Staking, Token Pricing, and Crypto Carry

(joint with Zhiheng He and Ke Tang)

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Blockchain & DeFi Innovations

- Blockchain as decentralized consensus:
  - Cong & He, (2019); Chen, Cong, & Xiao, (2020); etc.
  - Single point of failure, systemic risk, market power, data silos.
  - Supply chains, smart contracting, secure-MPC, etc.

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- Tokenomics:
  - Categories and Functions of Crypto-tokens (Cong & Xiao, 2020; Cong, Karolyi, Tang, & Zhao, 2021; Cong & Wilson, 2022).
  - General Payment Tokens, Platform Tokens, Ownership/Product Tokens, Security Tokens.
  - Dynamic adoption, token pricing, and token-based monetary policy on platforms: Cong, Li, & Wang (2021a,b).

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- Decentralized Finance (DeFI, Harvey, Ramachandran, & Santoro, 2021):
  - Stablecoins/payment; lending/yield farming, etc.
  - DeFi U.S. $ 120 B according to CoinMarketCap.

- Contributions:
  1. First studies on asset pricing of staking (also Saleh, John, and Rivera, 2021).
  2. Application of mean field game in finance beyond inequality/macro settings.

Institutional Background of Staking

- Proof-of-Stake (PoS) protocols; inefficiency and environmental costs of PoW (Cong, He, & Li, 2021; Saleh, 2021); Proof-of-Credit (POC) in NULS, etc.; US \$21 B $\rightarrow$ \$326 B (Oct 2021) in a year.
- 60 stakable DeFi assets, 27 masternodes and more than 50 mainstream cryto; 8%-15% on average.
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- (Bank) deposit, interest rate, and currency carry.

Outline

- Introduction and Background
- A Model of Consensus/DeFi Staking
- Empirical Findings
- Discussion and Conclusion

The Staking Economy

- Time is continuous infinite horizon: $t \in [0, \infty)$.
- Native platform token and consumption numeraire.
- Platform productivity (Cong, Li, & Wang, 2021a):

$$\frac{dA_t}{A_t} = \mu^A dt + \sigma^A dZ_t.$$

- Agents: unit measure; indexed by i, characterized by wealth $w_{i,t}$; allocate wealth onchain $(x_t + l_t)$, offchain (n_t), & consumption y_t.
- Token convenience (Cong, Li, & Wang, 2021a,b):

$$dv(x_t) = dv_t = x_t^{1-\alpha}(U(w_t)A_t)^\alpha dt.$$

- Transaction costs in consumption (numeraire convenience, Bansal & Coleman, 1996; Valchev, 2020):

$$\Psi_t = \Psi_t(y_t, n_t) \geq 0, \quad \frac{\partial \Psi}{\partial y} > 0 \quad \& \quad \frac{\partial \Psi}{\partial n} < 0.$$

Staking and Price Process

- Aggregate reward process:

$$R_t = \iota_t Q_t + F_t(Q_t, A_t) = \iota_t Q_t + \tau_t Q_t.$$

- Reward rate/yield:

$$r_t \equiv \frac{R_t}{L_t}, \quad \text{where} \quad L_t \text{ is the aggregate no. staked.}$$

- Slashing rate $c_t < r_t$.
- Endogenous token price:

$$dP_t = P_t \mu_t dt + P_t \sigma_t dZ_t.$$

- Staking ratio:

$$\Theta_t = \Theta(r_t) = \frac{L_t}{Q_t} = \frac{\int_W l(t, w_t; r_t) m(t, w_t) dw_t}{\int_W [x(t, w_t; r_t) + l(t, w_t; r_t)] m(t, w_t) dw_t}.$$

Agents' Optimal Allocation, Staking, and Consumption

- Agent's wealth process:

$$dw_t = [(x_t+l_t)\mu_t+l_t(r_t-c_t)+v_t-y_t-\Psi_t]dt+(x_t+l_t)\sigma_t dZ_t, \quad y_t \le w_t-l_t.$$

- Individual staking as optimal control:

$$\max_{\{y_s,x_s,l_s\}_{s=t}^{\infty}} t\int_t^{\infty} e^{-\phi(s-t)}\mathcal{U}(y_s)ds,$$

- Indirect utility and Hamilton-Jacobi-Bellman (HJB) equation:

$$\begin{aligned} J(t,w_t;r_t) = & \max_{\{y_s,x_s,l_s\}_{s=t}^{\infty}} t\int_t^{\infty} e^{-\phi(s-t)}\mathcal{U}(y_s)ds \\ 0 = & \max_{\{y_t,x_t,l_t\}} \Big\{\mathcal{U}(y_t) - \phi J + f(y_t,x_t,l_t;w_t,r_t)\frac{\partial J(t,w_t;r_t)}{\partial w} \\ & +\frac{\sigma_t^2}{2}(x_t+l_t)^2\frac{\partial^2 J(t,w_t;r_t)}{\partial w^2}\Big\}, \end{aligned}$$

where $f(y_t,x_t,l_t;w_t,r_t) = (x_t+l_t)\mu_t + l_t(r_t-c_t)+v_t-y_t-\Psi_t$.

Solving the Mean Field Game

- System dynamics a la Fokker-Planck; $m(0, w_0) = m_0$.

$$0 = \frac{\partial}{\partial t} m + \frac{\partial}{\partial w} \left[f(y_t, x_t, l_t; w_t, r_t) m \right] - \frac{1}{2} \frac{\partial^2}{\partial w^2} \left[(x_t + l_t)^2 \sigma^2 m \right],$$

where $\quad f(y_t, x_t, l_t; w_t, r_t) = (x_t + l_t)\mu_t + l_t(r_t - c_t) + v_t - y_t - \Psi_t.$

- Agents take reward rate as given but equilibrium belief is consistent: $r_t = \frac{R_t}{Q_t \Theta(r_t)} \rightarrow$ a fixed point problem.

- MFG equilibrium:
  - Agents' controls $\{y_t, x_t, l_t\}_{t=0}^{\infty}$ and law of motion for system states $\{P_t, r_t, \Theta_t, m_t\}_{t=0}^{\infty}$.
  - (i) Each agent solve her optimization, (ii) token markets clear, (iii) system satisfies Fokker-Planck, and (iv) reward rate solves fixed point problem.

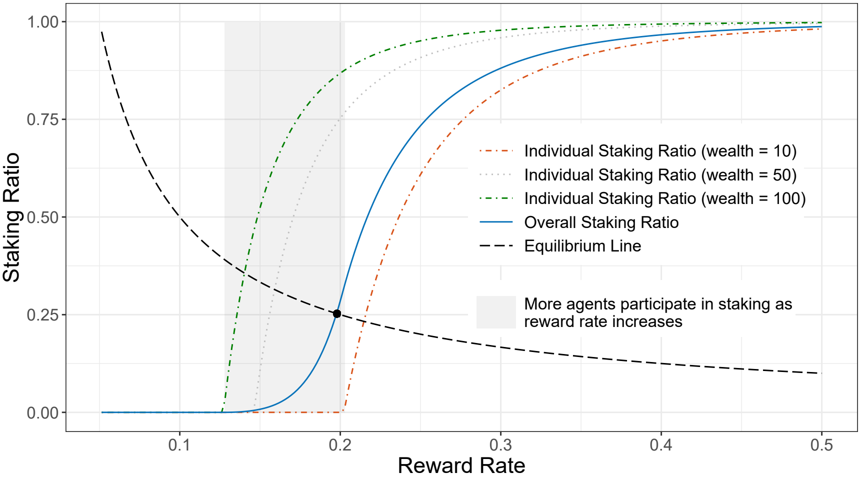
Model Implications

1. $\frac{\partial r_t}{\partial \Theta_t} < 0$. Higher staking ratio instantly decreases reward rate.
2. $\forall \rho_t' > \rho_t$, $\Theta_t'^* \geq \Theta_t^*$, with $\rho_t = R_t/Q_t$;
   $\Theta_t'^* = \Theta_t^*$ only if everyone's wealth is fully staked (unlikely).
   Higher reward rate attracts the investors to stake more.
3. Higher staking ratio predicts higher price appreciation.
4. UIP violated across tokens.
5. Dynamic token pricing formula:

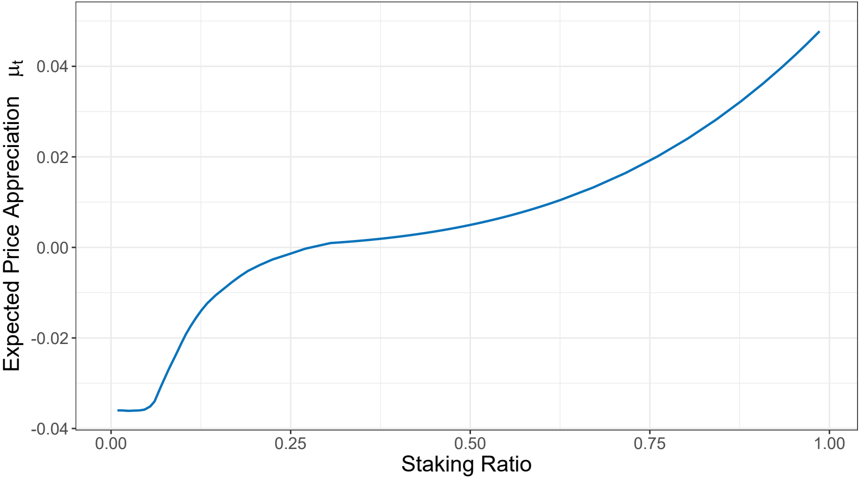
$$0 = \frac{\partial P}{\partial Q} Q_t \iota_t + \frac{\partial P}{\partial A} A \mu^A + \left(\frac{\partial P}{\partial A}\right)^2 \left(\frac{I^x}{P} + \frac{Q\Theta}{I}\right) \left(A\sigma^A\right)^2 + \frac{1}{2}\frac{\partial^2 P}{\partial A^2}\left(A\sigma^A\right)^2 + \left(\frac{\rho}{\Theta} - c + I^n\right) P, \tag{1}$$

where $I = \int_\Sigma \frac{\partial_w J}{\partial_{ww} J} m dw$, $I^x = \frac{A}{I}\left(\frac{1-\alpha}{r-c}\right)^{\frac{1}{\alpha}} \int_\Sigma U m dw$, and
$I^n = \frac{1}{I}\int_\Sigma \frac{\partial \Psi}{\partial n}\frac{\partial_w J}{\partial_{ww} J} m dw$.

Optimal Staking and Equilibrium Staking Ratio



Staking Ratio and Price Dynamics



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Data

Daily observations from stakingrewards.com, 30 stakable tokens, July 2018-Aug 2020, pan-PoS protocols and on-chain projects.

Table 1: Summary statistics: staking reward, staked ratio and crypto price

Token	Sample Start Date	Market Cap	Reward Rate (%, Annual)		Staking Ratio (%)		Daily Return (Logarithmic, %)	
		(Million, $)	Mean	Std.dev.	Mean	Std.dev.	Mean	Std.dev.
0x	2020-05-06	314.094	0.601	0.291	2.782	0.275	0.795	6.581
aion	2019-11-21	51.809	6.574	1.319	26.097	3.278	0.166	6.718
algorand	2019-07-06	336.657	7.929	4.455	65.531	5.420	−0.144	6.266
ark	2019-06-07	71.088	9.293	0.520	54.549	1.404	−0.036	6.267
bitbay	2019-07-13	179.673	2.249	0.504	45.859	6.679	0.960	19.706
celo	2020-05-23	1014.247	6.456	0.465	5.389	2.792	2.722	13.103
cosmos	2019-03-17	1176.824	9.197	1.404	68.287	8.473	−0.025	6.545
dash	2019-07-13	866.006	6.278	0.284	52.454	2.447	−0.085	5.615
decred	2019-07-31	194.972	8.513	0.717	50.160	0.704	−0.026	4.876
eos	2019-03-25	2979.338	1.848	0.283	55.837	3.514	0.138	5.116
fantom	2019-12-28	38.768	46.094	21.604	49.181	10.658	0.158	8.235
icon	2019-09-04	234.310	17.423	3.459	27.299	6.516	0.198	6.150
idex	2019-03-17	33.181	10.336	4.211	30.360	6.889	0.404	7.734
iotex	2019-03-25	47.516	10.154	3.624	41.815	6.170	0.092	6.616
irisnet	2019-09-20	80.689	10.637	0.592	33.828	1.921	0.403	8.120
livepeer	2019-08-16	15.460	63.628	31.169	63.636	4.704	−0.268	11.482
lto-network	2020-01-07	19.209	7.309	1.231	24.586	4.397	0.669	7.141
nem	2019-07-31	331.823	0.018	0.015	41.393	1.414	−0.165	4.780
neo	2018-10-12	1003.894	2.534	1.293			0.052	5.453
nuls	2019-10-17	46.269	9.165	0.967	44.259	4.049	0.169	6.885
polkadot	2020-06-19	3179.430	11.801	4.288	53.121	4.986	1.771	8.840
qtum	2019-09-01	294.424	5.677	0.867	16.862	2.495	0.093	5.260
smartcash	2019-09-26	9.699	2.486	0.532	7.771	0.766	0.118	6.402
snx	2019-09-11	535.644	57.595	8.265	80.308	4.503	0.925	6.721
terra	2020-01-18	220.947	11.358	1.797	26.138	2.242	0.445	5.899
tezos	2018-07-06	3020.387	7.028	1.642	68.769	10.568	0.091	6.111
tron	2019-08-16	932.897	2.430	1.448	19.823	6.000	−0.095	5.531
wanchain	2019-11-28	35.883	8.120	0.563	24.767	1.663	0.190	6.240
waves	2019-08-16	191.197	4.562	2.143	52.412	3.466	0.131	5.122
zcoin	2019-08-30	77.896	16.038	3.282	56.880	10.177	0.355	4.715

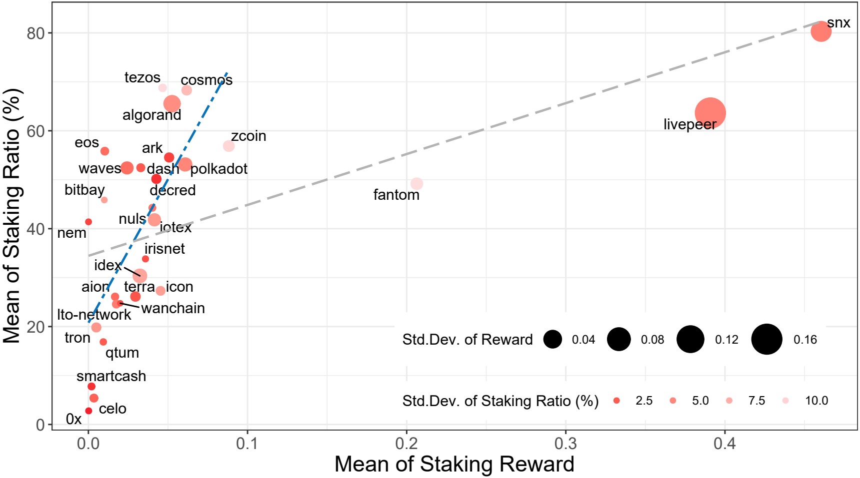
Reward Adjustment Mechanism

$$\Delta\text{Reward}_{i,t} = a_i + b_{t-1} + c\Delta\text{StakingRatio}_{i,t} + \epsilon_{i,t}$$

	$\Delta\text{Reward}_{i,t}$						
	7-day					14-day	30-day
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
$\Delta\text{StakingRatio}_{i,t}$	−0.363***	−0.359***	−0.380***	−0.375***	−0.385***	−0.343***	−0.246***
	(−16.388)	(−16.295)	(−16.076)	(−15.930)	(−15.824)	(−10.709)	(−6.377)
$\frac{1}{100}\log(\text{Cap})_{i,t}$					−0.382**	−0.455	−0.142
					(−2.182)	(−1.472)	(−0.399)
$\text{Volatility}_{i,t}$					0.023	0.056	−0.090
					(1.117)	(1.351)	(−1.348)
$\text{StakingRatio}_{i,t}$					0.024*	0.034	−0.008
					(1.691)	(1.287)	(−0.212)
Fixed Effects							
Token		Y		Y	Y	Y	Y
Time			Y	Y	Y	Y	Y
R^2	0.182	0.183	0.190	0.191	0.197	0.206	0.215

Notes: ***Significant at the 1 percent level.
**Significant at the 5 percent level.
*Significant at the 10 percent level.

Higher Reward Attracts More Staking



Staking Ratio Predicting Token Price Returns

Table 4: Higher staking ratio predicts larger token price appreciation.
This table presents the analyses of how staking ratio predicts token price appreciation. The main independent is the staking ratio of previous period, $StakingRatio_{i,t-1}$. The dependent $r_{price_{i,t}}$ is the log price change. The results show that the coefficient is significantly positive, which implies that higher staking ratio will predict higher token price appreciation. Considering that there exist market effect and market value effect in cryptocurrency market, we also add the market price return $r_{MKT i,t}$ and the market cap term $\log(Cap)_{i,t-1}$ as controls. After adding these controls, the estimated coefficient of staking ratio is still significant. We also do the test in different horizons and with fixed effects to show the robustness of the results.
***,**,* indicate statistical significance at the 1%, 5% and 10% respectively.

	$r_{price_{i,t}}$				
	7-day			14-day	30-day
	(1)	(2)	(3)	(4)	(5)
$StakingRatio_{i,t-1}$	0.191**	0.196**	0.219***	0.553***	1.034*
	(2.076)	(2.353)	(2.644)	(3.117)	(1.947)
$r_{MKT i,t}$		0.638***	0.624***	0.626***	0.782***
		(16.200)	(15.927)	(8.560)	(7.205)
$\log(Cap)_{i,t-1}$			−0.782***	−1.516***	−3.234***
			(−4.447)	(−3.969)	(−3.080)
Fixed Effects					
Crypto	Y	Y	Y	Y	Y
R^2	0.004	0.185	0.198	0.164	0.273

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Uncovered Interest Rate Parity Violations

$$\mathbb{E}_t\left[s_{t+1} - s_t\right] = i_t - i_t^*.$$

Fama (1984):

$$\begin{aligned} \lambda_{i,t+1} &= \alpha + \beta(r_t^f - r_{i,t} + c_{i,t}) + \epsilon_{i,t+1}, \\ \text{where} \quad \lambda_{i,t} &= \log P_{i,t+1} - \log P_{i,t} + (r_{i,t} - c_{i,t}) - r_t^f, \end{aligned} \tag{2}$$

Table 5: UIP Violation.

Local	Horizon: 7-day			Horizon: 30-day		
Currency	Coef., β	t-statistic	R^2	Coef., β	t-statistic	R^2
Currency & mainstream cryptocurrencies.						
US Dollar	−0.994	(−32.761)	0.470	−1.041	(−5.307)	0.102
Bitcoin	−0.985	(−39.458)	0.540	−0.942	(−7.822)	0.171
Ethereum	−0.984	(−38.653)	0.529	−0.938	(−8.658)	0.202
In-sample cryptocurrencies.						
0x	−1.214	(−16.900)	0.462	−1.703	(−6.493)	0.366
aion	−0.961	(−24.408)	0.397	−0.988	(−5.519)	0.139
algorand	−1.007	(−36.924)	0.549	−0.923	(−6.114)	0.185
ark	−0.985	(−34.314)	0.496	−0.934	(−8.516)	0.212
bitbay	−1.024	(−9.144)	0.171	−0.774	(−1.505)	0.018
celo	−1.312	(−7.554)	0.422	−1.260	(−2.491)	0.205

Crypto Carry and Predictability of Returns

Carry (Koijen et.al., 2018):

$$\text{return} \equiv \text{carry} + \text{E(price appreciation)} + \text{unexpected price shock.}$$

Crypto carry:

$$\text{carry}_t \equiv \frac{r_t - c_t - r^f}{1 + r^f}.$$

Strategy	Mean (Annual, %)	St.dev. (Annual, %)	Skewness	Kurtosis	Sharpe Ratio (Annual)
Equal Weighted	92.768	52.517	−1.511	8.952	1.766
Carry Trade	143.863	57.918	−1.042	4.247	2.484

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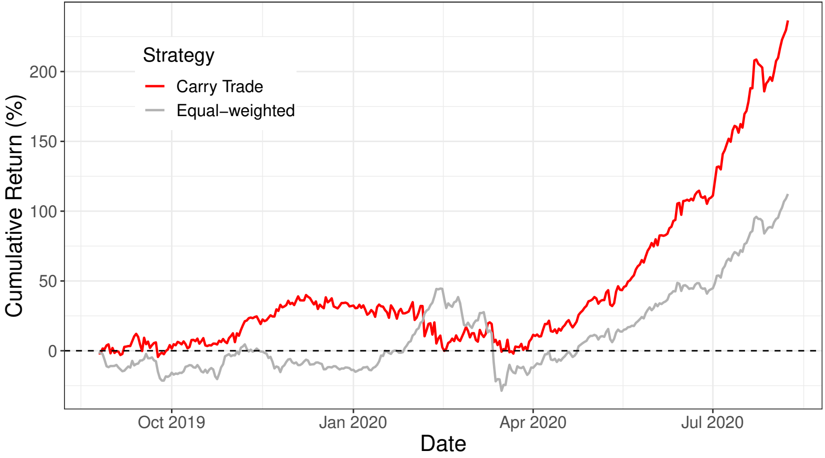
Return Predictability:

$$\text{ExcessReturn}_{i,t} = a_i + b_{t-1} + c\text{Carry}_{i,t-1} + \epsilon_{i,t}$$

Cumulative Returns of a Crypto Carry Strategy



Excess Return Predicted by Carry

Table 8: How does carry predicts excess return?

This table reports the results from the panel regression of Eq.(39), estimated c and t-statistics are reported. Without crypo and time fixed effects, c represents the total predictability of returns from carry from both its passive and dynamic components. Including crypto specific fixed effect will remove the predictable return component of carry coming from passive exposure to tokens with different unconditional average returns.

Panel A: 7-day	$\text{ExcessReturn}_{i,t}$			
	(1)	(2)	(3)	(4)
$\text{Carry}_{i,t-1}$	1.013***	0.612***	1.003***	0.700***
	(27.734)	(6.538)	(31.678)	(8.443)
Fixed Effects				
Crypto		Y		Y
Time			Y	Y
R^2	0.372	0.033	0.457	0.058

Panel B: 14-day	$\text{ExcessReturn}_{i,t}$			
	(1)	(2)	(3)	(4)
$\text{Carry}_{i,t-1}$	1.049***	0.163	1.023***	0.316
	(12.820)	(0.760)	(13.811)	(1.595)
Fixed Effects				
Crypto		Y		Y
Time			Y	Y
R^2	0.212	0.001	0.254	0.005

Conclusion

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- Emerging, economically intriguing, practically relevant, & exciting.