# Asset Pricing for the Shortfall Averse

Paolo Guasoni[1,2] Gur Huberman[3]

Boston University[1]

Dublin City University[2]

Columbia Business School[3]

Five Star Conference, NYU, December 2015

# Shortfall Aversion

- $e^{-\beta t} \frac{(c_t/\mathbf{h_t^\alpha})^{1-\gamma}}{1-\gamma}$ **;** $\quad h_t = \max\{c_s : s \leqq t\}$
- Maximize

$$E\left[\int_0^\infty e^{-\beta t} \frac{(c_t/h_t^\alpha)^{1-\gamma}}{1-\gamma} dt\right]; \quad h_t = \max\{c_s : s \leqq t\}$$

  over spending rate $c_t$ and fraction of wealth in stocks $\pi_t$.
- At $c_t = h_t$, marginal utility for increasing consumption is strictly lower than for decreasing consumption
- Past peak consumption is a REFERENCE POINT
- The intensity of shortfall aversion is $0 \leq \alpha < 1$

# Reference point/Discontinuous Marginal utility

- $e^{-\beta t}\frac{(c_t/h_t^{\alpha})^{1-\gamma}}{1-\gamma}$ ; $h_t = \max\{c_s : s \leq t\}$
- Inspired by Prospect Theory
- But here: Utility of consumption, and dynamic
- Choice of reference point inspired by
  - Peak-end rule
  - Business cycle dating
  - Definition of rare disaster
- Basic Premise: Fix current consumption; the higher past historical consumption, the lower the utility of current consumption
- Basic feature: As soon as consumption exceeds its past maximum, the maximum resets

# With a Utility Function

- $e^{-\beta t} \frac{(c_t/h_t^{\alpha})^{1-\gamma}}{1-\gamma}$ ; $\quad h_t = \max\{c_s : s \leq t\}$
- Either assume asset prices (i.e., processes for returns) & derive consumption/savings & portfolio choices
- Or assume a consumption process for the representative agent in an equilibrium framework & derive asset prices & their attributes, e.g., moments.

# Consumption & Dividend: Correlated Geometric Brownian Motions

| A: | Empirical Market Inputs | |
|---|---|---|
| | Average | S.D. |
| Consumption Growth | 1.93 | 2.13 |
| Dividend Growth | 1.15 | 11.05 |
| Correlation $\rho = 0.25$ | | |

source: Beeler & Campbell (2012), Benzoni et al (2011)

- $\frac{dc_t}{c_t} = \mu_c dt + \sigma_c dW_t^c$
- $\frac{dD_t}{D_t} = \mu_D dt + \sigma_D(\rho dW_t^c + \sqrt{1-\rho^2} dW_t^D)$

# Outline

- What is shortfall aversion?
- Numerical results & the state variable
- A benchmark model & interest rate results
- The equity premium

# Calibration & Confrontation with US Data

$$e^{-\beta t}\frac{(c_t/\mathbf{h}_t^{\alpha})^{1-\gamma}}{1-\gamma}\ ; \quad h_t = \max\{c_s : s \leqq t\}$$

| B: | | Calibrated Preference Parameters |
|---|---|---:|
| Discount Rate | $\beta$ | 0 |
| Risk Aversion | $\gamma$ | **4.220** |
| Shortfall Aversion | $\alpha$ | **0.498** |

| C: | Average | | S.D. | |
|---|---:|---:|---:|---:|
| | Data | Model | Data | Model |
| Equity Premium | 5.47 | 4.72 | 20.17 | 12.43 |
| Price/Dividend | 31.85 | 25.25 | 15.09 | 0.48 |
| 3-Month Real Rate | 0.56 | 0.55 | 2.89 | 2.25 |
| Long-Term Real Rate | ? | 4.83 | 0 | 0 |

# Sharpe Ratio with US Data

$$e^{-\beta t}\frac{(c_t/\mathbf{h}_t^{\alpha})^{1-\gamma}}{1-\gamma}\;; \qquad h_t = \max\{c_s : s \leqq t\}$$

| C: | Average | | S.D. | |
|---|---|---|---|---|
| | Data | Model | Data | Model |
| Equity Premium | 5.47 | 4.72 | 20.17 | 12.43 |
| Price/Dividend | 31.85 | 25.25 | 15.09 | 0.48 |
| 3-Month Real Rate | 0.56 | 0.55 | 2.89 | 2.25 |
| Long-Term Real Rate | ? | 4.83 | 0 | 0 |

- If standard procedure were used to estimate Sharpe ratio of model-generated data, it would be $\frac{4.72}{12.43} = .38$
- Compare with estimate from 1930-2011 data, $\frac{5.47}{20.17} = .27$ or with post war-based data, .32 or FF 1947-2012 .44.

# The State Variable $x_t = \frac{c_t}{h_t}$

- Its density distribution

$$Prob(x_t \in dx) = \begin{cases} \lambda x^{\lambda-1} & x \in (0,1) \\ 0 & x \notin (0,1) \end{cases} \qquad \text{where} \quad \lambda = 2\mu_c/\sigma_c^2 - 1$$

- This distribution has mean $\lambda/(\lambda+1)$, variance $\lambda/((\lambda+1)^2(\lambda+2))$, and its lower $p$-quantile is $p^{1/\lambda}$
- With $\lambda = 84$, highly skewed

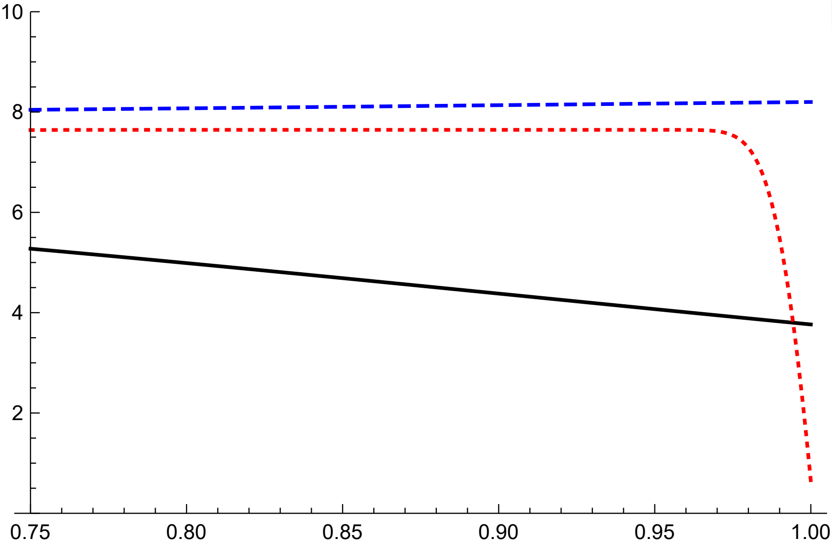
| Probability | 1% | 5% | 10% | 50% |
|---|---|---|---|---|
| Theoretical | 0.946 | 0.964 | 0.972 | 0.991 |
| Empirical | 0.987 | 0.994 | 0.999 | 1.000 |

Data: Q1:1952-Q1:2015

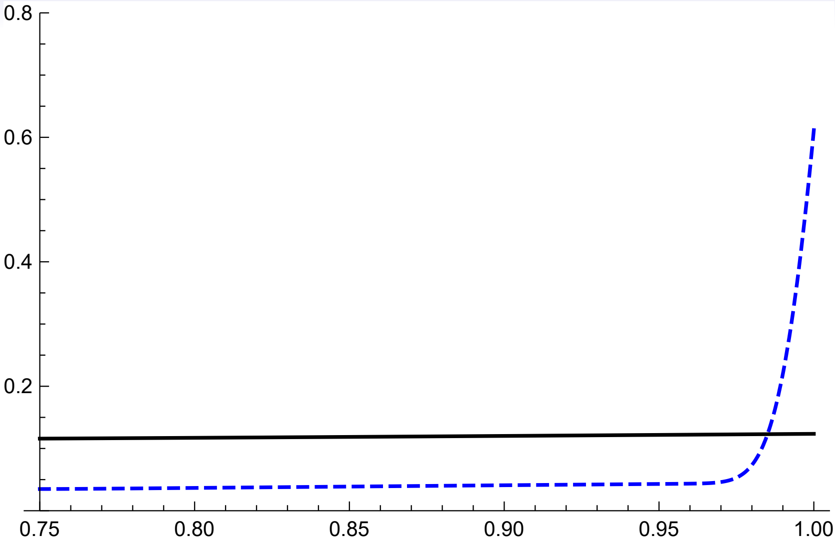
- Lowest post-war quarterly c/x: Q2’09: 98.4%; Q1, Q3 ’09 were 98.8%, 98.7%; also, Q1’58 was 98.8%

# When $c_t$ is very near its historical maximum $h_t$...

- Strong incentive to save rather than increase consumption
- that, plus market clearing =>
- Higher prices for the savings instruments, i.e.,
- Lower interest rate, lower expected stock return




Dividend yield (solid), stock return (dashed), and three-month rate (dotted), in percent per annum (vertical axis) against the state variable $x_t = c_t / h_t$ (horizontal axis).




Stock return volatility (solid) and Sharpe ratio (dashed) per annum (vertical axis) against the state variable $x_t = c_t/h_t$ (horizontal axis).

# Outline

- What is shortfall aversion?
- Numerical results & the state variable
- A benchmark model & interest rate results
- The equity premium

# A Convenient Benchmark: A Lucas (1978)-Based Model

- Continuous time; consumption & dividends correlated but not equal
- Our model with $\alpha = 0$
- Interest rate $r_0$, div yield $y_0$, expected equity return $e_0$ all constant, at
- $r_0 = \beta + \gamma\mu_c - \frac{\sigma_c^2}{2}\gamma(\gamma + 1) = 7.64\%$
- $y_0 = r_0 - \mu_D + \gamma\rho\sigma_c\sigma_D = 7.48\%$
- $e_0 = r_0 + \gamma\rho\sigma_c\sigma_D = 8.63\%$
- (Numbers based on calibration to our model)
- To make equity premium $(e_0 - r_0)$ large, need high $\gamma$
- To make $r_0$ small, need small $\gamma$

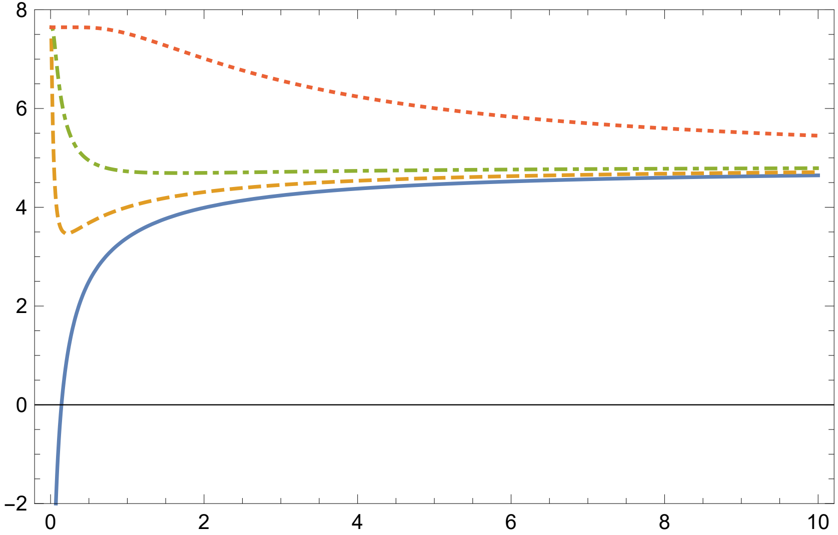
# $\frac{1}{\gamma}$ as Elasticity of Intertemporal Substitution

- Continuous time; no uncertainty; no shortfall aversion
- $r_0 = \beta + \gamma\mu_c$
  Lower $\gamma$ => Lower desire to smooth consumption
- That & $\mu_c > 0$ & equilibrium => Lower $r_0$
- Also, lower $\mu_c \implies$ lower int rate

# With $0 < \alpha < 1$

- Int rate, div yield, exp return depend on the state & the horizon
- The unconditional long-term int rate
  $R^{\infty} = \beta + \gamma^{*}\mu_c - \frac{\sigma_c^2}{2}\gamma^{*}(\gamma^{*} + 1) \qquad \gamma^{*} = \alpha + (1-\alpha)\gamma$
- Compare with $r_0 = \beta + \gamma\mu_c - \frac{\sigma_c^2}{2}\gamma(\gamma + 1) = 7.64\%$
- $\gamma^{*}$ is the $\alpha$-weighted average of 1 & $\gamma$
- delivers $R^{\infty}$ of 4.83%,
  - i.e., a drop of $\approx 2.8\%$

# Term Structure of Int Rate, for Different States $c_t/h_t$




$c_t/h_t = 99.9\%$ (bottom), 99.5%, 99%, to 95% (top)

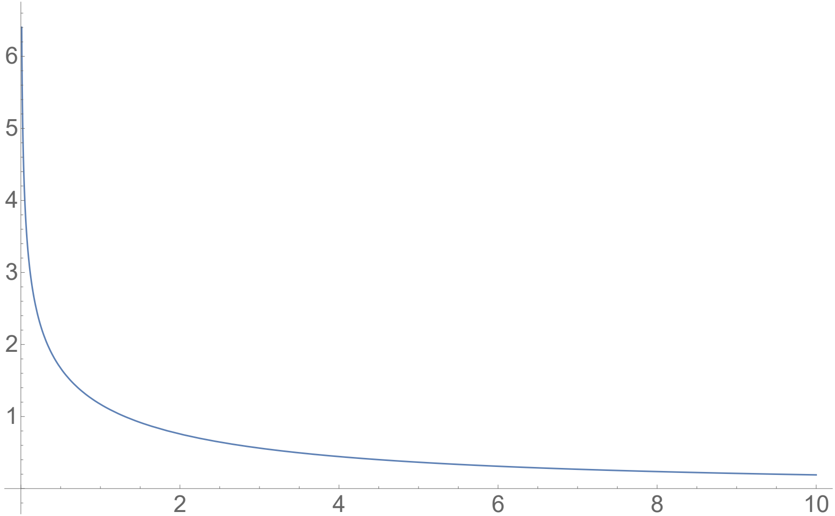
# Short Maturity Debt

- Given a state $c_t/h_t < 1$ & for very very short maturities,
- Virtually impossible for the state to reach 1 during the life of the bond
- Therefore rate = $r_0 = 7.64\%$
- The average instantaneous rate

  $E[R^0] = r_0 - \alpha(\gamma - 1)(\mu_c - \sigma_c^2/2) = 4.6\%$
- Caveat: Model unsuitable to produce an accurate instantaneous rate;

  In contrast, the model's predicted 3-month rate is a very reasonable .55%

# Term Structure of the Volatility of Interest Rates




- 3-month rate: Model-generated: 2.25%;
- Data: 1930 – 2008: 2.89%; 1947.2 – 2008.4: 1.82%

# Outline

- What is shortfall aversion?
- Numerical results & the state variable
- A benchmark model & interest rate results
- The equity premium

# Consol is the Fixed Income Analogue of Equity, with $\mu_D = \sigma_D = 0$

- Consol's average return $\approx R^\infty + \alpha^2(\gamma-1)^2\frac{\sigma_c^2}{2}$

$$\underbrace{\underbrace{\beta + \mu_c\gamma - \frac{\sigma_c^2}{2}\gamma(\gamma+1)}_{\text{Lucas Rate}} - \underbrace{\alpha(\gamma-1)\left(\mu_c - \sigma_c^2(\alpha(1-\gamma)+2\gamma+1)/2\right)}_{\text{Long-Term Spread}}}_{\text{Long-Term Rate} = R^\infty}$$

$$+ \underbrace{\alpha^2(\gamma-1)^2\frac{\sigma_c^2}{2}}_{\text{Yield-Return Spread}} + o(\sigma_c^2)$$

# Equity Expected Return

$$\underbrace{\underbrace{\beta + \mu_c\gamma - \frac{\sigma_c^2}{2}\gamma(\gamma+1)}_{\text{Lucas Rate}} - \underbrace{\alpha(\gamma-1)\left(\mu_c - \sigma_c^2(\alpha(1-\gamma)+2\gamma+1)/2\right)}_{\text{Long-Term Spread}}}_{\text{Long-Term Rate}}$$

$$+ \underbrace{\alpha^2(\gamma-1)^2\frac{\sigma_c^2}{2}}_{\text{Yield-Return Spread}} + \underbrace{\gamma\sigma_c\sigma_D}_{\text{Consumption-Risk Premium}} + o(\sigma_c^2)$$

- Or
  Consol's average return + $\gamma\sigma_c\sigma_D$

# Shortfall Aversion & The Equity Premium and the Safe Rate

- Interest rate is state & maturity dependent; its average is considerably lower than than the classic
- The model delivers a real 3-month rate of .55% – close to the average observed
- In bad states short-term rates are high. These states are rare and absent from post-war US data.

# Thank You!