest in costly internal control over financial reporting. Since firm A does not observe firm B's choice of internal control at the time of choosing q_A, it conjectures that firm B will choose q_B^* in equilibrium. Moreover, firm A anticipates that managers at $t = 1$ will respond to its actual choice of q_A through $m_A^*(q_A, q_B^*)$ and $m_B^*(q_B^*, q_A)$.

Based on these expectations, the firm value at $t = 0$ as a function of its internal control choice q_A is

$$V_{A0}(q_A; q_B^*) \equiv E_0[V_A(q_A, q_B^*)] = \theta_A - \Pr(s_A = 0)C_A(m_A^*(q_A, q_B^*)) - K_A(q_A). \qquad (10)$$

The firm value at $t = 0$ has three components. The first is the expected gross cash flow $E_{v_A}[s_A] = \theta_A$ in the absence of manipulation and internal control investment. Second, manipulation generates the expected deadweight loss $\Pr(s_A = 0)C_A(m_A^*(q_A, q_B^*))$. The existence of this deadweight loss means that the firm has a private incentive to invest in internal control to prevent manipulation. Finally, the internal control investment itself consumes resources and reduces the firm value by $K_A(q_A)$.

Firm A at $t = 0$ chooses q_A to maximizes $V_{A0}(q_A, q_B^*)$ subject to the managers' subsequent equilibrium manipulation responses $m_A^*(q_A, q_B^*)$ and $m_B^*(q_B^*, q_A)$. Differentiating $V_{A0}(q_A, q_B^*)$ in Equation 10 with respect to q_A, the first order condition is

$$\frac{\partial V_{A0}}{\partial m_A^*}\frac{\partial m_A^*}{\partial q_A} - K_A'(q_A) = 0. \qquad (11)$$

The first term $\frac{\partial V_{A0}}{\partial m_A^*}\frac{\partial m_A^*}{\partial q_A}$ captures the benefit of internal control q_A in reducing manipulation, which is balanced by the marginal cost $K'(q_A)$. Note that $\tilde{q}_A^*(q_B^*)$ implied by the first order condition is firm A's best response to its conjecture of firm B's equilibrium internal control choice q_B^*. We can solve firm B's internal control decision using the same procedure. Taking q_A^* as given, firm B's best response to q_A^*, $\tilde{q}_B^*(q_A^*)$, is characterized by a similar first order condition. Intercepting the two best responses, $q_A^* = \tilde{q}_A^*(q_B^*)$ and $q_B^* = \tilde{q}_B^*(q_A^*)$, we prove that there exists a unique equilibrium when the cost function of internal control investment is sufficiently convex. This completes the characterization of the entire equilibrium.

Proposition 3 *The unique equilibrium $\{q_A^*, q_B^*, m_A^*(s_A), m_B^*(s_B), P_A^*(r_A, r_B), P_B^*(r_B, r_A)\}$ is collectively characterized by Equation 11, Equation 6, $m_A^*(s_A = 1) = 0$, Equation 5 and their counterparts for firm B.*

After we have solved for the entire equilibrium, we evaluate the efficiency of the privately

optimal internal control investment. Proposition 3 shows that firms do have private incentives to invest in internal control. But are the privately optimal choices of internal control efficient from a social perspective? We use the Pareto efficiency criterion. In our context, two firms' internal control investment decisions are Pareto efficient if it is impossible to make any one firm better off without making the other worse off. In other words, the privately optimal internal control investment pair $\{q_A^*, q_B^*\}$ is not efficient if there exists a pair of internal control investments $\{q_A', q_B'\}$ such that both firms are strictly better off under $\{q_A', q_B'\}$ than under their private choices, i.e., $V_{A0}(q_A', q_B') > V_{A0}(q_A^*, q_B^*)$ and $V_{B0}(q_B', q_A') > V_{B0}(q_B^*, q_A^*)$. Moreover, the firms underinvest in internal control if $q_A' > q_A^*$ and $q_B' > q_B^*$.

Proposition 4 *The privately optimal choices of internal control $\{q_A^*, q_B^*\}$ characterized in Proposition 3 are not Pareto efficient. Moreover, firms underinvest in internal control.*

The key intuition behind Proposition 4 is that the internal control investment in firm A exerts a positive externality on firm B. More specifically, Proposition 2 suggests that a higher internal control by firm A reduces manager B's manipulation, $\frac{\partial m_B^*}{\partial q_A} < 0$. The reduction in m_B^* in turn improves the value of firm B. Because of this positive externality, the social planner can implement an upward deviation of q_A from firm A's private optimal choice q_A^*, which benefits firm B. Note that firm A is not hurt by this deviation because in the neighborhood of $q_A = q_A^*$, the marginal effect of increasing q_A on firm A's value is zero.

Proposition 4 suggests that a coordination failure could exist among firms' individual choices of internal control over financial reporting, which provides one rationale for intervention in firms' internal control investment. In the presence of peer pressure for manipulation, one firm's internal control investment has a positive externality on other firms. A floor of internal control investment could improve the firm value of all firms. Looking through the lens of Proposition 4, the proposal in Romano (2005) that the internal control mandates in SOX should be made optional is flawed. Competition among firms (or state laws) does not lead to a socially optimal investment in internal control.

Proposition 4 highlights one potential benefit of regulating internal control over financial reporting. It is silent on other costs of regulation. For example, since Stigler (1971), we have

known that regulators' incentives may be aligned with regulated firms and organized vested interest groups rather than with the social welfare. In another example, even well-intended regulators may not be able to achieve the full potential of regulations due to the information disadvantage in designing and implementing regulations. Yet another example is that a regulator may care about not only efficiency, but also redistribution among heterogenous firms. An intervention in internal control may increase a combined total firm value at the expense of reducing one firm's value. Thus, while our model provides one possible benefit of regulations such as SOX, a complete characterization of socially optimal regulation is a complicated enterprise and beyond the scope of this paper. We refer the readers to the recent surveys of Leuz and Wysocki (2007) and Coates and Srinivasan (2014).

4 Extensions

We have made a number of simplifying assumptions in the baseline model in order to highlight the economic forces behind our main result: namely there exists a positive externality among firms' internal control investments. The positive externality arises from the strategic complementarity of managers' manipulation decisions. A manager manipulates more when he expects that reports from his peer managers are also more likely to be manipulated. In this section, we discuss three extensions to the baseline model.

4.1 Continuous state and message spaces

The baseline model employs the binary state and message spaces. In this subsection, we extend our analysis to continuous state and message spaces. This extension serves two purposes. First, it helps to identify the broad conditions under which our main result–that internal control investment has a positive externality–holds. Second, it formalizes the difference between two related notions, strategic complementarity and spillover effect, and better connects our paper to the prior literature.

The continuous extension is the same as the baseline binary model except for the state and message spaces. First, we extend the state space to be continuous. The true profitability

of each firm, s_i, $i \in \{A, B\}$, is drawn for a normal distribution with mean $\bar{s}$ and variance σ_s^2. Moreover, s_A and s_B are correlated with a correlation coefficient $\rho \in [-1, 1]$. Second, we also extend the message space to be continuous and modify the manipulation technology accordingly. Specifically, at $t = 1$ after observing s_i, manager i chooses manipulation m_i at a cost $C_i(m_i)$ that affects the report r_i in the following way

$$r_i = s_i + m_i(1 - q_i) + h(m_i)\tilde{\varepsilon}_i.$$

$m_i \geq 0$ is the manager's manipulation choice. $q_i \in [0, 1]$ is the firm's internal control investment. $h(m_i)$ is an arbitrary function of m_i that satisfies $h(m_i) \geq 0$ and $h'(m_i) \geq 0$. $\tilde{\varepsilon}_i$ is a standard normal random variable that is independent of all other variables in the model.

Note that our continuous extension nests the setting in Stein (1989) as a special case with $h'(m_i) = 0$ for any $m_i \geq 0$ (or equivalently $h(m_i) = h_0$). In Stein (1989), manipulation increases the mean of reports but doesn't affect the variance. In contrast, our assumption that $h' \geq 0$ allows the possibility that manipulation degrades the report's informativeness. When $h'(m_i) > 0$, manipulation is assumed to increase the report's variance (or equivalently reduce the report's informativeness).

Proposition 5 *In a continuous setting,*

1. *if $h' > 0$ and $\rho \neq 0$, managers' manipulation decisions are strategic complements, and firms underinvest in internal control;*

2. *if $h' = 0$ or $\rho = 0$, managers' manipulation decisions are independent and firms' internal control investment decisions are Pareto efficient.*

Proposition 5 identifies the exact conditions under which strategic complementarity among managers' manipulation decisions and inefficiency in firms' internal control decisions arise in a continuous setting. As in the baseline model, the inefficiency in firms' internal control investment decisions arises from the investment's positive externality, which in turn results from the strategic complementarity of managers' manipulation decisions. The necessary and

sufficient condition for the strategic complementarity is that manipulation reduces the report's informativeness, a feature that rises naturally from the binary structure. Therefore, our main result is not driven by the binary structure per se, it is driven by the feature that manipulation reduces the report's informativeness. That our main result does not hold in a Stein setting (which has $h' = 0$) is an exception, which proves the rule. It highlights that the fundamental economic force behind our main result is that manipulation reduces the report's informativeness.

As such, the robustness of our main result is boiled down to the plausibility of the economic force that manipulation reduces the report's informativeness. In our view, this economic force is at least as reasonable as, if not more reasonable than, the opposite–that is, manipulation does not affect the report's informativeness. This seems particularly true in our context of studying such an internal control investment regulation as SOX. The motivation for SOX arose partly from concern about the adverse consequences of manipulation, of which the degradation in information quality resulting from manipulation was an important part. Therefore, that manipulation reduces the report's informativeness seems a reasonable feature as far as our research question is concerned.[8]

The continuous extension also highlights the distinction between two notions: spillover and strategic complementarity. Spillover refers to the effect that the presence of report r_B affects manager A's manipulation; that is, spillover effect exists if $m_A^*|_{\text{SingleFirm}} \neq m_A^*|_{\text{TwoFirms}}$. In contrast, strategic complementarity refers to the effect that a *change* in manager B's manipulation affects manager A's manipulation; that is, $\frac{\partial m_A^*}{\partial m_B^*} > 0$.

Proposition 6 *In the continuous setting,*

1. spillover effect exists if and only if $\rho \neq 0$;

2. strategic complementarity exists if and only if $\rho \neq 0$ and $h' > 0$;

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[8]In contrast, Stein (1989) is not concerned about whether manipulation reduces the report's informativeness. His model was motivated to demonstrate the possibility that myopic behavior (in the form of manipulation) can arise even if investors in the capital market have rational expectations, as an answer to Michael Jensen's claim that forward-looking capital market should discipline managers from taking short-term activities. The particular feature in Stein–that manipulation does not affect the report's informativeness–seems unimportant for his argument and makes his model more elegant in making his point. In other words, Stein could have made his point with a setting of our continuous model, even though the extra feature of EM reducing informativeness would be distracting to making his point.

3. if $\rho \neq 0$ and $h' = 0$, there exists spillover but no strategic complementarity.

Proposition 6 explains the relation between spillover and strategic complementarity. The two notions are related but different. The correlation of two firms' fundamentals is necessary for both the spillover and strategic complementarity. This correlation is also sufficient for the spillover effect, but insufficient for the strategic complementarity. In other words, a case could exist in which the spillover effect is present but the strategic complementarity is not. Such a case arises when $\rho \neq 0$ but $h' = 0$. As a result, in a multi-firm variant of Stein setting, there would be spillover but no strategic complementarity.

The distinction between spillover and strategic complementarity is important for our research question. It is strategic complementarity, not the spillover effect, that drives the positive externality of internal control investment that our paper focuses on. Internal control investment has a positive externality if and only if strategic complementarity exists between two managers' manipulation decisions. In contrast, spillover per se does not lead to the externality of internal control investment, as shown in Proposition 5. When there is spillover but no strategic complementarity (i.e., $\rho \neq 0$ and $h' = 0$), firm A's internal control reduces manager A's manipulation but does not affect manager B's manipulation. As a result, no externality exists in firms' internal control investment.

Since manipulation reduces the report's informativeness monotonically in the binary structure, both the spillover effect and the strategic complementarity arise simultaneously in our baseline model.[9] As a result, it is difficult to see their differences.

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[9]It is straightforward to verify that in our binary baseline model, spillover effect exists if and only if $\rho \neq 0$. To see this, comparing the first-order condition in the single-firm model (4) and that in the two-firm model (6) reveals that the two differs only in that (4) contains $\theta_A(1)$ while (6) contains $E_{r_B}[\theta_A(1, r_B)|s_A = 0]$. One can verify that

$$E_{r_B}[\theta_A(1, r_B)|s_A = 0] - \theta_A(1)$$
$$= \left[\frac{\Pr(r_B = 1|s_A = 0)}{\Pr(r_B = 1|r_A = 1)} - 1\right][\theta_A(1, \phi) - \theta_A(1, 0)]$$
$$= \rho\sqrt{\frac{(1 - \theta_B)\,\theta_B}{(1 - \theta_A)\,\theta_A}}[\theta_A(1, \phi) - \theta_A(1, 0)].$$

Thus $E_{r_B}[\theta_A(1, r_B)|s_A = 0] > \theta_A(1)$ if and only if $\rho \neq 0$. This in turn makes $m_A^*|_{\mathrm{TwoFirms}} > m_A^*|_{\mathrm{SingleFirm}}$.

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4.2   Alternative internal control technology

In the baseline model, the costly internal control may detect the manager's manipulation attempt and prevent him from manipulation. The resulting trade-off between the cost of internal control and the cost of manipulation seems realistic in the context of the debate regarding internal control regulations such as SOX.

An alternative approach to model internal control that has been used in the prior literature is that internal control directly increases the marginal cost of manipulation. We now check the robustness of our main result to this representation of internal control.

This extension is the same as our main setup with two differences. First, the probability that the bad manager issues a fraudulent good report, $\mu_i \equiv \Pr(r_i = 1 | s_i = 0, m_i) = m_i$, which depends only on the manipulation choice by the manager (not directly on the internal control quality, q_i). Second, the manager's cost of manipulation $C_i(m_i, q_i)$ depends on the internal control choice q_i, with the marginal cost of manipulation increasing in q_i, i.e., $\frac{\partial^2 C_i}{\partial q_i \partial m_i} > 0$ for all q_i and m_i. For illustrational purposes, we specify a functional form for $C_i(m_i, q_i) = \frac{q_i + c_0}{2} m_i^2$ where $c_0 \geq 0$ is a constant. We show in the proposition below that our main results hold under the condition that the manipulation cost C_i is sufficiently convex in m_i.

Proposition 7 *In this extension, managers' manipulation decisions are strategic complements. Moreover,*

1. *if c_0 is sufficiently large, firms underinvest in internal control.*

2. *if c_0 is sufficiently small, there is no demand for internal control.*

Proposition 7 shows that the strategic complementarity between managers' manipulation decisions is robust to this alternative internal control technology. However, the demand for internal control with this alternative technology depends on the parameters of such technology. If the cost parameter c_0 is sufficiently large, then the demand for internal control is interior, and our main result on the externality of internal control holds. On the other hand, if the cost parameter c_0 is sufficiently small, Proposition 7 shows that there is no demand for internal control. As a result, our research question regarding the efficiency of private internal

control investment becomes moot. This latter case is most transparent when considering an extreme case with $c_0 = 0$. If the cost of manipulation comes mainly from internal control and the firm's internal control is designed to reduce the cost of manipulation, then setting the internal control quality to 0 would minimize the cost of manipulation and thus optimal for both the firms and the social planner. As a result, the problem of optimal internal control degenerates. This feature has been often overlooked in the prior literature because prior studies often don't endogenize the choice of internal control. Since our research question concerns the optimal choice of internal control quality, we need a setup in which the demand for internal control is interior.

4.3 Private manipulation cost

Another assumption we make in our baseline model is that the cost of manipulation $C_i(m_i)$ reduces firm value. We now check the robustness of our results to an alternative interpretation, that is, C_i is the manager's private cost. We show that our main results hold if we accompany this change with an additional assumption that ex ante at $t = 0$, the firm would internalize at least part of the manager's ex-post manipulation cost through some sort of individual participation constraint for the manager.

More specifically, we make two modifications to our baseline model in order to examine the case that $C_i(m_i)$ is private. First, when the manipulation cost is completely private, it only affects the manager's payoff U_i, but does not affect the firm value V_i. This leads to changes in firm value and short-term prices with the new firm value, $V_i = s_i - K_i(q_i)$ and the new short-term price, $P_i^*(r_A, r_B) = E_2[V_i | r_A, r_B]$. Second, we assume that each manager has an outside opportunity that pays $\bar{U}$. Each firm offers its manager a compensation package that includes two components: a weighted average of the long-term firm value V_i, and the short-term stock price P_i as in the baseline model, $\delta_i P_i + (1 - \delta_i)V_i$, plus a fixed base wage U_{0i}. As a result, the payoff function of manager i is given by

$$U_i = \delta_i P_i + (1 - \delta_i)V_i + U_{0i} - C(m_i), \ i \in \{A, B\}.$$

Firms have all the bargaining power and choose the base wage U_{0i} ex ante to make managers break even in equilibrium, i.e., $E[U_i^*] = \bar{U}$. In addition, firms choose the internal control to maximize the firm value minus the wage paid to the managers. The timeline of the modified model is as follows.

1. $t = 0$, firm i publicly chooses its internal control quality q_i and the base wage U_{0i};

2. $t = 1$, manager i privately chooses manipulation m_i after privately observing s_i;

3. $t = 2$, investors set stock price P_i after observing both report r_A and r_B;

4. $t = 3$, cash flows are realized and paid out.

We show in the proposition below that our main results hold in this extension.

Proposition 8 *In this extension with the private manipulation cost, managers' manipulation decisions are strategic complements and firms underinvest in internal control.*

The key economic driving force behind Proposition 8 is that even though firms do not directly bear the ex-post manipulation cost, they ex ante internalize managers' private manipulation cost through the individual participation constraint. As a result, both the strategic complementarity and the positive externality of internal control in our baseline model are robust to the party directly bears the manipulation cost ex post.

It is useful to point out that this extension intends to check the robustness of our main mechanism to the alternative assumption regarding the manipulation cost. The extension is not designed to address the issue of optimal contracting for managers. We have included only the manager's individual participation condition. It is beyond the scope of this paper to explicitly model a moral hazard problem that generates the manager's utility function U_i.

5   Conclusion and Discussion

We have presented a model to show that a firm's investment in internal control has a positive externality on peer firms. The core of the mechanism is the strategic complementarity of

manipulation. A manager manipulates more if he expects peer firms' reports are more likely to be manipulated. As a result, a firm's investment in internal control benefits not only itself by reducing its own manager's manipulation, but also peer firms by mitigating the manipulation pressure on peer managers. Without internalizing this positive externality, firms underinvest in internal control over financial reporting. Regulations that provide a floor of internal control investment can mitigate the underinvestment problem.

We have presented a stylized model to deliver the intuition for the strategic complementarity of manipulation and the externality of a firm's internal control. In particular, the binary structure has dramatically simplified the exposition. However, we believe that the economic forces behind peer pressure for manipulation are more general. The strategic complementarity between the two managers' manipulation decisions is driven by two features of the model. First, manager A can forecast manager B's manipulation better than the investors. Second, manipulation reduces the report's informativeness. The first feature arises naturally in a setting when the managers have private information about their own fundamentals that are correlated. Thus, as long as the second feature is preserved in a richer model in which manipulation leads to information degradation, the strategic complementarity between the two managers' decisions is expected to be intact.

In addition to the extensions studied in Section 4, there could be other extensions to the baseline model. We have focused on a two-firm economy in the baseline model. It is straightforward to extend the model to $N > 2$ firms. A manager manipulates more if he suspects that any of his peer firms' reports is more likely to be manipulated. Conditional on the fundamental s_i, report r_i and r_j are independent of each other. As a result, the presence of any additional firms $k \in N\backslash\{i, j\}$ does not affect the interaction between firm i and j. In other words, investors can first use all firms' reports r_k, other than r_i and r_j, to update their belief about s_i. Treating this posterior as a prior, investors continue to use report r_i and r_j, as in our baseline model of two firms.[10] The peer pressure holds for any pair of firms within in the N firms as long as their fundamentals are correlated.

Another possible extension is to relax the assumption that the managers know their

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[10] See footnote 7 for the discussion of the implications of the conditional independence properties.

firms' fundamentals perfectly. If managers receive only noisy private information about their firms' fundamentals, there would be measurement errors in the reports in the absence of manipulation. In this scenario, manipulation may then help correct the measurement errors. we now assume that the fundamental or gross cash flow is $v_i \in \{0, 1\}$. Each manager receives a noisy signal $s_i \in \{0, 1\}$ about v_i : $\Pr(s_i = 1 | v_i = 1) = \Pr(s_i = 0 | v_i = 0) = \tau \in [\frac{1}{2}, 1]$. τ measures the quality of managers' signals and our baseline model is a special case of $\tau = 1$. With this specification, we can replicate Proposition 1 that manager A with $s_i = 0$ manipulates more if he expects that report B is more likely to be manipulated. The proof goes through essentially by redefining the fundamental s_i. Even though managers receive noisy signals about the fundamentals, they still know more than the investors and thus the information asymmetry persists in equilibrium.

Finally, we have focused on the capital market pressure as the driver for accounting manipulation, which appears empirically important (e.g., Graham, Harvey, and Rajgopal (2005)). As a result, we have assumed that the two firms are independent, with the exception of the correlation of their fundamentals. In practice, peer firms are likely to interact in other areas (such as product markets, labor markets, performance benchmarking, and regulation), which may also lead to interactions of their accounting decisions. As we discussed in the literature review, these other interactions are promising venues for future research.

6   Appendix

Proof. of Lemma 1: For notational ease, we use $C_A^* = C_A(m_A^*)$ whenever no confusion arises. For a given $q_A \in (0, 1)$, after we impose the rational expectations requirement, m_A^* is determined by the first order condition:

$$H^A(m_A^*) = \delta_A (1 - q_A) \theta_A (1) (1 + C_A^*) - (1 - \delta_A) C_A'(m_A^*) = 0.$$

We first verify that the equilibrium is unique, *i.e.*, $H^A(m_A^*) = 0$ has a unique solution. First, under our assumption that C_A is sufficiently convex, we have $\frac{\partial H^A(m_A^*)}{\partial m_A^*} < 0$. This is because

$$
\begin{aligned}
\frac{\partial H^A(m_A^*)}{\partial m_A^*} &= \delta_A (1 - q_A) \frac{\partial}{\partial m_A^*} [(1 + C_A^*) \theta_A (1)] - (1 - \delta_A) C_A''(m_A^*) \\
&= (\delta_A (1 - q_A) \theta_A (1))^2 \frac{1 + C_A^*}{1 - \delta_A} + \delta_A (1 - q_A) (1 + C_A^*) \frac{\partial \theta_A (1)}{\partial m_A^*} - (1 - \delta_A) C_A''(m_A^*).
\end{aligned}
$$

The sign of $\frac{\partial H^A(m_A^*)}{\partial m_A^*}$ is dominated by the sign of $C_A''(m_A^*)$. Second, at $m_A^* = 0$, $H^A(0) = \delta_A (1 - q_A) \theta_A (1; m_A^* = 0) > 0$. Finally, at $m_A^* = 1$, $H^A(1) = \delta_A (1 - q_A) \theta_A (1; m_A^* = 1) (1 + C_A(1)) - (1 - \delta_A) C_A'(1) = -\infty < 0$. Therefore, by the intermediate value theorem, the equilibrium m_A^* that satisfies $H^A(m_A^*) = 0$ is unique.

For any parameter $x \in \{\theta_A, \delta_A, q_A\}$, the application of the implicit function theorem generates

$$\frac{\partial m_A^*}{\partial x} = -\frac{\frac{\partial H^A(m_A^*; x)}{\partial x}}{\frac{\partial H^A(m_A^*)}{\partial m_A^*}}.$$

The denominator $\frac{\partial H^A(m_A^*)}{\partial m_A^*} < 0$. As a result, the sign of $\frac{\partial m_A^*}{\partial x}$ is the same as that of $\frac{\partial H^A(m_A^*; x)}{\partial x}$. In particular,

$$\frac{\partial H^A(m_A^*; \delta_A)}{\partial \delta_A} = (1 - q_A)(1 + C_A^*) \theta_A (1) + C_A'(m_A^*) > 0,$$

$$\begin{aligned}
\frac{\partial H^A(m_A^*; \theta_A)}{\partial \theta_A} &= \delta_A (1 - q_A)(1 + C_A^*) \frac{\partial \theta_A (1)}{\partial \theta_A} \\
&= \delta_A (1 - q_A)(1 + C_A^*) \frac{\mu_A^*}{\left[\theta_A + (1 - \theta_A) \mu_A^*\right]^2} > 0,
\end{aligned}$$

$$\begin{aligned}
\frac{\partial H^A(m_A^*; q_A)}{\partial q_A} &= -\delta_A (1 + C_A^*) \theta_A (1) + \delta_A (1 - q_A)(1 + C_A^*) \frac{\partial \theta_A (1)}{\partial q_A} \\
&= \delta_A (1 + C_A^*)(-\theta_A (1) + \theta_A (1)(1 - \theta_A(1))) \\
&= -\delta_A (1 + C_A^*) \theta_A^2 (1) < 0.
\end{aligned}$$

∎

Proof. of Lemma 2: We first use the Bayes rule to write out

$$\begin{aligned}
\theta_A (1, 1) - \theta_A (1, \phi) &= \frac{\theta_A (1, \phi)}{\theta_A (1, \phi) + (1 - \theta_A (1, \phi)) \frac{\Pr(r_B=1|s_A=0)}{\Pr(r_B=1|s_A=1)}} - \theta_A (1, \phi) \\
&= \frac{\theta_A (1, \phi)(1 - \theta_A (1, \phi)) \left[1 - \frac{\Pr(r_B=1|s_A=0)}{\Pr(r_B=1|s_A=1)}\right]}{\theta_A (1, \phi) + (1 - \theta_A (1, \phi)) \frac{\Pr(r_B=1|s_A=0)}{\Pr(r_B=1|s_A=1)}}.
\end{aligned}$$

$\theta_A (1, \phi) = \theta_A(1)$ is expressed in Equation 2.

Therefore, $\theta_A (1, 1) - \theta_A (1, \phi) > 0$ if and only if $\frac{\Pr(r_B=1|s_A=0)}{\Pr(r_B=1|s_A=1)} < 1$ or equivalently $\frac{\Pr(r_B=1|s_A=1)}{\Pr(r_B=1|s_A=0)} > 1$. Moreover, since $\theta_A (1, \phi)$ is independent of μ_B^*, $\theta_A (1, 1) - \theta_A (1, \phi)$ is increasing in μ_B^* if and only if $\frac{\Pr(r_B=1|s_A=1)}{\Pr(r_B=1|s_A=0)}$ is increasing in μ_B.

We then write out

$$\frac{\Pr(r_B = 1|s_A = 1)}{\Pr(r_B = 1|s_A = 0)} - 1$$

$$= \frac{\Pr(r_B = 1|s_B = 1, s_A = 1)\Pr(s_B = 1|s_A = 1) + \Pr(r_B = 1|s_B = 0, s_A = 1)\Pr(s_B = 0|s_A = 1)}{\Pr(r_B = 1|s_B = 1, s_A = 0)\Pr(s_B = 1|s_A = 0) + \Pr(r_B = 1|s_B = 0, s_A = 0)\Pr(s_B = 0|s_A = 0)} - 1$$

$$= \frac{\Pr(r_B = 1|s_B = 1)\Pr(s_B = 1|s_A = 1) + \Pr(r_B = 1|s_B = 0)\Pr(s_B = 0|s_A = 1)}{\Pr(r_B = 1|s_B = 1)\Pr(s_B = 1|s_A = 0) + \Pr(r_B = 1|s_B = 0)\Pr(s_B = 0|s_A = 0)} - 1$$

$$= \frac{\frac{\Pr(r_B=1|s_B=1)}{\Pr(r_B=1|s_B=0)}\Pr(s_B = 1|s_A = 1) + \Pr(s_B = 0|s_A = 1)}{\frac{\Pr(r_B=1|s_B=1)}{\Pr(r_B=1|s_B=0)}\Pr(s_B = 1|s_A = 0) + \Pr(s_B = 0|s_A = 0)} - 1$$

$$= \frac{\left(\frac{\Pr(r_B=1|s_B=1)}{\Pr(r_B=1|s_B=0)} - 1\right)\Pr(s_B = 1|s_A = 1) + 1}{\left(\frac{\Pr(r_B=1|s_B=1)}{\Pr(r_B=1|s_B=0)} - 1\right)\Pr(s_B = 1|s_A = 0) + 1} - 1$$

$$= \frac{\left(\frac{1}{\mu_B^*} - 1\right)}{\left(\frac{1}{\mu_B^*} - 1\right)\Pr(s_B = 1|s_A = 0) + 1}\left(\Pr(s_B = 1|s_A = 1) - \Pr(s_B = 1|s_A = 0)\right)$$

$$= \frac{\left(\frac{1}{\mu_B^*} - 1\right)}{\left(\frac{1}{\mu_B^*} - 1\right)\Pr(s_B = 1|s_A = 0) + 1}\rho\sqrt{\frac{(1 - \theta_B)\theta_B}{(1 - \theta_A)\theta_A}}.$$

$$\propto \rho$$

"$\propto$" reads as "has the same sign as." The second last equality holds because

$$\Pr(s_B = 1|s_A = 1) - \Pr(s_B = 1|s_A = 0)$$

$$= \frac{\Pr(s_B = 1, s_A = 1)}{\Pr(s_A = 1)} - \frac{\Pr(s_B = 1, s_A = 0)}{\Pr(s_A = 0)}$$

$$= \frac{\Pr(s_B = 1, s_A = 1)}{\Pr(s_A = 1)} - \frac{\Pr(s_B = 1) - \Pr(s_B = 1, s_A = 1)}{\Pr(s_A = 0)}$$

$$= \frac{\theta_A\theta_B + \rho\sqrt{(1 - \theta_A)\theta_A(1 - \theta_B)\theta_B}}{\theta_A} - \frac{\theta_B - \left(\theta_A\theta_B + \rho\sqrt{(1 - \theta_A)\theta_A(1 - \theta_B)\theta_B}\right)}{1 - \theta_A}$$

$$= \rho\sqrt{\frac{(1 - \theta_B)\theta_B}{(1 - \theta_A)\theta_A}}.$$

The last step holds because $\mu_B^* \in (0, 1)$ and thus $\frac{1}{\mu_B^*} > 1$. Thus, $\frac{\Pr(r_B=1|s_A=1)}{\Pr(r_B=1|s_A=0)} > 1$ if and only if $\rho > 0$.

Moreover, we can show that $\frac{\Pr(r_B=1|s_A=1)}{\Pr(r_B=1|s_A=0)}$ is increasing in μ_B^* if and only if $\rho < 0$, because

$$\frac{\partial}{\partial \mu_B^*}\frac{\Pr(r_B = 1|s_A = 1)}{\Pr(r_B = 1|s_A = 0)} \quad \propto \quad -\left(\Pr(s_B = 1|s_A = 1) - \Pr(s_B = 1|s_A = 0)\right)$$

$$\propto \quad -\rho. \tag{12}$$

The proof for the properties of $\theta_A(1, 1) - \theta_A(1, 0)$ is similar and hence omitted. Therefore, we have proved Lemma 2. ∎

Proof. of Proposition 1: For given interior q_A and μ_B^* and investors' conjecture m_A^*, manager A's best response $\tilde{m}_A^*(\mu_B^*)$ is determined by the first order condition:

$$H^A(\tilde{m}_A^*; \mu_B^*) \equiv \delta_A (1 - q_A) E_{r_B}[\theta_A(1, r_B)|s_A = 0](1 + C_A(\tilde{m}_A^*(\mu_B^*))) - (1 - \delta_A) C_A'(\tilde{m}_A^*) = 0.$$

The application of the implicit function theorem generates

$$\frac{\partial \tilde{m}_A^*(\mu_B^*)}{\partial \mu_B^*} = -\frac{\frac{\partial H^A(\tilde{m}_A^*; \mu_B^*)}{\partial \mu_B^*}}{\frac{\partial H^A}{\partial \tilde{m}_A^*}}.$$

The denominator $\frac{\partial H^A}{\partial \tilde{m}_A^*}$ is negative following a similar proof in Lemma 1. Thus, the sign of $\frac{\partial \tilde{m}_A^*(\mu_B^*)}{\partial \mu_B^*}$ is the same as that of $\frac{\partial H^A(\tilde{m}_A^*; \mu_B^*)}{\partial \mu_B^*}$. Note that μ_B^* shows up in $H^A(\tilde{m}_A^*; \mu_B^*)$ only through $E_{r_B}[\theta_A(1, r_B)|s_A = 0]$. Therefore, $\frac{\partial H^A(\tilde{m}_A^*; \mu_B^*)}{\partial \mu_B^*}$ has the same sign as $\frac{\partial E_{r_B}[\theta_A(1, r_B)|s_A = 0]}{\partial \mu_B^*}$. We now prove that $\frac{\partial E_{r_B}[\theta_A(1, r_B)|s_A = 0]}{\partial \mu_B^*} > 0$. In words, the bad manager becomes more optimistic about investors' belief as μ_B^* increases.

We first write out the investors' belief about $s_A = 1$ before they observe r_B but after they observe $r_A = 1$:

$$\begin{aligned}
\theta_A(1, \phi) &\equiv \Pr(s_A = 1|r_A = 1) \\
&= \Pr(r_B = 0|r_A = 1)\theta_A(1, 0) + \Pr(r_B = 1|r_A = 1)\theta_A(1, 1) \\
&= \theta_A(1, 0) + \Pr(r_B = 1|r_A = 1)(\theta_A(1, 1) - \theta_A(1, 0)).
\end{aligned}$$

The second step writes out the expectation and the third regroups the terms. This gives us Equation 7 in the text which we reproduce here:

$$\theta_A(1, 1) - \theta_A(1, 0) = \frac{\theta_A(1, \phi) - \theta_A(1, 0)}{\Pr(r_B = 1|r_A = 1)}. \tag{13}$$

We can similarly write out the bad manager's expectation about the investors' expectation of $s_A = 1$ as

$$\begin{aligned}
&E_{r_B}[\theta_A(1, r_B)|s_A = 0] \\
&= \Pr(r_B = 0|s_A = 0)\theta_A(1, 0) + \Pr(r_B = 1|s_A = 0)\theta_A(1, 1) \\
&= \theta_A(1, 0) + \Pr(r_B = 1|s_A = 0)(\theta_A(1, 1) - \theta_A(1, 0)) \\
&= \theta_A(1, 0) + \frac{\Pr(r_B = 1|s_A = 0)}{\Pr(r_B = 1|r_A = 1)}(\theta_A(1, \phi) - \theta_A(1, 0)) \\
&= \theta_A(1, 0) + \frac{\Pr(r_B = 1|s_A = 0)(\theta_A(1, \phi) - \theta_A(1, 0))}{\Pr(r_B = 1|s_A = 0)\Pr(s_A = 0|r_A = 1) + \Pr(r_B = 1|s_A = 1)\Pr(s_A = 1|r_A = 1)} \\
&= \theta_A(1, 0) + \frac{1}{\Pr(s_A = 0|r_A = 1) + \frac{\Pr(r_B=1|s_A=1)}{\Pr(r_B=1|s_A=0)}\Pr(s_A = 1|r_A = 1)}(\theta_A(1, \phi) - \theta_A(1, 0)).
\end{aligned}$$

Again the first step writes out the expectation and the second regroups the terms. The third step plugs in Equation 13. The last step writes out the total probability of $\Pr(r_B = 1|r_A = 1)$ and reorganize the terms. Note that μ_B^* only affects the likelihood ratio $\frac{\Pr(r_B=1|s_A=1)}{\Pr(r_B=1|s_A=0)}$ in the

last equality. Thus, we have

$$\frac{\partial E_{r_B}\left[\theta_A\left(1, r_B\right)|s_A = 0\right]}{\partial \mu_B^*} \quad \propto \quad -\left(\theta_A(1, \phi) - \theta_A(1, 0)\right) \frac{\partial}{\partial \mu_B^*} \frac{\Pr(r_B = 1|s_A = 1)}{\Pr(r_B = 1|s_A = 0)}$$
$$\propto \quad \left(\theta_A(1, \phi) - \theta_A(1, 0)\right) \rho$$
$$> \quad 0.$$

The second step relies on expression 12, the result from the proof in Lemma 1. Therefore, regardless of ρ, $E_{r_B}\left[\theta_A(1, r_B)|s_A = 0\right]$ is increasing in μ_B^*. Lastly, since $\mu_B^* = m_B^*\left(1 - q_B\right)$ is increasing in m_B^* and decreasing in q_B, $\tilde{m}_A^*\left(\mu_B^*\right)$ is increasing in m_B^* and decreasing in q_B. ∎

Proof. of Proposition 2: We first prove that the equilibrium $\left(m_A^*\left(q_A, q_B\right), m_B^*\left(q_B, q_A\right)\right)$ is unique in two steps. First, we solve for manager A's unique best response $\tilde{m}_A^*(m_B^*)$. This part is similar to the proof in Lemma 1 because manager A's best response problem (after imposing the investors' rational expectations) is essentially a single firm problem with given m_B^* and q_B.

Second, we plug manager A's best response into manager B's first order condition and show that manager B's optimization has a unique solution as well. By substituting the best response $\tilde{m}_A^*\left(m_B^*\right)$ into $H^B(\tilde{m}_B^*; m_A^*) = 0$ and obtain

$$H^B(\tilde{m}_B^*; \tilde{m}_A^*\left(\tilde{m}_B^*\right)) = 0.$$

We show that this Equation has a unique solution when the cost functions are sufficiently convex. At $\tilde{m}_B^* = 0$, $H^B(0; \tilde{m}_A^*\left(0\right)) = \delta_B\left(1 - q_B\right)\left(1 + C_B^*\left(0\right)\right) > 0$. At $\tilde{m}_B^* = 1$, $H^B(1; \tilde{m}_A^*\left(1\right)) = \delta_B\left(1 - q_B\right)\left(1 + C_B^*\left(1\right)\right) E_{r_A}\left[\theta_B\left(1, r_A; \tilde{m}_B^* = 1, \tilde{m}_A^* = \tilde{m}_A^*\left(1\right)\right)|s_A = 0\right] - C_B'(1) = -\infty < 0$. In addition, we verify that $H^B(\tilde{m}_B^*; \tilde{m}_A^*\left(\tilde{m}_B^*\right))$ is strictly decreasing in $\tilde{m}_B^*$.

$$\frac{dH^B(\tilde{m}_B^*; \tilde{m}_A^*\left(\tilde{m}_B^*\right))}{d\tilde{m}_B^*}$$

$$= \quad \frac{\partial H^B(\tilde{m}_B^*; \tilde{m}_A^*)}{\partial \tilde{m}_B^*} + \frac{\partial H^B(\tilde{m}_B^*; \tilde{m}_A^*)}{\partial \tilde{m}_A^*} \frac{\partial \tilde{m}_A^*\left(\tilde{m}_B^*\right)}{\partial \tilde{m}_B^*}$$

$$= \quad \frac{\partial H^B(\tilde{m}_B^*; \tilde{m}_A^*)}{\partial \tilde{m}_B^*} - \frac{\partial H^B(\tilde{m}_B^*; \tilde{m}_A^*)}{\partial \tilde{m}_A^*} \frac{\frac{\partial H^A(\tilde{m}_A^*; \tilde{m}_B^*)}{\partial \tilde{m}_B^*}}{\frac{\partial H^A(\tilde{m}_A^*; \tilde{m}_B^*)}{\partial \tilde{m}_A^*}}$$

$$= \quad \frac{\frac{\partial H^A(\tilde{m}_A^*; \tilde{m}_B^*)}{\partial \tilde{m}_A^*} \frac{\partial H^B(\tilde{m}_B^*; \tilde{m}_A^*)}{\partial \tilde{m}_B^*} - \frac{\partial H^B(\tilde{m}_B^*; \tilde{m}_A^*)}{\partial \tilde{m}_A^*} \frac{\partial H^A(\tilde{m}_A^*; \tilde{m}_B^*)}{\partial \tilde{m}_B^*}}{\frac{\partial H^A(\tilde{m}_A^*; \tilde{m}_B^*)}{\partial \tilde{m}_A^*}},$$

where the second step is from $\frac{\partial \tilde{m}_A^*\left(\tilde{m}_B^*\right)}{\partial \tilde{m}_B^*} = -\frac{\frac{\partial H^A(\tilde{m}_A^*; \tilde{m}_B^*)}{\partial \tilde{m}_B^*}}{\frac{\partial H^A(\tilde{m}_A^*; \tilde{m}_B^*)}{\partial \tilde{m}_A^*}}$. When C_A and C_B are sufficiently convex, it is easy (but tedious) to verify that the numerator is positive (the Hessian matrix of the objective function is negative definitive). The denominator is negative from the first step. Thus, $\frac{dH^B(\tilde{m}_B^*; \tilde{m}_A^*\left(\tilde{m}_B^*\right))}{d\tilde{m}_B^*} < 0$. Therefore, $H^B(\tilde{m}_B^*; \tilde{m}_A^*\left(\tilde{m}_B^*\right))$ is decreasing in $\tilde{m}_B^*$, and by the intermediate value theorem, $H^B(\tilde{m}_B^*; \tilde{m}_A^*\left(\tilde{m}_B^*\right)) = 0$ has a unique solution $m_B^*\left(q_B, q_A\right)$. In addition, $m_A^*\left(q_A, q_B\right) = \tilde{m}_A^*\left(m_B^*\left(q_B, q_A\right)\right)$ is also unique. Now we can write the first order

condition of manager A as $H^A(m_A^*(q_A, q_B); m_B^*(q_B, q_A))$.

To derive $\frac{\partial m_A^*}{\partial q_A}$ and $\frac{\partial m_B^*}{\partial q_A}$, the application of the multivariate implicit function theorem generates

$$\frac{\partial H^A(m_A^*; m_B^*)}{\partial m_A^*}\frac{\partial m_A^*}{\partial q_A} + \frac{\partial H^A(m_A^*; m_B^*)}{\partial q_A} + \frac{\partial H^A(m_A^*; m_B^*)}{\partial \mu_B^*}\frac{\partial \mu_B^*}{\partial m_B^*}\frac{\partial m_B^*}{\partial q_A} = 0,$$

$$\frac{\partial H^B(m_B^*; m_A^*)}{\partial m_B^*}\frac{\partial m_B^*}{\partial q_A} + \frac{\partial H^B(m_B^*; m_A^*)}{\partial \mu_A^*}\left(\frac{\partial \mu_A^*}{\partial m_A^*}\frac{\partial m_A^*}{\partial q_A} + \frac{\partial \mu_A^*}{\partial q_A}\right) = 0,$$

which can be simplified into

$$\frac{\partial H^A(m_A^*; m_B^*)}{\partial m_A^*}\frac{\partial m_A^*}{\partial q_A} + \frac{\partial H^A(m_A^*; m_B^*)}{\partial q_A} + (1 - q_B)\frac{\partial H^A(m_A^*; m_B^*)}{\partial \mu_B^*}\frac{\partial m_B^*}{\partial q_A} = 0,$$

$$\frac{\partial H^B(m_B^*; m_A^*)}{\partial m_B^*}\frac{\partial m_B^*}{\partial q_A} + \frac{\partial H^B(m_B^*; m_A^*)}{\partial \mu_A^*}\left((1 - q_A)\frac{\partial m_A^*}{\partial q_A} - m_A^*\right) = 0.$$

Solving the two equations gives

$$\frac{\partial m_A^*}{\partial q_A} = \frac{-\left[\frac{\partial H^A(m_A^*; m_B^*)}{\partial q_A}\frac{\partial H^B(m_B^*; m_A^*)}{\partial m_B^*} + (1 - q_B)\frac{\partial H^A(m_A^*; m_B^*)}{\partial \mu_B^*}\frac{\partial H^B(m_B^*; m_A^*)}{\partial \mu_A^*}\right]}{\frac{\partial H^A(m_A^*; m_B^*)}{\partial m_A^*}\frac{\partial H^B(m_B^*; m_A^*)}{\partial m_B^*} - (1 - q_B)(1 - q_A)\frac{\partial H^A(m_A^*; m_B^*)}{\partial \mu_B^*}\frac{\partial H^B(m_B^*; m_A^*)}{\partial \mu_A^*}},$$

$$\frac{\partial m_B^*}{\partial q_A} = \frac{\frac{\partial H^B(m_B^*; m_A^*)}{\partial \mu_A^*}\left(\frac{\partial H^A(m_A^*; m_B^*)}{\partial q_A}(1 - q_A) + \frac{\partial H^A(m_A^*; m_B^*)}{\partial m_A^*}m_A^*\right)}{\frac{\partial H^A(m_A^*; m_B^*)}{\partial m_A^*}\frac{\partial H^B(m_B^*; m_A^*)}{\partial m_B^*} - (1 - q_B)(1 - q_A)\frac{\partial H^A(m_A^*; m_B^*)}{\partial \mu_B^*}\frac{\partial H^B(m_B^*; m_A^*)}{\partial \mu_A^*}}.$$

We have shown that in the unique equilibrium, the denominator is positive. Hence the signs of $\frac{\partial m_A^*}{\partial q_A}$ and $\frac{\partial m_A^*}{\partial q_B}$ are determined by their numerators, respectively. First,

$$\frac{\partial m_A^*}{\partial q_A} \propto -\left[\frac{\partial H^A(m_A^*; m_B^*)}{\partial q_A}\frac{\partial H^B(m_B^*; m_A^*)}{\partial m_B^*} + (1 - q_B)\frac{\partial H^A(m_A^*; m_B^*)}{\partial \mu_B^*}\frac{\partial H^B(m_B^*; m_A^*)}{\partial \mu_A^*}\right].$$

From a proof similar to that of Lemma 1, we have $\frac{\partial H^A(m_A^*; m_B^*)}{\partial q_A} < 0$ and $\frac{\partial H^B(m_B^*; m_A^*)}{\partial m_B^*} < 0$. Proposition 1 shows $\frac{\partial H^A(m_A^*; m_B^*)}{\partial \mu_B^*} > 0$ and $\frac{\partial H^B(m_B^*; m_A^*)}{\partial \mu_A^*} > 0$. As a result, $\frac{\partial m_A^*}{\partial q_A} < 0$.

Similarly,

$$\frac{\partial m_B^*}{\partial q_A} \propto \frac{\partial H^B(m_B^*; m_A^*)}{\partial \mu_A^*}\left(\frac{\partial H^A(m_A^*; m_B^*)}{\partial q_A}(1 - q_A) + \frac{\partial H^A(m_A^*; m_B^*)}{\partial m_A^*}m_A^*\right),$$

where $\frac{\partial H^B(m_B^*; m_A^*)}{\partial \mu_A^*} > 0$, $\frac{\partial H^A(m_A^*; m_B^*)}{\partial q_A} < 0$, $\frac{\partial H^A(m_A^*; m_B^*)}{\partial m_A^*} < 0$. As a result, $\frac{\partial m_B^*}{\partial q_A} < 0$. ∎

Proof. of Proposition 3: The proof of the uniqueness of the internal control equilibrium is similar to that of the manipulation choice in Proposition 2. In short, when $K_A(q_A)$ is sufficiently convex, the LHS of the first order condition of q_A, $\frac{\partial V_{A0}}{\partial m_A^*}\frac{\partial m_A^*}{\partial q_A} - K_A'(q_A)$, is decreasing in q_A, making the best response $\tilde{q}_A^*(q_B^*)$ unique. Substituting $\tilde{q}_A^*(q_B^*)$ into manager B's best

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response gives

$$\frac{\partial V_{B0}}{\partial m_B^*}\frac{\partial m_B^*}{\partial \tilde{q}_B}\Big|_{q_A^*=\tilde{q}_A^*(q_B^*)} - K_B'(\tilde{q}_B) = 0.$$

When $K_B(q_B)$ is sufficiently convex, the LHS $\frac{\partial V_{B0}}{\partial m_B^*}\frac{\partial m_B^*}{\partial \tilde{q}_B}\Big|_{q_A^*=\tilde{q}_A^*(q_B^*)} - K_B'(\tilde{q}_B)$ is decreasing in $\tilde{q}_B$, making the solution of $q_B^* = \tilde{q}_B^*(q_B^*)$ unique. As a result, the equilibrium decisions $q_A^* = \tilde{q}_A^*(q_B^*)$, $m_A^*(q_A^*, q_B^*)$, and $m_B^*(q_B^*, q_A^*)$ are also unique. $\blacksquare$

Proof. of Proposition 4: We show that there exists a pair of internal control levels $\{q_A', q_B'\}$ with $q_A' > q_A^*$ and $q_B' > q_B^*$ such that $V_{A0}(q_A', q_B') > V_{A0}(q_A^*, q_B^*)$ and $V_{B0}(q_B', q_A') > V_{B0}(q_B^*, q_A^*)$. To see this, set $q_A' = q_A^* + \varepsilon$ and $q_B' = q_B^* + \varepsilon$, where $\varepsilon > 0$ is an arbitrarily small positive number. By a first order Taylor expansion,

$$\begin{aligned} V_{A0}(q_A', q_B') &= V_{A0}(q_A^*, q_B^*) + \frac{\partial V_{A0}(q_A^*, q_B^*)}{\partial q_A}(q_A' - q_A^*) + \frac{\partial V_{A0}(q_A^*, q_B^*)}{\partial q_B}(q_B' - q_B^*) \\ &= V_{A0}(q_A^*, q_B^*) + \frac{\partial V_{A0}(q_A^*, q_B^*)}{\partial q_B}\varepsilon. \end{aligned}$$

The second step is by the first order condition of q_A^*, i.e., $\frac{\partial V_{A0}(q_A^*, q_B^*)}{\partial q_A} = 0$. Recall that

$$\frac{\partial V_{A0}(q_A^*, q_B^*)}{\partial q_B} = \frac{\partial V_{A0}(q_A^*, q_B^*)}{\partial m_A^*}\frac{\partial m_A^*(q_A^*, q_B^*)}{\partial q_B} > 0.$$

The inequality is because $\frac{\partial V_{A0}(q_A^*, q_B^*)}{\partial m_A^*} = -\Pr(s_A = 0)C_A'(m_A^*) < 0$ and $\frac{\partial m_A^*(q_A^*, q_B^*)}{\partial q_B} < 0$ from Proposition 2. As a result, $\frac{\partial V_{A0}(q_A^*, q_B^*)}{\partial q_B}\varepsilon > 0$ and $V_{A0}(q_A', q_B') > V_{A0}(q_A^*, q_B^*)$. The proof for $V_{B0}(q_B', q_A') > V_{B0}(q_B^*, q_A^*)$ is similar. $\blacksquare$

Proof. of Proposition 5: We first characterize the managers' equilibrium manipulation decisions $\{m_A^*(q_A, q_B), m_B^*(q_B, q_A)\}$ given the two firms' internal control (q_A, q_B). We focus on manager A's manipulation decisions first. Upon observing both reports and given investors' conjecture of manipulation $(\hat{m}_A, \hat{m}_B)$, investors set a price for firm A,

$$P_A^*(r_A, r_B) = E[V_A|r_A, r_B, \hat{m}_A, \hat{m}_B] = E_{s_A}[s_A|r_A, r_B, \hat{m}_A, \hat{m}_B] - C_A(\hat{m}_A) - K_A(q_A).$$

We conjecture an equilibrium in which investors' conjecture $(\hat{m}_A, \hat{m}_B)$ is independent of the profitability (s_A, s_B) and verify this conjecture when we derive the equilibrium.

To characterize the price, we first derive $E_{s_A}[s_A|r_A, r_B, \hat{m}_A, \hat{m}_B]$. As in our baseline model, we decompose the derivation of $E_{s_A}[s_A|r_A, \hat{m}_A, r_B, \hat{m}_B]$ into two steps. First, we use $(r_B, \hat{m}_B)$ to obtain investors' posterior belief about s_A. Second, we treat this posterior as the prior about s_A and then use report r_A to update. Specifically, upon observing r_B and $\hat{m}_B$ the triplets (r_B, s_A, s_B) are jointly normally distributed. Using properties of multivariate normal distributions, the distribution of s_A conditional on report r_B and $\hat{m}_B$ is normal with mean

and variance:

$$E_{s_A}[s_A|r_B, \hat{m}_B] = \bar{s} + \beta_A(\hat{m}_B)(r_B - \hat{m}_B(1-q_B) - \bar{s}), \text{ with } \beta_A(\hat{m}_B) = \frac{\rho \sigma_s^2}{\sigma_s^2 + [h(\hat{m}_B)]^2},$$

$$Var[s_A|r_B, \hat{m}_B] = \sigma_s^2 - \frac{\rho^2 \sigma_s^4}{\sigma_s^2 + [h(\hat{m}_B)]^2}.$$

Treating this conditional distribution as the prior of s_A, investors use $(r_A, \hat{m}_A)$ to update their beliefs about s_A. We have

$$E_{s_A}[s_A|r_A, \hat{m}_A, r_B, \hat{m}_B] = \beta_A(\hat{m}_A, \hat{m}_B)(r_A - \hat{m}_A(1-q_A)) \quad (14)$$
$$+ (1 - \beta_A(\hat{m}_A, \hat{m}_B))E_{s_A}[s_A|r_B, \hat{m}_B],$$
$$\text{with } \beta_A(\hat{m}_A, \hat{m}_B) = \frac{Var[s_A|r_B, \hat{m}_B]}{Var[s_A|r_B, \hat{m}_B] + [h(\hat{m}_A)]^2}.$$

$\beta_A(\hat{m}_A, \hat{m}_B)$ is the response coefficient of firm A's price to report r_A conditional on investors' conjecture of both managers' manipulation choices $\hat{m}_A$ and $\hat{m}_B$.

Given the price $P_A^*(r_A, r_B)$, the payoff function of manager A is

$$U_A = \delta_A P_A^*(r_A, r_B) + (1 - \delta_A)V_A, \quad (15)$$

where the firm value

$$V_A = s_A - C_A(m_A) - K_A(q_A).$$

The first order condition on the manager's choice m_A is given by

$$\frac{\partial U_A}{\partial m_A} = \delta_A \beta_A(\hat{m}_A, \hat{m}_B)(1 - q_A) - (1 - \delta_A)C_A'(m_A) = 0.$$

m_A affects the price $P_A^*(r_A, r_B)$ only through affecting the report r_A and affects V_A only through affecting the manipulation cost $C_A(m_A)$. Also notice that from the first order condition, the optimal m_A depends on neither s_A or s_B, thus confirming our conjecture on the equilibrium in the beginning. Imposing the rational expectations conditions that $m_A^* = \hat{m}_A$ and $m_B^* = \hat{m}_B$ and defining $H^A(m_A, m_B) = \frac{\partial U_A}{\partial m_A}$, manager A's optimal response $\tilde{m}_A^*(m_B^*)$ is a solution to a fixed point problem:

$$H^A(m_A, m_B^*) = \delta_A \beta_A(m_A^*, m_B^*)(1 - q_A) - (1 - \delta_A)C_A'(m_A) = 0. \quad (16)$$

Equation 16 shows that the larger the impact of r_A on the price P_A^* (a larger $\beta_A(m_A^*, m_B^*)$), the larger the benefit of manipulation for manager A and the more he manipulates. In addition, since $\frac{\partial H^A}{\partial q_A} < 0$, $\frac{\partial m_A^*}{\partial q_A} < 0$.

Similarly, we can solve for the manager B's best response $\tilde{m}_B^*(m_A^*)$. In particular, $\tilde{m}_B^*(m_A^*)$ is a solution to the the following fixed point problem:

$$H^B(m_B, m_A^*) = \delta_B \beta_B(m_B^*, m_A^*)(1 - q_B) - (1 - \delta_B)C_B'(m_B) = 0. \quad (17)$$

The equilibrium manipulation choices (m_A^*, m_B^*) are the joint solutions to Equation 16 and 17.

We now prove the existence and uniqueness of the equilibrium. To prove the exis-

tence of the best response $\tilde{m}_A^*(m_B^*)$ from $H^A(\tilde{m}_A^*, m_B^*) = 0$, consider an $m_A \in [0, +\infty)$. $\lim_{m_A \to 0} H^A(m_A, m_B^*) = \delta_A \beta_A(0, m_B^*)(1 - q_A) > 0$ and $\lim_{m_A \to \infty} H^A(m_A) = \delta_A \beta_A(\infty, m_B^*)(1 - q_A) - \infty < 0$ since $\beta_A(\infty, m_B^*) < 1$ is finite. By the intermediate value theorem, $m_A^*(m_B^*)$ always exists. In addition, $\tilde{m}_A^*(m_B^*)$ is also unique if C_A' is sufficiently large and $H_A^A < 0$. For the other best response $\tilde{m}_B^*(m_A^*)$, following similar steps, we can also verify its existence and uniqueness given C_B' is sufficiently large and $H_B^B < 0$.

Now we prove the existence of the equilibrium $\{m_A^*(q_A, q_B), m_B^*(q_B, q_A)\}$. Substituting the best response $\tilde{m}_B^*(m_A^*)$ into $H^A = 0$ gives

$$H^A(m_A^*, \tilde{m}_B^*(m_A^*)) = 0.$$

At $m_A = 0$, $\tilde{m}_B^*(0) > 0$ and is finite, and $\lim_{m_A^* \to 0} H^A(m_A^*, \tilde{m}_B^*(m_A^*)) > 0$; at $m_A = \infty$, $\tilde{m}_B^*(\infty) > 0$ and is finite, $\delta_A \beta_A(\infty, \tilde{m}_B^*(\infty))(1 - q_A)$ is finite and smaller than ∞, i.e., $\lim_{m_A^* \to \infty} H^A(m_A^*, \tilde{m}_B^*(m_A^*)) < 0$. By the intermediate value theorem, an equilibrium always exists. In addition, m_A^* is also unique if $\frac{dH^A}{dm_A^*} = H_A^A + H_B^A \frac{\partial m_B^*}{\partial m_A^*} < 0$. This satisfies if C_A' is sufficiently large.

With the unique equilibrium characterized, we turn to the strategic relation between two managers' manipulation decisions. We focus on analyzing the effect of m_B^* on $\tilde{m}_A^*(m_B^*)$ and the other case is similar. From $H^A(\tilde{m}_A^*(m_B^*), m_B^*) = 0$ and by the implicit function theorem, we have $H_A^A \frac{\partial \tilde{m}_A^*}{\partial m_B^*} + H_B^A = 0$, which gives $\frac{\partial \tilde{m}_A^*}{\partial m_B^*} = -\frac{H_B^A}{H_A^A}$. Recall that for C_A' is sufficiently large, H^A is decreasing in $\tilde{m}_A^*$ in the unique equilibrium, i.e., $H_A^A \leq 0$, the sign of $\frac{\partial \tilde{m}_A^*}{\partial m_B^*}$ is the same as H_B^A, i.e.,

$$\frac{\partial \tilde{m}_A^*}{\partial m_B^*} \propto H_B^A = \delta_A(1 - q_A) \frac{\partial \beta_A(m_A^*, m_B^*)}{\partial m_B^*}.$$

That is, the sign of $\frac{\partial \tilde{m}_A^*}{\partial m_B^*}$ is determined by how m_B^* affects $\beta_A(m_A^*, m_B^*)$.

Consider first the case of $\rho = 0$. Since the two firms' profitability are uncorrelated, $Var[s_A | r_B, m_B^*] = \sigma_s^2$, which makes $\beta_A(m_A^*, m_B^*) = \frac{Var[s_A | r_B, m_B^*]}{Var[s_A | r_B, m_B^*] + [h(m_A^*)]^2} = \frac{\sigma_s^2}{\sigma_s^2 + [h(m_A^*)]^2}$. Therefore, $\beta_A(m_A^*, m_B^*)$ is independent of m_B^* and $H_B^A = 0$. That is, the two firms' manipulation choices are strategically independent.

Second, consider $\rho \neq 0$ and $h' = 0$, i.e., manipulation does not affect the informativeness of the report. We have the conditional variance $Var[s_A | r_B, m_B^*] = \sigma_s^2 - \frac{\rho^2 \sigma_s^4}{\sigma_s^2 + [h(m_B^*)]^2}$ independent of m_B^* and thus so is $\beta_A(m_A^*, m_B^*) = \frac{Var[s_A | r_B, m_B^*]}{Var[s_A | r_B, m_B^*] + [h(m_A^*)]^2}$. As a result, $\tilde{m}_A^*$ is independent of m_B^*. In sum, for $\rho = 0$ or $h' = 0$, internal control produces no externality with $\frac{\partial m_B^*}{\partial q_A} = 0$.

Lastly, consider $\rho \neq 0$ and $h' > 0$, i.e., manipulation reduces the informativeness of the report. We have the conditional variance $Var[s_A | r_B, m_B^*] = \sigma_s^2 - \frac{\rho^2 \sigma_s^4}{\sigma_s^2 + [h(m_B^*)]^2}$ increases in m_B^* since $h' > 0$ and thus so is $\beta_A(m_A^*, m_B^*) = \frac{Var[s_A | r_B, m_B^*]}{Var[s_A | r_B, m_B^*] + [h(m_A^*)]^2}$. As a result, $\tilde{m}_A^*$ is increasing in m_B^*. In addition, since $\frac{\partial m_A^*}{\partial q_A} < 0$, the strategic complementarity between the manipulation decisions implies that $\frac{\partial m_B^*}{\partial q_A} < 0$.

We now derive firms' internal control decisions. As in the baseline model, the firm's ex

ante payoff V_{i0} can be reduced into

$$V_{i0} = \bar{s} - C_i\left(m_i^*\right) - K_i\left(q_i\right).$$

Firm A at $t = 0$ chooses q_A to maximizes V_{A0}. Differentiating V_{A0} with respect to q_A, the first order condition is

$$\frac{\partial V_{A0}}{\partial m_A^*}\frac{\partial m_A^*}{\partial q_A} - K_A'(q_A) = -C_A'\left(m_A^*\right)\frac{\partial m_A^*}{\partial q_A} - K_A'\left(q_A\right) = 0,$$

which is similar to the one in the baseline model. Thus following similar proofs in Proposition 3, we show that there exists a unique pair of optimal private choices of internal control $\{q_A^*, q_B^*\}$ by the firms that solves the first order conditions.

If $h' > 0$ and $\rho \neq 0$, then given the positive externality result $\frac{\partial m_B^*(q_B, q_A)}{\partial q_A} < 0$, the proof for the existence of Pareto improvements is identical to the one in Proposition 4.

If $h' = 0$ or $\rho = 0$, $\frac{\partial m_B^*(q_B, q_A)}{\partial q_A} = 0$. That is, improving q_A produces no benefit to firm B as it affects neither m_B^* nor $C_B\left(m_B^*\right)$. As a result, the privately optimal choice q_A^* by firm A is also socially optimal and thus Pareto efficient. ∎

Proof. of Proposition 6: We have already derived the conditions for the strategic complementarity in Proposition 5. Therefore, we focus on examining the sufficient and necessary condition for the spillover effect. We find that the spillover effect exists if and only if the two firms' profitability are correlated, $\rho \neq 0$. To see this, we first compute a single manipulation A's manipulation decision. Following similar analyses in the two-firm case, one can verify that the manager A's equilibrium decision m_A^* is a solution to a fixed point problem:

$$H^A(m_A) = \delta_A \beta_A(m_A^*)(1 - q_A) - (1 - \delta_A)C_A'\left(m_A\right) = 0.$$

$\beta_A(m_A^*)$ is the response coefficient of firm A's price to report r_A given only firm A's report. It is straightforward to show that

$$\beta_A(m_A^*) = \frac{\sigma_s^2}{\sigma_s^2 + \left[h(m_A^*)\right]^2}.$$

To analyze the spillover effect, consider first the case $\rho = 0$, i.e., the two firms' profitability are independent. At $\rho = 0$, it is straightforward to verify that

$$Var[s_A | r_B, m_B^*] = \sigma_s^2 - \frac{\rho^2 \sigma_s^4}{\sigma_s^2 + \left[h(m_B^*)\right]^2} = \sigma_s^2,$$

that is, the additional report of firm B, r_B, provides no information about firm A's profitability s_A and thus does not affect investors' conditional variance about s_A. In addition,

$$\begin{aligned}
\beta_A(m_A^*, m_B^*) &= \frac{Var[s_A | r_B, m_B^*]}{Var[s_A | r_B, m_B^*] + \left[h(m_A^*)\right]^2} \\
&= \frac{\sigma_s^2}{\sigma_s^2 + \left[h(m_A^*)\right]^2},
\end{aligned}$$

that is, $\beta_A(m_A^*, m_B^*) = \beta_A(m_A^*)$. As a result, the first order condition in the two-firm case, Equation 16, reduces into the one in the single-firm case. The presence of firm B does not affect the equilibrium m_A^* and the spillover effect does not exist.

With $\rho \neq 0$, however, we have

$$Var[s_A | r_B, m_B^*] = \sigma_s^2 - \frac{\rho^2 \sigma_s^4}{\sigma_s^2 + \left[h(m_B^*)\right]^2} < \sigma_s^2,$$

that is, the additional report of firm B, r_B, provides information about firm A's profitability s_A and thus reduces investors' conditional variance about s_A compared with investors' prior variance σ_s^2. In addition,

$$
\begin{aligned}
\beta_A(m_A^*, m_B^*) &= \frac{Var[s_A | r_B, m_B^*]}{Var[s_A | r_B, m_B^*] + \left[h(m_A^*)\right]^2} \\
&< \frac{\sigma_s^2}{\sigma_s^2 + \left[h(m_A^*)\right]^2},
\end{aligned}
$$

that is, with the presence of firm B, firm A's report has a smaller impact on the price P_A^* ($\beta_A(m_A^*, m_B^*) < \beta_A(m_A^*)$) as firm A contains less incremental information. As a result, manager A's incentive to inflate r_A is weakened and m_A^* decreases with the presence of firm B.

Overall, the sufficient and necessary condition for the spillover effect is $\rho \neq 0$. ∎

Proof. of Proposition 7: It is straightforward to verify that the first order condition is the same as in the baseline model:

$$H^A(\tilde{m}_A^*; \mu_B^*) \equiv \delta_A E_{r_B}[\theta_A(1, r_B) | s_A = 0](1 + C_A(\tilde{m}_A^*)) - (1 - \delta_A) C_A'(\tilde{m}_A^*) = 0.$$

Plugging in the functional form of $C_A(m_A) = \frac{(q_A + c_0)}{2} m_A^2$ gives

$$H^A(\tilde{m}_A^*; \mu_B^*) = \delta_A E_{r_B}[\theta_A(1, r_B) | s_A = 0](1 + \frac{(q_A + c_0)}{2}(\tilde{m}_A^*)^2) - (1 - \delta_A)(q_A + c_0)\tilde{m}_A^* = 0.$$

Following similar proofs in the baseline model, we verify that for c_0 sufficiently large, there exists a unique best response of $\tilde{m}_A^*(\mu_B^*)$. In addition, since $E_{r_B}[\theta_A(1, r_B) | s_A = 0]$ is strictly increasing in μ_B^* (Proposition 1), $\tilde{m}_A^*(\mu_B^*)$ is also strictly increasing in μ_B^*.

Combining $H^A(\tilde{m}_A^*; \mu_B^*) = 0$ with its counter part for manager B, $H^B(\tilde{m}_B^*; \mu_A^*) = 0$, characterizes the managers' manipulation decisions $\{m_A^*(q_A, q_B), m_B^*(q_B, q_A)\}$. The proof for the existence and uniqueness is similar to that in the baseline model.

In addition, as shown in Proposition 2,

$$\frac{\partial m_A^*}{\partial q_A} = -\frac{\frac{\partial H^B(m_B^*; \mu_A^*)}{\partial m_B^*} \frac{\partial H^A(m_A^*; \mu_B^*)}{\partial q_A}}{\frac{\partial H^A(m_A^*; \mu_B^*)}{\partial m_A^*} \frac{\partial H^B(m_B^*; \mu_A^*)}{\partial m_B^*} - \frac{\partial H^A(m_A^*; \mu_B^*)}{\partial m_B^*} \frac{\partial H^B(m_B^*; \mu_A^*)}{\partial m_A^*}}.$$

The dominator is positive and $\frac{\partial H^B(m_B^*; \mu_A^*)}{\partial m_B^*} < 0$ (Proposition 2). Thus the sign of $\frac{\partial m_A^*}{\partial q_A}$ is

determined by $\frac{\partial H^A}{\partial q_A}$ which is given by

$$
\begin{aligned}
\frac{\partial H^A}{\partial q_A} &= \delta_A E_{r_B}[\theta_A(1, r_B)|s_A = 0]\frac{1}{2}(\tilde{m}_A^*)^2 - (1 - \delta_A)\tilde{m}_A^* \\
&= -\frac{\delta_A E_{r_B}[\theta_A(1, r_B)|s_A = 0]}{q_A + c_0} < 0.
\end{aligned}
$$

The second equality uses the first order condition $H^A = 0$. Thus $\frac{\partial m_A^*}{\partial q_A} < 0$. Because of the strategic complementarity between manipulation decisions, one can verify $\frac{\partial m_B^*}{\partial q_A} < 0$ following similar steps in Proposition 2. Thus our main result of positive externality of internal control is indeed robust to this alternative modelling of internal control.

We now discuss the firms' optimal internal control choices. The firm value at $t = 0$ is

$$
V_{A0}(q_A; q_B^*) \equiv E_0[V_A(q_A, q_B^*)] = \theta_A - \Pr(s_A = 0)C_A(m_A^*(q_A, q_B^*), q_A) - K_A(q_A).
$$

Differentiating $V_{A0}(q_A, q_B^*)$ with respect to q_A gives

$$
\frac{\partial V_{A0}}{\partial m_A^*}\frac{\partial m_A^*}{\partial q_A} + \frac{\partial V_{A0}}{\partial q_A} = -\Pr(s_A = 0)\left(\frac{\partial C_A}{\partial m_A^*}\frac{\partial m_A^*}{\partial q_A} + \frac{\partial C_A}{\partial q_A}\right) - K_A'(q_A) = 0.
$$

The first term $\frac{\partial V_{A0}}{\partial m_A^*}\frac{\partial m_A^*}{\partial q_A}$ captures the benefit of internal control q_A in reducing manipulation. It is balanced by two costs of internal control, a direct cost $K_A'(q_A)$ and an indirect cost in increasing the manipulation cost, $\frac{\partial V_{A0}}{\partial q_A} = -\Pr(s_A = 0)\frac{\partial C_A}{\partial q_A} < 0$. Define

$$
\begin{aligned}
G^A(q_A, q_B) &\equiv -\frac{\partial C_A}{\partial m_A^*}\frac{\partial m_A^*}{\partial q_A} - \frac{\partial C_A}{\partial q_A} - K_A'(q_A) \\
&= -(q_A + c_0)m_A^*\frac{\partial m_A^*}{\partial q_A} - \frac{(m_A^*)^2}{2} - K_A'(q_A).
\end{aligned}
$$

At $q_A = \infty$, since $\lim_{q_A \to \infty} K_A'(q_A) = \infty$, $\lim_{q_A \to \infty} G^A(q_A, q_B) < 0$. At $q_A = 0$,

$$
G^A(0, q_B) = -c_0 m_A^*\frac{\partial m_A^*}{\partial q_A} - \frac{(m_A^*)^2}{2}.
$$

We now verify that for c_0 sufficiently large, $G^A(0, q_B) > 0$. First, by the implicit function theorem,

$$
\begin{aligned}
\frac{\partial m_A^*}{\partial q_A} &= -\frac{\frac{\partial H^B(m_B^*;\mu_A^*)}{\partial m_B^*}\frac{\partial H^A(m_A^*;\mu_B^*)}{\partial q_A}}{\frac{\partial H^A(m_A^*;\mu_B^*)}{\partial m_A^*}\frac{\partial H^B(m_B^*;\mu_A^*)}{\partial m_B^*} - \frac{\partial H^A(m_A^*;\mu_B^*)}{\partial m_B^*}\frac{\partial H^B(m_B^*;\mu_A^*)}{\partial m_A^*}} \\
&< -\frac{\frac{\partial H^B(m_B^*;\mu_A^*)}{\partial m_B^*}\frac{\partial H^A(m_A^*;\mu_B^*)}{\partial q_A}}{\frac{\partial H^A(m_A^*;\mu_B^*)}{\partial m_A^*}\frac{\partial H^B(m_B^*;\mu_A^*)}{\partial m_B^*}} \\
&= -\frac{\frac{\partial H^A(m_A^*;\mu_B^*)}{\partial q_A}}{\frac{\partial H^A(m_A^*;\mu_B^*)}{\partial m_A^*}}.
\end{aligned}
$$

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The second inequality is by the strategic complementarity between m_A^* and m_B^*, $\frac{\partial H^A(m_A^*;\mu_B^*)}{\partial m_B^*}$, $\frac{\partial H^B(m_B^*;\mu_A^*)}{\partial m_A^*} > 0$ and thus $\frac{\partial H^A(m_A^*;\mu_B^*)}{\partial m_B^*}\frac{\partial H^B(m_B^*;\mu_A^*)}{\partial m_A^*} > 0$. In addition, recall that $\frac{\partial H^A(m_A^*;\mu_B^*)}{\partial q_A}$, $\frac{\partial H^B(m_B^*;\mu_A^*)}{\partial m_B^*} < 0$, thus $\frac{\partial m_A^*}{\partial q_A} < -\frac{\frac{\partial H^B(m_B^*;\mu_A^*)}{\partial m_B^*}\frac{\partial H^A(m_A^*;\mu_B^*)}{\partial q_A}}{\frac{\partial H^A(m_A^*;\mu_B^*)}{\partial m_A^*}\frac{\partial H^B(m_B^*;\mu_A^*)}{\partial m_B^*}}$. Plugging this inequality into $G^A(0,q_B)$ gives

$$G^A(0,q_B) > c_0 m_A^* \frac{\frac{\partial H^A(m_A^*;\mu_B^*)}{\partial q_A}}{\frac{\partial H^A(m_A^*;\mu_B^*)}{\partial m_A^*}} - \frac{(m_A^*)^2}{2}. \tag{18}$$

Plugging in the expressions of $\frac{\partial H^A(m_A^*;\mu_B^*)}{\partial m_A^*}|_{q_A=0} = \delta_A \frac{\partial E_{r_B}[\theta_A(1,r_B)|s_A=0]}{\partial m_A^*}(1+\frac{c_0}{2}(m_A^*)^2) + \delta_A E_{r_B}[\theta_A(1,r_B)|s_A=0]c_0 m_A^* - (1-\delta_A)c_0$ and $\frac{\partial H^A(m_A^*;\mu_B^*)}{\partial q_A}|_{q_A=0} = \delta_A E_{r_B}[\theta_A(1,r_B)|s_A=0]\frac{1}{2}(m_A^*)^2 - (1-\delta_A)m_A^*$, the RHS of (18) can be rewritten as:

$$c_0 m_A^* \frac{\delta_A E_{r_B}[\theta_A(1,r_B)|s_A=0]\frac{1}{2}(m_A^*)^2 - (1-\delta_A)m_A^*}{\delta_A \frac{\partial E_{r_B}[\theta_A(1,r_B)|s_A=0]}{\partial m_A^*}(1+\frac{c_0}{2}(m_A^*)^2)+\delta_A E_{r_B}[\theta_A(1,r_B)|s_A=0]c_0 m_A^*-(1-\delta_A)c_0} - \frac{(m_A^*)^2}{2}$$

$$= (m_A^*)^2\left\{\frac{(1-\delta_A)c_0 - \delta_A E_{r_B}[\theta_A(1,r_B)|s_A=0]\frac{c_0 m_A^*}{2}}{(1-\delta_A)c_0 - \delta_A E_{r_B}[\theta_A(1,r_B)|s_A=0]c_0 m_A^* - \delta_A \frac{\partial E_{r_B}[\theta_A(1,r_B)|s_A=0]}{\partial m_A^*}(1+\frac{c_0}{2}(m_A^*)^2)} - \frac{1}{2}\right\}.$$

To have $G^A(0,q_B) > 0$, a sufficient condition is then to have the RHS of (18) positive, i.e.,

$$\frac{(1-\delta_A)c_0 - \delta_A E_{r_B}[\theta_A(1,r_B)|s_A=0]\frac{c_0 m_A^*}{2}}{(1-\delta_A)c_0 - \delta_A E_{r_B}[\theta_A(1,r_B)|s_A=0]c_0 m_A^* - \delta_A \frac{\partial E_{r_B}[\theta_A(1,r_B)|s_A=0]}{\partial m_A^*}(1+\frac{c_0}{2}(m_A^*)^2)} > \frac{1}{2}.$$

One can simplify this inequality into

$$(1-\delta_A)c_0 + \delta_A \frac{\partial E_{r_B}[\theta_A(1,r_B)|s_A=0]}{\partial m_A^*}(1+\frac{c_0}{2}(m_A^*)^2) > 0. \tag{19}$$

From $H^A(m_A^*;\mu_B^*)|_{q_A=0} = 0$, $\delta_A(1+\frac{c_0}{2}(\tilde{m}_A^*)^2) = \frac{(1-\delta_A)c_0\tilde{m}_A^*}{E_{r_B}[\theta_A(1,r_B)|s_A=0]}$. Plugging this into (19), one can reduce (19) into:

$$\frac{\partial E_{r_B}[\theta_A(1,r_B)|s_A=0]}{\partial m_A^*}\frac{m_A^*}{E_{r_B}[\theta_A(1,r_B)|s_A=0]} > -1. \tag{20}$$

In addition,

$$E_{r_B}[\theta_A(1,r_B)|s_A=0] = E_{r_B}[\frac{\theta_A(\phi,r_B)}{\theta_A(\phi,r_B)+(1-\theta_A(\phi,r_B))m_A^*}|s_A=0],$$

$$\frac{\partial E_{r_B}[\theta_A(1,r_B)|s_A=0]}{\partial m_A^*} = -E_{r_B}[\frac{\theta_A(\phi,r_B)(1-\theta_A(\phi,r_B))}{[\theta_A(\phi,r_B)+(1-\theta_A(\phi,r_B))m_A^*]^2}|s_A=0].$$

$\theta_A(\phi,r_B) \equiv \Pr(s_A=1|r_B)$ is the conditional probability of $s_A=1$ given only r_B. Plugging

the expressions of $E_{r_B}[\theta_A(1,r_B)|s_A = 0]$ and $\frac{\partial E_{r_B}[\theta_A(1,r_B)|s_A=0]}{\partial m_A^*}$ into (20), one can simplify (20) into

$$\frac{E_{r_B}\left[\frac{\theta_A(\phi,r_B)(1-\theta_A(\phi,r_B))}{\left[\theta_A(\phi,r_B)+(1-\theta_A(\phi,r_B))m_A^*\right]^2}|s_A = 0\right]}{E_{r_B}\left[\frac{\theta_A(\phi,r_B)}{\theta_A(\phi,r_B)+(1-\theta_A(\phi,r_B))m_A^*}|s_A = 0\right]}m_A^* < 1.$$

One can verify that for c_0 sufficiently large (the cost of manipulation sufficiently high), m_A^* goes to zero. Therefore, for c_0 sufficiently large, the LHS of the above inequality becomes close to 0 and the inequality is satisfied. That is, we have shown that $G^A(0, q_B) > 0$ for c_0 sufficiently large. Combined with $\lim_{q_A \to \infty} G^A(\infty, q_B) < 0$, there exists a best response $\tilde{q}_A^*(q_B^*)$ that solves $G^A(\tilde{q}_A^*, q_B) = 0$.

Lastly, for K_A sufficiently convex, $-\Pr(s_A = 0)\left(\frac{\partial C_A}{\partial m_A^*}\frac{\partial m_A^*}{\partial q_A} + \frac{\partial C_A}{\partial q_A}\right) - K_A'(q_A)$ is decreasing in q_A, making the best response $\tilde{q}_A^*(q_B^*)$ unique. Plugging $q_A = \tilde{q}_A^*(q_B^*)$ into the first order condition of q_B, $G^B(q_B, q_A) = 0$, one can then prove the uniqueness of the equilibrium $\{q_A^*, q_B^*\}$ following similar steps as above.

Given the existence and uniqueness of the interior private optimal choices by the firms, and the positive externality result ($\frac{\partial m_B^*}{\partial q_A} < 0$) we have shown earlier, we follow similar steps in Proposition 4 and verify the Pareto improvement result.

For c_0 sufficiently small (close to 0), there is no demand for internal control. To see this, suppose c_0 is close to 0, thus the manipulation cost becomes $C_i(m_i, q_i) = \frac{q_i}{2}m_i^2$. It is obvious that firms' privately optimal internal control choice is $q_i^* = 0$ as this makes $C_i(m_i, q_i) = 0$. In addition, $q_i^* = 0$ is also Pareto efficient because the manipulation costs for both firms have already been minimized. ∎

Proof. of Proposition 8: We solve the model by backward induction. At $t = 3$, the equilibrium stock price $P_A^*(r_A, r_B)$ is set to be equal to investors' expectation of the firm value V_i conditional on r_A and r_B:

$$P_A^*(r_A, r_B) = \theta_A(r_A, r_B) - K_A(q_A).$$

Anticipating the investors' pricing response, manager A chooses manipulation at $t = 2$. Averaged over r_B, a bad manager A's expected payoff is given by

$$\delta_A\mu_A E_{r_B}[\theta_A(1, r_B)|s_A = 0] - K_A(q_A) + U_{0A} - C_A(m_A).$$

Notice that since firm A does not observe m_A at the time of setting U_{0A}, U_{0A} is independent of m_A. As before, manager A conjectures that manager B will choose manipulation m_B^* and thus succeed in issuing a fraudulent report with probability $\mu_B^* = m_B^*(1 - q_B)$. His best response to μ_B^*, denoted as $\tilde{m}_A^*(\mu_B^*)$, is determined by the following first order condition:

$$H^A(m_A; m_B^*)|_{m_A = \tilde{m}_A^*} \equiv \delta_A\frac{\partial \mu_A}{\partial m_A}E_{r_B}[\theta_A(1, r_B)|s_A = 0] - C_A'(m_A) = 0.$$

From the baseline model (Proposition 1), we have shown that $E_{r_B}[\theta_A(1, r_B)|s_A = 0]$ is strictly increasing in μ_B^*. As a result, $\tilde{m}_A^*(\mu_B^*)$ is also strictly increasing in μ_B^*, making the two firms' manipulation decisions strategic complements to each other. Similarly, the first order

condition for manager B is

$$H^B(m_B; m_A^*)|_{m_B=\tilde{m}_B^*} \equiv \delta_B (1 - q_B) E_{r_A}[\theta_B(1, r_A)|s_B = 0] - C_B'(m_B) = 0.$$

The two first order conditions jointly determine the managers' optimal choices of manipulation (m_A^*, m_B^*) as functions of the firms' choices of internal control (q_A, q_B). Notice that the two first order conditions is the same as the ones in the baseline model. Thus the proof for $\frac{\partial m_A^*(q_A, q_B)}{\partial q_A} < 0$ and $\frac{\partial m_B^*(q_B, q_A)}{\partial q_A} < 0$ is the same as that in Proposition 2.

At $t = 1$, the firms choose U_{0i} to make the managers break even on average. The manager i's expected payoff $E[U_i^*]$ given $\{m_A^*, m_B^*, q_A, q_B\}$, is

$$\begin{aligned} E[U_i^*] &= E[\delta_i P_i + (1 - \delta_i)V_i + U_{0i} - C_i(m_i^*)] \\ &= \theta_i - (1 - \theta_i) C_i(m_i^*) + U_{0i} - K_i(q_i). \end{aligned}$$

Thus the break even condition $E[U_i^*] = \bar{U}$ gives

$$U_{0i}^* = \bar{U} + K_i(q_i) + (1 - \theta_i) C_i(m_i^*) - \theta_i.$$

At $t = 0$, the firms choose q_i to maximize its payoff, the firm value less the wage paid to its manager, $E[V_i - \delta_i P_i - (1 - \delta_i)V_i - U_{0i}^*]$. Given U_{0i}^*, the firm's ex ante payoff V_{i0} can be reduced into

$$V_{i0} = \theta_i - (1 - \theta_i) C_i(m_i^*) - K_i(q_i) - \bar{U}.$$

Notice that the ex ante firm value differs from the one in our baseline model only by a constant $-\bar{U}$. Firm A at $t = 0$ chooses q_A to maximizes V_{A0}. Differentiating V_{A0} with respect to q_A, the first order condition is

$$\frac{\partial V_{A0}}{\partial m_A^*} \frac{\partial m_A^*}{\partial q_A} - K'(q_A) = -(1 - \theta_A) C_A'(m_A^*) \frac{\partial m_A^*}{\partial q_A} - K_A'(q_A) = 0,$$

which is the same as in the baseline model. Thus following similar proofs in Proposition 3, we show that there exists a unique pair of optimal private choices of internal control $\{q_A^*, q_B^*\}$ by the firms that solves the first order conditions.

Given the positive externality result $\frac{\partial m_B^*(q_B, q_A)}{\partial q_A} < 0$, the proof for the existence of Pareto improvements is identical to the one in Proposition 4. ∎

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