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Strategic Timing of IPOs and Disclosure: A Dynamic Model of Multiple Firms

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ABSTRACT: We study a dynamic timing game between multiple firms, who decide when to go public in the presence of possible information externalities. A firm's IPO pricing is a function of its privately observed idiosyncratic type and the level of investor sentiment, which follows a stochastic, mean-reverting process. Firms may wish to delay their IPOs in order to observe the market reception of the offerings of their peers. We characterize the unique symmetric threshold equilibrium, whereby pioneer firms with high idiosyncratic types endogenously emerge. The results provide novel implications regarding variation in IPO timing, sequential clustering, IPO droughts, the composition of new issues over time, and how IPO volume fluctuates over time. These include, among others, that in more populated industries, a *lower* proportion of firms emerge as industry pioneers, but follower IPO volume is intensified. Additionally, heightened uncertainty over investor sentiment exacerbates delay and leads to lower IPO volume.

Keywords: initial public offerings; information spillovers; investor sentiment; disclosure.

> [Tenable CEO Steve] Vintz added that when Tenable went public in July 2018, it bumped up its IPO plans by several quarters when it saw favorable investor sentiment around fellow cybersecurity player Zscaler (which had gone public a couple of months earlier) . . . Other companies eyeing an IPO this year will be watching Lyft's IPO and closely taking stock of investor appetites for certain types of stocks.
>
> —Gaus (2019)

I. INTRODUCTION

Private firms that wish to go public often face uncertainty as to the market reception of their initial public offerings (IPOs). This is particularly salient for firms in new industries for which no peers have gone public, leaving companies with significant uncertainty over investor sentiment. A pervasive finding in the empirical literature is that investor

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Supplemental material can be accessed by clicking the link in Appendix B.

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sentiment is one of the most significant determinants of IPO returns.[1] Naturally, given its importance, the potential to *learn* investor sentiment for new issuances in an industry becomes valuable for private firms, and may significantly influence their IPO timing. A well-known example is the 2012 IPO of Facebook, the pioneer firm to go public in the new social media industry. The price fall that ensued after Facebook's IPO reportedly pushed back the offering of Twitter for several months.[2] Lowry, Michaely, and Volkova (2017) additionally mention accounts by industry practitioners who have delayed their firms' IPO times in order to observe the market reaction to peer IPOs. These examples indicate that the ability to observe investor sentiment before going public is valuable to firms, and may lead firms to delay their IPO times. Delay, however, is costly, as this may result in the loss of investment and expansion opportunities, loss of competitive advantage, less public attention to the firm's products, and additional resources devoted to debt financing costs.

We seek to study the above trade-off in a strategic timing game of IPOs by multiple firms, in which each firm chooses when to disclose its private information and go public (or sell a project). Unlike existing models of IPO timing—which largely feature either a single firm, multiple firms whose actions are independent or irrelevant to one another, or multiple firms that act in an exogenously determined order—we consider multiple firms whose endogenous IPO timing decisions are *interdependent*. Our model demonstrates the endogenous emergence of pioneer firms (those that go public first) in the face of informational rents and characterizes the equilibrium distribution of IPOs over time. Our results offer new insights and testable implications concerning how the magnitude of industry concentration (i.e., the number of firms), the level of industry uncertainty over market sentiment, the extent of mean reversion in investor sentiment, and the cost of delaying the offering affects IPO volume, timing, and the variation in the quality of IPOs over time.

We study the following three-period, multi-firm setting. The realized pricing of a firm's IPO is determined by two components: the firm's idiosyncratic component, which we refer to as the firm's type, and the realization of the period's common component (investor sentiment), which affects all new issues in the industry in that period. The first ingredient of our model is that, given all else equal, each firm prefers to go public as early as possible. This assumption could reflect the above-mentioned costs of delaying the IPO. To capture this time preference, we assume that managers discount the future payoff from selling the firm (or the project). The second ingredient of our model pertains to the common component, which we often refer to as investor sentiment or the *state of nature*. Investor sentiment is assumed to follow a mean-reverting stochastic process.[3] Ritter (1984) documents that investor sentiment over IPOs varies by industry, while Ritter (1991) and Loughran and Ritter (1995) document evidence consistent with mean-reversion in investor sentiment.

Firms that wish to go public are, therefore, able to learn about the sentiment level from recent IPOs within their industry. The fewer the number of firms that go public in a given period, the less that can be learned about the state of nature. For example, if Facebook had not gone public in May 2012, then there would have been much greater uncertainty as to the market's sentiment toward the social media industry. To capture this key aspect of IPOs in the simplest, most tractable way, we assume in our baseline model that a period's sentiment is observed as long as at least one firm goes public in that period. We later relax this simplifying assumption and show that the results qualitatively hold in the more realistic setting where the precision of beliefs about the period's sentiment increases in the number of IPOs in that period (see the Online Appendix, Section B.2; see Appendix B for the link to the downloadable file).

The mean-reverting nature of investor sentiment gives rise to a real option from delaying the IPO in the first period. Information regarding sentiment is revealed upon an IPO by at least one other firm. Private firms that find the sentiment realization at the end of the first period to be sufficiently low continue to delay their IPO until the third period. The reason is that, following a poor IPO, i.e., low sentiment, the mean-reverting property implies an expected improvement in investor sentiment from the second to the third period. Likewise, if the information from the first period's IPOs indicate that sentiment was sufficiently high, then a firm that did not IPO in the first period may find it more profitable to go public in the second period rather than further delaying its IPO. When deciding whether to go public in the first period, the firm considers the trade-off between the direct costs of delaying the IPO and the benefit from the expected value of the real option from delaying the IPO. Each firm considers the probability that the other firms will go public in the first period, as this affects the likelihood of observing information about the first period's state and, thus, the option value from delaying the IPO. This introduces strategic interaction between firms, as the IPO strategy of one firm affects the payoffs and the optimal strategies of the other firms.

[1] See, e.g., Ritter (1984, 1991), Lerner (1994), Loughran and Ritter (1995), Pagano, Panetta, and Zingales (1998), Baker and Wurgler (2002), Lowry (2003), Cornelli, Goldreich, and Ljungqvist (2006), Kaustia and Knüpfer (2008), Dorn (2009), and Bajo and Raimondo (2017), among others. IPO initial returns are a common measure for investor sentiment (along with other measures). As noted in Baker and Wurgler (2006, 1656): "The IPO market is often viewed as sensitive to sentiment, with high first-day returns on IPOs cited as a measure of investor enthusiasm, and the low idiosyncratic returns on IPOs often interpreted as a symptom of market timing."

[2] Several other firms, such as Kayak, were also reported to have delayed their IPO dates specifically because of the market reaction to the Facebook IPO. See Tobak (2012). Strategic IPO delay and the importance of pioneer firms have been documented in recent media coverage, as well: "A successful debut could help break a logjam formed by many of those billion dollar startups that have been waiting in the wings" (Farrell and Driebusch 2016).

[3] The key trade-off that we identify does not rely on mean reversion, but rather exists for any serial correlation structure in the state of nature.

We analyze the above setting and show that there exists a unique symmetric equilibrium in which firms follow an IPO threshold strategy in each period. In particular, each firm goes public in the first period if and only if the realization of its idiosyncratic type is sufficiently high. We show this characterization by first establishing a critical property of the timing decision: the real option from delay is less valuable for high-type firms. Upon delaying the IPO in the first period, high-type firms require more pessimistic realizations of sentiment in the first period in order to take advantage of the option and further delay the IPO in the second period. This occurs, in part, due to the equilibrium property that discounting is relatively more punitive for high-type firms, which decreases the incentive to delay. Hence, a *lower* and, thus, more severe, realization of the first-period state is necessary for delay in the second period to be profitable for higher-type firms. This results in the value of the real option from first-period delay to be decreasing in firm type. As a result, high-type firms are less inclined to delay and endogenously emerge as industry IPO pioneers (i.e., firms that go public in the first period). Correspondingly, low- and intermediate-type firms are comparatively more inclined to delay their IPOs not only in the first period, but in the second period, as well.

The analysis gives rise to a number of novel insights and implications concerning the distribution of IPOs over time, and how this distribution varies with the cost of delay, uncertainty over investor sentiment, persistence in sentiment, and industry concentration. The delay cost (discount rate) affects the characteristics of the equilibrium in the following ways. An increase in the discount rate naturally leads to less delay in the first period and heavier early IPO volume. However, as the discount rate increases and delay becomes more costly, we find that the probability of an *IPO drought,* where no firms go public in the second period after observing at least one IPO in the first, *increases*. There are two drivers for this result. First, the IPO threshold in the first period decreases in the cost of delay, and hence there are more firms that IPO in the first period and fewer remaining firms that can potentially go public in the second period. Second, as the cost of delay increases, the firms that do not go public in the first period are of a lower type. As discussed above, lower-type firms are more inclined to delay their IPOs in the second period (i.e., these firms delay for milder first-period realizations of investor sentiment). Overall, the model predicts that a higher delay cost results in heavier expected initial IPO volume, which is followed by a higher likelihood of an IPO drought. Likewise, the results predict lower average issuer quality and greater variation in quality among pioneer IPOs when delay costs are high.

The model also provides predictions on the effect of changes in the level of uncertainty over investor sentiment. An increase in uncertainty increases the real option from delaying the IPO in the first period. This implies that firms are more inclined to delay their IPOs in the first period and, thus, we have lower initial IPO volume. However, pioneer firms that emerge are expected to be of higher quality and the distribution of early IPO firms exhibits less variation. Likewise, we are more likely to see delay in the second period by remaining firms after a pioneer has emerged, due to the heightened possibility of severely unfavorable investor sentiment. This results in a lower volume of firms that will have gone public by the end of the second period. These implications help to explain the results of Schill (2004), who finds greater delay and greater dispersion of IPOs during periods of heightened uncertainty.

Interestingly, the model predicts that the persistence of investor sentiment has a non-monotone effect on the IPO threshold and IPO timing properties. In particular, we find that first-period delay is hump-shaped in persistence (and, thus, IPO volume is U-shaped). To see this, consider the case in which sentiment is independent between periods. In this extreme case, there is no value from delaying the IPO, as information learned in the first period is irrelevant for the second. As persistence in sentiment increases from zero, so does the value of delay, as learning about the state becomes more valuable. However, for sufficiently high levels of persistence in sentiment, a further increase in persistence begins to decrease the incentive to delay. Under greater persistence, the expected change in sentiment between the second and third period is small, which implies that the option value from delay is also low. One interpretation of the persistence in investor sentiment is the average time between filing the IPO and the IPO completion date, which includes the Securities and Exchange Commission's (SEC) processing time and the firm's marketing and book-building efforts. A lower processing time of the IPO implies greater persistence in sentiment. The results imply that reducing the regulatory burdens for filing the IPO and, thus, shortening the approval time (as legislated in the 2012 Jumpstart Our Business Startups Act), may have the unintended consequence of rather *increasing* delay in IPO timing.

Our model also provides a novel opportunity to study the effect of the level of industry concentration on IPO timing and on how IPO volume fluctuates over time. An increase in the number of firms implies that there is a greater likelihood that at least one firm's type is sufficiently high to go public and, thus, a higher likelihood that investor sentiment will be revealed by the end of the first period.[4] At the same time, all firms recognize this situation, which strengthens the incentive for delay in the first period. As a result, the first-period IPO threshold increases and, thus, the probability that a given firm initially *delays* its IPO

[4] To see this, suppose by contradiction that increased industry concentration decreases the probability of at least one IPO in the first period. Then, each firm's real option from delaying its IPO would decrease, resulting in a lower IPO threshold and a higher likelihood of IPO by each firm. This implies a higher overall volume—a contradiction.

increases. Hence, paradoxically, an increase in industry concentration reduces initial IPO volume (as a proportion of firms in the industry), but increases the likelihood that a pioneer firm emerges.

However, conditional on the emergence of a pioneer firm in the first period, we expect heavier IPO volume in the second period in more concentrated industries. This stems from two effects. As the threshold increases, more high-type firms delay their offerings until the second period. As discussed above, these high-type firms need a more severe realization of first-period sentiment in order to further delay their IPO, which implies that remaining firms in the second period have a higher likelihood of going public. Consequently, we more often observe firms going public following pioneer IPOs, which we refer to as *sequential clustering,* and greater second-period IPO volume. In sum, more intense industry concentration implies a higher likelihood that a few high-quality firms go public in the first period, while a larger proportion of firms delay. However, upon the emergence of a pioneer, heavier IPO volume ensues. To the best of our knowledge, our model is the first to relate the salient feature of industry concentration to IPO timing and volume.

Related Literature

Our study is related to the work of Persons and Warther (1997), who consider a model of technology adoption where multiple firms must choose their adoption time. Each firm observes the noisy cash flow returns of firms that have already adopted and decides whether to adopt the innovation. They generate "booms" in the adoption of the new technology, as each additional firm that adopts the innovation may lead to another firm's subsequent adoption. Our setting differs from Persons and Warther (1997) primarily in that, in our model, agents have private information over their types, the state variable follows a stochastic mean-reverting process, and agents are discrete. Our contribution relative to Persons and Warther (1997) and the extant literature is as follows. First, we examine how firms' learning incentives map into and affect their IPO timing decisions when there is *uncertainty* about the types and IPO decisions of other firms. The presence of private information leads to a unique symmetric equilibrium that always exhibits strictly positive probability that no IPOs are observed in the first period, whereas this kind of overall *industry delay* is absent in Persons and Warther (1997). This helps to explain, for example, delay in the emergence of pioneer IPO firms in new industries (and potential variation in this delay across new industries). Hence, the economic forces of our setting and the resulting equilibrium characterization are qualitatively distinct from Persons and Warther (1997).

Second, our comparative statics analysis provides insights that are largely not captured in the framework of Persons and Warther (1997) or the other extant literature, particularly with regard to changes in the number of firms. These insights include predictions regarding IPO timing, IPO volume (and how volume fluctuates over time), the expected incidence of sequential clustering or an IPO drought, and the variation of issue quality over time. In particular, we provide testable implications for variation in these outcomes with respect to industry concentration, the cost of delay, uncertainty, and persistence in sentiment. Third, we show that the introduction of a stochastic, mean-reverting state variable results in non-monotone effects of the degree of mean-reversion on the incentive to delay. This leads to additional implications concerning the role of persistence in investor sentiment and helps to provide an explanation for delay *between* firms' IPO times. Moreover, our analysis more broadly contributes to the theoretical literature, as few papers consider information spillovers with a stochastic state, and hence our study provides insights on how learning incentives are affected when the underlying state evolves over time.

The present study is also related to the extant theoretical literature on IPO timing, which largely consists of models that include either a single firm or multiple firms that move in an exogenously determined order. Alti (2005) develops a model of information spillovers in an IPO setting, where information asymmetry decreases following an IPO and consequently lowers the cost of going public for the other firms. The cost of going public is due to adverse pricing by the market in a second price auction in the presence of informed traders. Among several differences, the IPO order in Alti (2005) critically depends on the assumption of a continuum of firms, where the cost of going public is implicitly assumed to be less for firms with a higher probability of discovering a project. As a result, a continuum of firms go public in commonly known order according to their benefit from going public. The current model differs in that the most salient feature is the uncertainty as to whether a firm will go public early. We are, thus, able to capture novel properties of timing and sequential clustering in issue markets. Hoffmann-Burchardi (2001) and Maksimovic and Pichler (2001) consider models where firms may delay their IPO plans. However, the order of moves and, thus, IPO timing, is exogenous in both studies as firms are pre-designated to move either first or second.[5]

Pástor and Veronesi (2005) model the strategic timing of an IPO as an inventor who faces a problem analogous to an American call option. The inventor can exercise the option to capitalize on abnormal profits, but sacrifices the possibility that

[5] Relatedly, Benveniste, Busaba, and Wilhelm (2002) examine a two-firm model that, similar to our setting, includes both an idiosyncratic component and an unknown common factor. The timing and IPO order, however, are assumed to be fixed, such that one firm is designated to move first and the other firm follows. In contrast, we allow the firms' timing of the IPO to be endogenously determined, and show that pioneer firms endogenously emerge and bear the (implicit) cost of information production (in terms of giving up their real option).

market conditions may worsen to cover the initial investment. Relatedly, Bustamante (2012) considers IPO timing as a real options problem with asymmetric information. She also finds that high-type firms go public earlier in some equilibria, and characterizes equilibria in which the presence of private information either speeds up or delays the IPO time relative to complete information. Our model varies from these studies in that we incorporate information spillovers between firms that affect the timing of IPOs, whereas strategic interaction between firms is absent in Pástor and Veronesi (2005) and Bustamante (2012). This leads to additional insights and predictions that are not captured by these prior studies.

Our model varies from the literature on dynamic voluntary disclosure (e.g., Dye and Sridhar 1995; Einhorn and Ziv 2008; Guttman, Kremer, and Skrzypacz 2014; Aghamolla and An 2021) in three ways. In our setting, (i) the manager receives information with probability one and disclosure is costless, (ii) the entrepreneur is only concerned with the firm's value in the period of disclosure and IPO, and (iii) there are multiple firms whose decisions are interrelated.[6] The following section presents the model, and Section III analyzes the equilibrium. Section IV examines comparative statics, while Section V presents empirical predictions. Section VI discusses extensions of the model, and the Section VII concludes. Proofs are relegated to Appendix A, unless otherwise stated. Additional details concerning extensions are included in the Online Appendix.

II. MODEL SETUP

We first briefly describe the IPO process in order to provide texture to the specific features of the model. In the U.S., any company that wishes to sell its shares on a public exchange must file a registration (S-1) statement with the Securities and Exchange Commission (SEC), which contains accounting information and the prospectus, among other information. The SEC then reviews the registration statement for accuracy and omissions. Generally, the registration disclosure has been carefully reviewed prior to filing by the firm and its underwriters, as the firm is held civilly liable for misstatements or omissions under the Securities Act of 1933.[7] Once the SEC issues clearance of the registration statement, the firm typically goes on non-deal "road shows" to promote the offering to institutional investors. The prospectus disclosure, thus, plays a crucial role in the IPO process as being the primary source of the company's accounting information, which also undergoes a rigorous review by the SEC.

In order to study the intertemporal IPO decisions among multiple firms, we analyze a setting with three periods, $t \in \{1, 2, 3\}$, and $N \geq 2$ firms. Each firm's pricing at the time of its IPO is a function of its idiosyncratic component and that period's level of investor sentiment. Prior to $t = 1$, each firm's manager privately observes the firm's idiosyncratic type, θ_i, which is the realization of a random variable θ with a cumulative distribution function $G(\theta)$ and probability density function $g(\theta)$. The support of θ is $[0, \infty)$ and $g(\theta)$ is positive over the entire support of θ.[8] For all $i \neq j$, the idiosyncratic components, θ_i and θ_j, are independent. We constrain θ to be non-negative to simplify the analysis, as well as to highlight that the timing results are driven by the strategic interaction between firms rather than by other exogenous factors; however, the results would not be qualitatively affected if the support of θ included negative values.[9] Firm managers are assumed to be risk-neutral.

Every firm must go public in one of the periods $t \in \{1, 2, 3\}$. As part of the IPO, the manager discloses the private information, θ_i (e.g., as part of the prospectus). While we assume that every firm must go public in our model, we note that our results are qualitatively unaffected if we rather allow firms not to go public and remain private.[10] Disclosure of the private type is assumed to be credible and costless. Managers optimally choose the timing of their IPO in order to maximize the firm's expected IPO market price. The firm's IPO price depends on its private type, as well as that period's realization of investor sentiment, denoted by s_t. Hence, the market IPO price at time τ for a firm i that discloses θ_i and goes public at $t = \tau$ equals the sum of the firm's type, θ_i, and the realized sentiment, s_τ. Each manager has a common time preference (discount), which we denote by r, such that the expected utility of the manager of firm i from going public and disclosing θ_i at $t = \tau$ is given by:

$$u_{i,\tau} = \frac{P_{i,\tau}}{(1+r)^{\tau-1}} = \frac{\theta_i + s_\tau}{(1+r)^{\tau-1}}.$$

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[6] The latter feature is present in Dye and Sridhar (1995).

[7] There is a sizable empirical literature concerning earnings management in IPOs, spearheaded by Teoh, Welch, and Wong (1998), although the findings in subsequent studies have been mixed (e.g., Ball and Shivakümar 2008; Boulton, Smart, and Zutter 2011; Cecchini, Jackson, and Liu 2012). To focus on the timing of disclosure and going public, we abstract away from earnings management; however, we note that this may be a promising avenue for future research.

[8] We consider the case of a bounded support in Section C of the Online Appendix.

[9] With negative values, firms would be compelled to delay disclosure since discounting works to improve the firm's payoff. We eliminate this case so as not to confound the results.

[10] The reason is that the main driver of our equilibrium, in which higher-type firms go public earlier, is the property that a firm's real option from delaying the IPO is decreasing in its type θ_i. Allowing firms not to go public introduces an additional real option from delaying the IPO. The value of this additional real option is also decreasing in the firm's type, and hence a qualitatively similar equilibrium prevails in this alternative setting. We restrict firms to go public by $t = 3$ mainly to keep the model parsimonious and more tractable.

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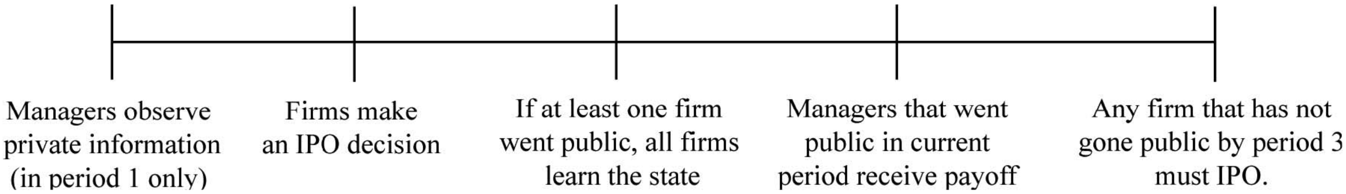
FIGURE 1
Sequence of the Stage Game



Discounting is meant to capture the costs associated with delaying the sale of a project or shares. Such costs could be due to, for example: costs of debt, the cost from forgoing positive net present value (NPV) investments or acquisition opportunities due to lack of financing, or the decrease in profitability due to an increase in competition and decrease in market share.

The state of nature in each period, s_t, is unobserved *ex ante;* however, upon an IPO by at least one of the firms, all of the firms learn s_t at the end of the period in which an IPO took place. We assume that the state of nature follows a mean-reverting $AR(1)$ process of the form:

$$s_t = \gamma s_{t-1} + \varepsilon_t,$$

where $\gamma \in (0, 1)$ and $\varepsilon_t \sim N(0, \sigma^2)$ with cumulative distribution function $F(\cdot)$ and density function $f(\cdot)$. The initial state is given by $s_0 = 0$, and so the first period's state is given by $s_1 = \varepsilon_1$. Hence, investor sentiment in the first period is simply a mean-zero error term.[11] As we discuss later, the degree of persistence in investor sentiment, γ, can reflect, for example, the average amount of time between filing the IPO and the actual IPO completion, with longer IPO processing times implying less persistence.

The sequence of events is as follows: Prior to $t = 1$, all managers privately observe their firm's value, θ_i. In $t = 1$, each firm simultaneously decides whether to go public in this period. If at least one firm went public in $t = 1$, the first-period state of nature, s_1, is publicly observed and firms that went public at $t = 1$ receive their IPO price. Firm managers receive their corresponding payoff and the remainder of the game is irrelevant for them. In $t = 2$, all firms that did not go public at $t = 1$ decide whether to undertake an IPO or delay the IPO to $t = 3$. If at least one firm went public at $t = 2$, the realization of the state of nature, s_2, is publicly revealed. The market price of firms that went public at $t = 2$ is determined and managers of these firms receive their payoff. Finally, at $t = 3$, which is the last period of the game, all firms that have not yet gone public must do so, and these managers obtain their payoff. The timeline of a generic period is given in Figure 1.

We note that the discrete time structure in this setting can be motivated by the fact that, as discussed above, there is indeed significant time between the IPO decision and the actual IPO completion date. Hence, the distance between periods can be interpreted as the SEC's registration processing time, and the firm's subsequent book-building and marketing efforts. We could, therefore, allow an equivalent setup where firms that delayed in the first period make the IPO filing decision at the end of the first period, but the actual IPO does not occur until the beginning of the second period.

We assume that all firms are *ex ante* homogeneous, that is, all firms have the same distribution of the value of the idiosyncratic component, θ_i, the same discount rate, r, and that the common factor, s_t, affects every firm's market value within that industry in the same way. The following section analyzes the equilibrium of the above timing game.

III. EQUILIBRIUM

Before we derive the equilibrium of our setting, note that in a two-period (rather than three-period) version of our model, all firms would go public in the first period. The reason is that firms cannot benefit from learning about investor sentiment, as firms must go public in the second period regardless of the realization of sentiment. Hence, the value of the real option from delay in the two-period setting is zero. As such, to capture the value of learning, the model requires at least three periods.

As we later show, in any equilibrium, firms follow a threshold IPO strategy in each period. That is, each firm goes public in the first period if and only if its type, θ_i, is greater than a threshold θ_1^*, which is a function of all the parameters of the model (the number of firms, the distribution of types, the distribution of the state of nature, the degree of mean-reversion, and the discount rate). At $t = 2$, if at least one firm $j \neq i$ went public at $t = 1$ and the state s_1 was revealed, firm i goes public if and only if $\theta_i > \theta_2^*(s_1)$. If there are no IPOs at $t = 1$, then the second period reduces to the two-period setting mentioned above, and hence,

[11] We assume normality of ε_t primarily for consistency with the literature and for tractability of some of the comparative statics. However, as we later show, the main results hold for any distribution of ε_t as long as mean-reversion of s_t is preserved.

all firms go public in $t=2$.[12] Given that there is positive probability of an IPO by at least one other firm in the first period, firm i has a real option from delaying the IPO at $t=1$, hoping to observe s_1 at the end of period 1. Upon observing the state, s_1, for sufficiently negative realizations, firm i would rather delay the IPO until $t=3$, as the state follows a mean-reverting process and is expected to increase toward zero at $t=3$.

In light of the above behavior in period 2, when deciding at $t=1$ whether to go public or delay, firms consider the trade-off between the benefit from the option value of delaying the IPO and the cost of delay. The cost of delay, due to discounting, is relatively higher for high-type firms. Moreover, as we show below, the value of the real option from delaying the IPO at $t=1$ is decreasing in the firm's type. As such, both of the above effects work in the same direction. That is, every firm follows a threshold strategy at $t=1$ such that, for realizations of θ_i that are sufficiently high, the firm prefers to go public at $t=1$, whereas for lower realizations, the firm is better off delaying the IPO at $t=1$. We solve for the unique symmetric threshold equilibrium. We begin by deriving the IPO strategy in the second period and then analyze the first-period decision.

Second-Period IPO Decision

As indicated above, if no firm went public at $t=1$, all firms go public at $t=2$.[13] Given an IPO by at least one firm at $t=1$ and the realization of s_1, firm i of type θ_i is indifferent between going public and delaying the IPO at $t=2$ if and only if the following indifference condition holds:

$$\frac{\theta_i + E(s_2|s_1)}{1+r} = \frac{\theta_i + E(s_3|s_1)}{(1+r)^2}.$$

The above has a unique solution. The unique optimal strategy in $t=2$, which we denote by $\theta_2^*(s_1)$, is as follows.

Lemma 1: In any equilibrium, the strategy of firm i that did not undertake an IPO at $t=1$ is as follows. If no firm went public at $t=1$, firm i goes public at $t=2$. If at least one firm went public at $t=1$ (and hence s_1 is observed), firm i follows a threshold strategy at $t=2$ such that it goes public if and only if[14]

$$\theta_i \geq \theta_2^*(s_1) \equiv -s_1((1+r)-\gamma)\left(\frac{\gamma}{r}\right). \tag{1}$$

After observing investor sentiment in the first period, s_1, firms will delay the IPO only for sufficiently negative values of s_1. Note that for all $s_1 \geq 0$, all firms that did not go public at $t=1$ will do so at $t=2$, as both effects (discounting and the reversal of investor sentiment) work in the same direction. When the realization of s_1 is negative (or, in general, lower than the mean of s) the mean-reversion property of s implies that s_3 is expected to be higher than both s_1 and s_2, which provides an incentive to delay the IPO to $t=3$. However, delaying the IPO is costly due to discounting, and hence the manager's IPO threshold at $t=2$ resolves the trade-off between these two effects.

To further demonstrate the intuition for the threshold at $t=2$, it is useful to consider extreme parameter values. For $\gamma=1$, such that the state of nature follows a random walk, manager i goes public at $t=2$ if and only if $\theta_i + s_1 > 0$. On the contrary, when $\gamma=0$, such that s_1 and s_2 are independent, the manager goes public immediately. For extreme values of the discount rate, it is easy to see that for $r=0$, firms go public at $t=2$ if and only if $\gamma s_1 > 0$ (or, equivalently, $s_1 > 0$) as the only effect in place is the reversal of the state. As the discount rate goes to infinity, all firms would have gone public at $t=1$ (and if manager i did not go public at $t=1$, she will do so at $t=2$ if and only if $\theta + s_1 > 0$).

Next, we analyze the equilibrium behavior at $t=1$.

Period 1 and the Option Value from Delay

We conjecture (and later prove in Corollary 1) a threshold strategy at $t=1$ such that firm i goes public in the first period if and only if $\theta_i \geq \theta_1^*$. Recall that if the manager of firm i goes public in $t=1$, her expected payoff is $\theta_i + E[s_1] = \theta_i$. If manager i does not IPO at $t=1$, then her payoff depends on whether at least one other firm went public at $t=1$. If there were no IPOs at t

[12] We note that this feature of the model can be relaxed by introducing additional information that arrives before $t=2$. For example, firms may receive a noisy signal of s_1 at the end of the first period even if no IPOs are observed. Similarly, firms may observe a public signal of s_1 at the beginning of $t=1$. Both assumptions do not qualitatively affect the main results, but lead to delay in $t=2$ even if no IPOs are observed in $t=1$. We examine the latter setting in the Online Appendix.

[13] In Online Appendix Section B.3, we introduce an additional signal of s_1 that all firms receive at the beginning of time 1. This leads to delay in $t=2$ even if no IPOs are observed in $t=1$.

[14] An alternative way to think about the IPO strategy is to take θ_i as given and to specify the realizations of s_1 for which the firm will and will not go public at $t=2$. This approach yields that for a given θ_i, firm i goes public at $t=2$ if and only if $s_1 < s_1^*(\theta_i) \equiv -\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}$.

American Accounting Association

$= 1$, firm i (as well as all other firms) will go public at $t = 2$ and each firm i will receive an expected payoff of $\frac{E(\theta_i + s_2)}{1+r} = \frac{\theta_i}{1+r}$. From Lemma 1, if at least one firm went public at $t = 1$, then firm i will IPO at $t = 2$ if and only if $\theta_i > \theta_2^*(s_1)$ Equivalently, firm i will go public at $t = 2$ if and only if $s_1 \geq s_1^*(\theta_i) \equiv -\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}$, where $s_1^*(\theta_i)$ is the realization of s_1 for which a firm of type θ_i is indifferent between going public at $t = 2$ or delaying the IPO to $t = 3$. Therefore, conditional on the realization of s_1 being sufficiently high to induce an IPO of firm i at $t = 2$, the expected payoff of the firm is $\frac{E\left(\theta_i + s_2 | s_1 > s_1^*(\theta_i)\right)}{1+r}$. If the realization of s_1 is sufficiently low, i.e., if $s_1 < s_1^*(\theta_i)$, firm i will delay the IPO until $t = 3$, in which case its expected payoff is $\frac{E\left(\theta_i + s_3 | s_1 < s_1^*(\theta_i)\right)}{(1+r)^2}$. In summary, the expected payoff of manager i from delaying the IPO at $t = 1$ is:

$$u_i = \Pr\left(ND_{j\neq i}^1\right)\left(\frac{\theta_i}{1+r}\right) + \left(1 - \Pr\left(ND_{j\neq i}^1\right)\right)\left(\begin{array}{c}\Pr\left(D_i^2\right)E\left[\text{payoff at } t = 2|\theta_i, D_{\mathrm{i}}^2\right] \\ +\Pr\left(ND_i^2\right)E\left[\text{payoff at } t = 3|\theta_i, ND_{\mathrm{i}}^2\right]\end{array}\right), \tag{2}$$

where $\Pr\left(ND_{j\neq i}^1\right)$ is the probability that no IPO is made by any other firm at $t = 1$, D_i^2 (ND_i^2) indicates that firm i goes public (does not IPO) at $t = 2$, and $\Pr\left(D_i^2\right)$ $(\Pr\left(ND_i^2\right))$ is the probability that firm i, which did not IPO at $t = 1$, will IPO (not IPO) at $t = 2$.

We analyze a symmetric equilibrium of $N \geq 2$ firms whose types are independent, so that the *ex ante* probability of observing at least one IPO is identical for all firms. Consequently, the probability that no IPO is made at $t = 1$ by any other firm is $\Pr\left(ND_{j\neq i}^1\right) = \left[G\left(\theta_1^*\right)\right]^{N-1}$. The probability that firm i with type θ_i that did not go public at $t = 1$ will IPO at $t = 2$, given that s_1 was revealed, is the probability that the realization of s_1 will be sufficiently high such that Equation (1) holds. That is, for any given θ_i, the firm will go public at $t = 2$ if and only if $s_1 \geq s_1^*(\theta_i) \equiv -\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}$. The probability of such an event is $F\left(\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)$. Substituting the above into the expected payoff of the manager of firm i from not going public at $t = 1$, given in Equation (2), yields:

$$u_i = \left[G\left(\theta_1^*\right)\right]^{N-1}\left(\frac{\theta_1^*}{1+r}\right) + \left(1 - \left[G\left(\theta_1^*\right)\right]^{N-1}\right)\left(\begin{array}{c}F\left(\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)E\left[\text{payoff at } t = 2|\theta_i,\, D_i^2\right] \\ +\left(1 - F\left(\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)\right)E\left[\text{payoff at } t = 3|\theta_i,\, ND_i^2\right]\end{array}\right). \tag{3}$$

Note that, unlike the threshold at $t = 2$, which depends on the manager's type and the realization of s_1, the IPO threshold of the first period, θ_1^*, depends only on the firm's type, θ_i (and all of the other parameters of the model).

In order to derive and analyze the equilibrium, it is useful to define and characterize the properties of the first-period value of the manager's real option from her ability to delay the IPO at $t = 2$. The option value arises from the firm's opportunity to determine the IPO decision at $t = 2$ after observing the realized value of s_1 (when at least one other firm went public at $t = 1$). As Lemma 1 illustrates, the firm prefers to take advantage of the real option and delay the IPO at $t = 2$ only for sufficiently low values of θ_i and s_1. To capture the option value that stems from not going public at $t = 1$, we first express the expected payoff of a type θ_i firm that is not strategic and always goes public at $t = 2$. The expected payoff of such a non-strategic firm, which we denote by $NS(\theta_i)$, is

$$NS(\theta_i) \equiv E[\text{Payoff if IPO at } t = 2] = E\left[\frac{\theta_i + s_2}{1+r}\right] = \frac{\theta_i}{1+r}.$$

The expected payoff of a type θ_i firm that never goes public at $t = 1$, but is strategic at $t = 2$, which we denote by $S\ (\theta_i)$, is given by:

$$S(\theta_i) \equiv E\left[\text{Payoff if follow IPO strategy } \theta_2^* \text{ at } t = 2\right].$$

Finally, we define the option value as the increase in the expected payoff of a manager who does not go public at $t = 1$ and is strategic at $t = 2$, relative to always going public at $t = 2$. The option value, which we denote by $V_2\ (\theta_i)$, is given by:

$$V_2(\theta_i) \equiv S(\theta_i) - NS(\theta_i) = \Pr\left(s_1 < s_1^*(\theta_i)\right)E\left[\frac{\theta_i + s_3}{(1+r)^2} - \frac{\theta_i + s_2}{1+r}|s_1 < s_1^*(\theta_i)\right].$$

The following Lemma describes a fairly intuitive property of the option value, which we later use to show existence and uniqueness of the symmetric threshold equilibrium.

Lemma 2: The manager's first-period value of her real option from being able to delay the IPO in $t=2$ is decreasing in θ_i, i.e.,

$$\frac{\partial V_2(\theta_i)}{\partial \theta_i} < 0.$$

The option value is decreasing in θ_i because of two effects. The first is that discounting is comparatively more punitive for higher-type firms, and hence, delaying the IPO is relatively more costly for high-type firms. The second and more salient effect is that the likelihood of taking advantage of the real option in period 2 is decreasing in θ_i. The reason for this can be seen from Lemma 1; a firm at $t=2$ only delays the IPO for sufficiently negative realizations of s_1. Moreover, higher-value firms require even lower realizations of s_1 in order to find it profitable to delay the IPO until $t = 3$. Thus, the likelihood of obtaining a sufficiently low realization of s_1, such that the firm can take advantage of the real option and delay the IPO at $t = 2$, is decreasing in the type θ_i. Both of the above effects consequently lead the option value to be decreasing in θ_i. The proof of Lemma 2 (found in Appendix A) provides a full and formal analysis.

Having established that the option value from delaying the IPO is decreasing in θ_i, and since the cost of delaying the IPO (due to discounting) is increasing in θ_i for any given strategy of the other firms, we can conclude that the optimal strategy in any equilibrium is a threshold strategy.

Corollary 1: In any equilibrium, each firm's optimal IPO timing strategy is characterized by a threshold at both $t=1$ and $t=2$, θ_1^* and θ_2^*.

We next solve for and analyze the unique symmetric equilibrium in which all firms follow the same strategy. In a symmetric equilibrium, each firm's best response to all other firms' strategies, which follow a threshold strategy θ_1^*, is consequently given by θ_1^*. The $t=1$ threshold level of all firms is such that a firm of the threshold type θ_1^* is indifferent between going public and not going public at $t=1$. Therefore, the threshold level is the type for which θ_1^* equals the expected payoff from not going public at $t=1$, given in Equation (3).

Lemma 3: The IPO threshold at $t=1$ is given by the solution to the following indifference condition at $t=1$:

$$\begin{aligned}\theta_1^* &= \left[G\left(\theta_1^*\right)\right]^{N-1}\left(\frac{\theta_1^*}{1+r}\right)\\ &\quad + \left(1-\left[G\left(\theta_1^*\right)\right]^{N-1}\right)\left[\begin{array}{c} F\left(\frac{\theta_1^*}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)\frac{\theta_1^*}{1+r}+\frac{1}{1+r}\gamma\sigma^2 f\left(-\frac{\theta_1^*}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)\\ +\left(1-F\left(\frac{\theta_1^*}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)\right)\frac{\theta_1^*}{(1+r)^2}-\frac{1}{(1+r)^2}\gamma^2\sigma^2 f\left(-\frac{\theta_1^*}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)\end{array}\right].\end{aligned} \tag{4}$$

The characterization of the first- and second-period IPO thresholds provides insight into the IPO timing of firms. Moreover, these results help to explain the IPO patterns found in the empirical literature and discussed in various media outlets. For example, a ubiquitous feature of IPO markets is industry-wide droughts of public offerings, where there is significant delay *between* IPO times, which can last for several months.[15] By an IPO *drought,* we mean that there are no IPOs in the second period following an IPO in the first period. The results above imply that this occurs when the observed sentiment in the first period, s_1, is sufficiently low, or when the pool of remaining firms have sufficiently low idiosyncratic types. The following corollary characterizes a sufficient condition for an IPO drought:

Corollary 2: An IPO drought occurs with probability one when the observed realization of s_1 is such that

$$s_1 \leq \frac{r}{\gamma}\cdot\frac{-\theta_1^*}{(1+r)-\gamma}.$$

The above Corollary follows from Lemmas 1 and 3. After the first period, the remaining firms must all be of a type less than θ_1^* as, otherwise, they would have gone public in the first period. From Lemma 1, we see that no firm would choose to go public in the second period for any state realization that would induce delay in the second period by the highest possible remaining type, which is bounded above by θ_1^*. Likewise, the probability of a drought occurring is given by $H\left(\theta_2^*\right)^n$, where $H(\cdot)$ is the truncated distribution of θ on $\left[0,\theta_1^*\right)$ and $n \leq N$ is the number of firms remaining after the first period.

[15] For example, see Driebusch and Huang (2014).

Unique Symmetric Equilibrium

In this section, we establish that there exists a unique equilibrium in which all firms follow the same threshold strategy, which we refer to as the symmetric equilibrium. Lemmas 1 and 3 tie down the IPO thresholds in a symmetric equilibrium. We use Lemma 2 to show that this equilibrium exists; any firm whose value is above the threshold indeed finds it optimal to go public at $t=1$. Moreover, we show that the thresholds characterized in Lemmas 1 and 3 are the unique threshold levels in the symmetric equilibrium.

Theorem 1: There exists a unique symmetric equilibrium in which firm i, $i \in \{1, 2, ..., N\}$, uses the following IPO threshold strategy:

(i) Firm i goes public at $t=1$ if and only if $\theta_i \geq \theta_1^*$, where θ_1^* is given by the solution to (4);
(ii) If there was at least one IPO at $t=1$, firm i (which did not go public at $t=1$) goes public at $t=2$ if and only if $\theta_i \geq \theta_2^*(s_1) \equiv -s_1((1+r)-\gamma)\left(\frac{\gamma}{r}\right)$;
(iii) If no IPO was made by any firm at $t=1$, firm i goes public at $t=2$ for all θ_i.

Proof. Given that the IPO strategy of firm i at $t=2$ does not depend on beliefs about θ_j, the IPO strategy at $t=2$ is given by (ii) and (iii) (note that if no firm went public at $t=1$, all firms go public in $t=2$). Under the assumption of the existence of a threshold equilibrium, any IPO threshold at $t=1$ should satisfy the first period's indifference condition in Equation (4).

At $t=2$, a firm will go public if and only if the expected payoff from going public is higher than if it delays the IPO, i.e., it will go public in $t=2$ if $\frac{\theta_i+E(s_2|s_1)}{1+r} \geq \frac{\theta_i+E(s_3|s_1)}{(1+r)^2}$, which holds for all $\theta_i \geq \theta_2^*(s_1) = -s_1((1+r)-\gamma)\left(\frac{\gamma}{r}\right)$. Therefore, no type has an incentive to deviate at $t=2$.

Next, we show that no type has an incentive to deviate at $t=1$. Assume that type $\theta_i = \theta_1^*$ is indifferent between going public and delaying the IPO at $t=1$. To show that all types higher (lower) than θ_1^* strictly prefer to IPO (not to IPO) at $t=1$, note that the marginal cost in delaying the IPO for higher (lower) θ_i is greater (smaller) due to discounting. In addition, the marginal benefit from delaying the IPO (captured by the option value) is lower for higher θ_i, as shown in Lemma 2. Hence, no type has an incentive to deviate at $t=1$.

Next, we show uniqueness of a symmetric IPO threshold. Assume by contradiction that there are two values of θ_1^*: θ_L and θ_H, where $\theta_H > \theta_L$, that are consistent with a symmetric equilibrium. If all firms move from θ_L to θ_H, the probability that the other firms will go public decreases, which, in turn, increases any firm's incentive to go public. That is, it decreases the best response IPO threshold. However, this contradicts the assumption of the existence of a higher equilibrium threshold, θ_H. A similar argument follows for a lower IPO threshold. More formally, the firm's indifference condition at $t=1$ is given by:

$$\begin{aligned}
\theta_i &= \Pr\left(ND_{j\neq i}^1\right)\left(\frac{\theta_i}{1+r}\right) + \left(1-\Pr\left(ND_{j\neq i}^1\right)\right)\left[\begin{array}{c}\Pr\left(D_i^2\right)E[\text{payoff at } t=2|\theta_i, \text{and IPO at } t=2] \\ +\Pr\left(ND_i^2\right)E[\text{payoff at } t=3|\theta_i, \text{and delay IPO at } t=2]\end{array}\right] \\
&= \Pr\left(ND_{j\neq i}^1\right)\left(\frac{\theta_i}{1+r}\right) + \left(1-\Pr\left(ND_{j\neq i}^1\right)\right)\left(\Pr\left(D_i^2\right)\frac{\theta_i}{1+r} + \Pr\left(ND_i^2\right)\left(\frac{\theta_i}{1+r} + V_2(\theta_i)\right)\right) \\
&= \frac{\theta_i}{1+r} + \left(1-\Pr\left(ND_{j\neq i}^1\right)\right)\Pr\left(ND_i^2\right)V_2(\theta_i).
\end{aligned}$$

If the IPO threshold of firm $j \neq i$ increases to θ_H, it has no effect on the option value (conditional on getting to $t=2$ when firm j went public at $t=1$ and firm i's type is $\theta_i > \theta_2^*$); however, the probability of this event decreases as the threshold of firm i increases. As such, the right-hand side of the above indifference condition decreases, which implies that, in order for firm i to be indifferent at $t=1$, the IPO threshold of firm i at $t=1$ must decrease as well—in contradiction to the assumption of the increased IPO threshold. ▪

Note that while we have assumed θ to have support over $[0, \infty)$, we have placed no restriction on the functional form of the distribution of θ. We also note that the results would be qualitatively unaffected if we had rather assumed $\theta_i \in (-\infty, \infty)$. Moreover, for tractability, we assume that ε_t is normally distributed; however, one can show that the above Theorem holds for other distributions of the error term, including distributions with bounded support such as the uniform distribution.

As is typical in games with multiple agents and a continuous type space, asymmetric equilibria may also exist. Under certain conditions of the distribution of θ, $G(\theta)$, we can shed some light on the potential asymmetric equilibria, as well. For the two-firm setting, we find that there exist, at most, three equilibria—the symmetric equilibrium and two asymmetric equilibria—when $G(\theta)$ has a nondecreasing hazard rate. This implies that the best response function for firm i is convex and, thus, there can be, at most, three intersections of the two agents' best response functions. We cannot confirm or preclude the existence of these two asymmetric equilibria; however, we can preclude the existence of any other equilibrium. The following Proposition formalizes this result.[16]

Proposition 1: When $N = 2$, $g'(\theta) < 0$, and $G(\theta)$ has a nondecreasing hazard rate, there are, at most, three equilibria: the symmetric equilibrium and two analogous asymmetric equilibria.

The condition that the density function is strictly decreasing and that $G(\theta)$ has a nondecreasing hazard rate holds for a wide variety of distributions, such as the exponential and certain parameterizations of the Chi-squared and generalized Weibull distributions.[17] Proposition 1 implies that the two possible asymmetric equilibria are analogous in the sense that the equilibrium first-period threshold pairs mirror one another, i.e., $\left(\theta^*_{1,i}, \theta^*_{1,j}\right) = \left(\theta^*_{1,j}, \theta^*_{1,i}\right)$. Hence, in any asymmetric equilibrium, one firm is more likely to IPO early (has a lower threshold) while the other firm delays more often (has a higher threshold).

IV. COMPARATIVE STATICS

We now analyze how the equilibrium is affected by the various parameters of the model. In particular, we generate comparative statics on the first-period threshold θ^*_1 with respect to changes in the following parameters of the model: the variance of the state, the number of firms, the discount rate, the rate of mean reversion in sentiment, and the variance of θ (when types are assumed to be exponentially distributed). We note that this analysis is with respect to the symmetric equilibrium. We explore additional properties that arise from this analysis in Section V.

We begin with the comparative statics regarding the variance of the state σ^2_ε. An increase in σ^2_ε imposes greater uncertainty on firms regarding investor sentiment in $t = 1$. This heightened uncertainty increases the value of the option from delaying the IPO as there is greater risk of a severely negative state. Consequently, firms have a stronger incentive to delay the offering and potentially observe the first-period realization of the state. This implies that the first-period threshold *increases* in uncertainty:

Proposition 2: The first-period IPO threshold is increasing in the variance of the state of nature, σ^2_ε, i.e., a higher variance induces less IPO in the first period:

$$\frac{d\theta^*_1}{d\sigma^2_\varepsilon} > 0.$$

We next consider the effect of an increase in the number of firms on the IPO threshold at $t = 1$. As more pre-IPO firms enter the industry, there is a higher likelihood that another firm's value is above the threshold level. Consequently, firms have a greater incentive to delay in the first period. In terms of the effect on the probability that at least one firm will go public, the above points to two opposite effects. The following proposition shows that the former effect always dominates and the IPO threshold increases in the number of firms.

Proposition 3: The first-period threshold is increasing in the number of firms, i.e.,

$$\frac{d\theta^*_1}{dN} > 0.$$

Recall that discounting (parameter r) is meant to capture the cost of delaying the offering for private firms (e.g., a higher cost of debt over equity or the loss due to a competitor's advantage). We first observe that the second-period threshold is decreasing in the cost of delay. This is natural as more severe realizations of s_1 are necessary to make delay worthwhile in $t = 2$ as r increases. Due to this effect, the possibility of taking advantage of the option in $t = 2$ becomes less likely, and hence the real option is comparatively less valuable as r increases. Moreover, r also has a direct effect on the first-period delay incentive. Both of these effects weaken the incentive to delay the offering in $t = 1$ and lead to a lower IPO threshold.

Proposition 4: The first-period IPO threshold is decreasing in the discount rate, r:

$$\frac{d\theta^*_1}{dr} < 0.$$

We now examine comparative statics for the persistence in investor sentiment γ. As we discuss further in Section V, one potential interpretation of the parameter γ is the expected time between filing the IPO and the actual IPO completion date.

[16] We can show a similar result also for uniform distributions of θ, where the difference is that the best response strategy is concave rather than convex.
[17] The nondecreasing hazard rate ensures that the distribution is not heavy-tailed.

Specifically, a greater average time to IPO completion implies less persistence in investor sentiment (i.e., lower γ). To gain insight into how the degree of mean-reversion affects the first-period IPO decision, we first consider the effect of γ on the second-period threshold θ_2^*:

$$\frac{d}{d\gamma}\theta_2^*(s_1) = -\frac{1}{r}s_1(r-2\gamma+1) = \left\{\begin{array}{ll} >0 & \text{for } \gamma < \frac{r+1}{2} \\ 0 & \text{for } \gamma = \frac{r+1}{2} \\ <0 & \text{otherwise} \end{array}\right\}.$$

We see that the effect of mean-reversion on the second-period threshold is non-monotone. For better intuition on this non-monotonicity, we consider separately the effect of the idiosyncratic component, θ, and the state, s_1, on the incentive to go public or delay the IPO at $t=2$. Since $\theta_i \geq 0$ and is constant over time, it always provides an incentive not to delay the IPO due to discounting. This incentive increases in θ_i. The incentive due to the state of nature is determined by two effects: (i) mean-reversion in the state provides an incentive to delay the IPO for low realizations of s_1; and (ii) the discount factor. To see this more clearly, consider first the limit case where $\gamma = 0$. In this case, realizations of s_t are independent and uncorrelated over time. Hence, $E(s_2|s_1) = 0$ and all managers go public at $t=2$ (if they did not go public at $t=1$). As γ increases from zero, realizations of s_1 become more informative about s_2 and s_3. Consequently, the value from delaying the IPO increases in γ for negative values of s_1. However, there is a second, mitigating effect that stems from the fact that mean-reversion of s_3 decreases as γ increases, which, in turn, decreases the benefit from delaying the IPO at $t=2$ for a given $\{\theta_i, s_1\}$. For sufficiently low γ, effect (i) dominates and the option value increases in γ, leading the IPO threshold to increase in γ. As γ further increases, the second effect becomes relatively more pronounced, such that the option value begins to decrease in γ. As γ approaches 1, the process of the state converges to a random walk and there is no mean-reversion. Hence, the value of the real option that stems from mean-reversion of low realizations of s_1 disappears, and the only reason the option is still valuable is that, when the expected firm value $\theta_i + s$ is negative, there is a benefit from delaying a negative payoff (due to discounting). The resulting pattern is an inverse-U shaped relation between γ and θ_2^*.

The preceding analysis provides a guide for how the option value from delay in the first period is affected with changes in γ. We find that the effect of γ on the first-period threshold closely resembles that of θ_2^*.

Proposition 5: The effect of the rate of mean-reversion, γ, on the IPO threshold at $t=1$, θ_1^*, is similar to its effect on the second period's threshold, $\theta_2^*(s_1)$. Specifically,

$$\frac{d\theta_1^*}{d\gamma} = \left\{\begin{array}{ll} 0 & \text{for } \gamma = \frac{r+1}{2} \\ >0 & \text{for } \gamma < \frac{r+1}{2} \\ <0 & \text{otherwise} \end{array}\right\}.$$

We find that the relationship is non-monotone and hump-shaped in γ. This is also illustrated in Figure 2. The intuition for the non-monotonicity is similar to that of the effect of γ on the second-period threshold. To see this, let us assume by contradiction that $\frac{d\theta_1^*}{d\gamma} < 0$ for $\gamma < \frac{r+1}{2}$. An increase in γ affects the expected option value from not going public at $t=1$ in several ways. First, conditional on another firm going public at $t=1$, the threshold at $t=2$ increases in γ, which consequently increases the expected value of the option. Moreover, under the contradictory assumption, the probability that another firm goes public at $t=1$ is increasing in γ, and hence the probability of taking advantage of the option value at $t=2$ is also increasing in γ. Overall, the expected option value increases. The manager, thus, has a stronger incentive to delay the IPO at $t=1$, which contradicts the assumption that θ_1^* is decreasing in γ. A symmetric argument applies for the case when $\gamma > \frac{r+1}{2}$. (See Appendix A for a formal proof.)

Finally, in the baseline setting, we did not assume a particular form for the distribution of θ in the solution. While this solution is quite general, to provide additional insights on the behavior of the first-period threshold, we consider the case where θ is exponentially distributed. This allows us to see how changes in the variance of θ affect the incentive to delay. An increase in the variance of θ implies that there is a higher likelihood that another firm has a value above the first-period threshold level. This strengthens the delay incentive as firms have a relatively greater likelihood of observing the first-period state, thus raising θ_1^*.

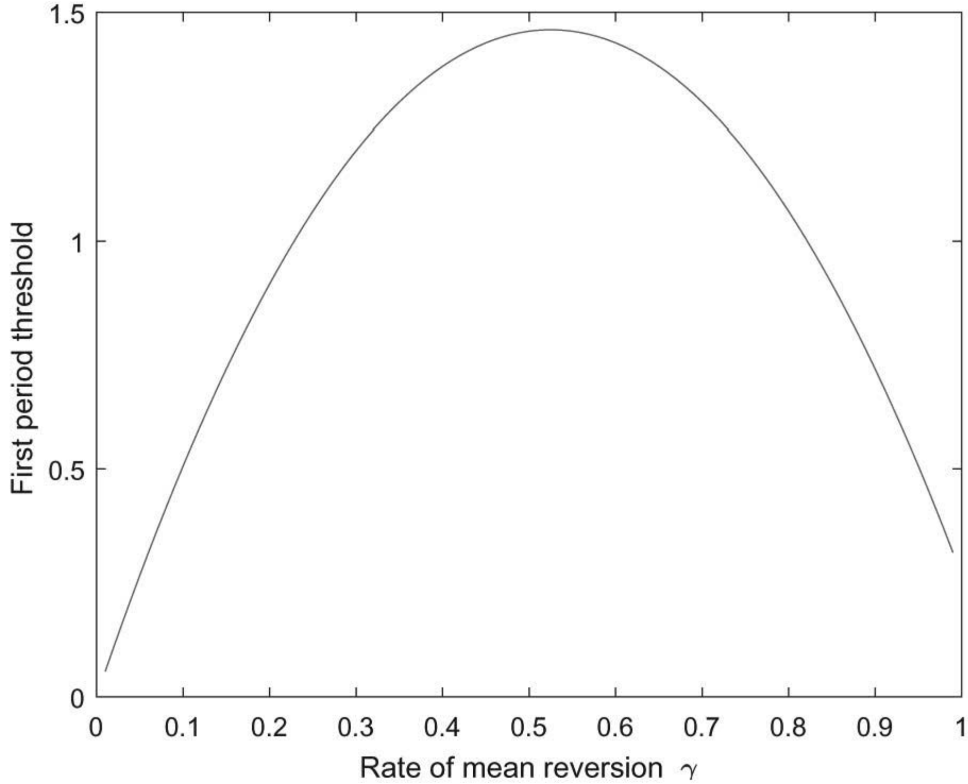
Proposition 6: For θ distributed exponentially, the first-period threshold is increasing in the variance of θ, i.e.,

$$\frac{d\theta_1^*}{d\sigma_\theta^2} > 0.$$

V. EMPIRICAL PREDICTIONS

Our analysis gives rise to a number of empirical predictions. The model offers implications for the timing—when firms decide to go public—as well as predictions for IPO volume, the likelihood of sequential clustering, and the composition of new

FIGURE 2
The First-Period Threshold θ_1^* as a Function of γ



The parameters used are $\sigma_\varepsilon^2 = 1$, $r = 0.05$, $N = 20$, and θ exponentially distributed with mean 1.

issuances over time.[18] By (information-based) *sequential clustering,* we mean that at least two firms go public within two consecutive periods, primarily with respect to the first two periods.[19] Similarly, we refer to firms that go public in the first period as *pioneers,* and firms that go public in the second or third period after a pioneer IPO in $t = 1$ as *followers*. Analogous to sequential clustering, we define an *IPO drought* as the situation where no firms go public in $t = 2$, conditional on at least one firm going public in $t = 1$. We also provide predictions for *IPO volume,* defined as the proportion of firms within an industry that go public in a particular period. Finally, *delay* in our context refers to a given firm postponing its IPO within a particular period. We note that for all of the parameters we consider, except for the number of firms, a given firm's propensity to delay in the first period changes in the opposite direction as the likelihood that a pioneer emerges (i.e., individual delay and overall industry delay are the same for all variables except for N). For brevity, our discussion focuses primarily on the delay of individual firms. We use both analytical, as well as numerical, analysis to determine the empirical implications.

The results of the model imply that the sequential clustering of IPOs is driven by informational spillovers from the IPOs of peer firms, which is consistent with the hypotheses of Lowry and Schwert (2002) and Benveniste, Ljungqvist, Wilhelm, and Yu (2003). Moreover, the results imply that IPO sequential clustering should disproportionately feature firms within a specific industry, as documented by Ritter (1984), and that sequential clustering and high IPO volume emerge following high initial returns of recent IPOs, as documented by Ibbotson and Jaffe (1975) and Lowry and Schwert (2002). The model also implies dispersion of IPOs under weak market conditions, which helps to explain bust patterns of IPOs, as documented in Loughran and Ritter (1995).

[18] A potentially suitable empirical methodology to test some of our predictions is duration analysis.
[19] Stated differently, sequential clustering refers to the situation where at least one firm goes public in $t = 2$, following the IPO of at least one firm in $t = 1$. We use this definition to highlight the learning effects of the timing decision. This usage is also consistent with the literature (e.g., Lowry and Schwert 2002) that discusses clustering in the context of sequential actions that may be attributed to learning.

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When firms have private information and IPO timing is endogenous, we see that there is always a positive amount of delay in the IPO times of some firms, but that other firms find it profitable to go public without delay and forgo potential informational rents. Hence, the first immediate prediction of the model is that firms with higher values (e.g., in terms of profitability or historical earnings) go public earlier than firms with comparatively lower values. Another implication is that following a "successful" IPO in the first period, in which the state of nature is revealed to be relatively high, we expect sequential clustering of IPOs. Our particular and stylized setting assumes that the distribution of the innovation in the state of nature is symmetric, which implies that all firms will go public following a state realization that is above the mean. However, under a more general distribution of the innovation in the state of nature, higher realizations of the state in the first period increase the expected number of firms that will go public in the second period. The following corollary summarizes these immediate predictions of the model:

Corollary 3: In the unique symmetric equilibrium:

- The higher a firm's type, the earlier it will disclose and go public.
- The expected number of IPOs in the second period, following an IPO in the first period, is increasing in the realization of the state of nature in the first period.

Relatedly, the results predict that average issuer quality is declining over time, as the highest-quality firms go public earlier. Additionally, as investor sentiment is mean-reverting, the results predict that during periods of high sentiment (i.e., $s_1 > 0$), initial (first day) returns should decline over time. Conversely, in periods of low investor sentiment ($s_1 < 0$), initial returns should be increasing over time. Benveniste et al. (2003) find evidence for this prediction, whereby follower firms receive lower returns during "hot" IPO markets, which corresponds to a realization of $s_1 > 0$ in our setting.

With respect to uncertainty, Proposition 2 implies that there are fewer pioneer IPOs in nascent industries when there is greater uncertainty over investor sentiment or market conditions. Thus, we expect more delay in the timing of pioneer IPOs, perhaps inefficiently so, and lower initial volume when there is higher industry-wide uncertainty. Moreover, since the likelihood of drawing a materially low state, s_1, is higher with a greater variance, σ_ε^2, there is a higher probability that firms that waited in the first period continue to delay their IPOs until the last period, after having observed an IPO in the first period. Hence, we expect more delay in the IPOs of firms when there is greater uncertainty. This prediction is stated in the following corollary:

Corollary 4: As the uncertainty over investor sentiment, σ_ε^2, increases,

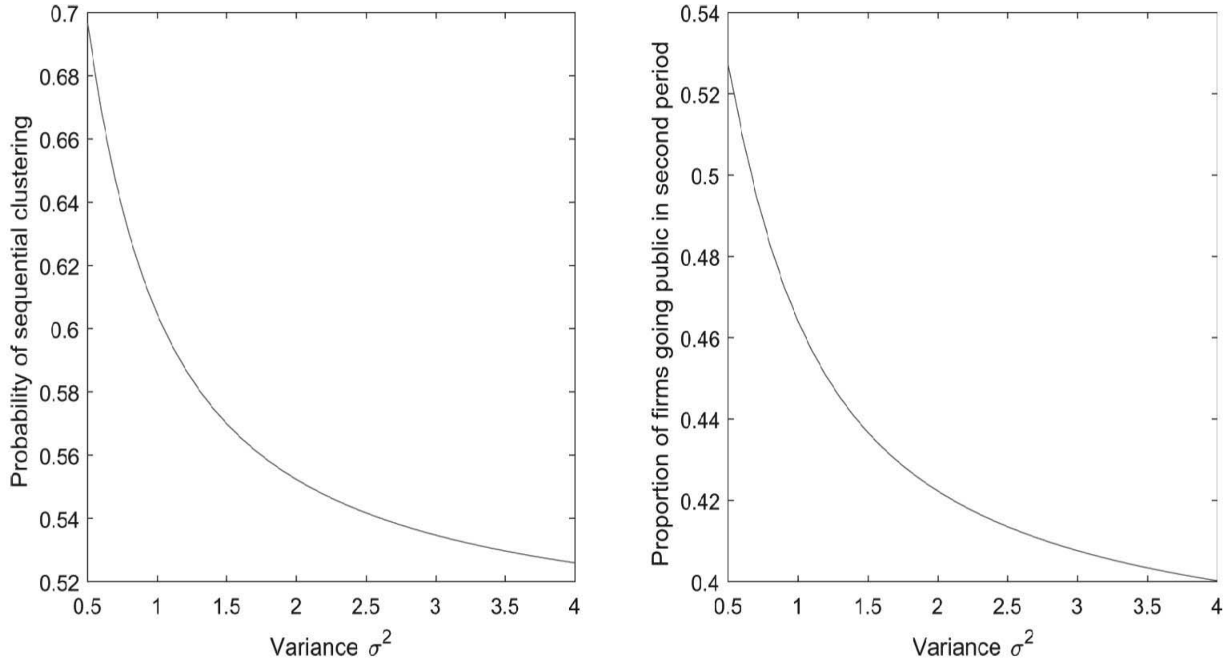
(i) Initial delay increases; i.e., the probability of delaying the IPO in the first period for a given firm increases.
(ii) First-period IPO volume decreases; i.e., a greater proportion of firms delay their IPO issues in the first period.
(iii) For a given firm that delayed in $t = 1$, the probability of delay in $t = 2$ increases, conditional on at least one IPO in the first period.
(iv) Total delay in periods 1 and 2 increases; a lower proportion of firms will have gone public by the end of the second period, conditional on at least one IPO in the first period.
(v) There is less variation in IPO values in the first period, and higher average value of IPOs in both the first and second periods.

The above corollary claims that we should see greater initial delay, lower initial IPO volume, and lower IPO volume by the end of the second period as uncertainty over investor sentiment increases. Some evidence for this has been documented by Schill (2004), who finds that there is lower IPO volume as market uncertainty rises. Benveniste et al. (2003) additionally find that firms that face greater uncertainty are more likely to withdraw their IPO filings. Relatedly, the results predict higher average issuer quality among early IPOs and lower variance in issuer quality among pioneer firms when uncertainty is high.

We obtain additional implications concerning sequential clustering and variation in IPO volume over time with changes in uncertainty through numerical analysis. There are two conflicting forces as σ_ε^2 increases. The first effect is due to the greater initial delay—as more high-type firms wait to potentially observe first-period investor sentiment, these high-type firms are more likely to go public in the second period. This follows from the fact that a more severe state, s_1, is necessary to induce delay for these high-type firms in the second period. The second effect is with regard to the distributional change; an increase in σ_ε^2 also increases the probability of severely pessimistic sentiment and, thus, increases the likelihood of delay in the second period. In Figure 3, we see that the latter effect dominates. In particular, an increase in uncertainty leads to a lower probability of sequential clustering, as well as lower second-period IPO volume. Hence, we predict *greater* dispersion in IPO timing and a lower likelihood of sequential clustering during times of heightened uncertainty.

With respect to the number of firms, the results of the model imply that a given firm is more inclined to delay its offering in the first period in more heavily populated industries. This prediction is implied from Proposition 3—the more concentrated the

FIGURE 3
Sequential Clustering and IPO Volume as a Function of Uncertainty over Sentiment, σ_ε^2



The left panel illustrates the likelihood of sequential clustering in the second period as a function of uncertainty, σ_ε^2, while the right panel shows the second-period IPO volume as a function of uncertainty. Both measures are conditional on at least one firm going public in the first period. The parameters used are $\gamma = 0.5$, $r = 0.05$, $N = 20$, and θ exponentially distributed with mean 1.

industry, the higher the threshold level of going public and, thus, greater delay for a given firm. Likewise, first-period expected IPO volume (measured as the proportion of total firms) is also lower under a greater number of firms. Interestingly, while first-period IPO volume decreases as N increases, there is a higher likelihood that at least one firm goes public in the first period. This somewhat paradoxical relation arises because, while an increase in N pushes the first-period threshold higher, there is also a greater probability that at least one of the firms will have a type realization above this threshold. Consequently, in contrast to our other parameters, the likelihood of a pioneer emerging increases along with the average propensity for delay.

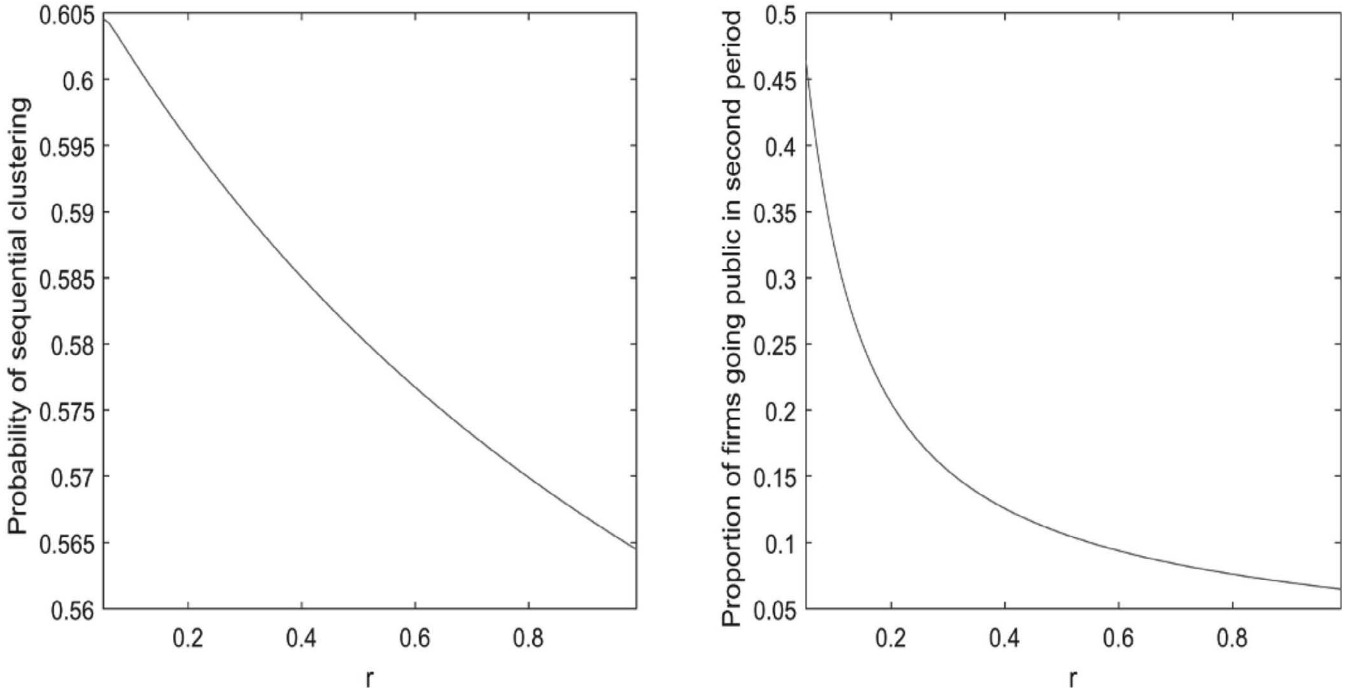
However, in the second period, the likelihood of sequential clustering is higher in industries with a comparatively large number of private firms. As more high-type firms delay their IPO time in the first period, the likelihood of sequential clustering increases as these high-type firms are also more likely to go public in the second period. This occurs since a lower state, s_1, is necessary to induce delay in the second period for high-type firms (as shown by Lemma 1). Moreover, second-period IPO volume, in terms of both the number and proportion of firms, is increasing in industry size. In sum, in more populated industries, a few very high-type pioneer IPOs are likely to emerge, but overall initial IPO volume declines. However, upon the emergence of a pioneer, heavy IPO volume ensues. Additionally, the more populous a (nascent) industry is, the more likely we are to observe sequential clustering of IPOs.

Corollary 5: As the number of firms in the industry increases, we have the following effects:

(i) The propensity to delay the IPO in the first period for a given firm increases.
(ii) First-period volume, as a proportion of total firms in the industry, decreases.
(iii) The probability that at least one firm goes public in $t = 1$ increases.
(iv) The probability of sequential clustering increases (the probability of an IPO drought decreases).
(v) Second-period IPO volume, in terms of the number and proportion of firms, increases.
(vi) There is lower variation of IPO values in the first period and greater variation in the second period. Average issuer value increases in both the first and second period.

The results predict that there should be less variation in issuer quality among pioneer firms (as well as higher average issuer quality), and greater variation in issuer quality among follower firms. Some empirical support for the predictions in Corollary 5

FIGURE 4
Sequential Clustering and IPO Volume as a Function of *r*



The left panel illustrates the likelihood of sequential clustering in the second period as a function of the discount rate, r, while the right panel shows the second-period IPO volume as a function of r. Both measures are conditional on at least one firm going public in the first period. The parameters used are $\gamma = 0.5$, $\sigma_{\varepsilon}^{2} = 1$, $N = 20$, and θ exponentially distributed with mean 1.

has been documented by Dunbar and Foerster (2008), who find that IPO completion likelihood is increasing in the number of previous filings. This suggests that follower firms are less likely to delay their IPOs after observing the IPOs of pioneer firms, resulting in a greater likelihood of sequential clustering.

We next consider the impact of an increased discount rate, which can be thought of as the relative cost of borrowing over equity financing as one interpretation. As shown in Proposition 4, as r increases, delay becomes more costly and hence we expect higher first-period volume. The effect of the delay cost on sequential clustering is less straightforward. An increase in r lowers the second-period threshold, which pushes firms to delay less in the second period, conditional on seeing the first-period state. However, the firms that delay in the first period are of lower quality as the delay cost increases, which implies that these firms require a relatively less severe realization of s_1 to continue delaying the IPO in the second period. Interestingly, we see in Figure 4 that the latter effect dominates, and that there is a *lower* likelihood of sequential clustering and, therefore, a higher chance of an IPO drought as the cost of delay increases. Hence, we have the following predictions: as the cost of delay (e.g., cost of borrowing) increases, we should see high initial volume, which is followed by greater dispersion in IPOs. This also implies that we should see greater variation in issuer quality of pioneer firms, and less variation in issuer quality of follower firms. The average issuer quality is lower for both pioneer and follower IPOs.

Last, we discuss the implications that arise with respect to the rate of mean reversion γ. This parameter measures the persistence in investor sentiment over time. As we see in Proposition 5, the first-period threshold is hump-shaped in γ, which implies that initial volume is U-shaped in γ. One possible interpretation for empirical analysis is that the persistence in sentiment reflects the average time between IPO filing and the actual IPO date.[20] As discussed in Section II, after the firm files the registration statement, the SEC must review the filing and approve the firm for IPO. A longer time between filing and IPO completion can be interpreted as less persistent sentiment, as the market, for example, has more time to correct its previous optimism. Similarly, a shorter time between filing and the completion date implies more persistent sentiment. We note that

[20] We thank an anonymous reviewer for this suggestion.

recent legislation has allowed for faster processing times of IPO registration statements for smaller firms and, thus, may provide a setting to test the implications with respect to the parameter γ.[21]

Corollary 6: A firm's propensity to delay the IPO in the first period is decreasing in the discount rate r and is inverse U-shaped in the rate of mean reversion γ. First-period IPO volume is increasing in r and is U-shaped in γ.

VI. EXTENSIONS: ROLE OF INFORMATION

Below, we discuss a few extensions to our main model regarding the role of information. We provide the technical details, as well as further discussion, in the Online Appendix. We also examine the case of a bounded support in the Online Appendix.

Increased Learning in the Number of IPOs

In the baseline model, we assume that all firms observe the realization of the state at $t = 1$, s_1, if there was at least one IPO in the first period. In this section, we discuss relaxing this feature in several ways. First, we may assume instead that firms receive an imperfect signal of s_1 upon an IPO by at least one firm at time 1, such as $q_1 = s_1 + \delta$, where $\delta \sim N(0, \sigma_\delta^2)$ and is independent of s_1. Our results are qualitatively unchanged under this specification as the only change is in the posterior distribution of s_1.

Second, we can further consider the situation where the precision of the signal of the state is increasing in the number of IPOs that took place in the previous period. This is natural, as observing the pricing or initial returns of a greater number of recent IPOs should provide more information regarding the level of investor sentiment. In particular, we can assume that each IPO firm discloses θ_i and receives a price $P_t^i = \theta_i + s_t + \xi_{t,i}$, where $\xi_{t,i}$ is a zero-mean, normally distributed error term that is independent of s_t, θ_i, and across firms, $\xi_{t,-i}$. For example, $\xi_{t,i}$ may be interpreted as an idiosyncratic demand shock to the firm's pricing. The remaining firms are unable to learn the state perfectly, but beliefs regarding s_t become more precise with a greater number of IPOs in that period.

This adds additional complexity to the model, as the benefit from delay and the resulting posterior distribution of s_1 now depend on the number of firms that go public in the first period. However, our main results are not substantively impacted under this alternative specification. In particular, the option value to delaying the IPO in the first period persists. Moreover, the monotone properties of the option value continue to hold as higher types continue to find delay less worthwhile relative to lower types for a given level of expected informativeness of s_1 from delay (see the Online Appendix for further details).

Additional Information about the State

In the baseline setting, we have assumed that firms learn about the state only by observing the IPOs of other firms. We can relax this feature and enrich the model with additional information about the state. For example, firms observe broader macroeconomic conditions or the IPO markets of other industries, and this may be correlated with the level of sentiment for new issuances in a pre-IPO firm's industry. To capture this, we may allow for the arrival of an imperfect public signal regarding s_1 at the beginning of the first period. The qualitative properties and structure of the equilibrium largely continue to be preserved with this extension (see the Online Appendix). One exception is that this feature introduces positive expected delay in the second period, even if no IPOs are observed in the first period, as the updated conditional mean of s_1 may be below zero.

A related extension is where firms receive an imperfect signal of the first-period sentiment at the end of the first period. As above, this would lead to additional delay in the second period even if no IPOs were observed in the first period. Moreover, this extra information would amplify the incentive to delay in the first period, leading to a higher threshold and greater initial delay. However, the qualitative properties of the equilibrium in the baseline setting would continue to hold.

Finally, we may consider a related setting that incorporates both additional information and increased learning from more IPOs. This setting also allows us to link the accounting reporting quality of firms to IPO decisions. Suppose that prior to the IPO decision in the first period, each firm privately observes only one signal that includes both the firm's idiosyncratic type, θ_i, and the common component, s_1. The signal that firm i observes is $x_i = \theta_i + s_1$, and each firm cannot perfectly disentangle the individual components of x_i. If firm i goes public at $t = 1$, the accounting process introduces some noise into the firm's disclosure, such that the market observes $y_i = \theta_i + s_1 + \eta_i$, where η_i is an error term with variance σ_η^2 and a mean of zero. The precision $1/\sigma_\eta^2$ can reflect the accounting quality of recent financial statements furnished to the SEC in the registration statement and prospectus. Upon observing the IPO of firm i, the remaining firms are, thus, unable to perfectly disentangle the state from firm i's idiosyncratic component. A higher level of accounting quality (lower σ_η^2) leads to more precise inference of

[21] This is in reference to the 2012 Jumpstart Our Business Startups Act, which included "de-burdening" provisions that lowered the filing requirements for firms with less than \$1 billion in revenues.

both the firm's idiosyncratic component and the common component, s_1. Similar to all of the settings we discussed previously, firms continue to have an option value from delaying the IPO, as the IPOs by other firms in the first period allow firms to update their beliefs about s_1. The precision in learning about the common factor s_1 is increasing in the accounting quality and in the number of IPOs. Note that the monotone properties of the option value in the firm's private information are preserved, as a firm's belief of θ_i is increasing in x_i.

While the above setting faces the same fundamental economic forces as our baseline model, it introduces an additional layer of complexity to analytical derivation of the equilibrium.[22] This model can provide insight into the relation between firm accounting quality, which is inversely related to σ_η^2, and the IPO decision. All else equal (in particular, when keeping constant the IPO threshold), an increase in accounting quality (decrease in σ_η^2) results in a more precise market inference about s_1 following a given firm's IPO. As such, the option value from delaying the IPO increases and a firm's IPO threshold in the first period increases. This increased incentive to delay the IPO leads to a secondary effect: the expected number of firms that IPO in the first period is decreasing in the accounting quality. The latter effect decreases the option value from delaying the IPO. One can see that the first effect always dominates. While an increase in the accounting quality always decreases the expected number of IPOs at $t=1$, the expected precision of the market beliefs about s_1 at the end of the first period is always increasing in the accounting quality. This implies that an increase in the accounting quality in the industry results in more delay of IPOs, but still increases learning about the state. While increased accounting quality generates more initial delay in going public, it always increases firms' welfare.[23]

VII. CONCLUDING REMARKS

In this study, we investigate a dynamic timing model where many firms must decide the time in which they disclose and sell shares of a firm or a project. Firms that delay their IPO times stand to gain from observing the IPOs of other firms. This informational rent is captured through a common component, such as investor sentiment, which is observed through the pricing of IPOs of other firms. The importance of investor sentiment in IPO pricing has been discussed in media outlets and is consistent with findings in the empirical literature (e.g., Cornelli et al. 2006). The main results of our model show that pioneer firms emerge endogenously, even in the face of information spillovers.

We present a number of novel results concerning IPO timing and volume. We find that the IPO order is determined by firm type: higher-value firms go public earlier and emerge as IPO pioneers, whereas low-value firms tend to delay their IPO times in order to first observe market conditions or benefit from information spillover. We show that sequential IPO clustering emerges after the realization of favorable investor sentiment. However, the results also show how IPOs can exhibit dispersion, where there is substantial delay between pioneer IPOs and followers—a ubiquitous phenomenon that has largely not been captured by previous theoretical studies. The results also provide predictions regarding how IPO volume fluctuates over time and can vary by industry concentration, uncertainty over the state, delay costs, and persistence in sentiment. We find that greater industry concentration (i.e., number of firms) increases the likelihood that a given firm delays its IPO in the first period, but this is met with a higher likelihood of sequential clustering and higher IPO volume in the second period.

Our model can also shed new light on the effect of accounting quality on the timing patterns of IPOs. In particular, higher accounting quality strengthens the incentive to delay the IPO in order to glean information from the IPOs of other firms, improves the information content of IPOs, and improves firms' welfare. Several extensions can be considered for future work. We have assumed that the firm's type (idiosyncratic component) is constant over time. A potentially interesting research question is to investigate a model where the firm's value is also evolving. Relatedly, our model can be extended to a continuous-time setting with a finite number of firms, where the market condition follows a Brownian diffusion process. We conjecture that in the continuous-time analog, there exists a symmetric equilibrium in which the IPO timing strategy is decreasing in firm-type and further delay is decreasing in realizations of the state. The continuous-time analog, thus, seems to share the main characteristics of our discrete-time model.

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[22] In the above setting, since the manager's private information, x_i, is correlated with the state, s_1, firms that delayed the IPO update their beliefs of s_1 not only from observing IPOs, but also from observing firms that *did not* go public at $t=1$. In particular, given the threshold strategy nature of the equilibrium, upon observing that firm i did not IPO at $t=1$, the other firms infer that x_i is lower than the IPO threshold. This is used in the updating about s_1, which depends on the joint distribution of all of the other information that the market has and θ_i and s_1. We note that posterior beliefs of each firm will vary based on the firm's private information. While this setting introduces tractability challenges, we conjecture that the qualitative characteristics of our baseline model continue to hold.

[23] Since accounting quality does not affect the *ex ante* expected sentiment or firm value, a revealed preference argument establishes that the increased delay in IPO is preferred by firms, and hence improves their welfare.

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APPENDIX A
Proofs

Proof of Lemma 1

By the second-period indifference condition, we have:

$$\frac{\theta_i + E(s_2|s_1)}{1+r} = \frac{\theta_i + E(s_3|s_1)}{(1+r)^2}$$

$$\frac{\theta_i + \gamma s_1}{1+r} = \frac{\theta_i + \gamma^2 s_1}{(1+r)^2}$$

$$\theta_i\left(\frac{r}{1+r}\right) = \frac{\gamma^2 s_1}{1+r} - \gamma s_1 = s_1\left(\frac{\gamma}{1+r} - 1\right)\gamma$$

$$\theta_2^*(s_1) = -s_1((1+r)-\gamma)\left(\frac{\gamma}{r}\right).$$

■

Proof of Lemma 2

The option value is equal to the likelihood that the firm that did not disclose at $t=1$ chooses not to disclose at $t=2$ times the increase in expected payoff due to the delay in the disclosure, which is

$$V_2(\theta_i) = S(\theta_i) - NS(\theta_i) = \Pr\left(S < s_1^*(\theta_i)\right)E\left[\frac{\theta_i + s_3}{(1+r)^2} - \frac{\theta_i + s_2}{1+r}|s_1 < s_1^*(\theta_i)\right], \tag{5}$$

where $s_1^*(\theta_i)$ is the value of s_1 such that the agent is indifferent between disclosing and not disclosing at $t=2$. Note that for any $s_1 < s_1^*(\theta_i)$, $\frac{\theta_i+s_3}{(1+r)^2} - \frac{\theta_i+s_2}{1+r} > 0$. From Equation (1), we have:

$$s_1^*(\theta_i) = -\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}$$

Note that

$$\frac{\partial s_1^*(\theta_i)}{\partial \theta_i} < 0,$$

which implies that also

$$\frac{\partial \Pr\left(S < s_1^*(\theta_i)\right)}{\partial \theta_i} < 0.$$

The derivative of the option value with respect to θ_i is:

$$\begin{aligned}\frac{\partial}{\partial \theta_i} V_2(\theta_i) &= \frac{\partial}{\partial \theta_i}\left[\Pr\left(S < s_1^*(\theta_i)\right) E\left[\frac{\theta_i + s_3}{(1+r)^2} - \frac{\theta_i + s_2}{1+r} | s_1 < s_1^*(\theta_i)\right]\right] \\ &= \frac{\partial}{\partial \theta_i}\left[F\left(s_1^*(\theta_i)\right) \cdot \left(\frac{1}{F\left(s_1^*(\theta_i)\right)} \int_{-\infty}^{s_1^*(\theta_i)} \left(\frac{\theta_i + E(s_3|s_1)}{(1+r)^2} - \frac{\theta_i + E(s_2|s_1)}{1+r}\right) f(s_1) ds_1\right)\right]\end{aligned}$$

Plugging in $E(s_2|s_1) = \int_{-\infty}^{\infty} (\gamma s_1 + \varepsilon_2) f(\varepsilon_2) d\varepsilon_2$ and $E(s_3|s_1) = \int_{-\infty}^{\infty} \left(\gamma \int_{-\infty}^{\infty} (\gamma s_1 + \varepsilon_2) f(\varepsilon_2) d\varepsilon_2 + \varepsilon_3\right) f(\varepsilon_3) d\varepsilon_3$, yields:

$$\frac{\partial}{\partial \theta_i} V_2(\theta_i) = \frac{\partial}{\partial \theta_i} \int_{-\infty}^{s_1^*(\theta_i)} \left[\begin{array}{c}\frac{\theta_i + \int_{-\infty}^{\infty}\left(\gamma \int_{-\infty}^{\infty} (\gamma s_1 + \varepsilon_2) f(\varepsilon_2) d\varepsilon_2 + \varepsilon_3\right) f(\varepsilon_3) d\varepsilon_3}{(1+r)^2} \\ - \frac{\theta_i + \int_{-\infty}^{\infty} (\gamma s_1 + \varepsilon_2) f(\varepsilon_2) d\varepsilon_2}{1+r}\end{array}\right] f(s_1) ds_1 = \frac{\partial}{\partial \theta_i} \int_{-\infty}^{s_1^*(\theta_i)} \left[\frac{\theta_i + \gamma^2 s_1}{(1+r)^2} - \frac{\theta_i + \gamma s_1}{1+r}\right] f(s_1) ds_1$$

Recall that $s_1^*(\theta_i)$ is the value of s_1 such that a firm of type θ_i is indifferent between disclosing in $t = 2$ or $t = 3$ upon the realization of s_1 in the beginning of $t = 2$. Hence, by definition, we have that $\frac{\theta_i + \gamma^2 s_1}{(1+r)^2} - \frac{\theta_i + \gamma s_1}{1+r} > 0$ for all $s < s_1^*(\theta_i)$ (i.e., it is more profitable to wait until $t = 3$ for even worse/more negative realizations of s_1. A marginal increase in θ_i thus has two effects. First, we see immediately that $\frac{\partial}{\partial \theta_i}\left(\frac{\theta_i + \gamma^2 s_1}{(1+r)^2} - \frac{\theta_i + \gamma s_1}{1+r}\right) = \frac{1}{1+r}\left(\frac{1}{1+r} - 1\right) < 0$ since $r > 0$. Moreover, $s_1^*(\theta_i)$ is decreasing in θ_i (i.e., the s_1 required for a higher θ_i to be indifferent must be even more negative) and, thus, the interval over which we integrate is truncated as θ_i increases. Hence, the integral $\int_{-\infty}^{s_1^*(\theta_i)} \left[\frac{\theta_i + \gamma^2 s_1}{(1+r)^2} - \frac{\theta_i + \gamma s_1}{1+r}\right] f(s_1) ds_1$ is decreasing in θ_i.

This can also be explicitly shown. Using Leibniz's rule, we have

$$\begin{aligned}&\frac{\partial}{\partial \theta_i} \int_{-\infty}^{s_1^*(\theta_i)} \left[\frac{\theta_i + \gamma^2 s_1}{(1+r)^2} - \frac{\theta_i + \gamma s_1}{1+r}\right] f(s_1) ds_1 \\ &= \int_{-\infty}^{s_1^*(\theta_i)} \frac{\partial}{\partial \theta_i}\left[\frac{\theta_i + \gamma^2 s_1}{(1+r)^2} - \frac{\theta_i + \gamma s_1}{1+r}\right] f(s_1) ds_1 + \frac{\partial s_1^*(\theta_i)}{\partial \theta_i}\left[\frac{\theta_i + \gamma^2 s_1^*(\theta_i)}{(1+r)^2} - \frac{\theta_i + \gamma s_1^*(\theta_i)}{1+r}\right] f\left(s_1^*\right) \\ &= \int_{-\infty}^{s_1^*(\theta_i)} \left[\frac{1}{(1+r)^2} - \frac{1}{1+r}\right] f(s_1) ds_1 + \left[\frac{\partial\left(-\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)}{\partial \theta_i}\right]\left[\frac{\theta_i + \gamma^2\left(-\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)}{(1+r)^2} - \frac{\theta_i + \gamma\left(-\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)}{1+r}\right] f\left(s_1^*\right) \\ &= \int_{-\infty}^{s_1^*(\theta_i)} \left[\frac{1}{(1+r)^2} - \frac{1}{1+r}\right] f(s_1) ds_1 - \left[\frac{r}{\gamma(r - \gamma + 1)}\right][0] f\left(s_1^*\right) = \int_{-\infty}^{s_1^*(\theta_i)} \frac{-r}{(1+r)^2} f(s_1) ds_1 < 0.\end{aligned}$$

Note that $s_1 = \varepsilon_1$ and we define the integral in terms of s_1 rather than ε_1 for presentational ease. Also note that the proof does not rely on a specific distribution of ε; as such, the result that the option value decreases in θ_i holds for any distribution of ε. ∎

Proof of Lemma 3

Starting from (2), given our disclosure threshold in $t = 2$, (2) becomes:

$$\left[G\left(\theta_1^*\right)\right]^{N-1}\left(\frac{\theta_i}{1+r}\right) + \left(1 - \left[G\left(\theta_1^*\right)\right]^{N-1}\right) \cdot \left[\begin{array}{c}\Pr\left[\theta_i > -s_1((1+r)-\gamma)\left(\frac{\gamma}{r}\right)\right] E\left[\frac{\theta_i + s_2}{1+r} | \theta_i > -s_1((1+r)-\gamma)\left(\frac{\gamma}{r}\right)\right] \\ + \Pr\left[\theta_i \le -s_1((1+r)-\gamma)\left(\frac{\gamma}{r}\right)\right] E\left[\frac{\theta_i + s_3}{(1+r)^2} | \theta_i \le -s_1((1+r)-\gamma)\left(\frac{\gamma}{r}\right)\right]\end{array}\right]. \tag{6}$$

Note that in any point in time, the agent knows the value of her θ. Next, we calculate each of the terms above:

$$\Pr\left[\theta_i > -s_1((1+r)-\gamma)\left(\frac{\gamma}{r}\right)\right] = \Pr\left[s_1 > -\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right] = F\left(\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right).$$

and:

$$E\left[\frac{\theta_i+s_2}{1+r}|\theta_i>-s_1((1+r)-\gamma)\left(\frac{\gamma}{r}\right)\right]=E\left[\frac{\theta_i+s_2}{1+r}|s_1>-\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right].$$

which becomes:

$$\begin{aligned}\frac{1}{F\left(\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)}\int_{-\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}}^{\infty}\frac{\theta_i+E(s_2|s_1)}{1+r}f(s_1)ds_1&=\frac{\theta_i}{1+r}+\frac{1}{1+r}\frac{1}{F\left(\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)}\int_{-\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}}^{\infty}[E(s_2|s_1)]f(s_1)ds_1\\&=\frac{\theta_i}{1+r}\\&\quad+\frac{1}{1+r}\frac{1}{F\left(\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)}\int_{-\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}}^{\infty}\left[\int_{-\infty}^{\infty}(\gamma s_1+\varepsilon_2)f(\varepsilon_2)d\varepsilon_2\right]f(s_1)ds_1\\&=\frac{\theta_i}{1+r}+\frac{1}{1+r}\frac{1}{F\left(\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)}\int_{-\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}}^{\infty}\gamma s_1 f(s_1)ds_1\\&=\frac{\theta_i}{1+r}+\frac{1}{1+r}\gamma E\left[s_1|s_1>-\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right].\end{aligned}$$

Recall that the formula for the expectation of the truncated normal distribution where $x\sim N(\mu_x,\sigma^2)$ is:[24]

$$E(x|x\in[a,b])=\mu_x-\sigma^2\frac{f(b)-f(a)}{F(b)-F(a)}.$$

Using the above formula, we have:

$$\begin{aligned}E\left[\frac{\theta_i+s_2}{1+r}|\theta_i>-s_1((1+r)-\gamma)\left(\frac{\gamma}{r}\right)\right]&=\frac{\theta_i}{1+r}+\frac{1}{1+r}\gamma\left(0-\sigma_\varepsilon{}^2\frac{-f(-\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)})}{1-F(-\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)})}\right)\\&=\frac{\theta_i}{1+r}+\frac{1}{1+r}\gamma\left(\sigma_\varepsilon{}^2\frac{f(-\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)})}{F(\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)})}\right).\end{aligned}$$

Finally:

$$\begin{aligned}E\left[\frac{\theta_i+s_3}{(1+r)^2}|\theta_i\le-s_1((1+r)-\gamma)\left(\frac{\gamma}{r}\right)\right]&=\frac{\theta_i}{(1+r)^2}+\frac{1}{(1+r)^2}\gamma^2E\left[s_1|s_1<-\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right]\\&=\frac{\theta_i}{(1+r)^2}+\frac{1}{(1+r)^2}\gamma^2\left(-\sigma_\varepsilon^2\frac{f(-\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)})-0}{F(-\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)})-0}\right)\\&=\frac{\theta_i}{(1+r)^2}+\frac{1}{(1+r)^2}\gamma^2\left(-\sigma_\varepsilon^2\frac{f(-\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)})}{\left(1-F(\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)})\right)}\right).\end{aligned}$$

Plugging this back into (2):

24 For $a=-\infty$, we have $E(x|x<b)=\mu_x-\sigma^2\frac{f(b)}{F(b)}$.

$$\left[G\left(\theta_1^*\right)\right]^{N-1}\left(\frac{\theta_i}{1+r}\right)+\left(1-\left[G\left(\theta_1^*\right)\right]^{N-1}\right)\left[\begin{array}{c} F\left(\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)\left(\frac{\theta_i}{1+r}+\frac{1}{1+r}\gamma\left(\sigma_\varepsilon^2\frac{f\left(-\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)}{F\left(\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)}\right)\right) \\ +\left(1-F\left(\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)\right)\left(\frac{\theta_i}{(1+r)^2}+\frac{1}{(1+r)^2}\gamma^2\left(-\sigma_\varepsilon^2\frac{f\left(-\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)}{\left(1-F\left(\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)\right)}\right)\right)\end{array}\right] \tag{7}$$

$$=\left[G\left(\theta_1^*\right)\right]^{N-1}\left(\frac{\theta_i}{1+r}\right)+\left(1-\left[G\left(\theta_1^*\right)\right]^{N-1}\right)\left[\begin{array}{c} F\left(\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)\frac{\theta_i}{1+r}+\frac{1}{1+r}\gamma\sigma_\varepsilon^2 f\left(-\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right) \\ +\left(1-F\left(\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)\right)\frac{\theta_i}{(1+r)^2}-\frac{1}{(1+r)^2}\gamma\sigma_\varepsilon^2 f\left(-\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)\end{array}\right] \tag{8}$$

The disclosure threshold for $t=1$, θ_1^*, is such that the agent is indifferent between disclosing at $t=1$ and obtaining $\theta_1^*+E[s_1]=\theta_1^*$ and the expected payoff from not disclosing at $t=1$, given in (8). So the candidate for a disclosure threshold is the solution to:

$$\theta_1^*=\left[G\left(\theta_1^*\right)\right]^{N-1}\left(\frac{\theta_1^*}{1+r}\right)+\left(1-\left[G\left(\theta_1^*\right)\right]^{N-1}\right)\left[\begin{array}{c} F\left(\frac{\theta_1^*}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)\frac{\theta_1^*}{1+r}+\frac{1}{1+r}\gamma\sigma_\varepsilon^2 f\left(-\frac{\theta_1^*}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right) \\ +\left(1-F\left(\frac{\theta_1^*}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)\right)\frac{\theta_1^*}{(1+r)^2}-\frac{1}{(1+r)^2}\gamma^2\sigma_\varepsilon^2 f\left(-\frac{\theta_1^*}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)\end{array}\right]$$

■

Proof of Proposition 1

Recall that $\theta\sim G(\theta)$. To simplify notation, we let the first-period threshold for player i, $\theta_{1,i}^*$, be denoted as θ_i^*, e.g., firm 2's first-period threshold is given by θ_2^*. Firm 1's best response function is defined by the indifference condition found in Lemma 3:

$$\theta_1^*=G\left(\theta_2^*\right)\left(\frac{\theta_1^*}{1+r}\right)+\left(1-G\left(\theta_2^*\right)\right)\left[\begin{array}{c} F_\varepsilon\left(\frac{\theta_1^*}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)\frac{\theta_1^*}{1+r}+\frac{1}{1+r}\gamma\sigma^2 f_\varepsilon\left(-\frac{\theta_1^*}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right) \\ +\left(1-F_\varepsilon\left(\frac{\theta_1^*}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)\right)\frac{\theta_1^*}{(1+r)^2}-\frac{1}{(1+r)^2}\gamma^2\sigma^2 f_\varepsilon\left(-\frac{\theta_1^*}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)\end{array}\right] \tag{9}$$

$$=G\left(\theta_2^*\right)\left(\frac{\theta_1}{1+r}\right)+\left(1-G\left(\theta_2^*\right)\right)\left[NS\left(\theta_1^*\right)+V_2\left(\theta_1^*\right)\right] \tag{10}$$

Let K (θ_1, θ_2) be defined as:

$$K(\theta_1,\theta_2)=G(\theta_2)\left(\frac{\theta_1}{1+r}\right)+(1-G(\theta_2))[NS(\theta_1)+V_2(\theta_1)]-\theta_1$$

And hence,

$$\begin{aligned}\frac{d\theta_1}{d\theta_2}&=-\frac{\frac{\partial K}{\partial\theta_2}}{\frac{\partial K}{\partial\theta_1}}=-\frac{g(\theta_2)\left(\frac{\theta_1}{1+r}\right)-g(\theta_2)[NS(\theta_1)+V_2(\theta_1)]}{G(\theta_2)\left(\frac{1}{1+r}\right)+(1-G(\theta_2))\frac{\partial[NS(\theta_1)+V_2(\theta_1)]}{\partial\theta_1}-1} \\ &=-\frac{g(\theta_2)\left(\frac{\theta_1}{1+r}\right)-g(\theta_2)[NS(\theta_1)+V_2(\theta_1)]}{G(\theta_2)\left(\frac{1}{1+r}\right)+(1-G(\theta_2))\left[\frac{1}{1+r}+\int_{-\infty}^{s_1^*(\theta_i)}\frac{-r}{(1+r)^2}f(s_1)ds_1\right]-1} \\ &=-\frac{-g(\theta_2)V_2(\theta_1)}{G(\theta_2)\left(\frac{1}{1+r}\right)+(1-G(\theta_2))\left[\frac{1}{1+r}+\frac{-r}{(1+r)^2}F\left(s_1^*(\theta_i)\right)\right]-1}=-\frac{-g(\theta_2)V_2(\theta_1)}{\left(\frac{1}{1+r}\right)+(1-G(\theta_2))\left[\frac{-r}{(1+r)^2}F\left(s_1^*(\theta_i)\right)\right]-1}<0\end{aligned}$$

Note that $\frac{-r}{(1+r)^2}F\left(s_1^*(\theta_i)\right)<0$ and hence the numerator is negative when $f(\cdot)>0$ for its support. This is expected as an increase in θ_2 results in a decrease in the probability that agent 1 observes s_1, which results in a lower threshold θ_1 necessary to satisfy indifference.

Next, we take the second derivative:

$$\frac{d^2\theta_1}{\left(d\theta_2^*\right)^2} = \frac{d^2 g\left(\theta_1, \theta_2^*\right)}{\left(d\theta_2^*\right)^2} = \frac{d}{d\theta_2^*}\left(\frac{g(\theta_2)V_2(\theta_1)}{\left(\frac{1}{1+r}\right) + (1 - G(\theta_2))\left[\frac{-r}{(1+r)^2}F\left(s_1^*(\theta_i)\right)\right] - 1}\right)$$

which is:

$$\frac{d^2\theta_1}{\left(d\theta_2^*\right)^2} = V_2(\theta_1)\frac{g'(\theta_2)\left(\left(\frac{1}{1+r}\right) + (1 - G(\theta_2))\left[\frac{-r}{(1+r)^2}F\left(s_1^*(\theta_i)\right)\right] - 1\right) + (g(\theta_2))^2\left[\frac{-r}{(1+r)^2}F\left(s_1^*(\theta_i)\right)\right]}{\left(\left(\frac{1}{1+r}\right) + (1 - G(\theta_2))\left[\frac{-r}{(1+r)^2}F\left(s_1^*(\theta_i)\right)\right] - 1\right)^2} \tag{11}$$

Note that the denominator in Equation (11) is always positive. Let $a = \frac{r}{(1+r)^2}F\left(s_1^*(\theta_i)\right)$. Examining the numerator, we have:

$$g' \cdot V_2(\theta_1)\left(\left(\frac{1}{1+r}\right) + (1 - G)[-a] - 1\right) + g^2[-a]V_2(\theta_1) > 0$$

$$g'\left(\left(\frac{1}{1+r}\right) + (1 - G)[-a] - 1\right) + g^2(-a) > 0$$

$$g'\left(\left(\frac{-r}{1+r}\right) + (1 - G)[-a]\right) + g^2(-a) > 0,$$

which holds if

$$g'\left(\left(\frac{-r}{1+r}\right) + (1 - G)[-a]\right) - g' \cdot (1 - G)(-a) > 0$$

$$\left(\left(\frac{-r}{1+r}\right) + (1 - G)[-a]\right) - (1 - G)(-a) < 0$$

$$-\left(\left(\frac{-r}{1+r}\right) + (1 - G)[-a]\right) + (1 - G)(-a) > 0$$

$$\frac{r}{1+r} - (1 - G)[-a] + (1 - G)(-a) > 0$$

$$\frac{r}{1+r} > 0$$

which always holds since $r > 0$. Note this uses the fact that $g^2 \geq -g' \cdot (1 - G)$, which can be seen by the nondecreasing hazard rate:

$$h'(\theta) = \frac{g' \cdot (1 - G) + g^2}{(1 - G)^2} \geq 0,$$

which implies

$$g' \cdot (1 - G) + g^2 \geq 0$$

$$g' \cdot (1 - G) \geq -g^2$$

$$-g' \cdot (1 - G) \leq g^2$$

Therefore,

$$\frac{d^2\theta_1^*}{\left(d\theta_2^*\right)^2} > 0$$

which implies that the best response is a decreasing convex function. Since the two best response functions are symmetric, there can only be three possible intersections. ∎

Proof of Proposition 2

Recall that the equilibrium first-period threshold is:

$$\theta_1^* = G(\theta_1^*)^{N-1}\left(\frac{\theta_1^*}{1+r}\right) + \left(1 - G(\theta_1^*)^{N-1}\right)\begin{bmatrix} F_\varepsilon\left(\frac{\theta_1^*}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)\frac{\theta_1^*}{1+r} + \frac{1}{1+r}\gamma\sigma^2 f_\varepsilon\left(-\frac{\theta_1^*}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right) \\ +\left(1 - F_\varepsilon\left(\frac{\theta_1^*}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)\right)\frac{\theta_1^*}{(1+r)^2} - \frac{1}{(1+r)^2}\gamma^2\sigma^2 f_\varepsilon\left(-\frac{\theta_1^*}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)\end{bmatrix} \tag{12}$$

$$= G(\theta_1^*)^{N-1}\left(\frac{\theta_1}{1+r}\right) + \left(1 - G(\theta_1^*)^{N-1}\right)\left[NS(\theta_1^*) + V_2(\theta_1^*)\right] \tag{13}$$

Let $K(\theta_1^*, r)$ be defined as:

$$K(\theta_1^*, r) = G(\theta_1^*)^{N-1}\left(\frac{\theta_1}{1+r}\right) + \left(1 - G(\theta_1^*)^{N-1}\right)\left[NS(\theta_1^*) + V_2(\theta_1^*)\right] - \theta_1^*$$

And hence,

$$\frac{d\theta_1^*}{d\sigma} = -\frac{\frac{\partial K}{\partial \sigma}}{\frac{\partial K}{\partial \theta_1^*}} = -\frac{\left(1 - G(\theta_1^*)^{N-1}\right)\frac{\partial}{\partial\sigma}\begin{bmatrix} F_\varepsilon\left(\frac{\theta_1^*}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)\frac{\theta_1^*}{1+r} + \frac{1}{1+r}\gamma\sigma^2 f_\varepsilon\left(-\frac{\theta_1^*}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right) \\ +\left(1 - F_\varepsilon\left(\frac{\theta_1^*}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)\right)\frac{\theta_1^*}{(1+r)^2} - \frac{1}{(1+r)^2}\gamma^2\sigma^2 f_\varepsilon\left(-\frac{\theta_1^*}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)\end{bmatrix}}{\begin{bmatrix} (N-1)G(\theta_1^*)^{N-2}g(\theta_1^*)\left(\frac{\theta_1^*}{1+r}\right) + G(\theta_1^*)^{N-1}\left(\frac{1}{1+r}\right) \\ -(N-1)G(\theta_1^*)^{N-2}g(\theta_1^*)\left[NS(\theta_1^*) + V_2(\theta_1^*)\right] + \left(1 - G(\theta_1^*)^{N-1}\right)\cdot\frac{\partial\left[NS(\theta_1^*)+V_2(\theta_1^*)\right]}{\partial\theta_1} - 1\end{bmatrix}} \tag{14}$$

Examining the numerator, we have that

$$\frac{\partial}{\partial\sigma}\begin{bmatrix} F_\varepsilon\left(\frac{\theta_1^*}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)\frac{\theta_1^*}{1+r} + \frac{1}{1+r}\gamma\sigma^2 f\left(-\frac{\theta_1^*}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right) \\ +\left(1 - F_\varepsilon\left(\frac{\theta_1^*}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)\right)\frac{\theta_1^*}{(1+r)^2} - \frac{1}{(1+r)^2}\gamma^2\sigma^2 f_\varepsilon\left(-\frac{\theta_1^*}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)\end{bmatrix}$$

$$= \frac{\partial}{\partial\sigma}\begin{bmatrix} \frac{1}{2}\left[1 + \operatorname{erf}\left(\frac{1}{\sigma\sqrt{2}}\frac{\theta_1^*}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)\right]\frac{\theta_1^*}{1+r} + \frac{1}{1+r}\gamma\sigma^2\frac{1}{\sigma\sqrt{2\pi}}\exp\left[-\frac{\left[\theta_1^*((1+r)-\gamma)^{-1}\left(\frac{\gamma}{r}\right)^{-1}\right]^2}{2\sigma^2}\right] \\ +\left(1 - \frac{1}{2}\left[1 + \operatorname{erf}\left(\frac{1}{\sigma\sqrt{2}}\frac{\theta_1^*}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)\right]\right)\frac{\theta_1^*}{(1+r)^2} - \frac{1}{(1+r)^2}\gamma^2\sigma^2\frac{1}{\sigma\sqrt{2\pi}}\exp\left[-\frac{\left[\theta_1^*((1+r)-\gamma)^{-1}\left(\frac{\gamma}{r}\right)^{-1}\right]^2}{2\sigma^2}\right]\end{bmatrix}$$

$$= -\frac{\exp\left[-\frac{r^2(\theta_1^*)^2}{2\sigma^2\gamma^2(1-\gamma+r)^2}\right]\gamma^2}{(1+r)^2\sqrt{2\pi}} + \frac{\exp\left[-\frac{r^2(\theta_1^*)^2}{2\sigma^2\gamma^2(1-\gamma+r)^2}\right]\gamma}{(1+r)\sqrt{2\pi}} + \frac{\exp\left[-\frac{r^2(\theta_1^*)^2}{2\sigma^2\gamma^2(1-\gamma+r)^2}\right]r(\theta_1^*)^2}{(1+r)^2\gamma(1-\gamma+r)\sigma^2\sqrt{2\pi}} - \frac{\exp\left[-\frac{r^2(\theta_1^*)^2}{2\sigma^2\gamma^2(1-\gamma+r)^2}\right]r(\theta_1^*)^2}{(1+r)\gamma(1-\gamma+r)\sigma^2\sqrt{2\pi}}$$

$$-\frac{\exp\left[-\frac{r^2(\theta_1^*)^2}{2\sigma^2\gamma^2(1-\gamma+r)^2}\right]r^2(\theta_1^*)^2}{(1+r)^2(1-\gamma+r)^2\sigma^2\sqrt{2\pi}} + \frac{\exp\left[-\frac{r^2(\theta_1^*)^2}{2\sigma^2\gamma^2(1-\gamma+r)^2}\right]r^2(\theta_1^*)^2}{(1+r)\gamma(1-\gamma+r)^2\sigma^2\sqrt{2\pi}}$$

$$= \frac{\exp\left[-\frac{r^2(\theta_1^*)^2}{2\sigma^2\gamma^2(1-\gamma+r)^2}\right]\gamma(1-\gamma+r)}{\sqrt{2\pi}(1+r)^2} > 0$$

Thus, the numerator in Equation (14) is positive. Equation (14) reduces to

$$-\frac{\left(1-G\left(\theta_1^*\right)^{N-1}\right)\frac{\exp\left[-\frac{r^2\left(\theta_1^*\right)^2}{2\sigma^2\gamma^2(1-\gamma+r)^2}\right]\gamma(1-\gamma+r)}{\sqrt{2\pi}(1+r)^2}}{-(N-1)G\left(\theta_1^*\right)^{N-2}g\left(\theta_1^*\right)V_2\left(\theta_1^*\right)-\left(1-G\left(\theta_1^*\right)^{N-1}\right)\cdot\left[\frac{r}{(1+r)^2}F\left(s_1^*(\theta_i)\right)\right]-\frac{r}{1+r}}.$$

The denominator is clearly negative. The entire term is multiplied by -1 and hence Equation (14) is positive. Thus, $\frac{d\theta_1^*}{d\sigma}>0$. ■

Proof of Proposition 3

Recall that the equilibrium first-period threshold is:

$$\theta_1^*=G\left(\theta_1^*\right)^{N-1}\left(\frac{\theta_1^*}{1+r}\right)+\left(1-G\left(\theta_1^*\right)^{N-1}\right)\left[\begin{array}{c}F_\varepsilon\left(\frac{\theta_1^*}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)\frac{\theta_1^*}{1+r}+\frac{1}{1+r}\gamma\sigma^2f_\varepsilon\left(-\frac{\theta_1^*}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)\\+\left(1-F_\varepsilon\left(\frac{\theta_1^*}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)\right)\frac{\theta_1^*}{(1+r)^2}-\frac{1}{(1+r)^2}\gamma^2\sigma^2f_\varepsilon\left(-\frac{\theta_1^*}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)\end{array}\right]\tag{15}$$

$$=G\left(\theta_1^*\right)^{N-1}\left(\frac{\theta_1}{1+r}\right)+\left(1-G\left(\theta_1^*\right)^{N-1}\right)\left[NS\left(\theta_1^*\right)+V_2\left(\theta_1^*\right)\right]\tag{16}$$

Let $K\left(\theta_1^*,N\right)$ be defined as:

$$K\left(\theta_1^*,N\right)=G\left(\theta_1^*\right)^{N-1}\left(\frac{\theta_1}{1+r}\right)+\left(1-G\left(\theta_1^*\right)^{N-1}\right)\left[NS\left(\theta_1^*\right)+V_2\left(\theta_1^*\right)\right]-\theta_1^*$$

And hence,

$$\frac{d\theta_1^*}{dN}=-\frac{\frac{\partial K}{\partial N}}{\frac{\partial K}{\partial\theta_1^*}}=-\frac{G\left(\theta_1^*\right)^{N-1}\ln\left(G\left(\theta_1^*\right)\right)\left(\frac{\theta_1^*}{1+r}\right)-G\left(\theta_1^*\right)^{N-1}\ln\left(G\left(\theta_1^*\right)\right)\left[NS\left(\theta_1^*\right)+V_2\left(\theta_1^*\right)\right]}{\left[\begin{array}{c}(N-1)G\left(\theta_1^*\right)^{N-2}g\left(\theta_1^*\right)\left(\frac{\theta_1^*}{1+r}\right)+G\left(\theta_1^*\right)^{N-1}\left(\frac{1}{1+r}\right)\\-(N-1)G\left(\theta_1^*\right)^{N-2}g\left(\theta_1^*\right)\left[NS\left(\theta_1^*\right)+V_2\left(\theta_1^*\right)\right]+\left(1-G\left(\theta_1^*\right)^{N-1}\right)\cdot\frac{\partial\left[NS\left(\theta_1^*\right)+V_2\left(\theta_1^*\right)\right]}{\partial\theta_1}-1\end{array}\right]}$$

This becomes

$$=-\frac{G\left(\theta_1^*\right)^{N-1}\ln\left(G\left(\theta_1^*\right)\right)\left(\frac{\theta_1^*}{1+r}\right)-G\left(\theta_1^*\right)^{N-1}\ln\left(G\left(\theta_1^*\right)\right)\left[NS\left(\theta_1^*\right)+V_2\left(\theta_1^*\right)\right]}{\left[\begin{array}{c}(N-1)G\left(\theta_1^*\right)^{N-2}g\left(\theta_1^*\right)\left(\frac{\theta_1^*}{1+r}\right)+G\left(\theta_1^*\right)^{N-1}\left(\frac{1}{1+r}\right)\\-(N-1)G\left(\theta_1^*\right)^{N-2}g\left(\theta_1^*\right)\left[NS\left(\theta_1^*\right)+V_2\left(\theta_1^*\right)\right]\\+\left(1-G\left(\theta_1^*\right)^{N-1}\right)\cdot\left[\frac{1}{1+r}+\int_{-\infty}^{s_1^*(\theta_i)}\frac{-r}{(1+r)^2}f(s_1)ds_1\right]-1\end{array}\right]}$$

which is

$$=-\frac{-G\left(\theta_1^*\right)^{N-1}\ln\left(G\left(\theta_1^*\right)\right)V_2\left(\theta_1^*\right)}{\left[\begin{array}{c}-(N-1)G\left(\theta_1^*\right)^{N-2}g\left(\theta_1^*\right)V_2\left(\theta_1^*\right)+G\left(\theta_1^*\right)^{N-1}\left(\frac{1}{1+r}\right)\\+\left(1-G\left(\theta_1^*\right)^{N-1}\right)\cdot\left[\frac{1}{1+r}+\frac{-r}{(1+r)^2}F\left(s_1^*(\theta_i)\right)\right]-1\end{array}\right]}$$

$$=\frac{G\left(\theta_1^*\right)^{N-1}\ln\left(G\left(\theta_1^*\right)\right)V_2\left(\theta_1^*\right)}{-(N-1)G\left(\theta_1^*\right)^{N-2}g\left(\theta_1^*\right)V_2\left(\theta_1^*\right)-\left(1-G\left(\theta_1^*\right)^{N-1}\right)\cdot\left[\frac{r}{(1+r)^2}F\left(s_1^*(\theta_i)\right)\right]-\frac{r}{1+r}}>0$$

We have that $G\left(\theta_1^*\right)^{N-1}\ln\left(G\left(\theta_1^*\right)\right)V_2\left(\theta_1^*\right)<0$ since $\ln\left(G\left(\theta_1^*\right)\right)<0$, and we can easily see that the denominator is negative. Thus $\frac{d\theta_1^*}{dN}>0$. ■

Proof of Proposition 4

We first examine the change in the option value, as this will be used in the proof of the claim. Recall that

$$V_2(\theta_i) = \int_{-\infty}^{s_1^*(\theta_i)} \left[\frac{\theta_i + \gamma^2 s_1}{(1+r)^2} - \frac{\theta_i + \gamma s_1}{1+r}\right] f(s_1) ds_1 = \frac{1}{1+r} \int_{-\infty}^{s_1^*(\theta_i)} \left[\frac{\theta_i + \gamma^2 s_1}{1+r} - (\theta_i + \gamma s_1)\right] f(s_1) ds_1.$$

Taking the derivative of $V_2(\theta_i)$ with respect to r and substituting $s_1^*(\theta_i) = -\frac{\theta_i r}{\gamma(1+r)-\gamma^2}$, we get

$$\begin{aligned}
& -\frac{1}{(1+r)^2} \int_{-\infty}^{s_1^*(\theta_i)} \left[\frac{\theta_i + \gamma^2 s_1}{1+r} - (\theta_i + \gamma s_1)\right] f(s_1) ds_1 \\
& + \frac{1}{1+r}\left[\int_{-\infty}^{s_1^*(\theta_i)} \left[\frac{-(\theta_i + \gamma^2 s_1)}{1+r}\right] f(s_1) ds_1 + \frac{\partial s_1^*(\theta_i)}{\partial r}\left[\frac{\theta_i - \frac{\theta_i r}{\frac{1}{\gamma}(1+r)-1}}{(1+r)} - \theta_i + \frac{\theta_i r}{(1+r)-\gamma}\right]\right]. \\
& = -\frac{1}{(1+r)^2} \int_{-\infty}^{s_1^*(\theta_i)} \left[\frac{\theta_i + \gamma^2 s_1}{1+r} - (\theta_i + \gamma s_1)\right] f(s_1) ds_1 \\
& + \frac{1}{1+r}\left[\int_{-\infty}^{s_1^*(\theta_i)} \left[\frac{-(\theta_i + \gamma^2 s_1)}{1+r}\right] f(s_1) ds_1 + \left[\frac{\theta_i(\gamma-1)}{(1-\gamma+r)^2\gamma}\right]\left[\frac{\theta_i - \frac{\theta_i r}{\frac{1}{\gamma}(1+r)-1}}{(1+r)} - \theta_i + \frac{\theta_i r}{(1+r)-\gamma}\right]\right] \\
& = -\frac{V_2(\theta_i)(1+r)}{(1+r)^2} + \left[\int_{-\infty}^{s_1^*(\theta_i)} \left[\frac{-(\theta_i + \gamma^2 s_1)}{1+r}\right] f(s_1) ds_1 + \left[\frac{\theta_i(\gamma-1)}{(1-\gamma+r)^2\gamma}\right][0]\right] \\
& = -\frac{V_2(\theta_i)(1+r)}{(1+r)^2} + \frac{1}{1+r}\left[\int_{-\infty}^{s_1^*(\theta_i)} \left[\frac{-(\theta_i + \gamma^2 s_1)}{1+r}\right] f(s_1) ds_1\right]
\end{aligned}$$

Now, to prove the claim, recall that the equilibrium first period threshold is:

$$\theta_1^* = G(\theta_1^*)^{N-1}\left(\frac{\theta_1^*}{1+r}\right) + \left(1 - G(\theta_1^*)^{N-1}\right)\left[\begin{array}{c} F_\varepsilon\left(\frac{\theta_1^*}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)\frac{\theta_1^*}{1+r} + \frac{1}{1+r}\gamma\sigma^2 f_\varepsilon\left(-\frac{\theta_1^*}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right) \\ +\left(1 - F_\varepsilon\left(\frac{\theta_1^*}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)\right)\frac{\theta_1^*}{(1+r)^2} - \frac{1}{(1+r)^2}\gamma^2\sigma^2 f_\varepsilon\left(-\frac{\theta_1^*}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right) \end{array}\right] \tag{17}$$

$$= G(\theta_1^*)^{N-1}\left(\frac{\theta_1}{1+r}\right) + \left(1 - G(\theta_1^*)^{N-1}\right)\left[NS(\theta_1^*) + V_2(\theta_1^*)\right] \tag{18}$$

Let $K(\theta_1^*, r)$ be defined as:

$$K(\theta_1^*, r) = G(\theta_1^*)^{N-1}\left(\frac{\theta_1}{1+r}\right) + \left(1 - G(\theta_1^*)^{N-1}\right)\left[NS(\theta_1^*) + V_2(\theta_1^*)\right] - \theta_1^*$$

And hence,

$$\frac{d\theta_1^*}{dr} = -\frac{\frac{\partial K}{\partial r}}{\frac{\partial K}{\partial \theta_1^*}} = -\frac{G(\theta_1^*)^{N-1}\left(\frac{-\theta_1^*}{(1+r)^2}\right) + \left(1 - G(\theta_1^*)^{N-1}\right)\left[\frac{-\theta_i}{(1+r)^2} + \frac{\partial}{\partial r}V_2(\theta_1^*)\right]}{\left[\begin{array}{c} (N-1)G(\theta_1^*)^{N-2}g(\theta_1^*)\left(\frac{\theta_1^*}{1+r}\right) + G(\theta_1^*)^{N-1}\left(\frac{1}{1+r}\right) \\ -(N-1)G(\theta_1^*)^{N-2}g(\theta_1^*)\left[NS(\theta_1^*) + V_2(\theta_1^*)\right] + \left(1 - G(\theta_1^*)^{N-1}\right) \cdot \frac{\partial\left[NS(\theta_1^*) + V_2(\theta_1^*)\right]}{\partial\theta_1} - 1 \end{array}\right]}$$

Plugging in for $\frac{\partial}{\partial r}V_2(\theta_1^*)$ and simplifying the denominator, we have:

$$= -\frac{G(\theta_1^*)^{N-1}\left(\frac{-\theta_1^*}{(1+r)^2}\right) + \left(1 - G(\theta_1^*)^{N-1}\right)\left[\frac{-\theta_i}{(1+r)^2} - \frac{V_2(\theta_i)(1+r)}{(1+r)^2} + \frac{1}{1+r}\left[\int_{-\infty}^{s_1^*(\theta_i)}\left[\frac{-\left(\theta_i + \gamma^2 s_1\right)}{1+r}\right] f(s_1) ds_1\right]\right]}{-(N-1)G(\theta_1^*)^{N-2}g(\theta_1^*)V_2(\theta_1^*) - \left(1 - G(\theta_1^*)^{N-1}\right) \cdot \left[\frac{r}{(1+r)^2}F\left(s_1^*(\theta_i)\right)\right] - \frac{r}{1+r}}$$

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$$= -\frac{\left(\frac{-\theta_1^*}{(1+r)^2}\right) + \left(1 - G(\theta_1^*)^{N-1}\right)\left[-\frac{V_2(\theta_i)}{(1+r)} + \left[\int_{-\infty}^{s_1^*(\theta_i)}\left[\frac{-(\theta_i+\gamma^2 s_1)}{(1+r)^2}\right]f(s_1)ds_1\right]\right]}{-(N-1)G(\theta_1^*)^{N-2}g(\theta_1^*)V_2(\theta_1^*) - \left(1 - G(\theta_1^*)^{N-1}\right)\cdot\left[\frac{r}{(1+r)^2}F(s_1^*(\theta_i))\right] - \frac{r}{1+r}}$$

$$= -\frac{\left(\frac{-\theta_1^*}{(1+r)^2}\right) + \left(1 - G(\theta_1^*)^{N-1}\right)\left[-\frac{V_2(\theta_i)}{(1+r)} + F(s_1^*(\theta_i))E\left[-\frac{\theta_i+\gamma^2 s_1}{(1+r)^2}|s_1 < s_1^*(\theta_i)\right]\right]}{-(N-1)G(\theta_1^*)^{N-2}g(\theta_1^*)V_2(\theta_1^*) - \left(1 - G(\theta_1^*)^{N-1}\right)\cdot\left[\frac{r}{(1+r)^2}F(s_1^*(\theta_i))\right] - \frac{r}{1+r}}$$

Note that $F(s_1^*(\theta_i))E\left[-\frac{(\theta_i+\gamma^2 s_1)}{(1+r)^2}|s_1 < s_1^*(\theta_i)\right]$ is positive since $\theta_i + \gamma^2 s_1 < 0$ for $s_1 < s_1^*(\theta_i)$. However, the magnitude of $\frac{-\theta_1^*}{(1+r)^2}$ is always greater than that of $E\left[-\frac{\theta_i+\gamma^2 s_1}{(1+r)^2}|s_1 < s_1^*(\theta_i)\right]$ since $\gamma^2 s_1 < 0$. Hence

$$\frac{-\theta_1^*}{(1+r)^2} + E\left[-\frac{\theta_i+\gamma^2 s_1}{(1+r)^2}|s_1 < s_1^*(\theta_i)\right] < 0,$$

and as a consequence

$$\left(\frac{-\theta_1^*}{(1+r)^2}\right) + \left(1 - G(\theta_1^*)^{N-1}\right)\left[-\frac{V_2(\theta_i)}{(1+r)} + F(s_1^*(\theta_i))E\left[-\frac{\theta_i+\gamma^2 s_1}{(1+r)^2}|s_1 < s_1^*(\theta_i)\right]\right] < 0$$

since $\left(1 - G(\theta_1^*)^{N-1}\right)F(s_1^*(\theta_i)) < 1$ and $-\frac{V_2(\theta_i)}{(1+r)} < 0$. The numerator is, thus, negative. The denominator is also clearly negative. Since the entire term is multiplied by -1, this implies that the entire term is negative. ∎

Proof of Proposition 5

From Lemma 2, we know that

$$V_2(\theta_i) = \int_{-\infty}^{s_1^*(\theta_i)}\left[\frac{\theta_i+\gamma^2 s_1}{(1+r)^2} - \frac{\theta_i+\gamma s_1}{1+r}\right]f(s_1)ds_1.$$

Since the discount rate is held constant, the first-period threshold changes in γ according to the change in the option value and the change in θ_2^*. Taking the derivative of $V_2(\theta_i)$ with respect to γ and substituting $s_1^*(\theta_i) = -\frac{\theta_i r}{\gamma(1+r)-\gamma^2}$, we get

$$\begin{aligned}
&\frac{\partial}{\partial\gamma}\int_{-\infty}^{s_1^*(\theta_i)}\left[\frac{\theta_i+\gamma^2 s_1}{(1+r)^2} - \frac{\theta_i+\gamma s_1}{1+r}\right]f(s_1)ds_1 \\
&\quad = \int_{-\infty}^{s_1^*(\theta_i)}\frac{\partial}{\partial\gamma}\left[\frac{\theta_i+\gamma^2 s_1}{(1+r)^2} - \frac{\theta_i+\gamma s_1}{1+r}\right]f(s_1)ds_1 + \frac{\partial s_1^*(\theta_i)}{\partial\gamma}\left[\frac{\theta_i+\gamma^2 s_1^*(\theta_i)}{(1+r)^2} - \frac{\theta_i+\gamma s_1^*(\theta_i)}{1+r}\right] \\
&\quad = \int_{-\infty}^{s_1^*(\theta_i)}\frac{\partial}{\partial\gamma}\left[\frac{\theta_i+\gamma^2 s_1}{(1+r)^2} - \frac{\theta_i+\gamma s_1}{1+r}\right]f(s_1)ds_1 + \frac{\partial s_1^*(\theta_i)}{\partial\gamma}\left[\frac{\theta_i - \frac{\theta_i r}{\frac{1}{\gamma}(1+r)-1}}{(1+r)^2} - \frac{\theta_i - \frac{\theta_i r}{(1+r)-\gamma}}{1+r}\right]
\end{aligned}$$

$$\begin{aligned}
\frac{\partial}{\partial\gamma}V_2(\theta_i) &= \int_{-\infty}^{s_1^*(\theta_i)}\left[\frac{2\gamma s_1}{(1+r)^2} - \frac{s_1}{1+r}\right]f(s_1)ds_1 + \left[\theta_i r\left(\gamma(1+r)-\gamma^2\right)^{-2}(1+r-2\gamma)\right]\left[\frac{\theta_i - \frac{\theta_i r}{\frac{1}{\gamma}(1+r)-1}}{(1+r)^2} - \frac{\theta_i - \frac{\theta_i r}{(1+r)-\gamma}}{1+r}\right] \\
&= \int_{-\infty}^{s_1^*(\theta_i)}\left[\frac{2\gamma s_1}{(1+r)^2} - \frac{s_1}{1+r}\right]f(s_1)ds_1 + \left[\theta_i r\left(\gamma(1+r)-\gamma^2\right)^{-2}(1+r-2\gamma)\right][0] \\
&= \int_{-\infty}^{s_1^*(\theta_i)}\left[\frac{2\gamma s_1}{(1+r)^2} - \frac{s_1}{1+r}\right]f(s_1)ds_1
\end{aligned}$$

Next, we show how the sign of $\frac{\partial\theta_1^*}{\partial\gamma}$ depends on the value of γ.

First, note that for $\gamma = \frac{r+1}{2}$,

$$\frac{\partial}{\partial \gamma} V_2(\theta_i) = \int_{-\infty}^{s_1^*(\theta_i)} \left[\frac{2\frac{r+1}{2} s_1}{(1+r)^2} - \frac{s_1}{1+r} \right] f(s_1) ds_1 = \int_{-\infty}^{s_1^*(\theta_i)} \left[\frac{s_1}{(1+r)} - \frac{s_1}{1+r} \right] f(s_1) ds_1 = 0.$$

For $\gamma > \frac{r+1}{2}$, we have that:

$$\int_{-\infty}^{s_2^*(\theta i)} \left[\frac{2\gamma s_1}{(1+r)^2} - \frac{s_1}{(1+r)} \right] f(s_1) ds_1 < 0.$$

And finally, for $\gamma < \frac{r+1}{2}$:

$$\int_{-\infty}^{s_2^*(\theta i)} \left[\frac{2\gamma s_1}{(1+r)^2} - \frac{s_1}{(1+r)} \right] f(s_1) ds_1 > 0.$$

Since θ_2^* follows the same direction as the change in the option value, the behavior of θ_1^* can be characterized by the above. For example, for $\gamma < \frac{r+1}{2}$, since $\frac{\partial \theta_2^*}{\partial \gamma} > 0$ and $\frac{\partial}{\partial \gamma} V_2(\theta_i) > 0$, then $\frac{\partial \theta_1^*}{\partial \gamma} > 0$. Put differently, since the option value increases in $\gamma < \frac{r+1}{2}$, the period 1 threshold will increase as waiting becomes more valuable, while the cost of waiting, *r*, remains the same. Likewise, since the second-period threshold increases in $\gamma < \frac{r+1}{2}$, the likelihood of taking advantage of the real option is increasing for fixed s_1, thus making the real option more valuable, resulting in an increased period 1 threshold for fixed *r*. Both of these effects work in the same direction and hence the θ_1^* is increasing in $\gamma < \frac{r+1}{2}$. A similar argument applies for $\gamma > \frac{r+1}{2}$ and $\gamma = \frac{r+1}{2}$. ■

Proof of Proposition 6

Recall that the equilibrium first period threshold is:

$$\theta_1^* = G(\theta_1^*)^{N-1} \left(\frac{\theta_1^*}{1+r} \right) + \left(1 - G(\theta_1^*)^{N-1}\right) \begin{bmatrix} F_\varepsilon\left(\frac{\theta_1^*}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)} \right) \frac{\theta_1^*}{1+r} + \frac{1}{1+r} \gamma \sigma^2 f_\varepsilon \left(- \frac{\theta_1^*}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)} \right) \\ + \left(1 - F_\varepsilon\left(\frac{\theta_1^*}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)} \right)\right) \frac{\theta_1^*}{(1+r)^2} - \frac{1}{(1+r)^2} \gamma^2 \sigma^2 f_\varepsilon \left(- \frac{\theta_1^*}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)} \right) \end{bmatrix} \tag{19}$$

$$= G(\theta_1^*)^{N-1} \left(\frac{\theta_1}{1+r} \right) + \left(1 - G(\theta_1^*)^{N-1}\right) \left[NS(\theta_1^*) + V_2(\theta_1^*) \right] \tag{20}$$

Let $K(\theta_1^*, N)$ be defined as:

$$K(\theta_1^*, \sigma_\theta) = G(\theta_1^*)^{N-1} \left(\frac{\theta_1}{1+r} \right) + \left(1 - G(\theta_1^*)^{N-1}\right) \left[NS(\theta_1^*) + V_2(\theta_1^*) \right] - \theta_1^*$$

And hence,

$$\frac{d\theta_1^*}{d\sigma_\theta} = -\frac{\frac{\partial K}{\partial \sigma_\theta}}{\frac{\partial K}{\partial \theta_1^*}} = -\frac{(N-1)G(\theta_1^*)^{N-2} \left(\frac{\theta_1}{1+r} \right) \left[\frac{\partial G(\theta_1^*)}{\partial \sigma_\theta} \right] - (N-1)G(\theta_1^*)^{N-2} \left[\frac{\partial G(\theta_1^*)}{\partial \sigma_\theta} \right] \left[NS(\theta_1^*) + V_2(\theta_1^*) \right]}{\begin{bmatrix} (N-1)G(\theta_1^*)^{N-2} g(\theta_1^*) \left(\frac{\theta_1^*}{1+r} \right) + G(\theta_1^*)^{N-1} \left(\frac{1}{1+r} \right) \\ -(N-1)G(\theta_1^*)^{N-2} g(\theta_1^*) \left[NS(\theta_1^*) + V_2(\theta_1^*) \right] + \left(1 - G(\theta_1^*)^{N-1}\right) \cdot \frac{\partial \left[NS(\theta_1^*) + V_2(\theta_1^*) \right]}{\partial \theta_1} - 1 \end{bmatrix}}$$

which becomes

$$= -\frac{-(N-1)G(\theta_1^*)^{N-2} \left[\frac{\partial G(\theta_1^*)}{\partial \sigma_\theta} \right] V_2(\theta_1^*)}{-(N-1)G(\theta_1^*)^{N-2} g(\theta_1^*) V_2(\theta_1^*) - \left(1 - G(\theta_1^*)^{N-1}\right) \cdot \left[\frac{r}{(1+r)^2} F(s_1^*(\theta_i)) \right] - \frac{r}{1+r}} \tag{21}$$

For θ exponential with parameter λ, the variance is given by $\sigma^2 = \frac{1}{\lambda^2}$, so $\lambda^2 = \frac{1}{\sigma^2}$, and hence $\lambda = \frac{1}{\sigma}$. The CDF is $1 - \exp\left[-\frac{1}{\sigma} x \right]$, and

$$\frac{\partial G(\theta_1^*)}{\partial \sigma_\theta} = -(N-1) \exp\left[-\frac{\theta_1^*}{\sigma_\theta} \right] \left[1 - \exp\left[-\frac{\theta_1^*}{\sigma_\theta} \right] \right]^{N-2} \left(\theta_1^* \sigma_\theta^{-2} \right) < 0,$$

and, thus, the numerator in Equation (21) is positive. The denominator is negative and the entire expression is multiplied by -1, so $\frac{d\theta_1^*}{d\sigma_\theta} > 0$. ■

Proof of Corollary 3

The claim follows immediately from Theorem 1. ■

Proof of Corollary 4

Claims (i) and (ii) follow from Proposition 2. For claim (iii), the probability of going public in the second period is given as

$$\Pr\left(\theta_i \geq \theta_2^*\right) = \Pr\left(\theta_i \geq -s_1((1+r)-\gamma)\frac{\gamma}{r}\right) = \Pr(\theta_i \geq -s_1\kappa) = \Pr\left(-\frac{\theta_i}{\kappa} \leq s_1\right) = \Pr\left(s_1 \leq \frac{\theta_i}{\kappa}\right) = F\left(\frac{\theta_i}{\kappa}\right),$$

where $\kappa = ((1+r)-\gamma)\frac{\gamma}{r}$. The penultimate step uses symmetry of the normal distribution. Now taking derivative with respect to σ_ε, we have

$$\begin{aligned}\frac{\partial F\left(\frac{\theta_i}{\kappa}\right)}{\partial \sigma_\varepsilon} &= \frac{\partial}{\partial \sigma_\varepsilon}\left[\frac{1}{2}\left(1+\operatorname{erf}\left(\frac{\frac{\theta_i}{\kappa}-\mu}{\sigma\sqrt{2}}\right)\right)\right] = \frac{\partial}{\partial \sigma_\varepsilon}\left[\frac{1}{2}\left(1+\frac{2}{\sqrt{\pi}}\int_0^{\frac{\frac{\theta_i}{\kappa}-\mu}{\sigma\sqrt{2}}} \exp\left(-t^2\right)dt\right)\right] \\ &= \frac{1}{\sqrt{\pi}}\exp\left[-\left(\frac{\frac{\theta_i}{\kappa}-\mu}{\sigma\sqrt{2}}\right)^2\right]\left(\frac{\frac{\theta_i}{\kappa}-\mu}{\sqrt{2}}\right)\left(-\sigma^{-2}\right) = \frac{1}{\sqrt{\pi}}\exp\left[-\left(\frac{\frac{\theta_i}{\kappa}}{\sigma\sqrt{2}}\right)^2\right]\left(\frac{\frac{\theta_i}{\kappa}}{\sqrt{2}}\right)\left(-\sigma^{-2}\right) < 0.\end{aligned}$$

Hence, $\Pr\left(\theta_i \geq \theta_2^*\right)$ is decreasing in σ_ε.

For claim (iv), denote the new equilibrium IPO threshold under increased σ_ε^2 as θ_1^{new}, and the previous threshold as θ_1^{old}. From Proposition 2, as σ_ε^2 increases, we have that $\theta_1^{new} > \theta_1^{old}$. For types $\theta \in \left[\theta_1^{old}, \theta_1^{new}\right]$, under the previous σ_ε^2, these types went public in $t=1$ with probability one. Under the increased σ_ε^2, the probability of going public before the end of $t=2$ is now strictly less than 1. Similarly, firms with type $\theta \in \left[0, \theta_1^{old}\right]$ delayed in $t=1$ under the previous σ_ε^2, and now have strictly lower probability of going public in $t=2$ under increased σ_ε^2. Hence, the total proportion of firms that will have gone public by the end of period 2 is decreasing in σ_ε^2. Finally, claim (v) follows from Proposition 2 and claims (iii) and (iv). ■

Proof of Corollary 5

The first two claims that first-period IPO volume is decreasing and delay increases in N follow immediately from Proposition 3. For claim (iii), we have

$$\frac{dG\left(\theta_1^*\right)^N}{dN} = \ln\left(G\left(\theta_1^*\right)\right)G\left(\theta_1^*\right)^N \frac{\partial G\left(\theta_1^*\right)}{\partial \theta_1^*}\frac{\partial \theta_1^*}{\partial N} < 0$$

since $\ln\left(G\left(\theta_1^*\right)\right) < 0$ and $\frac{\partial G\left(\theta_1^*\right)}{\partial \theta_1^*} > 0$. This implies that $1 - G\left(\theta_1^*\right)^N$ is increasing in N. Hence, there is a higher probability that at least one firm goes public in $t=1$.

For claim (iv), an IPO drought occurs with probability

$$\Pr\left(s_1 \leq \frac{r}{\gamma}\cdot\frac{-\theta_1^*}{(1+r)-\gamma}\right) = \Pr\left(s_1 \geq \frac{r}{\gamma}\cdot\frac{\theta_1^*}{(1+r)-\gamma}\right) = 1 - \Pr\left(s_1 \leq \frac{r}{\gamma}\cdot\frac{\theta_1^*}{(1+r)-\gamma}\right),$$

where the second step follows by the symmetry of the normal distribution. Hence

$$\frac{d\Pr\left(s_1 \leq \frac{r}{\gamma}\cdot\frac{\theta_1^*}{(1+r)-\gamma}\right)}{dN} = \frac{1}{\sqrt{2\pi\sigma^2}}\exp\left[\frac{-\left(\frac{r}{\gamma}\cdot\frac{\theta_1^*}{(1+r)-\gamma}\right)^2}{2\sigma^2}\right]\cdot\frac{d\theta_1^*}{dN}\cdot\frac{r}{(1+r-\gamma)\gamma} + \int_{-\infty}^{\frac{r}{\gamma}\cdot\frac{\theta_1^*}{(1+r)-\gamma}}\frac{\partial}{\partial N}\left[\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left[\frac{-x^2}{2\sigma^2}\right]\right]dx > 0$$

since θ_1^* is increasing in N and since N does not affect the distribution of s_1. Hence, $\frac{d\left[1-\Pr\left(s_1 \leq \theta_1^*\right)\right]}{dN} < 0$, which implies a lower likelihood of a drought and a higher likelihood of clustering given an IPO by at least one firm in $t=1$.

For claim (v), the change in the number of firms that go public in $t=2$ is given by

$$\frac{d}{dN}\left[N \cdot G\left(\theta_1^*\right) \cdot \Pr\left(\theta_i \geq -s_1((1+r)-\gamma)\frac{\gamma}{r}\right)\right] = \left[G\left(\theta_1^*\right) + N\frac{\partial G\left(\theta_1^*\right)}{\partial \theta_1^*}\frac{\partial \theta_1^*}{\partial N}\right]\Pr\left(\theta_i \geq -s_1((1+r)-\gamma)\frac{\gamma}{r}\right) > 0$$

Hence, the number of firms going public in $t=2$ is increasing in N. A similar calculation can be done to show that the proportion is also increasing in N.

Finally, claim (vi) follows from Proposition 3 and claim (v). ■

Proof of Corollary 6

The claim follows immediately from Propositions 4 and 5. ■

APPENDIX B

accr-53011_Online Appendix: http://dx.doi.org/10.2308/accr-53001.s01