Volatility, the Macroeconomy and Asset Prices*

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Abstract

In this paper we show that volatility movements have first-order implications for the consumption, the stochastic discount factor, and asset prices. Volatility shocks carry a risk-premium in our model. Accounting for volatility risks leads to a positive correlation between the return to human capital and the market return, while this correlation is negative when volatility risks is ignored. Our volatility-risk based asset pricing model can account for the levels and differences in the risk premia across value and size portfolios in the data, and that macroeconomic volatility shocks contribute significantly to return volatility.

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1 Introduction

Financial economists are interested in understanding risk and return and the underlying economic sources for movements in asset markets. In this paper we show that volatility news (news about economic uncertainty) is an important and separate source of risk which critically affects the aggregate economy (i.e., consumption) and asset prices. In particular, we show that volatility risks have first-order implications for the properties of the return to financial wealth, the return to human capital, as well as standard attribute-sorted portfolios. Our analysis leads us to consider a dynamic asset-pricing framework with *three* sources of risks: cash-flow, discount rate, and volatility news. We report three central results (i) volatility risks affects consumption; this impact is important for understanding the relation between return to human capital and equity return (ii) ignoring volatility news results in a misspecified stochastic discount factor (SDF) and distorted inference of the sources and magnitudes of economic risks (iii) volatility risk-premia associated with volatility risks are important for explaining the level and dispersion in the cross-section of assets.

Bansal and Yaron (2004) provide a basic framework to analyze volatility risk; they model consumption growth with time-varying volatility of underlying shocks and show that with Epstein and Zin (1989) preferences, risk-premia is affine in volatility, and that volatility shocks carry a separate risk-premia. In this article, we pursue an approach which builds on the aforementioned idea, as well as that of Campbell (1996), who provides a framework in which consumption is substituted-out by the equilibrium return to wealth; his approach assumes that all shocks are homoscedastic and therefore volatility risk is absent. We show that unlike the typical homoscedastic environments analyzed in the literature, expected consumption growth is no longer proportional to the expected return on wealth, but is also driven by the news about aggregate uncertainty. Specifically, following positive volatility news agents typically decrease their consumption when the intertemporal elasticity of substitution (IES) parameter is above one. It is this critical model implication that introduces volatility risk as one of the fundamental economic sources of risk. We show that ignoring volatility results in a misspecified stochastic discount factor at *any* value of the IES. In particular, we show that in the model with constant volatility, the discount rate news just reflect the revisions in future risk-free rate as all asset risk premia are constant.

We use a long-run risks model of Bansal and Yaron (2004) to quantitatively highlight and analyze the importance of volatility news. Using a calibrated economy that matches several key features of the data, we show the ramification of incorrectly assuming aggregate volatility is constant for inference regarding consumption and other components of the stochastic discount factor. We show that the volatility of the implied consumption shock will be significantly biased upwards in the specification which incorrectly ignores the variation in economic uncertainty. The correlations between the implied consumption innovations and the discount rate and volatility shocks are significantly negative, even though these correlations for the true consumption shock are zero. Ignoring the presence of aggregate uncertainty also biases downward the volatility of the implied stochastic discount factor and the level of the market risk premia. This is consistent with findings in the literature regarding consumption properties that are analyzed using financial market data (returns) under homoscedastic assumptions (e.g., Campbell (1996)). In all, this analysis underscores the significant misspecification of inference regarding the macroeconomic sources of risk when fluctuations in aggregate uncertainty are ignored.

The results above are based on the assumption that the return to wealth is readily available. In the actual data, the return on wealth is unobservable and is different from the observed market return. To implement our analysis on actual data, we assume that the wealth return can be written as a weighted average of the return to the stock market and the return to human capital. In the context of measuring expected returns and the correlation between human and equity return Lustig and Van Nieuwerburgh (2008) highlight a puzzle that the two returns are negatively related as observed consumption is very smooth. In this article we provide a potential resolution to their puzzling finding by highlighting the importance of time-varying volatility. Following Lustig and Van Nieuwerburgh (2008), we also assume that the expected return on human capital is linear in the economic states. This allows us to adopt a standard VAR-based methodology for extracting the underlying news to construct the implied shocks into consumption and stochastic discount factor. We find that considering stochastic volatility has important implications for the properties of the market, human capital, and wealth returns. In the model without volatility, as in Lustig and Van Nieuwerburgh (2008), the correlations between labor and market returns are very negative. They become positive once the volatility risks are introduced, making labor income now risky rather and than a hedge asset. Similarly, the correlations between market and wealth, and wealth and labor returns become closer to one once volatility risks are accounted for. At our values of preference parameters (risk aversion of 6.5 and IES of 2.5), the risk premium for the market portfolio is 9.7%, and it is equal to 4% and 2.6% for the returns to the wealth portfolio and the human capital, respectively. The volatility risks contribute about one-third of the overall risk premium for the human capital, and about a half for the wealth portfolio and the market. The inclusion of the volatility risks has important implications for the time-series properties of the underlying economic shocks. For example, in the model with volatility risks the implied discount rate news are high and positive in recent recessions of 2001 and 2008, which is consistent with a rise in economic volatility in those periods. The model without the volatility channel, however, produces discount rate news which are negative in those times.

To explore the importance of volatility risks further, we make the assumption that the return to aggregate wealth is perfectly correlated with the return to the observed market return (e.g., Epstein and Zin (1991), Campbell (1996)). In this case the market volatility is observed and can be used in our analysis. We further consider the asset-pricing implications for a broader cross-section of assets which includes 10 size and 10 value portfolios. We show that our model captures well the levels and differences in the risk premia across the assets: the R^2 in the cross-sectional regression of the risk premia in the data on their counterparts in the model is 65% for value and size portfolios. In our volatility-based model, volatility risks are important for the level of the risk-premia and contribute about 2% to the premium, cash-flow risks contribute the most to risk-premia. It is worth noting that when volatility risks are absent, and thus risk premia is constant, the discount rate news simply reflects risk free rate news. If the risk free rate is assumed constant, an empirically relevant assumption, there is no discount rate beta and all the risk premium in the economy should be be captured by the cashflow news. Empirically, we show that imposing the restriction that market premium is affine in market volatility helps in the identification of parameters suggesting there is an intimate link between discount rate and volatility news. Indeed, we find that discount rate shocks are highly correlated with volatility shocks —that is volatility risk drives discount rates significantly. The qualitative implications of these findings are consistent with the Bansal and Yaron (2004) model.

The rest of the paper is organized as follows. In Section 2 we present a theoretical derivation of the our generalized dynamic CAPM. We set up the long-run risks model in Section 3 to gain further understanding on how volatility affects inference about consumption innovations, cash-flow and discount rate variation. Based on the calibrated model, we highlight and quantify the mis-specification of consumption and the stochastic discount factor, which is presented in Section 3. In Section 5 we develop and implement an econometric framework to quantify the role of the volatility channel in the data. The model implications for the market, human capital and wealth portfolio are discussed in Section 5. In section 6 we estimate the role of volatility using the market return for explaining a broader cross-section of assets. Conclusion follows.

2 Theoretical Framework

In this section we consider a general economic framework with recursive utility and time-varying economic uncertainty, and derive the implications for the implied innovations into the current and future consumption growth, returns, and the stochastic discount factor. We show that ignoring the fluctuations in economic uncertainty can severely bias the inference on economic news, and alter the implications for the financial markets.

2.1 Consumption Innovation

We adopt a discrete-time specification of the endowment economy where the agent's preferences are described by a Kreps and Porteus (1978) recursive utility function of Epstein and Zin (1989) and Weil (1989). The life-time utility of the agent U_t satisfies

$$U_t = \left[(1-\delta)C_t^{1-\frac{1}{\psi}} + \delta\left(E_t U_{t+1}^{1-\gamma}\right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}}\right]^{\frac{1}{1-\frac{1}{\psi}}}, \tag{2.1}$$

where C_t is the aggregate consumption level, δ is a subjective discount factor, γ is a risk aversion coefficient, ψ is the intertemporal elasticity of substitution (IES), and for notational ease we denote $\theta = (1-\gamma)/(1-\frac{1}{\psi})$. When $\gamma = 1/\psi$, the preferences collapse to a standard expected power utility.

As shown in Epstein and Zin (1989), the stochastic discount discount factor M_{t+1} can be written in terms of the log consumption growth rate, $\Delta c_{t+1} \equiv \log C_{t+1} - \log C_t$, and the log return to the consumption asset (wealth portfolio), $r_{c,t+1}$. In logs,

$$m_{t+1} = \theta \log\delta - \frac{\theta}{\psi}\Delta c_{t+1} + (\theta-1) r_{c,t+1}. \tag{2.2}$$

A standard Euler condition

$$E_t M_{t+1} R_{t+1} = 1 \tag{2.3}$$

allows us to price any asset in the economy. Assuming that the stochastic discount factor and the consumption asset returns are jointly log-normal, the Euler equation for the consumption asset thus implies:

$$E_t \Delta c_{t+1} = \psi \log\delta + \psi E_t r_{c,t+1} - \frac{\psi-1}{\gamma-1} V_t, \tag{2.4}$$

where we defined V_t to be the conditional variance of the stochastic discount factor plus the consumption asset return:

$$V_t = \frac{1}{2} Var_t(m_{t+1} + r_{c,t+1}). \tag{2.5}$$

In general, the volatility component V_t reflects the conditional second moments of the underlying shocks in the economy which drive the stochastic discount factor and the fundamental return on the wealth portfolio of the agent. In this sense, we interpret V_t as a measure of the economic uncertainty. In our subsequent discussion we show that, under further model restrictions, the economic volatility V_t is proportional to the conditional variance of the future aggregate consumption; the proportionality coefficient is always positive and depends on the risk aversion and the relative magnitude of the news about future consumption.

The equilibrium restriction for the expected consumption in Equation (2.4) is the key to our analysis. It states that when IES parameter ψ is not equal to one, the fluctuations in expected consumption are driven both by the movements in expected returns, $E_t r_{c,t+1}$, and the aggregate volatility V_t. Specifically, when IES is above one, the substitution effect dominates the wealth effect, so agents respond to positive news about future expected returns by decreasing their current consumption and increasing their savings and investment, which increases expected consumption in the future. On the other hand, when $\psi > 1$, positive shock to economic uncertainty makes agents less willing to save and invest today, so the expected future consumption goes down. Notably, the fluctuations in expected returns and volatility are not independent of each other: a rise in economic volatility typically leads to a simultaneous increase in expected returns due to a rise in a risk premium. Ignoring volatility risks would imply, then, that these times of high expected returns correspond to periods of high expected consumption, while in fact future expected consumption goes down due to an increase in economic uncertainty. Thus, ignoring the volatility risks can lead to a severe bias and mis-measurement in consumption innovation and its response to the underlying economic news, and alter the implications for the financial markets.

The volatility shocks have no impact for the consumption innovation when there is no stochastic volatility in the economy (so V_t is a constant), or when the IES $\psi = 1$. These cases have been entertained in Campbell (1983), Campbell (1996), Campbell and Vuolteenaho (2004), and Lustig and Van Nieuwerburgh (2008). In the empirical section of the paper we argue for economic importance of the variation in aggregate uncertainty and $IES > 1$ to interpret financial markets.

We use the equilibrium restriction in the Equation (2.4) to derive the immediate consumption news. The return to the consumption asset $r_{c,t+1}$ which enters the equilibrium condition in Equation (2.4) is the return on the overall wealth portfolio of the agent which pays consumption as its dividends each time period. Using standard log-linearization approach, the immediate consumption innovation can be written as the revision in expectation of future returns on consumption asset, minus the revision in expectation of future cash flows:

$$c_{t+1} - E_t c_{t+1} = (E_{t+1} - E_t) \sum_{j=0}^{\infty} \kappa_1^j r_{c,t+1+j} - (E_{t+1} - E_t) \sum_{j=1}^{\infty} \kappa_1^j \Delta c_{t+j+1}, \tag{2.6}$$

where the linearization parameter κ_1 is related to the unconditional mean of the price-consumption ratio. Using the Equation (2.4), we further decompose the consumption shock into news in consumption return, $N_{R,t+1}$, revisions of expectation of future returns (discount rate news), $N_{DR,t+1}$, as well as the news about future volatility $N_{V,t+1}$:

$$N_{C,t+1} = N_{R,t+1} + (1-\psi) N_{DR,t+1} + \frac{\psi - 1}{\gamma - 1} N_{V,t+1}, \tag{2.7}$$

where for convenience we denoted

$$\begin{aligned}
N_{C,t+1} &\equiv c_{t+1} - E_t c_{t+1} & N_{R,t+1} &\equiv r_{c,t+1} - E_t r_{c,t+1}, \\
N_{DR,t+1} &\equiv (E_{t+1} - E_t) \left(\sum_{j=1}^{\infty} \kappa_1^j r_{c,t+j+1} \right), & N_{V,t+1} &\equiv (E_{t+1} - E_t) \left(\sum_{j=1}^{\infty} \kappa_1^j V_{t+j} \right) \\
N_{CF,t+1} &\equiv (E_{t+1} - E_t) \left(\sum_{j=0}^{\infty} \kappa_1^j \Delta c_{t+j+1} \right) = N_{DR,t+1} + N_{R,t+1}
\end{aligned} \tag{2.8}$$

The response of consumption to volatility is obfuscated by the fact that N_V and N_{DR} are highly correlated. To understand this response, it is instructive to put some additional minimal structure on the environment. We assume that risk premia is affine in volatility, that is

$$E_t[r_{c,t+1} - r_{f,t}] = \beta_0 + \beta_1 V_t \tag{2.9}$$

and that volatility shocks follow an AR(1) process with persistence ν_1.[1] Recognizing that $N_R = N_{CF} - N_{DR}$, using the fact that N_{DR} can be written as the sum of N_{RP} and N_{RF}, that is the risk premia and the risk free rate news, respectively, the innovation to consumption $N_{C,t+1}$ in equation (2.7), can be rewritten as:

$$\begin{aligned}
N_{C,t+1} &= N_{CF,t+1} - \psi N_{DR,t+1} + \frac{\psi - 1}{\gamma - 1} N_{V,t+1} \\
&= N_{CF,t+1} - \psi N_{RF,t+1} + \left[-\psi \beta_1 + \frac{\psi - 1}{\gamma - 1} \right] N_{V,t+1}.
\end{aligned} \tag{2.10}$$

Hence, when ψ is greater than one and $\beta_1 > 0$, and for plausible magnitudes of risk aversion, consumption will decline and savings will increase in response to volatility. Indeed, as depicted in Figure 1, in the data, based on a VAR (details of which are described in section 5) the response of consumption to an ex-ante volatility shock is negative. This evidence underscores the motivation for our analysis.

[1] This structure will turn out to be true in the LRR model we analyze further below.

2.2 Discount Factor

The innovation into the stochastic discount factor implied by the representation in Equation (2.2) is given by,

$$m_{t+1} - E_t m_{t+1} = -\frac{\theta}{\psi}(\Delta c_{t+1} - E_t \Delta c_{t+1}) + (\theta - 1)(r_{c,t+1} - E_t r_{c,t+1}). \qquad (2.11)$$

Substituting the consumption shock in the Equation (2.7), we obtain that the stochastic discount factor is driven by immediate return news, $N_{R,t+1}$, news about future discount rates, $N_{DR,t+1}$ and news about future economic volatility, $N_{V,t+1}$:

$$m_{t+1} - E_t m_{t+1} = -\gamma N_{R,t+1} - (\gamma - 1) N_{DR,t+1} + N_{V,t+1}. \qquad (2.12)$$

Thus, the key sources of risk in the economy include the news to current and future discount rates, and news to future volatility in the economy. The first risk factor, similar to a standard CAPM model and has a market price of risk equal to the risk aversion coefficient γ; the discount rate news has a market price risk of $\gamma - 1$, and the volatility component has a market price of risk of negative one.

An alternative decomposition of the innovation into the stochastic discount factor involves future expected cash flow news, $N_{CF,t+1}$, future discount rate news, $N_{DR,t+1}$, and volatility news, $N_{V,t+1}$:

$$m_{t+1} - E_t m_{t+1} = -\gamma N_{CF,t+1} + N_{DR,t+1} + N_{V,t+1}, \qquad (2.13)$$

where the future expected cash flow news are given by,

$$N_{CF,t+1} \equiv (E_{t+1} - E_t) \left(\sum_{j=0}^{\infty} \kappa_1^j \Delta c_{t+j+1} \right) \equiv N_{DR,t+1} + N_{R,t+1}. \qquad (2.14)$$

The last equality follows from the consumption innovation identity in Equation (2.6).

Using Euler equation, we obtain that the risk premium on any asset is equal to the negative covariance of asset return $r_{i,t+1}$ with the stochastic discount factor:

$$E_t r_{i,t+1} - r_{ft} + \frac{1}{2} Var_t r_{i,t+1} = Cov_t(-m_{t+1}, r_{i,t+1}). \qquad (2.15)$$

Hence, knowing the exposures (betas) of a return to the fundamental sources of risk, we can calculate the risk premium on this asset, and decompose it into the risk compensations for the future cash-flow, discount rate, and volatility news:

$$\begin{aligned} &E_t r_{i,t+1} - r_{ft} + \frac{1}{2} Var_t r_{i,t+1} \\ &= \gamma Cov_t(r_{i,t+1}, N_{CF,t+1}) - Cov_t(r_{i,t+1}, N_{DR,t+1}) - Cov_t(r_{i,t+1}, N_{V,t+1}). \end{aligned} \qquad (2.16)$$

2.3 Risk and Return with Constant Volatility

As shown in the stochastic discount factor Equations (2.12) and (2.13), the price of the volatility risks is equal to negative 1; notably, the volatility risks are present even if the IES parameter $\psi = 1$. Thus, even though with IES equal to one ignoring volatility does not lead to the mis-specification of the consumption residual, the inference on the stochastic discount factor is still incorrect and can cause significant changes in the interpretation of the asset markets.

Let us consider in a greater detail the case when the volatility is constant and all the economic shocks are homoscedastic. First, it immediately implies that the revision in expected future volatility news is zero, $N_{V,t+1} = 0$. Further, using accounting identity, let us rewrite discount factor news in terms of risk-free rate news and risk premia news:

$$\begin{aligned} N_{DR,t+1} &= (E_{t+1} - E_t)\left(\sum_{j=1}^{\infty} \kappa_1^j r_{c,t+j+1}\right) \qquad (2.17) \\ &= (E_{t+1} - E_t)\left(\sum_{j=1}^{\infty} \kappa_1^j \left[(E_{t+j} r_{c,t+j+1} - r_{f,t+j}) + r_{f,t+j}\right]\right) \\ &= (E_{t+1} - E_t)\left(\sum_{j=1}^{\infty} \kappa_1^j (E_{t+j} r_{c,t+j+1} - r_{f,t+j})\right) + (E_{t+1} - E_t)\left(\sum_{j=1}^{\infty} \kappa_1^j r_{f,t+j}\right). \end{aligned}$$

Following the Equation (2.15), the risk premium on consumption asset depends on the conditional covariance of consumption return with the stochastic discount factor, and the conditional variance of the consumption return. However, when all the economic shocks are homoscedastic, all the variances and covariances are constant, which implies that the risk premium on the consumption asset is constant as well. Thus, under homoscedasticity, the revision in future risk premia in Equation (2.17) is equal to zero, and the discount rate shocks just capture the innovations into the future expected risk-free rates:

$$N_{DR,t+1}^{NoVol} = (E_{t+1} - E_t)\left(\sum_{j=1}^{\infty} \kappa_1^j r_{f,t+j}\right) \equiv N_{RF,t+1}. \qquad (2.18)$$

Hence, under homoscedasticity, the economic sources of risks include the revisions in future expected cash flow, and the revisions in future expected risk-free rates:

$$m_{t+1}^{NoVol} - E_t m_{t+1}^{NoVol} = -\gamma N_{CF,t+1} + N_{RF,t+1}, \qquad (2.19)$$

Further, in homoscedastic economy the risk premium on any asset is constant and depend on the unconditional covariances of the asset return to the economic risks:

$$\begin{aligned} & E_t r_{i,t+1} - r_{ft} + \frac{1}{2} Var_t r_{i,t+1} \\ & = \gamma Cov(r_{i,t+1}, N_{CF,t+1}) - Cov(r_{i,t+1}, N_{RF,t+1}). \end{aligned} \tag{2.20}$$

Notably, the beta of returns to discount rate shocks, $N_{DR,t+1}$, should just be equal to the return beta to the future expected risk-free shocks, $N_{RF,t+1}$. In several empirical studies in the literature (see e.g., Campbell and Vuolteenaho (2004)), the risk-free rates are assumed to be constant. Following the above analysis, it implies, then, that the news to the future discount rates are exactly zero, so that there is no discount rate beta, and all the risk premium in the economy is captured just by the risks in the future cash-flows. Thus, ignoring volatility risks can significantly alter the interpretation of the risk and return in financial markets.

3 Long-Run Risks Model

To gain further understanding on how volatility affects inference about consumption innovations, cash-flow and discount rate variation, and more generally fluctuations in the stochastic discount factor, asset prices and risk premia, we utilize a standard long-run risks model of Bansal and Yaron (2004). This model captures many salient features of the asset market data and importantly ascribes a prominent role for volatility risk.[2]

In a standard long-run risks model consumption dynamics satisfies

$$\Delta c_{t+1} = \mu + x_t + \sigma_t \eta_{t+1}, \tag{3.1}$$

$$x_{t+1} = \rho x_t + \varphi_e \sigma_t \epsilon_{t+1}, \tag{3.2}$$

$$\sigma_{t+1}^2 = \sigma_c^2 + \nu(\sigma_t^2 - \sigma_c^2) + \sigma_w w_{t+1}, \tag{3.3}$$

where ρ governs the persistence of expected consumption growth x_t, and ν determines the persistence of the conditional aggregate volatility σ_t^2. η_t is a short-run consumption shock, ϵ_t is the shock to the expected consumption growth, and w_{t+1} is the shock to the conditional volatility of consumption growth; for parsimony, these three shocks are assumed to be $i.i.d$ Normal.

[2]See Bansal and Yaron (2004) and Bansal, Kiku, and Yaron (2007a) for a discussion of the long-run risks channels for the asset markets and specifically the role of volatility risks, Bansal, Khatchatrian, and Yaron (2005b) for early extensive empirical evidence on the role of volatility risks, and Eraker and Shaliastovich (2008), Bansal and Shaliastovich (2010), and Drechsler and Yaron (2011), for the importance of volatility risks for derivative markets.

The equilibrium model solution is derived in Bansal and Yaron (2004), and for convenience is reproduced in the Appendix. In particular, the equilibrium solution to the price-consumption ratio, pc_t, is linear in the expected growth and consumption volatility:

$$pc_t = A_0 + A_x x_t + A_\sigma \sigma_t^2, \tag{3.4}$$

and the innovation into the stochastic discount factor is determined by the short-run, expected consumption and the volatility news:

$$m_{t+1} - E_t m_{t+1} = -\lambda_c \sigma_t \eta_{t+1} - \lambda_x \varphi_e \sigma_t \epsilon_{t+1} - \lambda_\sigma \sigma_w w_{t+1}. \tag{3.5}$$

The equilibrium loadings for the price-consumption ratio A_0, A_x and A_σ, and the market prices of risks λ_c, λ_x and λ_σ depend on the preference parameters and the consumption dynamics, and are provided in the Appendix. In particular, when IES is bigger than one, positive shocks to expected consumption increase immediate and future expected consumption return news ($A_x > 0$), while positive shocks to consumption volatility decrease immediate consumption return shocks and increase future expected return shocks ($A_\sigma < 0$).

Given the model solution, we can provide explicit expressions for the immediate consumption returns news, $N_{R,t+1}$, the discount rate shocks, $N_{DR,t+1}$, and the volatility news shocks, $N_{V,t+1}$, in terms of the underlying economic structure.

The consumption return shock, $N_{R,t+1}$ is driven by all three shocks in the economy,

$$N_{R,t+1} = A_x \kappa_1 \varphi_x \sigma_t \epsilon_{t+1} + A_\sigma \kappa_1 \sigma_w w_{t+1} + \sigma_t \eta_{t+1}, \tag{3.6}$$

while the discount rate shocks, $N_{DR,t+1}$ is driven only by the expected growth and volatility innovations:

$$N_{DR,t+1} = \frac{1}{\psi} \frac{\kappa_1}{1 - \kappa_1 \rho} \varphi_e \sigma_t \epsilon_{t+1} - \kappa_1 A_\sigma \sigma_w w_{t+1}. \tag{3.7}$$

The economic volatility component, V_t, is directly related to the conditional variance of consumption growth:

$$V_t = \frac{1}{2} Var_t(r_{c,t+1} + m_{t+1}) = const + \frac{1}{2} \chi (1 - \gamma)^2 \sigma_t^2, \tag{3.8}$$

where the proportionality parameter χ is provided in the Appendix. Therefore, the innovation into the future expected volatility $N_{V,t+1}$ satisfies

$$N_{V,t+1} = \frac{1}{2} \chi (1 - \gamma)^2 \frac{\kappa_1}{1 - \kappa_1 \nu} \sigma_w w_{t+1}. \tag{3.9}$$

Notably, under the model restrictions, the volatility parameter χ is unambiguously positive and is equal to the ratio of variances of the long-run cash flows news, $N_{CF,t+1}$, to the immediate consumption news, $N_{C,t+1}$:

$$\chi = \frac{Var(N_{CF,t+1})}{Var(N_{C,t+1})}. \tag{3.10}$$

This restriction is useful in identifying χ in the empirical work.

Notice that all the three shocks, $N_{R,t+1}$, $N_{DR,t+1}$ and $N_{V,t+1}$, are correlated with each other as they depend on the underlying shocks in the economy. In particular, if IES is above one, the discount rate shocks and the volatility shocks are positively correlated, because the volatility is driving the risk premium which is an important component of discount rate innovations.

The expression for consumption innovations, $N_{C,t+1}$, and the stochastic discount factor can now be written in terms of the innovations to the consumption return, discount rate and volatility shocks, as shown in Equations (2.7) and (2.12):

$$\begin{aligned} c_{t+1} - E_t(c_{t+1}) &= N_{R,t+1} + (1-\psi)N_{DR,t+1} + \frac{\psi-1}{\gamma-1}N_{V,t+1} \\ m_{t+1} - E_t m_{t+1} &= -\gamma N_{R,t+1} - (\gamma-1)N_{DR,t+1} + N_{V,t+1}. \end{aligned} \tag{3.11}$$

Under the null of the model, the consumption shock is equal to $\sigma_t \eta_{t+1}$, and the innovation into the stochastic discount factor matches the expression in Equation (3.5).

It is important to recognize that volatility innovations are relevant for the correct inference on the consumption return, discount rate, and volatility shocks, as long as ψ is different from 1 (as $A_\sigma \neq 0$). Ignoring the volatility component can distort the measurement of $N_{R,t+1}$, and $N_{DR,t+1}$, and $N_{V,t+1}$. Even if the return news $N_{R,t+1}$ and $N_{DR,t+1}$ could be correctly estimated using flexible specification in the data, it is clear from equation (3.11) that the economic implications about the consumption innovations and the stochastic discount factor can be very misleading when the volatility channel is ignored.

To analyze the volatility risks implications for the equity returns, we introduce a generic dividend process,

$$\Delta d_{t+1} = \mu_d + \phi x_t + \pi \sigma_t \eta_{t+1} + \varphi_d \sigma_t u_{d,t+1}. \tag{3.12}$$

In the Appendix it is shown that under the model, the innovation in the dividend asset return can be written in terms of the future expected cash flow, consumption return, and volatility news:

$$(E_{t+1} - E_t) r_{d,t+1} = \beta_{CF} N_{CF,t+1} + \beta_{DR} N_{DR,t+1} + \beta_V N_{V,t+1} + \varphi_d \sigma_t u_{d,t+1}, \tag{3.13}$$

where $\varphi_d \sigma_t u_{d,t+1}$ is independent of the fundamental shocks, and β_{CF}, β_{DR}, and β_V capture the sensitivity of return to cash-flow, discount rate, and volatility news, respectively. These betas depend on the parameters governing the consumption and dividend dynamics; in particular, for reasonable parameter values considered in the literature, cash-flow betas are positive, $\beta_{CF} > 0$, while betas to discount rate and volatility news are negative: $\beta_{DR} < 0$ and $\beta_V < 0$.

One potential important aspect in conducting empirical work is the fact the consumption return itself is not observed and therefore the market return is often used instead. The discrepancy between these two assets can exacerbate the distortions and economic inference problems described above. In the next section we quantify these various issues in turn.

4 Volatility Risks and Mis-Measurement of Consumption Innovation

In this section we evaluate, based on the calibration of the the long-run risks model, the extent to which consumption innovations are mis-measured if one ignores the presence of volatility. The parameter configuration used in the model simulation is similar to Bansal, Kiku, and Yaron (2009b) and is given in Table 1. The model reproduces key asset market and consumption moments of the data and thus provides a realistic laboratory for our analysis – Table 2 reports these moments. Notice that the model produces a significant positive correlation between the discount rate news and the volatility news: it is 60% for the consumption asset, and 90% for the market. Further, for both consumption and market return, most of the risk compensation comes from the cash-flow and volatility news, while the contribution of the discount rate news is quite small. In terms of the sensitivity of asset return to the underlying sources of risks, note that the cash-flow beta is positive, while the discount rate and volatility betas are both negative in the model.

Table 3 reports the implied consumption innovations when volatility is ignored, that is when the term N_V is not accounted for in constructing the consumption innovations. In constructing the implied consumption innovations via equation (2.7) we use the analytical expressions for $N_{R,t+1}$, $N_{DR,t+1}$, and $N_{V,t+1}$ which are given in Equations (3.6), (3.7), and (3.9), respectively. In particular, we assume that the consumption return news $N_{R,t+1}$ and $N_{DR,t+1}$ can be identified correctly even if the volatility component is ignored, and we focus only on the mis-specification caused by an omission of the volatility news $N_{V,t+1}$. We consider the implications of the volatility news for the measurements of the consumption return innovations, $N_{R,t+1}$, $N_{DR,t+1}$ in the subsequent section.

Table 3 shows that when IES is not equal to one, the implied consumption innovations are distorted. In particular, when IES is equal to two, the volatility of consumption innovations is about twice that of the true consumption innovations. Furthermore, when volatility is ignored, the correlation between the true consumption shock and the implied consumption shock is only 0.5. In addition, the correlation of the implied consumption innovation and the discount rate and volatility news are very negative while in the model they should be zero when volatility is correctly accounted for. Similar distortions are present when the IES is less than one albeit by a smaller magnitude. Notably, Panel B of Table 3 confirms that when the model has no stochastic volatility, and thus constant risk premia, the implied consumption innovations and the true ones coincide for all IES values. In Table 4 we report the implications of ignoring volatility for the stochastic discount factor. When volatility is ignored, for all values of the IES the SDF's volatility is downward biased by about one-third. The market risk premium is almost half that of the true one, and the correlations of the SDF with the return, discount rate, and cash-flow news are distorted. Finally, it is important to note that even when the IES is equal to one, the SDF is still misspecified. In all, the evidence clearly demonstrates the potential pitfalls that might arise in interpreting asset pricing models and the asset markets sources of risks if the volatility channel is ignored.

The analysis above assumed the researcher has access to the return on wealth, $r_{c,t+1}$. In many instances, however, that is not the case (e.g., Campbell and Vuolteenaho (2004), Campbell (1996)) and the return on the market $r_{d,t+1}$ is utilized instead. In Table 5 we repeat the analysis above, except that $r_{d,t+1}$ replaces $r_{c,t+1}$ in the stochastic discount factor, and hence in the construction of N_R, N_{DR}, and N_V. The fact the market return is a levered asset relative to the consumption/wealth return exacerbate the inference problems shown earlier. In particular, Table 5 shows that when the IES is equal to two, the volatility of the implied consumption shocks is about 14.3%, relative to the true volatility of only 2.5%. Moreover, the correlation structure with various shocks is distorted in a significant manner. The correlation between the implied consumption shocks and the discount rate shocks and volatility shocks are very negative (in the model they should be zero), while the correlation with the immediate return shock is almost one whereas the true correlation should be 0.45. It is interesting to note that now even when IES is equal to one the consumption innovation shocks are misspecified. The columns marked 'Mkt vol' correspond to the case in which N_V is included in the definition of N_C but $r_{d,t+1}$ is used in the definition of V_t. The small difference between the case of ignoring volatility altogether and the case in which volatility is included but is based on the market return, indicates that much of the misspecification arise in the construction of the return and discount rate innovations, N_R, and N_{DR} respectively. The market return, being a levered return relative to the consumption return, yields much too volatile implied consumption innovations. Further, the distinction between $r_{d,t+1}$ and $r_{c,t+1}$ leads to a distorted innovation structure even when the underlying economy has constant volatility (see Panel B of Table 5).

Campbell (1996) (Table 9) reports the implied consumption innovations based on equation (2.7) when volatility is ignored and the return and discount rate shocks are read off a VAR using observed financial data. The volatility of the consumption innovations when the IES is assumed to be 2 is about 22%, not far from the quantity displayed in our simulated model in Table 5.[3] As in our case, lower IES values lead to somewhat smoother implied consumption innovations. While Campbell (1996) concludes that this evidence is more consistent with a low IES, the analysis here suggests that in fact this evidence is consistent with an environment in which the IES is greater than one and the innovation structure contains a volatility component.

5 Volatility Risks, Consumption and Labor Income

In this section we develop and implement an econometric framework to quantify the role of the volatility channel for the asset markets. As the consumption return is not directly observed in the data, we follow Lustig and Van Nieuwerburgh (2008) and Campbell (1996) and assume that it is equal to the weighted average of the return to the stock market and the return to human capital. This allows us to adopt a standard VAR-based methodology to extract the underlying innovations to consumption return and volatility, construct the implied shocks into consumption and stochastic discount factor, and assess the importance of the volatility channel for the inference about the returns to the human capital, the market and the wealth portfolio, as well as the size and value risk premia.

5.1 Econometric Specification

Let X_t a vector of state variables, which includes the real market return $r_{d,t}$, consumption growth rate Δc_t, labor income growth Δy_t, market price-dividend ratio pd_t, and the measure of the realized variance of aggregate consumption RV_t which we discuss in detail later:

$$X_t = \begin{bmatrix} r_{d,t} & \Delta c_t & \Delta y_t & pd_t & RV_t \end{bmatrix}'. \tag{5.1}$$

For parsimony, we focus on a minimal set of economic variables in our empirical analysis. We have checked that our results do not materially change if the vector X_t is extended to include other predictors, such as interest rate, term and default

[3]The data used in Campbell (1996) is from 1890-1990 which leads to slightly higher volatility numbers than the calibrated model produces.

spread, etc. We also entertained a co-integrating specification between consumption and labor income, which produced similar results.

We assume that the vector of state variables X_t follows an unrestricted VAR(1) specification:

$$X_{t+1} = \mu_X + \Phi X_t + u_{t+1}, \tag{5.2}$$

where Φ is a persistence matrix, μ_X is an intercept, and u_{t+1} is a vector of Normal shocks with mean zero and variance-covariance matrix Ω. As shown below, this specification allows us to extract the volatility news and the immediate and future expected revisions into the returns in a convenient way.

The key novel element in the state vector X_t is the realized variance RV_t, which is based on the square of consumption residual:

$$RV_t = (\Delta c_t - E(\Delta c))^2, \tag{5.3}$$

so that expectations of RV_{t+1} implied by the dynamics of the state vector capture the ex-ante uncertainty about future consumption in the economy; this way of extracting conditional aggregate volatility is similar to Bansal et al. (2005b), Bansal, Kiku, and Yaron (2007b), among others. Following the derivations in Section 3, the economic volatility V_t is assumed to be proportional to the ex-ante the expectation of the realized variance RV_{t+1} from the VAR(1):

$$\begin{aligned} V_t &= V_0 + \frac{1}{2}\chi(1-\gamma)^2 E_t RV_{t+1} \\ &= V_0 + \frac{1}{2}\chi(1-\gamma)^2 i_v' \Phi X_t, \end{aligned} \tag{5.4}$$

where V_0 is an unimportant constant which disappears in the expressions for shocks, i_v is a column vector which picks out the realized variance measure from X_t, and χ is a parameter which captures the link between the observed aggregate consumption volatility and V_t. In the model with volatility risks, we fix the value of χ to the ratio of the variances of the cash-flow to immediate consumption news, consistent with the restriction in Equation (3.10). In the specification where volatility risks are absent, the parameter χ is set to zero.

Following the above derivations, the revisions in future expectations of the economic volatility can be calculated in the following way:

$$N_{V,t+1} = \frac{1}{2}\chi(1-\gamma)^2 i_v' Q u_{t+1}, \tag{5.5}$$

where Q is the matrix of the long-run responses, $Q = \kappa_1 \Phi \left(I - \kappa_1 \Phi\right)^{-1}$.

The VAR specification implies that the shocks into immediate market return, $N^d_{R,t+1}$, and future market discount rate news, $N^d_{DR,t+1}$, are given by[4]

$$N^d_{R,t+1} = i'_r u_{t+1}, \quad N^d_{DR,t+1} = i'_r Q u_{t+1}, \tag{5.6}$$

where i_r is a column vector which picks out market return component from the set of state variables X_t; that is, i_r has 1 in the first row and zeros everywhere else.

While the market return is directly observed and the market return news can be extracted directly from the VAR(1), in the data we can only observe the labor income but not the total return on human capital. We make the following identifying assumption, identical to Lustig and Van Nieuwerburgh (2008), that expected labor income return is linear in the state variables:

$$E_t r_{y,t+1} = \alpha + b' X_t, \tag{5.7}$$

where b captures the loadings of expected human capital return to the economic state variables. Given this restriction, the news into future discounted human capital returns, $N^y_{DR,t+1}$, are given by,

$$N^y_{DR,t+1} = b' \Phi^{-1} Q u_{t+1}, \tag{5.8}$$

and the immediate shock to labor income return, $N^y_{R,t+1}$, can be computed as follows:

$$\begin{aligned} N^y_{R,t+1} &= (E_{t+1} - E_t) \left(\sum_{j=0}^{\infty} \kappa_1^j \Delta y_{t+j+1} \right) - N^y_{DR,t+1} \\ &= i'_y (I + Q) u_{t+1} - b' \Phi^{-1} Q u_{t+1}, \end{aligned} \tag{5.9}$$

where the column vector i_y picks out labor income growth from the state vector X_t.

To construct an aggregate wealth return, following Lustig and Van Nieuwerburgh (2008) and Campbell (1996), Lettau and Ludvigson (2001) among others, we make the assumption that the consumption return is given by the weighted average of the returns to human capital and the stock market:

$$r_{c,t} = (1 - \omega) r_{d,t} + \omega r_{y,t}. \tag{5.10}$$

The share of human wealth in total wealth ω is assumed to be constant. It immediately follows that the immediate and future discount rate news on the consumption

[4] In what follows, we use superscript "d" to denote shocks to the market return, and superscript "y" to identify shocks to the human capital return. Shocks without the superscript refer to the consumption asset, consistent with the notations in Section 2.

asset are equal to the weighted average of the corresponding news to the human capital and market return, with a weight parameter ω :

$$
\begin{aligned}
N_{R,t+1} &= (1-\omega)N^d_{R,t+1} + \omega N^y_{R,t+1}, \\
N_{DR,t+1} &= (1-\omega)N^d_{DR,t+1} + \omega N^y_{DR,t+1}.
\end{aligned}
\tag{5.11}
$$

These consumption return innovations can be expressed in terms of the VAR(1) parameters and shocks and the vector of the expected labor return loadings b following Equations (5.6)-(5.9).

Finally, we can combine the expressions for the volatility news, immediate and discount rate news on the consumption asset to back out the implied immediate consumption shock following the Equation (2.7):

$$
\begin{aligned}
c_{t+1} - E_t c_{t+1} &= N_{R,t+1} + (1-\psi)N_{DR,t+1} + \frac{\psi-1}{\gamma-1}N_{V,t+1} \\
&= \underbrace{\left[(1-\omega)i_r'Q + \omega(i_y'(I+Q) - b'\Phi^{-1}Q)\right] u_{t+1}}_{N_{R,t+1}} \\
&\quad + (1-\psi)\underbrace{\left[(1-\omega)i_r'Q + \omega b'\Phi^{-1}Q\right] u_{t+1}}_{N_{DR,t+1}} + \left(\frac{\psi-1}{\gamma-1}\right)\underbrace{\frac{1}{2}\chi(1-\gamma)^2 i_v' Q u_{t+1}}_{N_{V,t+1}} \\
&\equiv q(b)' u_{t+1}.
\end{aligned}
\tag{5.12}
$$

The vector $q(b)$ defined above depends on the model parameters, and in particular, it depends linearly on the expected labor return loadings b. On the other hand, as consumption growth itself is one of the state variables in X_t, it follows that the consumption innovation satisfies,

$$
c_{t+1} - E_t c_{t+1} = i_c' u_{t+1}, \tag{5.13}
$$

where i_c is a column vector which picks out consumption growth out of the state vector X_t. We impose this important consistency requirement that the model-implied consumption shock in Equation (5.12) matches the VAR consumption shock in (5.13), so that

$$
q(b) \equiv i_c, \tag{5.14}
$$

and solve the above equation, which is linear in b, to back out the unique expected human capital loadings b. That is, in our approach the specification for the expected labor return ensures that the consumption innovation implied by the model is identical the consumption innovation in the data. This can be compared to the approach in Lustig and Van Nieuwerburgh (2008) who numerically estimate the loading b to match a few selected moments of the model-implied consumption shock in the data.

5.2 Data and Estimation

In our empirical analysis, we use an annual sample from 1930 to 2010. Real consumption corresponds to real per capita expenditures on non-durable goods and services, and real income is the real per capita disposable personal income; both series are taken from the Bureau of Economic Analysis. Market return data is for a broad portfolio from CRSP. The realized consumption variance measure is constructed from the demeaned squares of real consumption, according to the Equation (5.3).

The summary statistics for these variables are presented in Table 6, and their time-series plots are depicted on Figure 3. The average labor income and consumption growth rate is about 2%. The labor income is more volatile than consumption growth, but the two series co-move quite closely in the data with the correlation coefficient of 0.80. The average log market return is 5.7%, and its volatility is almost 20%. The realized consumption variance is quite volatile in the data, and spikes up considerably in the recessions, as evident from Figure 3. Notably, the realized variance is negatively correlated with the price-dividend ratio: the correlation coefficient is about -0.30, which is consistent with a high (bigger than one) value of the IES parameter ψ.

The estimation results for the unrestricted VAR(1) specification are reported in Table 7. It is hard to interpret individual slope coefficients due to the correlations between all the variables, and quite a few of the slope coefficients are imprecisely estimated. Overall, future consumption, labor income and equity prices are expected to increase following positive shocks to the labor income, and decrease following a rise in aggregate consumption volatility. Fall in consumption and labor income, market returns and prices predicts an increase in the ex-ante volatility in the economy. The adjusted R^2 in these regressions vary from 4% for the market return to nearly 80% for the price-dividend ratio. Notably, the consumption growth is quite predictable with this rich setting, and the R^2 reaches almost 60%.

To highlight the significance of the volatility risks, we estimate an impulse response of the state variables to a one standard deviation shock in ex-ante consumption volatility, $E_t RV_{t+1}$; see Appendix for the details of the computations. One standard deviation volatility shock corresponds to an increase in ex-ante consumption volatility from its mean of $(2.2\%)^2$ to $(3.1\%)^2$. As shown in Figure 1, real consumption growth significantly declines by 0.7% following an increase the impact of volatility news, and remains negative up to three years in the future. The response of the labor income growth is similar and it declines by 1.1%. As for the asset markets, volatility shocks have a significant and persistent negative effect on price-dividend ratio, and significant positive effect on market equity premium –for brevity the graphs are omitted. Overall, volatility risks have considerable effect on the consumption and asset prices in the data, which motivates our analysis.

5.3 Labor, Market and Wealth Return Correlations

To derive the implications for the market, human capital, and wealth portfolio returns, we set the risk aversion coefficient γ to 6.5, and the IES parameter ψ to 2.5; we examine the sensitivity of model results to the preference parameters in our subsequent discussion. We fix the share of human wealth in the overall wealth ω to 0.792, as in Lustig and Van Nieuwerburgh (2008).

Table 8 reports the model-implied correlation structure between market, human capital and wealth portfolio returns. Without the volatility channel, shocks to market and human capital returns are significantly negatively correlated, which is consistent with the evidence in Lustig and Van Nieuwerburgh (2008). Indeed, as shown in the top panel of the Table, the correlation of immediate news to returns, $N^d_{R,t+1}$ and $N^y_{R,t+1}$, is -0.50; it is -0.78 for the discount rate news, $N^d_{DR,t+1}$ and $N^y_{DR,t+1}$, and it is -0.65 when we consider the future long-term (5-year) expected returns, $E_t r^d_{t\to t+5}$ and $E_t r^y_{t\to t+5}$. All these correlations turn positive when the volatility channel is present: the correlation of immediate return news increases to 0.36; for discount rate to 0.25, and for the expected 5-year returns to 0.51. Figure 4 plots the implied time-series of long-term expected returns on the market and human capital. A negative correlation between the two series is evident in the model specification which ignores volatility risks.

These effects for the co-movements of returns are also similar for the wealth and labor, and the market and wealth returns, as shown in the middle and lower panels of Table 8. Because the wealth return is a weighted average of the market and human capital returns, these correlations are in fact positive without the volatility channel, but the correlations become considerably larger and closer to one once the volatility risks are introduced. For example, all the correlations between the market and wealth returns increase to 90% with the volatility channel, while they are between 0 and 50% without it.

To understand conceptually the role of the volatility risks for these effects, it is helpful to re-write the consumption restriction in the Equation (2.7) in the following way:

$$N_{CF,t+1} - N_{C,t+1} = \psi N_{DR,t+1} - \frac{\psi - 1}{\gamma - 1} N_{V,t+1}. \qquad (5.15)$$

Hence, the revisions about future expected consumption, $N_{CF,t+1} - N_{C,t+1} \equiv (E_{t+1} - E_t)\left(\sum_{j=1}^{\infty} \kappa_1^j \Delta c_{t+j+1}\right)$, are positively related to the discount rate news to the wealth portfolio, and if $\psi > 1$, are negatively related to the news about future economic volatility. In the model without the volatility channel, $N_{V,t+1} = 0$, so all the revisions in the future expected consumption have to be proportional to the discount rate news on the wealth portfolio, magnified by the IES parameter ψ. However, empirically, the volatility of the expected consumption news is much smaller than the volatility of the discount rate news on the market, $N^d_{DR,t+1}$, which is one of the components of the discount rate news on the wealth portfolio (recall that $N_{DR,t+1} = (1-\omega)N^d_{DR,t+1} + \omega N^y_{DR,t+1}$). This means that the discount rate news on human capital must offset a large portion of the discount rate news on the market, which manifests itself in a strong negative correlation between market and labor return news documented in Table 8.

On the other hand, in the model with volatility risks, the variance of future expected consumption news depends on the variance of discount rate news, the variance of volatility news, as well as the covariance between the discount rate and the volatility news:

$$\begin{aligned} Var(N_{CF,t+1} - N_{C,t+1}) = \psi^2 Var(N_{DR,t+1}) + \left(\frac{\psi-1}{\gamma-1}\right)^2 Var(N_{V,t+1}) \\ - 2\psi\frac{\psi-1}{\gamma-1} Cov(N_{DR,t+1}, N_{V,t+1}). \end{aligned} \tag{5.16}$$

The discount rate and the volatility news are strongly positively correlated in the economic models and data (see calibrated model output in Table 2 and subsequent discussion). Thus, when IES is above one, the covariance term in the above equation can substantially reduce the right-hand side, which allows to the model to match the volatility of cash-flow news without forcing a negative correlation between the labor and market return dynamics.

5.4 Risk Sources and Risk Compensation

The model-implied current and future discounted consumption news, discount rate news on the wealth portfolio, and the volatility news are plotted on Figures 5-7, and the volatilities and cross-correlations for these shocks are shown in Table 11.

The future cash-flow news $N_{CF,t+1}$ and the immediate consumption news $N_{C,t+1}$ remain the same in the model specifications with and without the volatility channel. Indeed, in our approach the immediate consumption news from the model, $N_{C,t+1}$, are matched exactly to their VAR counterpart in the data. Similarly, future discounted cash-flow news, $N_{CF,t+1}$, under the model are equal to the weighted average of the future expected labor news and future expected dividend news, which are also extracted directly from the VAR. The cash-flow news are strongly counter-cyclical: the correlation of future discounted cash-flow news with NBER recession indicator is -22%, and it is -13% for the immediate consumption news. The future cash-flow news are more volatile than the immediate consumption innovations, and they drop significantly in the recessions. On average, future expected consumption growth is revised down by 1.3% in recessions, and these revisions in future expectations can go as low as -8.8% and -9.7% in the recessions of 2008 and 1974, and -22% in 1932.

The discount rate news on the wealth portfolio, in the model specifications with and without the volatility channel, are plotted on Figure 6. The volatility channel has a large impact on the measurement of the discount rate news. The two discount rates in the models with and without the volatility are virtually uncorrelated correlated with each other: the correlation coefficient is 0.09 for the whole sample and it drops to -0.05 post-war. In the model with volatility risks, the discount rates are more volatile and are strongly related to the volatility news, $N_{V,t+1}$. Indeed, the correlation of the volatility news with the discount rate news on the consumption asset is 0.84; it is 0.47 for the discount rate on human capital return and 0.79 for the market. These findings are consistent with the intuition of the long-run risks model where a significant component of the discount rate news comes from the volatility channel (see Section 3). On the other hand, without the volatility channel, the discount rate news no longer reflect the fluctuations in the volatility, but rather mirror the revisions in future expectations of consumption: as shown in Equation (5.15), when volatility channel is absent, the discount rate news become proportional to future consumption shocks $N_{CF} - N_C$. Consistent with this result, in the case without volatility news the correlation of discount rate news and the current and future cash flow news N_{CF} is 0.98, as documented in Table 11. The discount rate news exhibit quite a different time-series behavior in the models with and without volatility risks, as depicted in Figure 6. In the model with volatility risks, discount rate news on average are positive in the recessions: for example, the discount rate news are 6.4% in the latest recession of 2008. Without the volatility channel, however, it would appear that the discount rate news are negative at those times: the measured discount rate shock is -2.7% in 2008. Thus, ignoring the volatility channel, the discount rate on the wealth portfolio can be significantly biased due to the omission of the volatility component, which would alter the interpretation of the fundamental risk sources in the market.

The volatility news are plotted in Figure 7. The volatility news are quite volatile and strongly counter-cyclical, especially post-war. For example, In the last recession of 2008 the volatility news increased dramatically to 82%. In the model with volatility, volatility news drive a significant portion of the discount rate news and the innovations in the stochastic discount factor: as shown in Table 11, the correlations of future volatility news with discount rate shocks on the consumption asset and SDF are 0.84 and 0.87, respectively.

We use the extracted news components to identify the innovation into the stochastic discount factor, according to the Equation (2.13), and document the implications for the risk premia in Table 9. At our calibrated preference parameters, in the model with volatility, the risk premium on the market is 9.70%; it is 4.04% for the wealth portfolio, and 2.55% for the labor return. Without the volatility channel, the risk premia drop to 3.49%, 1.34%, and 0.78%, respectively. The contribution of the volatility risks to the overall risk premia vary from about one-third for the human capital, to about a half for the wealth portfolio and the market. These numbers are generally consistent with the output from the calibrated LRR model (see Table 2).

While the main results in the paper are obtained with preference parameters $\gamma = 6.5$ and $\psi = 2.5$, in Table 10 we document the model implications for the range of risk aversion (5, 6.5 and 8) and IES (from 0.5 to 3.0) parameters. Without the volatility channel, the correlations between labor and market returns are all negative at all considered values for the preference parameters, which is consistent with the evidence in Lustig and Van Nieuwerburgh (2008). The risk premia on the assets increases with the risk aversion and the IES. In the model with volatility risks, it is evident that one requires IES sufficiently above one to generate a positive link between labor and market returns – with IES below one these correlations are even lower than in the case without volatility risks. A higher than one value of the IES is also required to capture the drop in price-consumption ratio on the impact of volatility news. Indeed, as shown by the impulse response graphs in Figure 2, the model-implied price-consumption ratio declines in response to a rise in ex-ante volatility when $\psi = 2.5$; however, when $\psi < 1$, it increases in response to a rise in ex-ante consumption volatility. High values for risk aversion and IES also lead to high implied risk premium, that is why we chose moderate values of γ and ψ to explain positive correlation between labor and market returns, and generate market risk premium close to the data.

6 Market-based VAR Approach

To further highlight the importance of the volatility channel for understanding the dynamics of asset prices, we use a market-based VAR approach to news decomposition. As frequently done in the literature, here, we assume that the wealth portfolio corresponds to the aggregate stock market and extract the underlying risks in a GMM framework that exploits both time-series and cross-sectional moment restrictions.

6.1 Market-Based Setup

We describe the state of the economy by vector:

$$X_t \equiv (RV_{r,t},\ \Delta d_t,\ pd_t,\ r_{f,t},\ ts_t,\ ds_t)'$$

that comprises the realized volatility of the aggregate market portfolio ($RV_{r,t}$), continuously compounded dividend growth rates (Δd_t) and the log of the price-dividend ratio (pd_t) of the aggregate market, the log of the risk-free rate ($r_{f,t}$), the term spread (ts_t) defined as a difference in yields on the 10-year Treasury bond and three-month T-bill, and the yield differential between Moody's BAA- and AAA-rated corporate bonds (ds_t). The data are real, sampled on an annual frequency and span the period from 1930 till 2010. The realized volatility is constructed by summing up squared monthly real rates of return within a year. The real interest rate is measured by the yield on the 10-year Treasury bond adjusted by inflation expectations.

We model the dynamics of X_t via a first-order vector-autoregression and construct cash-flow, discount-rate and volatility news by iterating on the VAR. We use the same algebra as in Section 5.1 with a simplification that all the news components are now directly read from the VAR since the return on the market is assumed to represent the return on the overall wealth. We use the extracted news to construct the innovation in the stochastic discount factor and price a cross-section of equity returns by exploiting the Euler equation, i.e.,

$$E_t\big[r_{i,t+1} - r_{ft}\big] + \frac{1}{2}Var_t(r_{i,t+1}) = -Cov\big(m_{t+1} - E_t m_{t+1},\ r_{i,t+1} - E_t r_{i,t+1}\big), \tag{6.1}$$

where $r_{i,t+1} - E_t r_{i,t+1}$ is the innovation into asset-i return.

To extract return innovations for the cross section, we use an econometric approach similar to Bansal, Dittmar, and Lundblad (2005a) and Bansal, Dittmar, and Kiku (2009a) that allows for a sharper identification of long-run cash-flow risks in asset returns. In particular, for each equity portfolio, we estimate its long-run cash-flow exposure (ϕ_i) by regressing portfolio's dividend growth rate on the three-year moving average of the market dividend growth:

$$\Delta d_{i,t} = \mu_i + \phi_i \overline{\Delta d}_{t-2\to t} + \epsilon^d_{i,t}\ , \tag{6.2}$$

where $\Delta d_{i,t}$ is portfolio-i dividend growth, $\overline{\Delta d}_{t-2\to t}$ is the average growth in market dividends from time $t-2$ to t, and $\epsilon^d_{i,t}$ denotes idiosyncratic portfolio news. Using the log-linearization of return:

$$r_{i,t+1} = \kappa_{i,0} + \Delta d_{i,t+1} + \kappa_{i,1} z_{i,t+1} - z_{i,t}\ , \tag{6.3}$$

the innovation into asset-i return is then given by:

$$r_{i,t+1} - E_t r_{i,t+1} = \phi_i(\Delta d_{t+1} - E_t \Delta d_{t+1}) + \epsilon^d_{i,t+1} + \kappa_i \epsilon^z_{i,t+1}\ , \tag{6.4}$$

where $z_{i,t}$ is the price-dividend ratio of portfolio i, $\kappa_{i,0}$ and $\kappa_{i,1}$ are portfolio-specific constants of log-linearization, $(\Delta d_{t+1} - E_t \Delta d_{t+1})$ is the VAR-based innovation in the market dividend growth rate, and $\epsilon^z_{i,t+1}$ is the innovation in the portfolio price-dividend ratio obtained by regressing $z_{i,t+1}$ on the VAR state variables. We use the extracted innovation in the portfolio return to construct the risk-premium restriction given in equation (6.1).[5]

[5] Our empirical results remain similar if instead we rely on the cointegration-based specification of Bansal et al. (2009a).

To extract the news and construct the innovation in the stochastic discount factor, we estimate time-series parameters and the coefficient of risk aversion using GMM by exploiting two sets of moment restrictions. The first set of moments comprises the VAR orthogonality moments; the second set contains the Euler equation restrictions for the market portfolio and a cross-section of five book-to-market and five size sorted portfolios. To ensure that the moment conditions are scaled appropriately, we weight each moment by the inverse of its variance and allow the weights to be continuously up-dated throughout estimation.

The cross-sectional implications of the GMM estimation are given in Panel A of Table 12. The table presents sample average excess returns on the market portfolio and the cross section, risk premia implied the market-based VAR, and the contribution of cash-flow, discount-rate and volatility risks to the overall premia. The evidence reported in the table yields several important insights. First, we find that cash-flow risks play a dominant role in explaining both the level and the cross-sectional variation in risk premia. At the aggregate market level, cash-flow risks account for 4.8% or, in relative terms, for about 60% of the total risk premium. The contribution of cash-flow risks to risk premia is monotonically increasing in book-to-market characteristics and is monotonically declining with size. Value and small stocks in the data are more sensitive to persistent cash-flow risks than are growth and large firms, which is consistent with the evidence in Bansal et al. (2005a), Hansen, Heaton, and Li (2008) and Bansal et al. (2009a). Second, we find that discount-rate and volatility risks, each, account for about 20% of the overall market risk premium, and seem to affect the cross section of book-to-market sorted portfolios in a similar way. Both discount-rate and volatility risks matter more for the valuation of growth firms than that of value firms.

Our estimation evidence shows that discount-rate and volatility risks share similar dynamics in time series. Both tend to increase during recessions and decline during economic expansions; the correlation between discount-rate(volatility) news and the NBER-dated business cycle indicator is -0.27(-0.30). Consistent with the results reported in Table 11, we find that discount-rate and volatility news implied by the market-based VAR are strongly positively correlated. This evidence aligns well with economic intuition. As the contribution of risk-free rate news is generally small, discount-rate risks are mostly driven by news about future risk premia, and the latter is tied to expectations about future economic uncertainty. While theoretically sound, the documented tight link between discount-rate and volatility news makes it hard to fully understand the (distinct) contribution of volatility risks in the current setup that is free of any structural economic restrictions.

6.2 Incorporating Restrictions on Risk-Premia Variation

To facilitate the interpretation of risks and identify the role of the volatility channel, we make the following assumption:

$$E_t[r_{t+1} - r_{f,t}] = \alpha_0 + \alpha\sigma^2_{r,t} \ . \tag{6.5}$$

That is, we assume that risk premia in the economy are driven by the conditional variance of the market return, $\sigma^2_{r,t} \equiv Var_t(N_{R,t+1})$. We can now re-write the innovation into the stochastic discount factor in terms of cash-flow news, risk-free rate news and long-run news in $\sigma^2_{r,t}$. In particular, using the definition of V_t and the dynamics of the SDF (see equations (2.5) and (2.13)):

$$\begin{aligned} V_t &= \frac{1}{2}Var_t\big(m_{t+1} + r_{t+1}\big) = \frac{1}{2}Var_t\big(-\gamma N_{CF,t+1} + N_{DR,t+1} + N_{V,t+1} + N_{R,t+1}\big) \qquad (6.6) \\ &= \frac{1}{2}Var_t\big(-\gamma(N_{R,t+1} + N_{RP,t+1} + N_{RF,t+1}) + N_{RP,t+1} + N_{RF,t+1} + N_{V,t+1} + N_{R,t+1}\big) \\ &\approx 0.5(1-\gamma)^2\sigma^2_{r,t} \ . \end{aligned}$$

Note that the second line in equation (6.6) makes use of the decomposition of discount-rate news into risk-premia (N_{RP}) and risk-free rate (N_{RF}) news, and the last line exploits assumption (6.5) and homoscedasticity of volatility shocks. Since variation in the risk-free rate in the data is quite small, we ignore its contribution to the conditional variance and use equation (6.6) as an approximation. We can now express the innovation in the SDF as:

$$\begin{aligned} m_{t+1} - E_t[m_{t+1}] &= -\gamma N_{CF,t+1} + N_{DR,t+1} + N_{V,t+1} \qquad (6.7) \\ &\approx -\gamma N_{CF,t+1} + N_{RF,t+1} + \big(\alpha + 0.5(1-\gamma)^2\big)N_{\sigma^2,t+1} \ , \end{aligned}$$

where $N_{\sigma^2,t+1} \equiv (E_{t+1}-E_t)\left(\sum_{j=1}^{\infty}\kappa_1^j\sigma^2_{r,t+j}\right)$, and $N_{RF,t+1} \equiv (E_{t+1}-E_t)\left(\sum_{j=1}^{\infty}\kappa_1^j r_{f,t+j}\right)$. Note that if the volatility channel is shut down (i.e., risk premia are constant), the last term of the innovation in the stochastic discount factor disappears.

We exploit the same market-based VAR set-up as earlier. Note that the first equation in the VAR allows us to estimate the dynamics of the conditional variance, $\sigma^2_{r,t}$, which we then use to obtain the estimate of the market risk premium. We continue to rely on GMM in estimation of the VAR parameters, the parameters of the risk-premium dynamics (α_0 and α), and risk aversion. The set of moment restrictions is augmented by the two moments of the risk-premium regression implied by equation (6.5).

Panel B of Table 12 presents the asset pricing implications of the market-based VAR specification that incorporates restrictions on the dynamics of the risk premium. It reports the model-implied premia of the aggregate market and the cross section, and the decomposition of the total compensation into premia for cash-flow, volatility and risk-free rate risks. Consistent with the evidence presented above, cash-flow risks remain the key determinant of the level of the risk premia and its dispersion in the cross section. Still, volatility risks contribute significantly. At the aggregate level, about 2% premia is due to volatility risks, which accounts for almost 30% of the overall market risk premium. At the cross-sectional level, the contribution of volatility risks is fairly uniform across size-sorted portfolios, but displays some tangible heterogeneity in the book-to-market sort. Value firms in the data seem to be quite immune to volatility risks, and therefore carry an almost zero volatility risk premium. Growth firms, on the other hand, are relatively sensitive to news about future economic uncertainty. Overall, the market-based VAR specification accounts for almost 95% of the cross-sectional variation in risk premia, and implies a value premium of 6% and a size premium of about 7%. The estimates of the market prices of cash-flow and volatility risks are both statistically significant. The estimate of risk aversion is 2.85 (SE=0.45), and the estimate of the volatility-risk price is -1.92 (SE=0.91). The model is not rejected by the overidentifying restrictions: the χ^2 test statistic is equal to 6.19 with a p-value of 0.79.[6] Recently, Campbell, Giglio, Polk, and Turley (2011) also consider a market-based CAPM with time-varying volatility and highlight the role of volatility risks. However, they report a negative compensation for volatility risks in the post-1964 sample, which is hard to interpret.

The variance decomposition of the stochastic discount factor reveals that 52% of the overall variation in the SDF is due to cash-flow risks and about 12% is due to volatility risks. While the direct contribution of volatility risks may seem modest, they account for another 32% of the variation in the SDF through their covariation with cash-flow news.[7] Similar to the consumption-based evidence presented in Section 5, cash-flow news rises during expansions and falls in recessions, while news about future uncertainty exhibits strongly counter-cyclical dynamics. Volatility risks have a sizable effect on the dynamics of asset prices. A one-standard deviation increase in volatility news leads to a negative 11% fall in the return of the aggregate market portfolio.

If the volatility channel is shut down, the risk premium is constant and the variation in the stochastic discount factor is driven by cash-flow and risk-free rate news, with cash-flow risks playing a dominant role and explaining almost all the variance of the SDF. Note that when the conditional volatility is time-varying, cash-flow and volatility news are strongly negatively correlated (the correlation between the two time series is about -64%), which adds significantly to the variation in the stochastic discount factor. When the volatility is assumed to be constant, the now-absent co-

[6] Our empirical evidence is fairly robust to economically reasonable changes in the VAR specification, sample period or frequency of the data. For example, if we omit term and default spreads from the VAR, the χ^2 test yields a p-value of 0.71; the estimation of the model using the post-1964 quarterly-sampled data results in the χ^2 test of 8.87 with a corresponding p-value of 0.54.

[7] The remaining part is due to risk-free rate news and its covariation with the other two shocks.

variation channel gets compensated by higher volatility of cash-flow risks (in order to generate enough variation in the SDF). That is, in the homoscedastic specification, cash-flow news fill-in for both cash-flow and volatility risks, which significantly alters the interpretation of the extracted shocks and the implied risk premia.

To summarize, our empirical evidence highlights the importance of the volatility channel in understanding the underlying sources of risks and their identification. We show that revisions in expectations about future volatility contribute significantly to the overall variation in the stochastic discount factor and carry a sizable risk premium.

Conclusions

In this paper we show that volatility is a key and separate source of risk which affect the measurement and interpretation of underlying risks in the economy and financial markets. We show that ignoring volatility can lead to substantial biases in the stochastic discount factor (SDF). Using a calibrated long run risks model we quantify and show that ignoring volatility can have first order implications for the implied consumption innovations, the SDF, and other assets. Specifically, we show that the volatility of the implied consumption shock will be significantly biased upwards in the specification which incorrectly ignores the variation in economic uncertainty. The correlations between the implied consumption innovations and the discount rate and volatility shocks are significantly negative, even though these correlations for the true consumption shock are zero. Ignoring the presence of aggregate uncertainty also biases downward the volatility of the implied stochastic discount factor and the level of the market risk premia.

Using a VAR based approach we show that accounting for volatility leads to a positive correlation between the return to human capital and the market, while this correlation is negative when volatility is ignored. Similarly, the correlations between market and wealth, and wealth and labor returns become closer to one once volatility risks are accounted for. The model implied risk premium for the market portfolio is 9.7%, and it is equal to 4% and 2.6% for the returns to the wealth portfolio and the human capital, respectively. The inclusion of the volatility risks has important implications for the time-series properties of the underlying economic shocks. For example, in the model with volatility risks the implied discount rate news are high and positive in recent recessions of 2001 and 2008, which is consistent with a rise in economic volatility in those periods. The model without the volatility channel, however, produces discount rate news which are negative in those times. In all, this evidence highlights the importance of volatility risks to interpret financial markets and thus leads to consider an asset pricing framework that explicitly incorporates volatility risks.

A Long-Run Risks Model Solution

The discount rate parameters and market prices of risks satisfy

$$
\begin{aligned}
& m_x = -\frac{1}{\psi}, \quad m_\sigma = (1-\theta)(1-\kappa_1\nu)A_\sigma, \quad m_0 = \theta\log\delta - \gamma\mu - (\theta-1)\log\kappa_1 - m_\sigma\sigma_c^2, \\
& \lambda_c = \gamma, \quad \lambda_x = (1-\theta)\kappa_1 A_x, \quad \lambda_\sigma = (1-\theta)\kappa_1 A_\sigma.
\end{aligned} \tag{A.1}
$$

Equilibrium price-to-consumption ratio parameters satisfy

$$
A_x = \frac{1-\frac{1}{\psi}}{1-\kappa_1\rho}, \qquad A_\sigma = (1-\gamma)(1-\frac{1}{\psi})\left[\frac{1+\left(\frac{\kappa_1\varphi_x}{1-\kappa_1\rho}\right)^2}{2(1-\kappa_1\nu)}\right], \tag{A.2}
$$

and κ_1 is the log-linearization parameter.

The equilibrium return on consumption asset in this economy satisfies

$$
r_{c,t+1} = const + \frac{1}{\psi}x_t + A_\sigma(\kappa_1\nu-1)\sigma_t^2 + A_x\kappa_1\varphi_x\sigma_t\epsilon_{t+1} + A_\sigma\kappa_1\sigma_w w_{t+1} + \sigma_t\eta_{t+1}. \tag{A.3}
$$

Using the solution to the equilibrium economy, the proportionality coefficient χ satisfies,

$$
\chi = \left(\frac{\kappa_1\varphi_e}{1-\kappa_1\rho}\right)^2 + 1. \tag{A.4}
$$

The price-dividend ratio satisfies

$$
pd_t = H_0 + H_x x_t + H_\sigma\sigma_t^2, \tag{A.5}
$$

where

$$
H_x = \frac{\phi-\frac{1}{\psi}}{1-\kappa_{1d}\rho}, \quad H_\sigma = \frac{m_s + 0.5((\pi-\gamma)^2 + (\lambda_x - \kappa_{1d}H_x)^2\varphi_e^2 + \varphi_d^2)}{1-\kappa_{1d}\nu}, \tag{A.6}
$$

for a log-linearization parameter κ_{1d}

$$
\log\kappa_{1d} = m_0 + \mu_d + H_\sigma\sigma_0^2(1-\kappa_{1d}\nu) + 0.5(\lambda_\sigma - \kappa_{1d}H_\sigma)^2\sigma_w^2. \tag{A.7}
$$

long-run cash flow, discount rate, and volatility shocks: The multivariate return betas are given by,

$$
\begin{aligned}
& \beta_{CF} = \pi, \\
& \beta_{DR} = \psi\left(\left[\frac{\kappa_{1d}}{1-\kappa_{1d}\rho}\frac{1-\kappa_1\rho}{\kappa_1}\right]\left(\phi-\frac{1}{\psi}\right) - \pi\right),
\end{aligned} \tag{A.8}
$$

and β_V satisfies

$$
\frac{1}{2}(1-\gamma)^2\left(\left(\frac{\kappa_1\varphi_e}{1-\kappa_1\rho}\right)^2 + 1\right)\frac{\kappa_1}{1-\kappa_1\nu}\beta_V = \kappa_{1d}H_\sigma + \kappa_1 A_\sigma\beta_{DR}. \tag{A.9}
$$

B Impulse Response Computations

The VAR(1) dynamics for the state variables follows,

$$X_{t+1} = \mu + \Phi X_t + \Sigma \epsilon_{t+1}. \tag{B.1}$$

Primitive shocks ϵ are i.i.d. with mean zero and variance 1. Σ is lower-diagonal.

The ex-ante consumption variance is $\sigma_t^2 \equiv E_t RV_{t+1} = \alpha' X_t$, for $\alpha = i_v' \Phi$. Hence, ex-ante volatility shocks are $(E_{t+1} - E_t)\sigma_{t+1}^2 = \alpha' \Sigma \epsilon_{t+1}$. To generate a one-standard deviation ex-ante volatility shock, we choose a combination of primitive shocks $\tilde{\epsilon}_{t+1}$ proportional to their impact on the volatility:

$$\tilde{\epsilon}_{t+1} = \frac{(\alpha' \Sigma)'}{\sqrt{\alpha' \Sigma \Sigma' \alpha}}. \tag{B.2}$$

Based on the VAR, we can compute impulse responses for consumption growth, labor income growth, price-dividend ratio and expected market return in the data. Using the structure of the model and the solution to the labor return sensitivity b, we can also compute the impulse response of model-implied consumption return and price-consumption ratio to the volatility shocks;

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Tables and Figures

Table 1: **Configuration of Long-Run Risks Model Parameters**

Preferences	δ	γ	ψ	
	0.9984	10	2	
Consumption	μ	ρ	φ_e	
	0.0015	0.975	0.037	
Volatility	σ_g	ν	σ_w	
	0.0072	0.999	2.8e-06	
Dividend	μ_d	ϕ	φ_d	π
	0.0015	2.5	3.5	2.0

Baseline parameter values for the long-run risks model. The model is calibrated on monthly frequency.

Table 2: **Consumption and Asset Market Calibration**

	Mean	Std. Dev.	AR(1)
Consumption:	1.82	2.90	0.43
Dividend:	1.82	10.54	0.34
Risk-free Rate:	1.52	1.14	0.98

	Wealth	Market
Corr. of discount rate with vol shock	0.59	0.96
Total Risk Premium	2.28	6.01
Cash-flow Risk Premium	1.22	3.44
Discount Rate Risk Premium	0.03	0.03
Vol. Risk Premium	1.06	2.54
Cash flow beta β_{CF}	1	2
Discount rate beta β_{DR}	-1	-0.33
Volatility beta β_V	0	-0.11

Long-run risks model implications for consumption growth and asset market. Based on a long model sample of monthly data. Consumption is time-aggregated to annual frequency.

Table 3: **Consumption Innovation Ignoring Volatility Channel**

	IES = 2		IES= 1		IES = 0.75	
	Ignore Vol	True	Ignore Vol	True	Ignore Vol	True
Panel A: Model with Time-varying Volatility						
Vol of cons. shock	5.46	2.49	2.49	2.49	2.79	2.49
Consumption shock correlations:						
True cons. shock	0.46	1.00	1.00	1.00	0.89	1.00
Return shock N_R	0.85	0.64	1.00	1.00	0.93	0.78
Discount rate shock N_{DR}	-0.72	0.00	0.00	0.00	-0.15	0.00
Volatility shock N_V	-0.89	0.00	0.00	0.00	0.45	0.00
Panel B: Model with Constant Volatility						
Volatility of cons. shock	2.49	2.49	2.49	2.49	2.49	2.49
Consumption shock correlations:						
True cons shock	1.00	1.00	1.00	1.00	1.00	1.00

Implied consumption innovations computed from the model ignoring volatility contribution, versus the true short-run consumption shock. Population values in the full model with time-varying volatility (Panel A) and the model with constant volatility (Panel B), monthly frequency. first-order VAR containing true consumption return, price-consumption ratio and risk-free rate. Volatility is annualized, in percent.

Table 4: **IMRS Innovations Using Ignoring Volatility**

	IES = 2		IES= 1		IES = 0.75	
	Ignore Vol	True	Ignore Vol	True	Ignore Vol	True
Panel A: Model with Time-Varying Volatility						
Vol of IMRS shock	0.41	0.62	0.40	0.60	0.39	0.58
Market Risk Premium	3.48	6.02	2.75	4.96	2.28	3.84
IMRS shock correlations:						
True IMRS shock	0.71	1.00	0.67	1.00	0.64	1.00
Return shock N_R	-0.78	-0.96	-0.62	-0.46	-0.24	0.24
Discount rate shock N_{DR}	-0.41	0.30	-0.78	-0.52	-0.71	-0.74
Volatility shock N_V	0.06	0.74	0.00	0.74	-0.04	0.74
-0.99	-0.74					
Panel B: Model with Constant Volatility						
Vol of IMRS shock	0.42	0.42	0.40	0.40	0.39	0.39
Corr. with true IMRS shock	1.00	1.00	1.00	1.00	1.00	1.00

Implied IMRS innovations computed from the model ignoring volatility contribution versus the true IMRS shock. Population values in the full model with time-varying volatility (Panel A) and the model with constant volatility (Panel B), monthly frequency. first-order VAR containing true consumption return, price-consumption ratio and risk-free rate. Volatility is annualized, in percent.

Table 5: **Consumption Innovation Ignoring Volatility and Consumption Return**

	IES = 2			IES= 1			IES = 0.75		
	Ignore Vol	Mkt Vol	True	Ignore Vol	Mkt Vol	True	Ignore Vol	Mkt Vol	True
Panel A: Model with Time-varying Volatility									
Vol of cons. shock	14.32	12.73	2.49	11.53	10.87	2.49	8.67	8.82	2.49
Correlation of implied cons. shock with:									
True cons. shock	0.35	0.39	1.00	0.43	0.46	1.00	0.58	0.57	1.00
Return shock N_R^d	0.92	0.95	0.45	0.97	0.98	0.48	0.99	0.99	0.59
Discount rate shock N_{DR}^d	-0.70	-0.63	0.00	-0.49	-0.44	0.00	0.22	0.19	0.00
Volatility shock N_V^d	-0.81	-0.76	0.00	-0.69	-0.64	0.00	-0.30	-0.35	0.00
Panel B: Model with Constant Volatility									
Volatility of cons. shock	8.48	8.48	2.49	8.47	8.47	2.49	8.41	8.41	2.49
Correlation of implied cons. shock with:									
True cons shock	0.59	0.59	1.00	0.59	0.59	1.00	0.59	0.59	1.00
Return shock N_R^d	0.99	0.99	0.52	1.00	1.00	0.54	0.99	0.99	0.64
Discount rate shock N_{DR}^d	0.59	0.59	0.00	0.59	0.59	0.00	0.58	0.58	0.00
Volatility shock N_V^d	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Implied consumption innovations computed from the model ignoring volatility contribution and using dividend return in place of consumption return, versus the true short-run consumption shock. Population values in the full model with time-varying volatility (Panel A) and the model with constant volatility (Panel B), monthly frequency. Volatility is annualized, in percent.

Table 6: **Data Summary Statistics**

	Mean	Std. Dev.	AR(1)
Consumption growth	1.86	2.18	0.48
Labor income growth	2.01	3.91	0.39
Market return	5.70	19.64	-0.01
Price-dividend ratio	3.38	0.45	0.88
Realized variance	4.70	12.40	0.39

Summary statistics for real consumption growth, real labor income growth, real market return, price-dividend ratio and realized consumption variance. Annual observations from 1930 to 2010. Consumption growth, labor income growth and market return statistics are in per cent; realized variance is multiplied by 10000.

Table 7: **VAR Estimation Results**

	r_{dt}	Δc_t	Δy_t	pd_t	RV_t	R^2_{adj}
			Φ			
$r_{d,t+1}$	0.06	-3.73	0.97	-0.08	-30.18	0.04
	(0.08)	(1.08)	(0.36)	(0.03)	(23.55)	
Δc_{t+1}	0.06	0.19	0.12	-0.00	-1.67	0.57
	(0.01)	(0.10)	(0.04)	(0.01)	(1.07)	
Δy_{t+1}	0.08	-0.27	0.48	0.00	-0.17	0.31
	(0.03)	(0.26)	(0.14)	(0.01)	(1.57)	
pd_{t+1}	-0.24	-3.90	0.94	0.92	-14.16	0.79
	(0.14)	(1.15)	(0.70)	(0.05)	(23.41)	
RV_{t+1}	-0.001	-0.009	-0.001	-0.001	0.217	0.24
	(0.001)	(0.012)	(0.005)	(0.001)	(0.124)	
			$\Omega^{1/2}$			
$r_{d,t+1}$	0.18					
Δc_{t+1}	0.00	0.01				
Δy_{t+1}	0.00	0.02	0.02			
pd_{t+1}	0.18	-0.03	-0.03	0.08		
RV_{t+1}	-0.0001	-0.0003	-0.0002	-0.0000	0.0009	

Estimation results of the VAR(1) dynamics of the economic states, $X_{t+1} = \mu_X + \Phi X_t + u_{t+1}$, where u_t is Normal with variance-covariance matrix Ω. X_t includes real market return, r_{dt}, real consumption growth, Δc_t, real labor income growth, Δy_t, price-dividend ratio, pd_t, and realized consumption variance, RV_t. Annual observations from 1930 to 2010.

Table 8: **Model-Implied Correlations With and Without Volatility**

		Without Vol	With Vol
Market and Labor Return:			
Immediate Shocks	$Corr(N_R^d, N_R^y)$	-0.50	0.36
Discount Shocks	$Corr(N_{DR}^d, N_{DR}^y)$	-0.78	0.25
5-year Expectations	$Corr(E_t r_{t\to t+5}^d, E_t r_{t\to t+5}^y)$	-0.65	0.51
Market and Wealth Return:			
Immediate Shocks	$Corr(N_R^d, N_R)$	0.51	0.82
Discount Shocks	$Corr(N_{DR}^d, N_{DR})$	-0.00	0.89
5-year Expectations	$Corr(E_t r_{t\to t+5}^d, E_t r_{t\to t+5})$	0.23	0.90
Wealth and Labor Return:			
Immediate Shocks	$Corr(N_R, N_R^y)$	0.48	0.83
Discount Shocks	$Corr(N_{DR}, N_{DR}^y)$	0.63	0.66
5-year Expectations	$Corr(E_t r_{t\to t+5}, E_t r_{t\to t+5}^y)$	0.58	0.83

Model-implied correlations between market, human capital, and wealth returns, with and without the volatility risks. Risk aversion is set at $\gamma = 6.5$, and IES $\psi = 2.5$.

Table 9: **Model-Implied Risk Premia and Shock Correlations**

		Without Vol	With Vol
Market Return:			
Risk Premium	$Cov(-N_M, N_R^d)$	3.49	9.70
Vol Risk Premium	$Cov(-N_V, N_R^d)$	0	5.60
Vol of Immediate News	$Std(N_R^d)$	18.45	18.45
Vol of Discount News	$Std(N_{DR}^d)$	12.45	12.45
Wealth Return:			
Risk Premium	$Cov(-N_M, N_R)$	1.34	4.04
Vol Risk Premium	$Cov(-N_V, N_R)$	0	1.88
Vol of Immediate News	$Std(N_R)$	3.78	6.40
Vol of Discount News	$Std(N_{DR})$	2.10	3.34
Labor Return:			
Risk Premium	$Cov(-N_M, N_R^y)$	0.78	2.55
Vol Risk Premium	$Cov(-N_V, N_R^y)$	0	0.90
Vol of Immediate News	$Std(N_R^y)$	4.74	4.95
Vol of Discount News	$Std(N_{DR}^y)$	4.23	1.97

Model-implied risk premia and shock correlations, with and without volatility. Risk aversion is set at $\gamma = 6.5$, and IES $\psi = 2.5$.

Table 10: **Robustness Evidence on Correlations with and without Volatility**

	Lbr-Mkt Corr			Risk Premia			Lbr-Mkt Corr			Risk Premia		
ψ	N_R	N_{DR}	Er	Mkt	Lbr	Wealth	N_R	N_{DR}	Er	Mkt	Lbr	Wealth
	With Volatility						**Without Volatility**					
						$\gamma = 5$						
0.5	-0.91	-0.44	-0.25	4.01	-2.11	-3.72	-0.83	-0.25	0.02	1.79	-0.79	-1.47
1.0	-0.94	-0.45	-0.20	5.27	0.20	-1.13	-0.94	-0.45	-0.20	2.32	0.17	-0.39
1.5	-0.50	-0.44	-0.12	5.70	1.29	0.13	-0.77	-0.60	-0.39	2.50	0.62	0.12
2.0	-0.06	-0.41	-0.01	5.91	1.89	0.84	-0.61	-0.70	-0.54	2.58	0.86	0.41
2.5	0.15	-0.30	0.12	6.03	2.27	1.28	-0.50	-0.78	-0.65	2.64	1.02	0.59
3.0	0.25	-0.15	0.24	6.12	2.53	1.59	-0.44	-0.83	-0.74	2.67	1.12	0.71
						$\gamma = 6.5$						
0.5	-0.91	-0.49	-0.32	7.23	-4.40	-7.46	-0.83	-0.25	0.02	2.65	-1.18	-2.18
1.0	-0.94	-0.45	-0.20	8.78	0.28	-1.95	-0.94	-0.45	-0.20	3.17	0.23	-0.54
1.5	-0.33	-0.34	0.02	9.29	2.28	0.44	-0.77	-0.60	-0.39	3.35	0.82	0.16
2.0	0.18	-0.07	0.31	9.55	3.36	1.74	-0.61	-0.70	-0.54	3.44	1.14	0.54
2.5	0.36	0.25	0.51	9.70	4.04	2.55	-0.50	-0.78	-0.65	3.49	1.34	0.78
3.0	0.44	0.40	0.58	9.81	4.50	3.11	-0.44	-0.83	-0.74	3.53	1.49	0.94
						$\gamma = 8$						
0.5	-0.92	-0.52	-0.38	11.28	-7.85	-12.88	-0.83	-0.25	0.02	3.50	-1.57	-2.90
1.0	-0.94	-0.45	-0.20	13.10	0.37	-2.97	-0.94	-0.45	-0.20	4.03	0.29	-0.69
1.5	-0.12	-0.20	0.17	13.71	3.70	1.07	-0.77	-0.60	-0.39	4.20	1.03	0.20
2.0	0.37	0.31	0.55	14.01	5.47	3.23	-0.61	-0.70	-0.54	4.29	1.43	0.67
2.5	0.51	0.54	0.67	14.19	6.57	4.57	-0.50	-0.78	-0.65	4.35	1.67	0.97
3.0	0.57	0.59	0.69	14.31	7.32	5.48	-0.44	-0.83	-0.74	4.38	1.84	1.17

Model implications for the correlations between human capital and market return news (immediate and future discount rate) and 5-year expected returns, and risk premia for market, human capital and wealth return, at different risk aversion and IES parameters.

Table 11: **News Dynamics Implied by the Labor-Market Model**

	N_{CF}	N_{DR}	N_V	SDF
	With Volatility			
N_{CF}	5.81			
N_{DR}	0.09	3.34		
N_V	-0.45	0.84	35.08	
SDF	-0.84	0.46	0.87	63.62
NBER Recession	-0.22	0.05	0.16	0.22
	Without Volatility			
N_{CF}	5.81			
N_{DR}	0.98	2.10		
N_V	0.00	0	0.00	
SDF	-0.99	-0.97	0.00	35.72
NBER recession	-0.22	-0.22	0.00	0.22

Table 11 presents pairwise correlations of cash-flow (N_{CF}), discount rate (N_{DR}), volatility (N_V) and SDF (N_M) news, and their per cent volatilities (on the diagonal), measured using the labor-market model. The top panel corresponds to the case that accounts for the variation in the volatility, and the bottom panel refers to the case without volatility. The last row of each panel reports correlations of the news series with the NBER recession indicator.

Table 12: **Risk Premia Implied by Market-Based VAR**

Panel A: No Restrictions on Risk-Premia Variation

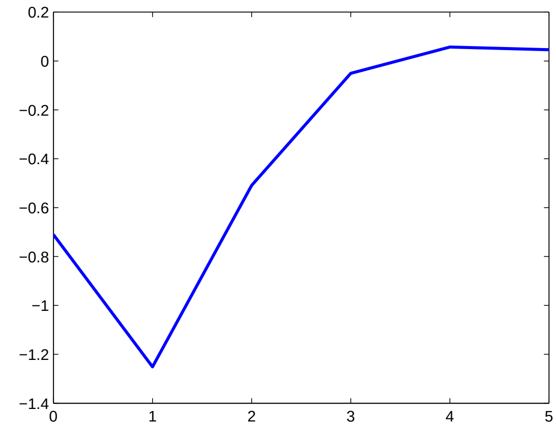
	Risk Premia		Decomposition		
	Data	Model	CF	DR	Vol
Market	7.9	7.7	4.8	1.7	1.2
BM1	7.2	7.2	4.5	1.5	1.1
BM2	7.6	8.3	5.6	1.6	1.1
BM3	9.4	10.0	7.0	1.6	1.4
BM4	10.8	11.3	8.7	1.6	1.0
BM5	13.1	13.2	12.2	0.9	0.1
Size1	14.8	14.2	12.1	1.5	0.6
Size2	13.0	12.4	9.1	2.0	1.3
Size3	11.6	11.2	7.8	1.9	1.4
Size4	10.4	9.7	6.4	1.9	1.4
Size5	7.4	7.4	5.0	1.4	1.0

Panel B: Incorporating Restrictions on Risk-Premia Variation

	Risk Premia		Decomposition		
	Data	Model	CF	Vol	Rfree
Market	7.9	7.6	5.8	2.1	0.1
BM1	7.2	7.0	5.5	1.8	-0.2
BM2	7.6	8.3	6.6	1.8	-0.1
BM3	9.4	10.2	8.0	2.1	0.1
BM4	10.8	11.4	9.7	1.6	0.2
BM5	13.1	13.0	12.9	0.1	0.0
Size1	14.8	14.1	13.2	1.0	-0.1
Size2	13.0	12.5	10.5	2.1	-0.1
Size3	11.6	11.3	9.1	2.2	0.0
Size4	10.4	9.8	7.5	2.3	0.1
Size5	7.4	7.4	5.8	1.5	0.0

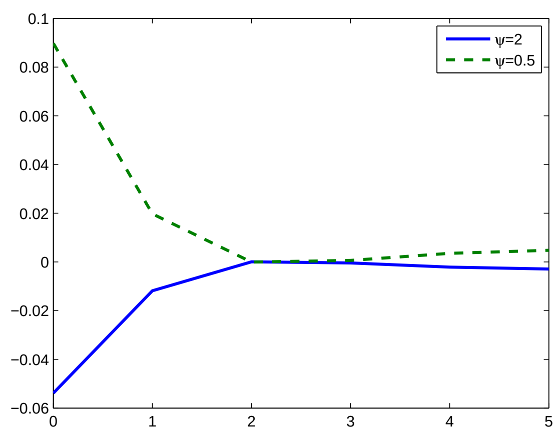
Table 12 shows risk premia implied by the market-based VAR for the aggregate market and a cross section of five book-to-market and five size sorted portfolios, and the contribution of various risk channels to the overall compensation. In Panel A, the dynamics of discount-rate news are unrestricted; in Panel B, discount-rate variation is decomposed into variation in risk premia (which is proportional to volatility news) and variation in the risk-free rate. "Data" column reports average excess returns in the 1930-2010 sample.

Figure 1: **Consumption Response to Volatility**



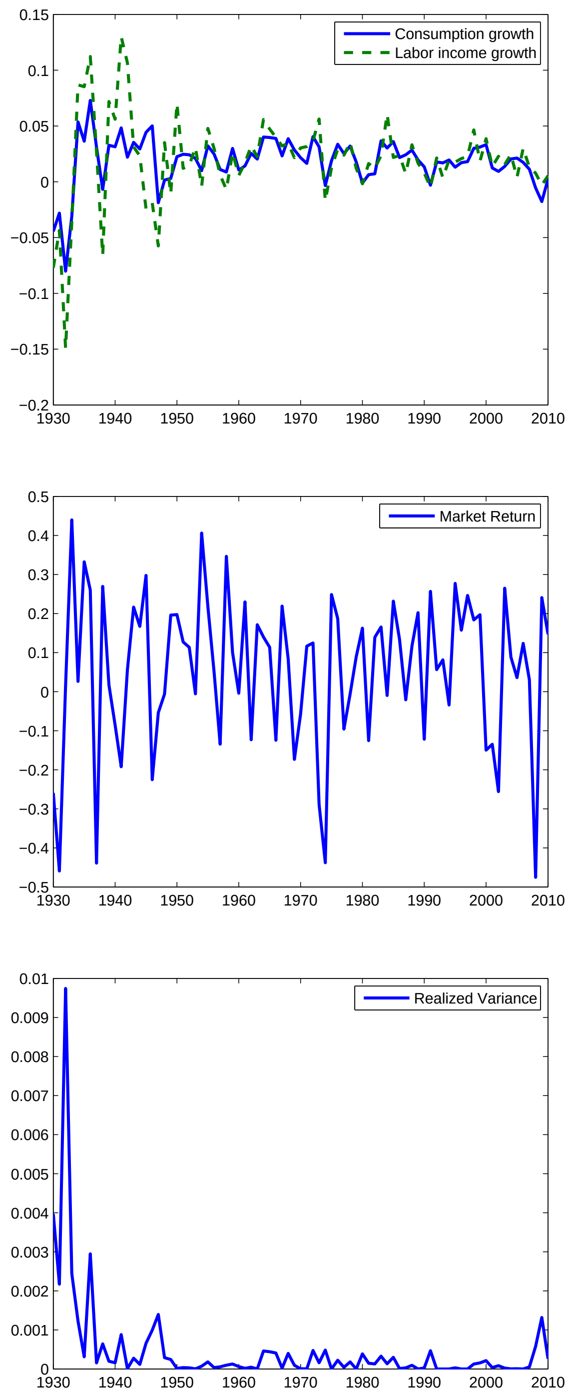
Impulse response of consumption growth to the one standard deviation shock in ex-ante volatility of consumption, implied by VAR(1) dynamics of the consumption, labor income growth and the market portfolio. One standard deviation volatility shock corresponds to an increase in ex-ante consumption variance from its mean of $(2.2\%)^2$ to $(3.1\%)^2$. Consumption growth is annual, in per cent.

Figure 2: **Price-Consumption Ratio Response to Volatility**



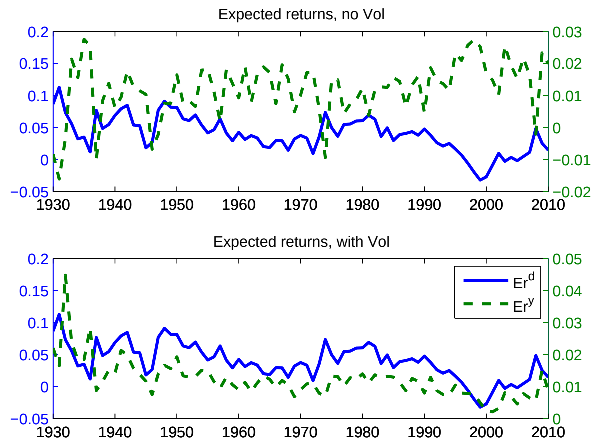
Impulse response of model-implied log price-consumption ratio to the one standard deviation shock in ex-ante volatility, implied by VAR(1) dynamics of the consumption, labor income growth and the market portfolio. One standard deviation volatility shock corresponds to an increase in ex-ante consumption variance from its mean of $(2.2\%)^2$ to $(3.1\%)^2$.

Figure 3: **Time Series of Variables**



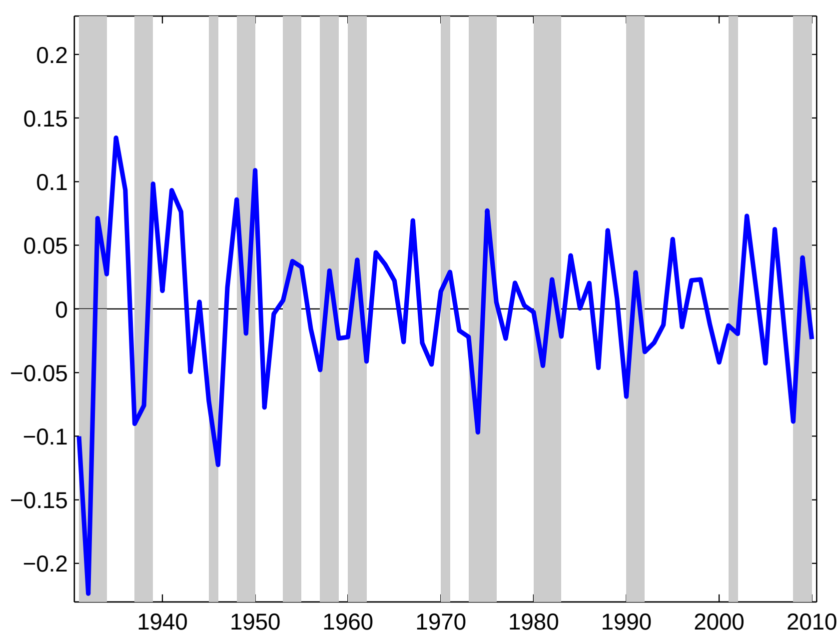
Real consumption and labor income growth rates (top panel), real market return (middle panel) and realized consumption variance (bottom panel). Annual data from 1930 to 2010.

Figure 4: **5-year Expected Market and Labor Returns**



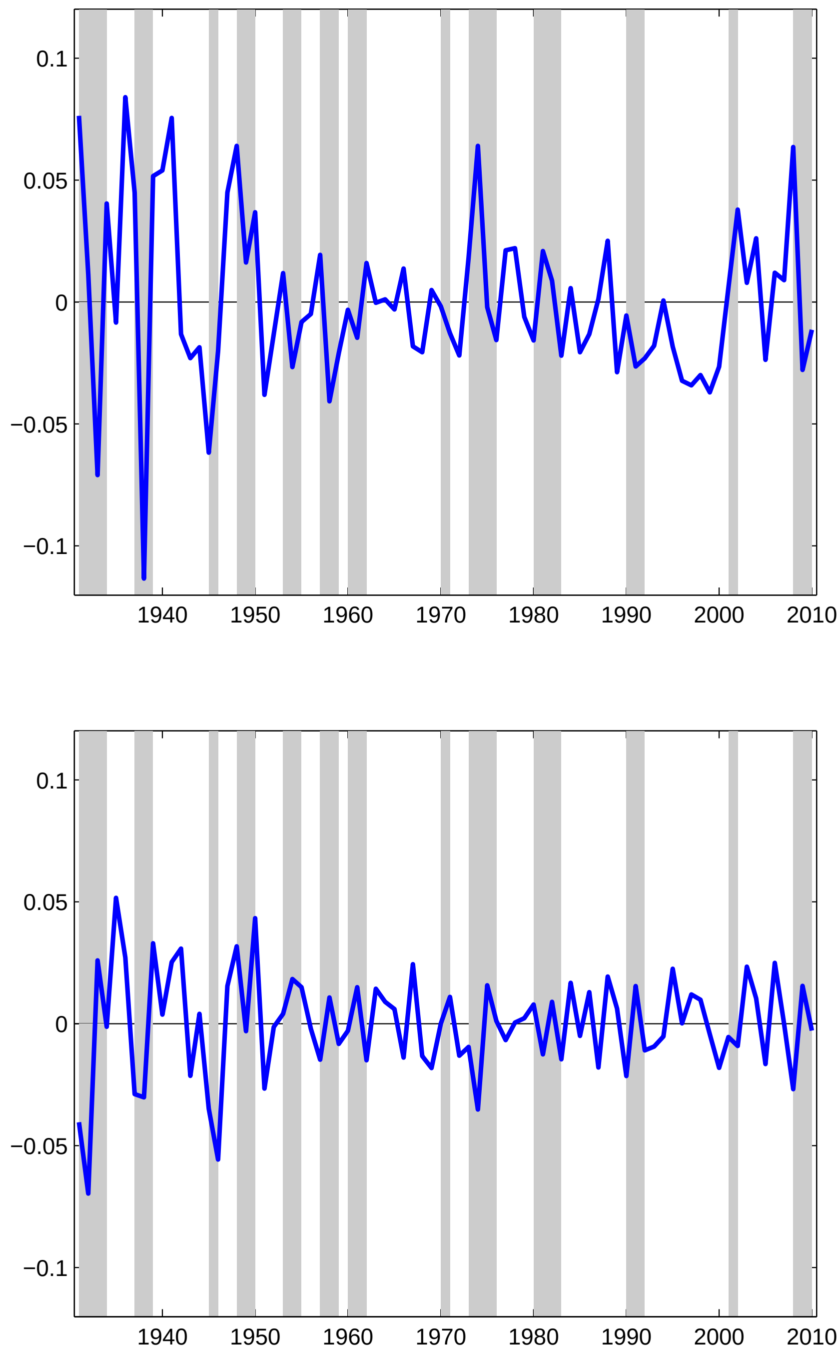
Five year DCAPM-implied expected returns on the market (solid line) and human capital (dashed line), in the specifications without volatility ($\chi = 0$) (top panel) and with volatility at the implied χ (bottom panel).

Figure 5: **Future Discounted Consumption News**



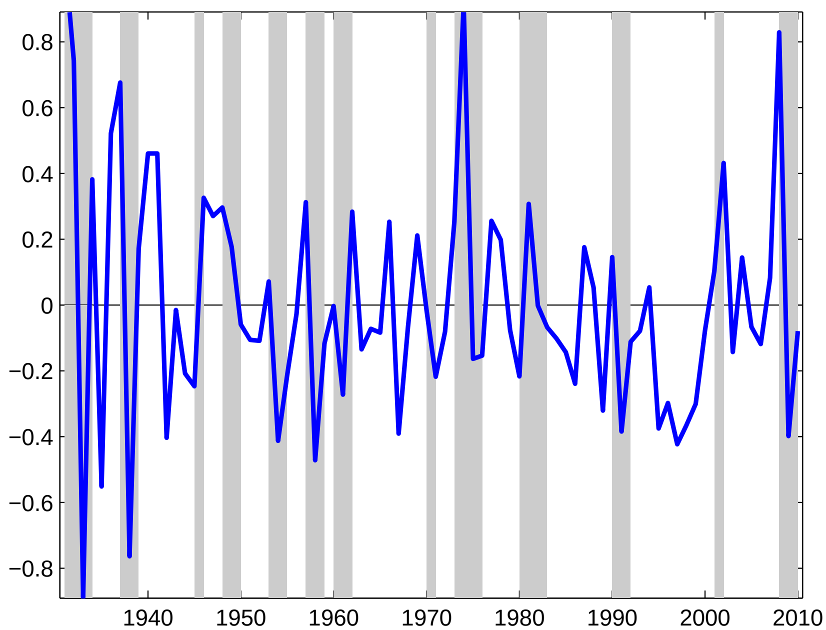
Current and future discounted consumption news N_{CF}. Grey bars indicate NBER recession years.

Figure 6: **Discount Rate News**



Discount rate news on the wealth portfolio N_{DR}. Top panel refers to the model with volatility news, the bottom panel is without volatility news. Grey bars indicate NBER recession years.

Figure 7: **Future Volatility News**



Future discounted volatility news N_V. Grey bars indicate NBER recession years.