Interest Rates and Prices in a State Dependent Inventory Model of Money with Costly Credit*

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Abstract

Inventory models of money demand dating back to Baumol (1952) and Tobin (1956) have a long and distinguished place in monetary economics. An important outgrowth of the initial literature is the recent development of more fully specified asset market segmentation. Using the segmented market models of Alvarez and Atkeson (1997) and Alvarez, Atkeson, and Kehoe (2002), we analyze the impact that access to credit has on interest rates and prices. In our formulation, a significant amount of transactions take place using credit, which is modeled along the lines of Schreft (1992) and Dotsey and Ireland (1996). We find that following a monetary shock is very difficult to generate either liquidity effects or significant price stickiness with segmented markets once an endogenous choice of transaction medium is allowed.

Keywords: Segmented markets, Credit, Money

JEL classification numbers:

1 Introduction

Inventory models of money demand dating back to Baumol (1952) and Tobin (1956) have a long and distinguished place in monetary economics. An important outgrowth of the initial literature is the recent development of more fully specified asset market segmentation. Seminal papers are those of Alvarez and Atkeson (1997) and Alvarez, Atkeson, and Kehoe (2002), which use the existence of fixed costs for transfering funds between assets and transaction media to explore a host of issues. Importantly, these models can account for sluggish movements in prices and a liquidity effect in interest rates[1]. However, a potentially key assumption in this literature is the restriction that money is the only available transactions vehicle. That restriction overlooks the fact that a meaningful amount of transactions take place using credit and thus, an important margin of choice is abstracted

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[1] The methodology has also been used to examine asset pricing behavior, such as the equity premium (for example see Gust and Lopez-Salido, 2010) and exchange rate behavior (see Atkeson et al., 2002).

from. To investigate the effects of allowing agents to use credit for transactions, we model credit use along the lines of Schreft (1992) and Dotsey and Ireland (1996). A main outgrowth of allowing agents to use credit for transactions is that it allows them to smooth consumption of not only goods bought with credit, but goods bought with cash. Thus, consumption profiles in the presence of credit are quite different from those that are obtained when transactions credit is unavailable. The relative smoothness of cash consumption occurs even in the presence of a significant degree of market segmentation. In turn, the change in behavior that results from including transactions credit has significant implications for the propagation of monetary shocks. Namely, it impairs the model's ability for generating either liquidity effects or price stickiness even in the precense of significant market segmentation. Also, because the use of transactions credit has meaningful implications for the allocation of consumption across different agents, impairing its use has significant negative effects on economic behavior. Further, the secular increase in the use of credit implies time varying behavior in the economy's response to monetary shocks as economies with little credit behave somewhat differently from economies that actively use transactions credit. Importantly, understanding the influence of money on economic activity also requires a careful consideration of how credit is used in transactions as the two media of exchange are intimately related.

Our work is most closely related to that of Alvarez, Atkeson, and Edmonds (2009) and especially to that of Khan and Thomas (2011).[2] Both papers analyze the effects that asset market segmentation has on the inventory behavior of money balances and the subsequent relationship between that behavior and the behavior of velocity, interest rates, and prices. Khan and Thomas take the significant step of making portfolio decisions state dependent using a methodology similar to the one developed by Dotsey, King, and Wolman (1999) in the state-dependent pricing literature. They, thus remove a potential weakness from much of the segmented market literature in that agents are allowed to adjust the timing of their asset market use in response to economic fluctuations. In this paper we take the additional step of realistically adding another form of transactions, namely credit. Doing so has significant implications for the way in which segmented markets influence economic activity. No longer are bond prices determined by the marginal utility of active agents separated across time, but are determined by the common marginal utility of consumption via credit.

We mostly concentrate on the model's implications for the behavior of velocity, inflation, and the existence of a liquidity effect. It is these fundamental elements that are most influenced by the behavior of transactions costs between money and bonds. The segmentation in the model also has implications for asset markets, but we downplay those; first because they are not very significant, but more importantly because other forms of segmentation may be important for asset market pricing, segmentation that has nothing to do with transactions on goods and services.

Our research also relates to the money demand models of Guerron-Quintana (2009; 2011). In his model, households save using a savings account and buy goods using cash from a checking account. The author uses a Calvo-style framework to model infrequent portfolio rebalancing between the two accounts. The staggered portfolio decision results in a Phillips-type money demand curve that

[2] Other papers that we have found informative are those of Occhino (2008), add more Grossman and Weiss (1997).

resembles the partial adjustment money demand models of Goldfeld (1976). The resulting model shares the property of segmented market models that not all agents are able to adjust money holdings in response to shocks. As a consequence, velocity is not constant and money is non-neutral.

In the next section we describe our model and in the following section we discuss our calibration. That calibration differs significantly from much of the literature and differences do not stem solely from the incorporation of credit, but from what we believe is a more justifiable interpretation of the transactions costs involved in managing money holdings. We then proceed to a description of our benchmark steady state and to an analysis our model economy's dynamics with respect to monetary shocks. In doing so, we are able to show the contribution that state-dependent portfolio behavior and endogenous use of credit play in those dynamics. We then proceed to examine the model's behavior with respect to shocks to the interest rate under a Taylor rule, income and shocks to the availability of credit. After that, we look at how changes in the availability of credit over time affect steady-state value of velocity and how they are likely to induce time varying behavior in the economy's response to shocks. The last section concludes.

2 The Model

The economy is populated by a household with a continuum of shoppers having meaure one. The household is run by a benevolent parent and this "super" household construct is shown by Khan and Thomas (2011) to replicate a more complicated environment in which each shopper operates in isolation, but has access to a complete set of state-contingent contracts as in Alvarez, Atkeson and Kehoe (2002). The timing of the model is as follows. First the goods market opens and the household receives an endowment, y_t, which is distributed evenly to all the shoppers. In the goods market, there are two basic types of shoppers: (1) *inactive* shoppers who did not replenish their transactions balances in the end-of-last period's asset market, and (2) *active* shoppers who did. Both types of shoppers can use either money or credit when purchasing a good and the precise decision for doing so is specified below. After the goods market closes, the asset market opens and the household rebalances its portfolio and also decides which shoppers should visit the asset market and replenish their money holdings. Visiting the asset market involves a fixed cost and thus the decision of whether or not to participate in the asset market is endogenous and state-dependent. As in Khan and Thomas (2011), the only idiosyncratic shocks faced by members of the household (the shoppers) are these transaction cost shocks. Alternatively, agents could be faced with iid income or preference shocks, but for ease of comparison we procede as in the manner of Khan and Thomas. In what follows we shall use money and cash interchangeably. We follow a similar convention when talking about assets and bonds.

2.1 Goods Market and Evolution of Money Balances

The period starts off with shopper's proceeding to the goods market. Shopper's are indexed by "j", which denotes how many periods have transpired since the shopper last visited the bond or asset market. The fraction of each type of shopper is denoted by θ_j ($j = 1, ..., J$). Because there is a maximun fixed cost of being active in the asset market, there will be a maximum number of periods that any shopper will remain inactive. That number is given by J, and it is endogenously determined. A type 1 shopper is a shopper whose money balances were replenished in last period asset market and these balances are denoted $M_{0,t}$. Similarly a shopper who visited the asset market at $t-2$, has $M_{1,t}$ balances and there are $\theta_{2,t}$ of them. Finally, a shopper who last visited the bond market $t-J$ periods ago enters with $M_{J-1,t}$ balances and there are $\theta_{J,t}$ of them. This shopper will leave the goods market with zero balances because he will visit the asset market with probability one in the second half of this period. All other shoppers probabilistically visit the asset market based on their draw of a transactions cost. Here, $M_{0,t}...M_{J-1,t}$ are state variables, as are the $\theta_{j,t}$.

The shopper also has the choice of buying a good with cash or credit. As in Dotsey and Ireland (1996), goods are indexed by $i \in [0,1]$ with the cost of using credit monotonically increasing in i. Thus, there will be an endogenously determined cutoff for each type of shopper, $i^*_{j,t}$, and goods whose index is below the cutoff will be purchased with credit and those with an index greater than the cutoff will be purchased with cash. The goods purchased with credit are paid for in the succeeding asset market. Further, each shopper is costlessly wired a fraction of the income earned from selling last period's endowment in last period's goods market. We think of this as an automatic deposit into a shopper's checking account. Thus, the cash in advance constraints can then be written as

$$\begin{aligned}
M_{0,t} + \phi P_{t-1} y_{t-1} &\geq P_t \int_{i^*_{0,t}}^{1} c_{0,t}(i)di + M_{1,t+1} \qquad (1) \\
M_{j,t} + \phi P_{t-1} y_{t-1} &\geq P_t \int_{i^*_{j,t}}^{1} c_{j,t}(i)di + M_{j+1,t+1} \text{ for } j = 1 \text{ to } J-2 \\
M_{J-1,t} + \phi P_{t-1} y_{t-1} &\geq P_t \int_{i^*_{J-1,t}}^{1} c_{J-1,t}(i)di, + M_{J,t+1},
\end{aligned}$$

where $\phi P_{t-1} y_{t-1}$ is money earned last period that is costlessly deposited in the shoppers transaction account. The Lagrange multipliers associated with each of these constraints will be denoted by $\mu_{j,t}$ $j = 0, ...J-1$. Also note that since type $J-1$ shoppers will go to the asset market for sure next period. As long as the interest rate is greater than zero they will not hold any money balances upon exiting the goods market, $M_{J,t+1} = 0$.

2.1.1 The Asset Market

Next the asset market meets and the household rebalances its portfolio as well as paying for the goods bought with credit in the goods market. The key decision is how many shoppers should visit the asset market and replenish their transaction balances. There are θ_{jt} fraction of shoppers who have not visited the bond market for j periods, and the probability that they will visit the asset market and replenish their cash balances by trading bonds for money is α_{jt}. These probabilities will be determined endogenously based on the draw of an exongenous fixed cost of entering the asset market. Those who visit the asset market are referred to as *active* shoppers. Below, we discuss how these probabilities and fractions are endogenously determined. Let each active type j shopper withdraw $X_{jt} = M_{0,t+1} - M_{j,t+1}$ balances for use in next period's goods market, where we note that the solution should imply that $M_{J,t+1} = 0$ because a current type $J-1$ shopper is visiting the bond market for sure. Given that credit is costly to use, it is optimal for this shopper to exhaust all of his money balances before turning to credit. Other shoppers, who may not end up visiting the bond market, will generally want to carry some money over into the next period. Bond holdings evolve according to

$$\begin{aligned} B_t \quad \leq \quad & R_{t-1}B_{t-1} + P_t(1-\phi)y_t + T_t - \sum_{j=1}^{J} \alpha_{jt}\theta_{j,t}(M_{0,t+1} - M_{j,t+1}) - \\ & P_t \sum_{j=1}^{J} \theta_{jt}\Xi_{j,t} - P_t \sum_{j=1}^{J} \theta_{j,t} \int_0^{i^*_{j-1,t}} [\widetilde{c}_{j-1,t}(i) + q_t(i)]di, \end{aligned} \tag{2}$$

where the first term on the right of the inequality represents the dollar value of last period's bonds plus interest income, the second term is the fraction of the nominal value of this period's endowment (sold to the other identical households in the time t goods market) that automatically is deposited in the asset market account. Recall that a fraction ϕ will be wired to shoppers in next period's goods market. The third term is net government lump sum transfers, the fourth term represents the withdrawals made by financially active shoppers, and the fifth term reflects the withdrawal needed to pay the nominal value of the financial transaction costs incurred by financially active shoppers. The last term is the total expenditure associated with credit, which includes the amount of consumption as well as the cost of using credit on each good i, $q(i)$.

In particular, each type j shopper draws a fixed cost $\xi_{j,t}$ from the distribution $H(\xi)$, and decides to visit the asset market if that cost is less than some endogenously determined cutoff, $\xi^*_{j,t}$. Thus, $\Xi_{j,t} = \int_0^{\xi^*_{j,t}} ah(a)da = \int_0^{H^{-1}(\alpha_{jt})} ah(a)da$ and the expected cost of going to the asset market conditional on actually going to the asset market is $\Xi_{j,t}/\ \alpha_{j,t}$. Further, the fraction of those drawing a cost less than $\xi^*_{j,t}$, and hence replenishing their money balances is given by $\alpha_{j,t} = H(\xi^*_{j,t})$. $\alpha_{j,t}$ also represents the probability that a type j shopper will visit the asset market. Denote the fraction of individuals who were last financially active j periods ago as $\theta_{j.t}$. Thus, the fraction of individuals at $t+1$ who were active at t is $\theta_{1,t+1} = \sum_{j=1}^{J} \alpha_{j,t}\theta_{j,t}$ and the transition of individual types who were inactive in the current period is given by

$$\theta_{j+1,t+1} = (1-\alpha_{j,t})\theta_{j,t} \text{ for } j = 1 \text{ to } J-1 \tag{3}$$

The Lagrange multipliers associated with the transitions are denoted $\gamma_{j,t}$ ($j = 0, ...J-1$), where $\gamma_{0,t}$ is associated with $\theta_{1,t+1} = \sum_{j=1}^{J} \alpha_{jt}\theta_{jt}$. Because the transactions costs of exchanging bonds for money is distributed iid, all agents who pay the cost and exchange bonds for money are identical. They, therefore, leave the bond market with the same amount of money, which implies that the withdrawals of money are different for each type of shopper.

In addition, there are goods that are bought with credit and there is a cost associated with using credit. The last term in (2) is the direct cost of the goods bought with credit and the cost of using credit itself. We use a "$\sim$" to indicate that good i is being bought with credit. Further, we follow the modeling strategy of Schreft (1992) and Dotsey and Ireland (1996), where there is a continuum of identical goods arranged on a unit circle, and i indexes the location of each good. The fixed cost of using credit, $q_t(i)$, is indexed by the location of the good and is a continuous monotonically increasing function. Thus, as the index increases, the shopper will be less likely to use credit. As in the case of portfolio rebalancing, there will be a cutoff value across goods for which a type j shopper will find credit too expensive and will instead use cash rather than incur that cost. The cutoff is endogenously determined and denoted as i_j^*

The Lagrange multiplier associated with (2) will be donoted by λ_t.

2.2 Recursive Household Problem

Given the preceeding description, the household's problem can be written recursively as

$$\begin{aligned} V(\{M_{jt}\}_0^{J-1}, \{\theta_{jt}\}_1^J, B_{t-1}, y_{t-1}, y_t) \quad = \quad & \max_{\{c_{jt}\},\{\widetilde{c}_{jt}\},\{\alpha_{jt}\},\{i_{j,t}\},\{M_{j,t+1}\}} \{\sum_{j=1}^{J} \theta_{jt}[\int_0^{i_{j-1,t}^*} u(\widetilde{c}_{j-1,t}(i))di \\ & + \int_{i_{j-1,t}^*}^{1} u(c_{j-1,t}(i))di] + \beta \mathbb{E}_t V(\{M_{j,t+1}\}, \{\theta_{j,t+1}\}, B_t, , y_t, y_{t+1}) \end{aligned} \tag{4}$$

subject to (1), (3), and (2).

2.3 Government Budget Constraint

The government's budget constraint is given by

$$R_{t-1}B_{t-1} + T_t \leq \overline{M}_{t+1} - \overline{M}_t + B_t,$$

where B are one period nominal bonds and $\overline{M}$ is the aggregate nominal money supply. We assume that the growth rate of money supply $g_{m,t} = \overline{M}_{t+1}/\overline{M}_t$ follows an AR(1) process

$$g_{m,t} = (1-\rho_m)\, g_m + \rho_m g_{m,t-1} + \sigma_m \varepsilon_{m,t},$$

where $\varepsilon_{m,t} \xrightarrow{d} N(0,1)$.

2.4 Market Clearing

Goods market clearing requires

$$\sum_{j=1}^{J} \theta_{jt} \{ \int_{0}^{i^*_{j-1,t}} [\widetilde{c}_{j-1,t}(i) + q_t(i)] di + \int_{i^*_{j-1t}}^{1} c_{j-1,t}(i) di \} + \sum_{j=1}^{J} \theta_{j,t} \Xi_{j,t} \leq y_t, \tag{5}$$

and end of periond money market clearing requires

$$\sum_{j=1}^{J} \alpha_{j,t} \theta_{jt} [(M_{0,t+1} - M_{j,t+1}) + P_t \phi y_t + \sum_{j=1}^{J-1} \theta_{jt} M_{j,t+1} \leq \overline{M}_{t+1}. \tag{6}$$

Alternatively, at the beginning of the period money market clearing is given by

$$\sum_{j=1}^{J} \theta_{j,t} M_{j-1,t} + \phi P_{t-1} y_{t-1} = \overline{M_t}. \tag{7}$$

The first term gives the money balances that shoppers bring into the goods market and the second term is the funds costlessly wired into each shoppers transaction accounts.

2.5 First Order Conditions

The solution to the model is found by linearizing around a non-stochastic steady state. In particular, we are solving for the J consumptions of cash goods, $\{c_{j,t}\}_{j=0}^{J-1}$, and the consumption of the credit good, $\{\widetilde{c_t}\}$ (it is shown in the appendix that the consumption of cash goods is independent of the index i and only depends on the index j). Further, no matter what type the shopper is, he consumes the same amount of each type i good with credit. The only difference is the measure of goods bought with credit. We also solve for the J nominal money stocks $\{M_{0,t+1}\}_{j=0}^{J}$, J fractions $\{\theta_{j,t+1}\}_{1}^{J}$, the J transaction cost cutoffs $\{\xi^*_{j,t})$, the J Lagrange multipliers $\{\gamma_{j,t}\}_{j=0}^{J-1}$ associated with the evolution of the $\theta's, (3)$, bonds B_t, the nominal interest rate R_t, and the price level P_t. Also, we must calculate the resources used by the household in going to the financial markets, $\{\Xi_{jt}\}_{j=1}^{J}$ as well as the J cutoffs, i^*_j and the implied cumulative cost of using credit for each shopper, $\int_0^{i^*_j} q(i) di = Q(j)$. Thus, we are solving for $8 * J + 3$ variables as well as the maximal value of J.

2.5.1 The Behavior of Consumption

The first order conditions for consumption of various shoppers depends on whether the good is bought with cash or credit. These are given by

$$u'(\widetilde{c}_t)/P_t = \beta \mathbb{E}_t u'(c_{0,t+1})/P_{t+1}, \tag{8}$$

and for the various goods bought with cash

$$\alpha_{j,t}(u'(\widetilde{c}_t)/P_t) + \beta(1-\alpha_{j,t})\mathbb{E}_t(u'(c_{j,t+1})/P_{t+1}) = u'(c_{j-1,t})/P_t \quad j = 1 \text{ to } J-1. \tag{9}$$

The first equation indicates the tradeoff between purchasing an extra good with credit today verse a cash good tomorrow, and the second equation trades off the value of buying the good with cash today (the right hand side) with the weighted average of using credit today or keeping an extra unit of money and buying the good with cash tomorrow.

2.5.2 Pricing Bonds

The first order condition for bonds is

$$u'(\widetilde{c}_t)/P_t = \beta \mathbb{E}_t(u'(\widetilde{c}_{t+1})/P_{t+1})R_t. \tag{10}$$

One immediately notes that this differs from the typical bond pricing condition in segmented markets, in that it depends on the common consumption of the credit goods between periods and is independent of which agents are active in different periods. Thus, the use of credit links the different types of shoppers stochastic discount factors and this link is absent in models where money is the sole transactions medium. Furthermore, this means that consumption of the credit good determines how interest rates react to monetary injections and hence the strength of liquidity effects.

2.5.3 Determining the Use of Credit

Having determined consumption, we next examine the condition determining the cutoff for whether a good is bought with credit or cash. This cutoff point will depend on the index j, which is associated with how long an individual shopper has been unable to replinish his cash. Differentiating the household's objective function (4) with respect to the various cutoffs, $i_{j,t}$ yields the following condition,

$$[u(\widetilde{c}_t) - u(c_{j,t})] + [u'(c_{j,t})c_{j,t} - u'(\widetilde{c}_t)\widetilde{c}_t] = u'(\widetilde{c}_t)q(i^*_{j,t}). \tag{11}$$

A good will be bought with credit as long as the LHS of (11), which represents the benefit of purchasing an additional type of good, i, with credit is less that the cost as depicted by the RHS of (11).

2.5.4 Determining Whether to be Active

We now turn to the determination of whether a shopper visits the asset market to replenish transactions balances. As long as

$$\lambda_t P_t \xi^*_{j,t} \leq (\gamma_{0,t} - \gamma_{j,t}) - \lambda_t[(M_{0,t+1} - M_{j,t+1})] \quad \text{for } j = 1 \text{ to } J-1 \tag{12}$$

the shopper will become active. The various Lagrange multipliers, $\gamma_{j,t}$ have the interpretation of the value to the household of having an additonal type j shopper. Thus, the right hand side of (12) depicts the value of of being active rather than inactive adjusted for the utility cost of changing money balances. In turn the values of being a type j shopper follow the recursive relationships depicted by

$$\begin{aligned}
\gamma_{j,t} &= \beta\mathbb{E}_t\alpha_{j+1,t+1}\gamma_{0,t+1} + \beta(1-\alpha_{j+1,t+1})\gamma_{j+1,t+1} + \\
&\quad \beta\mathbb{E}_t[\int_0^{i^*_{j,t+1}} u(\widetilde{c}_{t+1})di + \int_{i^*_{j,t+1}}^1 u(c_{j,t+1})di] - \\
&\quad \beta\mathbb{E}_t\lambda_{t+1}\alpha_{j+1,t+1}(M_{0,t+2} - M_{j+1,t+2}) - \beta\mathbb{E}_t\lambda_{t+1}P_{t+1}\int_0^{i^*_{j,t+1}}(\widetilde{c}_{t+1} + q(i))di \\
&\quad -\beta\mathbb{E}_t\lambda_{t+1}P_{t+1}\Xi_{j+1,t+1} \\
\text{for } j &= 0, ..., J-2,
\end{aligned} \tag{13}$$

and

$$\begin{aligned}
\gamma_{J-1,t} &= \beta\mathbb{E}_t\gamma_{0,t+1} + \beta\mathbb{E}_t[\int_0^{i^*_{J-1,t+1}} u(\widetilde{c}_{t+1})di + \int_{i^*_{J-1,t+1}}^1 u(c_{J-1,t+1})di] - \\
&\quad \beta\mathbb{E}_t\lambda_{t+1}[M_{0,t+2} - M_{J-1,t+1} - \phi P_t y_t + P_{t+1}\int_{i^*_{J-1,t+1}}^1 c_{J-1,t+1}(i)di] - \\
&\quad \beta\mathbb{E}_t\lambda_{t+1}P_{t+1}\int_0^{i^*_{J-1,t+1}}(\widetilde{c}_{t+1} + q(i))di - \beta\mathbb{E}_t\lambda_{t+1}P_{t+1}\Xi_{J,t+1}
\end{aligned} \tag{14}$$

2.6 Calculating the Steady State

Conditional on knowing the cutoff value for using credit for each type of shopper and the cutoff values for going to the asset market, which then determines the $\alpha_{j,t}$ we can determine the other variables. We have the J equations for determining consumption, (8) and (9), along with goods market clearing to determine the $J+1$ various values of consumption. The CIA constraints along with the first order condition for $M_{0,t+1}$ can then be used to determine the various money holdings. Given these solutions, the cutoff values for credit can be ascertained and the multipliers $\gamma_{j,t}$ can be solved for. In turn, the cutoff values for going to the bond market and the expected costs of doing so can be calculated. In turn, knowing the cutoffs for credit allows one to calculate the cost of using credit. Iterating on these conditions until convergence is attained in the credit and asset market cutoff values or solving all the equations nonlinearly can produce the steady state values of the economy.

3 Calibration and Steady State Properties of the Model

There are a number of challenges involved in calibrating the model. These involve parameterizing the cost of using credit, $q(i)$, to match data on credit card use that nets out the convenience use of credit cards. Convenience use refers to purchases that are paid off immediately, and using a credit card in this way is no different from using a debit card. To do this, we use information in the 2010 Survey of Consumer Finances (SCF), which indicates that roughly eight percent of appropriately defined consumption is accomplished through credit. Making this calculation involves translating the income reported in the SCF with income reported in the national income accounts and then relating this number to consumption. In defining consumption that is closely linked with our model concept we remove consumption of implicit housing services and consumption related to medical expenditures. We delete the former because that consumption is largely non-market, and the latter because it mostly involves third-party payments. In our benchmark model 7.9 percent of consumption is done with credit, which is in line with our empirical estimate of 8.0 percent (see appendix for details). We also choose the parameters of the cost of using credit so that the short-run interest semielasticity of money demand is 2.7, a value that is in line with many empirical studies (for example see Guerron-Quintana; 2009).

The other central calibration issue involves the cost of participating in the asset market, $\Xi_{j,t}$. Here we differ from the literature, which uses a study of transactions in risky assets by Vissing-Jorgensen (2002). Her study uses data on portfolio transactions from the Consumer Expenditure Survey (CEX). In this survey, participating households disclose their holdings of both risky assets (stocks, bonds, mutual funds, and other such securities) and riskless assets (savings and checking accounts). Vissing-Jorgensen finds that the probability of buying/selling assets is 0.29 for individuals in the lowest financial wealth decile and 0.53 for those in the highest decile. These numbers, in turn, indicate that households rebalance their portfolios of risky assets somewhere in between every 22 to 41 months.

The results from Vissing-Jorgensen's research motivates the calibrations found in Alvarez, Atkenson, and Edmond (2009) and in Khan and Thomas (2011). The latter need a maximum state-dependent fixed cost that exceeds twenty five percent of output. We find that value of costs improbable, especially when Vissing-Jorgenson estimates those costs at between \$50.00 and \$260.00 per quarter in 2000 dollars. This would translate to a fixed cost of at most 1.73 percent of an average workers personal income. And we wish to reiterate that this calculation is for risky assets and assigns the entire reason for infrequent trade to transaction costs.

Rather than adopt the approach of basing our transactions frequency for replenishing transactions accounts on that data, we adopt a more conservative calibration. We calibrate our costs so that the maximum length of time between rebalancing a shopper's transaction account is six months. When thinking of the relevant reallocation of transactions accounts as occuring due to a transfer from M2 type savings vehicles to M1 transaction accounts, this seems like a cautious approach to transactions frequencies. We obtain this calibration with a maximal fixed cost of 9.7 percent of income and a total fixed cost of transacting that is only 1.0 percent of income, which still appears rather large and indicates that the six-month calibration is fairly conservative. Further, we obtain an annualized velocity of money equal to 6.9, which is fairly consistent with actual average consumption velocity of M1 over the period 1990-2007 when one subtracts the fraction of U.S. currency that is estimated to be held oversees.[3] Calculating velocity in that manner yields a number very close to 7.0.

We follow Alvarez, Atkeson, and Edmond (2009) and Khan and Thomas (2011) in assuming that sixty percent of income is costlessly deposited into the transactions accounts of shoppers. This calibration assumes that approximately 90 percent or more of labor income is directly deposited. We set the discount factor to .9975, which yields an annual risk-free interest rate of 3.0 percent. Finally, we assume some functional forms. The utility function is taken to be logarithmic: $u(c) = log(c)$; the cost of buying with credit is

$$q(x) = \upsilon \left(\frac{x}{1-x} \right)^{\delta}, \tag{15}$$

and the fixed costs ξ_j are drawn from a beta distribution with parameters α_d and β_d. The persistence of money growth is taken from Khan and Thomas (2011). Table 1 summarizes the parameter values used in our simulation exercises.

Table 1: Parametrization

	β	$\xi_{\max}$	α_d	β_d	υ	δ	g_m	ρ_m	σ_m	y_s	ϕ	J
Benchmark	0.9975	0.097	1.5	0.5	28.6	3.05	1.0024	$0.57^{1/3}$	1	1	0.6	6
No Credit	0.9975	0.0455	1.5	0.5	-	-	1.0024	$0.57^{1/3}$	1	1	0.6	6

α_d and β_d correspond to the parameters of the beta distribution

For our benchmark economy, the consumption of goods bought with money (cash goods) and money balances by type of shopper are shown in Figure 1 in red circles. One sees that consumption (panel a) and money balances (panel c) are monotonically declining as the time remaining inactive increases. Consumption of each good that is bought using credit is slightly higher than those purchased with cash and the number of goods bought with credit increases with the length of time since last replenishing money balances. That is, the optimal index, i, that determines whether an individual good is bought with cash or credit is increasing with j. This is shown in panel b of Figure 1, along with the probability that a shopper will be active (panel d). Figure 2 in turn reports the fraction of each type of shopper (θ_j).

[3] We assume that two-thirds of U.S. currency is held oversees, a number that is informed by the work of Porter and Judson (1996).

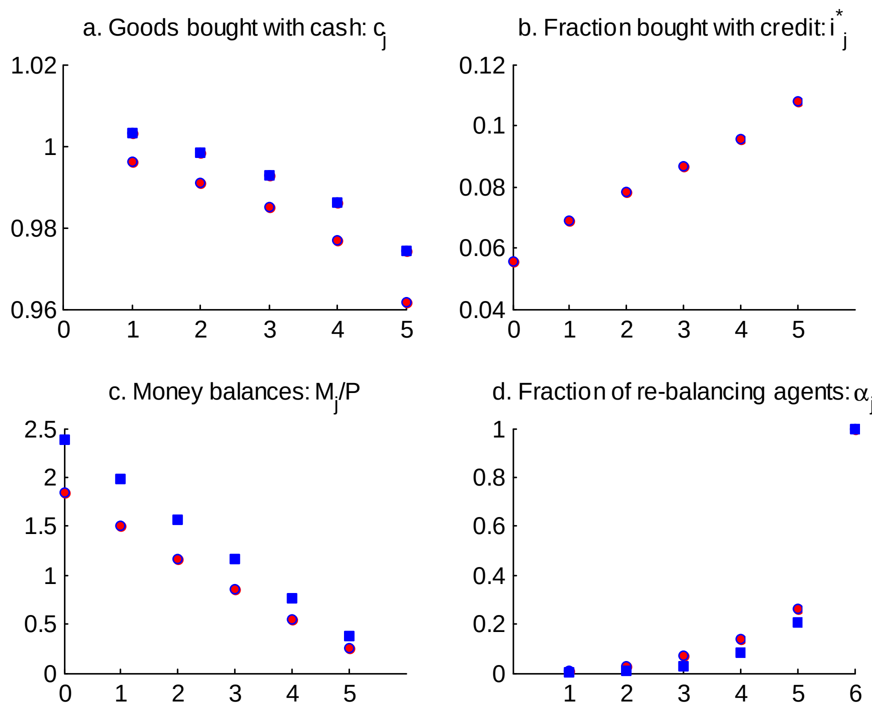


Figure 1: Steady State in the Benchmark and Nocredit Models

To compare our results with a model in which money is the only medium of transactions, we eliminate credit useage and calibrate the maximum fixed cost needed for a shopper to find it optimal to use financial markets at least once every six months. That model requires a maximal fixed cost of 4.55 percent of income. The comparable steady state values are shown in blue squares in Figures 1 and 2. It is interesting that in the benchmark model with credit a slightly greater fraction of shoppers choose to rebalance their portfolios. This is due the feature that in the credit economy money balances are somewhat lower and agents smooth consumption by both using credit and being a bit more active. Because shoppers in the no-credit economy hold somewhat larger money balances, this model yields a somwhat lower annual velocity of roughly six (see Figure 1 panel c).

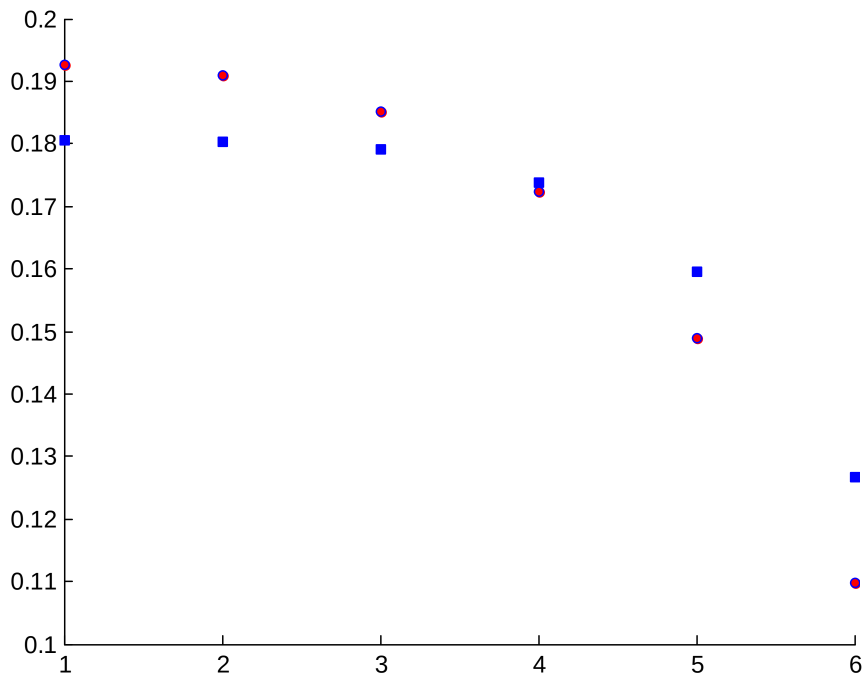


Figure 2: Fraction of Each Type of Shopper.

4 Dynamics in Response to Monetary Policy Shocks

In this section we analyze our benchmark model's dyanamics in response to both temporary and permanent money growth shocks and to a shock to the nominal interest rate. As a comparison, we also present results for a model in which agents have no access to transacting with credit.

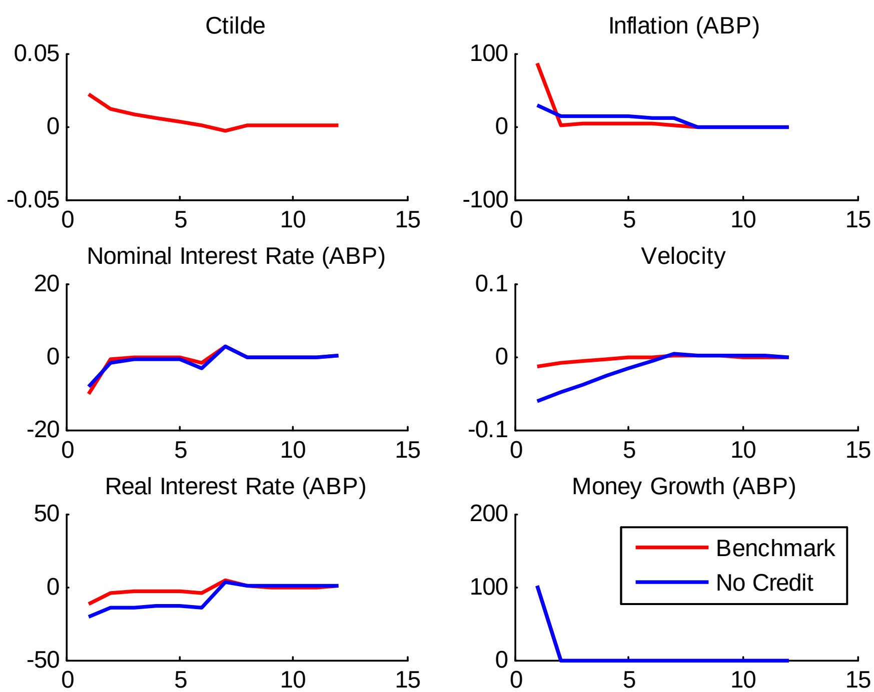
4.1 A temporary money growth shock

Figure 3 plots the impulse responses to a transitory 100 annualized basis points increase in the money growth rate. The red line corresponds to our benchmark economy while the blue line is the model without credit.[4] Consistent with previous studies, our benchmark model features a liquidity effect following a transitory monetary shock (Figure 3.a). There are two driving forces behind this result. First, the real interest rate is not very responsive to the money shock in the benchmark model because consumption of the credit good does not change dramatically, and hence the stochastic discount factor is roughly constant. Without access to transactions credit, active shoppers consumption jumps one period after the shock (Figure 3.b) as they are the only ones able to take advantage of the monetary injection. Recall that the asset market meets after the goods market and it is in the asset market that money is injected. Subsequent active shoppers do not get the same chance, and thus there is

[4] In the impulse responses, consumption and velocity are expressed in percentage deviation from steady state. Consumption corresponds to intensive consumption. Inflation, interest rates, and the money growth rate are in annualized basis points. The remaining variables are reported as deviations from steady state.

a large drop in consumption across consecutive active shoppers. As a result, the real interest rate, which is determined by the growth of active shopper's consumption between periods $t+1$ and $t+2$, declines significantly.[5] Further, one notices that the consumption of cash goods is much smoother over time and across shopper types, when credit is available for transactions use. We regard this as a key important aspect of the availability of transactions credit, and it will be a continuing theme in the experiments that follow. Second, as argued above, the ability to purchase goods using credit allows every shopper to respond instantaneously to the monetary injection. There is a large increase in nominal aggregate demand by all shoppers in the benchmark economy, and this results in an approximately one-for-one change in prices and current inflation. However, the rise in prices is temporary and there is not a large effect on expected inflation. Thus, the movements in the real interest rate dominates the change in the nominal rate.

Regarding velocity, because the price effects in the benchmark model resemble more closely a standard cash in advance economy than one with segmented markets, the almost one-to-one response of prices results in a muted response of velocity. In contrast, inflation in the model without credit rises by less than the monetary injection, which results in a delayed response of prices and the more pronounced decline in velocity. These findings confirm those from the creditless segmented market models of Atkeson et al. (2009) and Khan and Thomas (2011).



[5] The timing in the cash-only model implies that the Euler equation for bonds is given by

$$R_t = \beta^{-1} E_t \frac{u'(c_{0,t+1})}{u'(c_{0,t+2})} \pi_{t+2}.$$

Figure 3.a.: Response of Aggregate Variables to a Monetary Shock.

Figure 3.b shows that the consumption of cash goods has a small reaction regardless of the type of shopper. The response of cash goods in our benchmark model has a similar shape but is an order of magnitude smaller than that in the model without credit. The reason is that the sharp increase in inflation in our baseline economy makes it more difficult for households to consume using cash. Overall, the shoppers who have been recently active (c_0, c_1, c_2) benefit the most from the monetary injection. This comes at the expense of the cash-good consumption of shoppers who were active several periods ago (c_3, c_4, c_5), but the fall in consumption is much less dramatic than in the no-credit economy.

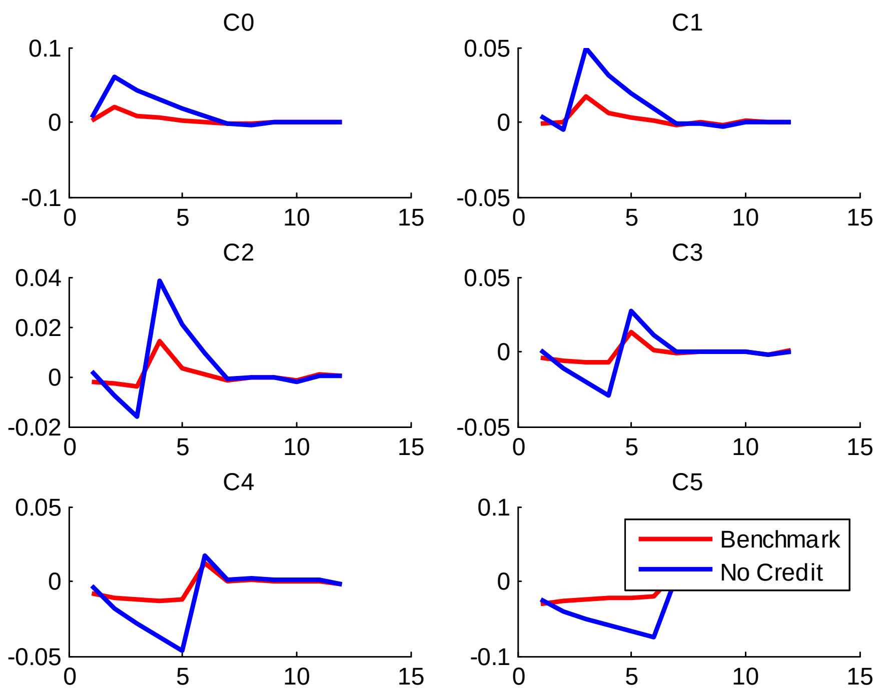


Figure 3.b.: Response of Consumption to a Monetary Shock.

Following the monetary innovation, all shoppers want to increase their consumption of credit goods (Figure 3.c). The reason is that the increase in inflation reduces the purchasing power of nominal money balances. Hence, shoppers use more credit to partially counterbalance the negative impact of declining real balances. However, since much of the money injection is consumed by inflation, there are no strong wealth effects in the transactions-credit economy and the increase in credit use is muted.

Interestingly, shoppers that went to the asset markets last period are the ones that increase their credit goods consumption the most (note the large i_0^*). The reason is that these shoppers know they are the less likely to go to the asset markets in the near future. So rather than depleting money balances to buy cash goods, they choose to smooth out consumption by relying more on credit.

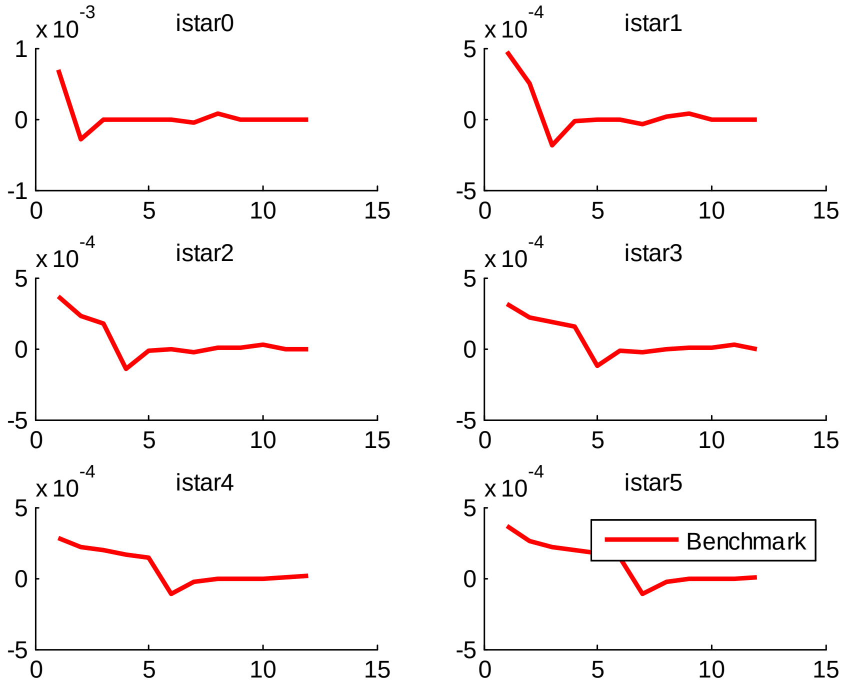


Figure 3.c.: Response of Fraction of Goods Bought with Credit to a Monetary Shock.

Figure 3.d shows that shoppers who were active several months ago are the ones becoming active following the monetary injection ($\alpha_3, ..., \alpha_5$) and that behavior is similar across both models. It occurs, however, for different reasons. With access to credit, the price level rises substantially eroding the purchasing power of the smaller money balances of shoppers who have not been recently active. They react by both increasing their use of transactions credit and by becoming active with an increasing frequency. In contrast, recently active shoppers have less incentive to transfer money holdings from the asset account to the checking account. The figure also shows that shoppers that re-balanced money balances yesterday have a muted response to the money shock in our economy with credit goods (α_1 with red color). The price response is more muted in the standard segmented markets model implying that inactive shoppers experience less of a decline in their real balances. They increase the frequency of going to the asset market because expected inflation induces them to take advantage of relatively low prices.

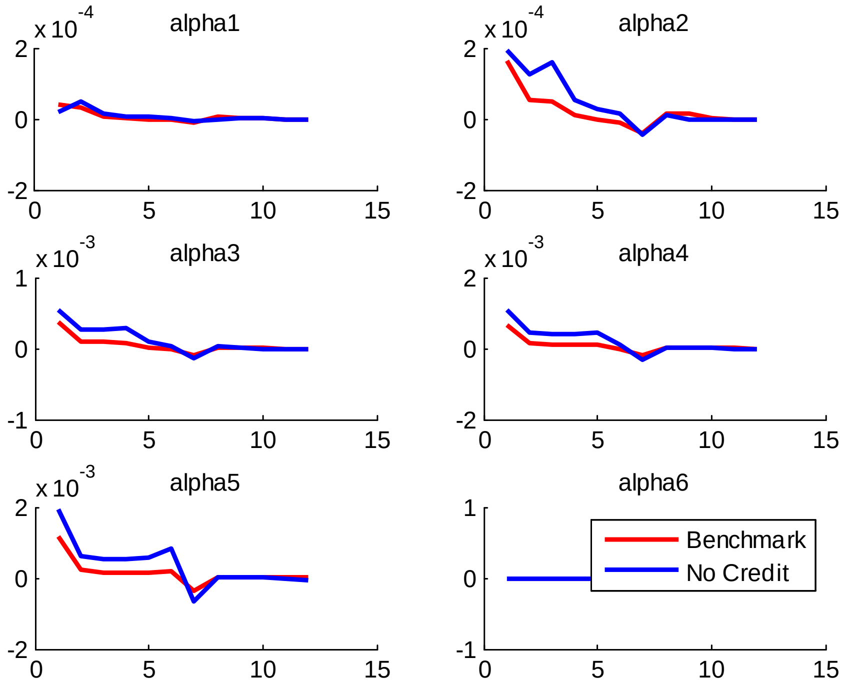


Figure 3.d.: Response Fraction Active Shoppers to a Monetary Shock.

Finally, Figure 3.e displays the response of the distribution of shoppers to the monetary expansion. Because the probability of becoming active behaves similarly across the two models, so does the evolution of the distribution of shopper types. The fraction of active shoppers increases a bit on impact and the fraction of long time inactive shoppers falls on impact. The impact behavior leads to an echo effect as the greater fraction of active shoppers makes its way through the distribution. After six months, which is longest period for which anyone would remain inactive, the distribution settles back to its steady state.

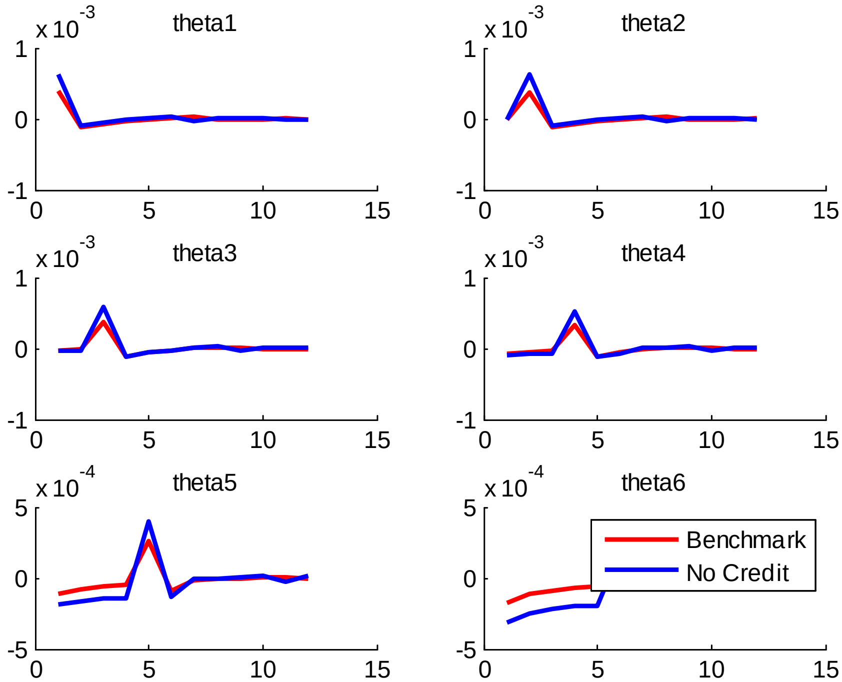


Figure 3.e.: Response Fraction of Shoppers to a Monetary Shock.

4.2 A persistent money growth shock

The responses to a persistence monetary shock are reported in Figure 4. The autocorrelation of money growth is set to .57 at a quarterly frequency, which is a fairly standard calibration in this literature. The money growth rate rises by 100 annualized basis points upon impact. The first striking result is that neither version of the model delivers a liquidity effect. In fact, the model without credit displays a greater increase in nominal rates. This greater increase occurs because expected future inflation is higher in the money-only model as price effects are more drawn out. As for the case with a temporary money shock, prices are very responsive in the benchmark model. They are much more responsive than in a standard cash-in-advance model, reflecting the fact that our benchmark calibration induces a relatively high *short-run* interest semi-elasticity of money demand (which is 2.7). In response to a persistent money growth shock, real money balances decline by 0.25 percent, which is responsible for the aggressive response of the current price level. In the long run, the price level and the money stock rise proportionately, but the rise in prices is front loaded and a majority of the price level increase occurs on impact. In the more standard segmented markets model, the price response occurs more gradually as different shoppers obtain the benefits of increased levels of transaction balances.

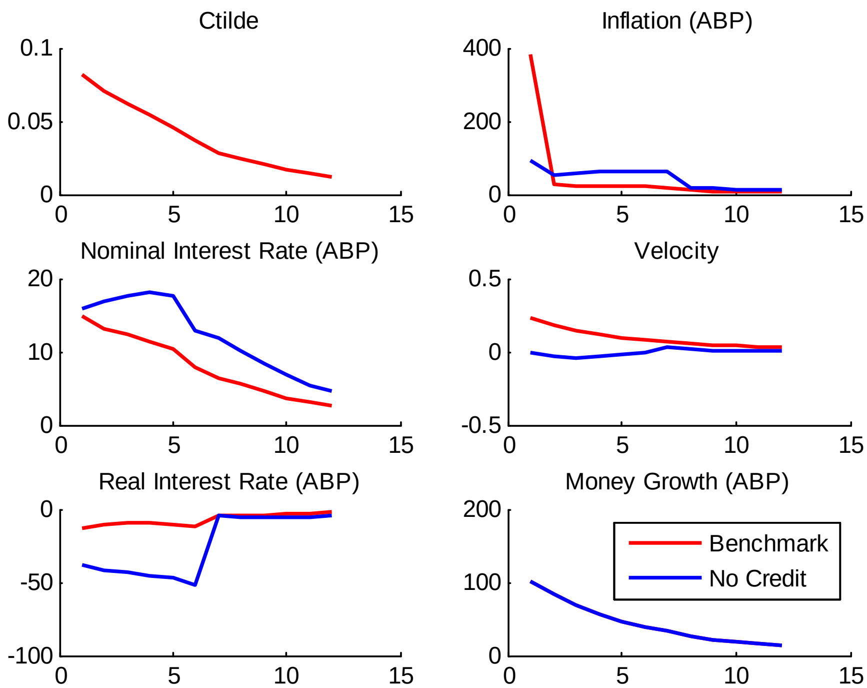


Figure 4.a.: Response Variables to Persistent Monetary Shock.

Further in the model without credit, each succeeding type of shopper, obtains less transactions balances because the size of the monetary injection is declining. Therefore, consumption by the active shoppers is declining leading to a sharp decline in the real interest rate (Figure 4.a and 4.b). The difference in consumption is much less dramatic in the economy with transactions credit and the consumption of the credit good falls slowly over time leading to much less of an effect on the real interest rate. However, the impact on consumption with credit is quantitatively much larger that in the case of a temporary money shock, and the greater credit use is reflected by the behavior of every type of shopper (Figure 4.c). The more aggressive use of transactions credit is due to the greater erosion in money balances under a persistent increase in the growth rate of money.

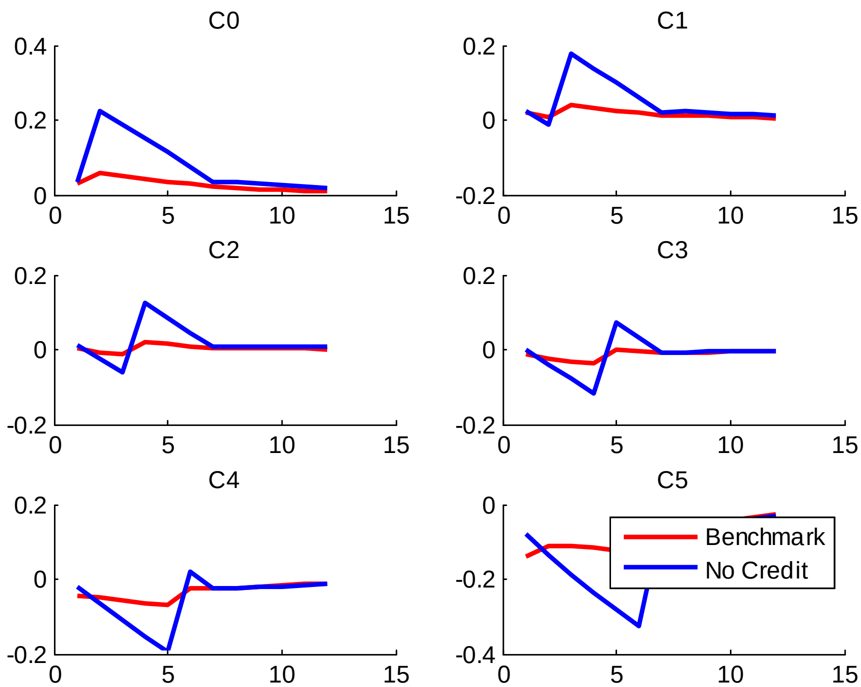


Figure 4.b.: Response Consumption to a Persistent Monetary Shock.

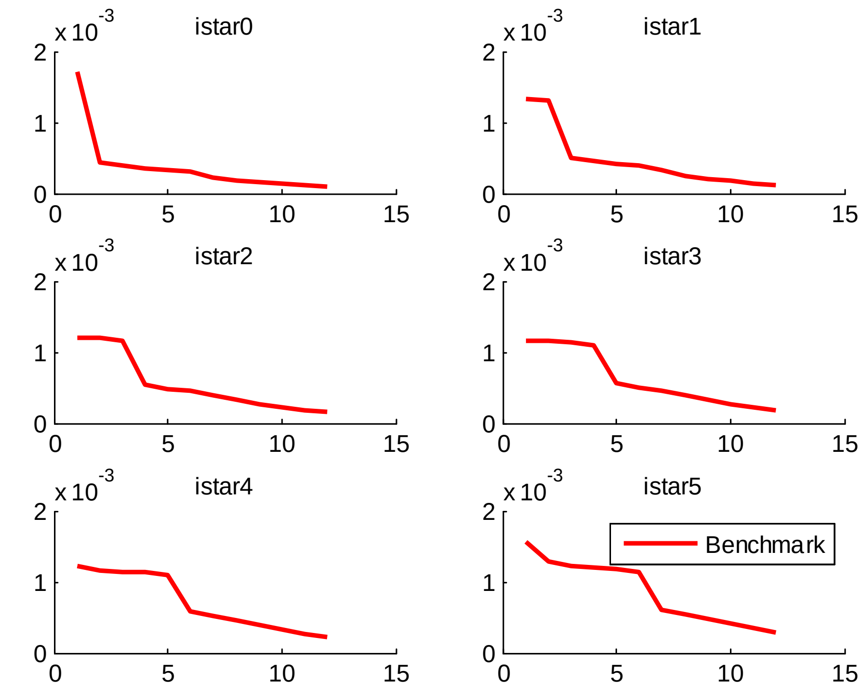


Figure 4.c.: Response Fraction Goods Bought with Credit to a Persistent Monetary Shock.

Under a persistent change to the growth of money, households understand that additional money balances will remain high today and in the near future as well. As a result, there is less incentives to become active in the asset markets since agents opt to wait to draw a more favorable fixed cost (Figure 4.d.). The dramatic rise in the price level reduces real balances on impact inducing households to consume less cash goods but simultaneously increasing the credit good consumption. The money-only segmented markets model shows behavior that is similar to that of the benchmark. Prices are not rising as aggressively, and combined with the continuing injection of money, this allows shoppers to delay becoming active (by more than in the benchmark model). This result mirrors that reported in Khan and Thomas (2011).

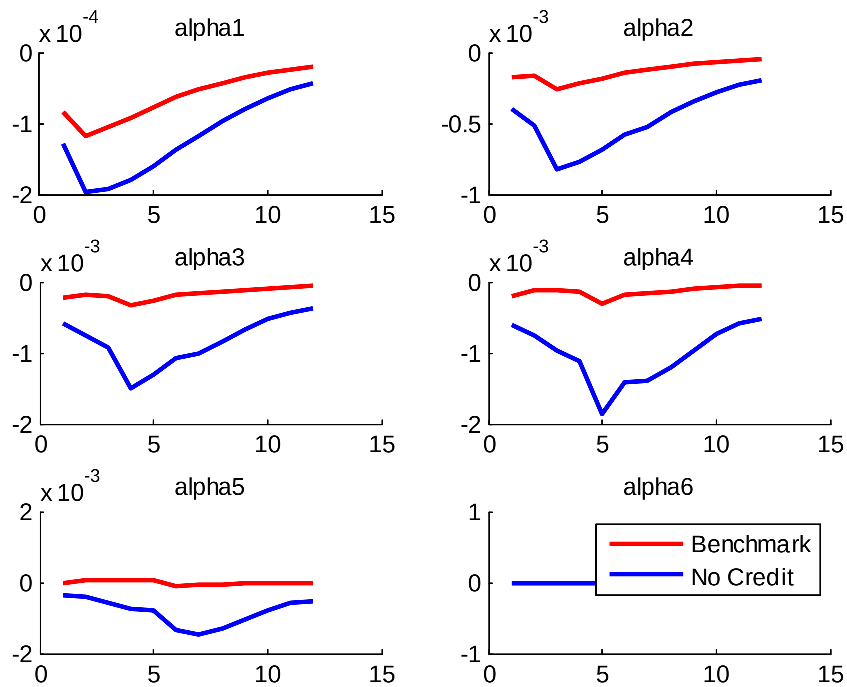


Figure 4.d.: Response Fraction of Active Shoppers to a Persistent Monetary Shock.

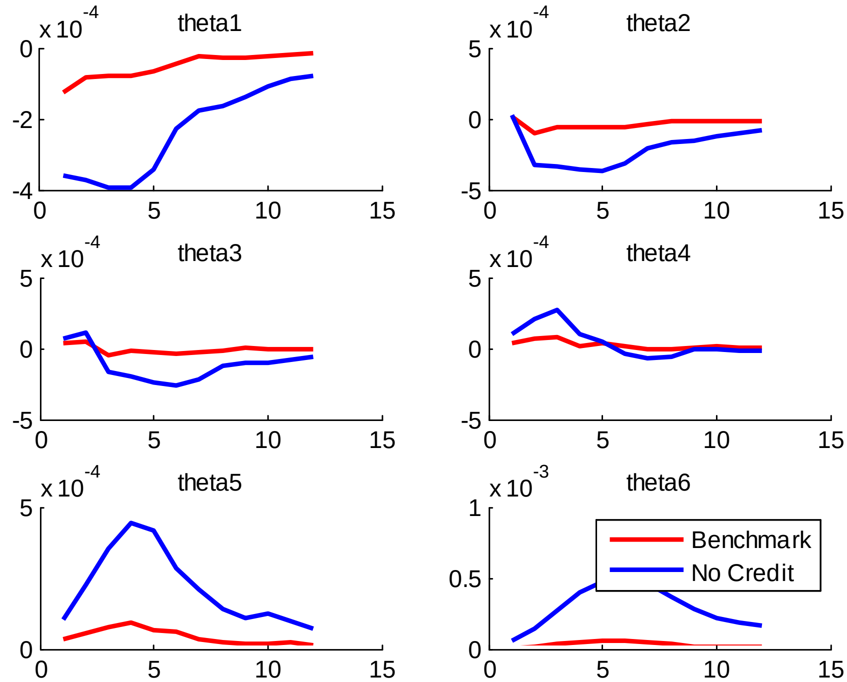


Figure 4.e.: Response of Fraction of Shoppers to a Monetary Shock.

4.3 An interest rate shock

Similar to Khan and Thomas (2011), we use the Taylor rule

$$R_t = R\,(\pi_t/\pi)^{1.5}\,\varepsilon_{r,t},$$

where R and π are the nominal interest rate and inflation, respectively. Figure 5 presents the responses to a monetary policy shock $\varepsilon_{r,t}$ that raises the nominal interest rate by 25 basis points upon impact in the baseline model as well as the one without credit. Aggregate demand falls as does the price level and inflation. Since goods bought with credit are paid in the asset markets, changes in interest rates distort the intertemporal consumption of these goods (see equation 10). As a consequence, the spike in interest rate in the model with credit induces a strong decline in consumption of credit goods, which leads to weaker demand and a sharper decline in inflation.

Due to the interest sensitivity of money demand in the benchmark economy, the demand for real money balances also decreases. The decline in real balances is concentrated in active shoppers, because the decline in the price level increases the level of real balances held by inactive households. With more real balances, inactive shoppers increase their consumption using cash (Figure 5.b.), and decrease the number of types of goods bought with credit (Figure 5.c). Further, the increase in the real balances of inactive shoppers reduces their need to replenish their money balances leading to a decline in the fraction of active shoppers (Figure 5.d). Thus, consumption with credit falls slightly on impact and then returns to steady state leading to a small increase in real rates as well. For the money-only model, the initial response of the real interest rate is governed by the relative consumption of active shoppers at $t+1$ and $t+2$. Thus, the real rate rises in this model economyas well. In the money-only economy, real balances also decline for active shoppers and increase for inactive shoppers leading to similar but more aggressive changes in consumption patterns. The effects are somewhat bigger because shoppers hold more real balances in this economy.

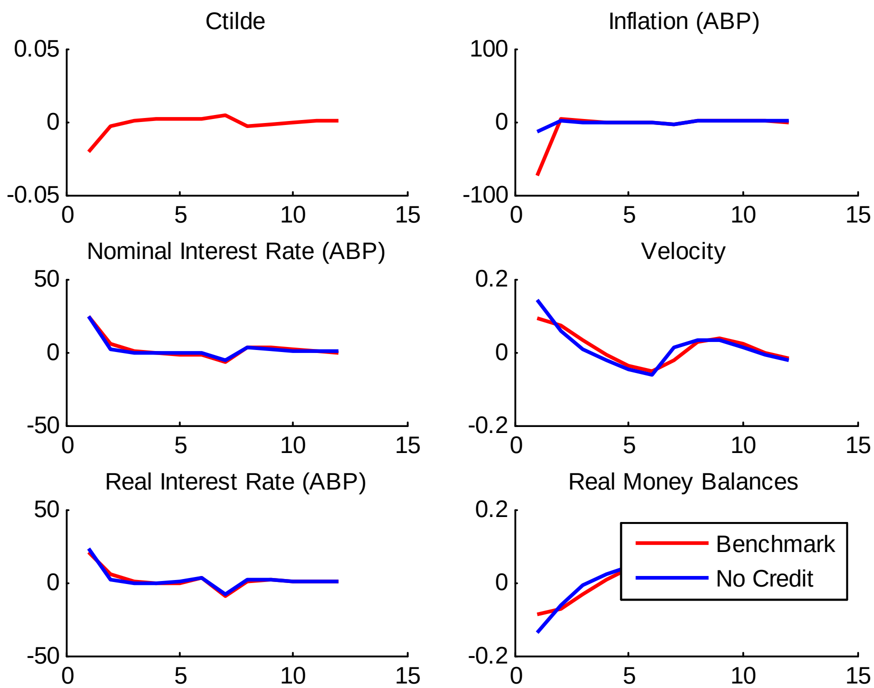


Figure 5.a: Response of Aggregate Variables to a Taylor Rule Shock.

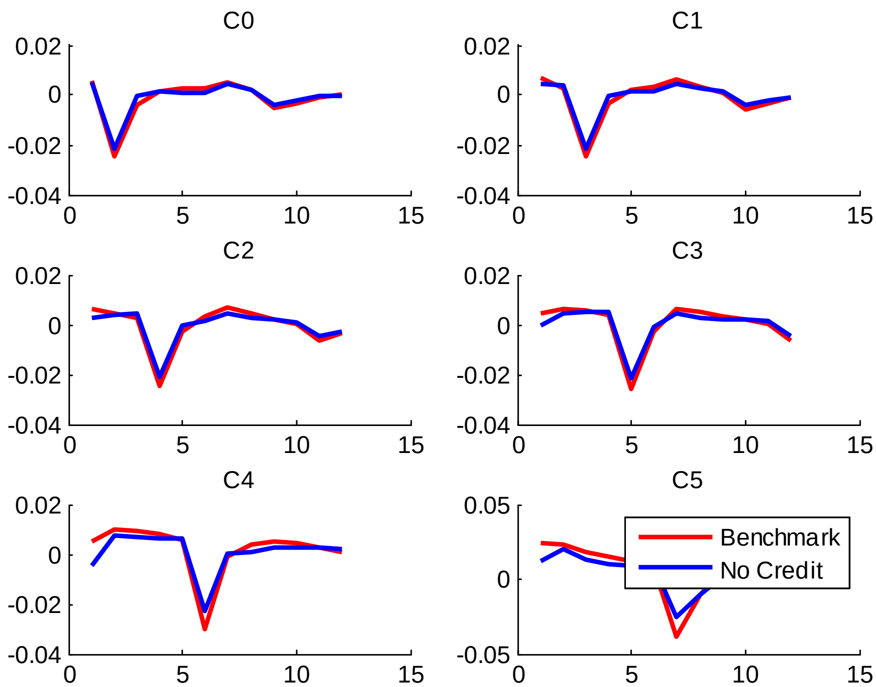


Figure 5.b: Response of Cash Goods to Taylor Rule Shock.

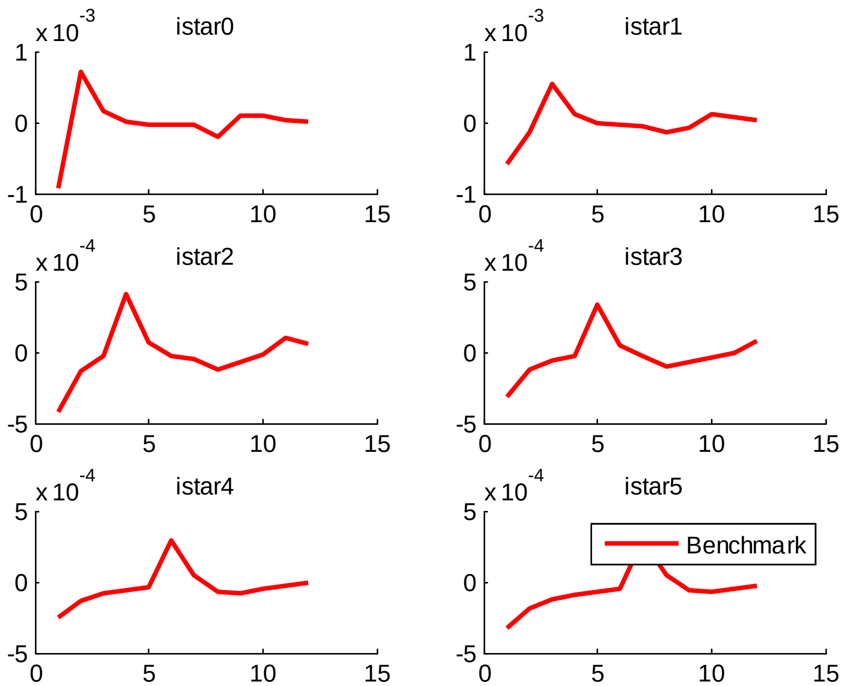


Figure 5.c.: Response of Fraction of Goods Bought with Credit to a Taylor Rule Shock.

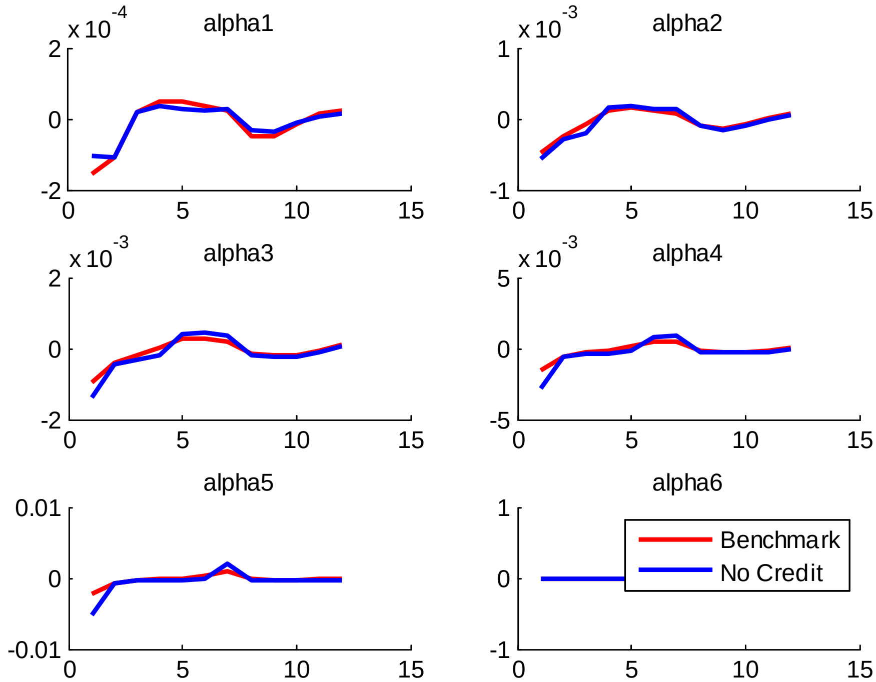


Figure 5.d.: Response of Fraction of Active Shoppers to a Taylor Rule Shock.

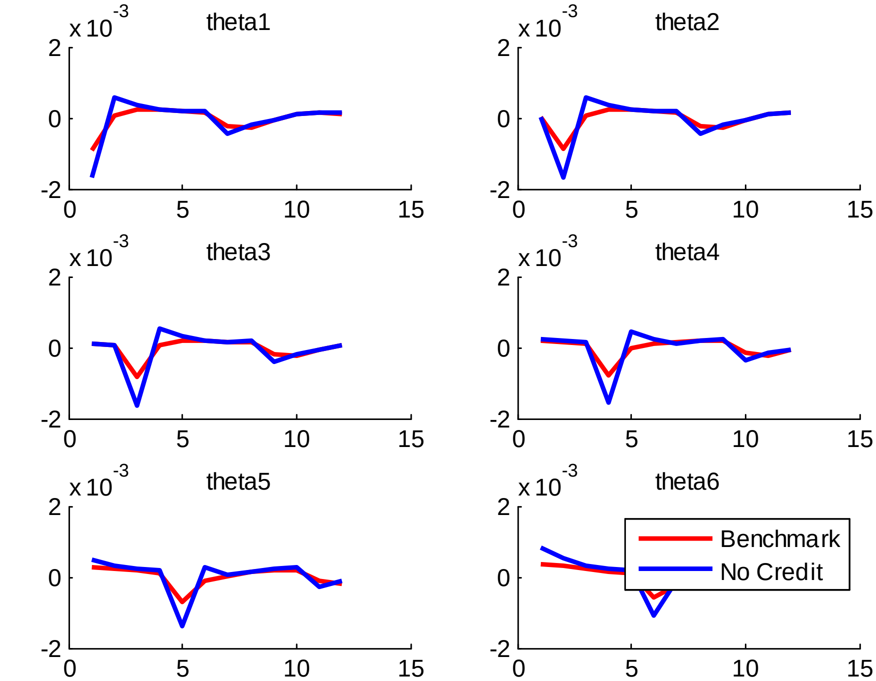


Figure 5.e.: Response of Fraction of Shoppers to a Taylor Rule Shock.

5 Dynamics in Response to Supply and Credit Shocks

Here we investigate the response of our model economies to an aggregate supply shock and a shock that increases the cost of using credit.

5.0.1 Supply Shock

The dynamic responses to an annualized 1% increase in output are plotted in Figure 6. The persistence is set to 0.91 on a monthly basis or 0.75 at a quarterly frequency. Note, also that we assume that a the money growth rule from the previous section is in place. In our benchmark economy, the consumption of the credit good behaves similarly to the exogenous behavior of output (Figure 6.a). As a result, the real interest rate declines. The increase in output also has the usual effect on inflation and inflation falls in both models. In particular the price level moves approximately one-for-one with the supply shock allowing existing money balances the ability to purchase the additional output. Expected future inflation is little changed and the movement in real rates dominates the movement in the nominal interest rate.

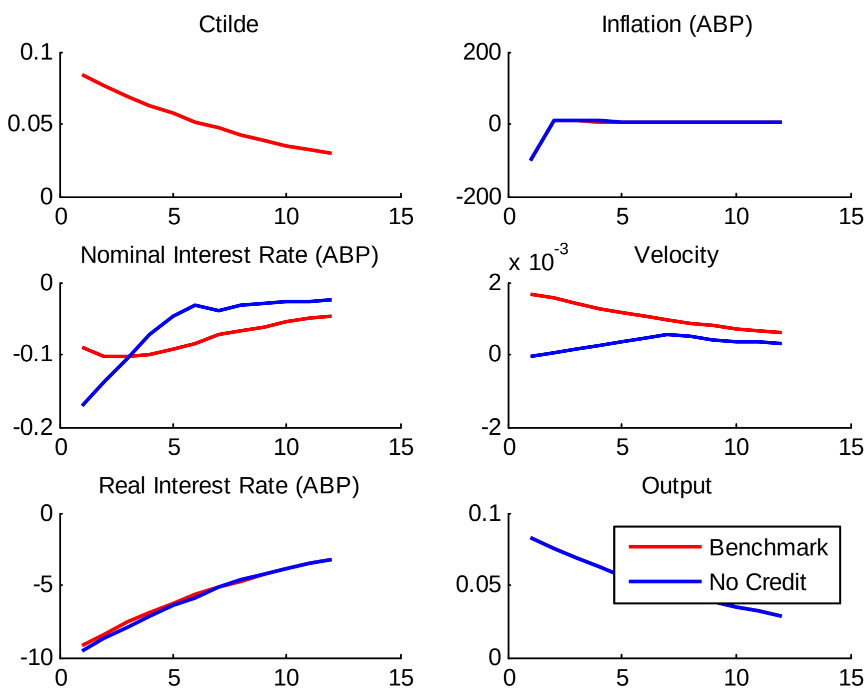


Figure 6.a: Response Aggregate Variables to a Persistent Supply Shock.

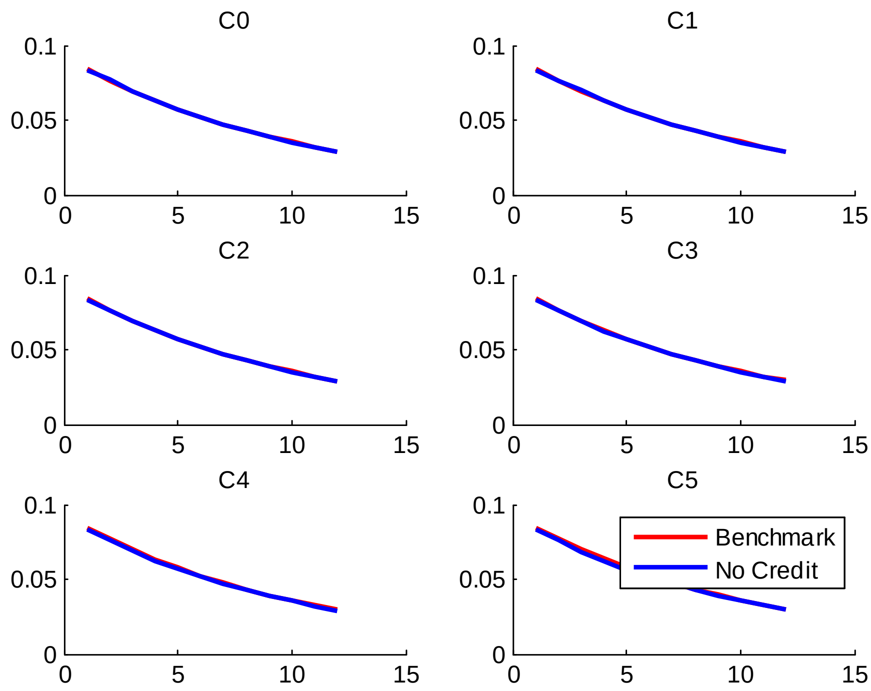


Figure 6.b: Response of Cash Goods to Persistent Supply Shock.

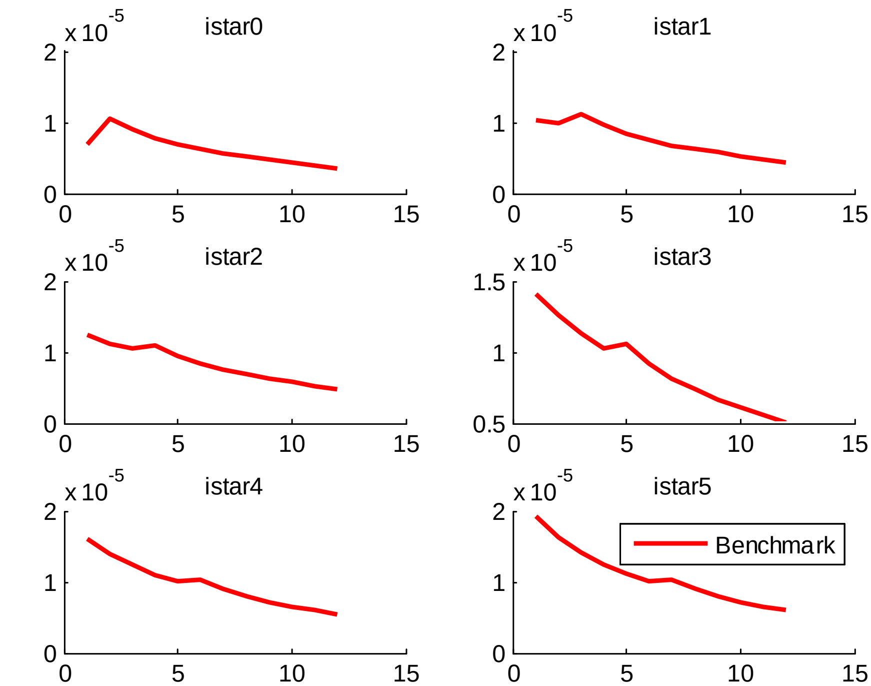


Figure 6.c: Response of Fraction of Goods Bought with Credit to Persistent Supply Shock.

Because of the one-to-one response of prices, consumption opportunities rise in step with income. As a result, the response of credit use is fairly muted in the benchmark economy. Basically, consumption behavior is largely scaled up at the intensive margin with little action at the extensive margin. The decline in the price level increases the value of real balances allowing shoppers to increase their purchase of both cash and credit goods simultaneously, leading to very little need for a change in the relative use of the two transactions media. The lag in the effect on consumption of cash goods results from the one period delay in greater household revenue being automatically sent to the shopper. This is true in both models. It is this lag in the access to greater income that motivates a relatively modest increase in the fraction of shoppers who become active, and the effect is again similar in both models (Figure 6.d.).

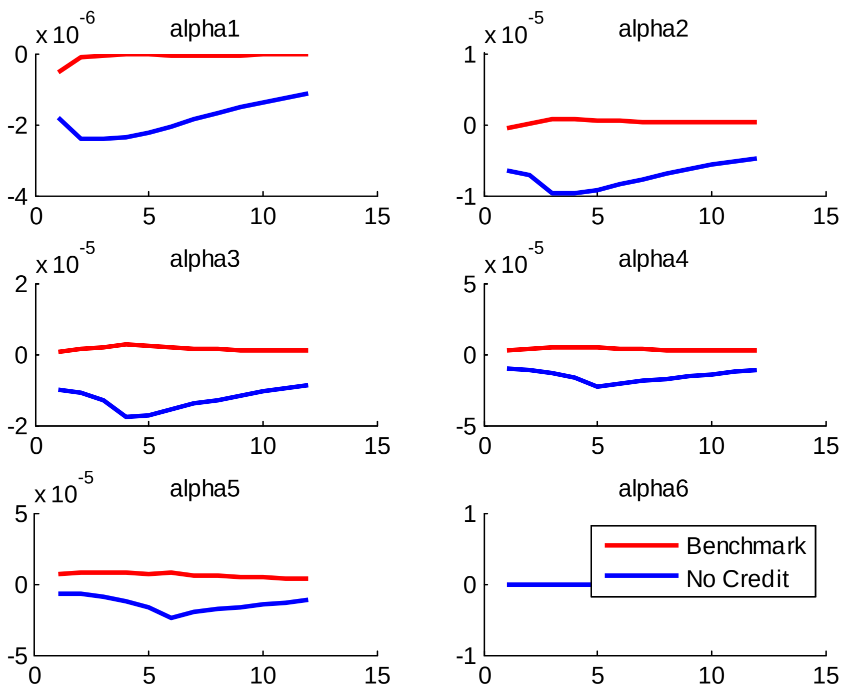


Figure 6.d: Response of Active Shoppers to Persistent Supply Shock.

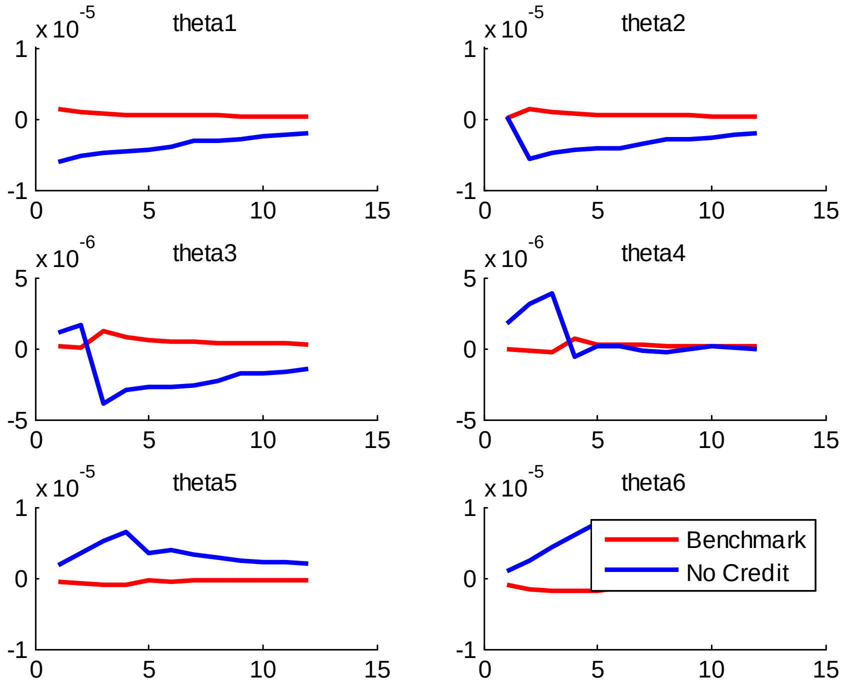


Figure 6.e.: Response Fraction Shoppers to a Persistent Supply Shock.

5.0.2 Credit Shock

Figure 7 displays the response of our benchmark model to a persistent shock to the credit cost function (15). We assume that the parameter υ is replaced by

$$\upsilon_t = (1 - \rho_\upsilon)\,\upsilon + \rho_\upsilon \upsilon_{t-1} + \sigma_\upsilon \varepsilon_{\upsilon,t}.$$

Here, the innovation $\varepsilon_{\upsilon,1}$ increases the cost υ_t by 10% above its steady state value and $\rho_\upsilon = 0.91$. The decrease in the efficiency of using credit causes shoppers to pull back on credit use along the extensive margin (Figure 7.c). They compensate by purchasing more of each credit good. Once the fixed cost of buying a type (i) good with credit is paid, there is no further direct effect on the amount of that good purchased. Recently active shopper's also respond by increasing the number of goods and the amount of each type i they purchase using cash, but shoppers who have been inactive for some time decrease consumption on the intensive margin (Figure 7.b.) Even though each of these shopper's real balance have increased due to the fall in prices, they must spread their money purchases over more goods and therefore, the purchase of each cash good declines (Figure 7.b). On net, more resources are spent using credit, overall aggregate demand falls, and with it inflation. The greater cost of using credit also spurs more shoppers to become active (Figure 7.d). The increase in the relative desirability of using money leads to more financial activity. On net, the decline in the efficiency of supplying credit, leads to what resembles a recession. Total consumption and thus velocity fall, and the real rate of interest and inflation decline.

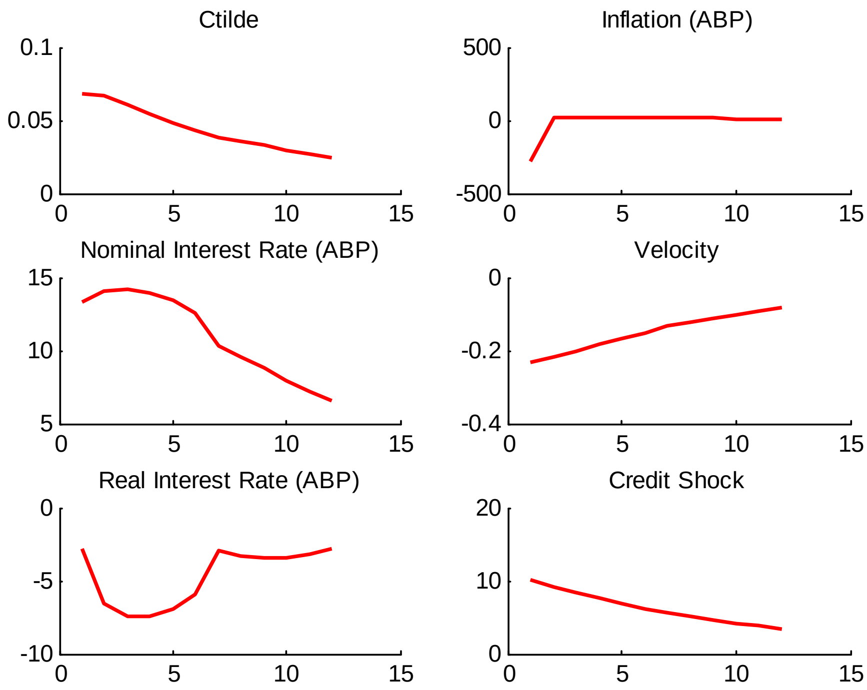


Figure 7.a: Response of Aggregate Variables to a Persistent Credit Cost Shock.

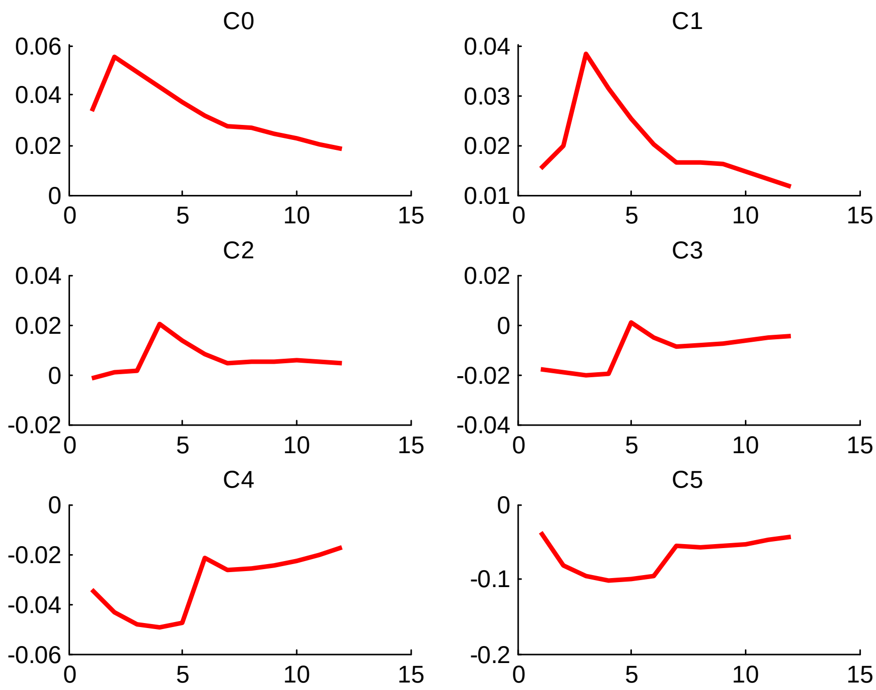


Figure 7.b: Response of Cash Goods to Persistent Credit Cost Shock.

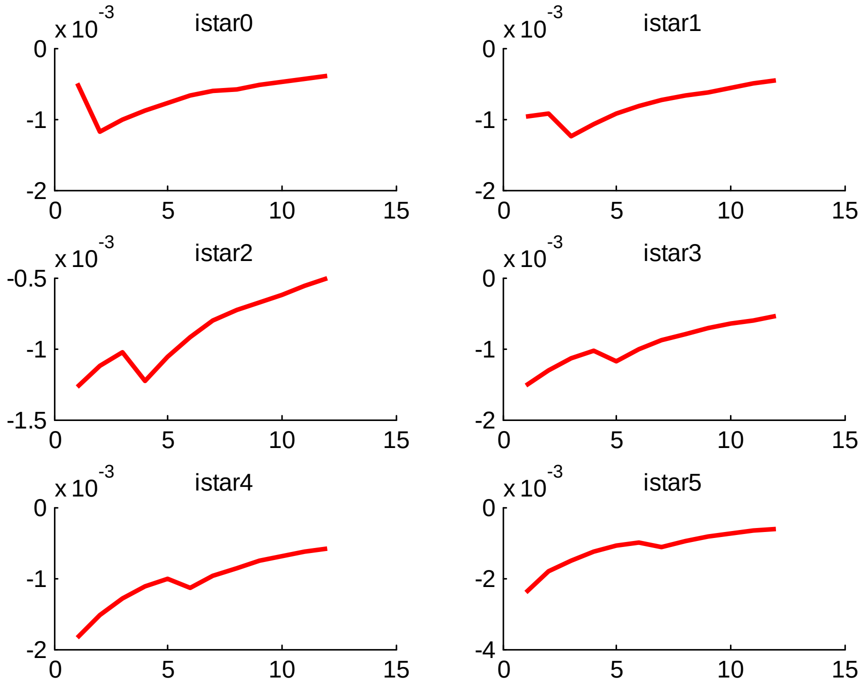


Figure 7.c: Response of Fraction of Goods Bought with Credit to Persistent Credit Cost Shock.

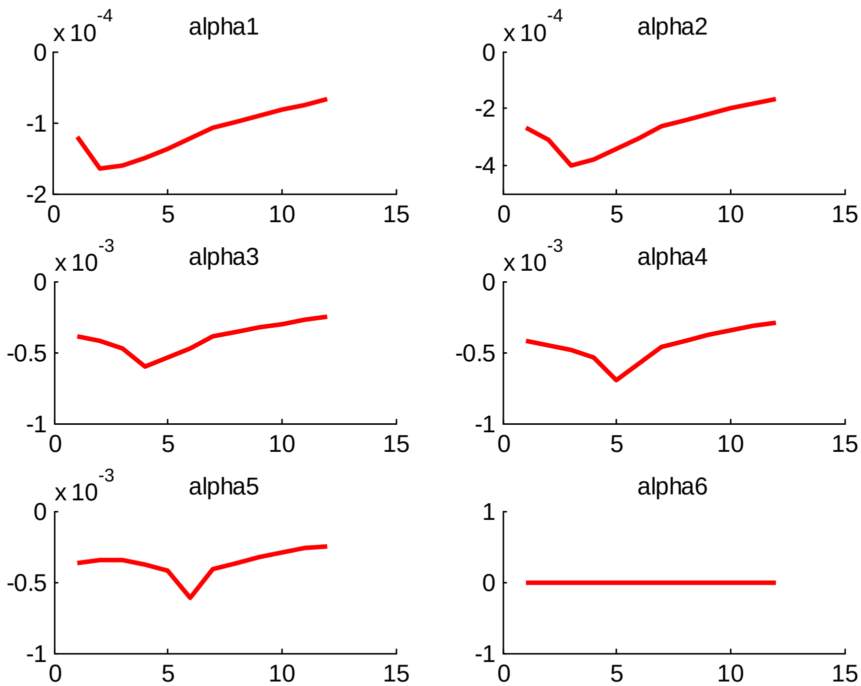


Figure 7.d: Response of Fraction of Active Shoppers to Persistent Credit Cost Shock.

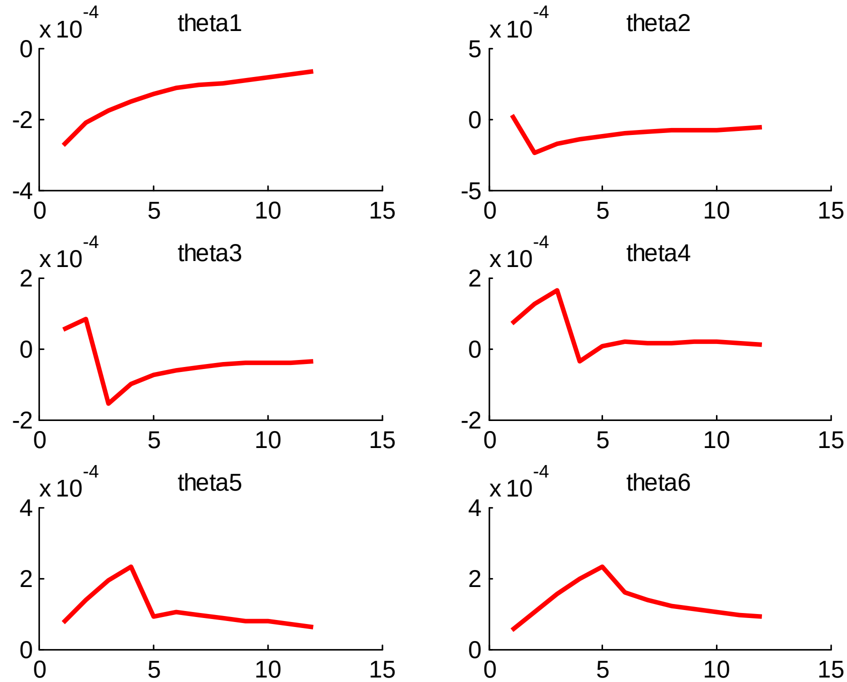


Figure 7.e: Response of Fraction of Shoppers to a Credit Shock.

5.1 A 18-Month Equilibrium

Figure 8 displays the response of some variables to a money growth rate shock when we calibrate the model so that households may wait up to 18 months before replenishing their money balances. We note that in order to induce such long financial inactivity requires a maximal fixed cost of 43% of income and total expenditures on transacting of 1.4% of income. We report both the response to an iid shock (Figure 8.a) and a persistent shock (Figure 8.b), where the persistence is set to the value in Table 1. We note that the results are qualitatively similar to those from our benchmark exercise. When the monetary shock is sufficiently persistent, there is no liquidity effect in the model with or without credit.

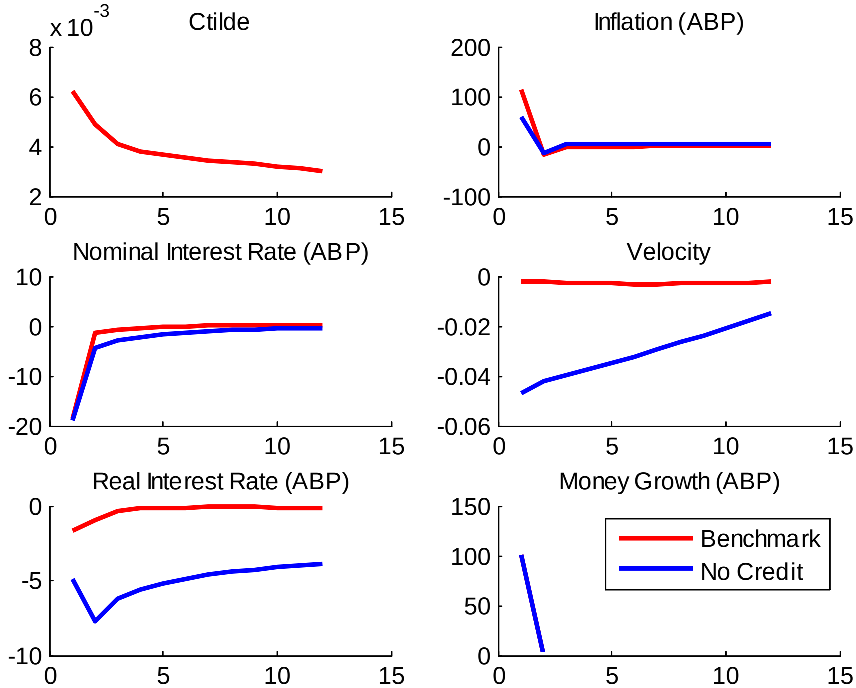


Figure 8.a: Response to a Monetary Shock.

Although it is not clear from Figure 8.b, inflation continues to display substantial persistence in response to the monetary innovation in the model without credit. In contrast, inflation quickly returns to steady state when we allow shoppers to also purchase goods using credit. Although there is no liquidity effect on the nominal interest rate, but the increase in the rate is muted relative the the benchmark example in figure 4.

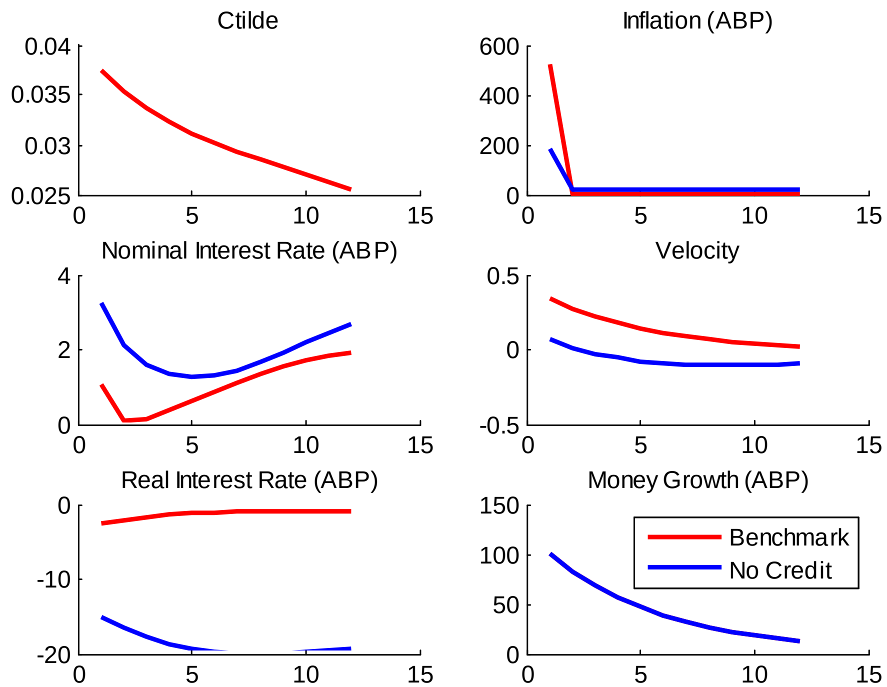


Figure 8.b: Response to a Persistent Monetary Shock.

6 An Investigation of Secular Movements in Credit Availability

In this section, we analyze how the availability of credit affects the transmission of a persistent monetary shock (Figures 9.a through 9.e). We use our baseline model as the starting point and vary the degree of access to credit from large (when people pay up to 16 percent of their purchases with credit) to low (when only 4 percent is paid with credit). For completeness, we also report the results from the model without credit. As a reference point, Table 2 provides the steady state values of total consumption bought with credit ($\widetilde{C}$), and consumption bought with cash by different groups ($c_0, \cdots, c_5$). The table also indicates that steady-state velocity is directly related to the accessibility of credit, increasing as the economy becomes more credit intensive.

Table 2: Steady State Consumption and Velocity

	$\widetilde{C}$	c_0	c_1	c_2	c_3	c_4	c_5	$velocity$
Benchmark	0.08	1.0013	0.9962	0.9910	0.9849	0.9767	0.9618	7.0
Large Credit	0.16	1.0068	1.0017	0.9966	0.9910	0.9837	0.9704	8.3
Low Credit	0.04	1.0081	1.0031	0.9981	0.9927	0.9859	0.9743	6.3
No Credit	NA	1.0083	1.0033	0.9982	0.9928	0.9860	0.9743	5.9

The results in Figure 9.a. indicate that consumption with credit is more responsive in the model with low credit. At first sight, this result seems counterintuitive since one would expect consumption to be more elastic in the economy with higher credit use. Note, however, that consumption with credit in steady state is larger in the economy with more access to credit. Once we factor in this observation, the change in the level of consumption bought with credit (rather than in percent as in Figure 9.a.) becomes more responsive with greater access to credit.

*W*ith respect to price flexibility, prices appear more flexible as the degree of credit use increases, and in turn velocity is more volatile as well. In response to the monetary shock, agents in the large-credit economy increase their use of credit by more (figure 9.c.) raising aggregate demand to a greater extent. Hence, prices must respond by more in order to clear the goods market. Also, the less costly is the use of credit the smoother is the consumption of cash goods (figure 9.b.) as well as credit goods. Essentially, when more types of goods are bought using credit, money balances are able to purchase more of each type of cash good. The greater smoothness in consumption implies less volatility in real and nominal interest rates as the affordability of credit increases. Finally, the greater volatility of velocity along with the lower volatility of nominal interest rates associated with greater credit use implies that the short-run interest elasticity of money demand increases in absolute value as credit usage increases. Thus, a changing availability of credit over time implies time varying behavior of economic variables in response to a monetary disturbance.

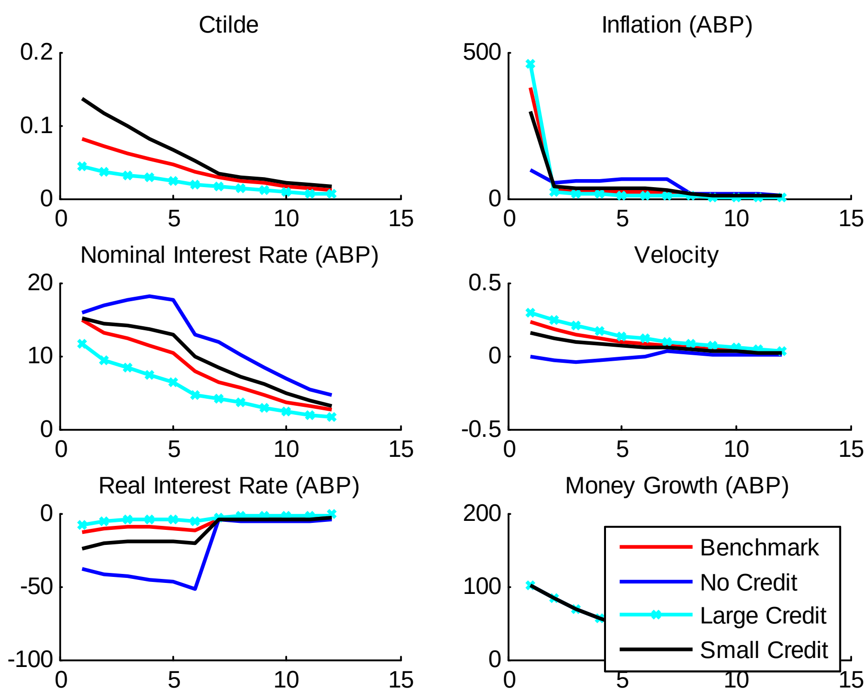


Figure 9.a.: Response of Aggregate Variables to a Monetary Shock.

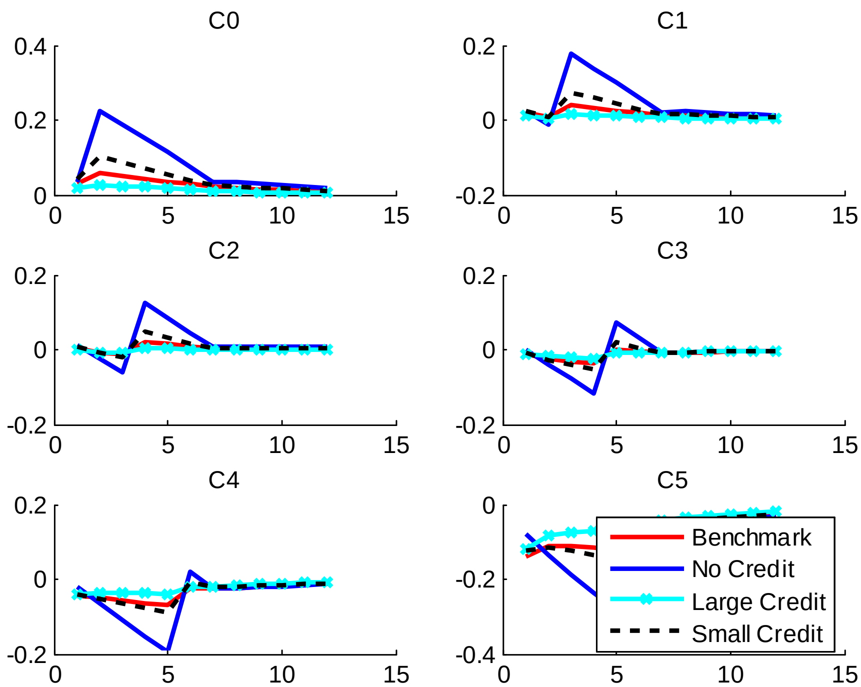


Figure 9.b.: Response of Consumption to a Monetary Shock.

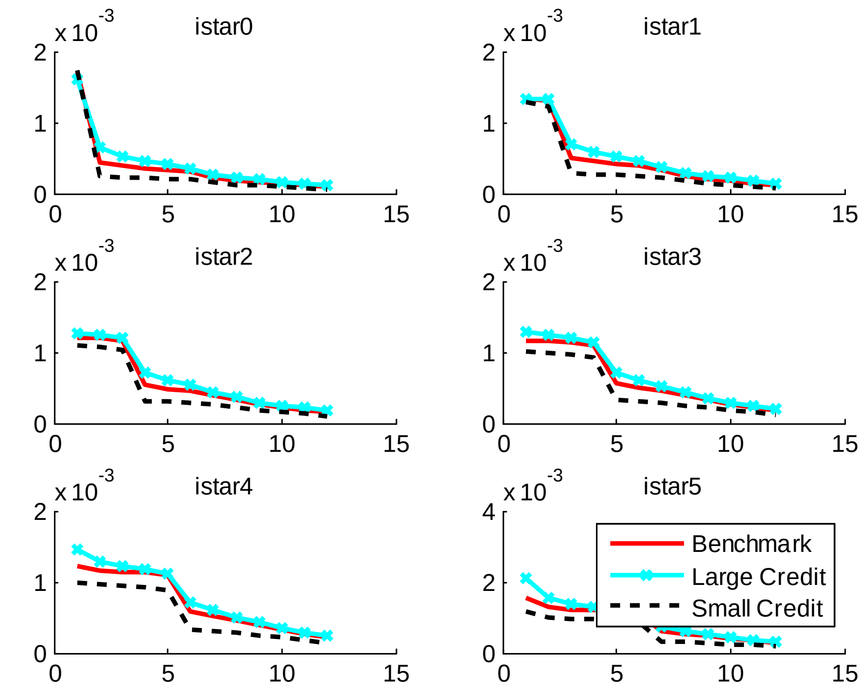


Figure 9.c.: Response of Fraction of Goods Bought with Credit to a Monetary Shock.

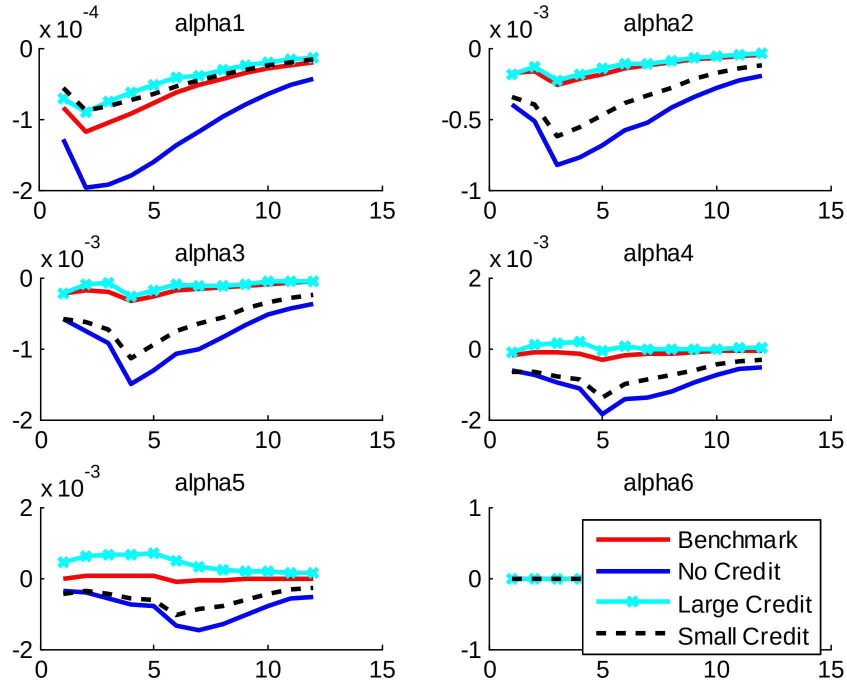


Figure 9.d.: Response of Fraction of Active Shoppers to a Monetary Shock.

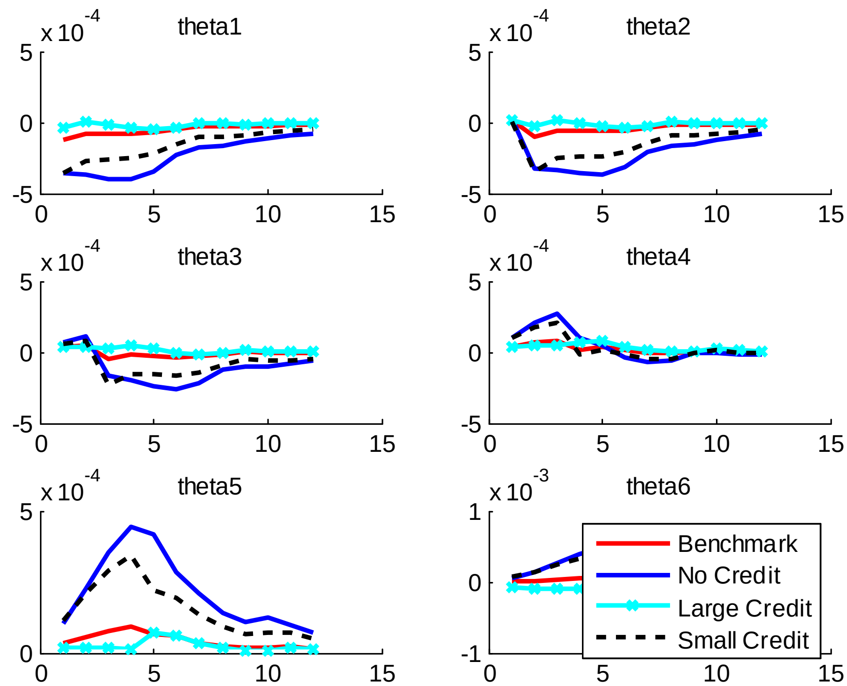


Figure 9.e.: Response of Fraction of Shoppers to a Monetary Shock.

7 Conclusion

Because the use of credit as a transactions medium is empirically relevant, it is important to investigate how its use affects behavior in the basic segmented markets model of money demand. We find that introducing credit goods drastically alters the predicitions of an endogenously segmented market economy and makes them closer to those obtained in a standard cash-in-advance model. Of importance is the effect that transactions credit has on consumption. Even though only roughly 8.0% of goods are purchased using credit, its use allows for significant consumption smoothing over time and across agents. Thus, increasing credit availability impairs the ability of market segmentation in generating sluggish nominal behavior and liquidity effects.

Access to credit also links interest rates to the behavior of each individual across time rather than to consumption of different individuals across time. As mentioned, access to credit allows shoppers another avenue for consumption smoothing by allowing them to by pass money when purchasing a good, which in turn frees up money balances to purchase more of each type of cash good. Thus, the presence of credit allows agents to smooth purchases of both types of consumption goods. And as credit becomes more available, consumption becomes smoother in response to monetary shocks and interest rates become less volatile. Therefore, the changing accessibility to credit over time has implications for time variabiltiy in economic behavior.

Importantly as well, disturbances to the accessibility of transactions-credit has implications for economic activity. A decline in credit availability whether it be an endogenous response of financial institutions to balance sheet stesses or government regulation can have negative implications for economic activity. Thus, studying in more detail the economics of credit provision and its implications for standard monetary theory is an avenue worth pursuing. The implications related to these two avenues of transaction behavior appear to be tightly linked, and the inclusion of transactions-type credit has first-order implications for thinking about monetary economics.

8 References

References

[1] Alvarez, F., A. Atkeson, and C. Edmonds (2009). Sluggish Response of Prices and Inflation to Monetary Shocks in an Inventory Model of Money Demand. Quarterly Journal of Economics 124, 911-967.

[2] Alvarez, F., A. Atkeson, and P. Kehoe (2002). Money, Interest Rates and Exchange Rates with Endogenously Segmented Markets. Journal of Political Economy 110, 73-112.

[3] Alvarez, F., A. Atkeson, and P. Kehoe (2009). Time Varying Risk, Interest Rates, and Exchange Rates in General Equilibrium. The Review of Economic Studies 76, 851-878.

[4] Dotsey, M. and P. Ireland (1996). The Welfare Cost of Inflation in General Equilibrium, Journal of Monetary Economics 37, 29-48.

[5] Dotsey ,M., R.G. King, and A.L. Wolman (1999). State Dependent Pricing and the General Equilibrium Dynamics of Money and Output, Quarterly Journal of Economics 114 (3), 655-690.

[6] Goldfeld, S. (1976). The Case of the Missing Money. Brookings Papers on Economic Activity 3, 683-740.

[7] Guerron-Quintana, P. (2009). Money Demand Heterogeneity and the Great Moderation. Journal of Monetary Economic 56, 255-266.

[8] Guerron-Quintana, P. (2011). The Implications of Inflation in an Estimated New-Keynesian Model. Journal of Economic Dynamics and Control 35, 947-962.

[9] Gust, C. and D. Lopez-Salido (2010). Monetary Policy and the Cyclicality of Risk. Board of Governors of The Federal Reserve System, International Finance Discussion Papers 2010-999.

[10] Khan, A. and J. Thomas (2011). Inflation and Interest Rates with Endogenous Market Segmentation. Mimeo Ohio State University.

[11] Porter, Richard D., and Ruth A. Judson (1996). The Location of U.S. Currency: How Much is Abroad? Federal Reserve Bulletin, October 1996, 883-903.

[12] Schreft, S. L. (1992). Welfare Improving Credit Controls, Journal of Monetary Economics (30), 57-72.

9 Appendix A: First Order Conditions

9.0.1 FOC wrt to c (2J equations)

The first order conditions for consumption of various shoppers depends on whether the good is bought with cash or credit. We will show that a good will be bought with credit if its index is less than some cutoff value, $i^*_{j,t}$. If a type $j = 0, ...J-1$ shopper buys good i with credit the foc is

$$u'(\widetilde{c}_{j,t}(i)) - \lambda_t P_t = 0 \text{ for } i \leq i^*_{j,t}, \tag{16}$$

and for type $j = 0, ...J-2$ if the good is bought with cash

$$\theta_{j+1,t} u'(c_{j,t}(i)) - \mu_{j,t} P_t = 0 \text{ for } i > i^*_{j,t}, \text{ and } j = 0, ...J-2 \tag{17}$$

Finally cash purchases for a type $J-1$ shopper

$$\theta_{J,t} u'(c_{J-1,t}(i)) - (\theta_{J,t}\lambda_t + \mu_{J-1,t}) P_t = 0 \text{ for } i > i^*_{J-1,t}. \tag{18}$$

From the above first order conditions it is clear that every good bought with credit will be purchased in equal amounts independent of the type of shopper and good, $\widetilde{c_{j,t}}(i) = \widetilde{c}_t$ and that these amounts will in general be different than those goods purchased with cash. Further, the amount of consumption of each cash good, indexed by i is independent of i, but depends on the time since the shopper last rebalanced his money balances.

9.0.2 FOC $\mathbf{M}_j's$ (J equations)

$$\beta \mathbb{E}_t(\partial V/\partial M_{0,t+1}) - \lambda_t \sum_{j=1}^{J} \alpha_{j,t}\theta_{j,t} = 0 \tag{19}$$

The first order conditions for $M_{j,t+1}$ $(j = 1..J-1)$ are

$$\beta \mathbb{E}_t(\partial V/\partial M_{j,t+1}) + \lambda_t \alpha_{j,t}\theta_{j,t} - \mu_{j-1,t} = 0 \tag{20}$$

Finally the Beneviniste-Sheinkman conditions for the states $M_{j,t}$ are for $j = 0, ..J-2$

$$\partial V/\partial M_{j,t} = \mu_{j,t}, \tag{21}$$

and for $M_{J-1,t}$

$$\partial V/\partial M_{J-1,t} = \mu_{J-1,t} + \theta_{J,t}\lambda_t. \tag{22}$$

Combing Equations (J conditions used in determining J+1 values of consumption) Updating the B-S conditions (21) and (22) and substituting into the foc for $M_{j,t+1}$ along with the foc for consumption yields the following J equations that along with goods market clearing determine the various values of consumption. For the consumption of goods purchased with credit we have

$$u'(\widetilde{c}_t)/P_t = \beta \mathbb{E}_t u'(c_{0,t+1})/P_{t+1}, \tag{23}$$

and for the various goods bought with cash

$$\alpha_{j,t}(u'(\widetilde{c}_t)/P_t) + \beta(1-\alpha_{j,t})\mathbb{E}_t(u'(c_{j,t+1})/P_{t+1}) = u'(c_{j-1,t})/P_t \quad j = 1 \text{ to } J-1. \tag{24}$$

It will subsequently be shown that, as in Dotsey and Ireland (1997), credit goods are bought in greater quantities than cash goods.

9.0.3 First order condition for bonds

The first order condition for bonds can be obtained by combining the first order condition for B_t along with B-S condition for B_{t-1} to obtain

$$u'(\widetilde{c}_t)/P_t = \beta \mathbb{E}_t(u'(\widetilde{c}_{t+1})/P_{t+1})R_t. \tag{25}$$

9.0.4 Determining the Cutoff

Having determined consumption, we next examine the condition determining the cutoff for whether a good is bought with credit or cash. This cutoff point will depend on the index j which is associated with how long an individual shopper has been unable to replinish his cash. Differentiating the household's objective function (4) with respect to the various cutoffs, $i_{j,t}$ and substituting out the Lagrange multipliers yields the following condition,

$$[u(\widetilde{c}_t) - u(c_{j,t})] + [u'(c_{j,t})c_{j,t} - u'(\widetilde{c}_t)\widetilde{c}_t] = u'(\widetilde{c}_t)q(i^*_{j,t}). \tag{26}$$

A good will be bought with credit as long as the LHS of (26) is less than or equal to the RHS.

9.0.5 FOC alphas (J-1 equations)

We now turn to determining when a shopper visits the asset market to replenish transactions balances.

$$\gamma_{0,t} - \gamma_{j,t} - \lambda_t[(M_{0,t+1} - M_{j,t+1}) + P_t\xi^*_{j,t}] = 0 \tag{27}$$
$$\text{for } j = 1 \text{ to } J-1$$

where I have used $\partial\Xi_{j,t}/\partial\alpha_{j,t} = \xi^*_{j,t}$ .Thus, once we have expressions that determined the various Lagrange multipliers we can uniquely determine the cutoff costs associated with visiting the asset market.

9.0.6 First order conditions for $\theta_{j,t+1}$(J equations)

The J first order conditions for the $\theta_{j,t+1}$ are given by

$$\beta\mathbb{E}_t\partial V(t+1)/\partial\theta_{j,t+1} = \gamma_{j-1,t} \tag{28}$$

9.0.7 Benveniste-Sheinkmen conditions for thetas

The B-S conditions for the first $J-1$ $\theta's$ are

$$\begin{gathered}\partial V/\partial\theta_{j,t} = \alpha_{j,t}\gamma_{0,t} + (1-\alpha_{j,t})\gamma_{j,t} + \\ [\int_0^{i^*_{j-1,t}} u(\widetilde{c}_t)di + \int_{i^*_{j-1,t}}^1 u(c_{j-1,t})di] \\ -\lambda_t\alpha_{j,t}(M_{0,t+1} - M_{j,t+1}) - \lambda_t P_t \int_0^{i^*_{j-1,t}} (\widetilde{c}_t + q(i))di \\ -\lambda_t P_t \Xi_{j,t} \\ \text{for } j = 1 \text{ to } \ J-1.\end{gathered} \tag{29}$$

For the J^{th} θ.

$$
\begin{aligned}
\partial V / \partial \theta_{J,t} &= \gamma_{0,t} + [\int_0^{i^*_{J-1,t}} u(\widetilde{c}_t)di + \int_{i^*_{J-1,t}}^1 u(c_{J-1,t})di] - &(30)\\
&\lambda_t[M_{0,t+1} - M_{J-1,t} - \phi P_{t-1}y_{t-1} + P_t \int_{i^*_{J-1,t}}^1 c_{J-1,t}(i)di] &(31)\\
&-\lambda_t P_t \int_0^{i^*_{J-1,t}} (\widetilde{c}_t + q(i))di - \lambda_t P_t \Xi_{J,t}
\end{aligned}
$$

Updating these two equations and using the first order condition for next period's thetas yields the following recursive relationships that determine the $\gamma' s$.

$$
\begin{aligned}
\gamma_{j,t} &= \beta E \alpha_{j+1,t+1}\gamma_{0,t+1} + \beta(1-\alpha_{j+1,t+1})\gamma_{j+1,t+1} + &(32)\\
&\beta \mathbb{E}_t[\int_0^{i^*_{j,t+1}} u(\widetilde{c}_{t+1})di + \int_{i^*_{j,t+1}}^1 u(c_{j,t+1})di] - \\
&\beta \mathbb{E}_t \lambda_{t+1}\alpha_{j+1,t+1}(M_{0,t+2} - M_{j+1,t+2}) - \beta \mathbb{E}_t \lambda_{t+1} P_{t+1} \int_0^{i^*_{j,t+1}} (\widetilde{c}_{t+1} + q(i))di \\
&-\beta \mathbb{E}_t \lambda_{t+1} P_{t+1} \Xi_{j+1,t+1} \\
\text{for } j &= 0, ..., J-2,
\end{aligned}
$$

and

$$
\begin{aligned}
\gamma_{J-1,t} &= \beta \mathbb{E}_t \gamma_{0,t+1} + \beta \mathbb{E}_t[\int_0^{i^*_{J-1,t+1}} u(\widetilde{c}_{t+1})di + \int_{i^*_{J-1,t+1}}^1 u(c_{J-1,t+1})di] - &(33)\\
&\beta \mathbb{E}_t \lambda_{t+1}[M_{0,t+2} - M_{J-1,t+1} - \phi P_t y_t + P_{t+1} \int_{i^*_{J-1,t+1}}^1 c_{J-1,t+1}(i)di] - \\
&\beta \mathbb{E}_t \lambda_{t+1} P_{t+1} \int_0^{i^*_{J-1,t+1}} (\widetilde{c}_{t+1} + q(i))di - \beta \mathbb{E}_t \lambda_{t+1} P_{t+1} \Xi_{J,t+1}
\end{aligned}
$$

9.1 Summing up

Conditional on knowing the cutoff value for using credit for each type of shopper and the cutoff values for going to the asset market, which determine the $\alpha_{j,t}$ we can determine the other variables. We have the J equations for determining consumption,(17) and (23), along with goods market clearing to determine the $J+1$ various values of consumption. The CIA constraints along with the first order condition for $M_{0,t+1}$ can then be used to determine the various money holdings. Given these solutions the cutoff values for credit can be ascertained and the multipliers $\gamma_{j,t}$ can be solved for. In turn, the cutoff values for going to the bond market and the expected costs of doing so can be calculated. In turn, knowing the cutoffs for credit allows one to calculate the cost of using credit. Iterating on these conditions until convergence is attained in the credit and asset market cutoff values or solving all the equations nonlinearly can produce the steady state values of the economy.

10 Appendix B: Steady State routine

The steady state for all variables can be solved if one knows the cutoffs ξ_j^* and the i_j^* for a given selection of J. Thus one must use numerical methods (nonlinear equation solver, hill climber, of bisection in a Gauss-Seidel setting to find these two vectors, and then one must determine if J is optimal (ie. do there exist any shoppers who would rather not go the bond market if not forced to do so). Thus, I will first indictate how to calculate the steady state consumptions, money balances, alphas, $\Xi' s$, costs of using credit, fractions of types, and the gamma's as functions of the two types of cutoffs. I will then describe the conditions determining the two cutoffs.

10.1 Probabilities, fractions, and costs

The alpha's are given by $\alpha_j = H(\xi_j^*)$ and the expected costs of transacting in the asset market is $\Xi_j = \int\limits_0^{\xi_j^*} ah(a)da$. The evolution of the steady state fractions are given by $\theta_{j+1} = (1 - \alpha_j)\theta_j$ for $j = 1$ to $J - 1$ and using the fact that the thetas sum to one yields an expression for each θ_j in terms of the alphas. Specifically, $\theta_1 = \frac{1}{\sum\limits_{j=0}^{J-1} \prod\limits_{i=0}^{j} (1-\alpha_i)}$ where $\alpha_0 \equiv 0$. The remaining $\theta' s$ can be calculated recursively.

The costs of using credit are given by $Q_j = \int\limits_0^{i_{jt}^*} q_t(i)]di$ for each $j = 0, ...J - 1$.

10.2 Consumptions

To calculate the consumptions first use the first order conditions (9) and (23) to determine the ratio of various $c_j' s$ to $\widetilde{c}$. Define

$$rmu_j = u'(c_j)/u'(\widetilde{c}).$$

Then for c_0, we have

$$rmu_0 = (\widetilde{c}/c_0)^\sigma = \mu/\beta. \tag{34}$$

Note that except at the Friedman rule the credit good is consumed in greater quantitites. The remaining ratios can be solved recursively,

$$rmu_j = (\widetilde{c}/c_j)^\sigma = (\mu/(\beta(1 - \alpha_j))[rmu_{j-1} - \alpha_j] \tag{35}$$

Thus, the consumption of cash goods is monotonically decreasing in j. With these expressions in hand, we have the ratio of the cash goods to the credit good for each shopper, $rc_j = rmu_j^{1/\sigma}$..

Substituting into goods market clearing yields an expression for consumption of the credit good, which in turn can now be used to calculate the consumption of each type of cash good.

$$\widetilde{c} = \frac{y - \sum_{j=1}^{J} \theta_j (Q_{j-1} + \Xi_j)}{\sum_{j=1}^{J} \theta_j (i_{j-1}^* + (1 - i_{j-1}^*)(1/rc_{j-1}))} \tag{36}$$

10.3 Calculating steady state money balances

We next use the CIA constraints to derive steady state money balances. We do this is real terms, defining $m_{j,t} = M_{j,t}/P_{t-1}$ and thus the $m's$ are predetermined variables. This is done by recursively working back from the $J-1$ shopper.

$$m_{J-1} = \mu(1 - i_{J-1}^*)c_{J-1} - \phi y \tag{37}$$

and

$$m_j = \mu(1 - i_j^*)c_j + \mu m_{j+1} - \phi y. \tag{38}$$

10.4 Calculating the steady state gammas

We next use (14) and (13) to calculate the steady state gammas. In particular,

$$\begin{aligned} \gamma_{J-1} &= \beta\gamma_0 + \beta[i_{J-1}^* u(\widetilde{c})di + (1 - i_{J-1}^*)u(c_{J-1})] - \\ &\quad \beta u'(\widetilde{c})[m_0 - m_{J-1}/\mu - \phi y/\mu] - \beta(1 - i_{J-1}^*)c_{J-1} - \\ &\quad \beta u'(\widetilde{c})i_{J-1}^* \widetilde{c}_{t+1} - \beta u'(\widetilde{c})Q_{J-1} - \beta u'(\widetilde{c})\Xi_J \end{aligned} \tag{39}$$

and

$$\begin{aligned} \gamma_j &= \beta\alpha_{j+1}\gamma_0 + \beta(1 - \alpha_{j+1})\gamma_{j+1,t+1} + \\ &\quad \beta[i_j^* u(\widetilde{c}) + (1 - i_j^*)u(c_j)] - \\ &\quad \beta u'(\widetilde{c})\alpha_{j+1}(m_0 - m_{j+1}) - \beta u'(\widetilde{c})i_j^*\widetilde{c} - \beta u'(\widetilde{c})Q_j \\ &\quad -\beta u'(\widetilde{c})\Xi_{j+1} \\ \text{for } j &= 0, ..., J-2. \end{aligned} \tag{40}$$

10.5 Determining the steady state cutoffs

With the above steady state values, which depend on the cutoffs we can now solve the functional equations for the cutoffs. For going to asset market each type j shopper's cutoff is given by

$$\gamma_0 - \gamma_j - u'(\widetilde{c})[(m_0 - m_j) = u'(\widetilde{c})\xi_j^* \quad (41)$$
$$\text{for } j = 1 \text{ to } J - 1$$

The cutoff for using credit is depicted by

$$[u(\widetilde{c}) - u(c_j)] + [u'(c_j)c_j - u'(\widetilde{c})\widetilde{c}] = u'(\widetilde{c})q(i_j^*). \quad (42)$$

One iterates on the cutoffs until convergence.

10.6 Sufficient condition for J

From the cutoff condition we can see that you prefer to go to the asset market if $\xi_j^* \leq [(\gamma_0/u'(\widetilde{c})) - m_0] - [(\gamma_j/u'(\widetilde{c})) - m_j]$. Suppose we let a single shopper's stay away from the asset market for one more period. That shopper would have no money balances to take into next period and thus his consumption would be given by $c_J = \phi y/(\mu(1 - i_J^*))$ and i_J^* can be calculated for using (42). We can then use an equation similar to (39), where we assume that $\alpha_{J+1} = 1$, that is the shopper will fall asleep for two periods. Thus, $\Xi_{J+1} = \int\limits_0^{\xi_{\max}} ah(a)da$. Then, we define a value function for this shopper to be

$$\begin{aligned}
\gamma_J \quad = \quad & \beta\gamma_0 + \beta[i_J^* u(\widetilde{c})di + (1 - i_J^*)u(c_J)] - \\
& \beta u'(\widetilde{c})[m_0 - \phi y/\mu] - \beta(1 - i_J^*)c_J - \\
& \beta u'(\widetilde{c})i_J^*\widetilde{c} - \beta u'(\widetilde{c})Q_J - \beta u'(\widetilde{c})\Xi_{J+1}.
\end{aligned} \quad (43)$$

This is the value of shopper who stays away from asset market one period to long with no money balances. If

$$\gamma_0 - u'(\widetilde{c})m_0 - \xi_{\max} \geq \gamma_J$$

then this shopper will regret not going to the asset market. That is, if the value of having gone to the asset market and having paid the maximal fixed cost makes one better off than having fallen asleep, then the guess for J is correct. If there exists shoppers who would not regret having fallen asleep, then the guess of J is too small.

11 Appendix B: Computing the Fraction of Goods Bought with Credit

We are after the fraction of goods that are bought in the economy using credit card (debt). We attack this problem as follows.

1. We use the Survey of Consumer Finances 2010 to recover Total New Charges to credit cards (NC). These new charges are separated into those that are paid fully (this reflect convenience use of credit cards; Conv) and those that are revolved (Revol).

2. Convenience charges are computed summing up new charges made by households that report that after paying their last monthly bill they have zero outstanding balances. The remaining households, therefore, have some revolving balances.

3. After annualizing and using the survey's weights to transform the survey results to national figures, we obtain NC = \$1,342.59 billion, Revol = \$415.88 billion.

4. Annual income from the survey is Income = 9,212.61 billion.

5. NIPA we get the average personal income and personal consumption (\$12,321.875 and 6,966.13, respectively). The consumption figure excludes housing expenditures that we think are paid in cash:

   (a) Rental of tenant-occupied nonfarm housing.

   (b) Imputed rental of owner-occupied nonfarm housing.

   (c) Rental value of farm dwellings.

   (d) Group housing.

   (e) Health service, which are to a large extent paid by insurance companies.

6. With these numbers, we conclude that 8.0 percent of goods are bought with credit cards, i.e. using revolving debt: $8.0\% = (415.88)/(9{,}212.61)\ (12{,}321.875)/(6{,}966.13)$.