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Debt contracts in the presence of performance manipulation

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Abstract Empirical evidence suggests that firms often manipulate reported numbers to avoid debt covenant violations. We study how a firm's ability to manipulate reports affects the terms of its debt contracts and the resulting investment and manipulation decisions that the firm implements. Our model generates novel empirical predictions regarding the use and the level of debt covenant, the interest rate, the efficiency of investment decisions, and the likelihood of covenant violations. For example, the model predicts that the optimal debt contract for firms with relatively strong (weak) corporate governance (i.e., cost of manipulation) induces overinvestment (underinvestment). Moreover, for firms with strong (weak) corporate governance, an increase in corporate governance quality leads to tighter (looser) covenant, more (less) frequent covenant violations and lower (higher) interest rate. Our model highlights that the interest rate, which is a common proxy for the cost of debt, neither accounts for the distortion of investment efficiency nor the expected manipulation costs arising under debt financing. We propose a measure of cost of debt capital that accounts for these effects.

Keywords Asymmetric information · Debt contracts · Earnings management

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1 Introduction

The finance, economics and accounting literature has studied debt contracts extensively, both theoretically and empirically. Unlike the empirical literature, the theoretical literature has overlooked an important and prevalent friction: managers can, and often do, manipulate financial reports to avoid debt covenant violations.[1] In this paper, we study the design of debt contracts when managers (borrowers) can manipulate the reported measure over which the covenant is written.

Given the vast evidence of performance manipulation to avoid covenant violations, it is important to understand its effect on the design of debt contracts and the likelihood of covenant violation, as well as on firm's investment policies. Absent a theory that considers the ability to manipulate reports, it is difficult to understand some features of debt contracts and to interpret the evidence relating debt contracts to firms' information environment.[2] For example, it seems intuitive that when the firm's information system is less reliable —namely, when the manager can more easily manipulate reports— the interest rate should be higher, to compensate the lender for the expected loss of control rights caused by the manager's potential manipulation. On the other hand, one might think that a less reliable reporting system should lead to tighter covenants, that is, greater control rights assigned to the lender, to offset the manager's reduced cost of misreporting. While these intuitions have inspired empirical research —and led to conflicting findings (see e.g., Kim et al. 2011; Costello and Wittenwerg-Moerman 2011)—, our model demonstrates that once the debt contract is optimally designed neither of the above hypotheses always hold. Moreover, these hypotheses cannot hold at the same time, or else the firm would be leaving money on the table.

We study how a cash constrained firm/entrepreneur, who needs to raise capital to pursue a positive NPV project, optimally designs a debt contract. The main innovation in our model is that the manager has the ability to manipulate financial reports to avoid a covenant violation. Manipulating the report is costly to the firm's manager.

We analyze how the aspects of the optimal debt contract are affected by the firm's ability to manipulate the performance measure upon which the covenant is written.[3] These aspects of the optimal debt contract include the level of the covenant, the

[1] For empirical evidence of misreporting to avoid covenant violations, see DeFond and Jiambalvo (1994), Sweeney (1994), Dichev and Skinner (2002), Graham et al. (2008), and Dyreng et al. (2011).

[2] There is a large empirical literature in accounting documenting the impact of corporate governance and accounting quality on debt contracts. See Bharath et al. (2008), Ball et al. (2008), and Costello and Wittenwerg-Moerman (2011).

[3] In this paper we restrict attention to debt financing and derive the optimal debt contract. While pure debt financing is not the optimal financing method in our setting, in additional analysis of an extended setting that includes hidden effort, we allow the firm to optimally choose the mix of debt and equity funding. Our numerical analysis demonstrates that the optimal mix includes both debt and equity. Hence the trade-off we identify in the baseline model qualitatively holds even under the optimal mix of debt and equity.

interest rate (face value), the efficiency of the investment termination/continuation decision, and the tightness of the covenant (defined as the probability of covenant violation). To gain further insight, we study how these aspects vary with firms' characteristics, including (i) how costly it is for the manager to manipulate the report, which may capture the quality of the firm's corporate governance or the reliability of the accounting system; and (ii) the precision of the firm's private information about future cash flows, which may capture the relevance of the firm's private information. Our model demonstrates that the answer to these questions is often counterintuitive.

Our model has three periods. In the first period, a cash-constrained manager offers a debt contract to a lender to finance the firm's investment project. The debt contract is characterized by a covenant and a face value. In the second period, the firm's manager privately observes a noisy signal of the profitability of the investment project and reports it to the lender. The manager can manipulate the report to avoid a covenant violation. Manipulating the report is costly, and the cost is increasing in the magnitude of manipulation. If the report is lower than the covenant, there is a covenant violation and the control rights are transferred to the lender, who may terminate the project. Termination of the project allows the lender to recover a fraction of the loan. If the project is continued, the firm's terminal cash flow is realized in the third period. Upon realization of the cash flow in the third period, the lender receives the minimum of the face value and the realized cash flows, while the equity holders receive the residual cash flow.

Given the structure of a debt contract, sometimes the manager has an ex-post incentive to manipulate the report to avoid liquidation, as the liquidation value would accrue to the lender.[4] When the manager's private signal is higher than the covenant, there is no need to manipulate the report. When the private signal is lower than the covenant, unless the manager manipulates the report upward, the covenant is violated and the project is liquidated. In such cases, the manager will manipulate upward if the cost of the required manipulation is lower than the manager's expected benefit from continuing the project. Thus, the presence of a covenant leads to a positive expected manipulation cost.

Liquidation is efficient when the signal indicates that the expected cash flow from the project is lower than liquidation value. In designing the debt contract, the firm considers the tension between investment efficiency (efficiency of the termination decision) and the expected cost of manipulation induced by the contract. In general, this tension is resolved by using a covenant that implements sub-optimal investment termination decisions. While a debt contract that implements the first-best continuation decision is feasible, it is suboptimal because it induces excessive expected manipulation costs.

[4] In the equilibrium of our model the manager's payoff following liquidation is zero. However, our main results hold even if the manager obtains positive payoff following liquidation, as long as liquidation is ex-post costly to the manager compared to continuation of the project. Beneish and Press (1993) document that, in their sample, following technical covenant violation firms experience increased interest costs ranging between 0.84 and 1.63 percent of the market value of firms' equity, and that costs of restructuring debt represent an average of 3.7% of the market value of equity.

The ability to manipulate the report distorts the firm's investment policy and induces expected manipulation costs. When the misreporting friction is mild (acute) the debt contract induces overinvestment (underinvestment) due to the over-continuation (termination) of the project. In particular, when the cost of manipulation is high the debt contract gives rise to over-continuation of the investment project. By contrast, when the cost of manipulation is low and the precision of firm's private signal is sufficiently high, the contract induces over-termination of the investment. Finally, when both the marginal manipulation cost and the precision level of the firm's private signal are low, the cost of implementing a covenant exceeds the benefit of the real option to terminate a bad project, resulting in an optimal debt contract that does not include a covenant. Our analysis suggests that one could empirically quantify the investment distortions caused by misreporting, by comparing the actual investment policy with a counterfactual policy arising in the absence of misreporting.

We study how cross-sectional variation in firm characteristics, notably in the marginal manipulation cost, affects the optimal debt contract. Cross-sectional variation in marginal manipulation costs can arise, for instance, due to differences in the quality of internal control systems. Costello and Wittenwerg-Moerman (2011) document that when a firm experiences a material internal control weakness (ICW) (Section 404 of the Sarbannes Oxley act), debt contracts are less likely to include covenants. Also, Kim et al. (2011) find that "loan spreads increase significantly after an ICW disclosure". We study the circumstances under which these predictions hold. In particular, we find that while an increase in the cost of manipulation unambiguously reduces the covenant level, its effect on the frequency of covenant violation and face value is non-monotone. Our analysis indicates that interest rates and the lender's control rights must be substitutes, that is, when the interest rate goes up the lender's control rights must decrease, or else the firm would leave money on the table. For firms with a relatively high marginal manipulation cost, an increase in the cost of manipulation increases the likelihood of covenant violation but reduces the face value. For firms with a relatively low marginal manipulation cost, the model predictions reverse. For such firms, an increase in the marginal manipulation cost decreases the likelihood of covenant violation but increases the face value.

While the model offers novel empirical predictions, it also highlights an important insight regarding measurement of the cost of debt capital. The debt contracting literature has often used the interest rate as a proxy for the cost of debt (see, e.g., Kim et al. 2011). This measure ignores two aspects of debt contracts that are simultaneously determined along with the face value: the economic value of control rights transferred to the lender via covenants, and the extent of the efficiency loss in investment decisions caused by the possibility of manipulation. To address these common measurement problems, we propose a measure of the cost of debt that captures the difference between the firm's return in an ideal scenario (without manipulation) and the firm's return in the presence of manipulation. Estimating our measure of the costs of debt would allow one to quantify separately the two sources of inefficiency caused by the possibility of manipulation: expected manipulation costs and investment distortions.

In practice, covenant violations are frequent but often renegotiated and waived. Indeed, Roberts and Sufi (2009) find that over 90% of long-term debt contracts are renegotiated prior to their stated maturity. In an extension, we study the effect of renegotiation. We find that the ability to renegotiate a debt contract can mitigate some of the consequences of manipulation, particularly when the manipulation friction is acute (low marginal manipulation cost). In such cases, covenants are set extremely tight, which leads to frequent violations. For a fraction of these violations, the lender waives the decision to terminate the project but, in exchange, negotiates a higher interest rate.

1.1 Related literature

We follow the Grossman-Hart-Moore property rights program by studying the optimal assignment of control rights given contractual incompleteness (see Grossman and Hart 1986; Hart 1995; Hart and Moore 1990). The incomplete contracting literature (see Aghion and Bolton 1992) considers the use of financial contracts to assign control rights across different states of the world. This strand of the literature takes the information structure as exogenous. In our setting, by contrast, information is contractible but potentially manipulated by the firm. In addition, given that our model introduces the ability to manipulate the report, the likelihood and magnitude of misreporting depend on how control rights are assigned.

A closely related and concurrent paper is Laux (2017), which studies a binary debt contracting setting with moral hazard and renegotiation. He demonstrates that the presence of a covenant can backfire and reduce both the probability that poor projects are liquidated and the manager's effort incentive. Another closely related paper is Sridhar and Magee (1996). Their primary focus is debt contracting in the presence of non-verifiable information. In an extension they study misreporting, assuming that misreporting is not costly.

Gao (2013) studies optimal debt contracts in a binary setting in which the manager can artificially increase the probability of a positive report and the lender can commit to verifying the report, in the spirit of Townsend (1979). The optimal debt contract prescribes verification of positive reports, consistent with a conservative accounting system. Caskey and Hughes (2012) study the impact of fair value measures on the efficiency of project selection and continuation in a setting where manipulation is not possible. They find that covenants based on a conservative fair value measure tend to perform best. The accounting literature has focused on the benefits of conservatism for debt contracting. For example, Gigler et al. (2009) show that a liberal accounting system is more efficient than a conservative system, because the former reduces the incidence of inefficient termination. Similarly (Li 2013) compares the benefits of conservative versus liberal accounting systems. Neither of these papers considers manipulation and both take the information system as given. We assume that the information system is an outcome of the contracting process. Beyer (2013) studies conservatism in a debt contracting setting without misreporting and shows that the maximum capital a firm can raise via a debt contract is higher under a conservative regime than under a fair value regime. Goex and Wagenhofer (2009) study optimal impairment rules. In their setting, the information system is designed ex-ante to maximize the probability that the lender will finance the firm's project when the firm's pledgeable assets may be insufficient to guarantee financing.

There is a large literature in accounting studying the causes and consequences of earnings manipulation. This literature has followed two strands: the first strand takes the manager incentives as exogenous and focuses on the market reaction to earnings reports (see, e.g., Dye 1988; Fischer and Verrecchia 2000; Guttman et al. 2006). The second strand studies optimal incentive contracts to the manager, who has the ability to manipulate the reports (see, e.g., Liang 2000; Beyer et al. 2014; Dutta and Fan 2014; Stein 1989). Our model is more closely related to the latter strand, where we study the contracting between a borrower and lender, rather than between shareholders and the manager.

Dessein (2005) studies the optimal allocation of control rights as a function of the severity of information asymmetries. Garleanu and Zwiebel (2009) also study the design and renegotiation of covenants in a setting where the lender has private information at the contracting stage. In their settings, the informed party gives up control rights to the lender to signal congruent preferences. Like Garleanu and Zwiebel (2009), we focus on debt contracts and do not address the more general security design question. The optimality of debt contracts in moral hazard settings under limited liability was first established by Innes (1990). More recently, Hebert (2015) proves the optimality of debt when managers' effort and risk choices are unobservable.

Cornelli and Yosha (2003) study stage financing in settings where managers can engage in ex-ante manipulation that shifts the distribution of non-contractible signals. They find that the optimal contract is a convertible debt contract, which eliminates manipulation. We consider a very different setting, where signals are "hard" and privately observed by the firm, but manipulable at some cost. As such, firms' reports are contractible.

The paper proceeds as follows. Section 2 describes the model. Section 3 presents our main results, which characterize the optimal debt contract and offer the main comparative statics. In Section 3.2 we provide the intuition for the main results. Section 4 discusses empirical implications. Section 5 considers the effect of renegotiation. Section 6 generalizes the model, and Section 7 concludes.

2 Model

We study a debt contracting setting in which the borrower can bias his report to avoid a covenant violation.

A liquidity constrained entrepreneur/firm has access to an investment opportunity that requires an initial investment of I. If the investment project is completed, it pays out a stochastic cash flow $\tilde{x}$, where $\tilde{x}$ is a random variable with a pdf $f(x)$ and cdf $F(x)$. In order to finance the investment opportunity, the firm needs to raise an amount of \$$I$ through a debt contract, where $I < E(\tilde{x})$. The debt contract specifies a covenant, z (as explained below), and a face value, K, which the borrower promises to pay the lender at the project's maturity.[5]

If the lender accepts the debt contract offered by the firm, the project is funded at $t = 1$ and undertaken. Then, at $t = 2$, the manager privately observes the realization of a noisy signal about the project's future cash flows. We denote this private signal by $\tilde{s}$. Given the realized signal s, the manager issues a (potentially biased) report of his private signal. The manager's report is denoted by r. The manager is not confined to truthfully reporting his signal; however, manipulating the report is costly.[6] We assume the manager's misreporting cost equals $c\,|r - s|$. [7] In the main analysis we assume, for simplicity, that the manipulation costs are personally borne by the manager (as commonly assumed in the theoretical accrual management literature, e.g., Fischer and Verrecchia (2000) and Guttman et al. (2006)). Our results continue to hold when the cost of earnings management is in part borne by the firm, which could reflect litigation costs incurred by the firm for earnings management activities or the cost of real earnings management (e.g., Stein 1989).

If at $t = 2$ the manager's report about his signal is lower than the contract's covenant, i.e., $r < z$, there is a covenant violation. When the covenant is violated, the lender receives the project's control rights and can terminate the project. The termination/liquidation proceeds are assumed to be L, where $L < I$. That is, upon covenant violation the lender can recoup part of her investment by terminating the project, in which case the manager obtains zero (this is without loss of generality: all we need for our results to hold is that the loss of control rights be costly to the firm/manager).[8]

If the project is not terminated, then at $t = 3$ the cash flow of the project is realized and payoffs are allocated according to the contract. Upon realization of the cash flows (at the maturity of the project at $t = 3$) the lender receives $\min(K, x)$ and the borrower retains the residual cash, that is, the manager gets $\max(x - K, 0)$. Figure 1 summarizes the timeline of the game.

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[5] A debt contract is not optimal in our setting, but there are frictions that would make it optimal. We have not modeled those frictions because we are most interested in debt contract design as opposed to broader financing contract design.

[6] The empirical literature has provided ample evidence that managers take (costly) actions to avoid covenant violation. Some examples are DeFond and Jiambalvo (1994) which finds that "managers of firms approaching default respond with income-increasing accounting changes and that the default costs imposed by lenders and the accounting flexibility available to managers are important determinants of managers' accounting responses."

[7] The specific cost function does not play an important role in the analysis. All the results qualitatively go through under any strictly increasing function of the magnitude of manipulation, e.g., quadratic cost function.

[8] Violating a covenant is costly to the firm. The cost of a covenant violation can vary substantially across firms in terms of the type of cost and its magnitude. For example, covenant violation costs can be due to transfer of control rights; increased interest rate (that may lead to refinancing costs); lenders' demand for partial or full repayment (which may lead to restructuring costs and modification of operations); and increased lender control and restrictions on assets sale, dividend payment, and investment activities (see e.g., Beneish and Press 1993).

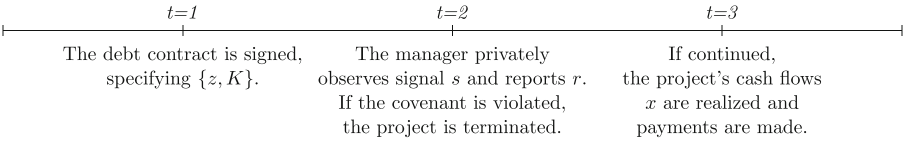


Fig. 1 Timeline

Both the lender and borrower are risk neutral. Cash flows are not discounted. The debt market is competitive such that the lender breaks even. Both the manager and lender maximize their expected payoff and obtain zero payoffs when the project is not financed. The model structure is common knowledge.

2.1 Information structure and distributional assumptions

The objective of this paper is to study the impact of the manager's ability to misreport on the design of optimal debt contracts, i.e., whether to use a covenant, and if so, what is the optimal level of the covenant. To focus on this effect, we design a setting that abstracts from other confounding effects. One such effect is the variation of the density of the distributions of cash flows and the manager's private signal. As indicated in the introduction, given a debt covenant, there will be an interval of signal realizations below the debt covenant, such that for all signals within this interval the manager will manipulate his report upwards in order to meet the covenant and avoid violating it. When designing the optimal contract in the first period, the manager considers the expected manipulation cost, which depends on the distribution of the private signal and the cash flows. For any distribution other than a uniform distribution, a shift in the location of a misreporting interval of a given size affects the expected manipulation cost. As such, when designing the optimal debt contract, the manager considers the particular properties of the distribution. For some distributions, the manager may wish to shift the manipulation interval towards the left end of the distribution (resulting in over-continuation of the project), whereas for other distributions there is an incentive to shift this interval towards the right end (resulting in over-termination of the project).

To abstract from such an effect of the distribution on the optimal debt contract, we assume that the cash flows are uniformly distributed, i.e., $\tilde{x} \sim U[0, h]$. This guaranties that for a given size of misreporting interval, the location of the interval does not affect the expected misreporting cost.

While the assumption of uniformly distributed cash flows is helpful in isolating the effect of misreporting on the design of the optimal debt contract, it has two major downsides. First, for many investment projects, a nonuniform distribution, such as a hump-shape distribution, may be more representative. Second, given that a uniform distribution is bounded from above, without any exogenous constraints the manager may issue a report that exceeds the upper bound of the distribution of cash flows.

While in our main analysis we assume a uniform distribution, we verify that all of our main results are not driven by the uniform distribution. In particular, we verify that our results are qualitatively robust to settings with common unbounded distributions such as Log-Normal, truncated Normal, and Exponential distributions.

Given that $\tilde{x} \sim U[0, h]$, a natural way to introduce the manager's noisy private signal is to assume that it is a mixture of the realized cash flows and the prior distribution. Specifically, we assume that with probability ρ the signal is equal to the realized cash flow x and with probability $1-\rho$ the signal is pure noise sampled from the prior distribution of $\tilde{x}$. Thus, ρ represents the precision of the manager's private signal. Given this information structure, the manager's private signal is also uniformly distributed, i.e., $\tilde{s} \sim U[0, h]$. The conditional expectation of $\tilde{x}$ given a realized signal s is linear in s and given by

$$E(\tilde{x}|\tilde{s} = s) = \rho s + (1-\rho)\mu,$$

where $\mu \equiv E(x)$.

2.2 First best benchmark

Before deriving the equilibrium, we consider the first-best project termination decision. This is a useful benchmark to understand the effect of manipulation on the design of debt contracts, and to quantify the efficiency loss caused by the presence of manipulation.

The first-best (labelled FB) continuation strategy is the one that maximizes the expected cash flows. In our model, it is attained, in the limit, as misreporting becomes prohibitively costly ($c \to \infty$). Since the expected cash flow of the project given continuation increases in the signal s, and the payoff given termination L is independent of s , the first-best continuation strategy is a threshold strategy. We denote this threshold signal by τ^{FB}. Under the first-best, for all $s < \tau^{FB}$ the project is terminated; otherwise the project is continued. τ^{FB} is the signal realization for which the expected cash flow given termination, L, equals the expected cash flow given continuation, $E\left(x|\tilde{s} = \tau^{FB}\right)$. Hence, the first best continuation threshold, τ^{FB}, is given by

$$\tau^{FB} = \frac{L-(1-\rho)\mu}{\rho}. \tag{1}$$

Denote the expected cash flows for a continuation threshold τ by $V(\tau)$. The expected cash flows under the first-best continuation policy is given by:

$$V\left(\tau^{FB}\right) = \Pr\left(s < \tau^{FB}\right) L + \Pr\left(s \geq \tau^{FB}\right) E\left(\tilde{x}|s > \tau^{FB}\right).$$

Due to the option to terminate the project, the firm value is greater than $E(\tilde{x})$. Naturally, if the signal is not sufficiently informative, then the project is never terminated. A signal is sufficiently informative to generate a real option if there are realizations of $\tilde{s}$, such that $E(\tilde{x}|s) < L$, or equivalently if $(1-\rho)E(\tilde{x}) < L$.

3 Equilibrium

We begin our derivation of the optimal contract by characterizing the manager's reporting strategy under any arbitrary debt contract. Given the ex-post reporting strategy, we then solve for the ex-ante optimal debt contract and provide the main results of our paper: the characterization of the optimal debt contract and the main comparative statics. The above is done in Section 3.1. In Section 3.2 we provide the intuition to our main results.

3.1 The optimal debt contract

A debt contract can be defined as a pair $\{K, z\}$, consisting of a face value K and a covenant z, where $K, z \in \mathbb{R}^+$.[9] The existence of a covenant z implies that the control rights are transferred to the lender whenever the manager's report, r, violates the covenant, that is, whenever $r < z$.

As an intermediate step, before deriving the optimal debt contract, we study the manager's misreporting behavior for any given exogenous contract $\{K, z\}$. Given the contract $\{K, z\}$, for any signal realization s that is higher than the covenant, i.e., any $s \geq z$, the manager has no incentive to manipulate the report. When the signal realization is lower than the covenant, that is, $s < z$, the manager needs to consider the cost of manipulating the report upward to avoid covenant violation versus his expected cash flow if the project is continued (the manager's continuation value). For $s < z$, the manipulation cost to avoid covenant violation is decreasing in s, whereas the manager's continuation value is (weakly) increasing in s. As such, any contract $\{K, z\}$ gives rise to a threshold signal $\tau(z, K)$, such that for any lower signal $s < \tau(z, K)$ it is too costly to manipulate the report. For all $s < \tau(z, K)$, the manager prefers to report truthfully, violating the covenant and transferring control rights to the lender (who, as we later show, in equilibrium optimally terminates the project). The termination threshold $\tau(z, K)$ is the signal $s = \tau(z, K)$ that makes the manager indifferent between manipulating the report to meet the covenant and continue the project, and reporting his signal truthfully to avoid the manipulation costs while letting the lender terminate the project.

Formally, the manager's termination threshold for a given contract $\{K, z\}$ is the signal value for which the manager's continuation value equals his manipulation cost, namely: [10]

$$E\left[(x-K)^{+} \mid s=\tau(z, K)\right]=c(z-\tau(z, K)). \tag{2}$$

Solving for the termination threshold yields:

$$\tau(z, K)=z-\frac{(1-\rho)(h-K)^{2}}{2 c h}. \tag{3}$$

[9] We are allowing z to exceed the upper bound of the support of cash flows. While it is not natural to have a covenant that exceeds the highest possible outcome, this assumption does not drive any of the results in our model. As indicated earlier, the uniform distribution allows us to abstract from variation in the density of outcomes and simplifies the analysis.

[10] We currently implicitly assume, and later show, that in equilibrium the lender always terminates the project following a covenant violation.

Figure 2 depicts the manager's reporting strategy at $t = 2$ for a given contract $\{K, z\}$ and the resulting termination threshold $\tau(z, K)$.

It is immediate from Eq. (3) that given all else equal, the termination threshold $\tau(z, K)$ is increasing in z, c, K, and ρ. Indeed, following an increase in the contract's covenant z, a manager needs to manipulate by more in order to meet the covenant; hence, fewer managers are willing to manipulate. Thus, the termination threshold $\tau(z, K)$ and likelihood of violation $F(\tau)$ increase in the covenant z. Following an increase in the manipulation cost, c, it becomes more costly to manipulate the report; hence, the termination threshold also increases in c. Following an increase in the face value, K, the manager's residual cash flow, if the project is continued, is lower. That is, the manager has less "skin in the game" as K increases. This decreases the manager's continuation value and his willingness to manipulate. Therefore, the termination threshold increases in K. Finally, an increase in the precision of the signal implies that conditional on $s = \tau(z, K)$, it is more likely that the cash flow will not be sufficient to fully pay the face value and leave a residual cash flow to the manager. Hence, the manager's continuation value is lower, which again increases the termination threshold (Figure 3).

The manager can always design a contract that implements efficient termination, i.e., $\tau(z, K) = \tau^{FB}$. However, any contract implementing a covenant ($z > 0$) induces positive expected manipulation costs. In particular, given an arbitrary contract $\{z, K\}$, the expected misreporting cost is given by:

$$C(z, K) \equiv \int_{\tau(z,K)}^{z} c(z - s) f(s) ds.$$

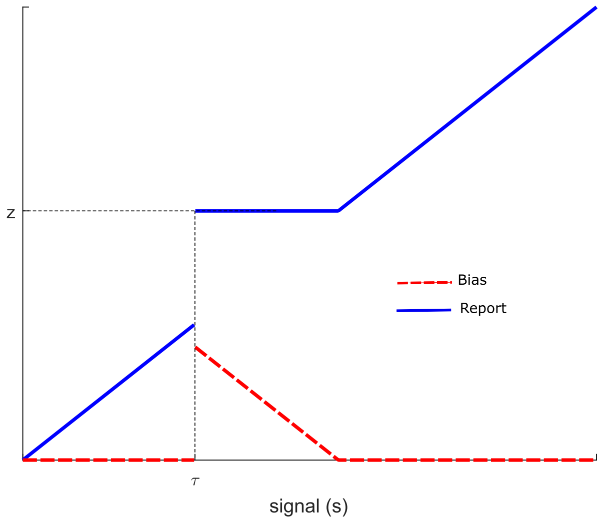


Fig. 2 Reporting strategy for an arbitrary contract $\{K, z\}$ and the resulting termination threshold τ. Below τ and above z the manager reports his signal truthfully. When $s \in [\tau, z]$, the manager reports exactly z, thereby over-reporting his signal s

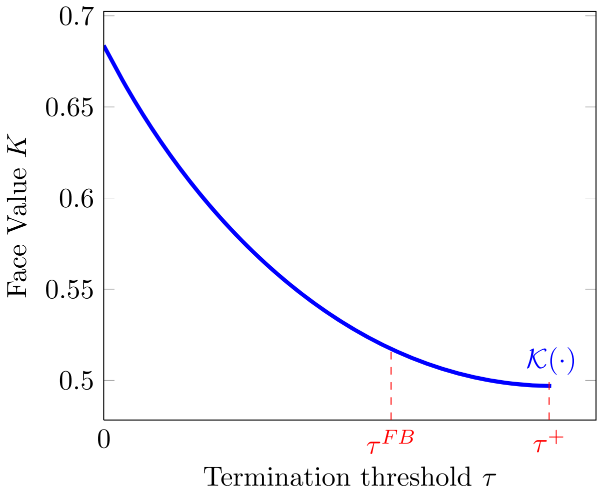


Fig. 3 The lender's participation constraint. Parameters: $\rho = .95$, $I = .45$, $L = .4$. In equilibrium, the termination threshold τ^* is always below τ^+

The manager could also design a contract that induces no manipulation by including no covenant ($z = 0$). However, such a contract induces over-continuation ($\tau = 0$). When designing the contract, the manager needs to optimally resolve a trade-off between efficiency of the termination decision and the magnitude of expected misreporting costs.

Consider how the contract's parameters $\{z, K\}$ affect the expected manipulation costs. First, consider the effect of changes in z, holding K constant, when $z < h$.[11] Increasing z induces a one-to-one increase in $\tau(z, K)$, thus shifting to the right the location of the interval of signals for which the manager chooses to manipulate the report. Given the uniform distribution, this shift does not affect the probability of manipulation. Therefore, for a given face value, the probability of misreporting and the expected misreporting cost are independent of the covenant level when $z < h$ (when keeping K constant). Next, consider the case when $z > h$. Increasing z leads again to a one-to-one increase in $\tau(z, K)$. However, now the probability of manipulation —and the expected misreporting cost— decreases in z, because the upper bound of the interval of signals for which the manager manipulates is fixed at h. Setting tighter (i.e., higher) covenants has the ability to reduce the likelihood of misreporting, *ceteris paribus*.

Similar to the effect of z, the effect of K is very intuitive. Increasing the face value K, for a given z, reduces the manager's *skin in the game*, thus weakening her ex-post manipulation incentive and lowering the expected misreporting costs. The following corollary summarizes this result.

[11] In general, the effect of z on expected misreporting costs is non-monotone. This is intuitive: the contract can eliminate misreporting costs by setting $z = 0$, i.e., no covenant. Alternatively, the contract can mitigate misreporting by setting a very high covenant, so that misreporting is always too costly to the manager. (At the limit, such a contract awards all control rights to the lender.)

Corollary 1 *For a given face value K, a marginal increase in the contract's covenant z weakly reduces expected manipulation costs, i.e.,*

$$\frac{dC(z,K)}{dz}=\begin{cases}0 & \text{if } z<h\\ <0 & \text{if } z\geq h\end{cases}.$$

For a fixed covenant z, the expected manipulation cost decreases in the face value, i.e.,

$$\frac{dC(z,K)}{dK}<0.$$

Given the above characterization of the manager's reporting strategy for an exogenously given debt contract, we next derive the optimal debt contract chosen by the manager ex-ante, at $t=1$.

Formally, the optimal debt contract is given by the pair $\{K,z\}$ that solves the following program

$$\max_{z,K} F(\tau(z,K))L+\int_{\tau(z,K)}^{h} E(x|s)f(s)\,ds-C(z,K),$$

subject to the lender's participation constraint:[12]

$$F(\tau(z,K))L+\int_{\tau(z,K)}^{h} E(\min(x,K)|s)f(s)\,ds\geq I.$$

The optimal debt contract maximizes expected cash flows net of manipulation costs, subject to the lender's participation constraint. As previously mentioned, when designing the debt contract, the manager considers the trade-off between the expected manipulation cost and the firm's investment efficiency (the extent to which the termination threshold deviates from the first best threshold τ^{FB}). To optimally balance this trade-off, the debt contract must optimize over two levers: the face value, K, and the lender control rights, captured by z. By controlling these aspects of the contract, the firm effectively determines not only the termination threshold, $\tau(z,K)$, but also the expected manipulation costs, $C(z,K)$.

It is convenient to reformulate the above optimization program as a single variable optimization problem, where the control variable is the termination threshold, τ. Before doing so, we introduce two definitions. Given that in equilibrium the lender participation constraint must bind, we can define $\mathcal{K}(\tau)$ as the face value that satisfies the lender's participation constraint when the project is terminated for $s<\tau$, and continued for $s\geq\tau$. $\mathcal{K}(\tau)$ solves

$$F(\tau)L+\int_{\tau}^{h} E(\min(x,\mathcal{K})|s)f(s)\,ds=I.$$

[12]One can generalize the model to allow for positive lender bargaining power by modifying the participation constraint such that instead of receiving I the lender gets $R>I$. In general, this would alter the debt contract's efficiency. For example, in the limit as the manager's residual surplus vanishes, his incentive to manipulate also vanishes, ensuring more efficient outcomes.

Lemma 1 *The face value that satisfies the lender's participation constraint when the contract $\{K, z\}$ induces a termination threshold τ, denoted by $\mathcal{K}(\tau)$, is given by*

$$\mathcal{K}(\tau) = h - \sqrt{\frac{2h(\tau L - Ih) + (h - \tau)\left(\rho\tau h + h^2\right)}{\rho\tau + h - \tau}}. \tag{4}$$

For any $\tau \in \left[0, \hat{\tau}\right]$ the face value $\mathcal{K}(\tau)$ is a continuous U-shaped function with the following characteristics:

$$\mathcal{K}(\tau) > I;$$
$$\mathcal{K}(0) = h - \sqrt{h^2 - 2Ih};$$

and the unique minimum of $\mathcal{K}(\cdot)$ over the interval $\left[0, \hat{\tau}\right]$, denoted τ^+, is given by

$$\tau^+ \equiv \arg\min_{s \in [0\ \hat{\tau}]} \mathcal{K}(s).$$

The value of $\hat{\tau}$ is defined by $\mathcal{K}\left(\hat{\tau}\right) = h$, and represents the termination threshold that makes the lender indifferent between accepting the contract or not, when she has 100% rights over the firm cash flows upon covenant violation when $s < \hat{\tau}$.

A lender accepts a contract that implements a threshold τ only if $K \geq \mathcal{K}(\tau)$. Of course, not all $\tau \in [0, \hat{\tau}]$ can be implemented, because the lender prefers not to terminate the project when the signal satisfies $s > \tau^+$. The U-shape of $\mathcal{K}(\tau)$ means that for low τ the face value and control rights are substitute means of compensating the lender: the manager can increase the lender's payoff by either increasing the face value or the covenant. However, when τ is large, face value and control rights are complements. [13]

Any optimal debt contract satisfies $\left.\frac{\partial \mathcal{K}(\tau)}{\partial \tau}\right|_{\tau=\tau^*} < 0$, so the increasing region of $\mathcal{K}$ is never part of an equilibrium. This is intuitive: if we decrease the lender's expected control rights by reducing the termination threshold τ, we must compensate the lender via higher face value, so that he continues to break-even. This implies that in equilibrium the face value and termination threshold are substitutes. This also implies that, when given control rights ($s < \tau^*$), the lender is always willing to terminate the project, as required.

Lemma 2 *The equilibrium threshold signal, τ^*, satisfies $\left.\frac{d\mathcal{K}(\tau)}{d\tau}\right|_{\tau=\tau^*} \leq 0$.*

[13] To gain some intuition for the U-shape of $\mathcal{K}(\tau)$, note that a low value of τ (i.e., when the probability of continuation is high) means the lender is unlikely to get control rights. As such, for low values of τ the lender demands a large face value to break even. As τ increases, the lender gets extra control rights: we add more signals under which the project is terminated —which is consistent with the lender's ex-post incentive for $s = \tau$— and hence the lender is willing to accept a lower face value. This continues up to a certain value of τ for which the termination threshold, τ, is ex-post optimal from the lender's standpoint, that is, for $s = \tau$ the lender is indifferent between continuation and termination. As we further increase τ, the contract starts inducing excessive termination, even from the lender's ex-post standpoint; hence, the lender demands additional compensation in terms of face value (recall $\mathcal{K}(\cdot)$ is derived assuming the lender can commit to terminating the project when he acquires control rights). As such, the face value for which the lender breaks even is a U-shape function of τ.

The second definition needed to reformulate the optimization program as a one-dimensional problem, is the covenant that induces a termination threshold τ when the lender is granted a face value $\mathcal{K}(\tau)$. We denote such covenant by $\zeta(\tau)$. Thus, ζ is the covenant that solves the misreporting indifference condition for a given τ when the face value is set at $\mathcal{K}(\tau)$. Formally, ζ is given by the solution to:

$$E\left[(x - \mathcal{K}(\tau))^{+} | s = \tau\right] = c\left(\zeta - \tau\right). \tag{5}$$

Armed with these definitions, we can reformulate the optimal debt contract as follows:

$$\max_{\tau} F(\tau) L + \int_{\tau}^{h} E(x|s) f(s) ds - \chi(\tau),$$

where $\chi(\tau) \equiv C(\zeta(\tau), \mathcal{K}(\tau))$ is the expected manipulation cost when the contract induces termination threshold τ and the lender breaks even.

Solving for the optimal debt contract is thus a two-step process. First, the manager considers the best contract with a covenant ($z > 0$) when the misreporting constraint is binding. Second, he compares the performance of such a contract to that of a no-covenant contract, where $z = \tau = 0$. If a no-covenant contract is selected, then there never is termination and the manager's expected payoff is $E(\tilde{x} - I)$.

We can write the optimization problem as follows:

$$\Pi^{*} = \max\left\{\max_{\tau}\left\{V(\tau) - \chi(\tau)\right\}, E(\tilde{x})\right\} - I, \tag{6}$$

where Π^{*} is the manager's expected payoff in equilibrium.

The existence of a maximum to the objective function Π^{*} is immediate given the bounded support and continuity of Π^{*}. Uniqueness is not obvious, even though $V(\tau)$ is single peaked. The reason is that the expected manipulation cost, $\chi(\tau)$, varies with τ in a non-monotone way.

We now turn to develop the paper's main result, Proposition 1, which describes the optimal debt contract. Then, in Proposition 2, we offer the main comparative statics describing how the contract is affected by changes in the cost of manipulation, c, and the precision of the signal, ρ. In Section 3.2 we discuss the intuition for the main result.

We begin by introducing two thresholds, $\hat{c} = \hat{c}(\rho)$ and $\hat{\rho} = \hat{\rho}(c)$, as implicitly defined by the following equations:

$$\max_{\tau}\left\{V\left(\tau|\hat{\rho}\right) - \chi\left(\tau|\hat{\rho}\right)\right\} = E(x), \tag{7}$$

and

$$\tau^{FB} = \arg\max_{\tau}\left\{V(\tau) - \chi\left(\tau|\hat{c}\right)\right\}. \tag{8}$$

$\hat{\rho}$ is the level of precision, for a given c, such that the optimal contract with a covenant results in the same expected payoff to the manager as a no-covenant contract. $\hat{c}$ is the level of manipulation cost, given ρ, such that the optimal debt contract induces the first-best termination threshold τ^{FB}.[14] Using the above definitions of $\hat{c} = \hat{c}(\rho)$ and $\hat{\rho} = \hat{\rho}(c)$, we state the paper's main result.

[14]Naturally, $\hat{\rho}$ depends on c and $\hat{c}$ depends on ρ. See Fig. 4.

Proposition 1 *There exists a unique optimal debt contract characterized as follows:*

1. *If $\rho < \hat{\rho}$ the contract does not include a covenant. Hence, $z^* = 0$ and the contract induces over-continuation.*
2. *If $\rho \geq \hat{\rho}$ the contract does include a covenant (i.e., $z^* > 0$) and features one of the following two patterns:*

   (a) *If $c \leq \hat{c}$ the contract entails over-termination, namely, $\tau^* \geq \tau^{FB}$,*
   (b) *If $c > \hat{c}$ the contract entails over-continuation, namely, $\tau^* < \tau^{FB}$.*

The above proposition reveals that both over- and under-termination can arise in equilibrium. When the manipulation friction is severe, i.e., when manipulation cost c is low, the contract induces over-termination (if $\rho > \hat{\rho}$). Covenants are set tight, leading to a high likelihood of covenant violation. This is an optimal contractual response, aimed not so much at compensating the lender from the manager's future "expropriation" but instead at mitigating both the likelihood and the expected cost of misreporting. By contrast, when the misreporting friction is mild (high c), the more intuitive outcome of over-continuation prevails. In this case, covenants are loose and less likely to be violated than in the absence of misreporting.

Figure 4 illustrates the results of the proposition. It shows how the properties of the accounting system and the informational environment (i.e., precision ρ and reliability c) affect the firm's investment choices. Broadly speaking, the accounting system not only modifies the debt contract design but, more importantly, alters the firm's real

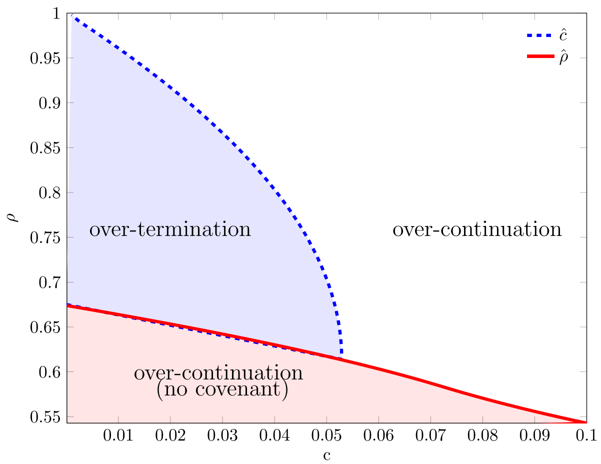


Fig. 4 **Accounting properties and investment efficiency**. The dotted curve is defined as the set of c, ρ such that the contract implements efficient termination, $\tau^* = \tau^{FB}$. The solid curve is defined as the set of c, ρ such that the expected payoff of the manager, with and without covenant, is the same, i.e., $\max_\tau \{V(\tau) - \chi(\tau) - E(x)\} = 0$. Parameters: $I = .45$, $L = .4$. Notice that the over-termination region is very small. This is due to our assumption that x is uniformly distributed over [0, 1]. If the density of s were decreasing over its support, as in the case of exponential distributions, over-termination would be more prevalent for a given c

choices, consistent with the so-called "real effects" literature in accounting. Over-termination is present when the cost of manipulation, c, is very low and precision is moderate. Observe that, for very low c, as we increase precision ρ we may transition from a contract that does not use a covenant (thus inducing over-continuation) to a contract that induces over-termination (for moderate ρ) and finally to a contract that induces over-continuation again (for high ρ). Otherwise, when c is large, the effect of precision ρ on the contract is more straightforward: as ρ increases, we transition from a contract without covenant to a contract with covenant but always inducing over-continuation.

Let us turn to the main comparative statics. The debt contracting literature in accounting studies whether and how disclosure quality influences the cost of debt (see, e.g., Bharath et al. 2008 and Sengupta 1998). Next we consider the impact of two qualitative aspects of an accounting system: the precision ρ of the manager's private information, and the cost of manipulation c. Specifically, the next proposition describes how the qualitative nature of the optimal debt contract varies with the main parameters of the model. The proposition focuses on the termination threshold τ, which captures the probability of covenant violation, and on the face value K of the debt contract.

Proposition 2 *Suppose $\rho \geq \hat{\rho}$ such that the optimal debt contract includes a covenant (i.e., $z^* > 0$). Then,*

1. *(effect of c) If $c \geq \hat{c}$, the likelihood of covenant violation $F(\tau^*)$ increases in c, and the face value K^* decreases in c. If $c < \hat{c}$, the likelihood of covenant violation $F(\tau^*)$ decreases in c, and the face value K^* increases in c.*
2. *(effect of ρ) The likelihood of covenant violation $F(\tau^*)$ may increase or decrease in ρ. If $c > \hat{c}$, τ^* increases in ρ as $\rho \to 1$. By contrast, when both c and $I - L$ are sufficiently small, the probability of violation $F(\tau^*)$ decreases in ρ as $\rho \to 1$.*

This result predicts that, once c is sufficiently high, covenant violations are more frequent among firms with higher manipulation cost, c. If we think of c as a proxy of the firm's corporate governance quality, then this result says that covenant violations are more likely among firms characterized by better governance. To the best of our knowledge, this is a prediction that has not been tested empirically. Dichev and Skinner (2002) report that they"find an unusually small number of loan/quarters with financial measures just below covenant thresholds and an unusually large number of loan/quarters with financial measures at or just above covenant thresholds," and interpret this fact as evidence of manipulation, but also recognize that they "cannot definitively rule out an explanation due to ex ante contracting that lenders systematically set covenant thresholds just below actual values." Our study suggests that the way contracts are written, and the extent to which covenants are tight, are inherently related to the properties of the firm's reporting system.

More generally, the proposition demonstrates that the effect of the parameters on the probability of violation and face value vary, qualitatively, with c. In particular, when the cost of manipulation is relatively low, the likelihood of covenant violation decreases in c, and, in response to less control rights, the face value increases in c. Hence for firms with relatively low manipulation costs, an increase in the manipulation cost will lead to a higher interest rate. By contrast, when c is large, an increase in c increases the likelihood of violations and leads to a lower face value.

The likelihood of manipulation is also non-monotone in c, as Fig. 5 shows. In particular, for c sufficiently small there is over-termination, and the covenant value z^* is greater than h. Since the probability of manipulation is proportional to the size of the manipulation interval, i.e., $\left[\min\{h, z^*\} - \tau^*\right]$ the probability of manipulation is relatively low when c is low. Following a small increase in c, the termination threshold τ^* decreases, and thus gets closer to the first-best threshold. However, we still have $z^* > h$ and hence the probability of manipulation increases. This process continues until $z^* = h$. As c further increases, the covenant decreases and the probability of covenant violation decreases as well, eventually vanishing as c grows large. This non-monotonicity implies that manipulation may thus be more likely among firms featuring higher quality of corporate governance than among firms featuring lower quality of corporate governance, when the debt contract is endogenously determined.

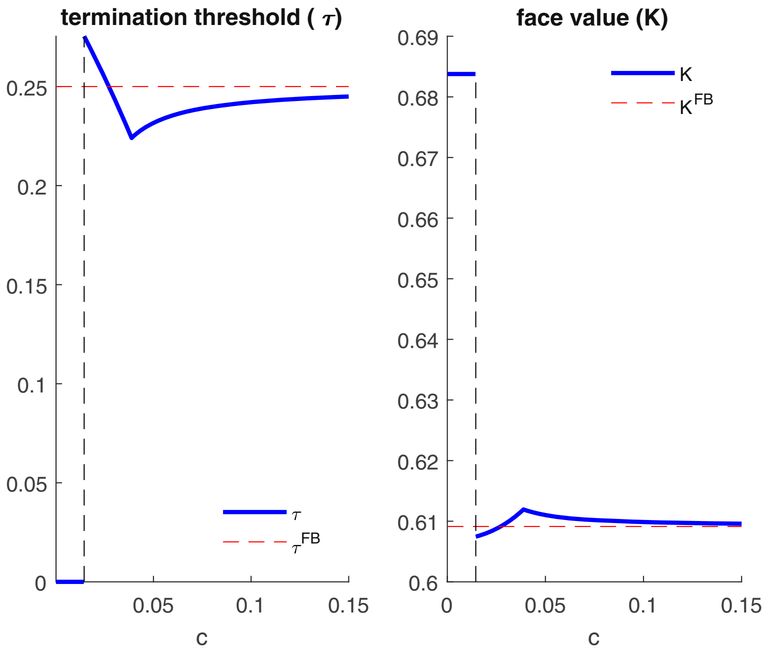


Fig. 5 The effect of manipulation cost, c. Parameters: $\rho = .6$, $L = .35$, $I = .45$. The left panel shows the evolution of the termination threshold τ^* as c increases. For very low c the contract does not include a covenant and $\tau^* = 0$. Initially, as c increases the threshold jumps above the first-best level τ^{FB}, and the contract induces over-termination. As c increases further the equilibrium threshold goes down and the contract eventually induces over-continuation. Finally, as c grows large the threshold attains first-best. The evolution of the face value (left panel) is also non-monotone and mirrors that of the threshold. This is a consequence of the substitution between control rights and face value

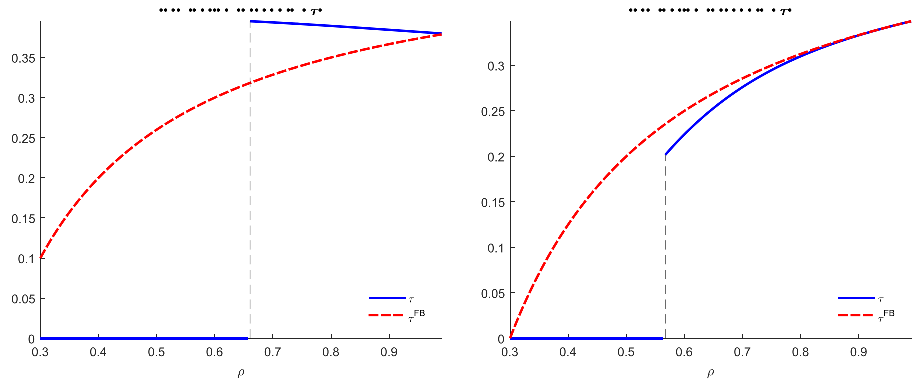
The effect of the precision of the manager's signal, ρ, on the debt contract also depends on the level of manipulation cost c. For firms with high manipulation cost (high c), an increase in the precision of the signal ρ increases the likelihood of covenant violation (and decreases the face value). However, for firms with low manipulation cost (low c), an increase in precision ρ may decrease the likelihood of covenant violation (see Fig. 6).

The non-monotone effect of the cost of misreporting, c, and precision, ρ, on the various aspects of the optimal debt contract (likelihood of covenant violation and face value) demonstrates the need for a theory to better understand the empirical consequences of manipulation on debt contracts and the cost of capital. Univariate analysis and linear relations regressions may lead to inconclusive and somewhat puzzling results.

To address this problem, one could consider measuring the cost of capital as given by

$$\frac{V\left(\tau^{FB}\right)-I-\Pi^{*}}{V\left(\tau^{FB}\right)-I}. \tag{9}$$

This measure captures the value destroyed by the misreporting friction, which has two components: i) investment distortions, given by $V\left(\tau^{FB}\right)-V\left(\tau^{*}\right)$, and ii) expected misreporting costs $\chi\left(\tau^{*}\right)$ (relevant when the contract includes a covenant). In principle, one could estimate the model's parameters, and the above measure of the cost of debt capital, using data on interest rates, covenants, reports and frequency of violations.



(a) The effect of precision ρ for small manipulation cost. $c = .001$ and $I = .4, L = .38$.

(b) The effect of precision ρ for large manipulation cost. $c = 0.1$ and $I = .4, L = .35$.

Fig. 6 The figure shows the evolution of the termination threshold τ^* as precision ρ increases. For low ρ, the contract does not include a covenant and $\tau^* = 0$, so there is over-continuation. Though the first best threshold τ^{FB} increases in ρ, the equilibrium threshold may sometimes decrease in ρ, when c and $I - L$ are small (left panel)

3.2 Intuition

The formal derivation of the equilibrium requires us to first establish some properties that must hold in any equilibrium and to use these properties to solve for the optimal debt contract. Given the length of the formal derivation, we first provide economic intuition for the main results, and defer the full formal derivation to the Appendix.

The Main trade-off As indicated earlier, the manager can always offer a contract that implements the first-best continuation decision, τ^{FB}, by adjusting the covenant z. However, implementing such a contract is in general too costly in terms of expected manipulation cost. On the other hand, the manager can also avoid any manipulation cost by omitting a covenant; however, a no-covenant contract forgoes the value of the real option to terminate the project based on the signal s. As such, when designing the contract, the manager considers the trade-off between the efficiency of the continuation decision and the expected manipulation cost. The choice of z and K determines the termination threshold $\tau(z, K)$, which itself determines the efficiency of the investment continuation $|\tau(z, K) - \tau^{FB}|$ and the expected manipulation cost $C(z, K)$.

Termination threshold and face value are substitutes Given that the capital market is efficient and the lender has no private information, the manager extracts all the rents from the lender so that the lender breaks even. That is, the lender's participation constraint is binding under the optimal debt contract. Since the manager always has an ex-post incentive to continue the project, any optimal debt contract must induce over-termination from the lender's standpoint. To see that, let us assume by contradiction that there is over-continuation from the lender's perspective, namely, that upon observing the threshold signal, the lender prefers to continue the project rather than terminate it. Given that the manager always prefers to continue the project, the manager can decrease the covenant without being required to compensate the lender for it —in contradiction to the assumption. Hence, under the optimal debt contract, the lender strictly prefers to terminate the project when he receives the control rights (i.e., when the covenant is violated). As such, if the manager wants to increase the threshold (by increasing the covenant), the lender will require a higher face value. This implies that under the optimal debt contract, face value and covenant are substitutes; they are alternative ways of paying the lender.

Over-continuation and over-termination As Proposition 1 indicates, the optimal debt contract can induce over-continuation or over-termination (relative to the first-best). When manipulation is sufficiently costly, i.e., c is sufficiently high, the contract induces over-continuation, whereas when c is low the contract either induces over-termination or includes no covenant thereby resulting in over-continuation. Note that the substitution of K and z, discussed above, implies that if the firm increases the termination threshold (by increasing z), the lender will accept a lower face value. A lower face value increases the manager's continuation value and manipulation incentives. Hence, it increases the size of the manipulation interval, $[z - \tau(z, K)]$. Even when the distribution is uniform, increasing the covenant, while keeping the lender's participation constraint binding, affects the manager's expected manipulation cost.

When c is high the manipulation interval $[z - \tau(z, K)]$ is relatively small. This also implies that implementing the first-best termination policy requires a covenant z lower than h. If the manager increases the covenant, such that the termination threshold is higher than the first-best, the face value will be lower, leading to stronger manipulation incentives and a larger manipulation interval. Under a uniform distribution, this will increase the expected manipulation cost and decrease the termination efficiency. Therefore, for high c the manager will never implement a termination threshold that is higher than the first-best. On the other hand, by decreasing the covenant (and the resulting termination threshold) below the one that induces first best, the manager will decrease the manipulation interval and the expected manipulation cost. Decreasing the termination threshold below the first-best impairs the efficiency of the continuation decision. However, around the first best policy τ^{FB}, the investment efficiency loss is second order, relative to the magnitude of expected manipulation costs. Hence, the optimal threshold is lower than the first-best. That is, for high values of c we obtain over-continuation. As c goes to infinity, the manipulation interval vanishes and the manipulation cost is no longer an issue. As a consequence, the optimal debt contract implements the first-best termination threshold.

When c is low, the manipulation interval is large and the manipulation friction is severe. For low c, setting a tight covenant becomes an effective way of curbing manipulation. When c is small, the manipulation interval $[z - \tau(z, K)]$ is relatively large and the expected manipulation cost is high. To implement the first-best termination threshold, the manager needs to set a covenant z that is greater than h. If the manager increases the covenant beyond the one that implements first-best termination, this will also increase the termination threshold and, in response, the face value will go down. This in turn, will increase the manager's incentive to manipulate, and the magnitude of the manipulation by the threshold type $s = \tau$ will be higher. However, since the support of signal is bounded by h, shifting the termination threshold to the right will decrease the size of the manipulation interval, $[h - \tau(z, K)]$, mitigating the likelihood of manipulation and the expected cost of manipulation. Notice that this result does not require that the distribution of the signal s be bounded: the result is present under unbounded distributions. The reason is that as long as the density of the signal vanishes when the signal grows large, an effective way to minimize the expected manipulation cost is to set a high covenant to push the manipulation interval toward the right tail and thereby reduce the likelihood (and expected cost) of manipulation. Here again, there is a trade-off between investment efficiency and expected manipulation costs. Around the first best threshold, τ^{FB}, the investment efficiency loss is second order relative to the magnitude of expected manipulation costs, hence it is beneficial to increase the termination threshold and decrease investment efficiency, relative to first-best, so as to reduce expected manipulation costs. Of course, if the signal is relatively uninformative, for very low values of c it may be too costly to implement a covenant. In such cases, the manager will sacrifice investment efficiency to eliminate any manipulation cost (the case $\rho < \hat{\rho}$).

Comparative statics Proposition 2 demonstrates that the effect of c on the debt contract varies qualitatively based on the cost of manipulation c. Let us consider first the case of large c, for which the optimal debt contract implements over-continuation. An increase in c decreases the likelihood of manipulation and hence decreases the expected manipulation cost (i.e., $\frac{\partial \chi(\tau)}{\partial c} < 0$.). This does not imply the threshold will go up following an increase in c. What matters is the effect of c on the marginal cost of implementing τ, i.e., $\frac{\partial \chi'(\tau)}{\partial c}$, which is negative for low values of τ and positive otherwise (i.e., $\chi(\cdot)$ is an inverted *U-shape*). For large c, the marginal cost of implementing a certain τ , in terms of manipulation costs, decreases in c (i.e., χ becomes flatter) because the manager's propensity to manipulate goes down. In contrast, the marginal benefit of τ in terms of the expected cash flow is independent of the manipulation cost ($\frac{\partial V'(\tau)}{\partial c} = 0$). As such, following an increase in c the optimal debt contract includes a higher covenant (higher termination threshold and likelihood of violation) and a lower face value. This mitigates over-continuation. Now consider the case of small c, for which the optimal debt contract induces over-termination. Similar to the case of large c, an increase in c decreases the likelihood of manipulation required to induce a given termination threshold τ (the function $\chi(\tau)$ becomes more "flat"). The marginal cost of inducing τ is negative for large τ (i.e., $\chi'(\tau) < 0$), but an increase in c makes it less negative $\left(\frac{\partial \chi'(\tau)}{\partial c} > 0\right)$. The reason is that as we increase τ the face value goes down ($\mathcal{K}'(\tau) < 0$), which in turn incentivizes manipulation. However this extra manipulation incentive via the reduction in face value associated with a higher τ becomes weaker as c increases. As such, following an increase in c, the debt contract implements a higher τ^*, thereby reducing the over-termination inefficiency. In other words, a higher manipulation cost c decreases the likelihood of covenant violation and increases the face value K^*.

Consider the effect of the precision of the signal, ρ. Broadly speaking, a precision improvement has two effects: First, it incentivizes termination, because the signal becomes more "useful." Since the signal becomes more informative, it also becomes more attractive, from an investment perspective, to terminate the project conditional on bad news. Second, other things equal, a higher ρ mitigates manipulation incentives because the manager's option value from continuation, conditional on bad news, goes down; after all, the benefit of manipulation is predicated on the possibility that the cash flows are potentially higher than the signal indicates. Now the qualitative effect of precision ρ on the debt contract depends on c. When c is large, such that there is over-continuation, the effect of ρ is intuitive: a higher ρ leads to a higher probability of termination and a lower face value. The intuition is as follows: since the signal becomes more informative, from an investment standpoint, it is efficient to terminate the project more often (τ^{FB} goes up). Indeed, for a given face value, a marginal shift to the left in the manipulation interval $z - \tau$ reduces investment efficiency (relative to a marginal shift to the right) without reducing expected misreporting costs, for a given K. Consistent with this, the contract implements a higher probability of termination to avoid increasing the over-continuation inefficiency, namely the gap between τ^{FB} and the actual probability of termination τ^*. In turn, since the contract provides the lender with greater control rights (and more valuable average continuation decisions) the manager can lower the face value. When c is very small, such that there is over-termination, the effect of precision ρ on τ^* is ambiguous: the probability of termination may go down as the signal becomes more precise, to mitigate the over-termination inefficiency; this is the case when liquidation proceeds L are large. This does not mean the face value will increase in response to a lower likelihood of termination, because ρ also increases the average continuation value of the lender. However, we have not found examples where the face value increases in ρ.

4 Empirical implications

Our paper predicts that when managers have access to sufficiently precise information about future cash flows, the optimal debt contract includes a covenant and induces manipulation. Using measures of "discretionary" accruals, DeFond and Jiambalvo (1994) find that managers use abnormal accruals to avoid debt covenant constraints. Sweeney (1994) finds that managers of firms in technical default make income-increasing accounting changes in periods before the violation, consistent with the debt covenant hypothesis. Dichev and Skinner (2002) find an unusually small number of loan/quarters with financial measures just below covenant thresholds and an unusually large number of loan/quarters with financial measures at, or just above, covenant thresholds. Similar results are found by Dyreng et al. (2011).

Our model suggests that, while the likelihood of covenant violation depends on the quality of a firm's internal control (cost of manipulation), this relationship is ambiguous. This may help explain the seemingly conflicting evidence in Costello and Wittenwerg-Moerman (2011), who find that firms with internal control weaknesses are less likely to use financial covenants and in Kim et al. (2011), who document exactly the opposite.

The literature finds that the consequences of covenant violations vary across firms. Violations for financially healthy firms are typically resolved with low-cost waivers, while troubled firms face serious consequences, including increased interest rates and tighter covenant restrictions. Dyreng et al. (2011) find evidence that firms engage in both real and accruals-based earnings management in order to avoid violating covenants, and document that covenant violations are costly for shareholders: bank intervention following covenant violations appear to change the firm's operations in a way that is suboptimal for equity holders. Our model studies how the possibility of misreporting to avoid the consequence of covenant violations impairs the ability of covenants to induce efficient investment decisions by properly allocating decision rights between debt-holders and shareholders.

Our analysis reveals that the relation between the reliability of the firm's accounting system and debt contract design is subtle and often counterintuitive. On some level, one would think that the possibility of misreporting should reduce the extent to which contracts will rely on financial covenants. Costello and Wittenwerg-Moerman (2011) study the impact of reporting quality, measured by Sarbanes-Oxley internal control reports, on debt contract design, and find that when a firm experiences a material internal control weakness, lenders decrease their use of financial covenants and financial-ratio-based performance pricing provisions. On the other hand, it is natural to think that the possibility of misreporting should tighten covenants up as a means of offsetting the manager's tendency to manipulate the firm's true performance in order to avoid a violation. The tension between these intuitions introduces some ambiguity as to how the accounting system's reliability affects, for example, covenant tightness (i.e., the probability of covenant violations). Our prediction is that, when the accounting system is relatively reliable, a further increase in the accounting system's reliability will lead to tighter covenants and a higher probability of covenant violation. Theoretically, one could use the probability of violation as a measure of the firm's accounting reliability, ceteris paribus. However, when the accounting system is relatively unreliable, improvements in reliability reduce covenant tightness. This non-monotone relation may make it difficult to draw empirical inferences and could weaken the power of empirical tests: tight covenants can, in principle, be a sign of a very high reliability or a very low one.

Another common intuition challenged by our theory is the relation between accounting reliability and interest rates. The evidence, for the most part, suggests that misreporting leads to higher interest rates. For example, Bharath et al. (2008) find that accounting quality has a significant effect on contract design but the effect differs across debt markets. In the case of private debt, both the price and non-price (i.e., maturity and collateral) terms are more stringent for poorer (accrual-based) accounting quality borrowers; this is unlike public debt, where only the price terms are more stringent. The impact of accounting quality on interest spreads of public debt is 2.5 times that of private debt, since the price terms alone reflect the variation in accounting quality. Graham et al. (2008) study the effect of financial restatement on bank loan contracting. Compared with loans initiated before restatement, loans initiated after restatement have significantly higher spreads, shorter maturities, higher likelihood of being secured, and more covenant restrictions. These results are consistent with banks using tighter contract terms to overcome the risk and information problems arising from misreporting. Similarly, Kim et al. (2011) use a sample of firms that disclosed internal control weaknesses (ICW) under Section 404 of the Sarbanes-Oxley Act, and compare the loan contracts of firms with ICW versus firms without ICW. They find that the interest rate and collateral requirements are higher for ICW firms than non-ICW firms by about 28 basis points.

This idea that lower reliability leads to higher interest rates is appealing—insofar as it seems strongly supported by intuition— given that misreporting is in essence an expropriation to debt-holders. To compensate for this potential expropriation —the intuition goes— debt contracts should adjust the interest rate upwards to ensure the lender is willing to provide funding. Sengupta (1998) finds evidence that the interest rate is negatively associated with firms' disclosure quality as evaluated by Financial Analysts; Yu (2005) finds a similar effect on the term structure of credit spreads). Despite its intuitive appeal, this intuition overlooks the fact that debt contracts are multi-dimensional, and the price and non-price terms (covenants) are substitutes. The higher the interest rate given to the lender, the less need there is to give him control rights via tight covenants. And though the misreporting possibility requires that we compensate the lender, it is not clear a a priori whether the most efficient way to do so is by tightening covenants or by increasing the interest rate. If misreporting calls for tighter covenants as the most efficient means to compensate the lender, it will at the same time lead to lower interest rates.

A striking empirical regularity is that covenants are remarkably tight. For example, Chava and Roberts (2006) document that, at inception, the average covenant threshold is only about one standard deviation away from the current value of the accounting ratio in question, so 15%–20% of outstanding loans are in violation during a typical quarter, and conditional on violating a covenant, a loan is delinquent about 40% of the time. Similarly, Roberts and Sufi (2009) find, using a large sample of private credit agreements between U.S. publicly traded firms and financial institutions, that over 90% of long-term debt contracts are renegotiated prior to their stated maturity. Our paper offers some explanation for this observation: tight covenants can be an efficient way to resolve the consequences of misreporting if they lead to renegotiation and effectively to performance pricing.

5 Renegotiation

Garleanu and Zwiebel (2009) argue that "given tight initial covenants, loans often fall into violation very quickly. One direct implication of this tightness is that covenants are frequently renegotiated".

In the baseline model we assume that both parties have full commitment. But the debt contract in the baseline model is not renegotiation-proof. For example, when the contract induces excessive termination, the lender would like to renegotiate the contract by proposing the manager to continue the project in exchange for a higher face value. Since the manager's default option yields zero payoff, he would accept the lender's proposal.

The effect of renegotiation depends on who has the bargaining power at the renegotiation stage. Consider the case where, upon a violation, the lender has the bargaining power and can make a take-it-or-leave-it offer (TIOLI) to the firm. In this case, renegotiation has bite only if the contract entails excessive termination (for if the contract prescribes excessive continuation, then, upon a covenant violation, there is no surplus from renegotiation).[15] From the lender's standpoint, the optimal TIOLI is to offer the manager the possibility to continue —when the expected cash flows from continuation are higher than the liquidation proceeds L— but, in exchange, retain full rights over the firm's cash flows. The firm is indifferent, given that its payof,f conditional on termination, is zero.

This renegotiation possibility, and the associated efficiency gains that it generates, allow the firm to reduce the face value K of the contract. To analyze this case, suppose there is renegotiation on the equilibrium path. In other words, suppose the equilibrium covenant z would lead to excessive termination in the absence of renegotiation, as is the case when c is low. Since renegotiation leads to efficient termination, the optimal contract now solves

$$\max_{\tau} V\left(\tau^{FB}\right) - \hat{\chi}\left(\tau\right),$$

[15] Allowing for renegotiation when the manager meets the covenant —and is thus allowed to continue the project— seems empirically less realistic.

where $\hat{\chi}$ is the expected misreporting cost under the possibility of renegotiation, taking into account that the participation constraint of the lender now becomes

$$F\left(\tau^{FB}\right) L + \int_{\tau^{FB}}^{\tau} E\,(x|s)\,ds + \int_{\tau}^{1} E\left(\min\left(x, \hat{\mathcal{K}}\right)|s\right) ds = I.$$

The lender gets L if the reported signal is below the first best threshold. Otherwise, the lender obtains either all the future cash flow if there is a violation leading to renegotiation, or the minimum of the realized cash flows and the face value in the absence of a violation. Since termination decisions are always efficient under renegotiation, the optimal contract's objective is to minimize expected misreporting costs. If $\hat{\chi}\,(\cdot)$ is decreasing in τ (namely for $\tau > \frac{3\rho-1}{4\rho}$), the threshold is such that $\hat{\mathcal{K}}\,(\tau^*) = 0$ (given the lender's limited liability), so the contract becomes an all-or-nothing contract.[16] Such a contract leads to a renegotiation termination threshold:

$$\tau^* = \frac{\sqrt{L\,(1-\rho) - L^2 + 2I\rho} - \frac{1}{2}\,(1-\rho)}{\rho}.$$

The expected cost of misreporting induced by such a contract is

$$\hat{\chi}\left(\tau^*\right) = \frac{(1-\tau^*)\,(c\tau^* + 2\rho\tau^* + 2\mu - 2\mu\rho - c)}{2},$$

which approaches zero when τ^* is close to one. This contract is optimal when I is large and c is small.

Renegotiation leads here to extremely tight covenants as a way to minimize the probability of misreporting, but violations are waived often, when $s \in \left[\tau^{FB}, \tau^*\right]$. Notice that renegotiation has the potential to mitigate both sources of inefficiency: not only will termination be efficient, but the likelihood of misreporting will decrease with renegotiation (under some circumstances). However, renegotiation will not eliminate misreporting altogether, as long as the manager retains a positive continuation value in equilibrium.

In summary, renegotiation can mitigate manipulation by inducing tighter covenants without inducing excessive termination. The ability to renegotiate the contract may explain why in practice covenants are set tight and often violated, but violations are frequently waived.

6 Unbounded support

The bounded support of the Uniform distribution simplifies the analysis but does not play a significant role in the results. To illustrate this claim, we provide below an example in which the cash flows x and signal s are Log-normally distributed.[17] The

[16] A positive face value would mean the contract has room for increasing τ^*—thereby decreasing the cost of manipulation— while still satisfying the lender's participation constraint.

[17] One could think of x as the firm's true net worth, and r as a report about the firm's net worth.

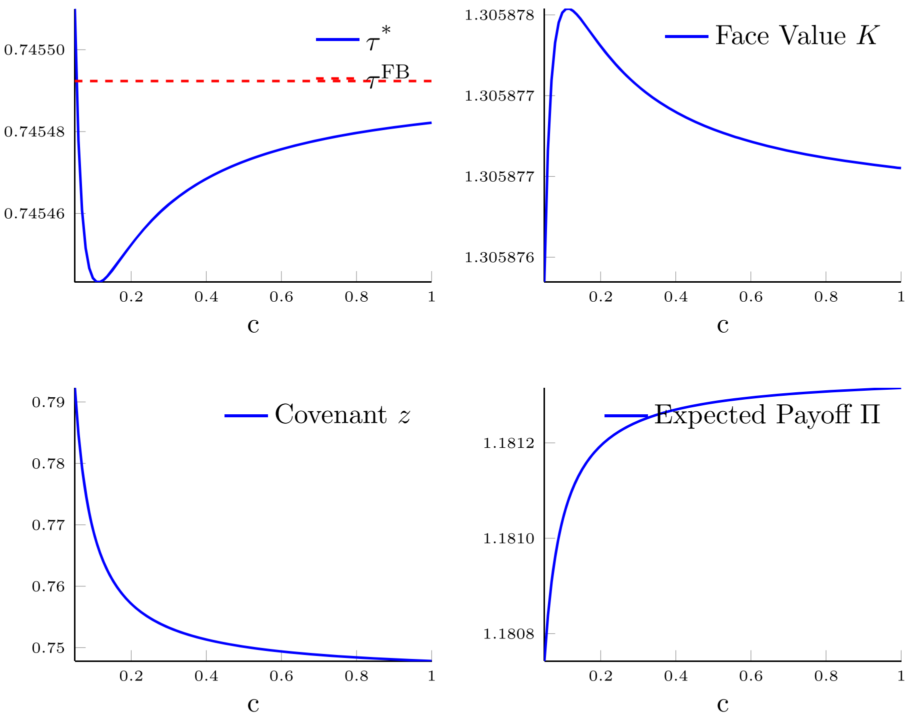


Fig. 7 Effect of misreporting cost c under Log-normal cash flows x. Parameters: $\mu = 0, \sigma = .5, \rho = .95, I = 0.9 * E(x), L = 0.75 * I$

Log-Normal distribution has desirable properties: its support is non-negative (consistent with limited liability) and its pdf is single-peaked (consistent with empirical evidence).

Under the Log-normal distribution the properties of the debt contract qualitatively mirror those under the Uniform distribution. Figure 7 shows that for relatively high (low) cost of misreporting c, there is over-continuation (over-termination). The face value is non monotone in c. By contrast, the covenant z and the manager' expected payoffs Π are monotone in c.

Additional numerical analyses reveal that qualitatively similar results are obtained under truncated normal distribution and exponential distribution.

7 Conclusion

While there is ample empirical evidence that firms can, and often do, manipulate reports in order to avoid costly covenant violation, the theoretical literature has, by and large, overlooked the study of optimal design of debt contracts in the presence of performance manipulation. As has been demonstrated in other contracting settings, the ability to manipulate a report may have a significant qualitative affect on the optimal contract. This paper tries to fill this gap by studying how debt contracts are set when the manager (borrower) can manipulate the report over which the covenant is written.

We consider a setting in which covenants are based on a report that can be manipulated by the manager in order to avoid a covenant violation. In the absence of ability to manipulate the report, the covenant would transfer control rights to the lender only when termination is efficient. However, implementing efficient termination is not necessarily optimal in the presence of manipulation. The presence of manipulation costs introduces a trade-off between expected manipulation costs and investment efficiency: a contract that aims at implementing efficient termination results in excessive manipulation costs. Our model shows that the optimal resolution of this trade-off can implement either excessive continuation or excessive termination, depending on the parameter values —the precision of the private information and the manipulation costs.

In our model, the manipulation costs, which can be proxied by the firm's corporate governance quality, affect the probability of covenant violations in a non-monotone fashion. When manipulation costs are low (even vanishingly low) and the manager's private information is relatively precise, the optimal debt contract leads to excessive termination as a means of mitigating manipulation incentives. In those cases, perhaps surprisingly, a higher manipulation cost leads to higher interest rates and looser covenants. When manipulation costs are relatively high, the optimal debt contract leads to excessive termination. We show that a lender's bargaining power often mitigates misreporting and leads to more efficient investment choices.

Our model demonstrates that the effect of the firm's environment on the characteristics of the optimal debt contract (tightness of covenant and the face value) is complex. As such, the model can guide future empirical research that studies debt contract sheds new light on the existing literature.

The complexity of the design of debt contracts calls for additional theoretical work to further our understanding of how to optimally design debt contracts. For example, to the best of our knowledge, there exists no theoretical guidance on how to optimally design contracts that include multiple covenants written over different performance measures.

First we prove a number of intermediate results.

Appendix

Proof of Lemma 1 Observe that

$$E\left(\min\left(x,K\right)|s\right)=\begin{cases}\rho s+(1-\rho)\left(\int_0^K \frac{x}{h}dx+\int_K^h \frac{K}{h}dx\right) & \text{if } s<K\\ \rho K+(1-\rho)\left(\int_0^K \frac{x}{h}dx+\int_K^h \frac{K}{h}dx\right) & \text{if } s>K\end{cases}.$$

Using the lender's participation constraint, while assuming $\tau<K$, yields

$$\frac{\tau}{h}L+\int_{\tau}^{h}E\left[\min\left(x,\mathcal{K}\right)|s\right]f\left(s\right)ds=I.$$

Solving for $\mathcal{K}$ yields

$$\mathcal{K}(\tau) = h - \sqrt{\frac{2h(\tau L - Ih) + (h-\tau)\left(\rho\tau h + h^2\right)}{\rho\tau + h - \tau}}.$$

The function $\mathcal{K}(\cdot)$ is defined over $[0, \hat{\tau}]$ where $\hat{\tau} \in (0, h)$ is given by the solution to

$$\mathcal{K}\left(\hat{\tau}\right) = h.$$

Furthermore, observe that $\frac{d\mathcal{K}(\tau)}{d\tau}|_{\tau=0} = h - \sqrt{h^2 - 2Ih} < 0$. Also, it's easy to verify that $\frac{d\mathcal{K}(\tau)}{d\tau}|_{\tau=\hat{\tau}} = \infty$. The equation $\frac{d\mathcal{K}(\tau)}{d\tau} = 0$ has two solutions. We argue by contradiction that only one of the two solutions is in $[0, \hat{\tau}]$. By the Intermediate Value Theorem, at least one solution must lie in this interval, given that $\frac{d\mathcal{K}}{d\tau}|_{\tau=0} < 0$ and $\frac{d\mathcal{K}}{d\tau}|_{\tau=\hat{\tau}} > 0$. If both solutions lay in this interval, then the equation

$$\frac{d\mathcal{K}(\tau)}{d\tau} = 0$$

would have at least three solutions, which is a contradiction. Hence, the function $\mathcal{K}(\tau)$ has a unique minimum over the interval $\left[0, \hat{\tau}\right]$, denoted τ^+ , given by

$$\tau^+ \equiv \arg\min_{\tau\in[0,\hat{\tau}]} \mathcal{K}(\tau).$$

□

Remark 1 In equilibrium $\tau^* \leq K^*$.

Proof of Remark 1 In equilibrium, the lender's participation constraint is binding (otherwise, slightly decreasing the contract's face value K, while holding constant the termination threshold τ, would satisfy the lender's participation constraint and increase the manager's expected payoff, despite increasing the expected manipulation cost). Hence,

$$I = \Pr\left(s < \tau^*\right) L + \Pr\left(s > \tau^*\right) E\left(\min\{x, K\}|s > \tau^*\right).$$

For any $s > K$ the continuation value to the lender is independent of s since

$$E(\min\{x, K\}|s) = \rho K + (1-\rho)\int_0^h \min\{x, K\} f(x)\,dx.$$

Suppose $\tau^* > K$. This implies that the lender's expected payoff from continuation, given τ^*, is strictly greater than his payoff upon termination, which is L. That is

$$E\left(\min\{x, K\}|s = \tau^*\right) > I > L.$$

Therefore, the lender strictly prefers to continue the project when $s = \tau^*$. The manager always prefers to continue the project. This implies there is a feasible contract that offers the lender the same face value and a lower threshold, and does not induce higher expected manipulation cost. Such a contract Pareto dominates the assumed contract; hence, a contract with $\tau^* > K^*$ is suboptimal. □

Proof of Lemma 2 We need to show that in any equilibrium $\mathcal{K}'(\tau) \leq 0$. Let $\{z^*, K^*\}$ be the equilibrium threshold and face value, then the manager's expected payoff can be written as

$$\int_{\tau^*}^{h} \left(\mathbb{E}\left[\max\left(x - K^*, 0\right)|s\right] f(s)\, ds - c\max\left(z^* - s, 0\right)\right) ds, \tag{10}$$

where z^* is the solution to the manager's misreporting constraint,

$$c\left(z^* - \tau^*\right) = \mathbb{E}\left(\left(x - K^*\right)^+ |s = \tau^*\right). \tag{11}$$

Let us consider an alternative contract $\{z^o, K^*\}$ such that $z^o = z^* - \varepsilon$ for small $\varepsilon > 0$. Define s^o as the solution to Eq. (11) such that $c\,(z^o - s^o) = \mathbb{E}\,(\max(x - K^*)\,|s^o)$. From Eq. (11), we see that $z^o < z^*$ implies $s^o < \tau^*$. If $\mathcal{K}'(\tau^*) > 0$ then the contract $\{z^o, K^*\}$ is feasible. Indeed, recall that the set of feasible contracts is defined by

$$\left\{\{\tau, K\} : K \geq \mathcal{K}(\tau), \tau \in \left[0, \hat{\tau}\right]\right\},$$

where, as shown before, $\mathcal{K}(\cdot)$ is a U-shaped function over $\left[0, \hat{\tau}\right]$. We will consider two cases: $\tau^* \leq K$ and $\tau^* > K$.

When $\tau^* \leq K$ a manager with a signal $s = \tau^*$ obtains positive expected payoff only if the signal is wrong, in which case his payoff from continuation is independent of τ^*. This implies that $z^* - \tau^*$ is independent of τ^*. In such a case, offering the contract $\{z^o, K^*\}$ is preferable to the lender, increases the likelihood of continuation (which is beneficial to the manager), and will have no effect on the manager's expected manipulation cost. As such, the contract $\{z^o, K^*\}$ is feasible and strictly dominates the contract $\{z^*, K^*\}$ for which $\mathcal{K}'(\tau^*) > 0$.

When $\tau^* > K$ a manager with a signal $s = \tau^*$ obtains positive expected payoff both when the signal is informative and when it is uninformative. In this case, the manager's expected payoff from continuation is increasing in τ^*. This implies that $z^* - \tau^*$ is also increasing in τ^*. To show that the contract $\{z^o, K^*\}$ dominates $\{z^*, K^*\}$ for which $\mathcal{K}'(\tau^*) > 0$ note that decreasing τ^* to s^0 decreases the magnitude of the manipulation for each $s \in \left[\tau^*, h\right]$; hence, the expected payoff of these types is higher under the contract $\{z^o, K^*\}$. All types $s \in \left(s^0, \tau^*\right)$ (who didn't manipulate and terminated the project under $\{z^*, K^*\}$) prefer to manipulate and continue the project and hence prefer $\{z^o, K^*\}$ over $\{z^*, K^*\}$. Since the lender also prefers the contract $\{z^o, K^*\}$, this contract is feasible and strictly dominates the contract $\{z^*, K^*\}$ for which $\mathcal{K}'(\tau^*) > 0$. □

Lemma 3 $z^* > h \rightarrow \frac{dz^*}{dc} < 0$.

Proof of Lemma 3 Let $\zeta(\tau)$ be the covenant that induces a cutoff τ when the face value is $\mathcal{K}(\tau)$, or:

$$\zeta(\tau) = \frac{(1-\rho)}{2c} \frac{(\mathcal{K}(\tau) - h)^2}{h} + \tau.$$

Now, $\frac{\partial \zeta(\tau)}{\partial c} < 0$. Also we know that $\mathcal{K}'(\tau^*) \leq 0$, which implies that $\left.\frac{d\zeta(\tau)}{d\tau}\right|_{\tau=\tau^*} > 0$ (given Lemma 2). Finally, by Lemma 4, we know that $z^* > h \Rightarrow \frac{d\tau^*}{dc} < 0$. Taken together these results imply that when $z^* = h$,

$$\frac{dz^*}{dc} = \left.\frac{\partial \zeta(\tau)}{\partial c}\right|_{\tau=\tau^*} + \left.\frac{\partial \zeta(\tau)}{\partial \tau}\right|_{\tau=\tau^*} \frac{d\tau^*}{dc} < 0.$$

□

Lemma 4 *Suppose $\rho \geq \hat{\rho}$, then there is a unique $\tilde{c} > \hat{c}$ such that $z^* > h$ if and only if $c < \tilde{c}$.*

Proof of Lemma 4 Clearly $\lim_{c\to\infty} z^* = \tau^{FB} < h$. Also, when ρ is large and c is small we have $z^* > h$ and

$$\tau^* = \arg \max_{\tau\in[0,\hat{\tau}]} \left\{ V(\tau) - \int_{\tau}^{h} c(\zeta(\tau) - s) f(s) ds \right\}.$$

Taking limits $\lim_{c\to 0} \tau^* = \frac{h - \sqrt{\frac{h\left(h\rho - 4\rho I + 2\rho L - 2L + 2\rho^2 I + 2I\right)}{\rho}}}{1-\rho} > \tau^{FB}$, hence $\lim_{c\to 0} z^* = \infty$.

Finally, Lemma 3 proves that $z^* = h \to \frac{dz^*}{dc} < 0$, so there is only one value $\tilde{c}$ such that $z^* = h$, and $z^* < h$ if and only if $c > \tilde{c}$. Now, when $z^* = h$, the optimal threshold satisfies

$$\begin{aligned} V'\left(\tau^*\right) &= \chi'\left(\tau^*\right) \\ &= -2c \frac{\left(\frac{1-\rho}{2c}\right)^2 (h - \mathcal{K}(\tau^*))^3}{h^3} \mathcal{K}'\left(\tau^*\right) > 0, \end{aligned}$$

so at $c = \tilde{c}$ there is over-continuation, i.e., $\tau^*(\tilde{c}) < \tau^{FB}$. □

Lemma 5 *If $c \geq \tilde{c}$ the equilibrium entails over-continuation. Formally $\tau^* \leq \tau^{FB}$.*

Proof of Lemma 5 See proof of Proposition 1. □

Corollary 2 *There is $\bar{\rho}$ such that for all $\rho \geq \bar{\rho}$ and all $c > 0$ the optimal debt contract includes a covenant.*

Proof of Corollary 2 First, we show that using a covenant is optimal when $\rho \in (1-\varepsilon, 1)$, even as $c \to 0$. The optimal threshold, in the absence of manipulation, is defined as:

$$\tau^{FB} \equiv \arg\max_{\tau} \left\{ F(\tau) L + \int_{\tau}^{h} E(x|s) f(s) ds \right\}.$$

By assumption $V^{FB} > \mathbb{E}(x)$. We argue that for ρ high enough, the debt contract includes a covenant, even as $c \to 0$. Suppose we implement τ^{FB} as the contract's threshold (perhaps sub-optimally) and set the face value accordingly at $\mathcal{K}\left(\tau^{FB}\right)$ to satisfy the lender's participation constraint. Assuming the contract leads to $z\left(\tau^{FB}\right) > h$ (which we can always guarantee by making c small enough), then the expected payoff of the manager is

$$\begin{aligned}\Pi\left(\tau^{FB}|\rho\right)+I &= V^{FB}-\chi\left(\tau^{FB}\right)\\ &> V^{FB}-(h-\tau)(1-\rho)\frac{\left(2h(\tau L-Ih)+(h-\tau)\left(\rho\tau h+h^2\right)\right)}{2h^2(\rho\tau+h-\tau)}\end{aligned}$$

Now, since

$$\begin{aligned}&\lim_{\rho\to 1}\left\{V^{FB}-(h-\tau)(1-\rho)\frac{\left(2h(\tau L-Ih)+(h-\tau)\left(\rho\tau h+h^2\right)\right)}{2h^2(\rho\tau+h-\tau)}\right\}\\ &= V^{FB} > E(x).\end{aligned}$$

This means we can select ρ close to 1, denoted ρ^+, to ensure

$$\Pi\left(\tau^{FB}|\rho^+\right)+I > E(x).$$

Of course, for a fixed c, a large ρ may lead to $\zeta\left(\tau^{FB}\right) < h$. So to ensure $\zeta\left(\tau^{FB}\right) > h$, we pick c small enough, say c_0, such that

$$\begin{aligned}\Pi\left(\tau^{FB}|\rho^+\right)+I &= \underbrace{V^{FB}-\left(h-\tau^{FB}\right)\left(1-\rho^+\right)\frac{\left(2h\left(\tau^{FB}L-Ih\right)+\left(h-\tau^{FB}\right)h\left(\rho^+\tau^{FB}+h\right)\right)}{2h^2\left(\rho^+\tau+h-\tau^{FB}\right)}}_{>E(x)}\\ &+\frac{1}{2}\frac{c_0\left(h-\tau^{FB}\right)^2}{h} > \mathbb{E}(x).\end{aligned}$$

This proves that using a covenant is optimal for sufficiently high ρ, even as $c \to 0$. □

Lemma 6 $\lim_{c\to 0}\tau^* = \frac{h-\sqrt{\frac{h\left(h\rho-4\rho I+2\rho L-2L+2\rho^2 I+2I\right)}{\rho}}}{1-\rho} > \tau^{FB}$ *and when* $z^* > h$, $\lim_{\rho\to 1}\tau^* = \frac{L-ch}{1-c} < \tau^{FB}$.

Proof of Lemma Lemma 6 To obtain the $\lim_{c\to 0}\tau^*$ we take the first order condition of the manager's optimization program with respect to τ, assuming $\tau^* > h$:

$$V'(\tau)-\chi'(\tau)=0,$$

which leads to the first order condition

$$\frac{\left(2c\rho^2 - 4c\rho + 2c\right)\tau^3 + \left(h\rho^2 - h\rho^3 - 6ch + 8ch\rho - 2ch\rho^2\right)\tau^2}{2h\left(\rho\tau + h - \tau\right)^2} + \frac{\left(6ch^2 - 4ch^2\rho - 2h^2\rho^2\right)\tau + \left(2h^2\rho^2 I - 2ch^3 + 2Lh^2\rho - 2h^2\rho I\right)}{2h\left(\rho\tau + h - \tau\right)^2} = 0$$

Now, taking the limit as $c \to 0$ and solving for τ yields

$$\lim_{c\to 0}\tau^* = \frac{h - \sqrt{\frac{h\left(h\rho - 4\rho I + 2\rho L - 2L + 2\rho^2 I + 2I\right)}{\rho}}}{1-\rho}.$$

We argue that $\lim_{c\to 0}\tau^* > \tau^{FB}$. In effect,

$$\lim_{\rho\to 1}\left[\lim_{c\to 0}\tau^* - \tau^{FB}\right] = 0.$$

and

$$\lim_{\rho\to 1}\left[\frac{d\left(\lim_{c\to 0}\tau^* - \tau^{FB}\right)}{d\rho}\right] = -\frac{1}{2}\frac{h^2 + L^2 - 2Ih}{h} < 0.$$

When $\rho > \hat{\rho}$, and $c \le \tilde{c}$, the optimal debt contract includes a covenant. Taking the limit of the FOC as $\rho \to 1$ and solving for τ^* yields

$$\lim_{\rho\to 1}\tau^* = \frac{L - ch}{1 - c} < \lim_{\rho\to 1}\tau^{FB} = L.$$

□

Corollary 3 *When $z^* \le h$, $\frac{\partial\tau^*}{\partial c} > 0$ and $\frac{\partial K^*}{\partial c} < 0$.*

Proof of Corollary 3 From Lemma 5, we know that $z^* \le h \Rightarrow \chi'(\tau^*) > 0$. Hence $\frac{\partial\chi'(\tau^*)}{\partial c} = -\frac{1}{c}\chi'(\tau^*) < 0$. Now the first order condition is

$$\Pi^*_\tau = 0 \Rightarrow \frac{\partial\tau^*}{\partial c} = -\frac{\Pi^*_{\tau c}}{\Pi^*_{\tau\tau}} = -\frac{-\frac{\partial\chi'(\tau^*)}{\partial c}}{\Pi^*_{\tau\tau}}$$
$$\Rightarrow \operatorname{sign}\left(\frac{\partial\tau^*}{\partial c}\right) = \operatorname{sign}\left(\frac{-\partial\chi'(\tau^*)}{\partial c}\right) > 0$$

This in turn implies that the face value decreases in c. □

Corollary 4 *When $z^* \ge h$, $\frac{\partial\tau^*}{\partial c} < 0$ and $\frac{\partial K^*}{\partial c} > 0$.*

Proof of Corollary 4 For a given τ (such that $\zeta(\tau) \geq h$) we can write the manager's expected payoff as

$$\Pi(\tau|c) + I \equiv V(\tau) - \frac{1}{2}c(h-\tau)\frac{\frac{(1-\rho)}{c}\frac{\frac{2h(\tau L - Ih)+(h-\tau)\left(\rho\tau h+h^2\right)}{\rho\tau+h-\tau}}{h} + \tau - h}{h}.$$

The cross partial derivative is

$$\Pi_{\tau c} = -\frac{h-\tau}{h} < 0,$$

which means that in equilibrium

$$\frac{\partial\tau^*}{\partial c} = -\frac{\Pi^*_{\tau c}}{\Pi^*_{\tau\tau}} < 0.$$

□

Proof of Proposition 1 First we show that the contract includes a covenant if and only if precision ρ is high enough. Recall that $\hat{\rho}$ is defined by

$$\max_{\tau\in[0,\tau^+]} V\left(\tau|\hat{\rho}\right) - \chi\left(\tau|\hat{\rho}\right) = E(x).$$

$\hat{\rho}$ is the precision level such that the firm is indifferent between using a covenant and not using one. Next we argue that the firm will use a covenant if and only if $\rho \geq \hat{\rho}$. Suppose, by contradiction, that for some $\rho \in \left(\hat{\rho}, 1\right)$ the firm does not use a covenant. Then we have that

$$\max_{\tau}\{V(\tau|\rho) - \chi(\tau|\rho)\} < E(x).$$

To prove that this leads to a contradiction, we construct a feasible contract that yields payoffs above $E(x) - I$. Indeed, consider a contract implementing the same threshold $\tau^*\left(\hat{\rho}\right)$ and face value $K^*\left(\hat{\rho}\right)$ as under precision $\hat{\rho}$. This new contract strictly satisfies the lender's participation constraint. Furthermore,

$$\begin{aligned}\int_{\tau^*(\hat{\rho})}^{h} E^{\rho}\left[(x-K)^+|s\right] f(s)\,ds - \int_{\tau^*(\hat{\rho})}^{z(\hat{\rho}|\rho)} c\left(z\left(\hat{\rho}|\rho\right)-s\right) f(s)\,ds &> \max_{\tau\in[0,\tau^+]} V\left(\tau|\hat{\rho}\right) - \chi\left(\tau|\hat{\rho}\right)\\ &\geq E(x),\end{aligned}$$

Hence, the contract we have constructed yields both higher continuation cash flows and lower misreporting costs under ρ than the optimal debt contract under $\hat{\rho}$, hence it must dominate a no-covenant contract. To prove the other direction suppose $\rho < \hat{\rho}$, but the debt contract includes a covenant, so

$$\max_{\tau} V(\tau|\rho) - \chi(\tau|\rho) > E(x).$$

Then consider using $\{\tau^*(\rho), K^*(\rho)\}$ under precision $\hat{\rho}$. If this contract was feasible under precision ρ it must also be feasible under $\hat{\rho}$ given that $\hat{\rho} > \rho$. Clearly, this contract must give the manager a higher payoff under $\hat{\rho}$ than under ρ, hence

$$\begin{aligned}\int_{\tau^*(\rho)}^{h} E^{\hat{\rho}}\left[(x-K)^+ |s\right] f(s)\, ds - \int_{\tau^*(\rho)}^{z(\rho|\hat{\rho})} c\left(z\left(\rho|\hat{\rho}\right)-s\right) f(s)\, ds &> \max_{\tau} V(\tau|\rho) - \chi(\tau|\rho) \\ &> E(x).\end{aligned}$$

which is a contradiction. Next, we show that when $\rho \geq \hat{\rho}$ there is over-continuation if and only if the cost of misreporting is higher than $\hat{c}$. Define $\tau^*(c)$ as

$$\tau^*(c) \equiv \arg\max_{\tau} \{V(\tau) - \chi(\tau|c)\}.$$

First, Corollary 2 demonstrates that $\lim_{c\downarrow 0} \tau^*(c) > \tau^{FB}$. Also, Corollary 3 and Corollary 4 prove that $\tau^*(c)$ decreases (resp. increases) in c for $c \geq \tilde{c}$ (resp. $c < \tilde{c}$) where $\tilde{c}$ is defined as $\zeta(\tau^*(\tilde{c})) = h$ (i.e, the value of c such that the equilibrium covenant is equal to h). Finally, $\lim_{c\to\infty} \tau^*(c) = \tau^{FB}$. Taken together these observations establish that there is a unique $\hat{c}$ defined

$$\tau^*\left(\hat{c}\right) = \tau^{FB}$$

such that if $c \leq \hat{c}$ (resp. $c > \hat{c}$) there is over-termination (resp. over-continuation).

Consider uniqueness of the optimal threshold τ^*. When $z^* \geq h$ the proof follows by contradiction. In this case, the first order condition of $V'(\tau) = \chi'(\tau)$ is a third order polynomial and has at most three real solutions, but the smallest solution is negative. The other two consecutive solutions cannot be both maxima, hence the maximum must be unique. When $z^* < h$ the first order condition is a fourth order polynomial, so there can be (at most) two local maxima of $\Pi(\tau)$. Now, we will show that one of the maxima lies on $[\frac{h}{1-\rho}, \infty)$, being outside the relevant range. Indeed,

$$\lim_{\tau\downarrow\frac{h}{1-\rho}} \Pi(\tau) = -\infty$$

and

$$\lim_{\tau\to\infty} \Pi(\tau) = -\infty.$$

This means there is at least one local maxima in $[\frac{h}{1-\rho}, \infty)$ which proves the optimal threshold is unique. □

Proof of Proposition 2 First we prove the comparative statics with respect to c. As before, we define

$$\tau^*(c) \equiv \arg\max_{\tau\in[0,h]} \{V(\tau) - \chi(\tau|c)\}.$$

(Note that $\tau^*(c)$ is not necessarily the optimal threshold since the optimal debt contract may use no covenant, in which case $z^* = \tau^* = 0$.)

Lemma 3 proves there is a unique value of c, denoted $\tilde{c}$, such that the covenant is $z^* = h$. If $c < \tilde{c}$ (resp. $c > \tilde{c}$) the covenant is larger (smaller) than h. On the other hand,

$$\frac{\partial \tau^*(c)}{\partial c} = -\frac{\Pi_{\tau c}}{\Pi_{\tau\tau}}.$$

Hence, $\text{sign}\left\{\frac{\partial \tau^*(c)}{\partial c}\right\} = \text{sign}\left(\frac{-\partial \chi'(\tau^*(c))}{\partial c}\right)$. Now when $c < \tilde{c}$, we have $\frac{-\partial \chi'(\tau^*(c))}{\partial c} < 0$, hence $\frac{\partial \tau^*(c)}{\partial c} < 0$. When $c > \tilde{c}$, we have $\frac{-\partial \chi'(\tau^*(c))}{\partial c} = -\frac{1}{c}\chi'(\tau^*(c)) < 0$, and $\chi'(\tau^*(c)) > 0$, hence $\frac{\partial \tau^*(c)}{\partial c} > 0$. Also, since $\frac{\partial K^*}{\partial c} = \mathcal{K}'(\tau^*)\frac{\partial \tau^*(c)}{\partial c}$ the sign $\left(-\frac{\partial K^*}{\partial c}\right) = \text{sign}\left(\frac{\partial \tau^*(c)}{\partial c}\right)$, since $\mathcal{K}'(\tau^*) < 0$. Consider the comparative statics with respect to ρ. Consider the $z^* > h$ case. We have:

$$\lim_{\rho\to 1} \tau^* = \frac{L - hc}{1 - c}$$

$$\lim_{\rho\to 1} \frac{\partial \tau^*}{\partial \rho} = \lim_{\rho\to 1}\left(-\frac{\Pi_{\tau\rho}}{\Pi_{\tau\tau}}\right) = \frac{\frac{-4\tau L + 2Ih + 2hL + 3\tau^2 - 4\tau h}{2h^2}}{-\Pi_{\tau\tau}}.$$

For $c \approx 0$, we have $\tau^* \approx L$. Plugging $\tau^* \approx L$ above yields $\lim_{\rho\to 1}\frac{\partial \tau^*}{\partial \rho} \approx \frac{-L^2 + 2h(I-L)}{-\Pi_{\tau\tau} 2h^2}$ which is negative as $L \to I$. This shows that τ^* may decrease in ρ, for sufficiently high ρ and small c. However, we can verify that even in this case, we have $\lim_{\rho\to 1, c\to 0}\frac{\partial K^*}{\partial \rho} < 0$. Consider the $z^* < h$ case. It's easy to verify $\lim_{\rho\to 1} \Pi^*_{\tau\rho} = \frac{E(x) - \tau^*}{4h}$. Since $\tau^* < E(x)$, this implies that $\lim_{\rho\to 1}\frac{\partial \tau^*}{\partial \rho} > 0$. $\square$

References

Aghion, P., & Bolton, P. (1992). An incomplete contracts approach to financial contracting. *The Review of Economic Studies*, *59*(3), 473–494.

Ball, R., Bushman, R.M., Vasvari, F.P. (2008). The debt-contracting value of accounting information and loan syndicate structure. *Journal of Accounting Research*, *46*(2), 247–287.

Beneish, M.D., & Press, E. (1993). Costs of technical violation of accounting-based debt covenants. *The Accounting Review*, *68*(2), 233–257.

Beyer, A. (2013). Conservatism and aggregation: The effect on cost of equity capital and the efficiency of debt contracts. Rock Center for Corporate Governance at Stanford University Working Paper, (120).

Beyer, A., Guttman, I., Marinovic, I. (2014). Optimal contracts with performance manipulation. *Journal of Accounting Research*, *52*(4), 817–847.

Bharath, S.T., Sunder, J., Sunder, S.V. (2008). Accounting quality and debt contracting. *The Accounting Review*, *83*(1), 1–28.

Caskey, J., & Hughes, J.S. (2012). Assessing the impact of alternative fair value measures on the efficiency of project selection and continuation. *The Accounting Review*, *87*(2), 483–512. https://doi.org/10.2308/accr-10201.

Chava, S., & Roberts, M.R. (2006). Is financial contracting costly? an empirical analysis of debt covenants and corporate investment. Rodney l white center for financial research-working papers-, 19.

Cornelli, F., & Yosha, O. (2003). Stage financing and the role of convertible securities. *The Review of Economic Studies*, *70*(1), 1–32.

Costello, A., & Wittenwerg-Moerman, R. (2011). The impact of financial reporting quality on debt contracting: evidence from internal control weakness reports. *Journal of Accounting Research*, *49*(1), 97–136.

DeFond, M.L., & Jiambalvo, J. (1994). Debt covenant violation and manipulation of accruals. *Journal of Accounting and Economics*, *17*(1), 145–176.
Dessein, W. (2005). Information and control in ventures and alliances. *The Journal of Finance*, *60*(5), 2513–2549.
Dichev, I.D., & Skinner, D.J. (2002). Large–sample evidence on the debt covenant hypothesis. *Journal of Accounting Research*, *40*(4), 1091–1123.
Dutta, S., & Fan, Q. (2014). Equilibrium earnings management and managerial compensation in a multiperiod agency setting. *Review of Accounting Studies*, *19*(3), 1047–1077.
Dye, R. (1988). Earnings management in an overlapping generations model. *Journal of Accounting research*, *26*, 195–235.
Dyreng, S., Hillegeist, S., Penalva, F. (2011). Earnings management to avoid debt covenant violations and future performance. Technical report Working Paper, Duke University, Arizona State University.
Fischer, P., & Verrecchia, R. (2000). Reporting bias. *The Accounting Review*, *75*(2), 229–245.
Gao, P. (2013). A measurement approach to conservatism and earnings management. *Journal of Accounting and Economics*, *55*(2), 251–268.
Garleanu, N., & Zwiebel, J. (2009). Design and renegotiation of debt covenants. *Review of Financial Studies*, *22*(2), 749–781.
Gigler, F., Kanodia, C., Sapra, H., Venugopalan, R. (2009). Accounting conservatism and the efficiency of debt contracts. *Journal of Accounting Research*, *47*(3), 767–797.
Goex, R.F., & Wagenhofer, A. (2009). Optimal impairment rules. *Journal of Accounting and Economics*, *48*(1), 2–16.
Graham, J.R., Li, S., Qiu, J. (2008). Corporate misreporting and bank loan contracting. *Journal of Financial Economics*, *89*(1), 44–61.
Grossman, S.J., & Hart, O.D. (1986). The costs and benefits of ownership: a theory of vertical and lateral integration. *Journal of Political Economy*, *94*(4), 691–719.
Guttman, I., Kadan, O., Kandel, E. (2006). A rational expectations theory of kinks in financial reporting. *The Accounting Review*, *81*(4), 811–848.
Hart, O.D. (1995). *Firms, contracts, and financial structure*. Oxford: Clarendon. ISBN: 978-0-19-828881-7.
Hart, O., & Moore, J. (1990). Property rights and the nature of the firm. *Journal of Political Economy*, *98*(6), 1119–1158.
Hebert, B. (2015). Moral hazard and the optimality of debt. Available at SSRN 2185610.
Innes, R.D. (1990). Limited liability and incentive contracting with ex-ante action choices. *Journal of Economic Theory*, *52*(1), 45–67.
Kim, J.B., Song, B.Y., Zhang, L. (2011). Internal control weakness and bank loan contracting: evidence from sox section 404 disclosures. *The Accounting Review*, *86*(4), 1157–1188.
Laux, V. (2017). Debt covenants, renegotiation, and accounting manipulation.
Li, J. (2013). Accounting conservatism and debt contracts: efficient liquidation and covenant renegotiation. *Contemporary Accounting Research*, *30*(3), 1082–1098.
Liang, P.J. (2000). Accounting recognition, moral hazard, and communication*. *Contemporary Accounting Research*, *17*(3), 458–490.
Roberts, M.R., & Sufi, A. (2009). Renegotiation of financial contracts: evidence from private credit agreements. *Journal of Financial Economics*, *93*(2), 159–184.
Sengupta, P. (1998). Corporate disclosure quality and the cost of debt. *The Accounting Review*, *73*(4), 459–474.
Sridhar, S.S., & Magee, R.P. (1996). Financial contracts, opportunism, and disclosure management. *Review of Accounting Studies*, *1*(3), 225–258.
Stein, J.C. (1989). Efficient capital markets, inefficient firms: a model of myopic corporate behavior. *The Quarterly Journal of Economics*, *104*(4), 655–669.
Sweeney, A.P. (1994). Debt-covenant violations and managers' accounting responses. *Journal of accounting and Economics*, *17*(3), 281–308.
Townsend, R.M. (1979). Optimal contracts and competitive markets with costly state verification. *Journal of Economic Theory*, *21*(2), 265–293.
Yu, F. (2005). Accounting transparency and the term structure of credit spreads. *Journal of Financial Economics*, *75*(1), 53–84.