Optimal Contracts and Career Concerns with Window-Dressing

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Xiaojing Meng
Stern Business School

Haresh Sapra
Chicago-Booth Business School

Tiago da Silva Pinheiro
Norwegian School of Economics

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1 Introduction

Accounting numbers are frequently used as performance measures to evaluate managers, but are often subject to manipulation. Incentives to manipulate the performance measures typically arise from a manager's explicit compensation being dependent on that measure of performance. A less explored reason for manipulation (but no less important) is a manager's career concerns. Explicit compensation and career concerns are different in that the latter are not under the control of shareholders, and must be carefully considered when designing explicit compensation. This paper studies how a manager's career concerns affect the optimal explicit compensation contracts, their efficiency and managerial behavior in the presence of window-dressing.

To address this question we develop a two-period model with two agents: a manager and a principal. We assume that in each period the manager can exert hidden and costly effort to increase the firm's output, which is not directly observable to the principal. Instead, the only available (contractible) performance measure is subject to manipulation, which is personally costly to the manager and constitutes a social loss. In other words, costly manipulation constitutes an important friction in the model, driving the optimal contract away from the first-best benchmark. To capture career concerns we follow Holmstrom (1999) and assume that the manager's talent (productive ability) is symmetrically unknown both to the manager and to the principal. Career concerns arise in our model because the future contracts the manager gets optimally depend on the principal's beliefs about the manager's talent, and these beliefs are influenced by the past realization of the performance measure and therefore provide effort and manipulation incentives to the manager. In addition, we follow Dewatripont, Jewitt and Tirole (1999b) and assume that the manager's talent and effort are complements in production.

In this setting both career concerns and explicit compensation incentives are based on the same performance measure, and both sources of incentives induce the manager to exert effort and to manipulate. That is, career concerns and explicit compensation incentives are substitutes and, on this account, more career concerns should lead to less explicit incentives. However, due to the complementarity between the manager's talent and effort, career concerns and explicit compensation incentives are not perfect substitutes. Depending on whether the manager's future payoff is concave or convex in the manager's talent, career concerns may be less effective or more effective in inducing effort than explicit incentives, which in turn determines how the strength of career concerns affects the optimal pay-for-performance, the efficiency of the principal-manager relation, and managerial behavior.

When career concerns are more effective in inducing effort than explicit incentives, the optimal pay-for-performance decreases with more career concerns. The reason being that in addition of career concerns substituting for explicit incentives, career concerns induce less manipulation than explicit incentives for each additional unit of effort, thus also leading the principal to rely less on explicit incentives. As a consequence of relying less on the explicit incentives, which induce more manipulation, the manager's overall manipulation decreases. Contract efficiency, on the other hand, may increase or decrease with the strength of the manager's career concerns. To understand this result, it is helpful to consider a hypothetical case where the principal could control the weight on the manager's career concerns. When career concerns are more effective in inducing effort than explicit incentives, the principal would choose a positive weight on career concerns. When the strength of the manager's career concerns is below that weight, contract efficiency increases as career concerns become stronger because the manager's career concerns are closer to that "optimal" level. The opposite occurs when the strength of the manager's career concerns is above that "optimal" level.

When career concerns are less effective in inducing effort relative to manipulation, more career concerns have an ambiguous effect on the optimal pay-for-performance: while the substitution effect works as before, the fact that career concerns induce more manipulation for each additional unit of effort than explicit incentives would call for the principal to rely more on the explicit incentives. These two countervailing effects lead to the ambiguous result on the optimal pay-for-performance. In contrast with the previous case, manipulation and contract efficiency decreases with more career concerns. The principal would optimally put a negative weight on the less effective career concerns if she were able to control the weight. However, given that the weight on career concerns is always positive, increasing such weight will only take it further away from the negative "optimal" level, and thus leads to less contract efficiency and more manipulation.

More career concerns lead to more effort. To understand this result, it is helpful to take the cost approach to the principal's problem. Viewing manipulation as the indirect cost of effort, we show that more career concerns always decreases the marginal indirect cost of effort (no matter whether the career concerns are more or less effective in inducing effort), which eventually lead to higher optimal effort.

Related Literature Career concerns were first introduced by Fama (1980), who argued that explicit incentive contracts are not necessary to resolve moral hazard problem because market force (labor market) alone will provide efficient implicit effort incentives. Holmstrom (1982, 1999) showed that although such labor market discipline can have substantial effects, explicit contracts are still need to provide optimal effort provision due to the principal's lack of control on the labor market force: in the absence of explicit contracts, agents typically work too hard in early years and not hard enough in later years. Gibbons and Murphy (1992) added explicit contracts in career concerns model and showed that implicit incentives provided by career concerns and explicit incentives provided by formal contracts are *perfect* substitute, and the explicit incentive contracts can be chosen (as a plug) to provide the optimal total effort incentives. In this paper, we emphasize that in the presence of window-dressing, when the agent's ability and action are complements, the explicit contracts and implicit incentives are substitutes, but not perfect. As a result, adding explicit contracts in career concerns models cannot fully resolve the lack of control problem in Holmstrom (1982, 1999).

Other related papers include Dewatripont, Jewitt and Tirole (1999b), Feltham and Xie (1994), and Goldman and Slezak (2006).

2 Model Setup

Consider a manager who works for two periods. In each period, the manager chooses productive effort e_t, and the firm's output is determined by the manager's effort e_t and the manager's non-negative productive ability μ in the following way. We assume that the manager's ability and productive effort are complements:

$$y_t = \mu e_t \tag{1}$$

The principal (or employer) cannot directly observe the firm's output, instead the principal (and all prospective employers) observe a performance measure which is subjected to manager's manipulation m_t[1]. Specifically, the performance measure is:

$$z_t = y_t + m_t \tag{2}$$

Both productive and manipulation efforts are costly to the manager with $c(e_t)$ and $r(m_t)$ being the cost functions. We assume that both $c(e_t)$ and $r(m_t)$ are quadratic function with $c(e_t) = \frac{1}{2}ce_t^2$ and $r(m_t) = \frac{1}{2}rm_t^2$.

At the beginning of the game, there is symmetric (but imperfect) information about the manager's ability: The managers' productive ability follows a p.d.f. $f(\mu)$ with a prior mean μ and is unknown to the manager and the principal alike. In addition, we assume that the manager's ability μ has a lower bound support $\sqrt{c/r}$, which implies that the effort is more effective in influencing the performance measure than manipulation after taking into consideration the respective costs. At the beginning of the second period, the manager and the principal update their expectations about the manager's productive ability after observing the first period performance measure z_1. In updating, the principal uses her belief about the manager's unobserved actions e_1 and m_1. The manager, on the other hand, uses his actual actions e_1 and m_1:

$$\bar{\mu}_{2,S} = \frac{z_1 - \hat{m}_1}{\hat{e}_1} \qquad \textit{and} \qquad \bar{\mu}_{2,M} = \frac{z_1 - m_1}{e_1} = \mu \tag{3}$$

[1] Whether the manager can observe the firm's output or not is irrelevant. The reason is that the manager's manipulation affects the performance measure deterministically. Therefore even if the manager can only observe the performance measure, he can perfectly infer the firm's output.

Note that the manager perfectly learns his own productive ability after the first period, while the perceived manager's ability by the principal (at the beginning of the second period) is increasing in the first period performance measure z_1.[2]

To keep our analysis simple, we exclude long-term contracts and focus only on short-term linear contracts. Specifically, we assume that the principal offers the manager a contingent wage contract $w(z_t)$ which is linear in the concurrent performance measure z_t: $w(z_t) = a_t + b_t z_t$.

The manager is assumed to be risk neutral with preferences given by the following function:

$$U(z, e, m) = w(z_1) - c(e_1) - r(m_1) + \beta[w(z_2) - c(e_2) - r(m_2)] \tag{4}$$

Where β is the discount factor.

Following Gibbons and Murphy (1992), we assume that the manager has the bargaining power and at the beginning of each period always chooses the most attractive contract simultaneously offered by the prospective employers.- That is, in each period, the employer will break even in expectation and the manager takes all the surplus.

2.1 Second Period

We solve the model by backward induction. Taking the second period compensation contract as given, the manager chooses effort and manipulation to:

$$\max_{e_2, m_2} E_{2,M}(a_2 + b_2 z_2) - c(e_2) - r(m_2)$$

Note that the manager perfectly learns his own ability after the first period. Hence the manager's second-period effort, $e_2^*(b_2, \mu)$ and manipulation, $m_2^*(b_2)$ satisfy the first-order

[2] Under the current assumptions the manager's ability is known after one period, and reputation concerns are short-lived. To make reputation concerns long-lived and extend the model to multiple periods one can simply assume that ability is imperfectly correlated across time, so that $\mu_{t+1} = \rho\mu_t + \varepsilon$.

condition, respectively[3]:

$$\mu b_2 = c'(e_2) \quad and \quad b_2 = r'(m_2) \tag{5}$$

Given that the costs of effort and manipulation are convex (c'', $r'' > 0$), the manager's second-period effort and manipulation choices increase in the degree of incentive-pay b_2.

At the beginning of the second-period, all the prospective employers simultaneously offer the manager the second-period contracts, among which the manager chooses the most attractive one. Competition among the prospective employers not only drives their expected profits to be zero, but also leads them to offer contracts which, *in their expectation*, maximize the manager's second-period payoff:

$$\max_{a_2, b_2} E_{2,S}[a_2 + b_2 z_2 - c(e_2^*(b_2, \mu)) - r(m_2^*(b_2))] \tag{6}$$

The zero expected profit condition on the employers implies that:

$$E_{2,S}(y_2 - a_2 - b_2 z_2) = 0 \Longleftrightarrow E_{2,S}(a_2 + b_2 z_2) = E_{2,S}(y_2) = E_{2,S}[\mu e_2^*(b_2, \mu)]$$

Therefore the optimization problem (6) can be reduced to:

$$\begin{aligned} &\max_{b_2} E_{2,S}[\mu e_2^*(b_2, \mu) - c(e_2^*(b_2, \mu)) - r(m_2^*(b_2))] \\ \Leftrightarrow &\max_{b_2} \bar{\mu}_{2,S} e_2^*(b_2, \bar{\mu}_{2,S}) - c(e_2^*(b_2, \bar{\mu}_{2,S})) - r(m_2^*(b_2)) \end{aligned} \tag{7}$$

The following result characterizes the second-period optimal slope coefficient.

Lemma 1. *The optimal second-period slope coefficient b_2^* is increasing in the employer's perception of the manager's productive ability $\bar{\mu}_{2,S}$:*

[3]The second-order conditions are satisfied because the costs of effort and manipulation are convex (c'', $r'' > 0$).

$$b_2^* = \frac{\bar{\mu}_{2,S}}{\bar{\mu}_{2,S} + \frac{m_2^{*\prime}(b_2^*)}{e_2^{*\prime}(b_2^*)}} = \frac{1}{1 + \frac{c}{r}\frac{1}{\bar{\mu}_{2,S}^2}} \tag{8}$$

As is standard in the window-dressing literature, the optimal second-period slope coefficient is positive but less than 1, its first-best level. The lack of congruence between the performance measure and output explains the result: In addition to inducing effort, pay-for-performance b_2 also induces manipulation, which has no effect on output but is costly, thus generating a deadweight loss. The optimal slope coefficient is then chosen to trade-off the benefits of inducing more productive effort e_2 with the cost of inducing more manipulation m_2. This trade-off implies that the optimal incentive-pay weight b_2^* is adjusted by the relative sensitivity of manipulation and effort to pay-for-performance $\frac{m_2^{*\prime}(b_2)}{e_2^{*\prime}(b_2)} = \frac{c}{r}\frac{1}{\bar{\mu}_{2,S}}$. Intuitively, the higher the relative sensitivity of manipulation and effort to pay-for-performance, the more the manager engages in costly manipulation for additional incentives and the lower the optimal b_2^* should be. Note that, higher perceived ability $\bar{\mu}_{2,S}$ increases the sensitivity of effort to pay-for-performance $e_2^{*\prime}(b_2)$ and thus decreases the relative sensitivity of manipulation and effort to pay-for-performance. At the same time, higher perceived ability $\bar{\mu}_{2,S}$ also increases the productivity of effort. Both forces lead the optimal b_2^* to increase with perceived ability $\bar{\mu}_{2,S}$.

For an arbitrary b_2, the manager's fixed wage follows from the zero profit condition :

$$a_2(\mu_{2,S}, b_2) = E_{2,S}\left[y_2 - b_2 z_2\right] = \bar{\mu}_{2,S}\left(1 - b_2\right) e_2^*(b_2, \bar{\mu}_{2,S}) - b_2 m_2^*(b_2)$$

Competition drives the employer to set, in her expectation, the manager's total compensation equal to expected output. Since the manager's pay includes an incentive component, the fixed wage is adjusted accordingly. This adjustment implies that the higher the expected productive effort $e_2^*(\cdot)$, the higher the fixed wage, because the optimal incentive-pay b_2^* is less than 1. Also, higher expected manipulation $m_2^*(\cdot)$ leads to lower fixed wage to offset the compensation paid to the manager on his manipulation through the incentive component.

Holding b_2 constant, it is immediate that:

$$\frac{\partial a_2(\bar{\mu}_{2,S}, b_2)}{\partial \bar{\mu}_{2,S}} = (1 - b_2)\left[e_2^* + \frac{\bar{\mu}_{2,S} b_2}{c''(e_2^*)}\right] > 0$$

That is, all else equal, the fixed wage is increasing in the manager's perceived ability. Higher ability implies higher output both because each unit of effort is now more productive and because the manager increases effort in response. Higher output then implies a higher fixed wage because the incentive part of the manager's wage has a sensitivity less than 1 ($b_2 < 1$) to output.

Given the optimal contract $(a_2(\bar{\mu}_{2,S}, b_2^*), b_2^*)$, the manager's second-period payoff will be:[4]

$$\pi_2(\bar{\mu}_{2,S}) = a_2(\bar{\mu}_{2,S}, b_2^*) + b_2^*[\mu e_2^*(b_2^*, \mu) + m_2^*(b_2^*)] - c(e_2^*(b_2^*, \mu)) - r(m_2^*(b_2^*)).$$

The following result shows how the manager's second-period payoff is affected by his own reputation.

Lemma 2. *Along the equilibrium path the manager's second-period payoff is increasing in the employers perception of the manager's productive ability $\bar{\mu}_{2,S}$:*

$$\frac{d\pi_2}{d\bar{\mu}_{2,S}} = [\mu e_2^*(b_2^*, \mu) - \bar{\mu}_{2,S} e_2^*(b_2^*, \bar{\mu}_{2,S})]\frac{db_2^*}{d\bar{\mu}_{2,S}} + \frac{\partial a_2(\bar{\mu}_{2,S}, b_2^*)}{\partial \bar{\mu}_{2,S}} = \frac{\partial a_2(\bar{\mu}_{2,S}, b_2^*)}{\partial \bar{\mu}_{2,S}} > 0$$

where the last equality follows from beliefs being consistent, $\bar{\mu}_{2,S} = \mu$, along the equilibrium path.

An important implication of Lemma (2) is that even if b_2^* is affected by the manager's perceived ability $\bar{\mu}_{2,S}$, along the equilibrium path the effect of $\bar{\mu}_{2,S}$ on the manager's second-period payoff is limited to the *direct* effect of $\bar{\mu}_{2,S}$ on the fixed wage $a_2(\cdot)$. This result is an application of the envelope theorem and of the fact that beliefs about the manager's ability are consistent along the equilibrium path.

[4] Note that the manager perfectly learns his own type at the beginning of the second period.

The manager's career concerns arise from the effect of the manager's perceived ability on his second-period payoff. Specifically, in order to improve his contracting terms in the second period, the manager has an incentive to improve the principal's beliefs about his ability, which by (3) is increasing in the first period performance measure z_1. Given that z_1 is affected by both effort and manipulation, such career concerns motivate the manager to exert more effort and, at the same time, more manipulation in the first period.

2.2 First Period

At the beginning of the first period, anticipating the impact on the second period and taking the first period contract as given, the manager chooses effort and manipulation to:[5]

$$\max_{e_1,m_1} E_1[a_1 + b_1 z_1 - c(e_1) - r(m_1) + \beta\pi_2(\bar{\mu}_{2,S})]$$

At the beginning of the first period, the manager doesn't know his own ability realization and only knows the prior mean $\bar{\mu}$. Therefore the manager's first-period effort and manipulation now have to satisfy:

$$c'(e_1) = \bar{\mu} b_1 + \frac{\beta}{\hat{e}_1} E_1\left[\mu \frac{d\pi_2}{d\bar{\mu}_{2,S}}\right]$$
$$r'(m_1) = b_1 + \frac{\beta}{\hat{e}_1} E_1\left[\frac{d\pi_2}{d\bar{\mu}_{2,S}}\right]$$

Note that the manager treats the principal's belief $(\hat{e}_1, \hat{m}_1)$ constant when deriving his best response above. In equilibrium, the principal's belief has to be correct, therefore after imposing the equilibrium condition $\hat{e}_1 = e_1$ and $\hat{m}_1 = m_1$, the manager's first-period equilibrium

[5] We suppress the subscript denoting different parties because both parties have the same information set at the beginning of the first period.

effort and manipulation, $e_1(b_1, \beta)$ and $m_1(b_1, \beta)$, for arbitrary b_1 and β, are determined by:[6]

$$\begin{aligned} c'(e_1) &= \bar{\mu} b_1 + \frac{\beta}{e_1} E_1 \left[\mu \frac{d\pi_2}{d\bar{\mu}_{2,S}} |_{\bar{\mu}_{2,S}=\mu} \right] && (9) \\ r'(m_1) &= b_1 + \frac{\beta}{e_1} E_1 \left[\frac{d\pi_2}{d\bar{\mu}_{2,S}} |_{\bar{\mu}_{2,S}=\mu} \right] && (10) \end{aligned}$$

For later reference, we denote $R_e \equiv E_1 \left[\mu \frac{d\pi_2}{d\bar{\mu}_{2,S}} |_{\bar{\mu}_{2,S}=\mu} \right]$ and $R_m \equiv E_1 \left[\frac{d\pi_2}{d\bar{\mu}_{2,S}} |_{\bar{\mu}_{2,S}=\mu} \right]$. The manager's first-period total effort incentive is the sum of the explicit incentive from the first-period compensation contact, $\bar{\mu} b_1$, and the implicit incentive from career concerns, $\frac{\beta}{e_1} R_e$. Similarly, the total manipulation incentive is the sum of the explicit incentive from the first period contract b_1, and the implicit incentive arising from career concerns $\frac{\beta}{e_1} R_m$. Both R_e and R_m are positive, which implies that the manager's career concerns provide both effort and manipulation incentives.

In this multi-tasking setting it is useful to think of the manager's reputation concerns $E_1 [\pi_2(\bar{\mu}_{2,S})]$ as an additional performance measure that depends on the manager's actions. This additional measure contrasts with the first-period performance measure z_1. To avoid confusion we call the former the reputation-based measure and the latter the explicit-incentive measure. In this context, the ratio $1/\bar{\mu}$ then captures the incongruity between the explicit-incentive measure z_1 and the output, while the ratio R_m/R_e measures the incongruity between the reputation-based measure and the output. When $R_e/R_m = \bar{\mu}$, the reputation-based measure and the explicit-incentive measure z_1 are equally congruent, and therefore are perfect substitutes. If $R_e/R_m > \bar{\mu}$, the reputation-based measure is more congruent with output than the explicit-incentive measure z_1 thus implying that implicit incentives are more effective in inducing effort (causing less manipulation) than explicit incentives. The opposite holds when $R_e/R_m < \bar{\mu}$. Unlike traditional multi-tasking models with multiple performance measures, the principal in this model does not control the weight

[6]For the second-order conditions please refer to the Appendix. The second-order conditions are globally satisfied along the equilibrium path, and so conditions (9) and (10) are necessary and sufficient to characterize the equilibrium.

on the reputation-based performance measure.

As before, competition among employers leads them to offer a contract which maximize the manager's payoff. Using the zero profit condition, the employer's problem is:[7]

$$\max_{b_1} E_1[y_1|e_1(b_1,\beta)] - c(e_1(b_1,\beta)) - r(m_1(b_1,\beta)) + \beta E_1[\pi_2(\bar{\mu}_{2,S})]$$

$$s.t. \hat{m}_1 = m_1 \;\; and \;\; \hat{e}_1 = e_1$$

The condition that the employer's beliefs have to be consistent with the manager's behavior implies that the employer's perception of the manager's ability is correct, i.e., $\bar{\mu}_{2,S} = \mu$. The following result characterizes the first-period optimal slope coefficient.

Proposition 1. *The optimal first-period slope coefficient b_1^* is determined by the following implicit function:*

$$b_1^* = \frac{\bar{\mu} - \frac{\beta}{e_1^*}[R_e - \bar{\mu} R_m]}{\bar{\mu} + \frac{\partial m_1/\partial b_1}{\partial e_1/\partial b_1}\,|_{b_1^*}} - \frac{\beta}{e_1^*} R_m \tag{11}$$

where $e_1^* \equiv e_1(b_1^*, \beta)$.

Comparing (11) with (8), it is clear that the first term in (11) is similar to the expression in (8). As before, since the explicit-incentive measure z_1 is incongruent with output and induces costly manipulation, the optimal weight on such measure is adjusted by the relative sensitivity of manipulation and effort to b_1 , i.e., $\frac{\partial m_1/\partial b_1}{\partial e_1/\partial b_1}\,|_{b_1^*}$. In addition, the numerator in the first term in (11) reflects the maximizing-congruity effect: the employer is choosing the weight on the explicit-incentive performance measure (b_1) to maximize the congruity between the manager's compensation and output. If $R_e/R_m > \bar{\mu}$, i.e, if the reputation-based performance measure is more congruent with the output, then the weight b_1 on the performance measure z_1 is lower when compared with the case in which both measures are

[7]Note that we impose the equilibrium condition of consistent beliefs about effort e_1 and manipulation m_1 at this point in which the employer is offering the first-period contact. The reason being that each b_1 corresponds to an equilibrium pair $(e_1(b_1,\beta), m_1(b_1,\beta))$. The employer's choice of the optimal contract amounts to selecting the best equilibrium from all the pairs (e_1, m_1) that satisfy the manager's incentive-compatibility constraints and the condition of consistent beliefs.

equally congruent with output. The opposite argument holds when $R_e/R_m < \bar{\mu}$. The second term in (11) represents the substitution effect between implicit and explicit incentives. Since reputation concerns provide effort incentives then less explicit incentives are necessary to induce a given level of effort.

Before proceeding with the analysis and for future reference let $X \equiv R_e - \bar{\mu}R_m$ and note that $X > 0$ means that the reputation-based performance measure is more congruent with output, and vice-versa. Further, consider the following result.

Lemma 3. The relative sensitivity of manipulation and effort to b_1 in the first period is given by:[8]

$$\frac{{}^{\partial m_1}/_{\partial b_1}}{{}^{\partial e_1}/_{\partial b_1}} = \frac{c + \frac{\beta}{e_1^2}X}{\bar{\mu}r} \tag{12}$$

The relative sensitivity increases with β if $X > 0$ and decreases with β when $X < 0$.

To understand this result combine equations (9) and (10) to obtain $r'(m_1) = \frac{1}{\bar{\mu}}\left[c'(e_1) - \frac{\beta}{e_1}X\right]$. Keeping effort constant, it follows that manipulation decreases with the strength of career concerns (higher β) when the reputation-based measure is more congruent with output ($X > 0$). This effect, however, is weakened when effort e_1 is higher, thus making manipulation decrease less with career concerns, i.e. $\frac{\partial}{\partial e_1}\left(\frac{\partial m_1}{\partial \beta}\right) \equiv \frac{\partial}{\partial \beta}\left(\frac{\partial m_1}{\partial e_1}\right) > 0$.[9] Our result then follows from the fact that $\frac{\partial m_1}{\partial e_1} = \frac{{}^{\partial m_1}/_{\partial b_1}}{{}^{\partial e_1}/_{\partial b_1}}$, i.e., the relative sensitivity of manipulation and effort to b_1 is equal to the sensitivity of manipulation to effort.

In this argument we used the fact that higher effort decreases the effect of reputation concerns on manipulation. We want to stress that this result is due to the production technology in which effort and talent are complements. Since effort increases the return to the manager's talent, a certain performance measure z_1 can be achieved with less talent the higher the manager's effort e_1. In turn, with higher effort e_1, employers increasingly discount the manager's performance when updating his reputation and thus the effect of

[8]This result follows from differentiating equations (9) and (10) w.r.t. b_1.

[9]Mathematically we have that $\frac{\partial^2 m_1}{\partial \beta \partial e_1} = \frac{1}{r''\bar{\mu}}\frac{1}{e_1^2}X - \frac{r'''}{r''}\frac{\partial m_1}{\partial \beta}\frac{\partial m_1}{\partial e_1} = \frac{1}{r''\bar{\mu}}\frac{1}{e_1^2}\mathrm{X}$ with the last term dropping with quadratic costs. The sign of the derivative $\frac{\partial^2 m_1}{\partial \beta \partial e_1}$ then depends on X.

career concerns in the manager's incentives is weakened.

Let $e_1^* \equiv e_1(b_1^*, \beta)$ and $m_1^* \equiv m_1(b_1^*, \beta)$ denote the first-period optimal effort and manipulation when the employer chooses optimal b_1^*. Define first-period contract efficiency as the first-period expected social surplus $CE_1(\beta) \equiv E_1[y_1 - c(e_1) - r(m_1)|e_1^*, m_1^*]$.

The following results show how the manager's career concerns parameter β affects the first-period optimal slope coefficient b_1^*, optimal effort e_1^*, optimal manipulation m_1^*, and contract efficiency CE_1. It is useful to separate the following three cases: (1) The implicit incentive and the explicit incentive are perfect substitutes ($X = 0$); (2) The implicit incentive is *more* efficient in inducing effort than the explicit incentive ($X > 0$); and (3) The implicit incentive is *less* efficient in inducing effort than the explicit incentive ($X < 0$).

Proposition 2. *If the implicit incentive and the explicit incentive are perfect substitutes, i.e., $X \equiv R_e - \bar{\mu} R_m = 0$, then:*

- *The optimal first-period slope coefficient b_1^* is decreasing in β.*
- *The first-period optimal effort e_1^*, optimal manipulation m_1^*, and contract efficiency CE_1 are constant in β. In addition, $0 < e_1^* < e_1^{FB}$, and $m_1^* > 0$.*

To understand this result, note that when $X = 0$, both the explicit-incentive measure z_1 and the reputation-based measure $E_1[\pi_2(\mu_{2,S})]$ are equally congruent with the output, and thus implicit and explicit incentives are perfect substitutes. It follows that a higher weight β on the reputation-based measure leads to a lower weight b_1^* on the explicit-incentive measure z_1. Since the congruence of the manager's compensation with output is independent of β then changes in β leave unaffected the first-period optimal effort, optimal manipulation, and contract efficiency. The optimal first-period effort is below first-best and identical to the optimal effort in the second-period.

Proposition 3. *If the reputation-based measure is more congruent with output than the explicit-incentive measure z_1, i.e., $X > 0$, then:*

- *The optimal first-period slope coefficient* b_1^* *is decreasing in* β.

- *The first-period optimal effort* $e_1^* > 0$ *is increasing in* β. *If* $\beta = \beta^o \equiv \frac{\bar{\mu}^2}{cX}$, *the first-period optimal effort reaches first-best benchmark,* $e_1^* = e_1^{FB}$. *If* $\beta > \beta^o$, *the optimal effort will be greater than the first-best benchmark,* $e_1^* > e_1^{FB}$.

- *The first-period optimal manipulation* m_1^* *is decreasing in* β. *If* $\beta < \beta^o$, *the first-period optimal manipulation is positive,* $m_1^* > 0$*; If* $\beta > \beta^o$, *the first-period optimal manipulation is negative,* $m_1^* < 0$.

- *The first-period contract efficiency* CE_1 *is increasing in* β *for* $\beta < \beta^o$, *and decreasing in* β *for* $\beta > \beta^o$. CE_1 *reaches the first-best benchmark at* $\beta = \beta^o$.

To understand how optimal b_1^* changes with β, note that by Proposition 1, the optimal b_1^* is determined by the following three factors: the relative sensitivity effect, the congruity-maximization effect, and the substitution effect. The substitution effect always implies that as the weight (β) on the reputation-based measure increases, less explicit incentives are necessary to induce a given level of effort. The congruity-maximization effect implies that less weight should be put on the less congruent measure, which is the explicit-incentive measure in the case of $X > 0$. Finally, the relative sensitivity effect suggests that the higher the relative sensitivity of manipulation and effort to pay-for-performance, the more the manager engages in costly manipulation for additional incentives and hence the lower the optimal b_1^* should be. By Lemma 3, the relative sensitivity of manipulation and effort to pay-for-performance increases with β for $X > 0$. Combining all three effects together, the optimal b_1^* decreases with β for $X > 0$.

To understand the contract efficiency result, we can think of the problem of a principal who chooses the weights on two performance measures to maximize the manager's compensation congruence with output. In such a setting, it is optimal to put a positive weight on the measure that is relatively more sensitive to effort, and vice-versa. In addition, it is possible to achieve perfect congruence and first-best effort.[10] When $X > 0$, the reputation-based measure is more congruent with output than the explicit-incentive measure, and the principal would choose a positive value of β that achieves first-best. We denote that positive value of β by β^o. In our setting, however, the weight on the reputation-based measure (β) is not under the control of the principal. Instead, it is exogenously determined by how the manager values the future. Since values of β closer to β^o imply higher contract efficiency than values of β further away, a higher weight β on the future payoff when $\beta < \beta^o$, leads to more contract efficiency. In contrast, increasing β when $\beta > \beta^o$, leads to less contract efficiency.

To better understand the effort result, it is helpful to take the cost approach to the principal's problem and to consider the cost of inducing a certain level of effort e_1. This cost is made of a direct cost $c(e_1)$ and an indirect cost associated with the manager's manipulation. Specifically, due to the non-observability of output, undesired manipulation is also induced when the principal tries to induce productive effort, and the amount of manipulation induced depends on the level of effort. In that sense, we can view the cost of the manager's manipulation $r(m_1(e_1))$ as the indirect cost of effort. When determining the optimal effort level, the principal needs to take into consideration both the direct cost and indirect cost of effort. Specifically, the optimal effort is determined by equating the expected marginal benefit of effort $\bar{\mu}$ with the expected marginal cost of effort $c'(e_1)+r'(m_1)m_1'(e_1)$, where $c'(e_1)$ represents the marginal *direct* cost and $r'(m_1)m_1'(e_1)$ represents the marginal *indirect* cost. Both the marginal benefit and the marginal direct cost of effort are independent of β. In contrast, each component $r'(m_1)$ and $m_1'(e_1)$ of the marginal indirect cost of effort depends

[10] In a traditional multi-tasking model the employer would choose the weights b_1 and β on their respective measures z_1 and $E_1[\pi_2(\bar{\mu}_{2,S})]$ to maximize the congruence of the worker's payoff with output. If the two measures had different congruence, first-best effort and manipulation would be achieved. Specifically, if measure z_1 was more congruent than $E_1[\pi_2(\bar{\mu}_{2,S})]$, then the employer would optimally set $b_1 = -\frac{\beta}{e_1} R_m > 0$ and $\beta = \bar{\mu}\frac{e_1^{FB}}{R_e - \bar{\mu}R_m} < 0$ so that the worker's incentive-constraints (9) and (10) would satisfy $c'(e_1) = \bar{\mu}$ and $r'(m_1) = 0$. Similarly, if the the measure z_1 was less congruent than $E_1[\pi_2(\bar{\mu}_{2,S})]$, then the employer would set b_1 and β as before, but the signs would be reversed. If both measures had the same congruence and were thus perfect substitutes first-best effort and manipulation would not be achievable.

on β. As we have seen before, holding effort constant, manipulation and the marginal cost of manipulation, $r'(m_1) = \frac{1}{\bar{\mu}}\left[c'(e_1) - \frac{\beta}{e_1}X\right]$, decrease with the weight β on the reputation-based measure when this measure is more congruent with output ($X > 0$). At the same time, as we have argued in Lemma 3, the sensitivity of manipulation to effort $m_1'(e_1)$ increases with β when $X > 0$. With quadratic costs, the former effect dominates, hence the marginal indirect cost of effort $r'(m_1)m_1'(e_1)$ decreases with β, which in turn implies that the optimal effort monotonically increases with β.

Manipulation, on the other hand, decreases with the strength of career concerns (β). As we discussed before, more weight β on the more congruent reputation-based measure ($X > 0$) leads to less manipulation holding effort constant. We call this the direct effect of β on manipulation. Manipulation is also affected by β through effort, and since effort increases with β, manipulation also increases with β. We call this the indirect effect of β. It turns out that the indirect effect is dominated by the direct effect of β, and hence the overall effect is such that manipulation decreases with β.

Proposition 4. *If the implicit incentive is less effective in inducing effort than the explicit incentive, i.e., $X < 0$, then:*

- *The effect of β on the optimal first-period slope coefficient b_1^* is ambiguous..*
- *The first-period optimal effort $e_1^* > 0$ is increasing in β. If $\beta = \beta^{oo} \equiv -\frac{\bar{\mu}^2}{cX}$, the first-period optimal effort reaches first-best benchmark, $e_1^* = e_1^{FB}$. If $\beta > \beta^{oo}$, the optimal effort will be greater than the first-best benchmark, $e_1^* > e_1^{FB}$.*
- *The first-period optimal manipulation m_1^* is always greater than 0 and increasing in β.*
- *The first-period contract efficiency CE_1 is decreasing in β.*

If the reputation-based measure is less congruent with the output than the explicit-incentive measure ($X < 0$), both the relative sensitivity effect and the congruity-maximization effect imply that the optimal b_1^* should increase with the weight on the reputation-based measure β. However, the substitution effect implies the opposite, and thus the net effect on b_1^* is ambiguous.

To understand the effect of β on contract efficiency note that the explicit-incentive measure z_1 is now relatively more congruent with output than the reputation-based measure, and thus a principal who were able to freely control the weights on both measures would choose an optimal negative weight on the reputation-based measure and achieve first-best. However, the manager's reputation concerns β is always positive and when it increases it is further away from the optimal weight, which leads the contract efficiency to decrease.

The effect of β on effort is, as argued before, dependent on how the marginal indirect cost of effort $r'(m_1)m_1'(e_1)$ changes with β. When $X < 0$ the effects of β on the marginal cost $r'(m_1)$ and on the sensitivity of manipulation to effort $m_1'(e_1)$ are reversed. However, the latter effect now dominates the former, and thus the marginal indirect cost of effort $r'(m_1)m_1'(e_1)$ are still decreasing in β, which leads the optimal effort to increase with career concerns β.

Manipulation is now always increasing with career concerns β. More weight β on the reputation-based measure when this measure is less congruent with output $(X < 0)$ leads to more manipulation holding effort constant. In addition, the indirect effect of β on manipulation m_1 through effort e_1 goes in the same direction, and therefore the overall effect of β on manipulation m_1 is positive.

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3 Appendix A

Proof of Lemma 1. The employer chooses second-period incentive pay b to:

$$\max_{b_2} \bar{\mu}_{2,S} e_2^*(b_2, \bar{\mu}_{2,S}) - c(e_2^*(b_2, \bar{\mu}_{2,S})) - r(m_2^*(b_2))$$

The first-order condition of the problem is:

$$(\bar{\mu}_{2,S} - c'(e_2^*(\cdot))) \frac{\partial e_2^*(\cdot)}{\partial b_2} - r'(m_2^*(\cdot)) \frac{\partial m_2^*(\cdot)}{\partial b_2} = 0 \tag{13}$$

Recall that the manager's effort choice and manipulation choice are determined by:

$$\mu b_2 = c'(e_2^*) \quad and \quad b_2 = r'(m_2^*) \tag{14}$$

Taking derivative with respect to b_2 on (14), we get:

$$c''(e_2^*) \frac{\partial e_2^*(\cdot)}{\partial b_2} = \mu \quad and \quad r''(m_2) \frac{\partial m_2^*(\cdot)}{\partial b_2} = 1 \tag{15}$$

Substituting (14) and (15) into (13), we get

$$b_2^* = \frac{1}{1 + \frac{c''}{r''} \frac{1}{\bar{\mu}_{2,S}^2}} = \frac{1}{1 + \frac{c}{r} \frac{1}{\bar{\mu}_{2,S}^2}}$$

Last, let's check the second-order condition:

$$(\bar{\mu}_{2,S} - c'(e_2^*)) \frac{\partial^2 e_2^*(\cdot)}{\partial b_2^2} - r'(m_2^*) \frac{\partial m_2^*(\cdot)}{\partial b_2^2} - c''(e_2^*) \left(\frac{\partial e_2^*(\cdot)}{\partial b_2} \right)^2 - r''(m_2^*) \left(\frac{\partial m_2^*(\cdot)}{\partial b_2} \right)^2 < 0$$

because $\frac{\partial^2 e_2^*(\cdot)}{\partial b_2^2} = \frac{\partial^2 m_2^*(\cdot)}{\partial b_2^2} = 0$ for quadratic functions. □

Proof of Lemma 2. The manager's second-period payoff

$$\pi_2(\bar{\mu}_{2,S}) = \underbrace{a_2(\bar{\mu}_{2,S}, b_2^*) + b_2^*\left[\mu e_2^*(b_2^*,\mu) + m_2^*(b_2^*)\right] - c\left(e_2^*(b_2^*,\mu)\right) - r\left(m_2^*(b_2^*)\right)}_{P}$$
$$= \underbrace{\bar{\mu}_{2,S} e_2^*(b_2^*, \bar{\mu}_{2,S}) - c\left(e_2^*(b_2^*, \bar{\mu}_{2,S})\right) - r\left(m_2^*(b_2^*)\right)}_{M}$$
$$+ \underbrace{\left[b_2^*\mu e_2^*(b_2^*,\mu) - c\left(e_2^*(b_2^*,\mu)\right)\right]}_{N(\mu)} - \underbrace{\left[b_2^*\bar{\mu}_{2,S} e_2^*(b_2^*,\bar{\mu}_{2,S}) - c\left(e_2^*(b_2^*,\bar{\mu}_{2,S})\right)\right]}_{N(\bar{\mu}_{2,S})}$$

Differentiating w.r.t. $\bar{\mu}_{2,S}$:

$$\begin{aligned}
\frac{d\pi_2}{d\bar{\mu}_{2,S}} &= \frac{\partial P}{\partial \bar{\mu}_{2,S}} + \frac{dP}{db_2^*}\frac{db_2^*}{d\bar{\mu}_{2,S}} \\
&= \frac{\partial a_2(\cdot)}{\partial \bar{\mu}_{2,S}} + \left[\frac{dM}{db_2^*} + \frac{dN(\mu)}{db_2^*} - \frac{dN(\bar{\mu}_{2,S})}{db_2^*}\right]\frac{db_2^*}{d\bar{\mu}_{2,S}}
\end{aligned}$$

Notice that

$$\begin{aligned}
\frac{dM}{db_2^*} &= 0 \qquad\qquad by\ \ (13) \\
\frac{dN(\mu)}{db_2^*} &= \mu e_2^*(b_2^*,\mu) \quad\ \ by\ the\ Envelope\ Theorem \\
\frac{dN(\bar{\mu}_{2,S})}{db_2^*} &= \bar{\mu}_{2,S} e_2^*(b_2^*,\bar{\mu}_{2,S})\ \ by\ the\ Envelope\ Theorem \\
\frac{\partial a_2(\cdot)}{\partial \bar{\mu}_{2,S}} &= (1-b_2^*)\left[e_2^*(b_2^*,\bar{\mu}_{2,S}) + \bar{\mu}_{2,S}\frac{\partial e_2^*(\cdot)}{\partial \bar{\mu}_{2,S}}\right]
\end{aligned}$$

Therefore

$$\begin{aligned}
\frac{d\pi_2}{d\bar{\mu}_{2,S}} &= (1-b_2^*)\left[e_2^*(b_2^*,\bar{\mu}_{2,S}) + \bar{\mu}_{2,S}\frac{\partial e_2^*(\cdot)}{\partial \bar{\mu}_{2,S}}\right] + \left[\mu e_2^*(b_2^*,\mu) - \bar{\mu}_{2,S} e_2^*(b_2^*,\bar{\mu}_{2,S})\right]\frac{db_2^*}{d\bar{\mu}_{2,S}} \\
&= (1-b_2^*)\frac{2\bar{\mu}_{2,S} b_2^*}{c} + \left[\mu^2 - \bar{\mu}_{2,S}^2\right]\frac{b_2^*}{c}\frac{db_2^*}{d\bar{\mu}_{2,S}}
\end{aligned}$$

The second equation is due to the quadratic cost function. □

Proof of Proposition 1.

The employer chooses slope coefficient b_1 to:

$$\max_{b_1} E_1 \left[y_1 | e_1(b_1, \beta) \right] - c\left(e_1(b_1, \beta)\right) - r\left(m_1(b_1, \beta)\right) + \beta E_1 \left[\pi_2 \left(\bar{\mu}_{2,S} \right) \right]$$
$$s.t. \hat{m}_1 = m_1 \ \ and \ \ \hat{e}_1 = e_1$$

The equilibrium condition imply that $\bar{\mu}_{2,S} = \mu$. Therefore the last term is independent of b_1. Then the optimal b_1^* has to satisfy:

$$\begin{aligned}
& \left[\bar{\mu} - c'\left(e_1\right) \right] \frac{\partial e_1}{\partial b_1} - r'\left(m_1\right) \frac{\partial m_1}{\partial b_1} = 0 \\
\Leftrightarrow & \bar{\mu} - c'\left(e_1\right) - r'\left(m_1\right) \frac{\partial m_1 / \partial b_1}{\partial e_1 / \partial b_1} = 0 \qquad (16) \\
\Leftrightarrow & \bar{\mu} - \bar{\mu} b_1 - \frac{\beta}{e_1} R_e - \left(b_1 + \frac{\beta}{e_1} R_m \right) \frac{\partial m_1 / \partial b_1}{\partial e_1 / \partial b_1} = 0 \ by \ (9) \ and \ (10) \\
\Rightarrow & b_1^* = \frac{\bar{\mu}}{\bar{\mu} + \frac{\partial m_1 / \partial b_1}{\partial e_1 / \partial b_1} |b_1^*} - \frac{\beta}{e_1(b_1^*, \beta)} R_m - \frac{\beta}{e_1(b_1^*, \beta)} \frac{\left[R_e - \bar{\mu} R_m \right]}{\bar{\mu} + \frac{\partial m_1 / \partial b_1}{\partial e_1 / \partial b_1} |b_1^*} \qquad (17)
\end{aligned}$$

Another way to solve the problem is to take the implementation approach. Specifically, we first solve the optimal effort the employer wants to implement, and then find out the slope coefficient b_1^* to implement the optimal effort e_1^*. The optimal effort e_1^* is chosen by the employer to:

$$\max_{b_1} E_1 \left[y_1 | e_1 \right] - c\left(e_1\right) - r\left(m_1(e_1)\right) + \beta E_1 \left[\pi_2 \left(\bar{\mu}_{2,S} \right) \right]$$
$$s.t. \hat{m}_1 = m_1 \ \ and \ \ \hat{e}_1 = e_1$$

Again, the last term is independent of b_1 due to $\bar{\mu}_{2,S} = \mu$ on the equilibrium path. Then the optimal e_1^* has to satisfy:

$$\begin{aligned}
&\bar{\mu} - c'(e_1) - r'(m_1)m_1^{'}(e_1) = 0 \\
\Leftrightarrow &\bar{\mu} - c'(e_1) - \frac{\left[c'(e_1) - \frac{\beta}{e_1}X\right]}{\bar{\mu}} \frac{\left[c + \frac{\beta}{e_1^2}X\right]}{\bar{\mu}r} = 0 \ by\ ()\ and\ (\textbf{??}) \\
\Rightarrow &\bar{\mu} - c'(e_1^*) - \frac{c^2 e_1^* - \frac{\beta^2 X^2}{e_1^{*3}}}{\bar{\mu}^2 r} = 0 \\
\Leftrightarrow &cr\bar{\mu}e_1^{*4} + c^2 e_1^{*4} - \bar{\mu}^3 r e_1^{*3} = \beta^2 X^2 \qquad (18)
\end{aligned}$$

Let's check the second-order condition:

$$-c''(e_1) - \frac{c^2}{\bar{\mu}r} - 3\frac{\beta^2 X^2}{\bar{\mu}r e_1^4} < 0$$

Therefore b_1^* is the slope coefficient implementing e_1^*:

$$\bar{\mu}b_1^* + \frac{\beta}{e_1^*}R_e = c'(e_1^*)$$

which is exactly the same as equation (17).■

Preliminaries for Proposition 2, 3 and 4.

- **First-period optimal effort** e_1^*.

  As shown in the proof of Proposition 1, the first-period optimal effort e_1^* is determined by equation (18). Therefore,

$$\frac{de_1^*}{d\beta} = \frac{2\beta X^2 e_1^*}{cr\bar{\mu}e_1^{*4} + c^2 e_1^{*4} + 3\beta^2 X^2} \qquad (19)$$

  Note that $\mu \geq \sqrt{c/r}$, therefore $\bar{\mu} > \sqrt{c/r}$, and $e_1^*(\beta = 0) = \frac{\bar{\mu}^3 r}{cr\bar{\mu}^2 + c^2} > \frac{\bar{\mu}}{2c} > 0$. As a result, $e_1^*(\beta)$ is always positive and $\frac{de_1^*}{d\beta} > 0$.

- **First-period optimal manipulation** m_1^***.**

By (10) and (17), the first-period optimal manipulation is determined by:

$$\begin{aligned}
& r'(m_1^*) = b_1^* + \frac{\beta}{e_1^*} R_m \\
\Leftrightarrow\, & r m_1^* = \frac{\bar{\mu} - \frac{\beta}{e_1^*} X}{\bar{\mu} + \frac{\partial m_1/\partial b_1}{\partial e_1/\partial b_1} |b_1^*} \\
\Leftrightarrow\, & \left[r\bar{\mu}^2 + c + \frac{\beta}{e_1^{*2}} X \right] m_1^* = \bar{\mu} \left[\bar{\mu} - \frac{\beta}{e_1^*} X \right]
\end{aligned} \tag{20}$$

Taking derivative w.r.t. β, we get:

$$\left[r\bar{\mu}^2 + c + \frac{\beta}{e_1^{*2}} X \right] \frac{\partial m_1^*}{\partial \beta} = -X \left[\bar{\mu} \frac{d\left(\frac{\beta}{e_1^*}\right)}{d\beta} + m_1^* \frac{d\left(\frac{\beta}{e_1^{*2}}\right)}{d\beta} \right] \tag{21}$$

Note that by (19),

$$\frac{\beta}{e_1^*} \frac{de_1^*}{d\beta} = \frac{2\beta^2 X^2}{cr\bar{\mu}e_1^{*4} + c^2 e_1^{*4} + 3\beta^2 X^2}$$

Therefore,

$$\begin{aligned}
\frac{d\left(\frac{\beta}{e_1^*}\right)}{d\beta} &= \frac{1}{e_1^*} \left[1 - \frac{\beta}{e_1^*} \frac{de_1^*}{d\beta} \right] = \frac{1}{e_1^*} \frac{cr\bar{\mu}e_1^{*4} + c^2 e_1^{*4} + \beta^2 X^2}{cr\bar{\mu}e_1^{*4} + c^2 e_1^{*4} + 3\beta^2 X^2} \\
&= \frac{1}{e_1^*} \frac{\bar{\mu}^3 r e_1^{*3} + 2\beta^2 X^2}{cr\bar{\mu}e_1^{*4} + c^2 e_1^{*4} + 3\beta^2 X^2} > 0
\end{aligned} \tag{22}$$

$$\begin{aligned}
\frac{d\left(\frac{\beta}{e_1^{*2}}\right)}{d\beta} &= \frac{1}{e_1^{*2}} \left[1 - 2\frac{\beta}{e_1^*} \frac{de_1^*}{d\beta} \right] = \frac{1}{e_1^*} \frac{cr\bar{\mu}e_1^{*4} + c^2 e_1^{*4} - \beta^2 X^2}{cr\bar{\mu}e_1^{*4} + c^2 e_1^{*4} + 3\beta^2 X^2} \\
&= \frac{\bar{\mu}^3 r e_1^*}{cr\bar{\mu}e_1^{*4} + c^2 e_1^{*4} + 3\beta^2 X^2} > 0
\end{aligned} \tag{23}$$

where the last step in both expressions uses (18). Substituting (22), (23) and (**??**) into

(21), we get

$$\underbrace{\left[r\bar{\mu}^2 + c + \frac{\beta}{e_1^{*2}} X\right]}_{A} \frac{dm_1^*}{d\beta} = -X \underbrace{\left[\frac{\bar{\mu}\left(2\beta^2 X^2 + \bar{\mu}^3 r e_1^{*3}\right) + \left(c e_1^{*2} - \beta X\right) e_1^* \bar{\mu}^2}{e_1^* \left[c r \bar{\mu} e_1^{*4} + c^2 e_1^{*4} + 3\beta^2 X^2\right]}\right]}_{B}$$

The following arguments show that both term A and term B are positive.

– If $X > 0$, then clearly $A > 0$. It is also immediate that $B > 0$ for $\beta X < c e_1^{*2}$. The more complicated case occurs when $\beta X > c e_1^{*2}$. If $\beta X > c e_1^{*2}$, then

$$\begin{aligned} Numerator(B) &> \bar{\mu}\beta X \left[2\beta X - \bar{\mu} e_1^*\right] \\ &> \bar{\mu}\beta X e_1^* \left[2 c e_1^* - \bar{\mu}\right] \\ &> \bar{\mu}\beta X e_1^* \left[2 c e_1^*(\beta = 0) - \bar{\mu}\right] \\ &> 0 \end{aligned}$$

Therefore $B > 0$.

– If $X \leq 0$, then clearly $B > 0$. By (**??**), $m_1^* > 0$, which by (20), implies that $A > 0$. Because both term A and term B are positive regardless of X, it is immediate that

$$Sign\left(\frac{dm_1^*}{d\beta}\right) = -Sign(X)$$

- **First-period contract efficiency CE_1.**

The first-period contract efficiency is defined as

$$CE_1(\beta) \equiv E_1[y_1 - c(e_1) - r(m_1)|e_1^*, m_1^*] = \bar{\mu} e_1(b_1^*, \beta) - c(e_1(b_1^*, \beta)) - r(m_1(b_1^*, \beta)).$$

Taking derivative w.r.t. β, and by Envelope Theorem (b_1^* is chosen optimally), we get:

$$\begin{aligned}\frac{dCE_1}{d\beta} &= [\bar{\mu} - c'(e_1^*)]\frac{\partial e_1(b_1^*,\beta)}{\partial\beta} - r'(m_1^*)\frac{\partial m_1(b_1^*,\beta)}{\partial\beta} \\ &\Leftrightarrow r'(m_1^*)\left[\frac{\partial e_1(b_1^*,\beta)}{\partial\beta}\frac{\partial m_1/\partial b_1}{\partial e_1/\partial b_1}|b_1^* - \frac{\partial m_1(b_1^*,\beta)}{\partial\beta}\right] \; by \; (16)\end{aligned} \tag{24}$$

Taking derivative w.r.t. β on (9) and (10), we can get:

$$\frac{\partial e_1(b_1^*,\beta)}{\partial\beta} = \frac{R_e/e_1^*}{c + \frac{\beta}{e_1^{*2}}R_e} \tag{25}$$

$$\frac{\partial m_1(b_1^*,\beta)}{\partial\beta} = \frac{cR_m}{re_1^*\left(c + \frac{\beta}{e_1^{*2}}R_e\right)} \tag{26}$$

Substituting (**??**), (25) and (26) into (24), and after algebra simplification, we get

$$\frac{dCE_1}{d\beta} = \frac{m_1^*}{e_1^*}\left(\frac{R_e}{\bar{\mu}} - R_m\right) = \frac{m_1^*}{e_1^*\bar{\mu}}X$$

- **First-period optimal slope coefficient b_1^*.**

There are three different ways to write out b_1^*:

$$\begin{aligned}b_1^* &= \frac{\bar{\mu}}{\bar{\mu} + \frac{\partial m_1/\partial b_1}{\partial e_1/\partial b_1}|b_1^*} - \frac{\beta}{e_1(b_1^*,\beta)}R_m - \frac{\beta}{e_1(b_1^*,\beta)}\frac{[R_e - \bar{\mu}R_m]}{\bar{\mu} + \frac{\partial m_1/\partial b_1}{\partial e_1/\partial b_1}|b_1^*} \\ b_1^* &= rm_1^* - \frac{\beta}{e_1^*}R_m \\ b_1^* &= \frac{ce_1^* - \frac{\beta}{e_1^*}R_e}{\bar{\mu}}\end{aligned}$$

Proof of Proposition 2. If $X = 0$, then it is immediate that:

$$e_1^* = \frac{\bar{\mu}^3 r}{cr\bar{\mu}^2 + c^2} < e_1^{FB} = \frac{\bar{\mu}}{c}$$
$$m_1^* = \frac{ce_1^*}{\bar{\mu}r} = \frac{\bar{\mu}^2}{r\bar{\mu}^2 + c} > 0$$
$$\frac{dCE_1}{d\beta} = 0$$
$$\frac{db_1^*}{d\beta} = r\frac{dm_1^*}{d\beta} - \frac{d\left(\frac{\beta}{e_1^*}\right)}{d\beta}R_m < 0$$

Proof of Proposition 3. If $X > 0$, then

$$\frac{de_1^*}{d\beta} > 0$$
$$\frac{dm_1^*}{d\beta} < 0$$
$$Sign\left(\frac{dCE_1}{d\beta}\right) = Sign\left(\frac{m_1^*}{e_1^*}\right)$$
$$\frac{db_1^*}{d\beta} = r\frac{dm_1^*}{d\beta} - \frac{d\left(\frac{\beta}{e_1^*}\right)}{d\beta}R_m < 0$$

If $\beta = \beta^o \equiv \frac{\bar{\mu}^2}{cX}$, then $e_1^* = e_1^{FB} = \frac{\bar{\mu}}{c}$, and $m_1^* = 0$. Hence

- If $\beta < \beta^o$, then $e_1^* < e_1^{FB}$, $m_1^* > 0$, and $\frac{dCE_1}{d\beta} > 0$;
- If $\beta > \beta^o$, then $e_1^* > e_1^{FB}$, $m_1^* < 0$, and $\frac{dCE_1}{d\beta} < 0$.

Proof of Proposition 3.

If $X < 0$, then

$$\begin{aligned}
\frac{de_1^*}{d\beta} &< 0 \\
\frac{dm_1^*}{d\beta} &> 0 \\
\frac{dCE_1}{d\beta} &< 0 \\
\frac{db_1^*}{d\beta} &= r\frac{dm_1^*}{d\beta} - \frac{d\left(\frac{\beta}{e_1^*}\right)}{d\beta} R_m
\end{aligned}$$

If $\beta = \beta^{oo} \equiv -\frac{\bar{\mu}^2}{cX}$, then $e_1^* = e_1^{FB} = \frac{\bar{\mu}}{c}$. Hence if $\beta > \beta^o$, then $e_1^* > e_1^{FB}$.