

# On the Stability of Money Demand

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## 1. INTRODUCTION

The failure of Lehman Brothers in September, 2008 immediately led to a severe banking panic, a rush by banks to exchange privately issued cash substitutes for government issued or government guaranteed cash. The Federal Reserve responded to this situation by increasing the level of bank reserves from some \$40 billion on September 1 to \$800 b. by New Years Day. This single action was surely the main factor in the resolution of the liquidity crisis by early 2009 and the ending of the decline in production after two quarters.

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\*We thank Fernando Alvarez, Espen Henriksen, Finn Kydland, Francesco Lippi, Nancy Stokey and Pedro Teles for helpful discussions and Manuel Macera and Serginio Sylvain for outstanding assistance.

It is a remarkable feature of these events that none of the leading macroeconomic models—including the model in use by the Fed itself—had anything to contribute to the analysis of this liquidity crisis or of the Fed’s response to it. None of these models had any role for bank reserves or for any other monetary aggregate or measure of liquidity. Bankers, as always, used short interest rates as the only indicator of the stance of monetary policy but sometime in the 1990s they were joined by most influential monetary economists. A broad consensus was reached that no measure of “liquidity” in an economy was of any value in conducting monetary policy.

There were good reasons behind this consensus. Long standing empirical relations connecting monetary aggregates like M1, M2 and the monetary base to movements in prices and interest rates began to fall apart in the 1980s and have not been restored since. Our first objective in this paper is to offer a diagnosis of this empirical breakdown. Our second is to propose a fix, to construct a new monetary aggregate that offers a unified treatment of monetary facts preceding and following 1980.<sup>1</sup>(See Figures 1,2 and 3.)

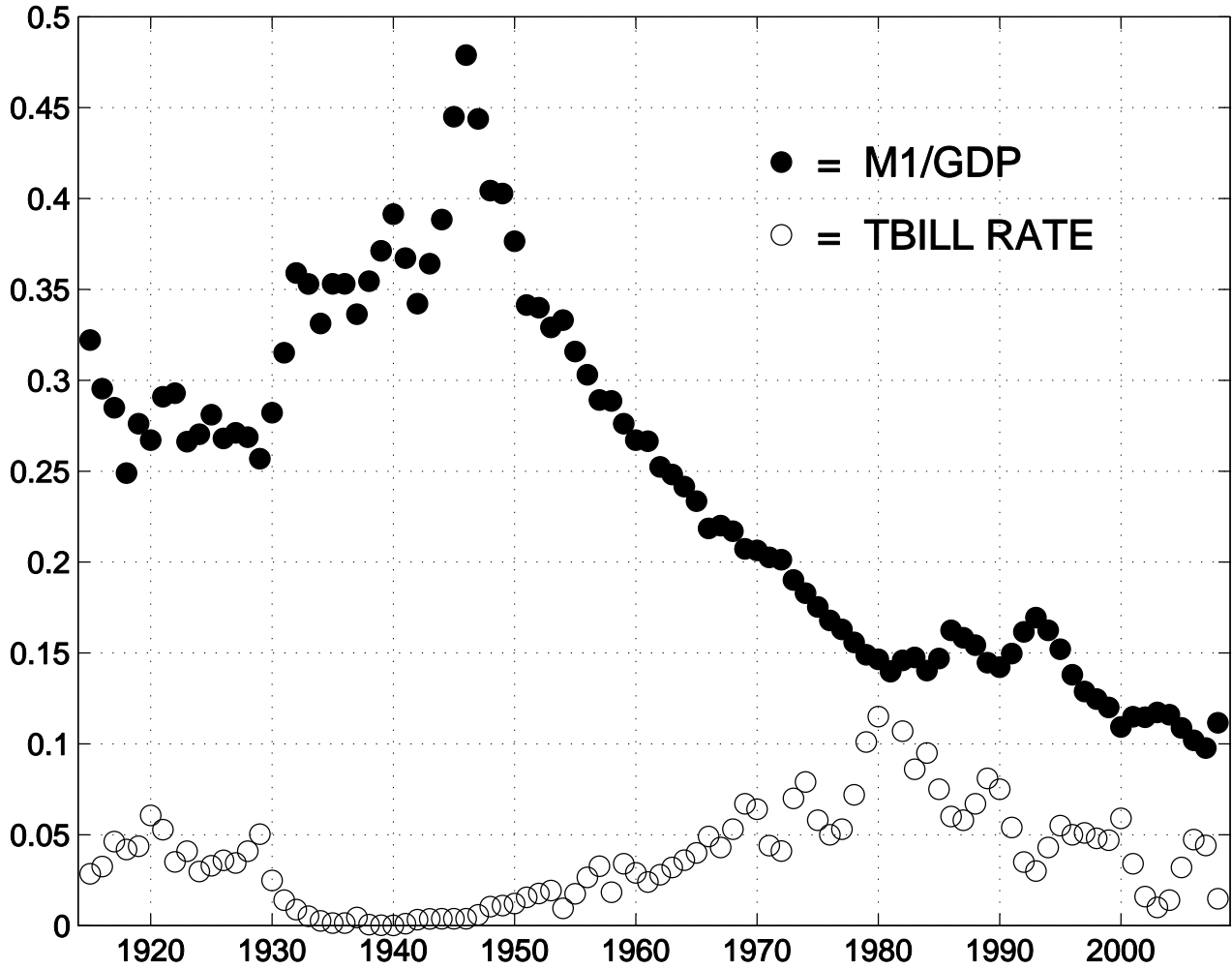
To do this we need to get behind such broad aggregates as M1 and M2 and model the role of banks in the payment system in an explicit way. For this purpose we adapt the model of Freeman and Kydland (2000), based on earlier work by Prescott (1987), to consider the distinct roles of currency, reserves, and commercial bank deposits. This model treats currency and demand deposits as distinct assets and can readily be adapted to include other forms of liquidity, as we will show. It proposes a banking “technology” that rationalizes the adding up of different assets to form aggregates like M1, and does so with some realism. Section 2 develops the theory. Section 3 provides a calibration and some simulations.

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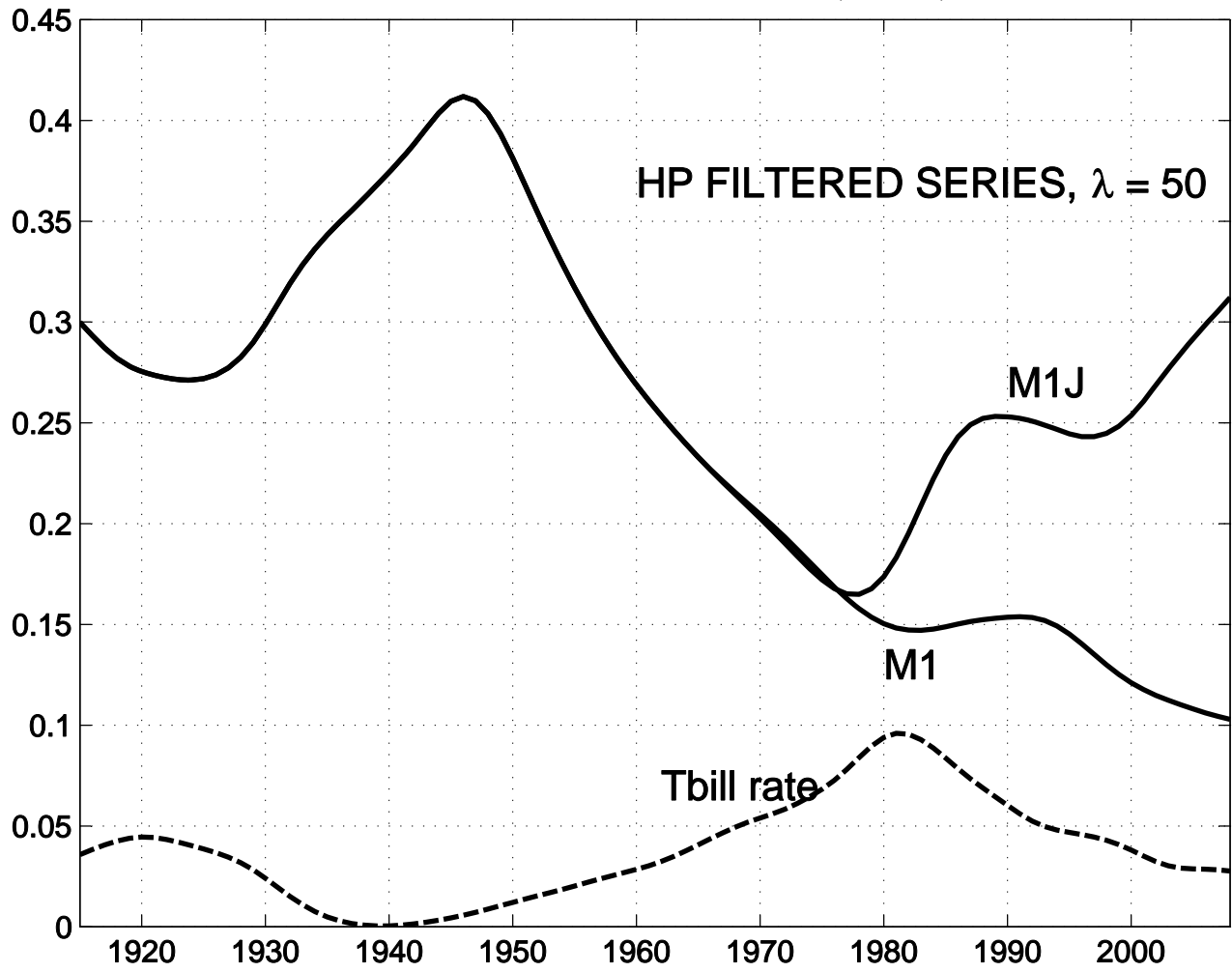
<sup>1</sup>Several economists have offered useful diagnoses of the behavior of M1 and M2 since 1980 and proposed other monetary measures. These include Motley (1988), Poole (1991), Reynard (2004). Teles and Zhou (2005) and Ireland (2009).



# U.S. MONEY/GDP RATIOS AND INTEREST RATES, 1915-2008



### TREND COMPONENTS OF M1, M1J, and r



In our adaptation of Freeman and Kydland, we treat banking activities as though they were conducted within each household, in order to situate banking activities within a general equilibrium. In Section 4 we decentralize this equilibrium, obtaining a model of banks that we then use to analyze the distorting effects of Regulation Q: the Glass-Steagall prohibition of interest on commercial bank deposits. We argue that this distortion, interacting with the inflation of the 1970s, was the main factor behind the empirical difficulties of M1 used as the single measure of the money supply.

In Section 5 we introduce another monetary asset, money market deposit accounts (MMDAs), as a third important means of payment along side currency and demand deposits. This involves a straightforward extension of the Freeman-Kydland model. Section 6 provides a calibration and simulations. The extended model introduces a new, single monetary aggregate—we call it M1J—that coincides with M1 prior to 1980 and includes MMDAs for the years since. We show that M1J gives us a model over the entire 1915-2008 that compares well the the behavior of M1 over the 1915-1980 period. Section 7 contains concluding remarks.

## 2. Basic Model

Freeman and Kydland considered a cash-in-advance model with two means of payment: currency and checks. Their model was designed to introduce erratic money supply behavior into a real business cycle model. Our objective is to understand longer run or lower frequency relations. We construct a deterministic stationary equilibrium with a constant technology for producing goods and a constant, perfectly foreseen growth rate in the supply of outside money. In this case the nominal interest rate  $r$  will be the Fisherian sum  $\rho + \pi$  of the subjective, real discount rate and the money growth rate  $\pi$ . Since these components are exogenous in our set-up, we can express equilibrium relations in the steady state as functions of  $r$ . We view episodes of U.S. monetary history as temporary steady states that differ in a systematic way

with differences in  $r$ . Then we compare the model's predictions to the actual behavior of prices and monetary aggregates determined by actual interest rate movements.

We turn to the details, beginning with preferences. Households consume a continuum of different goods in fixed proportions. Goods come in different “sizes“, with production costs and prices that vary in proportion to size. Let the cdf  $F(z)$ ,  $z > 0$ , be the fraction of goods of size less than or equal to  $z$  and let  $f(z)$  be the corresponding density. Denote the mean by  $\mu = \int_0^\infty z f(z) dz$ . All households have the common preferences, of the form

$$\sum_{t=0}^{\infty} \beta^t U(x_t), \quad \beta = \frac{1}{1 + \rho}. \quad (2.1)$$

Goods are not storable. Each unit of  $x_t$  comprises the full spectrum of goods  $z$  in the proportions given by the density  $f$ . Consuming  $x$  means purchasing  $x$  units of each good.

Each household has one unit of labor each period, to be divided between  $1-s$  used in producing goods and  $s$  used in cash management. For now, we treat both activities as carried out by the household. A household must buy goods from other households and sell what it produces itself to others. There are two payment technologies available to agents. One is a simple cash (currency) in advance technology. The second is a checking technology: the seller accepts a note instructing the buyer's bank to provide cash when the seller delivers the note. Both technologies are here assumed to be reliable and sellers are indifferent between the two modes of payment. But there is a constant fixed cost of processing a check so only the more expensive goods are purchased by check. There will be some cutoff good size  $\gamma > 0$  such that sizes larger than  $\gamma$  are paid for by check and the rest are paid for in cash. Check processing entails a labor cost proportional to the number (not the value) of checks:  $k[1 - F(\gamma)]$ . Changes in  $x$  do not alter the number of checks written.

We assume, in the manner of Baumol (1952) and Tobin (1956), that households

choose the number  $n$  of “trips to the bank” they take during a period (a year, in this paper). For us, the availability of two payment methods complicates the nature of these trips. We follow Freeman and Kydland and assume that each period is divided into  $n$  stages that are identical in all respects. At the beginning of a year, a household begins with  $M$  dollars. These holdings are the economy’s entire stock of base or outside money. These dollars are divided into currency  $C$  and bank deposits,  $\theta D$ , where  $\theta$  is a required reserve ratio. The deposits are augmented by loans by the bank to increase the household’s deposits, against which checks can be written, to a level  $D$ . During the first of the  $n$  subperiods, all of this currency and bank deposits will be spend on consumption goods. During this same initial period another member of the household produces and sells goods in exchange for cash and checks. At the end of the subperiod, producers visit a bank, checks are cleared, base money is divided as before, reserves, loans and deposits are renewed, and the situation at the beginning of the second subperiod replicates exactly the situation at the beginning of the first. This process is repeated exactly  $n$  times during the year.<sup>2</sup>

A household has one unit of time each year, which it divides among bank trips  $\phi n$ , check processing time  $k(1 - F(\gamma))$ , and goods production time,  $1 - \phi n - k(1 - F(\gamma))$ . The marginal product of labor is a constant  $y$  so production is

$$x = [1 - \phi n - k(1 - F(\gamma))] y. \tag{2.2}$$

There are no factors of production other than labor.

We construct a deterministic equilibrium in which the monetary base  $M$  grows at a constant rate  $\pi$  by means of lump sum transfers  $\pi M$  to each household. We

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<sup>2</sup>Notice the only payments in this model are for household purchases of final goods. The model omits the use of cash to pay employees and suppliers of intermediate goods and to clear asset exchanges. We are implicitly treating all these payments—together *much* larger than final goods payments—as proportional to final goods payments. This will require introducing a constant of proportionality as another free parameter in the calibration of the model.



let  $m$  denote a households *relative* holdings of base money, so that in equilibrium  $m = 1$ . We renormalize relative money holdings every period. Let  $p = P/M$  be the normalized price of goods and similarly for other dollar-denominated goods, like deposits  $d = D/M$  and currency  $c = C/M$ . Required reserves are  $\theta d$  so base money is divided into

$$m = c + \theta d. \quad (2.3)$$

(Here we view each household as operating its own “bank” but still subject to a government-imposed reserve requirement. In the Section 4 we will decentralize and explicitly assign different functions to households and banking firms.) The cash constraints facing this consolidated household/bank are

$$nc \geq px\Omega(\gamma) \quad (2.4)$$

and

$$nd \geq px [1 - \Omega(\gamma)], \quad (2.5)$$

where

$$\Omega(\gamma) = \frac{1}{\mu} \int_0^\gamma z f(z) dz$$

is the fraction of total purchases paid for in cash, expressed as a function of the cutoff level  $\gamma$ . Notice that the function  $F$  measures numbers of transactions while  $\Omega$  measures numbers of dollars.

The household Bellman equation is

$$v(m) = \max_{x,n,c,d,\gamma} \left\{ U(x) + \beta v \left( \frac{m + \pi + p(1 - \phi n - k(1 - F(\gamma)))y - px}{1 + \pi} \right) \right\}$$

subject to (2.3)-(2.5). Let  $\lambda$ ,  $\lambda_c$  and  $\lambda_d$  be the multipliers associated with (2.3)-(2.5) respectively. The first order and envelope conditions are then

$$U'(x) = \frac{\beta}{1 + \pi} v'(m') p + \lambda_c p \Omega(\gamma) + \lambda_d p [1 - \Omega(\gamma)],$$

$$\frac{\beta}{1+\pi}v'(m')\phi py = \lambda_c c + \lambda_d d,$$

$$\lambda = \lambda_c n,$$

$$\theta \lambda = \lambda_d n,$$

$$\frac{\beta}{1+\pi}v'(m')k f(\gamma)py = \lambda_c \frac{px}{\mu} \gamma f(\gamma) - \lambda_d \frac{px}{\mu} \gamma f(\gamma)$$

and

$$v'(m) = \frac{\beta}{1+\pi}v'(m') + \lambda.$$

We consider only steady state equilibria in which  $m = m' = 1$ . In this case we can use the envelope condition to conclude

$$v'(1) \simeq \frac{1+r}{r}\lambda \quad \text{and} \quad \frac{\beta}{1+\pi}v'(m') \simeq \frac{1}{r}\lambda, \quad (2.6)$$

where  $r = \rho + \pi$  is the equilibrium nominal interest rate.

Applying (2.6), eliminating the three multipliers, and setting aside the first-order condition for  $x$  (which serves only to determine the value of  $\lambda$ ) we obtain the equilibrium conditions for  $n$  and  $\gamma$ :

$$\frac{1}{r}\phi py = \frac{c + \theta d}{n} \quad (2.7)$$

and

$$\frac{1}{r}k = \frac{1}{n} \frac{x}{y} \frac{\gamma}{\mu} (1 - \theta). \quad (2.8)$$

We assume that all constraints bind, so that

$$c + \theta d = 1, \quad (2.9)$$

$$nc = px\Omega(\gamma), \quad (2.10)$$

and

$$nd = px[1 - \Omega(\gamma)]. \quad (2.11)$$

Finally goods production equals consumption:

$$(1 - \phi n - k(1 - F(\gamma)))y = x. \quad (2.12)$$

These are six equations in  $x, n, c, d, \gamma$  and  $p$ .

We reduce this system to two equations in  $n$  and  $\gamma$ , First multiply (2.10) by  $\theta$ , add the result to (2.11) and apply (2.9) to get

$$n = [\Omega(\gamma) + \theta [1 - \Omega(\gamma)]] px \quad (2.13)$$

Substitute into (2.12) for  $px$  using equation (13) and  $py$  using (2.7) and (2.9) to obtain

$$r [\Omega(\gamma) + \theta [1 - \Omega(\gamma)]] (1 - \phi n - k(1 - F(\gamma))) = \phi n^2. \quad (2.14)$$

Equation (2.14) involves only the two endogenous variables  $\gamma$  and  $n$ . To get a second equation in these two variables we use (2.8), the first order condition for the cutoff variable  $\gamma$ , and (2.12) to eliminate  $x/y$  and obtain

$$(k + r\phi) n = r (1 - k(1 - F(\gamma))) \frac{\gamma}{\mu} (1 - \theta). \quad (2.15)$$

With (2.14) and (2.15) solved for the functions  $n(r)$  and  $\gamma(r)$ , (2.10) and (2.11) together imply

$$n(r)(c + d) = px.$$

Defining  $m_1 = c + d$  as a normalized M1 and  $px$  as GDP, the theoretical counterpart to the ratio of M1 to GDP is

$$\frac{m_1}{px} = \frac{1}{n(r)}.$$

As acknowledged in Note 2, money in the real economy is used in many more payments than it does in this model: We are implicitly treating all these payments as proportional to final goods payments. This requires the addition of a constant of proportionality, and we write

$$\frac{m_1}{px} = \frac{A}{n(r)}. \quad (2.16)$$

Finally, we use (2.10) and (2.11) again to obtain the currency-deposit ratio

$$\frac{c}{d} = \frac{\Omega(\gamma(r))}{1 - \Omega(\gamma(r))}. \quad (2.17)$$

The left sides of (2.16) and (2.17) are observable so both of these two equations provide testable restrictions on calibrated versions of this model. This will be the subject of the next section.

### 3. Calibration and Simulation-1

In order to get some idea of how the predictions of our version of the Freeman/Kydland model match up to U.S. time series we need to put some numbers on the table. In this section we calibrate the model of the last section using as a benchmark 1960 data on currency, demand deposits, the required reserve ratio, the short term interest rate, the size of the banking sector, and nominal GDP.

We also need to specify an exact distribution  $F(z)$  for transaction sizes. There are many possibilities, but for this section we illustrate a calibration procedure applying the rectangular distribution on  $[0, 1]$  that Freeman and Kydland used:  $F(\gamma) = \gamma$ ,  $\mu = 1/2$ , and  $\Omega(\gamma) = \gamma^2$ . (A more general class is considered in Sections 5 and 6.) In this case, (2.14) and (2.15) become

$$r [\gamma^2 + \theta [1 - \gamma^2]] (1 - \phi n - k(1 - \gamma)) = \phi n^2 \quad (3.1)$$

and

$$(k + r\phi) n = 2r (1 - k(1 - \gamma)) \gamma (1 - \theta). \quad (3.2)$$

We need numerical estimates for the parameters  $\omega$ ,  $\phi$ ,  $k$  and  $\theta$ , all of which are now treated as constants. We use 1960 data to benchmark the model. The interest rate in this year averaged  $r = .029$ . The currency deposit ratio in 1960 was .256 so for this year

$$.256 = \frac{\gamma(r)^2}{1 - \gamma(r)^2},$$

implies  $[\gamma(.029)]^2 = 0.2$  and so  $\gamma(.029) = 0.45$ . Setting the reserve ratio  $\theta$  at 0.1, (3.1) and (3.2) imply

$$(.029) (.28) (1 - \phi n - k(.55)) = \phi n^2 \quad (3.3)$$

and

$$(k + (.029)\phi)n = 2(.029)(1 - k(.55))(.45)(.9). \quad (3.4)$$

The sum  $\phi n + k(.8)$  is the fraction of the economy's labor endowment that is used in the payments process. Here we think of  $\phi n$  as household time, not included in measured gdp, and  $k(.55)$  as bank employee time. We can think of Phillipon's (2008) estimate of .05 as a bound on the fraction of gdp generated in the financial sector, but the fraction devoted to payment activities will be much less. We assume  $k(.55) = .02$  so  $k = .54$  and for the household in 1960,  $\phi n = .02$ . Then (3.3) implies that for 1960

$$\phi n^2 = (.02)n(.029) = (.029)(.28)(1 - .04)$$

yielding the benchmark value of  $n(.029) = 0.39$ . The constant  $\phi$  is then estimated at  $\phi = (.02)/(.39) = .05$ .

The parameters of the model, specialized in this way, imply that for all interest rates  $r$  the functions  $n(r)$  and  $\gamma(r)$  estimated, equations (3.1) and (3.2) imply

$$r[\gamma^2 + (0.1)(1 - \gamma^2)](1 - (.05)n - (.54)(1 - \gamma)) = (.05)n^2 \quad (3.5)$$

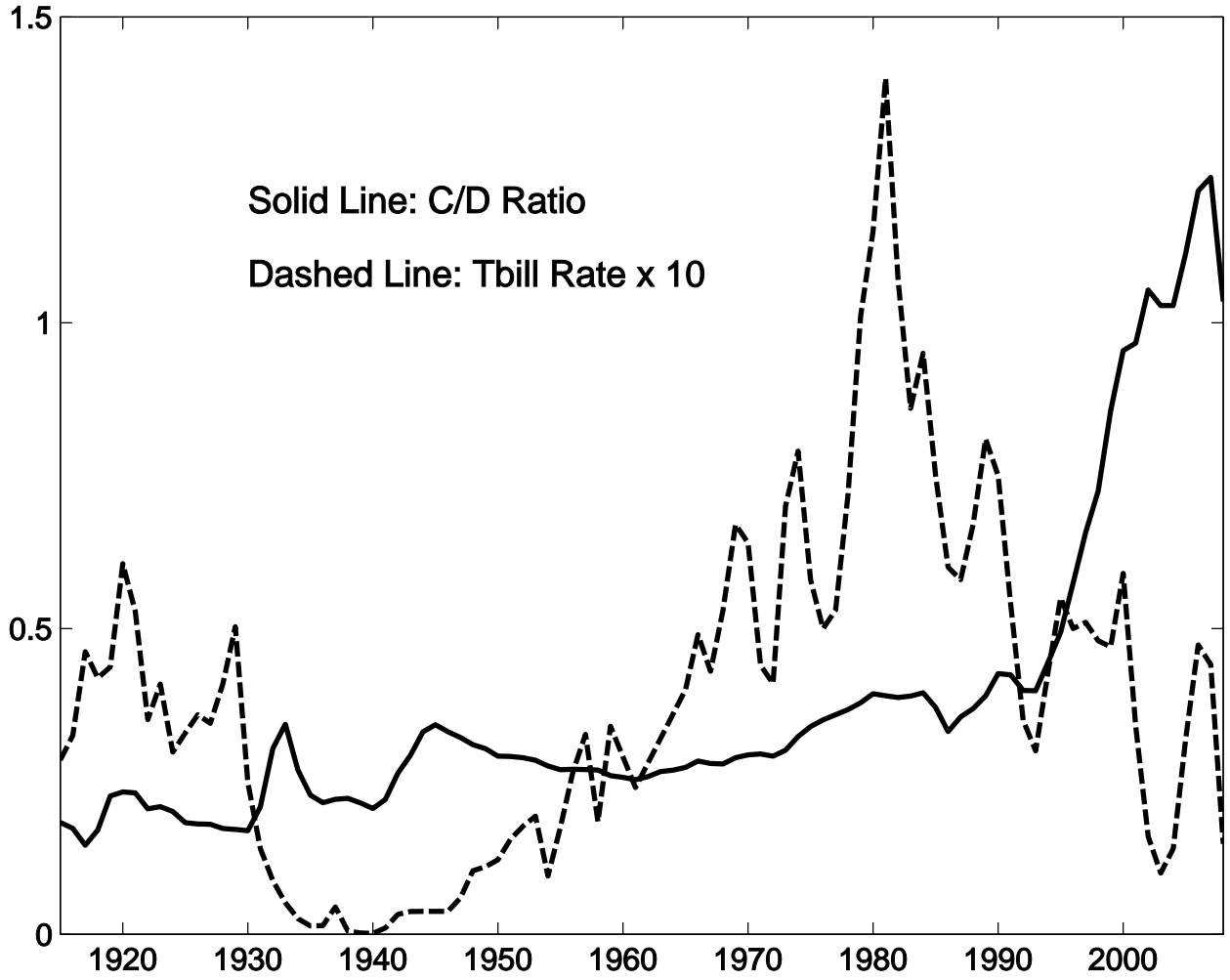
and

$$[.54 + (.05)r]n = 2r(1 - (.54)(1 - \gamma))(.9)\gamma. \quad (3.6)$$

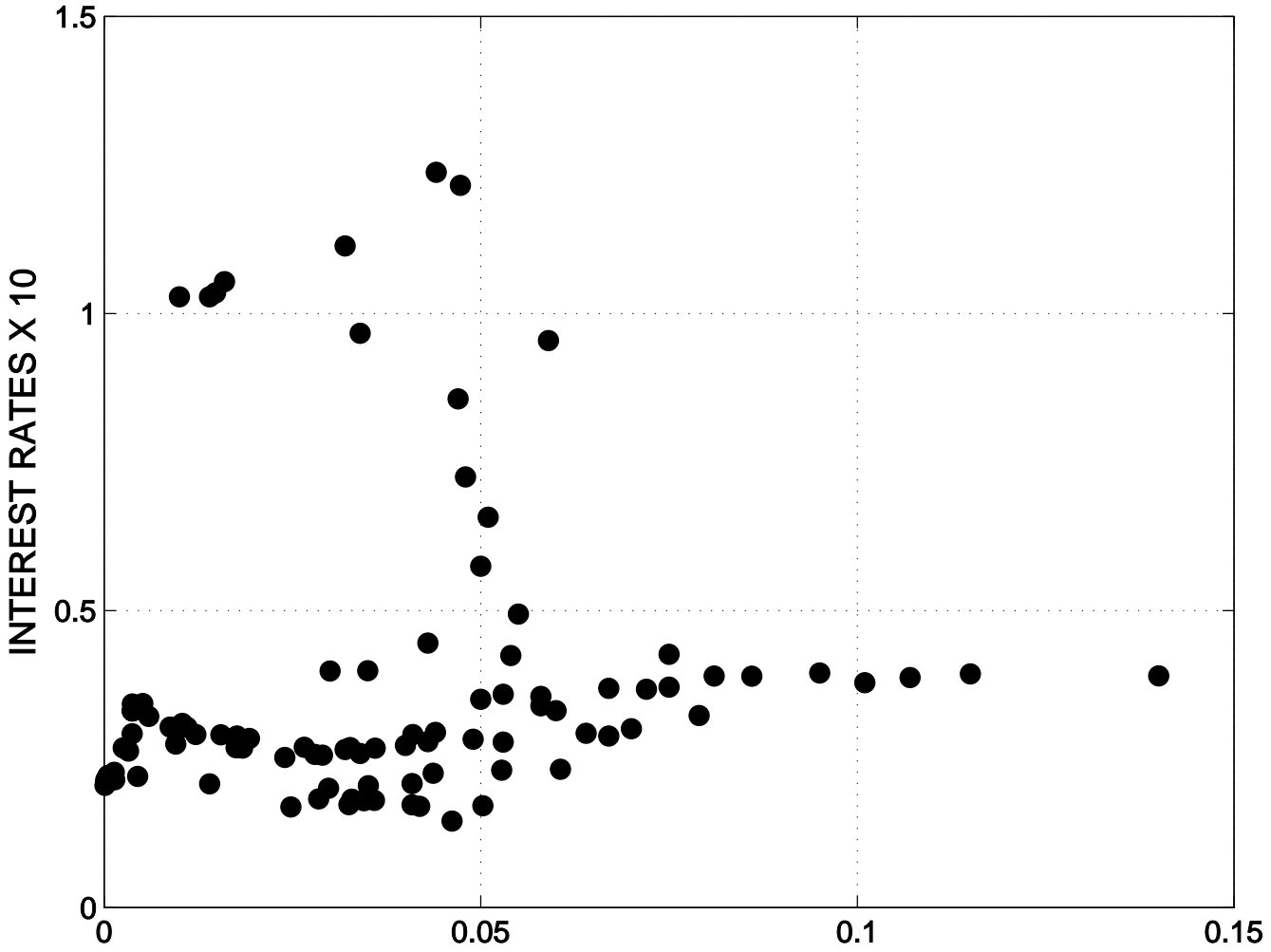
To test this model against the predictions (2.16) and (2.17), we also need a numerical value for the parameter  $A$ . For the benchmark year of 1960, the ratio of M1 to annual nominal GDP was .27. The implied value for this parameter is  $A = (.27)(.39) = .105$ .

We tried to fit this model, so calibrated, to the 1915-1980 period in the U.S., when commercial bank deposits were essentially the only checkable deposits. It performed very poorly. The rectangular distribution implies a function  $\gamma(r)$  that is much too sensitive to interest rate movements to be consistent with (2.17). In fact, currency/deposit ratio did not vary much over this period and the variations that did occur were not closely related to interest rate movements. See Figures 4 and 5.

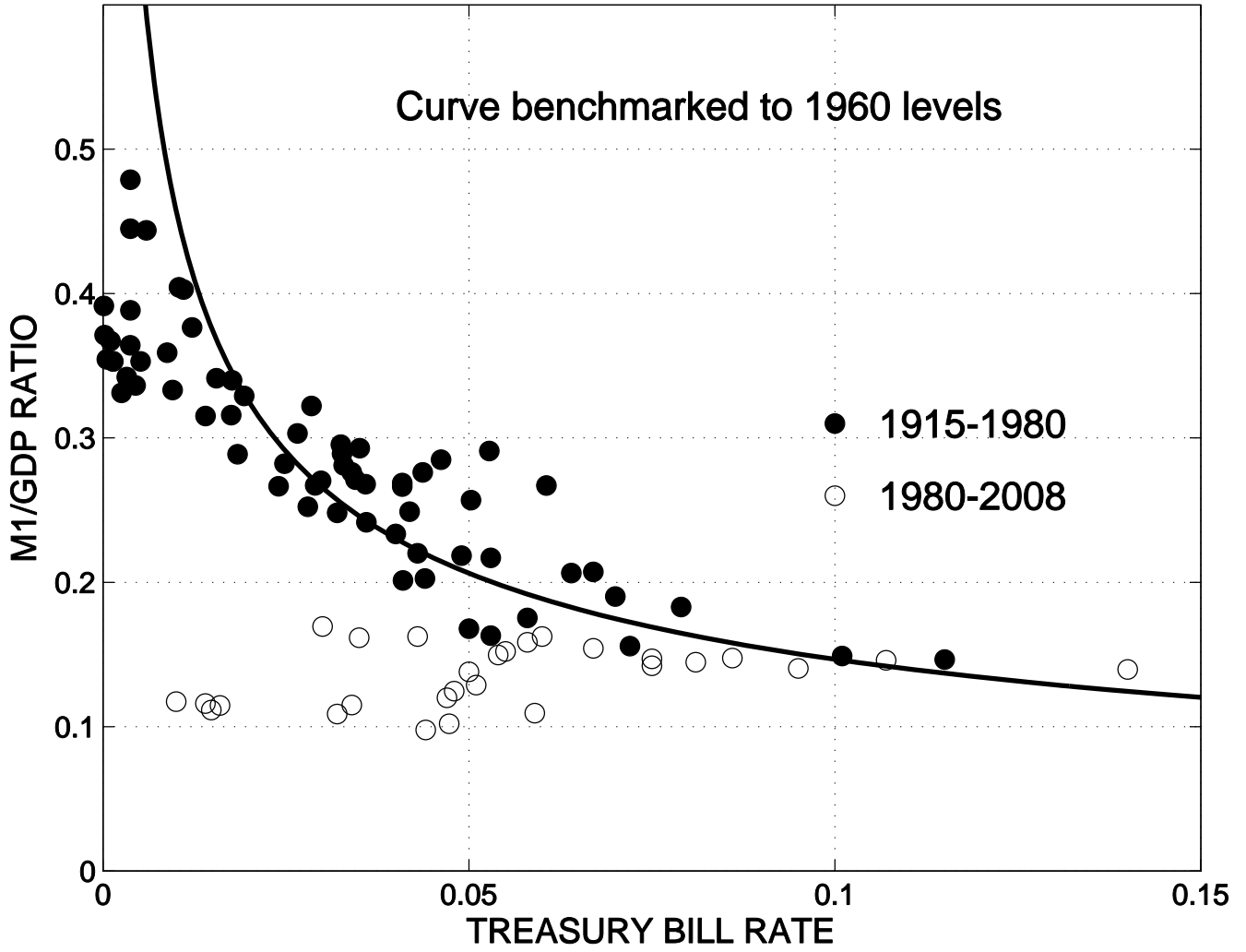
# CURRENCY/DEPOSIT RATIOS AND INTEREST RATES, 1915-2008



CURRENCY/DEPOSIT RATIOS AND INTEREST RATES, 1915-2008



# U.S. MONEY DEMAND, 1915-2008





There are many other possibilities for the distribution  $F(\gamma)$  that one might explore, but here we simply report results for the extreme case when  $\gamma$  is taken to be constant at the benchmark value  $(0.2)^{1/2} = 0.45$ . (The distribution on  $[0,1]$  that actually delivers this is one that concentrates .2 of its mass at 0 and the other .8 at 1.) In this case (3.5) is reduced to

$$r(.28)(.98 - (.05)n) = (.05)n^2, \quad (3.7)$$

a simple quadratic to be solved for  $n(r)$ , See Figure 6,

#### 4. Decentralization and Regulation Q

We have developed an equilibrium by treating production and banking services as different functions carried out within a single household. But in this constant returns environment, we can scale these activities up or down and decentralize them into banks and households. With free entry and competitive pricing—including interest rates on demand deposits—the decentralized equilibrium coincides with the results of Section 2, We then use the decentralized model to consider the effects of Regulation Q: the prohibition of interest payments on commercial bank deposits.

To carry this out, we need to allocate the two payment activities, “trips to the bank”  $\phi n$  and “check processing”  $k(1 - F(\gamma))$  between banks and households. As in Section 3 we assume that “trips” are a household activity that does not appear in the national accounts or as employment, and that banks process checks using labor hired from households at the equilibrium wage  $Py$ . Households pay the bank a fee  $Q = Mq$  per check processed. The technology of check processing is unchanged.

As in the basic model, the normalized monetary base is divided into currency and reserves held against deposits, (2.3). The deposit of  $\theta d$  entitles the depositor to withdraw  $d$  in checks. We treat this as a loan of  $(1 - \theta)d$  to the household by the bank, entitling the bank to  $r(1 - \theta)d$  at the end of the period. If the bank may pays

interest at rate  $r_d$  on deposits the net interest cost to the household of setting up a deposit of size  $d$  is  $(r(1-\theta) - r_d)d$ . Finally, if the bank should make a profit this would be paid as a lump sum  $\Pi$  to households in their capacity as shareholders. The numbers  $(q, r_d, \Pi)$  are taken as given by depositors. Their equilibrium values will be determined below.

The flow budget constraint facing the household is then

$$m' = \frac{m + \pi + (r_d - r(1-\theta))d + \Pi + py(1-\phi n) - px - qx(1-F(\gamma))}{1 + \pi}. \quad (4.1)$$

The household's Bellman equation is

$$v(m) = \max_{x,c,d,\gamma,n} \{U(x) + \beta v(m')\}$$

subject to (2.3) and the cash constraints (2.4) and (2.5).

The analysis of the household's problem closely parallels the argument of Section 2. The steady state first-order and envelope conditions imply

$$\frac{1}{r}q = \frac{p}{\mu} \frac{\gamma}{n} \frac{r_d}{r} \quad (4.2)$$

and

$$\frac{n}{r}py\phi = c + \left(1 - \frac{r_d}{r}\right)d. \quad (4.3)$$

The cash constraints (2.3)-(2.5) must all bind. Given prices  $(q, r_d, \Pi)$ , then, we have five equations in the household's decision variables  $x, c, d, \gamma$  and  $n$ .

In an equilibrium, we must have  $m' = m = 1$  which implies that

$$(r_d - r(1-\theta))d + \Pi + py(1-\phi n) = px + qx(1-F(\gamma)). \quad (4.4)$$

As before, market clearing in goods and cash requires

$$x = [1 - \phi n - k(1 - F(\gamma))]y \quad (4.5)$$

and

$$c + \theta d = 1. \quad (4.6)$$

Banking has a constant returns technology in this model. With free entry, we seek a zero profit equilibrium:  $\Pi = 0$ . We note that if  $\Pi = 0$ , if  $r_d = (1 - \theta)r$ , and if  $pyk(1 - F(\gamma)) = qx(1 - F(\gamma))$  (or  $pyk = qx$ ), then (4.2) and (4.3) become

$$\frac{1}{r}k = \frac{x}{y} \frac{1}{\mu} \frac{\gamma}{n} (1 - \theta)$$

and

$$\frac{n}{r}py\phi = c + \theta d = 1,$$

replicating exactly (2.7) and (2.8). The prices  $(q, r_d, \Pi) = (pyk/x, r(1 - \theta), 0)$  thus implement the equilibrium of the basic model. At these prices, (4.4) follows from (4.5).

This equivalence between a decentralized equilibrium and a relatively centralized one is familiar, but of course it depends on prices being market-determined. In our application to the U.S. monetary system, though, this assumption seem's dubious. A notable feature of the 1933 Glass-Steagall Act was Regulation Q: the prohibition of interest payments on commercial bank deposits. This prohibition was in force from 1933 until 2011, or almost the entire period covered in the figures we have reviewed. This complication needs to be discussed.

If the inequality (4.7) does not hold and Regulation Q is enforced, setting  $r_d = 0$  and preventing entry of other suppliers of checkable, deposits banks make positive profits. Banks will compete for deposits by offering services. In the current model, that can only be done by reducing the cutoff level  $\bar{\gamma}$  above which checking is free. In any such equilibrium, households will always use checks for transactions above  $\bar{\gamma}$  and cash for smaller transactions. The constraints (2.4) and (2.5) will bind and be fully determined by  $\bar{\gamma}$  and the households choices of  $n$  and  $px$  :

$$c = \frac{px}{n}\Omega(\bar{\gamma}) \quad \text{and} \quad d = \frac{px}{n}[1 - \Omega(\bar{\gamma})]. \quad (4.9)$$

Equation (2.14), evaluated at  $\gamma = \bar{\gamma}$ , will continues to hold

$$r [\Omega(\bar{\gamma})(\bar{\gamma}) + \theta (1 - \Omega(\bar{\gamma})(\bar{\gamma}))] (1 - \phi n - k (1 - F(\bar{\gamma}))) = \phi n^2 \quad (4.10)$$

but in general  $\bar{\gamma}$  will not be the same as the  $\gamma(r)$  determined in (2.14)-(2.15) and the  $n$  that satisfies (4.10) will differ from the  $n(r)$  that satisfies (2.14)-(2.15).

We have not fully characterized the equilibrium possibilities, but it is clear that in the absence of entry the induced distortion will be to increase deposits relative to currency. Indeed, service competition did occur increasingly as interest rates rose from the 1960s on, taking the forms of free checking and other inducements. But it is also clear that the net effect of responses to Regulation Q was a decrease in deposits relative to currency from about 1990 on. We think this must be attributed to the emergence of other assets or arrangements that assumed part of the function in the payments system that commercial bank deposits has served earlier.

## 5. MMDAs

In this section we incorporate money market deposit accounts, the most important new addition to the stock of checkable deposits. As suggested by Figure 3 and illustrated in Figure 6 we will form a new aggregate by simply adding MMDAs to M1. We call this M1J. But MMDAs and ordinary demand deposits coexist—they are not perfect substitutes—so we need to model the distinct roles that these two assets play in the payment system. We do this by applying the Freeman-Kydland approach to three rather than two means of payment and two cutoff points: one between currency and deposits and another between the two deposit types. Moreover, as we argued in Section 3, the first of these cutoffs does not appear to be interest sensitive and we can do as well by thinking of a fixed fraction of payments  $\omega$  in cash and the remaining  $1 - \omega$  in checks, either on commercial banks or on MMDAs. The transaction size distribution  $F$  and its density  $f$  now refer only to the fraction  $1 - \omega$  of total purchases  $px$  that are paid for by some kind of check.

On the two assets, we use  $Pd$  and  $Pa$  for their dollar values,  $k_d$  and  $k_a$  for the labor costs per check and  $\theta_d$  and  $\theta_a$  for their reserve requirements. The natural assumptions

are that  $k_a > k_d$  and  $\theta_a < \theta_d$ . If the cutoff transaction size is  $\gamma$  (now serving a different function from our use in Sections 2-4), equilibrium production will be

$$x = [1 - \phi n - k_d F(\gamma) - k_a(1 - F(\gamma))] y. \quad (5.1)$$

We maintain the assumption of a constant growth rate  $\pi$  in the monetary base  $m$  and consider an equilibrium steady state in which  $\pi$  is the economy's inflation rate. The associated nominal interest rate  $r = \rho + \pi$ . Using the same notation and normalization as in Section 2, the cash constraints facing this consolidated household/bank are

$$m \geq c + \theta_d d + \theta_a a, \quad (5.2)$$

$$nc \geq \omega px, \quad (5.3)$$

$$nd \geq (1 - \omega) px \frac{1}{\mu} \int_0^\gamma z f(z) dz = (1 - \omega) \Phi(\gamma) px \quad (5.4)$$

and

$$na \geq (1 - \omega) px \frac{1}{\mu} \int_0^\gamma z f(z) dz = (1 - \omega) (1 - \Phi(\gamma)) px, \quad (5.5.)$$

where  $\Phi(\gamma)$  is the fraction of dollar purchases paid for from demand deposits and  $1 - \Phi(\gamma)$  is the fraction paid for from an MMDA.

The household Bellman equation is

$$v(m) = \max_{x,n,c,d,a,\gamma} \left\{ U(x) + \beta v \left( \frac{m + \pi + p(1 - \phi n - k_d F(\gamma) - k_a(1 - F(\gamma))) y - px}{1 + \pi} \right) \right\}.$$

We use the first order and envelope conditions to characterize the steady state. In the steady state  $m = 1$  and (5.1)-(5.5) hold with equality. The two other equilibrium conditions are found to be

$$\frac{1}{r} \phi p y = \frac{c + \theta_d d + \theta_a a}{n} \quad (5.6)$$

and

$$\frac{1}{r} (k_a - k_d) y = (1 - \omega) \frac{x}{n} \frac{1}{\mu} \gamma (\theta_d - \theta_a). \quad (5.7)$$

For  $r > 0$  all constraints bind, so that

$$c + \theta_d d + \theta_a a = 1, \quad (5.8)$$

$$nc = \omega px, \quad (5.9)$$

$$nd = (1 - \omega) px \Omega(\gamma), \quad (5.10)$$

and

$$na = (1 - \omega) px (1 - \Omega(\gamma)) \quad (5.11)$$

Then (5.1) and (5.6)-(5.11) are seven equations in  $x, n, c, d, a, \gamma$  and  $p$ .

From (5.6) and (5.8) we obtain

$$\frac{1}{r} \phi py = \frac{1}{n}. \quad (5.12)$$

Add (5.9) to (5.10) times  $\theta_d$  and (5.11) times  $\theta_a$  to obtain

$$n = px [\omega + (1 - \omega) [\theta_d \Omega(\gamma) + \theta_a (1 - \Omega(\gamma))]] \quad (5.13)$$

Now (5.1), (5.7), (5.12) and (5.13) involve only  $px, py, n$  and  $\gamma$ . Eliminating  $px$  and  $py$  gives two equations in  $n$  and  $\gamma$

$$n = \frac{(k_a - k_d) \mu}{\phi \gamma} \left[ \frac{\omega + (1 - \omega) \theta_a}{(1 - \omega) (\theta_d - \theta_a)} + \Omega(\gamma) \right] \quad (5.13)$$

$$n = \left[ \frac{(k_a - k_d) \mu}{(\theta_d - \theta_a) (1 - \omega) r \gamma} + \phi \right]^{-1} [1 - k_a + F(\gamma) (k_a - k_d)] \quad (5.14)$$

which can be solved for  $n(r)$  and  $\gamma(r)$ .

Then, the ratio of money to output can be written as

$$\frac{c + d + a}{px} = \frac{\omega px + (1 - \omega) px \Omega(\gamma) + (1 - \omega) px (1 - \Omega(\gamma))}{npx} = \frac{1}{n(r)}$$

As in Section 2, we require an addition constant of proportionality, and we write

$$\frac{m_1}{px} = \frac{A}{n(r)} \quad (5.15)$$

The ratio of cash to money is assumed to be 0.2, and the ratio of demand deposits to total deposits is

$$\frac{d}{d + a} = \Omega(\gamma(r)) \quad (5.16)$$

## 6. Calibration and simulation-2

We now calibrate the parameters of the model, following the logic described in Section 3. We need numerical values for the constants  $\phi, k_d, k_a, \theta_d, \theta_a$  and  $\omega$ , and we need to specify the form and parameters of the transaction size distribution  $F$  and the scaling parameter  $\mu$ . Beginning with  $F$ , let  $z \in [0, \infty)$  and

$$f(z) = \frac{\eta - 1}{(1 + z)^\eta}, \quad \eta > 1.$$

Then the functions  $F$  and  $\Omega$  are

$$F(\gamma) = \int_0^\gamma f(z) dz = \int_0^\gamma \frac{\eta - 1}{(1 + z)^\eta} dz = \left(1 - \frac{1}{(1 + \gamma)^{\eta-1}}\right)$$

and

$$\Omega(\gamma) = \frac{1}{\mu} \int_0^\gamma z f(z) dz = \frac{1}{\mu} (\eta - 1) \int_0^\gamma \frac{z}{(1 + z)^\eta} dz = \frac{1}{\mu} \frac{1}{(\eta - 2)} \left( \frac{\gamma(1 - \eta) - 1 + (1 + \gamma)^{\eta-1}}{(1 + \gamma)^{\eta-1}} \right)$$

where

$$\mu = \int_0^\infty z f(z) dz.$$

Equations (5.13) and (5.14) of the model, given the solutions above, can be written

$$n = \frac{(k_a - k_d) \mu}{\phi} \frac{1}{\gamma} \left[ \frac{\omega + (1 - \omega) \theta_a}{(\theta_d - \theta_a) (1 - \omega)} + \frac{1}{\mu} \frac{1}{(\eta - 2)} \left( \frac{\gamma(1 - \eta) - 1 + (1 + \gamma)^{\eta-1}}{(1 + \gamma)^{\eta-1}} \right) \right] \quad (6.1)$$

$$n = \left[ \frac{(k_a - k_d) \mu}{(\theta_d - \theta_a) (1 - \omega) r \gamma} + \phi \right]^{-1} \left[ 1 - k_a + \left(1 - \frac{1}{(1 + \gamma)^{\eta-1}}\right) (k_a - k_d) \right] \quad (6.2)$$

We call the right side of (6.1)  $\psi(\gamma)$ . Abbreviating some constants, we write  $\psi(\gamma)$  as

$$\psi(\gamma) = \frac{A}{\gamma} + B \left( \frac{(1 - \eta)}{(1 + \gamma)^{\eta-1}} + \frac{1}{\gamma} \left[ 1 - \frac{1}{(1 + \gamma)^{\eta-1}} \right] \right)$$

Provided  $\eta > 2$  (which is the case in the examples we compute),  $\psi'(\gamma) < 0$ ,  $\psi(\gamma) \rightarrow \infty$  as  $\gamma \rightarrow 0$ , and  $\psi(0) \rightarrow \infty$  as  $\gamma \rightarrow \infty$ .

The right side of equation (6.2) defines an increasing function  $n = g(\gamma)$ , where  $\lim_{\gamma \rightarrow 0} g(\gamma) = 0$  and  $\lim_{\gamma \rightarrow \infty} g(\gamma) = (1 - k_d) / \phi$ . In addition, increases in  $r$  shift the

$g$  function upwards, reducing  $\gamma$  and increasing  $n$ . This functional form adds one free parameter. We will solve the model for several values of  $\eta$ .

We benchmark the model to the year 1984, which is the first year for which we have separate data on MMDA's. It is also two years after the MMDA's were allowed in the US, which provides enough time for the desired substitution between demand deposits and MMDA's to occur. Following the discussion in Section 3, we set the ratio of currency to deposit equal to 0.25, so the ratio of currency to money is  $\omega = 0.2$ .

The short term interest rate on Tbills in 1984 was 9.5%. The ratio of MMDA to other deposits is 1, so this means that

$$\frac{\Omega(\gamma(.095))}{1 - \Omega(\gamma(.095))} = 1$$

As before, we assume that total household time spent on transactions is 1% of GDP, and that total output spent on check clearing at banks for both deposit types is 2% of GDP. Thus, the following two equations must hold for 1985

$$n\phi = 0.01 \tag{6.3}$$

$$k_d F(\gamma(.095)) + k_a(1 - F(\gamma(.095))) = 0.02 \tag{6.4}$$

As before, we set the reserve requirement  $\theta^d = 0.1$ . We also set  $\theta^a = 0.01$ .

Then, given a value for  $\eta$ , equations (6.1)-(6.4) imply

$$n(0.095)\gamma(0.095, \eta) = \frac{(k_a - k_d)}{\phi} \left[ \frac{0.2 + 0.8 * 0.01}{(0.1 - 0.01) * 0.8} + \Omega(\gamma(0.095, \eta)) \right]$$

and

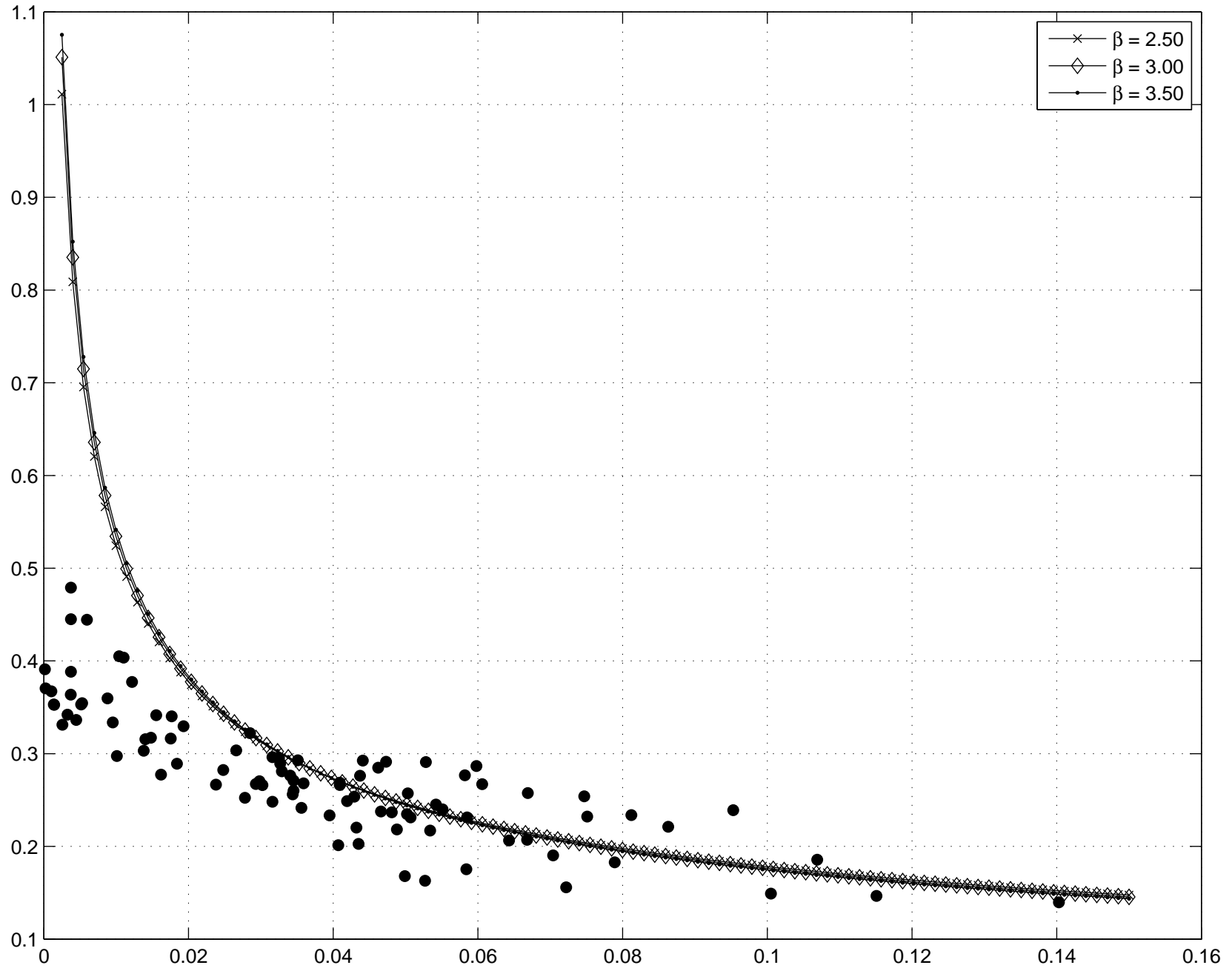
$$n(0.095) \left[ \frac{(k_a - k_d)}{(0.1 - 0.01) * 0.8 * r\gamma(0.095, \eta)} + \phi \right] = [1 - k_a + (k_a - k_d) F(\gamma(0.095, \eta))],$$

which pin down the values for  $\phi, n(0.095), k^d, k^a$ . Finally, we pick  $A$  so the money to output ratio goes through the grand-mean of the period 1915-2008.

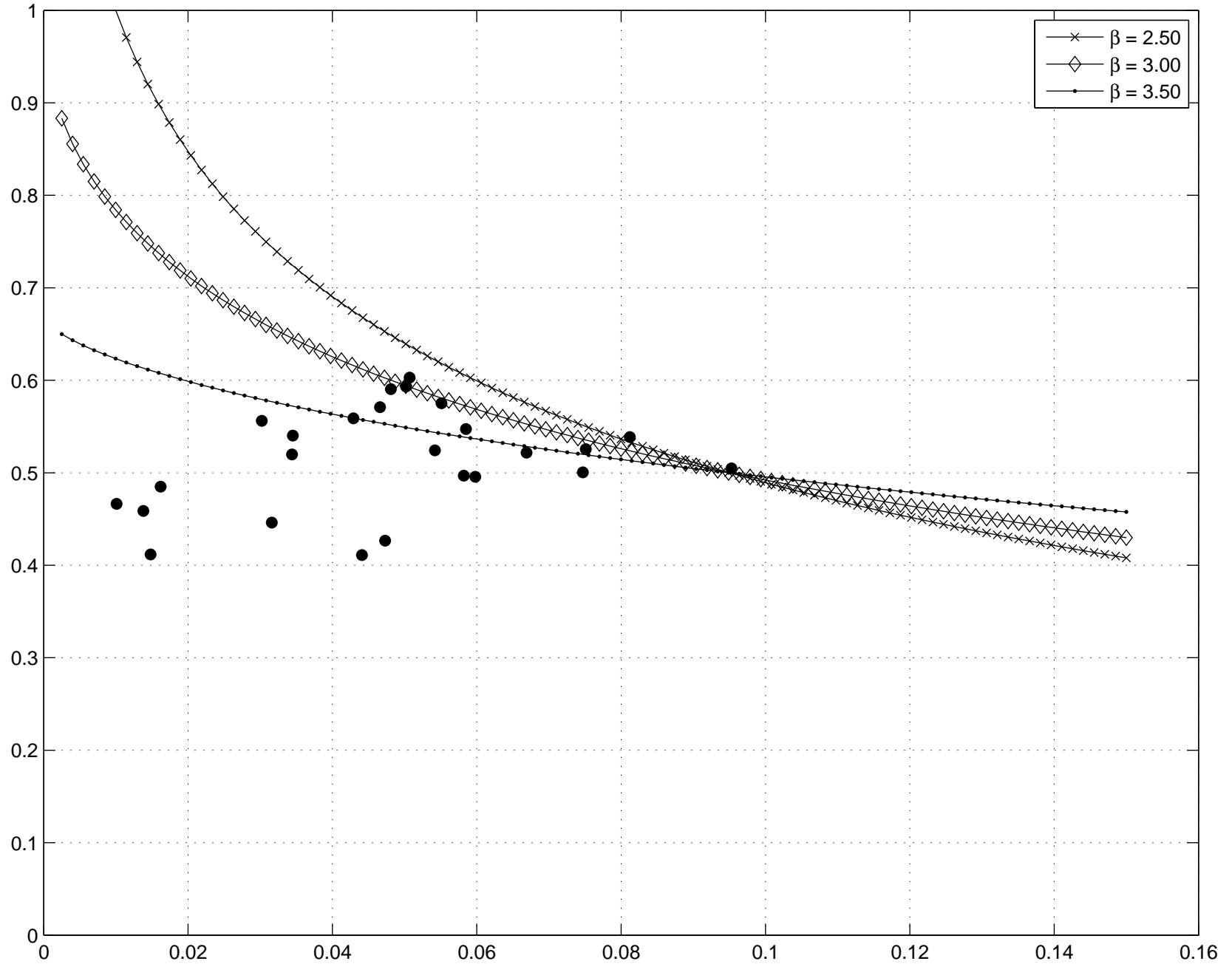
We solve the model for three different values of the parameter  $\eta$  (2.5, 3 and 3.5). The resulting values for the parameters  $\phi, k^d, k^a$  change very little when we vary the parameter  $\eta$ , and are given in the following Table.



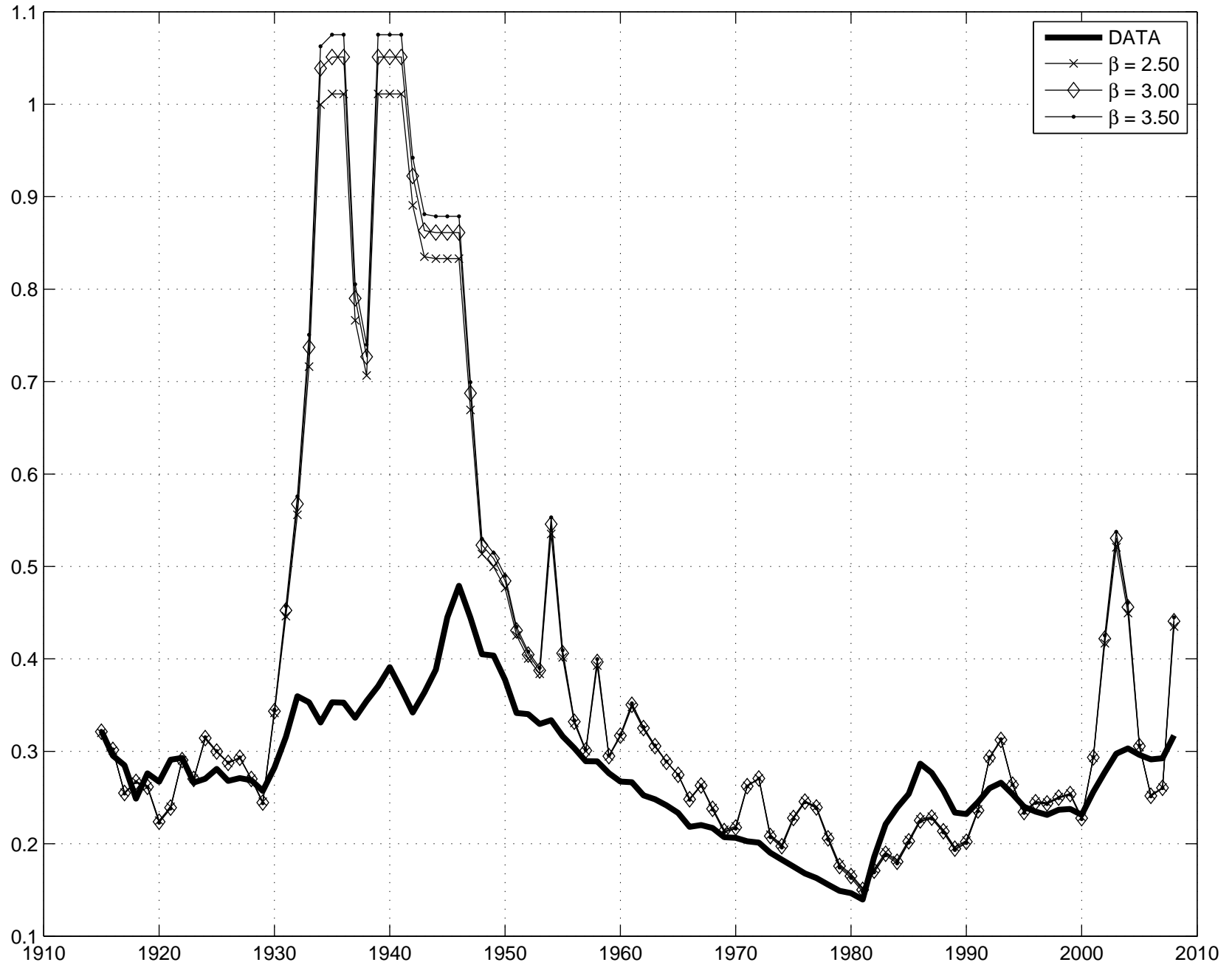
MONEY BALANCES AS A % OF GDP VS. INTEREST RATE 3MTBILL, 1915 – 2008



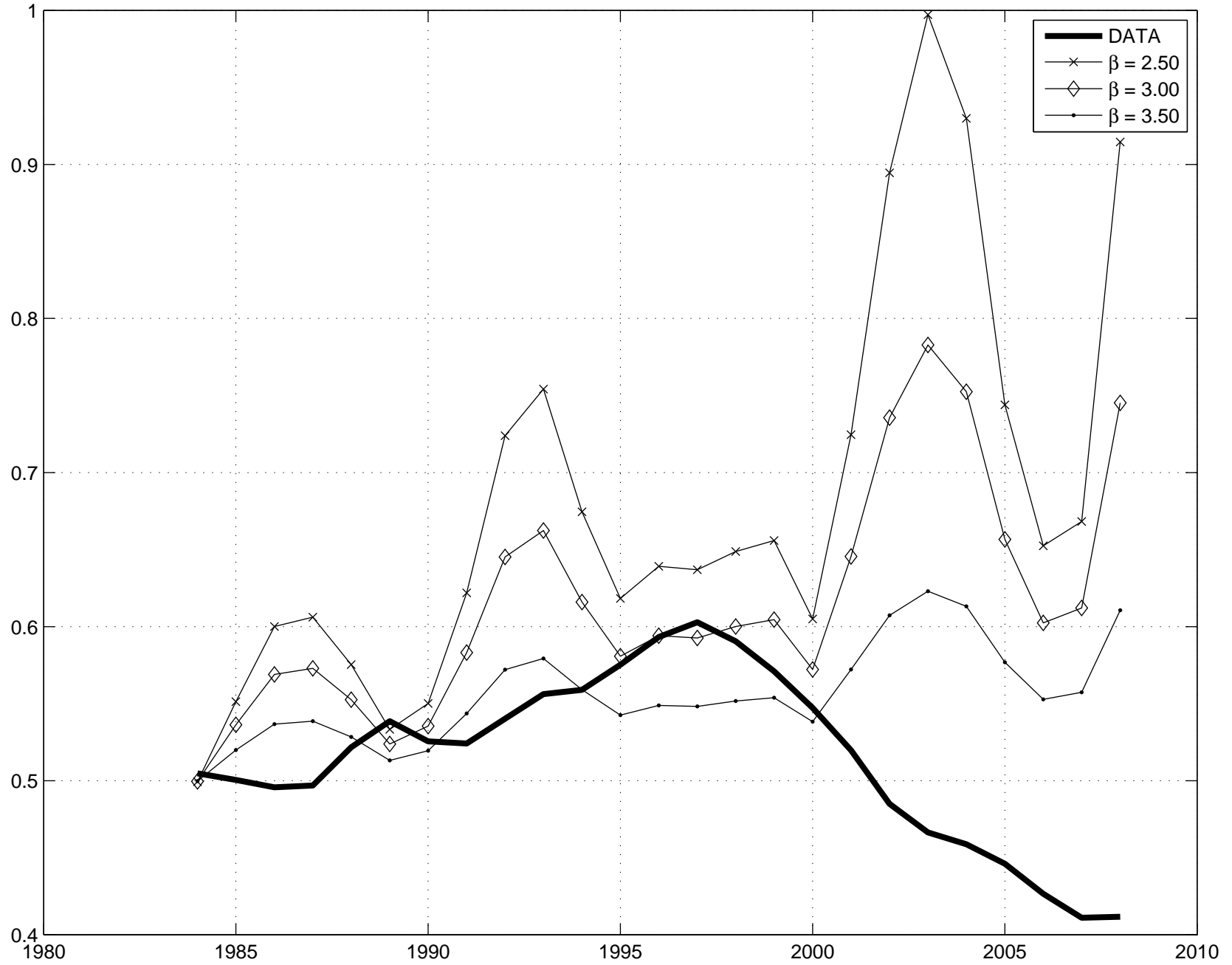
DEP / (DEP + MMDAS) VS INTEREST RATE 3MTBILL, 1984 – 2008



MONEY BALANCES AS A % OF GDP, 1915 – 2008



DEP / (DEP + MMDA), 1984 –2008



$\eta$	$\phi$	$k^a$	$k^d$
2.5	0.0044	0.0254	0.0189
3	0.0044	0.0265	0.0194
3.5	0.0044	0.0291	0.0197

The results are shown in Figures 7 - 10. The first two plot the theoretical curves for the M1J to GDP ratio and the interest rate, together with the data. The first graph has data since 1915, the second graph has data only since 1984 (the denominator is zero before that year). The other two, have the time series of the same data. As can be seen, the behavior of the monetary aggregate barely changes when we we change the shape of the density  $f$ . On the other hand, the behavior of each component depends crucially on that parameter.

The time series of the DD to MMDA ratio and the interest rate shown on Figure 10 clearly misses on the high frequency movements, just as we have seen in the plot of the M/GDP ratio. The curve for  $\eta = 3.5$ , captures reasonably well the trend until the end of the 90's. This is the time in which sweeps become very important. For all parameter values shown, the model implies that the ratio of DD to MMDA should trend upwards from 1984 till 2008, when the TBill rate was trending downwards. The model will clearly miss the behavior of the ratio since the end of the 90's.

## 7. Conclusions

We view the new aggregate M1J as a step toward doing for M1 what Motley (1988) and Poole (1991) did for M2 with their new aggregate MZM. Both are attempts to define monetary aggregates by the *functions* they have in the payments system rather than by the institutions whose liabilities they are. Roughly speaking, the deposit component of M1J includes and is limited to deposits you can write checks on, on paper or electronically, a measure of the same thing that M1 measured in the

past. The new aggregate does about as well on low and medium frequencies over the period 1915-2008 as M1 did from 1915 to 1990, as about as poorly on high frequencies.

Our application of the Freeman/Kydland model of the components of these broader aggregates, reported in Sections 3 and 6, is frankly exploratory. The whole question of the substitutability among various means of payment is captured by the distribution of transaction sizes. We have just begun to think through and test the many possibilities.

Finally, since we are writing in 2012, we should emphasize that the evidence we used is all pre-crisis data and the analysis is all based on theoretical steady states. We are trying to get the quantity theory of money back to where it seemed to be in 1980, but after all the older theories were as silent on financial crises as is this one is.

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