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The Effect of Voluntary Disclosure on Investment Inefficiency

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ABSTRACT: We introduce real decisions (a project choice decision, an investment scale decision, and an information acquisition decision) to the Dye (1985) voluntary disclosure framework and examine how the prospect of voluntary disclosure affects managers' real decisions. Riskier projects lead to more volatile environment and hence entail higher efficiency loss at the subsequent investment scale decision stage if managers are uninformed. If managers are informed, they can withhold bad information, and the value of this option is higher for riskier projects. We show that the voluntary nature of managers' disclosure may lead to two types of inefficiencies: (1) managers may choose riskier projects, which generate lower expected cash flow due to the higher efficiency loss at the subsequent decision stage, and (2) managers may over-invest in information acquisition, because informed managers with bad information have the option to pool with uninformed managers and benefit from being overpriced.

Keywords: investment efficiency; voluntary disclosure; real effects.

I. INTRODUCTION

Firms' investment and disclosure decisions are often intertwined. In practice, corporate investment decisions often include multiple stages: firms first need to choose between different projects. These types of decisions will help establish firms' strategic direction and determine the riskiness of the business environment in which firms operate. Later, firms may learn new information about the business environment (e.g., the efficacy of the technology, its market potential) and make subsequent decisions to adapt to the environment. We refer to this information about the business environment as "productive information." A substantial part of the productive information reaches the capital market through firms' voluntary disclosures in the form of press releases, conference calls, company events, internet sites, mission statements, etc. A question then naturally arises: whether and to what extent firms' voluntary disclosure of productive information affects their investment decisions?

While the effects of mandatory disclosure on economic efficiency have been extensively studied, the literature on the effect of voluntary disclosure on economic efficiency has been scant. In this paper, we examine how the prospect of voluntary disclosure (regarding productive information) affects a firm's multistage investment decisions and the resulting potential inefficiencies. The setting of our model is as follows. A firm's manager cares about both the (long-term) cash flow and the market's beliefs in the short-term, as reflected in our model by the firm's price (but could also be reflected by the cost of equity/debt capital).[1] The firm/manager first has to choose between projects with different levels of risk. The riskiness of the chosen project is unobservable to the market. After the project has been chosen, there is a positive probability that the manager will

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[1] The short-term incentive is typically more prominent for firms that start a new investment project, since the market's beliefs about the prospect of the project affect the cost of raising funding (via debt, equity, or mergers and acquisitions).

learn new information about the productivity of the project. If the manager is informed about the productivity, she can voluntarily disclose this information (a la Dye [1985] and Jung and Kwon [1988]). Following the manager's disclosure, or lack of such disclosure, the market sets the firm's price to equal the expected cash flow from the project. Finally, based on her information set, the manager chooses the level of resources to invest in the project, which we refer to as the "investment scale decision."

To fix ideas, consider a decision that many automakers (e.g., Ford, General Motors, Volkswagen) have been recently facing—whether to focus their R&D investment on electric vehicles or on hybrids. After the manager has chosen the firm's strategic direction, she collects further information about the efficacy of the technology chosen, its market potential, and/or the relevant regulations. If informed, the manager can disclose the information in a voluntary fashion. The manager then makes subsequent decisions (i.e., the investment scale decisions), based on her information set.[2]

If the manager learns the productive information privately, this information is used both for the potential disclosure and for internal decision-making.[3] The subsequent investment scale decisions will be adapted to this realized productivity. If the manager does not have the private information about productivity, she cannot tailor the investment scale to the realized productivity but can only make an uninformed decision. Therefore, the expected cash flow generated by an uninformed manager is lower than that generated by a manager with private productive information. We refer to this difference in the expected cash flow between informed and uninformed firms as "cost of ignorance." If the chosen project is a riskier one, then the environment is more volatile and the cost of ignorance is higher as well.

At the disclosure stage, as standard in the voluntary disclosure literature (Dye 1985; Jung and Kwon 1988), the uninformed manager has no choice but to stay silent, and the informed manager follows a threshold disclosure strategy: withholding information if the realized productivity turns out to be lower than a certain threshold. The voluntary nature of disclosure gives the informed manager an option to conceal bad information and pool with the uninformed firm. Given that the riskiness of the chosen project is unobservable to the capital market, the disclosure threshold will be the same across different projects. Because the riskier project will have more productivity realizations that fall below the disclosure threshold—that is, a larger nondisclosure region—this option value of strategic nondisclosure will be higher for the riskier project.

When deciding which type of project to choose, the manager faces two conflicting incentives. On one hand, choosing a safer project leads to a less volatile environment, which makes the subsequent decisions less inefficient when the manager is uninformed. That is, the cost of ignorance is lower for the safer project. On the other hand, the option value of strategic nondisclosure is higher for the riskier project, and hence incentivizes the manager (who faces capital market pressure) to choose the riskier project (as in Ben-Porath, Dekel, and Lipman 2018). This incentive exists only when the manager is informed and its importance is increasing in the extent to which the manager cares about the short-term price. The manager's optimal project choice balances these two conflicting incentives. We show that there always exists a unique equilibrium.

Our model predicts that the stronger the capital market pressure faced by the manager (the myopia measure), the more likely that the manager will choose the riskier project. In addition, the higher the probability the manager is informed about the realized productivity, the more likely is the manager to choose the riskier project. The reason is that the higher the likelihood that the manager is informed about the realized productivity, (1) the lower the likelihood that she will incur the cost of ignorance and (2) the higher the likelihood that she can benefit from the option of strategic nondisclosure. Both of these direct effects incentivize the manager to choose the riskier project.[4]

As a useful benchmark, consider a scenario in which there is no information asymmetry or, equivalently, in which the (privately informed) manager can commit to full disclosure. In this case, the manager will always choose the safer project, because the safer project induces a lower cost of ignorance and hence generates a higher expected cash flow. However, with information asymmetry and voluntary disclosure, the manager has an incentive to choose the riskier project. Given that the manager is more likely to choose the riskier project when myopia is stronger, the investment inefficiency will increase in managerial myopia. Therefore, our model suggests that, as in other settings (e.g., Stein [1989] in the context of real earnings management), managerial myopia has negative real effects and decreases firms' expected cash flow.

With technology and data revolutionizing business, the managers are more likely (than before) to obtain information about the properties of their new projects. How does this improved information endowment affect investment efficiency? Our analysis shows that the effect is more nuanced. On one hand, higher probability of information endowment (weakly) increases the manager's probability of choosing the riskier project (which is inefficient). On the other hand, the difference in the expected cash flow between the safer and riskier projects, which captures the inefficiency embedded in choosing the riskier project, is

[2] Another example might be a pharmaceutical firm that decides which drug to develop. If the clinical trial of the developed drug is completed on time, the firm can voluntarily disclose the result.

[3] A similar feature is also modeled in Hemmer and Labro (2019). However, managers in Hemmer and Labro (2019) never engage in any strategic reporting behavior, whereas in our model, managers' strategic disclosure behavior is the key driving force.

[4] There exists an opposite and more subtle indirect effect, which is always dominated by the direct effects.

decreasing in the probability of information endowment. The reason is that only when the manager is uninformed will the safer and riskier projects generate different cash flow due to the difference in cost of ignorance. Analysis of these two conflicting forces reveals that the investment inefficiency is nonmonotonic in the probability of information endowment.

In practice, managers can actively allocate resources to improve the information they obtain about the properties of their new investments/projects. To capture this, we extend our model and study a setting in which the manager could increase her likelihood of obtaining information by exerting unobservable costly effort after the project choice is determined. We show that regardless of which project the manager chooses, the manager will overinvest in information acquisition, relative to the first-best information acquisition benchmark. The reason is that voluntary disclosure essentially allows an *informed* manager with low realization of productivity to be overpriced by pooling with the uninformed type. Therefore, to the extent that the manager cares about short-term price, she will overinvest in information acquisition. That is, besides tilting the manager's project choice toward the riskier one, the voluntary nature of disclosure induces another inefficiency: over-investment in the manager's information acquisition. We also show that the manager will acquire more information when choosing the riskier project than when choosing the safer project. The reason is that the marginal benefit of becoming informed is higher for the riskier project because (1) from the (short-term) price perspective, becoming informed allows the manager to exercise the option of strategic nondisclosure, whose value is higher for the riskier project, and (2) from the (long-term) cash flow perspective, becoming informed helps the manager save the cost of ignorance, which again is higher for the riskier project.

Related Literature

A large body of research has examined whether and why financial reporting and disclosure affect investment. As Roychowdhury, Shroff, and Verdi (2019) emphasize in their survey paper, "a fundamental question in accounting is whether and to what extent financial reporting facilitates the allocation of capital to the right investment projects." Although the effects of accounting measurement and its mandatory disclosure on corporate investment have been extensively studied (for a survey of this literature, see Kanodia 2006; Kanodia and Sapra 2016; Roychowdhury et al. 2019), there is scant research on how voluntary disclosure affects corporate investment decisions. The extensive theoretical literature on voluntary disclosure has focused mainly on pure-exchange economy settings, in which the firm's value/cash flow is given and the manager needs to decide whether to disclose her private information about the firm value or conceal it.

An emerging strand of the literature has examined voluntary disclosure settings with real decisions. For example, Beyer and Guttman (2012) investigate a setting in which a manager who is privately informed about the value of the firm's assets in place may issue equity to finance a profitable investment opportunity.[5] Their model considers the interdependencies between firms' disclosure and investment decisions (where the firm can manipulate the disclosed information) and shows that firms with intermediate signals sometimes forego the new profitable investment opportunity. Wen (2013) assumes that the investment cash flow is correlated with the firm's ongoing activities and that the firm can make a disclosure about the investment cash flow if and only if the investment is undertaken. This (mechanical) link between investment and disclosure induces distorted investment decisions. Dye and Sridhar (2002) and Langberg and Sivaramakrishnan (2010) demonstrate that feedback from financial markets, triggered by managers' voluntary disclosures, can guide managers' real actions. Kumar, Langberg, and Sivaramakrishnan (2012) examine a setting in which voluntary disclosure serves both a valuation role and a resource allocation role in that the corporate investment is chosen by the uninformed activist shareholders, who rely on the manager's voluntary disclosure.[6]

Whereas the existing literature investigates how voluntary disclosure affects the subsequent (or concurrent) investment decisions, our paper takes a different perspective. We examine how the prospect of a future voluntary disclosure decision affects *ex ante* real decisions, in particular, how it affects the choice between projects with different risk characteristics. In that regard, our paper is closely related to Ben-Porath et al. (2018). In their main analysis, Ben-Porath et al. (2018) show that if all projects generate the same expected cash flow, then the myopic manager will choose the riskier project. In our model, there is a second-stage investment scale decision. This modeling feature leads the informed and uninformed managers to generate different expected cash flows. In Ben-Porath et al. (2018), in contrast, the expected cash flows generated by the informed and uninformed managers are the same. This distinction gives us the flexibility to establish the strict preference for the riskier project (from the price perspective) without constraining the expected cash flow to be the same across projects. Furthermore, because Ben-Porath et al. (2018) allow for general distributions, they cannot establish the uniqueness of the equilibrium, which prevents them from conducting comparative statics. Our paper demonstrates the existence and uniqueness of the equilibrium,

[5] Goex and Wagenhofer (2009) derive the optimal impairment rule that firms commit to when they want to raise debt capital for investment purposes.

[6] Baldenius and Meng (2010) explore a setting in which start-up firms (entrepreneurs) signal their type when seeking financing from potentially active investors. They show that the signaling by high-type entrepreneurs serves the dual role of securing a fair share price (valuation role) and guiding the active investors' action (resource allocation role).

which enables us to conduct comparative statics with respect to the manager's information endowment and the extent of managerial myopia. In Section V, we endogenize information acquisition, allowing the manager to improve on her likelihood of being informed after the project is invested. This analysis highlights another inefficiency that is caused by the voluntary nature of disclosure: the manager will overinvest in information acquisition.

Our paper is also related to other approaches to studying real effects of (mandatory and voluntary) disclosure. One stream of research has focused on firms' disclosure about market conditions to competitors in a setting of product market competition (e.g., Darrough and Stoughton 1990; Wagenhofer 1990; Feltham and Xie 1992; Darrough 1993; Kanodia, Mukherji, Sapra, and Venugopalan 2000; Hughes, Kao, and Williams 2002; Fischer and Verrecchia 2004). Another stream of research examines the relation between firms' disclosure and cost of capital. For example, Gao (2010) examines how disclosure quality (a prespecified disclosure policy) affects cost of capital in a setting in which disclosure influences firms' investment decisions. Cheynel (2013) investigates the association between firms' voluntary disclosures and their cost of capital in a general equilibrium model, where the cost of capital captures the real effects of voluntary disclosure in terms of investment and risk-sharing efficiency.[7] Dye and Hughes (2018) examine a manager's voluntary disclosure decisions and the associated asset pricing, cost of capital, and information transfer effects in a model where risk-averse investors trade multiple securities. Heinle, Smith, and Verrecchia (2018) analyze the impact of a commitment to provide factor-exposure disclosure on a firm's cost of capital. Finally, some studies have focused on the endogenous nature of information disclosed. For example, Friedman, Hughes, and Michaeli (2019) introduce information acquisition into disclosure models and examine how mandatory disclosure affects firms' incentives to gather, and voluntarily disclose, private information.

Our paper also contributes to the strand of literature demonstrating that managerial myopia has negative real effects. Starting from Stein (1989), the prior theoretical work (e.g., Bebchuk and Stole 1993; Ewert and Wagenhofer 2005) has investigated how managerial myopia and different types of information asymmetries distort managers' investment decisions. Edmans, Fang, and Lewellen (2017) provide direct empirical evidence that the CEO's short-term stock price incentives affect real decisions.

II. MODEL SETUP

The model entails two risk-neutral players: a manager and the capital market. The manager cares about both the short-term stock price and the long-term firm value, and makes the following decisions: (1) at the beginning of the game, the manager has to choose between two mutually exclusive projects with different levels of risk—a decision influencing the firm's strategic direction; (2) after choosing the project/strategy, the manager may receive additional information about the profitability of the project, and will decide whether to disclose it to the market; (3) the manager needs to make subsequent decisions to adapt to the realized productivity of the project, that is, the investment scale decision. The market prices the firm to be equal to the expected cash flow, based on the manager's disclosure, or lack of disclosure, at time $t = 2$.

We now provide a more detailed description of the model. The manager's payoff, denoted by U, is a convex combination of the short-term stock price, denoted by P, and the firm's long-term cash flow, denoted by CF:

$$U(P, CF) = \alpha P + (1 - \alpha) CF,$$

where α is the weight the manager assigns to the short-term stock price. We interpret α as the extent of managerial myopia. As is common in the literature, we take α as exogenously given and commonly known. This assumed managerial myopia could be the result of the agency problem between the manager and the firm; alternatively, it could simply reflect the capital market pressure the firm faces due to various reasons such as raising funding to finance the projects, debt covenant, performance pricing, etc.[8]

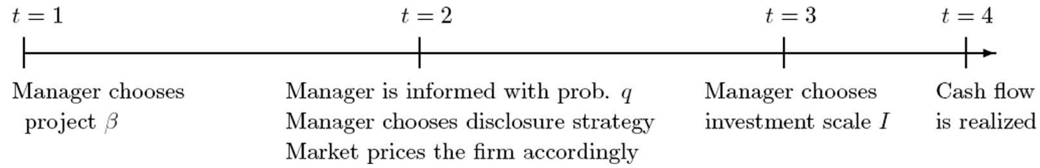
At $t = 1$, the manager has to choose between two mutually exclusive projects/strategies with different levels of risk. The projects are mutually exclusive for various reasons—for example, the products produced by the two projects are substitutes in the product market, or the two projects represent different strategic directions and thereby require very different personnel and human capital investments. The riskiness of the manager's chosen project is unobservable to the market. This initial project choice determines the distribution of a productivity parameter. In order to isolate the effect of voluntary disclosure on the investment decisions, we assume that both projects have the same *expected* productivity; however, the variance of the productivity is higher under the riskier project.[9] Specifically, the project type/riskiness is characterized by $\beta \in \{L, H\}$, where L

[7] Relatedly, Zhang (2013) studies how mandatory accounting standards affect the financial and real sectors of a large economy.

[8] In this paper, we don't intend to endogenize managerial myopia. Instead, we take some degree of myopia as given and study the consequence of it.

[9] The main trade-off and the results of our model are qualitatively unaffected if we allow the expected productivity of the two projects to vary, as long as the difference is not too large. If the expected productivity of one project is sufficiently larger than the other, then the project with the higher expected productivity will always be chosen for any parameter values of α and q.

FIGURE 1
Timeline



denotes the low-risk project and H denotes the high-risk project. Without loss of generality, we assume $0 \le L < H \le 1$. For simplicity, we assume that the productivity parameter, which we denoted by $\tilde{x}_\beta$, follows a uniform distribution:

$$\tilde{x}_\beta \sim U[K - \beta K, K + \beta K].$$

Here K is a commonly known positive constant that reflects the expected productivity level. The above specification guarantees that the realized productivity is always positive.

At $t=2$, after the initial project choice, the manager learns the realization of productivity x_β with probability $q \in (0,1)$. For most of the paper we assume that q is exogenously given and commonly known. In Section V, we relax this assumption and endogenize q by allowing the manager to acquire information. If the manager is informed about x_β, she needs to decide whether to truthfully disclose x_β or to withhold this information (a la Dye [1985] and Jung and Kwon [1988]). If the manager does not learn the realized productivity, she cannot credibly convey that she is uninformed. Upon observing the manager's disclosure of x_β, or the lack of such disclosure, the market sets the stock price to equal the expected cash flow, given all the available information.

At $t=3$, the manager needs to make the investment scale decision based on her information set. We denote the investment scale by I.[10] To focus on the effect of voluntary disclosure on investment decisions, we assume that the firm has sufficient capital to invest any amount that it finds optimal, and hence there is no need for external financing.

Finally, at $t=4$, the firm's cash flow is realized based on the productivity parameter x_β and the investment scale I, according to the following Cobb-Douglas production function:[11]

$$CF(x_\beta, I) = 2\sqrt{x_\beta I} - I. \tag{1}$$

Figure 1 summarizes the timeline of the model.

III. ANALYSIS

We solve the game by backward induction. We start by deriving the manager's optimal investment scale decision at $t=3$, given her information set. Next, we derive the manager's optimal disclosure decision at $t=2$ and the resulting market price for any given market belief about the manager's project choice. Then, we solve for the manager's optimal project choice at $t=1$ for any given market pricing function. Finally, we impose the equilibrium condition that the market's belief about the manager's project choice is consistent with the manager's optimal project choice and solve for the equilibrium.

The Manager's Optimal Investment Scale $I^*(\Omega)$

In this subsection, we analyze the manager's optimal choice of investment scale, $I \in R$. We denote the manager's information set at $t=3$ by $\Omega \in \{x_\beta, \varnothing\}$, where x_β indicates that the manager is informed about productivity and $\varnothing$ indicates that the manager is uninformed.[12] The manager makes the investment scale decision $I^*(\Omega)$ based on her information set. Because the market price is not influenced by the actual investment scale I, the manager chooses I to maximize the expected

10 An interesting extension would be to consider the case in which the manager's investment scale decisions/expected cash flows are also disclosed. That would give rise to additional signaling incentives which may affect the equilibrium.

11 All results are qualitatively the same for a more general concave production function $CF(x_\beta, I) = a x_\beta^{(1-c)} I^c - bI$, where $c \in (0,1)$ and $\frac{a}{b}\left(\frac{b}{ac}\right)^{-(1-c)^2} > 1$. The parameter restrictions guarantee that the expected cash flow given the optimal investment level is positive. If the production function is convex, then both the expected cash flow and the option value of concealment are higher for the riskier project. Hence, the riskier project will always be chosen in equilibrium.

12 Note that the manager is always informed about her project choice β. Therefore, the manager's information set should also include β. For simplicity, we slightly abuse the notation and only emphasize the manager's information about x_β in Ω.

cash flow. The following lemma summarizes the manager's optimal investment scale decision and the resulting expected cash flow for each information set.

Lemma 1: The manager's optimal investment scale decision $I^*(\Omega)$ maximizes the expected cash flow, and is given by:

(1) If the manager is informed about x_β, then $I^*(x_\beta) = x_\beta$, and the expected cash flow is $E[CF(x_\beta, I^*(x_\beta))] = E[x_\beta]$.
(2) If the manager is uninformed about x_β, then $I^*(\varnothing) = \left(E\left[\sqrt{x_\beta}\right]\right)^2$, and the expected cash flow is $E\left[CF\left(x_\beta, I^*(\varnothing)\right)\right] = \left(E\left[\sqrt{x_\beta}\right]\right)^2$.

If the manager is informed about the realized productivity, the investment scale decision will be first best. In contrast, if the manager is uninformed, she chooses the investment scale based on the prior. That is, endowment of information enables the manager to better tailor the investment scale to the realized productivity and, hence, increases the expected cash flow compared to the cases in which the manager is uninformed. We refer to the expected increase in cash flow resulting from being informed as the "value of information" or, alternatively, the "cost of ignorance." Note that for a given project choice, the value of information is an *ex ante* measure. The following lemma quantifies the expected value of information for any chosen project β.

Lemma 2: For any given project choice β, the expected value of information (cost of ignorance) is given by:

$$E\left[CF\left(x_\beta, I^*(x_\beta)\right)\right] - E\left[CF\left(x_\beta, I^*(\varnothing)\right)\right] = E\left[x_\beta\right] - \left(E\left[\sqrt{x_\beta}\right]\right)^2 = Var\left[\sqrt{x_\beta}\right] > 0. \tag{2}$$

The cost of ignorance is higher for the riskier project.

It is intuitive that as the economic environment becomes riskier, people will value their information more. Therefore, the value of information (cost of ignorance) increases with the riskiness of the project.

Which project will generate a higher expected cash flow *ex ante*? Denote by $ECF(\beta)$ the expected cash flow generated by project β:

$$ECF(\beta) = qE\left[x_\beta\right] + (1-q)\left(E\left[\sqrt{x_\beta}\right]\right)^2. \tag{3}$$

With probability q, the manager is informed and makes the first-best investment scale decision, generating expected cash flow $E\left[x_\beta\right]$. With probability $1-q$, the manager is uninformed and chooses investment scale $I^*(\varnothing)$. The expected cash flow in this case is $\left(E\left[\sqrt{x_\beta}\right]\right)^2$. Note that both projects generate the same expected cash flow $E\left[x_\beta\right] = K$ if the manager is informed but that the riskier project entails a higher cost of ignorance if the manager is uninformed. Therefore,

Lemma 3: The riskier project yields a lower expected cash flow, that is,

$$ECF(\beta = H) < ECF(\beta = L).$$

Disclosure Equilibrium for Given Market Belief about the Manager's Project Choice

This subsection serves as another building block for deriving the equilibrium of our model. Taking the market belief about the manager's project choice at $t=1$ as given, we solve the equilibrium of the disclosure subgame. That is, we derive the manager's equilibrium disclosure strategy and the resulting market price at $t=2$.

To describe the market belief about the manager's project choice, we first formally introduce the manager's project choice at $t=1$, which is denoted by $\sigma \in [0,1]$, where σ is the probability that the manager chooses the riskier project. We denote by $\hat{\sigma}$ the market belief about σ. Note that a pure strategy is characterized by $\sigma = 0$ or $\sigma = 1$, whereas $\sigma \in (0,1)$ represents a mixed strategy.

If the manager discloses x_β, the market perfectly learns the realization of productivity and correctly anticipates that the investment scale will also be x_β (see Lemma 1). As such, the price given disclosure of x_β, denoted by $P^D(x_\beta)$, is given by

$$P^D(x_\beta) = E\left[CF(x_\beta, I^*(x_\beta))|x_\beta\right] = x_\beta.$$

As is common in voluntary disclosure settings, the manager's optimal disclosure strategy is a threshold strategy. Recall that the manager's optimal investment scale decision is independent of the manager's disclosure decision (Lemma 1). Therefore, at $t=2$, when the manager makes the disclosure decision, she only considers the effect of the disclosure decision on the market price. Specifically, the manager discloses if and only if the disclosure price $P^D(x_\beta)$ is higher than the price given no disclosure, denoted by P^{ND}. Given that the market price given disclosure is $P^D(x_\beta) = x_\beta$ and that the price given no disclosure P^{ND} is independent of x_β, the disclosure condition is equivalent to $x_\beta > P^{ND}$.

Given the manager's optimal disclosure strategy, when the manager does not disclose, the market is uncertain whether the manager is informed about x_β but strategically chooses to conceal her information or the manager is simply uninformed. Therefore, the price given no disclosure is the weighted average of the expected cash flows over two events: (i) the manager is informed and chooses to conceal it (because $x_\beta < P^{ND}$)—we refer to it as "strategic non-disclosure" case and (ii) the manager is uninformed and hence cannot disclose x_β—we refer to it as "no information" case. The weights are assigned according to the conditional probabilities of each event given no disclosure. That is, the price given no disclosure is

$$\begin{aligned} P^{ND} &= E\left[CF(x_\beta, I^*(\Omega))|ND, \hat{\sigma}\right] \\ &= \Pr\left(\Omega = x_\beta|ND, \hat{\sigma}\right) \cdot E\left[CF\left(x_\beta, I^*\left(x_\beta\right)\right)|\Omega = x_\beta, x_\beta < P^{ND}, \hat{\sigma}\right] + \Pr(\Omega = \varnothing|ND, \hat{\sigma}) \cdot E\left[CF\left(x_\beta, I^*(\varnothing)\right)|\Omega = \varnothing, \hat{\sigma}\right]. \end{aligned} \tag{4}$$

To summarize, holding constant the market belief about the manager's project choice, at the disclosure stage, the disclosure and pricing equilibrium (of the subgame) is as follows.

Lemma 4: At the disclosure stage, the informed manager follows a threshold strategy: disclosing x_β *for* $x_\beta > P^{ND}$ and withholding information otherwise. The market prices the firm correctly in expectation. Upon disclosure, $P^D(x_\beta) = x_\beta$. Upon nondisclosure, P^{ND} is determined according to (4). Moreover, P^{ND} depends on the market belief about the manager's project choice $\hat{\sigma}$ and the manager's probability of being informed q in the following way:[13]

(a) $P^{ND}(\hat{\sigma}, q) < K$.
(b) $P^{ND}(\hat{\sigma}, q)$ is continuous and monotonically decreasing in $\hat{\sigma}$.
(c) $P^{ND}(\hat{\sigma}, q)$ is continuous and monotonically decreasing in q.

The first property of P^{ND} states that $P^{ND}(\hat{\sigma}, q) < K$. In our setting, K is not only the expected productivity for both projects, but also the expected cash flow when the manager is informed. Although the fact that the price given no disclosure is lower than the prior mean is standard in voluntary disclosure settings with uncertainty about information endowment, in our setting there is an additional reason for this result. When investors set $P^{ND}(\cdot)$, they take a weighted average of the expected cash flow over (i) the strategic non-disclosure case and (ii) the no information case. The conditional expected cash flow in both cases is strictly smaller than K, although for different reasons. In the strategic non-disclosure case, the high realizations of x_β are truncated; hence, the expected cash flow is lower than the prior mean of the distribution, which equals K. In the no information case, due to the cost of ignorance, the expected cash flow of an uninformed manager is less than the expected cash flow of an informed manager, which again equals K.

One can also show that the conditional expected cash flow of an informed manager choosing not to disclose (the strategic non-disclosure case) is lower than that of an uninformed manager (the no information case). The reason is simple. The voluntary nature of disclosure gives the informed manager the option to conceal information and pool with the uninformed manager. Hence, by revealed preferences, an informed manager who chooses to pool with the uninformed manager must be a low type whose cash flow is lower than the expected cash flow of the uninformed manager.

Changes in $\hat{\sigma}$ affect both the conditional probabilities of either case (the strategic non-disclosure case and the no information case) and the conditional expected cash flow for either case. First, an increase in $\hat{\sigma}$ shifts mass symmetrically to the two extreme ends of the distribution of x_β (the riskier project is a mean preserving spread of the safer project). Given that $P^{ND} < K$, this shifting increases the probability that $x_\beta < P^{ND}$ and hence the market's belief (upon no disclosure) that the manager is informed (the strategic non-disclosure case). This increase in the conditional probability of the strategic non-disclosure case, combined with the fact that the conditional expected cash flow of the strategic non-disclosure case is smaller than that of the no information case, implies a negative effect of $\hat{\sigma}$ on $P^{ND}(\cdot)$. In addition to the effect on the conditional probabilities, an increase in $\hat{\sigma}$ also decreases the conditional expected cash flow for both cases. Conditional on the strategic non-disclosure case, an increase in $\hat{\sigma}$ shifts additional mass to the left tail of the distribution of the cash flow and therefore decreases the conditional expected cash flow. Conditional on the no information case, an increase in $\hat{\sigma}$ also decreases the conditional expected cash flow, because the riskier project incurs a higher cost of ignorance than the safer project (see Lemma 2). Because all the above effects work in the same direction, we can conclude that an increase in $\hat{\sigma}$ decreases $P^{ND}(\cdot)$.

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[13] In the proof of Lemma 4, instead of using the direct definition of P^{ND} as in (4), we take an indirect approach to determining P^{ND}. The indirect approach relies on the market's correct pricing of the firm in expectation. That is, there may be mispricing for each type of nondisclosing firm, but overall the expected mispricing is zero. Specifically, P^{ND} is determined by $(1-\hat{\sigma})\left\{(1-q)\left(P^{ND} - \left(E\left[\sqrt{x_L}\right]\right)^2\right) + q\, O(L, P^{ND})\right\} + \hat{\sigma}\left\{(1-q)\left(P^{ND} - \left(E\left[\sqrt{x_H}\right]\right)^2\right) + q\, O(H, P^{ND})\right\} = 0$.

The effect of q on $P^{ND}(\cdot)$ is simpler. Although changes in q do not affect the conditional expected cash flow for either case, an increase in q increases the market's belief upon no disclosure that the manager is informed. Because the conditional expected cash flow of an informed firm choosing not to disclose is lower than that of an uninformed firm, an increase in q decreases $P^{ND}(\cdot)$.

The Manager's Optimal Project Choice for Given Market Pricing

In this subsection, we develop the last building block of the full analysis of the equilibrium. We analyze the manager's project choice at $t=1$ for a given market pricing function. For brevity, we suppress the arguments $(\hat{\sigma}, q)$ in $P^{ND}(\cdot)$ when there is no scope for confusion. The manager anticipates the future market price and chooses the project $\beta \in \{L, H\}$ to maximize her expected payoff, which is given by

$$EU(\beta) = \alpha\left[(1-q)P^{ND} + qE\left[P|\Omega = x_\beta, \beta\right]\right] + (1-\alpha)ECF(\beta). \tag{5}$$

The manager's project choice, β, affects both the expected price (with weight α) and the expected cash flow (with weight $1-\alpha$). In terms of the expected price, with probability $(1-q)$, the manager is uninformed and cannot disclose, always obtaining a price P^{ND}. With probability q, the manager is informed and obtains an expected price $E\left[P|\Omega = x_\beta, \beta\right]$, which is computed below.

When the manager is informed, she can choose whether to truthfully disclose x_β or conceal it. The price she obtains is therefore $\max\{P^D, P^{ND}\}$. The expected price is then

$$E\left[P|\Omega = x_\beta, \beta\right] = \int_{K-\beta K}^{K+\beta K} \max\left\{P^D, P^{ND}\right\}\frac{1}{2\beta K}dx_\beta = \int_{K-\beta K}^{K+\beta K} \max\left\{x_\beta, P^{ND}\right\}\frac{1}{2\beta K}dx_\beta = E\left[x_\beta\right] + O\left(\beta, P^{ND}\right). \tag{6}$$

The expected price that an informed manager obtains equals $E\left[x_\beta\right]$ (the expected fair price) plus $O(\beta, P^{ND})$, which measures the expected overpricing that an informed manager (who has chosen project β) receives by having the option to strategically conceal bad realizations of productivity. We thereby refer to $O(\beta, P^{ND})$ as the "option value" of strategic nondisclosure, and it is given by

$$O\left(\beta, P^{ND}\right) \equiv \begin{cases} \int_{K-\beta K}^{P^{ND}} \frac{P^{ND}-x_\beta}{2\beta K}dx_\beta & \text{for } K-\beta K < P^{ND} \\ 0 & \text{for } K-\beta K \geq P^{ND} \end{cases}. \tag{7}$$

To understand the option value $O(\beta, P^{ND})$, note that when the informed manager conceals information, she obtains a price P^{ND}, whereas absent information asymmetry (or if the manager were to always disclose), the price should have been x_β. Therefore, by withholding information, the manager benefits from an overpricing of $P^{ND} - x_\beta$. Integrating the price difference over the nondisclosure region, $x_\beta < P^{ND}$, the option value is computed as in Equation (7). Note that in the case of $K - \beta K \geq P^{ND}$, even the worst realization of productivity under the chosen project β is higher than the price given no disclosure. In this case, the informed manager will always disclose and the option value will be zero.[14] The following lemma examines the properties of the option value.

Lemma 5: The manager's option value of strategic nondisclosure, $O(\beta, P^{ND})$, has the following properties:

(1) The option value $O(\beta, P^{ND})$ is (weakly) increasing in the price given no disclosure P^{ND}.
(2) The riskier project has a higher option value than the safer project, that is,

$$O\left(H, P^{ND}\right) > O\left(L, P^{ND}\right).$$

Recall that the option value captures the expected overpricing that an informed manager receives by concealing bad information. Hence, all else being equal, the higher the price given no disclosure P^{ND}, the higher the overpricing and thereby the option value $O(\beta, P^{ND})$. Moreover, because the option value kicks in only upon nondisclosure, the ranking of the option values between the two projects will depend on the nondisclosure region (and the distribution of x_β) under each project. Given that the manager's disclosure strategy is to conceal information below a certain threshold and that this threshold is the same across different projects (due to the unobservability of project choice), it is clear that the riskier project is more likely to conceal bad information and hence will have a higher option value.

[14] This may occur only for the safer project.

We now use the option value $O(\beta, P^{ND})$ to express the manager's expected payoff. Plugging Equation (6) into (5), we obtain

$$EU(\beta) = \alpha\left[(1-q)P^{ND} + q\left(E\left[x_\beta\right] + O\left(\beta, P^{ND}\right)\right)\right] + (1-\alpha)ECF(\beta). \tag{8}$$

The manager's choice between the riskier and the safer project optimally balances the following trade-off. On one hand, the riskier project yields a higher option value (Lemma 5) and hence a higher expected price. On the other hand, from the cash flow perspective, the riskier project entails a higher cost of ignorance and hence yields a lower expected cash flow (Lemma 3). Define $\Delta(q, \alpha, \hat{\sigma})$ as the difference of the manager's expected utility from choosing the riskier project and from choosing the safer project:

$$\Delta(q, \alpha, \hat{\sigma}) = EU(\beta = H) - EU(\beta = L) = q\alpha\left(O\left(H, P^{ND}\right) - O\left(L, P^{ND}\right)\right) + (1-q)(1-\alpha)\left((E[\sqrt{x_H}])^2 - (E[\sqrt{x_L}])^2\right). \tag{9}$$

Note that the option value depends on the price given no disclosure, P^{ND}, which is in turn influenced by the investors' belief about the manager's project choice $\hat{\sigma}$. Therefore $\Delta(\cdot)$ is a function of $\hat{\sigma}$.

The manager's best response in terms of project choice, denoted by σ^{BR}, maximizes her expected payoff, and is given by

$$\sigma^{BR} = \left\{ \begin{array}{lll} 1 & if & \Delta(q, \alpha, \hat{\sigma}) > 0 \\ 0 & if & \Delta(q, \alpha, \hat{\sigma}) < 0 \\ \sigma_m & if & \Delta(q, \alpha, \hat{\sigma}) = 0 \end{array} \right\}.$$

The above is only a partial analysis, as we take the price given no disclosure P^{ND}—or equivalently, the investors' belief about the manager's project choice $\hat{\sigma}$—as given. In equilibrium, the investors' belief about the manager's strategy should be consistent with her actual strategy; hence, an equilibrium consists of a fixed point where $\sigma^{BR} = \hat{\sigma} = \sigma^*$.

The Equilibrium

In this subsection, we impose the equilibrium condition that $\sigma^{BR} = \hat{\sigma}$ and solve for the equilibrium, using the building blocks we developed in previous sections. The following proposition establishes the existence and uniqueness of the equilibrium.

Proposition 1: There exists a unique equilibrium in which:

1. At the project choice stage, the manager chooses the riskier project with probability σ^*. σ^* is given by:

$$\sigma^* = \left\{ \begin{array}{ccc} 1 & if & \alpha > \frac{\left(E\left[\sqrt{x_L}\right]\right)^2 - \left(E\left[\sqrt{x_H}\right]\right)^2}{\left(E\left[\sqrt{x_L}\right]\right)^2 - (1-H)K} \ and \ q > \bar{q}(\alpha) \\ 0 & if & q < \underline{q}(\alpha) \\ \sigma_m^*(q, \alpha) & & otherwise \end{array} \right\},$$

   where $\sigma_m^*(q, \alpha)$ is determined by

$$\Delta(q, \alpha, \sigma_m^*(q, \alpha)) = 0,$$

   $\bar{q}(\alpha)$ is determined by

$$\Delta(\bar{q}(\alpha), \alpha, \hat{\sigma} = 1) = 0,$$

   and $\underline{q}(\alpha)$ is determined by

$$\Delta(\underline{q}(\alpha), \alpha, \hat{\sigma} = 0) = 0.$$

   Moreover, $0 \leq \underline{q}(\alpha) \leq \bar{q}(\alpha) \leq 1$. For the special case of $\alpha = 0$, we have $\underline{q}(\alpha) = 1$. That is, the non-myopic manager always chooses the safer project for all levels of q. For the special case of $\alpha = 1$, we have $\underline{q}(\alpha) = \bar{q}(\alpha) = 0$. That is, the completely myopic manager always chooses the riskier project for all levels of q.
2. At the disclosure stage, the informed manager follows a threshold disclosure strategy: disclosing x_β if and only if $x_\beta > P^{ND}$. The market prices the firm correctly in expectation. Upon disclosure, $P^D(x_\beta) = x_\beta$, and upon nondisclosure, P^{ND*} is determined according to (4).

3. At the investment scale decision stage, the manager chooses the optimal investment scale $I^*(\Omega)$ as characterized in Lemma 1.

We have established existence of the equilibrium for the disclosure subgame given an assumed project choice.[15] We still need to show the existence and uniqueness of the equilibrium for the project choice. An equilibrium is a fixed point such that the manager's best response to a market pricing function is consistent with the market belief about the project choice, that is, $\sigma^{BR} = \hat{\sigma}$. Recall that $\Delta(q, \alpha, \hat{\sigma})$, defined in (9), captures the manager's preference between the riskier and the safer projects for a given market belief $\hat{\sigma}$. We argue that a sufficient condition for the existence and uniqueness of the equilibrium is that $\Delta(q, \alpha, \hat{\sigma})$ is monotonically decreasing in $\hat{\sigma}$. This monotonicity implies that increasing $\hat{\sigma}$ increases the manager's preference toward the safer project compared to the riskier project. Therefore, (1) if the parameters are such that the manager prefers the safer project for market belief $\hat{\sigma} = 0$, then for all market belief $\hat{\sigma} \in [0, 1]$, the manager also prefers the safer project—that is, for such parameter values, there is a unique pure strategy equilibrium in which $\sigma^* = 0$; (2) if the parameters are such that the manager prefers the riskier project for market belief $\hat{\sigma} = 1$, then for all market belief $\hat{\sigma} \in [0, 1]$, the manager also prefers the riskier project—that is, for such parameter values, there is a unique pure strategy equilibrium in which $\sigma^* = 1$; and (3) if the parameters are such that the manager prefers the riskier project for market belief $\hat{\sigma} = 0$ but prefers the safer project for market belief $\hat{\sigma} = 1$, then for such parameter values, there exists a unique market belief $\hat{\sigma}_m \in (0, 1)$ such that the manager is indifferent between the riskier and the safer projects. Therefore, the unique equilibrium is a mixed-strategy in which $\sigma^* = \hat{\sigma}_m$.

To complete the argument, we need to show that $\Delta(q, \alpha, \hat{\sigma})$ is indeed monotonically decreasing in $\hat{\sigma}$. Note that $\hat{\sigma}$ affects $\Delta(q, \alpha, \hat{\sigma})$ solely through its effect on P^{ND}. An increase in $\hat{\sigma}$ decreases the price given no disclosure P^{ND} (Lemma 4), which in turn decreases the option value of strategic nondisclosure (Lemma 5). Given that the riskier project benefits more from the option of strategic nondisclosure (Lemma 5), the decrease in the option value weakens the manager's preference for the riskier project.

IV. COMPARATIVE STATICS

We now conduct comparative statics with respect to the parameters of our model, which include: the probability of information endowment q and the level of managerial myopia α.

Proposition 2: The equilibrium project choice $\sigma^*(q, \alpha)$ has the following properties:

(1) $\sigma^*(q, \alpha)$ is continuous and (weakly) increasing in α.
(2) $\sigma^*(q, \alpha)$ is continuous and (weakly) increasing in q.

To understand the effect of managerial myopia α on the equilibrium project choice $\sigma^*(\cdot)$, recall that the manager faces two countervailing forces when deciding which type of project to invest in—see Equation (9). On one hand, the option value of strategic nondisclosure is higher for the riskier project, and hence incentivizes the manager to choose the riskier project. This incentive is stronger if the manager cares more about the price. On the other hand, the manager has an incentive to choose the safer project, because the safer project generates higher expected cash flow due to the lower cost of ignorance. This incentive decreases in managerial myopia α (increases in $1 - \alpha$). Both effects work in the same direction. Therefore, the more the manager cares about the price, the more likely she will choose the riskier project in equilibrium. In the limit case in which the manager only cares about the price, that is, $\alpha = 1$, the manager always chooses the riskier project.

The effect of information endowment q on the equilibrium project choice $\sigma^*(\cdot)$ is more subtle. The higher the likelihood that the manager is informed about the realized productivity: (1) the lower the likelihood that she will incur the cost of ignorance and (2) the higher the likelihood that she can benefit from the option of strategic nondisclosure. Both of these direct effects incentivize the manager to choose the riskier project. However, on top of these direct effects, there is a more subtle indirect effect: all else being equal, higher q reduces $P^{ND}(\cdot)$ (Lemma 4), which in turn reduces the manager's option value of strategic nondisclosure. Given that the riskier project benefits more from the option value, this decrease in the option value weakens the manager's incentives to choose the riskier project. Our analysis shows that the direct impacts are the dominant forces and hence the higher q is, the more likely the manager will choose the riskier project in equilibrium.

The Equilibrium Market Price upon No Disclosure

Having analyzed the equilibrium project choice, we now examine the properties of the equilibrium market price given no disclosure, $P^{ND^*}(q, \alpha) = P^{ND}(\sigma^*(q, \alpha), q)$.

[15] The uniqueness of the disclosure equilibrium is immediate from the minimum principle (Acharya, DeMarzo, and Kremer 2011; Guttman, Kremer, and Skrzypacz 2014).

Corollary 1: The equilibrium market price given no disclosure, $P^{ND^*}(q, \alpha)$, has the following properties:

(1) $P^{ND^*}(\cdot)$ is continuous and monotonically decreasing in α.
(2) $P^{ND^*}(\cdot)$ is continuous and monotonically decreasing in q.

Note that managerial myopia α does not directly affect the equilibrium market price given no disclosure $P^{ND^*}(\cdot)$; rather, it affects $P^{ND^*}(\cdot)$ indirectly through its impact on the manager's equilibrium project choice σ^*. As Proposition 2 shows, the more the manager cares about the price, the more likely she will choose the riskier project in equilibrium, which in turn will drive down the market price given no disclosure (Lemma 4).

The manager's information endowment q, in contrast, will affect $P^{ND^*}(\cdot)$ both directly and indirectly. On one hand, all else being equal, the higher the likelihood that the manager is informed, the lower will be the market price given no disclosure (Lemma 4). On the other hand, the higher the likelihood that the manager is informed, the more likely she will choose the riskier project in equilibrium (Proposition 2), which in turn decreases the market price given no disclosure (Lemma 4). Both forces work in the same direction, making $P^{ND^*}(\cdot)$ a decreasing function in q.

The Manager's Equilibrium Expected Payoff

Because the market prices the firm correctly in expectation, the manager's equilibrium expected payoff is simply the expected cash flow generated. In equilibrium, the manager chooses the riskier project with probability σ^* as characterized in Proposition 1. Therefore, the manager's equilibrium expected payoff is

$$EU^*(q, \alpha) = \sigma^*(q, \alpha) \cdot ECF(H) + (1 - \sigma^*(q, \alpha)) \cdot ECF(L). \tag{10}$$

The following proposition characterizes how the manager's equilibrium expected payoff changes with managerial myopia α and the probability of information endowment q.

Proposition 3: In equilibrium, the manager's expected payoff $EU^*(q, \alpha)$ has the following properties:

(1) $EU^*(\cdot)$ is continuous and monotonically (weakly) decreasing in α.
(2) $EU^*(\cdot)$ is continuous in q.
- If the risk differential between the two projects is sufficiently small, that is, L is sufficiently close to H, then $EU^*(\cdot)$ is monotonically increasing in q.
- If the risk differential between the two projects is sufficiently large, that is, L is sufficiently further away from H, then $EU^*(\cdot)$ is nonmonotonic in q. There exists a nonempty region of q where $EU^*(\cdot)$ is locally decreasing in q.

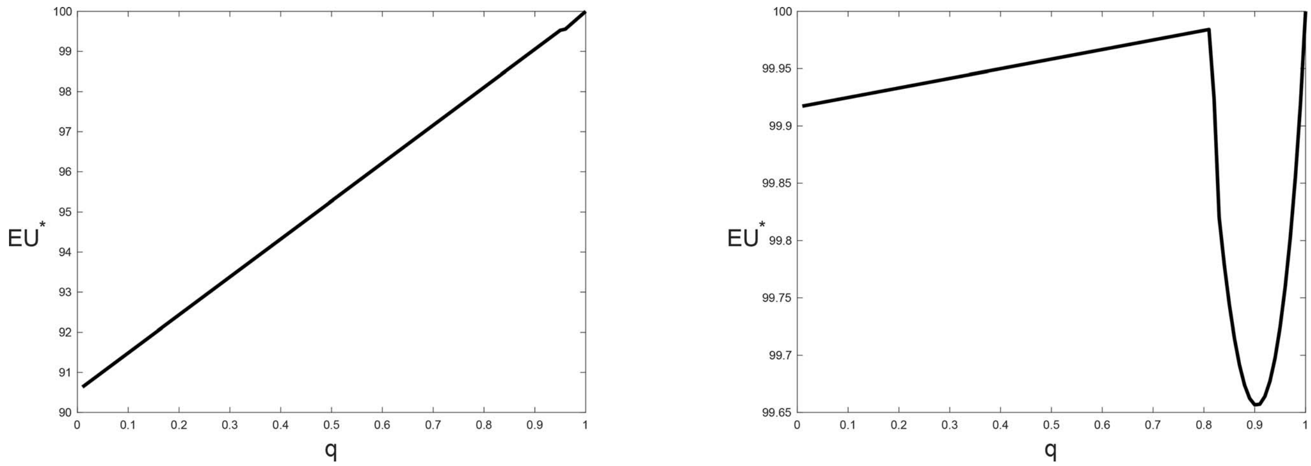
The first result of Proposition 3 states that the more the manager cares about the price, the lower the manager's equilibrium expected payoff will be. The intuition is relatively straightforward. As Proposition 2 shows, the more the manager cares about the short-term price, the more likely she will choose the riskier project in equilibrium. However, the riskier project generates lower expected cash flow than the safer one due to the higher cost of ignorance (Lemma 3). It is worth mentioning that because the market prices the firm correctly in expectation, it is the manager who eventually bears the consequence of her own myopic behavior.

The effect of q on the manager's equilibrium expected payoff is twofold. On one hand, regardless of the project choice, the higher the likelihood that the manager is informed, the more likely she will make an informed investment scale decision rather than an uninformed one. This will increase the expected cash flow for either project ($ECF(\beta)$, $\beta \in \{H, L\}$) and directly increase the manager's expected payoff. On the other hand, as Proposition 2 shows, the higher the likelihood that the manager is informed, the more likely she will choose the riskier project, which generates lower expected cash flow due to the higher cost of ignorance. That is, the indirect impact of q on the manager's equilibrium payoff is negative. If the risk differential between the two projects is sufficiently small, then the indirect negative impact is dominated and the overall impact of q on the manager's equilibrium expected payoff will be positive. If the risk differential between the two projects is sufficiently large, then the negative impact of q caused by choosing the riskier project may become the dominant force and render the overall impact negative. See Figure 2 for an illustration.

Investment Inefficiency

As a useful benchmark, consider a scenario in which there is no information asymmetry or, equivalently, in which the (privately informed) manager can commit to full disclosure. In this case, the manager will always choose the safer project, because the safer project generates a higher expected cash flow (Lemma 3). With information asymmetry and voluntary disclosure, the manager chooses the riskier project with probability σ^*—as characterized in Proposition 1. Define the

FIGURE 2
Manager's Equilibrium Expected Payoff $EU^*(\cdot)$ as a Function of q (for $\alpha = 0.1$, $H = 1$, and $K = 100$)



(a) For $L = 0.95$ (b) For $L = 0.1$

investment inefficiency as the difference in expected cash flow between the benchmark case and the equilibrium:

$$\text{Inv. Inefficiency } (q, \alpha) = \sigma^*(q, \alpha) \cdot [ECF(L) - ECF(H)] = \sigma^*(q, \alpha) \cdot (1 - q) \cdot \left[(E[\sqrt{x_L}])^2 - (E[\sqrt{x_H}])^2\right]. \tag{11}$$

As Equation (11) suggests, the investment inefficiency arises from the (potentially) inefficient project choice $\sigma^*(\cdot)$. At the same time, the difference in the expected cash flow between the safer and riskier projects is due to the difference in cost of ignorance under the two projects, which occurs only when the manager is uninformed. The following result characterizes how the investment inefficiency is affected by managerial myopia α and the probability of information endowment q.

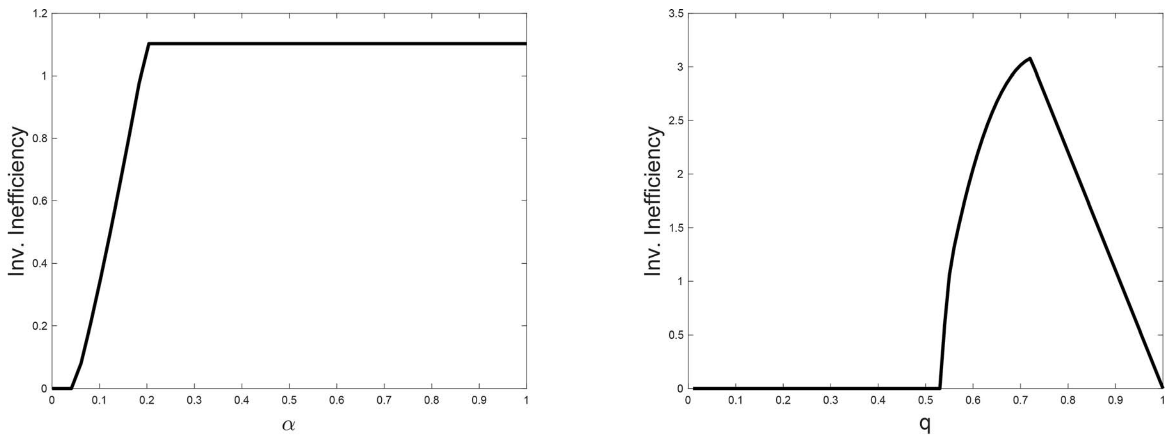
Proposition 4

(1) The investment inefficiency is continuous and monotonically (weakly) increasing in α.
(2) The investment inefficiency is continuous in q.
  - If $\alpha = 0$, the manager always chooses the safer project. The resulting investment inefficiency is zero and hence independent of q.
  - If $\alpha = 1$, the manager always chooses the riskier project. The resulting investment inefficiency is decreasing in q.
  - If $\alpha \in (0, 1)$, the investment inefficiency is nonmonotonic in q: it (weakly) increases in q when q is small and decreases in q when q is large.

As expected, the higher the likelihood that the manager chooses the riskier project in equilibrium, the higher the investment inefficiency is. As Proposition 2 shows, higher managerial myopia increases the manager's probability of choosing the riskier project in equilibrium, and hence causes higher investment inefficiency.

As suggested by Equation (11), the probability of information endowment q affects the investment inefficiency through two channels: (i) higher q (weakly) increases the manager's probability of choosing the riskier project (which is inefficient), but (ii) higher q also decreases the cash flow differential between the safer and riskier projects. In the special case of no managerial myopia, the manager always chooses the safer project, which leads to no investment inefficiency. In the other special case in which the manager only cares about the short-term price, the manager always chooses the riskier project for all levels of q. That is, higher q does not affect the manager's probability of choosing the riskier project, but decreases the inefficiency level of choosing the riskier project. Therefore, in this case, the resulting investment inefficiency is decreasing in q. If the manager cares about both price and cash flow, that is, $\alpha \in (0, 1)$, then both effects are present: higher q (weakly) increases the likelihood that the manager chooses the riskier project in equilibrium, but at same time decreases the inefficiency level of choosing the riskier project. For relatively low levels of q, the manager chooses the riskier project with zero probability. Hence, the investment inefficiency is zero. As q increases, the manager increases the probability of choosing the riskier project. For sufficiently low probability, the first effect dominates the second. As a result, the investment inefficiency is increasing in q. For sufficiently high

FIGURE 3
Investment Inefficiency



(a) Investment inefficiency as a function of α (for $q = 0.9$, $L = 0.1$, $H = 1$, and $K = 100$)

(b) Investment inefficiency as a function of q (for $\alpha = 0.3$, $L = 0.1$, $H = 1$, and $K = 100$)

q, the manager chooses the riskier project with very high probability, which strengthens the second effect and eventually leads the investment inefficiency to decrease in q. See Figure 3 for an illustration.

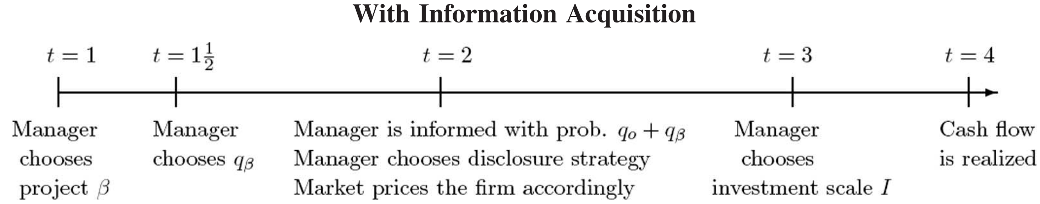
V. INFORMATION ACQUISITION

Up to this point, we have taken the manager's information endowment as exogenously given. In this section, we endogenize the manager's information endowment by considering information acquisition. Specifically, we assume that the manager's baseline probability of being informed is q_o and that the manager can increase this probability by q_β at some cost $C(q_\beta) = \frac{c}{2}q_\beta^2$, where c is a commonly known positive coefficient. The manager's information acquisition effort q_β is unobservable to the market. The timeline for this setting is shown in Figure 4.

The manager's optimal investment scale decision $I^*(\Omega)$ at $t = 3$ is the same as in the main setting: the manager chooses the optimal investment scale to maximize the cash flow based on her information set. At $t = 2$, the informed manager again follows a threshold disclosure strategy, and the market prices the firm similarly as in the main setting—with the exception that in the current setting the nondisclosure price will depend not only on the market belief about the manager's project choice $\hat{\sigma}$ but also on the market belief about the manager's information acquisition effort $\hat{q}_\beta$. Specifically, P^{ND} is determined by the following equation, which implies that the market prices the nondisclosing firm correctly in expectation.

$$\begin{aligned}&\hat{\sigma}\Big\{(1-q_o-\hat{q}_H)\Big(P^{ND}-(E[\sqrt{x_H}])^2\Big)+(q_o+\hat{q}_H)O\big(H,P^{ND}\big)\Big\}\\ &+(1-\hat{\sigma})\Big\{(1-q_o-\hat{q}_L)\Big(P^{ND}-(E[\sqrt{x_L}])^2\Big)+(q_o+\hat{q}_L)O\big(L,P^{ND}\big)\Big\}=0. \qquad (12)\end{aligned}$$

FIGURE 4
Timeline
With Information Acquisition



To understand Equation (12), recall that upon observing nondisclosure, investors do not know whether the manager is uninformed or is informed but strategically conceals her information. Because these types are pooled together and obtain the same price P^{ND}, there is mispricing for each type. Generally speaking, because the informed manager has the option to conceal bad information and pool with the uninformed manager, the informed manager on average gets overpriced whereas the uninformed manager gets underpriced. The underpricing that an uninformed manager gets is $P^{ND} - \left(E\left[\sqrt{x_\beta}\right]\right)^2$, because the fair price the uninformed manager should have received (without information asymmetry) is $\left(E\left[\sqrt{x_\beta}\right]\right)^2$, which is the expected cash flow generated. As argued before, the overpricing that an informed manager (who chose a project β) gets is $O(\beta, P^{ND})$. Because the market prices the nondisclosing firm correctly in expectation, the expected mispricing should be zero after taking expectations over the distribution of the nondisclosing firms (which is affected by the conjectured project choice $\hat{\sigma}$ and conjectured information acquisition $\hat{q}_\beta$). That is, $P^{ND}(\hat{\sigma}, \hat{q}_H, \hat{q}_L, q_o)$ is the price P^{ND} for which Equation (12) holds.

At $t = 1\frac{1}{2}$, the manager chooses the information acquisition effort q_β^{BR}, according to the following first-order condition:[16]

$$C'(q_\beta^{BR}) = K - \left(E\left[\sqrt{x_\beta}\right]\right)^2 + \alpha\left\{O\left(\beta, P^{ND}\right) - \left(P^{ND} - \left(E\left[\sqrt{x_\beta}\right]\right)^2\right)\right\}. \tag{13}$$

The left-hand side is the marginal cost of information acquisition, and the right-hand side is the marginal benefit of information acquisition. The marginal benefit of information acquisition includes two parts: (1) being informed helps the manager make informed investment scale decisions, which increases cash flow by $K - \left(E\left[\sqrt{x_\beta}\right]\right)^2$, and (2) being informed enables the manager to conceal bad information and get overpriced: the overpricing the informed manager gets is $O(\beta, P^{ND})$, whereas if uninformed, the manager would be underpriced by $(P^{ND} - \left(E\left[\sqrt{x_\beta}\right]\right)^2)$. Note that the second part of the marginal benefit only matters to the extent that the manager cares about price, that is, with weight α. Imposing the equilibrium fixed point that $q_\beta^{BR} = \hat{q}_\beta = q_\beta^*$, the manager's information acquisition effort q_β^* is determined by the following system of equations: for $\beta \in \{H, L\}$,

$$C'(q_\beta^*) = K - \left(E\left[\sqrt{x_\beta}\right]\right)^2 + \alpha\left\{O\left(\beta, P^{ND}(\hat{\sigma}, q_H^*, q_L^*, q_o)\right) - \left(P^{ND}(\hat{\sigma}, q_H^*, q_L^*, q_o) - \left(E\left[\sqrt{x_\beta}\right]\right)^2\right)\right\}. \tag{14}$$

The following lemma characterizes some features of the manager's information acquisition effort.

Lemma 6: With information acquisition,

(1) The manager acquires more information when choosing the riskier project, that is, $q_H^* > q_L^*$.
(2) Regardless of which project the manager chooses, for all $\alpha > 0$ the manager overinvests in information acquisition relative to the first-best benchmark.

Rearranging the terms in Equation (13), we get another way to express the manager's marginal benefit of getting informed

$$C'(q_\beta^*) = (1 - \alpha)\left[K - \left(E\left[\sqrt{x_\beta}\right]\right)^2\right] + \alpha\left[O\left(\beta, P^{ND}\right) - P^{ND} + K\right].$$

First, getting informed helps the manager save the cost of ignorance, which is higher for the riskier project; second, getting informed allows the manager to exercise the option of strategically concealing bad information, whose value is (again) higher for the riskier project. Combining both effects, the marginal benefit of getting informed is higher for the riskier project and hence will lead to more information acquisition for the riskier project. In that sense, project riskiness and information acquisition can be viewed as strategic complements.

To understand the overinvestment in information acquisition, it is useful to refer back to Equation (13). The first-best information acquisition level, conditional on project β, is q_β^{FB} that maximizes firm's expected cash flow, which is given by $q_\beta^{FB} = \frac{K - \left(E\left[\sqrt{x_\beta}\right]\right)^2}{c}$. To the extent that the manager cares about short-term price, the feature of our model (as standard in Dye [1985] settings) that the uninformed manager is underpriced and the informed manager is on average overpriced introduces an additional incentive for the manager to become informed—that is, to achieve overpricing and avoid underpricing—and thereby leads to overinvestment in information acquisition. Without information asymmetry, or alternately, if the manager could commit to full disclosure, the first-best information acquisition would have prevailed. That is, managerial myopia combined with the voluntary nature of disclosure leads to a new inefficiency: overinvestment in information production.

[16] The first-order condition is taken with respect to q_β on the manager's expected payoff on Date $1\frac{1}{2}$, which is calculated analogously to (8): $EU(q_\beta \mid \beta) = \alpha\left[(1 - q_o - q_\beta)P^{ND} + \left(q_o + q_\beta\right)(K + O(\beta, P^{ND}))\right] + (1 - \alpha)\left[(1 - q_o - q_\beta)\left(E\left[\sqrt{x_\beta}\right]\right)^2 + (q_o + q_\beta)K\right] - C(q_\beta)$.

At $t = 1$, the manager chooses the optimal project choice. In equilibrium, the manager's optimal project choice has to be consistent with the market belief about the project choice. The following proposition characterizes the equilibrium project choice, information acquisition, disclosure, and investment scale decisions.[17]

Proposition 5: There always exists an equilibrium in which

1. At $t = 1$, depending on the parameter values, the manager's project choice is either a pure-strategy one or a mixed-strategy one.
2. At $t = 1\frac{1}{2}$, the manager acquires information according to (14).
3. At $t = 2$, an informed manager follows a threshold strategy: disclosing x_β for all $x_\beta > P^{ND}$ and withholding information otherwise. The price given disclosure of x_β is $P^D(x_\beta) = x_\beta$ and the price given no disclosure P^{ND*} is determined according to (12).
4. At $t = 3$, the manager chooses the optimal investment scale $I^*(\Omega)$ that maximizes the expected cash flow, as characterized in Lemma 1.

VI. CONCLUSION

In this paper, we examine how the prospect of *ex post* voluntary disclosure affects managers' real decisions. We introduce real decisions (a project choice decision, an investment scale decision, and an information acquisition decision by managers) to the Dye (1985) voluntary disclosure framework with uncertainty about information endowment. When managers can choose among projects with various levels of risk, the option value embedded in voluntary disclosure gives rise to an incentive for the managers to engage in excessive risk taking. We show that the voluntary nature of managers' disclosure may lead to two types of inefficiencies: (1) managers may choose riskier projects, which generate lower expected cash flow (than safer projects) *ex ante* due to higher cost of ignorance at a later stage, and (2) managers may overinvest in information acquisition. If the manager or a regulator could implement a full disclosure policy, these inefficiencies would not exist in equilibrium.

Our study provides a novel channel through which voluntary disclosure has a real effect. In our setting, the same productive information that is voluntarily disclosed to the capital market is also used for internal decision-making. Our paper assumes that these internal decisions are not publicly observable. An interesting direction for future research would be to consider the case in which the internal decisions are publicly observable. In addition to the direct effect on stock price, the observability of internal decisions generates managerial incentives that affect the *ex ante* project choice as well as *ex post* internal investment decisions. We leave these for future work.

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[17] The existence of the equilibrium is easy to demonstrate (see the proof in Appendix A). However, given the complexity of the setting, to establish the uniqueness of the equilibrium, we would need to introduce a technical condition that the marginal cost of acquiring information c is sufficiently large.

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APPENDIX A

Proof of Lemma 1

(1) If the manager is informed about x_β, she chooses the investment scale I that maximizes her payoff:

$$\max_{I \in R} \alpha P + (1-\alpha) CF(x_\beta, I) = \begin{cases} \alpha P^{ND} + (1-\alpha)\left(2\sqrt{x_\beta I} - I\right) & \text{No Disclosure} \\ \alpha P^{D}(x_\beta) + (1-\alpha)\left(2\sqrt{x_\beta I} - I\right) & \text{Disclosure} \end{cases}.$$

Since I is unobservable and its choice does not affect the price, the manager chooses I to maximize cash flow, i.e., maximizing $\left(2\sqrt{x_\beta I} - I\right)$, resulting in

$$I^*(x_\beta) = x_\beta.$$

The expected cash flow generated is

$$E[CF(x_\beta, I^*(x_\beta))] = E[x_\beta].$$

(2) If the manager is uninformed, she chooses I to maximize:

$$\max_{I \in R} E[\alpha P^{ND} + (1-\alpha)CF(x_\beta, I)] = \alpha P^{ND} + (1-\alpha)E[CF(x_\beta, I)] = \alpha P^{ND} + (1-\alpha)\left[2E\left[\sqrt{x_\beta}\right] \cdot \sqrt{I} - I\right].$$

The FOC with respect to I gives rise to

$$I^*(\emptyset) = \left(E\left[\sqrt{x_\beta}\right]\right)^2.$$

The expected cash flow generated is: $E\left[CF(x_\beta, I^*(\emptyset))\right] = \left(E\left[\sqrt{x_\beta}\right]\right)^2$. ∎

Proof of Lemma 2

We need to show that the cost of ignorance, defined in Equation (2), is higher for the riskier project. To show that, it is necessary and sufficient to show that $E[\sqrt{x_\beta}]$ is lower for the riskier project.

Under the uniform distribution, the expectation of $\sqrt{x_\beta}$ is given by

$$E\left[\sqrt{x_\beta}\right] = \int_{K-\beta K}^{K+\beta K} \frac{\sqrt{x_\beta}}{2\beta K} dx_\beta = M(\beta)\sqrt{K}, \tag{15}$$

where

$$M(\beta) \equiv \frac{\left[(1+\beta)^{\frac{3}{2}} - (1-\beta)^{\frac{3}{2}}\right]}{3\beta}.$$

The first derivative of $M(\beta)$ is:

$$M'(\beta) = \frac{(2+\beta)\sqrt{1-\beta} - (2-\beta)\sqrt{1+\beta}}{6\beta^2}.$$

The denominator is positive. At the limit where $\beta = 0$, the numerator equals zero. To further characterize the numerator, we take its derivative with respect to β, yielding:

$$\frac{d\left[(2+\beta)\sqrt{1-\beta} - (2-\beta)\sqrt{1+\beta}\right]}{d\beta} = \frac{3\beta\left(\sqrt{1-\beta} - \sqrt{1+\beta}\right)}{2\sqrt{1-\beta^2}} < 0.$$

Therefore, for any $\beta > 0$, the numerator is negative. Since the denominator is positive, we have $M'(\beta) < 0$. Given $M'(\beta) < 0$, the lemma is immediate from (15). ∎

Proof of Lemma 4

We have shown in the main text that an informed manager always follows a threshold disclosure strategy. We next prove parts (a)–(c) of the Lemma.

Part (a)

Recall that the price given no disclosure (Equation (4)) is given by:

$$P^{ND} = Pr\left(\Omega = x_\beta \mid ND\right) \cdot E\left[CF(x_\beta, I^*(x_\beta)) \mid \Omega = x_\beta, x_\beta < P^{ND}, \hat{\sigma}\right] + Pr(\Omega = \emptyset \mid ND) \cdot E\left[CF(x_\beta, I^*(\emptyset)) \mid \Omega = \emptyset, \hat{\sigma}\right].$$

Note that

$$E\left[CF(x_\beta, I^*(x_\beta)) \mid \Omega = x_\beta, x_\beta < P^{ND}, \hat{\sigma}\right] < E\left[CF(x_\beta, I^*(x_\beta)) \mid \Omega = x_\beta, \hat{\sigma}\right] = K,$$

$$E\left[CF(x_\beta, I^*(\emptyset)) \mid \Omega = \emptyset, \hat{\sigma}\right] < E\left[CF(x_\beta, I^*(x_\beta)) \mid \Omega = x_\beta, \hat{\sigma}\right] = K.$$

Therefore, $P^{ND}(\cdot) < K$.

Part (b)

We can further refine the representation of the price given no disclosure. There are four types of managers who may not disclose: (1) an uninformed manager who has chosen the safer project; (2) an uninformed manager who has chosen the riskier project; (3) an informed manager who has chosen the safer project; (4) an informed manager who has chosen the riskier project. Let us define the *mispricing* as the difference between the prevailing price and the price the firm should have gotten under full disclosure. The following computes the expected mispricing for each type of managers:

$$\text{Expected Mispricing for uninformed } \beta \text{ type} = P^{ND} - \left(E\left[\sqrt{x_\beta}\right]\right)^2,$$

$$\text{Expected Mispricing for informed } \beta \text{ type} = O\left(\beta, P^{ND}\right) = \max\left\{\int_{(1-\beta)K}^{P^{ND}} \frac{|\, P^{ND} - x \,|}{2\beta K} dx, 0\right\}.$$

Note that, the market does not observe the manager's project choice and whether she is informed or not, hence there is mispricing for each of the above four types of managers. However, the market prices the firm correctly in expectation, which implies that the expected mispricing is zero. That is,

$$(1-\hat{\sigma})\underbrace{\left\{(1-q)\left(P^{ND} - (E[\sqrt{x_L}])^2\right) + q\,O\left(L, P^{ND}\right)\right\}}_{G(L)} + \hat{\sigma}\underbrace{\left\{(1-q)\left(P^{ND} - (E[\sqrt{x_H}])^2\right) + q\,O\left(H, P^{ND}\right)\right\}}_{G(H)} = 0. \quad (16)$$

$G(\beta)$, where $\beta = \{L, H\}$, represents the *ex ante* (before the realization of information endowment) mispricing the β-type firm gets. Because $O(H, P^{ND}) > O(L, P^{ND})$ (see the proof of Lemma 5) and $\left(E\left[\sqrt{x_H}\right]\right)^2 < \left(E\left[\sqrt{x_L}\right]\right)^2$, it is straightforward that $G(L) < G(H)$. Furthermore,

$$\frac{dG(\beta)}{dP^{ND}} = 1 - q + q \cdot \max\left\{\frac{P^{ND} - (1-\beta)K}{2\beta K}, 0\right\} > 0.$$

Note that $\frac{dG(\beta)}{dP^{ND}}$ represents the probability of non-disclosure by a manager who chooses project β.

We prove Part (b) indirectly, by taking the derivative with respect to $\hat{\sigma}$ of Equation (16):

$$\underbrace{G(H) - G(L)}_{+} + \underbrace{\left\{(1-\hat{\sigma})\frac{dG(L)}{dP^{ND}} + \hat{\sigma}\frac{dG(H)}{dP^{ND}}\right\}}_{+}\frac{\partial P^{ND}}{\partial \hat{\sigma}} = 0 \Rightarrow \frac{\partial P^{ND}}{\partial \hat{\sigma}} < 0.$$

Part (c)

Again we prove this claim indirectly, by taking the derivative with respect to q of Equation (16):

$$(1-\hat{\sigma})O\left(L, P^{ND}\right) + \hat{\sigma}O\left(H, P^{ND}\right) + (1-\hat{\sigma})(E[\sqrt{x_L}])^2 + \hat{\sigma}(E[\sqrt{x_H}])^2 - P^{ND} + \underbrace{\left\{(1-\hat{\sigma})\frac{dG(L)}{dP^{ND}} + \hat{\sigma}\frac{dG(H)}{dP^{ND}}\right\}}_{+}\frac{\partial P^{ND}}{\partial q} = 0.$$

From Equation (16), we get

$$\begin{aligned} &(1-\hat{\sigma})O\left(L, P^{ND}\right) + \hat{\sigma}O\left(H, P^{ND}\right) + (1-\hat{\sigma})(E[\sqrt{x_L}])^2 + \hat{\sigma}(E[\sqrt{x_H}])^2 - P^{ND} \\ &= \frac{1}{1-q}\left[(1-\hat{\sigma})O\left(L, P^{ND}\right) + \hat{\sigma}O\left(H, P^{ND}\right)\right]. \end{aligned}$$

Therefore,

$$\begin{aligned} \frac{\partial P^{ND}}{\partial q} &= -\frac{(1-\hat{\sigma})O(L, P^{ND}) + \hat{\sigma}O(H, P^{ND}) + (1-\hat{\sigma})\left(E\left[\sqrt{x_L}\right]\right)^2 + \hat{\sigma}\left(E\left[\sqrt{x_H}\right]\right)^2 - P^{ND}}{(1-\hat{\sigma})\frac{dG(L)}{dP^{ND}} + \hat{\sigma}\frac{dG(H)}{dP^{ND}}} \\ &= -\frac{1}{1-q}\frac{(1-\hat{\sigma})O(L, P^{ND}) + \hat{\sigma}O(H, P^{ND})}{(1-\hat{\sigma})\frac{dG(L)}{dP^{ND}} + \hat{\sigma}\frac{dG(H)}{dP^{ND}}} < 0. \end{aligned} \quad (17)$$

■

Proof of Lemma 5

By Lemma 4, the smallest $P^{ND}(\cdot)$ is achieved when $\hat{\sigma}=1$ and $q=1$. That is, $P^{ND}(\hat{\sigma},q)\geq P^{ND}(\hat{\sigma}=1,q=1)=(1-H)K$. Therefore,

$$O\left(H,P^{ND}\right)=\int_{(1-H)K}^{P^{ND}}\frac{P^{ND}-x}{2HK}dx=\frac{\left[P^{ND}-(1-H)K\right]^2}{4HK}. \tag{18}$$

Note that $P^{ND}(\cdot)$ could be greater or smaller than $(1-L)K$, therefore,

$$O\left(L,P^{ND}\right)=\begin{cases}\int_{(1-L)K}^{P^{ND}}\frac{P^{ND}-x}{2LK}dx=\frac{\left[P^{ND}-(1-L)K\right]^2}{4LK} & \text{for } P^{ND}(\cdot)\geq(1-L)K\\ 0 & \text{for } P^{ND}(\cdot)<(1-L)K\end{cases}.$$

In the case of $P^{ND}(\cdot)\geq(1-L)K$, we have $O(\beta,P^{ND})=\frac{\left[P^{ND}-(1-\beta)K\right]^2}{4\beta K}$ for $\beta=H,L$. It is readily verified that

$$\frac{dO(\beta,P^{ND})}{dP^{ND}}=\frac{[P^{ND}-(1-\beta)K]}{2\beta K}>0.$$
$$\frac{dO(\beta,P^{ND})}{d\beta}=\frac{[P^{ND}-(1-\beta)K][K-P^{ND}+K\beta]}{4K\beta^2}>0.$$

■

Proof of Proposition 1

Our proof of the Proposition consists of six steps. To avoid clutter, we write P^{ND} instead of $P^{ND}(\hat{\sigma},q)$ when there is no scope for confusion. In Steps 1–5 we consider the interior $\alpha\in(0,1)$. In Step 6 we consider the special cases of $\alpha=0$ and $\alpha=1$.

Step 1

There always exists a unique equilibrium in which

$$\sigma^*=\left\{\begin{array}{ccc}1 & \text{if} & \Delta(q,\alpha,\hat{\sigma}=1)>0\\ 0 & \text{if} & \Delta(q,\alpha,\hat{\sigma}=0)<0\\ \sigma_m^*(q,\alpha) & & \text{otherwise}\end{array}\right\},$$

where $\sigma_m^*(q,\alpha)$ is determined by $\Delta(q,\alpha,\sigma_m^*(q,\alpha))=0$.

Proof of Step 1

We first show below that $\Delta(q,\alpha,\hat{\sigma})$ is continuous and decreasing in $\hat{\sigma}$.

Recall that $\Delta(q,\alpha,\hat{\sigma})$ is defined in (9) as follows:

$$\Delta(q,\alpha,\hat{\sigma})=(1-q)(1-\alpha)\left[(E[\sqrt{x_H}])^2-(E[\sqrt{x_L}])^2\right]+q\alpha\left[O\left(H,P^{ND}\right)-O\left(L,P^{ND}\right)\right].$$

Clearly, $\Delta(\cdot)$ is a function of $\hat{\sigma}$ through $P^{ND}(\cdot)$. It is straightforward that,

$$\frac{d\Delta(\cdot)}{d\hat{\sigma}}=q\alpha\frac{d[O(H,P^{ND})-O(L,P^{ND})]}{dP^{ND}}\frac{\partial P^{ND}(\cdot)}{\partial\hat{\sigma}}.$$

Furthermore, note that

$$\frac{d[O(H,P^{ND})-O(L,P^{ND})]}{dP^{ND}}=\begin{cases}\frac{(H-L)(K-P^{ND})}{2KHL}>0 & \text{for } P^{ND}(\cdot)\geq(1-L)K\\ \frac{P^{ND}-(1-H)K}{2KH}>0 & \text{for } P^{ND}(\cdot)<(1-L)K\end{cases}.$$

From Lemma 4, we know that $\frac{\partial P^{ND}(\cdot)}{\partial\hat{\sigma}}<0$. Therefore, $\frac{d\Delta(\cdot)}{d\hat{\sigma}}<0$.

If $\Delta(q,\alpha,\hat{\sigma}=1)>0$, then $\Delta(q,\alpha,\hat{\sigma})>0$ for all $\hat{\sigma}$ and hence $\sigma^{BR}=1$. That is, the unique equilibrium is $\sigma^*=\hat{\sigma}=\sigma^{BR}=1$.

If $\Delta(q,\alpha,\hat{\sigma}=0)<0$, then $\Delta(q,\alpha,\hat{\sigma})<0$ for all $\hat{\sigma}$ and hence $\sigma^{BR}=0$. That is, the unique equilibrium is $\sigma^*=\hat{\sigma}=\sigma^{BR}=0$.

If $\Delta(q,\alpha,\hat{\sigma}=1)\leq 0$ and $\Delta(q,\alpha,\hat{\sigma}=0)\geq 0$, then by the continuity and monotonicity of $\Delta(\cdot)$ in $\hat{\sigma}$, there must exist a unique σ_m such that $\Delta(q,\alpha,\hat{\sigma}=\sigma_m)=0$. ■

Steps 2–4 aim to find joint conditions on q and α for the pure strategy equilibrium to exist. That is, we find thresholds on q and α for which $\Delta(q, \alpha, \hat{\sigma} = 0) < 0$ or $\Delta(q, \alpha, \hat{\sigma} = 1) > 0$ hold. To prove the uniqueness of these thresholds, we show the concavity of $\Delta(q, \alpha, \hat{\sigma})$ (for $\hat{\sigma} \in \{0, 1\}$) in Step 2 and use it in Steps 3 and 4.

Step 2

We show that both $\Delta(q, \alpha, \hat{\sigma} = 0)$ and $\Delta(q, \alpha, \hat{\sigma} = 1)$ are concave in q.

Proof of Step 2

Taking the derivative of $\Delta(q, \alpha, \hat{\sigma})$ (Equation (9)) with respect to q, yields[18]

$$\begin{aligned}\frac{d\Delta(q, \alpha, \hat{\sigma})}{dq} &= -(1-\alpha)\left[(E[\sqrt{x_H}])^2 - (E[\sqrt{x_L}])^2\right] + \alpha\left[O\left(H, P^{ND}\right) - O\left(L, P^{ND}\right)\right] \\ &\quad + q\alpha \frac{d[O(H, P^{ND}) - O(L, P^{ND})]}{dP^{ND}} \frac{\partial P^{ND}(\cdot)}{\partial q}.\end{aligned} \tag{19}$$

The second-order derivative with respect to q is:

$$\begin{aligned}\frac{d^2\Delta(q, \alpha, \hat{\sigma})}{dq^2} &= 2\alpha \frac{d[O(H, P^{ND}) - O(L, P^{ND})]}{dP^{ND}} \frac{\partial P^{ND}(\cdot)}{\partial q} + q\alpha \frac{d^2[O(H, P^{ND}) - O(L, P^{ND})]}{dP^{ND^2}} \left(\frac{\partial P^{ND}(\cdot)}{\partial q}\right)^2 \\ &\quad + q\alpha \frac{d[O(H, P^{ND}) - O(L, P^{ND})]}{dP^{ND}} \frac{\partial^2 P^{ND}(\cdot)}{\partial q^2}.\end{aligned} \tag{20}$$

We want to show that $\frac{d^2\Delta(q,\alpha,\hat{\sigma})}{dq^2} < 0$ for $\hat{\sigma} \in \{0, 1\}$. To that end, we need to characterize $\frac{\partial P^{ND}(\cdot)}{\partial q}$ and $\frac{\partial^2 P^{ND}(\cdot)}{\partial q^2}$ for $\hat{\sigma} \in \{0, 1\}$. We do so indirectly. We first examine the case of $\hat{\sigma} = 1$. From Equation (16), $P^{ND}(\hat{\sigma} = 1, q)$ is determined by

$$(1-q)\left(P^{ND} - (E[\sqrt{x_H}])^2\right) + q\, O\left(H, P^{ND}\right) = 0. \tag{21}$$

Taking derivative with respect to q on both sides of Equation (22), we get

$$O\left(H, P^{ND}\right) + (E[\sqrt{x_H}])^2 - P^{ND} + \left[1 - q + q\frac{dO(H, P^{ND})}{dP^{ND}}\right] \frac{\partial P^{ND}(\cdot)}{\partial q} = 0. \tag{22}$$

Taking further derivative with respect to q on both sides of (22), we get,

$$2\underbrace{\left[\frac{dO(H, P^{ND})}{dP^{ND}} - 1\right]}_{-} \underbrace{\frac{\partial P^{ND}(\cdot)}{\partial q}}_{-} + q\underbrace{\frac{d^2O(H, P^{ND})}{dP^{ND^2}}}_{+} \underbrace{\left(\frac{\partial P^{ND}(\cdot)}{\partial q}\right)^2}_{+} + \underbrace{\left[1 - q + q\frac{dO(H, P^{ND})}{dP^{ND}}\right]}_{+} \frac{\partial^2 P^{ND}(\cdot)}{\partial q^2} = 0 \Rightarrow \frac{\partial^2 P^{ND}(\cdot)}{\partial q^2} < 0. \tag{23}$$

Analogous argument shows that $\frac{\partial P^{ND}(\cdot)}{\partial q} < 0$ and $\frac{\partial^2 P^{ND}(\cdot)}{\partial q^2} < 0$ for $\hat{\sigma} = 0$.

Next, note that

(i) for those q values such that $P^{ND}(\cdot) \geq (1 - L)K$, we have

$$\frac{d[O(H, P^{ND}) - O(L, P^{ND})]}{dP^{ND}} = \frac{(H - L)(K - P^{ND})}{2KHL} > 0, \tag{24}$$

$$\frac{d^2[O(H, P^{ND}) - O(L, P^{ND})]}{dP^{ND^2}} = -\frac{(H - L)}{2KHL} < 0. \tag{25}$$

Therefore, from (20) and the results that $\frac{\partial P^{ND}(\cdot)}{\partial q} < 0$ and $\frac{\partial^2 P^{ND}(\cdot)}{\partial q^2} < 0$ (for $\hat{\sigma} \in \{0, 1\}$), it is immediate that $\frac{d^2\Delta(q,\alpha,\hat{\sigma})}{dq^2} < 0$ for $\hat{\sigma} \in \{0, 1\}$ in those q ranges satisfying case (i).

[18] Note that for $\Delta(q, \alpha, \hat{\sigma})$, $\frac{d\Delta(q,\alpha,\hat{\sigma})}{dq} = \frac{\partial\Delta(q,\alpha,\hat{\sigma})}{\partial q}$.

(ii) for those q values such that $P^{ND}(\cdot) < (1-L)K$, we have $O(L, P^{ND}) = 0$, then

$$\frac{d[O(H,P^{ND}) - O(L,P^{ND})]}{dP^{ND}} = \frac{dO(H,P^{ND})}{dP^{ND}} = \frac{P^{ND} - (1-H)K}{2KH} > 0, \tag{26}$$

$$\frac{d^2[O(H,P^{ND}) - O(L,P^{ND})]}{dP^{ND^2}} = \frac{d^2O(H,P^{ND})}{dP^{ND^2}} = \frac{1}{2KH} > 0. \tag{27}$$

In this case, signing $\frac{d^2\Delta(q,\alpha,\hat{\sigma})}{dq^2}$ (Equation (20)) is more involved than in case (i). First, it is clear that $P^{ND}(\hat{\sigma}=0,q) \geq (1-L)K$ for all q. That is, $P^{ND}(\cdot) < (1-L)K$ (case (ii)) can only occur when $\hat{\sigma}=1$. For $\hat{\sigma}=1$, from (17), we have

$$\frac{\partial P^{ND}(\hat{\sigma}=1,q)}{\partial q} = -\frac{1}{1-q}\frac{O(H,P^{ND})}{\frac{dG(H)}{dP^{ND}}} = -\frac{1}{1-q}\frac{O(H,P^{ND})}{\left(1-q+q\frac{dO(H,P^{ND})}{dP^{ND}}\right)}. \tag{28}$$

At the same time, from (23), we have

$$\frac{\partial^2 P^{ND}(\hat{\sigma}=1,q)}{\partial q^2} = -\frac{2\left[\frac{dO(H,P^{ND})}{dP^{ND}} - 1\right]\frac{\partial P^{ND}(\cdot)}{\partial q} + q\frac{d^2O(H,P^{ND})}{dP^{ND^2}}\left(\frac{\partial P^{ND}(\cdot)}{\partial q}\right)^2}{\left[1-q+q\frac{dO(H,P^{ND})}{dP^{ND}}\right]}. \tag{29}$$

Plugging (29) in to (20) and simplifying the terms, we get, (note that $O(L, P^{ND}) = 0$ for those q values such that $P^{ND}(\cdot) < (1-L)K$)

$$\frac{d^2\Delta(q,\alpha,\hat{\sigma}=1)}{dq^2} = \frac{\alpha}{\left[1-q+q\frac{dO(H,P^{ND})}{dP^{ND}}\right]}\frac{\partial P^{ND}(\cdot)}{\partial q}\left[2\frac{dO(H,P^{ND})}{dP^{ND}} + q(1-q)\frac{d^2O(H,P^{ND})}{dP^{ND^2}}\frac{\partial P^{ND}}{\partial q}\right].$$

Since $\frac{\partial P^{ND}(\cdot)}{\partial q} < 0$, it is clear that,

$$Sign\left(\frac{d^2\Delta(q,\alpha,\hat{\sigma}=1)}{dq^2}\right) = -Sign\left(2\frac{dO(H,P^{ND})}{dP^{ND}} + q(1-q)\frac{d^2O(H,P^{ND})}{dP^{ND^2}}\frac{\partial P^{ND}}{\partial q}\right). \tag{30}$$

From (28), we can simplify the terms as follows:

$$\begin{aligned} 2\frac{dO(H,P^{ND})}{dP^{ND}} + q(1-q)\frac{d^2O(H,P^{ND})}{dP^{ND^2}}\frac{\partial P^{ND}}{\partial q} &= 2\frac{dO(H,P^{ND})}{dP^{ND}} - q\frac{d^2O(H,P^{ND})}{dP^{ND^2}}\frac{O(H,P^{ND})}{1-q+q\frac{dO(H,P^{ND})}{dP^{ND}}} \\ &\geq 2\frac{dO(H,P^{ND})}{dP^{ND}} - \frac{d^2O(H,P^{ND})}{dP^{ND^2}}\frac{O(H,P^{ND})}{\frac{dO(H,P^{ND})}{dP^{ND}}} = \frac{3}{4}\frac{P^{ND}-(1-H)K}{KH} \geq 0. \end{aligned}$$

The first inequality holds because $1-q+q\frac{dO(H,P^{ND})}{dP^{ND}} \geq q\frac{dO(H,P^{ND})}{dP^{ND}}$ and the last equality follows from substituting $O(H, P^{ND})$, $\frac{dO(H,P^{ND})}{dP^{ND}}$ and $\frac{d^2O(H,P^{ND})}{dP^{ND^2}}$ (Equations (18), (26) and (27)). Therefore, $\frac{d^2\Delta(q,\alpha,\hat{\sigma}=1)}{dq^2} < 0$. That is, $\Delta(q,\alpha,\hat{\sigma}=1)$ is concave in q in those q ranges satisfying case (ii). ■

Step 3

We show that the necessary and sufficient conditions for $\Delta(q,\alpha,\hat{\sigma}=1) > 0$ are $\alpha > \frac{\left(E\left[\sqrt{x_L}\right]\right)^2 - \left(E\left[\sqrt{x_H}\right]\right)^2}{\left(E\left[\sqrt{x_L}\right]\right)^2 - (1-H)K}$ and $q > \bar{q}(\alpha)$, where $\bar{q}(\alpha)$ is determined by $\Delta(\bar{q}(\alpha),\alpha,\hat{\sigma}=1) = 0$.

Proof of Step 3

In this step we deal with the case of $\hat{\sigma}=1$. First we check the limit case of $q=0$. By (9),

$$\Delta(q=0,\alpha,\hat{\sigma}=1) = (1-\alpha)\left[(E[\sqrt{x_H}])^2 - (E[\sqrt{x_L}])^2\right] < 0.$$

For the limit case of $q=1$, unravelling will prevail such that $P^{ND}(\hat{\sigma}=1,q=1) = (1-H)K$. That is, the informed manager (no matter which project he has chosen) will always disclose. Hence, $\Delta(q=1,\alpha,\hat{\sigma}=1) = 0$.

Given the concavity of $\Delta(q,\alpha,\hat{\sigma}=1)$ in q and the fact that $\Delta(q=0,\alpha,\hat{\sigma}=1) < 0$ and $\Delta(q=1,\alpha,\hat{\sigma}=1) = 0$, there is either no interior value of q such that $\Delta(q,\alpha,\hat{\sigma}=1) = 0$ or there exists a unique such value of q. A sufficient and necessary condition for the existence of such interior q is that $\Delta(q,\alpha,\hat{\sigma}=1)$ is decreasing in q at the limit $q=1$. Evaluating Equation (19) at $q=1$ and $\hat{\sigma}=1$ yields

$$\begin{aligned}
&\lim_{q\to 1}\frac{d\Delta(q,\alpha,\hat{\sigma}=1)}{dq}\\
&= -(1-\alpha)\Big[(E[\sqrt{x_H}])^2-(E[\sqrt{x_L}])^2\Big]+\lim_{q\to 1}\alpha O\big(H,P^{ND}\big)+\lim_{q\to 1}\alpha\frac{d[O(H,P^{ND})]}{dP^{ND}}\frac{\partial P^{ND}(\cdot)}{\partial q}\\
&= -(1-\alpha)\Big[(E[\sqrt{x_H}])^2-(E[\sqrt{x_L}])^2\Big]-\lim_{q\to 1}\alpha\frac{O(H,P^{ND})}{1-q}\\
&= -(1-\alpha)\Big[(E[\sqrt{x_H}])^2-(E[\sqrt{x_L}])^2\Big]-\alpha\Big[(E[\sqrt{x_H}])^2-(1-H)K\Big].
\end{aligned}$$

The second equation arises from the fact that $\lim_{q\to 1} O(H,P^{ND})=0$ and $\lim_{q\to 1}\frac{\partial P^{ND}(\hat{\sigma}=1,q)}{\partial q}=-\frac{1}{1-q}\frac{O(H,P^{ND})}{\frac{dO(H,P^{ND})}{dP^{ND}}}$ (by Equation (28)). When calculating $\lim_{q\to 1}\frac{O(H,P^{ND})}{1-q}$, we use the closed-form solution of $P^{ND}(\hat{\sigma}=1,q)$, which is derived from (16).

For $\alpha>\frac{\left(E\left[\sqrt{x_L}\right]\right)^2-\left(E\left[\sqrt{x_H}\right]\right)^2}{\left(E\left[\sqrt{x_L}\right]\right)^2-(1-H)K}$ we have $\lim_{q\to 1}\frac{d\Delta(q,\alpha,\hat{\sigma}=1)}{dq}<0$. This implies that there exists a unique $\bar{q}(\alpha)<1$, such that $\Delta(\bar{q}(\alpha),\alpha,\hat{\sigma}=1)=0$. Given the concavity of $\Delta(q,\alpha,\hat{\sigma}=1)$ in q and the fact that $\Delta(q=0,\alpha,\hat{\sigma}=1)<0$ and $\Delta(q=1,\alpha,\hat{\sigma}=1)=0$, it is straightforward that $\Delta(q,\alpha,\hat{\sigma}=1)>0$ for all $q>\bar{q}(\alpha)$. ■

Step 4

We show that the necessary and sufficient conditions for $\Delta(q,\alpha,\hat{\sigma}=0)<0$ is $q<\underline{q}(\alpha)$, where $\underline{q}(\alpha)$ is determined by $\Delta(\underline{q}(\alpha),\alpha,\hat{\sigma}=0)=0$.

Proof of Step 4

For $\hat{\sigma}=0$, $P^{ND}(\hat{\sigma}=0,q)\geq(1-L)K>(1-H)K$. Therefore, using Equation (9) for the limit cases, we get

$$\begin{aligned}
&\Delta(q=1,\alpha,\hat{\sigma}=0)=\alpha\big[O\big(H,P^{ND}\big)-O\big(L,P^{ND}\big)\big]>0,\\
&\Delta(q=0,\alpha,\hat{\sigma}=0)=(1-\alpha)\Big[(E[\sqrt{x_H}])^2-(E[\sqrt{x_L}])^2\Big]<0.
\end{aligned}$$

By the continuity and concavity of $\Delta(q,\alpha,\hat{\sigma}=0)$ in q, there must exist a unique $\underline{q}(\alpha)$ such that $\Delta(\underline{q}(\alpha),\alpha,\hat{\sigma}=0)=0$. Hence, for all $q<\underline{q}(\alpha)$ we have $\Delta(q,\alpha,\hat{\sigma}=0)<0$. ■

Step 5

We show that $\underline{q}(\alpha)\leq\bar{q}(\alpha)$.

Proof of Step 5

Remember that $\underline{q}(\alpha)$ is determined by $\Delta(\underline{q}(\alpha),\alpha,\hat{\sigma}=0)=0$. Because $\Delta(\cdot)$ is decreasing in $\hat{\sigma}$ (Step 1), $\Delta(\underline{q}(\alpha),\alpha,\hat{\sigma}=1)\leq\Delta(\underline{q}(\alpha),\alpha,\hat{\sigma}=0)=0$. Recall that $\bar{q}(\alpha)$ is determined by $\Delta(\bar{q}(\alpha),\alpha,\hat{\sigma}=1)=0$. Hence by the definition and uniqueness of $\bar{q}(\alpha)$ (Step 3), it is clear that $\underline{q}(\alpha)\leq\bar{q}(\alpha)$. ■

Step 6

We examine the special cases of $\alpha=0$ and $\alpha=1$.

Proof of Step 6

For the special case of $\alpha=0$, Equation (9) can be reduced to

$$\Delta(q,\alpha,\hat{\sigma})=(1-q)\Big((E[\sqrt{x_H}])^2-(E[\sqrt{x_L}])^2\Big)\leq 0.$$

Therefore, the manager will always choose the safer project. For the special case of $\alpha=1$, Equation (9) can be reduced to

$$\Delta(q,\alpha,\hat{\sigma})=q\big(O\big(H,P^{ND}\big)-O\big(L,P^{ND}\big)\big)\geq 0.$$

Hence the manager will always choose the riskier project. ■

Proof of Proposition 2

In the proof, we change the order: first we prove that σ^* is increasing in q; then we prove that σ^* is increasing in α.

(1) As Proposition 1 shows, for q sufficiently small, $\sigma^*=0$. For q sufficiently large, $\sigma^*=1$. Otherwise, $\sigma^*=\sigma_m^*(q,\alpha)$. We just need to prove that $\sigma_m^*(q,\alpha)$ is increasing in q. For that purpose, we take derivative with respect to q on both sides of $\Delta(q,\alpha,\sigma_m^*(q,\alpha))=0$ and get

$$\frac{\partial\Delta(q,\alpha,\sigma_m^*)}{\partial q}+\frac{d\Delta(\cdot)}{d\hat{\sigma}}|_{\hat{\sigma}=\sigma_m^*}\frac{d\sigma_m^*(\cdot)}{dq}=0.$$

By step 1 of the proof of Proposition 1, we know that $\frac{d\Delta(\cdot)}{d\hat{\sigma}}<0$. Hence to prove that $\frac{d\sigma_m^*(\cdot)}{dq}>0$, it is sufficient to show that $\frac{\partial\Delta(q,\alpha,\sigma_m^*)}{\partial q}>0$.
Note that,

$$\Delta(q,\alpha,\sigma_m^*)=0\Leftrightarrow-(1-\alpha)\left[(E[\sqrt{x_H}])^2-(E[\sqrt{x_L}])^2\right]=\frac{q}{1-q}\alpha\left[O(H,P^{ND})-O(L,P^{ND})\right].\tag{31}$$

Plugging (31) into (19), we get

$$\frac{\partial\Delta(q,\alpha,\sigma_m^*)}{\partial q}=\frac{\alpha}{1-q}\left(O(H,P^{ND})-O(L,P^{ND})\right)+q\alpha\left(\frac{dO(H,P^{ND})}{dP^{ND}}-\frac{dO(L,P^{ND})}{dP^{ND}}\right)\frac{\partial P^{ND}(\cdot)}{\partial q}.$$

Recall that from (17),

$$\begin{aligned}\frac{\partial P^{ND}(\cdot)}{\partial q}&=-\frac{1}{1-q}\frac{(1-\sigma_m^*)O(L,P^{ND})+\sigma_m^*O(H,P^{ND})}{(1-\sigma_m^*)\frac{dG(L)}{dP^{ND}}+\sigma_m^*\frac{dG(H)}{dP^{ND}}}\\&=-\frac{1}{1-q}\frac{(1-\sigma_m^*)O(L,P^{ND})+\sigma_m^*O(H,P^{ND})}{\left(1-q+(1-\sigma_m^*)q\frac{dO(L,P^{ND})}{dP^{ND}}+\sigma_m^*q\frac{dO(H,P^{ND})}{dP^{ND}}\right)}.\end{aligned}$$

Therefore,

$$\begin{aligned}\frac{\partial\Delta(q,\alpha,\sigma_m^*)}{\partial q}\propto&\left(1-q+(1-\sigma_m^*)q\frac{dO(L,P^{ND})}{dP^{ND}}+\sigma_m^*q\frac{dO(H,P^{ND})}{dP^{ND}}\right)\left(O(H,P^{ND})-O(L,P^{ND})\right)\\&-q\left(\frac{dO(H,P^{ND}}{dP^{ND}}-\frac{dO(L,P^{ND})}{dP^{ND}}\right)\left((1-\sigma_m^*)O(L,P^{ND})+\sigma_m^*O(H,P^{ND})\right)\\&=(1-q)\left(O(H,P^{ND})-O(L,P^{ND})\right)+q\left(\frac{dO(L,P^{ND})}{dP^{ND}}O(H,P^{ND})-\frac{dO(H,P^{ND})}{dP^{ND}}O(L,P^{ND})\right)\\&=(1-q)\left(O(H,P^{ND})-O(L,P^{ND})\right)+\begin{cases}q\frac{(P^{ND}-(1-H)K)}{2KH}\frac{(P^{ND}-(1-L)K)}{2KL}\frac{(H-L)K}{2} & for\ P^{ND}(\cdot)>(1-L)K\\0 & for\ P^{ND}(\cdot)\le(1-L)K\end{cases}>0.\end{aligned}$$

This completes that proof of $\frac{d\sigma_m^*(\cdot)}{dq}>0$.

(2) As Proposition 1 shows, for $q<\underline{q}(\alpha)$, $\sigma^*=1$. For α sufficiently large and $q\geq\bar{q}(\alpha)$, $\sigma^*=1$. Otherwise, $\sigma^*=\sigma_m^*(q,\alpha)$. Therefore, to prove that σ^* is (weakly) increasing in α, we need to show that both cutoffs, $\underline{q}(\alpha)$ and $\bar{q}(\alpha)$, are increasing in α and the mixed strategy $\sigma_m^*(q,\alpha)$ is increasing in α as well.

As $\bar{q}(\alpha)$ is determined by $\Delta(\bar{q}(\alpha),\alpha,\hat{\sigma}=1)=0$ and $\underline{q}(\alpha)$ is determined by $\Delta(\underline{q}(\alpha),\alpha,\hat{\sigma}=0)=0$, similar proof as in part (1) would show that both $\underline{q}(\alpha)$ and $\bar{q}(\alpha)$ are increasing in α.

For the mixed strategy $\sigma_m^*(q,\alpha)$, recall that $\sigma_m^*(q,\alpha)$ is determined by $\Delta(q,\alpha,\sigma_m^*(q,\alpha))=0$. To prove that $\frac{d\sigma_m^*(\cdot)}{d\alpha}>0$, we take derivative with respect to α on both sides of $\Delta(q,\alpha,\sigma_m^*(q,\alpha))=0$,

$$\frac{\partial\Delta(q,\alpha,\sigma_m^*)}{\partial\alpha}+\frac{d\Delta(\cdot)}{d\hat{\sigma}}|_{\hat{\sigma}=\sigma_m^*}\frac{d\sigma_m^*(\cdot)}{d\alpha}=0.$$

By step 1 of the proof of Proposition 1, we know that $\frac{d\Delta(\cdot)}{d\hat{\sigma}}<0$. Hence to show that $\frac{d\sigma_m^*(\cdot)}{d\alpha}>0$, we just need to prove that $\frac{\partial\Delta(q,\alpha,\sigma_m^*)}{\partial\alpha}>0$. From (9),

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$$\frac{\partial \Delta(\cdot)}{\partial \alpha} = -(1-q)\left[(E[\sqrt{x_H}])^2 - (E[\sqrt{x_L}])^2\right] + q\left[O\left(H, P^{ND}\right) - O\left(L, P^{ND}\right)\right] > 0.$$

This completes that proof of $\frac{d\sigma_m^*(\cdot)}{d\alpha} > 0$. ■

Proof of Proposition 3

Part (1): Taking derivative with respect to α on (10):

$$\frac{dEU^*(q,\alpha)}{d\alpha} = (1-q)\underbrace{\left[(E[\sqrt{x_H}])^2 - (E[\sqrt{x_L}])^2\right]}_{-}\underbrace{\frac{d\sigma^*}{d\alpha}}_{+} \le 0.$$

Part (2): Taking derivative with respect to q on (10):

$$\begin{aligned}\frac{dEU^*(q,\alpha)}{dq} &= K - \sigma^*(E[\sqrt{x_H}])^2 - (1-\sigma^*)(E[\sqrt{x_L}])^2 + (1-q)\left[(E[\sqrt{x_H}])^2 - (E[\sqrt{x_L}])^2\right]\frac{d\sigma^*}{d\alpha}\\ &= \underbrace{K - (E[\sqrt{x_L}])^2}_{+} + \underbrace{\left[(E[\sqrt{x_L}])^2 - (E[\sqrt{x_H}])^2\right]}_{+}\left(\sigma^* - (1-q)\frac{d\sigma^*}{dq}\right).\end{aligned}$$

By Proposition 1, for $q < \underline{q}(\alpha)$ or $q > \bar{q}(\alpha)$, the equilibrium is a pure-strategy one. That is, $\frac{d\sigma^*}{dq} = 0$. Therefore,

$$\frac{dEU^*(q,\alpha)}{dq} = K - (E[\sqrt{x_L}])^2 + \left[(E[\sqrt{x_L}])^2 - (E[\sqrt{x_H}])^2\right]\sigma^* > 0.$$

For $q \in [\underline{q}(\alpha), \bar{q}(\alpha)]$, $\sigma^* = \sigma_m$. Taking the limit and note that $\lim_{L\to 0}\left(E\left[\sqrt{x_L}\right]\right)^2 = K$,

$$\lim_{L\to H}\frac{dEU^*(q,\alpha)}{dq} = K - (E[\sqrt{x_H}])^2 > 0,$$

$$\lim_{L\to 0}\frac{dEU^*(q,\alpha)}{dq} = \left[K - (E[\sqrt{x_H}])^2\right]\left(\sigma^* - (1-q)\frac{d\sigma^*}{dq}\right).$$

For $q \to \underline{q}(\alpha)^+$, $\sigma^* = \sigma_m^* \to 0$. Therefore,

$$\lim_{L\to 0, q\to \underline{q}(\alpha)^+}\frac{dEU^*(q,\alpha)}{dq} = -(1-q)\left[K - (E[\sqrt{x_H}])^2\right]\frac{d\sigma^*}{dq} < 0.$$

■

Proof of Proposition 4

Part (1): Taking derivative with respect to α on (11):

$$\frac{d\ \text{Inv. Inefficiency}(q,\alpha)}{d\alpha} = (1-q)\left[(E[\sqrt{x_L}])^2 - (E[\sqrt{x_H}])^2\right]\frac{d\sigma^*}{d\alpha} \ge 0.$$

Part (2): Taking derivative with respect to q on (11):

$$\frac{d\ \text{Inv. Inefficiency}(q,\alpha)}{dq} = \left[(E[\sqrt{x_L}])^2 - (E[\sqrt{x_H}])^2\right]\left(-\sigma^* + (1-q)\frac{d\sigma^*}{dq}\right).$$

- For $\alpha = 0$, $\sigma^*(\cdot) = 0$ for all q. Hence $\left(-\sigma^* + (1-q)\frac{d\sigma^*}{dq}\right) = 0$.
- For $\alpha = 1$, $\sigma^*(\cdot) = 1$ for all q. Hence $\left(-\sigma^* + (1-q)\frac{d\sigma^*}{dq}\right) = -1 < 0$.
- For $\alpha \in (0,1)$,
  - if $q < \underline{q}(\alpha)$, $\sigma^*(\cdot) = 0$. In this region, $\left(-\sigma^* + (1-q)\frac{d\sigma^*}{dq}\right) = 0$.
  - if $q \to \underline{q}(\alpha)^+$, $\sigma^*(\cdot) = \sigma_m^* \to 0$. In this region, $\left(-\sigma^* + (1-q)\frac{d\sigma^*}{dq}\right) > 0$.
  - if $q \to 1$, $\sigma^*(\cdot) > 0$. In this region, $\left(-\sigma^* + (1-q)\frac{d\sigma^*}{dq}\right) < 0$.

■

Proof of Lemma 6

At the information acquisition stage, holding constant the price, the manager chooses q_β to maximize its expected payoff:

$$EU(q_\beta|\beta) = (1 - q_o - q_\beta)\Big[\alpha P^{ND} + (1-\alpha)\big(E\big[\sqrt{x_\beta}\big]\big)^2\Big] + \big(q_o + q_\beta\big)\big[E\big[x_\beta\big] + \alpha O\big(\beta, P^{ND}\big)\big] - C(q_\beta).$$

The FOC gives rise to

$$C'\Big(q_\beta^{BR}\Big) = K - (E[\sqrt{x_H}])^2 + \alpha\Big[O\big(H, P^{ND}(\hat{\sigma}, \hat{q}_H, \hat{q}_L)\big) - \Big(P^{ND}(\hat{\sigma}, \hat{q}_H, \hat{q}_L) - (E[\sqrt{x_H}])^2\Big)\Big].$$

Imposing the fixed point that $q_\beta^{BR} = \hat{q}_\beta = q_\beta^*$, the equilibrium information acquisition is determined by the following system of equations:

$$\begin{cases} C'(q_H^*) = K - \big(E\big[\sqrt{x_H}\big]\big)^2 + \alpha\Big[O\big(H, P^{ND}(\hat{\sigma}, q_H^*, q_L^*)\big) - \Big(P^{ND}(\hat{\sigma}, q_H^*, q_L^*) - \big(E\big[\sqrt{x_H}\big]\big)^2\Big)\Big] \\ C'(q_L^*) = K - \big(E\big[\sqrt{x_L}\big]\big)^2 + \alpha\Big[O\big(L, P^{ND}(\hat{\sigma}, q_H^*, q_L^*)\big) - \Big(P^{ND}(\hat{\sigma}, q_H^*, q_L^*) - \big(E\big[\sqrt{x_L}\big]\big)^2\Big)\Big] \end{cases}. \tag{32}$$

Part (1): It is clear that

$$C'(q_H^*) - C'(q_L^*) = -(1-\alpha)\underbrace{\Big[(E[\sqrt{x_H}])^2 - (E[\sqrt{x_L}])^2\Big]}_{-} + \alpha\underbrace{\big[O\big(H, P^{ND}(\hat{\sigma}, q_H^*, q_L^*)\big) - O\big(L, P^{ND}(\hat{\sigma}, q_H^*, q_L^*)\big)\big]}_{+} > 0.$$

Therefore, $q_H^* > q_L^*$.

Part (2): To show that $q_\beta^* > q_\beta^{FB} = \frac{K - \left(E\left[\sqrt{x_\beta}\right]\right)^2}{c}$, it is sufficient to show that $O(\beta, P^{ND}) - P^{ND} + \big(E\big[\sqrt{x_\beta}\big]\big)^2 > 0$. It is easy to verify that

$$\frac{d\Big(O(\beta, P^{ND}) - P^{ND} + \big(E\big[\sqrt{x_\beta}\big]\big)^2\Big)}{d\,P^{ND}} < 0.$$

Given $P^{ND} < K$ (Lemma 4),

$$\begin{aligned} O\big(\beta, P^{ND}\big) - P^{ND} + \big(E\big[\sqrt{x_\beta}\big]\big)^2 &> O\big(\beta, P^{ND}\big) - P^{ND} + \big(E\big[\sqrt{x_\beta}\big]\big)^2\Big|_{P^{ND}=K} \\ &= \frac{\beta K}{4} - K + \big(E\big[\sqrt{x_\beta}\big]\big)^2 = K\left[\frac{\beta}{4} - 1 + M^2(\beta)\right] \overset{\text{for } \beta\in[0,1]}{>} 0. \end{aligned}$$

■

Proof of Proposition 5

Similar to (9), define

$\Delta(\hat{\sigma}) = EU(\beta = H) - EU(\beta = L)$.

(a) If $\Delta(\hat{\sigma} = 1) > 0$, then the equilibrium project choice is $\sigma^* = \sigma^{BR} = \hat{\sigma} = 1$.
(b) If $\Delta(\hat{\sigma} = 0) < 0$, then the equilibrium project choice is $\sigma^* = \sigma^{BR} = \hat{\sigma} = 0$.
(c) If $\Delta(\hat{\sigma} = \sigma_m) = 0$, then the equilibrium project choice is $\sigma^* = \sigma^{BR} = \hat{\sigma} = \sigma_m$.

To prove the existence of the equilibrium, we need to show that at least one of the above three cases exist. To that purpose, note that $\Delta(\hat{\sigma} = 1)$ is either positive or (weakly) negative. Same for $\Delta(\hat{\sigma} = 0)$. If $\Delta(\hat{\sigma} = 1) > 0$ **or** $\Delta(\hat{\sigma} = 0) < 0$, then case (a) or case (b) holds. If $\Delta(\hat{\sigma} = 1) \leq 0$ **and** $\Delta(\hat{\sigma} = 0) \geq 0$, then by continuity of $\Delta(\cdot)$, there must exist some $\sigma_m \in [0, 1]$ such that $\Delta(\sigma_m) = 0$, that is, the mixed strategy equilibrium. ■