Capital Requirements in a Quantitative Model of Banking Industry Dynamics (Preliminary and Incomplete)*

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September 6, 2012

Abstract

We develop a model of banking industry dynamics to study the relation between commercial bank market structure, risk taking, bank failure, and capital requirements. We propose a market structure where dominant banks interact with many small competitive fringe banks. A nontrivial size distribution of banks arises out of endogenous entry and exit. The paper extends our previous work by letting banks accumulate securities like treasury bills and to undertake short-term borrowing when there are cash flow shortfalls. We find that a 33% rise in capital requirements leads to a 15% drop in big bank exit rates and a less concentrated industry. In a different counterfactual, we show that strategic interaction between dominant banks and the competitive fringe generates more volatility (a riskier environment) than in a perfectly competitive banking system. This results in higher capital ratios in the benchmark economy than in the one with only competitive banks. The loan supply strategy by dominant banks results in an important source of amplification of business cycles.

1 Introduction

Capital requirements are intended to ensure that banks are not making investments that increase the risk of failure and that they have enough capital to sustain operating losses

*The authors wish to thank Gianni DeNicolo, Matthias Kehrig, Skander Van Den Heuvel, as well as seminar participants at the Board of Governors, University of Maryland, the Conference on Money and Markets at the University of Toronto, the Conference on Monetary Economics to honor the contributions of Warren Weber, the CIREQ Macroeconomics Conference, and the Econometric Society for helpful comments.

while still honoring withdrawals. In this paper we develop a structural model of banking industry dynamics to answer the following question: how does an increase in capital requirements affect failure rates and market shares of large and small banks, bank risk taking and aggregates?

While we endogenized market structure in an earlier paper (Corbae and D'Erasmo [10]), we limited the asset side of the bank balance sheet to loans and the liabilities side to deposits. While these are clearly the largest components of each side of the balance sheet of U.S. banks, this simplification does not admit ways for banks to insure themselves at a cost through holdings of securities like T-bills and borrowings in the interbank market.

In this paper, we extend the portfolio of bank assets from the prior paper to include securities like treasury bills and to undertake short-term borrowing when there are cash flow shortfalls. Further we assume that banks are randomly matched with a quantity of deposits and that these matches follow a markov process which is independently distributed across banks. Thus, we add liquidity shocks to the model of the first paper.

At the end of the period, banks can choose to exit if their net charter value is less than what they would obtain after repaying deposits and their net securities. The exit value takes into account that there is limited liability. To keep the state space reasonable in our original environment, banks were not allowed to hold net securities and the exit decision depended only on ex-post profits and the cost of issuing equity. In this extension, banks can use a positive stock of net securities as a buffer and borrow (whenever possible) to avoid being liquidated or issuing "expensive" equity. Thus, the extension allows us to consider banks undertaking precautionary savings in the face of idiosyncratic shocks as in Huggett [19], but with a strategic twist. Capital and liquidity requirements ensure that banking institutions are not making investments that increase the risk of default and that banks have enough capital to sustain operating losses while still honoring withdrawals. A benefit of our structural framework is that we can conduct policy counterfactuals. For example, it allows us to study questions like those posed in Kashyap and Stein [20]; whether the impact of Fed policy on lending behavior is stronger for banks with less liquid balance sheets (where liquidity is measured by the ratio of net securities to loans plus net securities). We can also assess the quantitative impact on borrower default frequencies and bank failure of a rise in capital requirements like that proposed in Basel III.

In our main policy counterfactual (Section 7.2), we find that a 33% rise in capital requirements leads to a 15% drop in big bank exit rates and a less concentrated industry. To understand the interaction between market structure and policy, we conduct a counterfactual (Section 7.1) where we increase the entry cost for dominant banks to a level that prevents their entry. Since our benchmark model nests a dominant bank model with a perfectly competitive sector, this counterfactual implies that we move endogenously to an environment with only perfectly competitive banks. We find that capital ratios are larger in the benchmark economy than in the one with only competitive banks. The reason is that the environment with dominant banks is riskier (a higher default frequency and volatility of all aggregates). The loan supply strategy by dominant banks results in an important source of amplification of business cycles. We also find, as one would expect, that in a perfectly competitive industry the loan interest rate is lower than in the benchmark, resulting in lower default frequencies and higher output.

This leads naturally to our final counterfactual (Section 7.3), where we study the differences in the predictions of the model derived from increasing the capital requirement between our benchmark model and the economy with only perfectly competitive banks. For instance, we find that an increase in capital requirements with perfectly competitive banks generates much less entry (i.e. the measure of banks falls by 6% as opposed to rising by 16% in the benchmark) since the capital requirement raises the shadow cost of running the bank.

The computation of this model is a nontrivial task. In an environment with aggregate shocks, all equilibrium objects, such as value functions and prices, are a function of the distribution of banks. The distribution of banks is an infinite dimensional object and it is computationally infeasible to include it as a state variable. Thus, we solve the model using an extension of the algorithm proposed by Krusell and Smith [21] or Farias et. al. [16] adapted to this environment. This entails approximating the distribution of banks by a finite number of moments. We use the mean asset and deposit levels of fringe banks joint with the asset level of the big bank since the dominant bank is an important player in the loan market. Furthermore, when making loan decisions, the big bank needs to take into account how changes in its behavior affects the total loan supply of fringe banks. This reaction function also depends on the industry distribution. For the same reasons as stated above, in the reaction function we approximate the behavior of the fringe segment of the market with the dynamic decision rules (including entry and exit) of the "average" fringe bank, i.e. a fringe bank that holds the mean asset and deposit levels.[1]

Some related literature follows. Van Den Heuvel [26] was one of the first quantitative general equilibrium models to study the welfare impact of capital requirements with perfect competition.[2] In a similar environment, Aliaga-Diaz and Olivero [1] analyze whether capital requirements can amplify business cycles. Also in a competitive environment, Repullo and Suarez [24] compare the relative performance of several capital regulation regimes and study their cyclical implications. In these papers, constant returns and perfect competition implies that there is an indeterminate distribution of bank sizes, so they do not examine the differential effect on big and small banks and how the strategic interaction affects outcomes when implementing a tighter capital regulation.

Other recent quantitative general equilibrium papers by Gertler and Kiyotaki [17] and Cociuba et. al. [8] consider the effects of credit policies and macro prudential policies on financial intermediation and risk taking incentives also with an indeterminate size distribution. In a paper more closely related to ours, De Nicolo et. al. [9] study the bank decision problem in a more general model than ours.[3] On the other hand, since they study only a

[1]An appendix to this paper states the algorithm we use to compute an approximate Markov perfect industry equilibrium.

[2]This paper follows in the tradition of quantitative general equilibrium models of banking beginning with Diaz Gimenez et. al. [12].

[3]Other papers that analyze the individual bank decision problem and capital requirements are Zhu [27] and Estrella [14]

decision problem, they do not consider the impact of such policies on interest rates on loans, the equilibrium bank size distribution, etc. Another related paper is that of Allen et.al. [2]. In their static framework, banks may hold levels of capital above the levels required by regulation due to incentives to monitor. They analyze how exogenous changes in market structure (i.e. the level of competition) affects the predictions of the model. Our paper is the first one with an endogenous bank size distribution. This allows us to to quantify how capital requirements affect failure rates and market shares of large and small banks and the importance of strategic interactions between large and small banks.

The paper is organized as follows. While we have documented a large number of banking facts that are relevant to the current paper in our previous work [10], Section 2 documents a new set of banking data facts relevant to this paper. Section 3 lays out a simple model environment to study bank risk taking and loan market competition. Section 4 describes a markov perfect equilibrium of that environment. Section 5 discusses how the preference and technology parameters are chosen and section 6 provides results for the simple model. Section 7 conducts our three counterfactuals: (i) differences in outcomes between our benchmark economy and a perfectly competitive model; (ii) the effects of an increase in bank capital requirements on business failures and banking stability in our benchmark economy with imperfect competition; and (iii) the interaction between policy changes and bank market structure.

2 Banking Data Facts

In our previous paper [10], we documented the following facts for the U.S. using data from the Consolidated Report of Condition and Income (known as Call Reports) that insured banks submit to the Federal Reserve each quarter.[4]. Entry is procyclical and exit by failure is countercyclical (correlation with detrended GDP equal to 0.62 and -0.25, respectively). Almost all entry and exit is by small banks. Loans and deposits are procyclical (correlation with detrended GDP equal to 0.58 and 0.10, respectively). Bank concentration has been rising; the top 4 banks have 35% of the loan market share. There is evidence of imperfect competition: the net interest margin is 4.6%; markups exceed 70%; the Lerner Index exceeds 35%; the Rosse-Panzar H statistic (a measure of the sensitivity of interest rates to changes in costs) is significantly lower than the perfect competition number of 100% (specifically, $H = 52$). Loan Returns, margins, markups, delinquency rates and charge-offs are countercyclical.[5]

[4] The number of institutions and its evolution over time can be found at http://www2.fdic.gov/hsob/SelectRpt.asp?EntryTyp=10
. Balance Sheet and Income Statements items can be found at https://cdr.ffiec.gov/public/.

[5] The countercyclicality of margins and markups is important. Building a model consistent with this is a novel part of our previous paper [10]. The endogenous mechanism in our papers works as follows. During a downturn, there is exit by small and medium size banks. This generates less competition among existing banks which raises the interest rate on loans. But since loan demand is inversely related to the interest rate, the ensuing increase in interest rates (barring government intervention) lowers loan demand even more thereby amplifying the downturn. In this way our model is the first to use imperfect competition to produce

Before turning to a set of new facts that this paper is intended to study, we first present the balance sheet of commercial banks (as a fraction of total assets) by bank size in years 1990 and 2010.

Table 1: Bank's Balance Sheet

Fraction Total Assets (%)	1990		2010	
	Bottom 99%	Top 1%	Bottom 99%	Top 1%
Cash	7.25	10.98	7.95	7.66
Securities	18.84	13.30	18.37	15.79
Loans	49.28	53.20	55.08	41.06
Deposits	69.70	62.75	64.37	56.02
Fed Funds and Repos	4.17	7.54	1.30	1.20
Equity Capital	6.20	4.66	9.94	10.66

Note: Data corresponds to commercial banks in the US. Source: Consolidated Report of Condition and Income.

We note that loans (which we will denote ℓ_t^θ for a bank of size θ in period t) and deposits (denoted d_t^θ) represent the largest asset and liability category for both bank sizes. Securities are the second largest asset component and it is larger for small banks than for big banks. For the model which follows, one can think of lumping cash and securities together (denoted a_t^θ), fed funds borrowed and repos (denoted B_t^θ) and equity capital (denoted e_t^θ).

Since we are interested in the effects of capital and liquidity requirements on bank behavior and loan rates, we organize the data to understand differences in capital holdings across banks of different sizes. Prior to 1980, no formal uniform capital requirements were in place. In 1981, the Federal Reserve Board and the Office of the Comptroller of the Currency announced a minimum total capital ratio (equity plus loan-loss reserves to total assets) of 6 percent for community banks and 5 percent for larger regional institutions. In 1985, a unified minimum capital requirement was set at 5.5% for all banks (see International Lending Supervision Act of 1983).

In 1988, central bank governors of the Group of Ten (G10) adopted the Basel Capital Accord (Basel I) which imposed binding capital requirements in the U.S. in 1992. One of the innovations in Basel I was the introduction of risk-weighted capital ratios. Assets are risk-weighted based on their perceived credit risk. For example, commercial loans carry a 100 percent risk weight while securities carry a zero weight. Basel I categorizes bank capital into Tier 1 (core) capital and Tier 2 capital.[6] Tier 1 capital is composed of common and preferred equity shares (a subset of total bank equity). Tier 2 capital includes subordinated

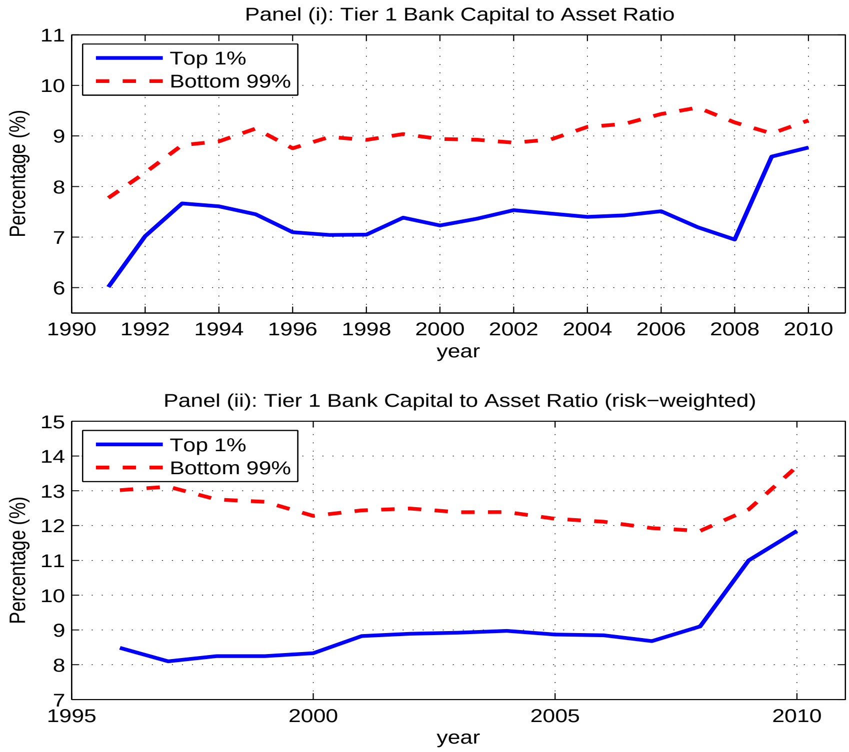
endogenous loan amplification in the banking sector.

[6]See Section 2.1 of DSC Risk Management Manual of Examination Policies (FDIC) for the complete description of U.S. bank capital regulation.

debt and hybrid capital instruments such as mandatory convertible debt. Total capital is calculated by summing Tier 1 capital and Tier 2 capital. Each individual bank, each Bank Holding Company (BHC), and each bank within a BHC is subject to three basic capital requirements: (i) Tier 1 Capital to Total Assets must be above 4% (if greater than 5% banks are considered well capitalized); (ii) Tier 1 Capital to Risk-Weighted Assets must exceed 4% (if greater than 6% banks are considered well capitalized); and (iii) Total Capital to Risk-Weighted Assets must be larger than 8% (if greater than 10% banks are considered well capitalized).

Given the timing in our model, we can express the risk-weighted capital ratio as e_t^θ/ℓ_t^θ and the capital to asset ratio as $e_t^\theta/(\ell_t^\theta + a_{t+1}^\theta)$. Table 1 documents that equity to asset ratios are larger for small banks in the early sample and the relation changes for the latest year in our sample. Further, since we are interested in bank capital ratios by bank size, Figure 1 presents the evolution of Tier 1 Capital to Asset Ratio and Tier 1 Capital to risk-weighted Asset Ratio for Top 1% and Bottom 99% banks when sorted by assets.

Figure 1: Bank Capital by Size

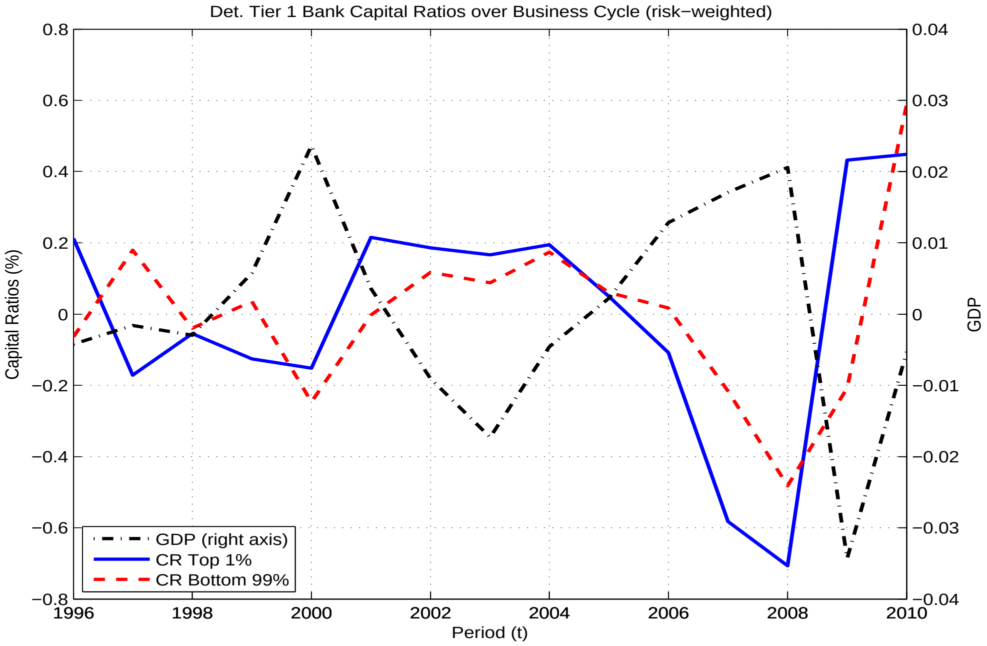


Note: Data corresponds to commercial banks in the US. Source: Consolidated Report of Condition and Income. GDP (det) refers to detrended real log-GDP. The trend is extracted using the H-P filter with parameter 6.25.

We note that the Total Capital Ratio has a slight upward trend even when separated by bank size. In all periods, capital ratios are lower for large banking institutions than those for small banks. The fact that capital ratios are above what regulation defines as well capitalized is suggestive of a precautionary motive.

Figure 2 presents the evolution of detrended Tier 1 Capital to risk-weighted Asset Ratio over time against detrended GDP.

Figure 2: Bank Capital and Business Cycles



Note: Data corresponds to commercial banks in the US. Source: Consolidated Report of Condition and Income. GDP (det) refers to detrended real log-GDP. The trend is extracted using the H-P filter with parameter 6.25.

The correlation of the Tier 1 capital ratio and GDP is -0.76 and -0.36 for top 1% and bottom 99% banks respectively. The fact that the correlation for small banks is less countercyclical than for large banks is suggestive that small banks try to accumulate capital during good times to build a buffer against bank failure in bad times. In fact, the correlation between Tier 1 Capital to total Assets and GDP is -0.45 for the top 1% banks and 0.48 for the bottom 99% banks.

3 Environment

Our dynamic banking industry model is based upon the static framework of Allen and Gale [3] and Boyd and DeNicolo [7]. In those models, there is an exogenous number of banks that are Cournot competitors either in the loan and/or deposit market.[7] We endogenize the number of banks by considering dynamic entry and exit decisions and apply a version of the Markov Perfect equilibrium concept in Ericson and Pakes [15] augmented with a competitive fringe as in Gowrishankaran and Holmes [18].

[7] Martinez-Miera and Repullo [22] also consider a dynamic model, but do not endogenize the number of banks.

Specifically, time is infinite. Each period, a mass N of one period lived ex-ante identical borrowers and a mass Ξ of one period lived ex-ante identical households (who are potential depositors) are born.

3.1 Borrowers

Borrowers demand bank loans in order to fund a project. The project requires one unit of investment at the beginning of period t and returns at the end of the period:

$$\begin{cases} 1 + z_{t+1}R_t & \text{with prob } p(R_t, z_{t+1}) \\ 1 - \lambda & \text{with prob } [1 - p(R_t, z_{t+1})] \end{cases} \tag{1}$$

in the successful and unsuccessful states respectively. Borrower gross returns are given by $1 + z_{t+1}R_t$ in the successful state and by $1 - \lambda$ in the unsuccessful state. The success of a borrower's project, which occurs with probability $p(R_t, z_{t+1})$, is independent across borrowers but depends on the borrower's choice of technology $R_t \geq 0$ and an aggregate technology shock at the end of the period z_{t+1} (the dating convention we use is that a variable which is chosen/realized at the end of the period is dated $t + 1$).

The aggregate technology shock is denoted $z_t \in \{z_b, z_g\}$ with $z_b < z_g$ (i.e. good and bad shocks). This shock evolves as a Markov process $F(z', z) = prob(z_{t+1} = z'|z_t = z)$.

At the beginning of the period when the borrower makes his choice of R_t, z_{t+1} has not been realized. As for the likelihood of success or failure, a borrower who chooses to run a project with higher returns has more risk of failure and there is less failure in good times. Specifically, $p(R_t, z_{t+1})$ is assumed to be decreasing in R_t and $p(R_t, z_g) > p(R_t, z_b)$. While borrowers are ex-ante identical, they are ex-post heterogeneous owing to the realizations of the shocks to the return on their project. We envision borrowers either as firms choosing a technology which might not succeed or households choosing a house that might appreciate or depreciate.

There is limited liability on the part of the borrower. If r_t^L is the interest rate on bank loans that borrowers face, the borrower receives $\max\{z_{t+1}R_t - r_t^L, 0\}$ in the successful state and 0 in the failure state. Specifically, in the unsuccessful state he receives $1 - \lambda$ which must be relinquished to the lender. Table 2 summarizes the risk-return tradeoff that the borrower faces if the industry state is ζ.

Table 2: Borrower's Problem

Borrower chooses R	Receive	Pay	Probability
			$\quad - \quad +$
Success	$1 + z'R$	$1 + r^L(\zeta, z)$	$p \ (R, \ z')$
Failure	$1 - \lambda$	$1 - \lambda$	$1 - p \ (R, \ z')$

Borrowers have an outside option (reservation utility) $\omega_t \in [\underline{\omega}, \overline{\omega}]$ drawn at the beginning of the period from distribution function $\Omega(\omega_t)$.

3.2 Depositors

Households are endowed with 1 unit of the good and have strictly concave preferences denoted $u(C_t)$. Households have access to a risk free storage technology yielding $1+\overline{r}$ with $\overline{r} \geq 0$ at the end of the period. They can also choose to supply their endowment to a bank or to an individual borrower. If the household deposits its endowment with a bank, they receive r_t^D whether the bank succeeds or fails since we assume deposit insurance. If they match with a borrower, they are subject to the random process in (1). At the end of the period they pay lump sum taxes τ_{t+1} which are used to cover deposit insurance for failing banks.

3.3 Banks

We assume there are two types of banks $\theta \in \{b, f\}$ for "big" and small "fringe" respectively. For simplicity, we assume there can be at most one big bank. If active, the big bank is a Stackelberg leader each period choosing a level of loans before fringe banks make their choice of loan supply. Consistent with the assumption of Cournot competition, the dominant bank understands that its choice of loan supply will influence interest rates. Fringe banks take the interest rate as given when choosing loan supply.

At the beginning of each period banks are matched with a random number of depositors. Specifically, in period t, bank i of type θ chooses how many deposits $d_{i,t}^{\theta}$ to accept up to a capacity constraint δ_t, i.e. $d_{i,t}^{\theta} \leq \delta_t$ where $\delta_t \in \{\delta^1, \ldots, \delta^n\} \subseteq \mathbb{R}_+$. The capacity constraint evolves according to a Markov process given by $G^{\theta}(\delta_{t+1}, \delta_t)$.

We denote loans made by bank i of type θ to borrowers at the beginning of period t by $\ell_{i,t}^{\theta}$. Bank i can also choose to hold securities $a_{i,t+1}^{\theta} \in \mathbb{R}_+$. We think of securities as associated with T-bills plus loans to other banks. We assume net securities have return equal to r_t^a. If the bank begins with $\tilde{a}_{i,t}^{\theta}$ net securities, the bank's feasibility constraint at the beginning of the period is given by:

$$\tilde{a}_{i,t}^{\theta} + d_{i,t}^{\theta} \geq \ell_{i,t}^{\theta} + a_{i,t+1}^{\theta}. \tag{2}$$

In Corbae and D'Erasmo [10] we document differences in bank cost structure across size. We assume that banks pay proportional non-interest expenses (net non-interest income) that differ across banks of different sizes, which we denote c_i^{θ}. Further, as in the data we assume a fixed cost κ_i^{θ}.

Let $\pi_{i,t+1}^{\theta}$ denote the end-of-period profits (i.e. after the realization of z_{t+1}) of bank i of type θ as a function of its loans $\ell_{i,t}^{\theta}$, deposits $d_{i,t}^{\theta}$ and securities $a_{i,t+1}^{\theta}$ given by

$$\pi_{i,t+1}^{\theta} = \Big\{p(R_t, z_{t+1})(1+r_t^L) + (1-p(R, z_{t+1}))(1-\lambda)\Big\}\ell_{i,t}^{\theta} + r_t^a a_{i,t+1}^{\theta} - (1+r_t^D)d_{i,t}^{\theta} - \Big\{\kappa_i^{\theta} + c_i^{\theta}\ell_{i,t}^{\theta}\Big\}. \tag{3}$$

The first two terms represent the gross return the bank receives from successful and unsuccessful loan projects respectively, the third term represents returns on securities, the fourth represents interest expenses (payments on deposits), and the fifth represents non-interest expenses.

After loan, deposit, and asset decisions have been made at the beginning of the period, we can define bank equity capital $e^{\theta}_{i,t}$ as

$$e^{\theta}_{i,t} \equiv \underbrace{a^{\theta}_{i,t+1} + \ell^{\theta}_{i,t}}_{\text{assets}} - \underbrace{d^{\theta}_{i,t}}_{\text{liabilities}} . \tag{4}$$

If banks face a capital requirement, they are forced to maintain a level of equity that is at least a fraction φ^{θ} of risk weighted assets (with weight w on the risk free asset). Thus, banks face the following constraint

$$e^{\theta}_{i,t} \geq \varphi^{\theta}(\ell^{\theta}_{i,t} + wa^{\theta}_{i,t+1}) \Rightarrow \ell^{\theta}_{i,t}(1-\varphi^{\theta}) + a^{\theta}_{i,t+1}(1-w\varphi^{\theta}) - d^{\theta}_{i,t} \geq 0. \tag{5}$$

If w is small, as called for in the BIS Basel Accord, then it is easier to satisfy the capital requirement the higher is $a^{\theta}_{i,t+1}$ and the lower is φ^{θ}. Securities relax the capital requirement constraint but also affect the feasibility condition of a bank. This creates room for a precautionary motive for net securities and the possibility that banks hold capital equity above the level required by the regulatory authority (i.e. $e^{\theta}_{i,t} > \varphi^{\theta}(\ell^{\theta}_{i,t} + wa^{\theta}_{i,t+1})$).

Another policy proposal is associated with bank liquidity requirements. Basel III [5] proposed that the liquidity coverage ratio, which is the stock of high-quality liquid assets (which include government securities) divided by total net cash outflows over the next 30 calendar days, should exceed 100%. In the context of a model period being one year, this would amount to a critical value of 1/12 or roughly 8%.[8] For the model, we assume

$$\gamma^{\theta} d^{\theta}_{i,t} \leq a^{\theta}_{i,t+1} \tag{6}$$

where γ^{θ} denotes the (possibly) size dependent liquidity requirement.

Following the realization of z_{t+1}, bank i of type θ can either borrow short term to finance cash flow deficiencies or store its cash/lend short term until next period. Specifically, denote short term borrowings by $B^{\theta}_{i,t+1} > 0$ and short term loans/cash storage by $B^{\theta}_{i,t+1} < 0$. The net rate at which banks borrow or lend is denoted $r^B_t(B_{i,t+1})$. For instance, if the bank chooses to hold cash over to the next period, then $r^B_t(B_{i,t+1}) = 0$.

Bank borrowing must be repaid at the beginning of the next period, before any other actions are taken. We assume that borrowing is subject to a collateral constraint:[9]

$$B^{\theta}_{i,t+1} \leq \frac{a^{\theta}_{i,t+1}}{(1+r^B_t)}. \tag{7}$$

[8] This is also close to the figure for reserve requirements which is bank size dependent, anywhere from zero to 10%. Since reserves now pay interest, bank liquidity requirements are similar in nature to current reserve requirement policy in our model.

[9] Along with limited liability, the collateral constraint can arise as a consequence of a commitment problem as in Gertler and Kiyotaki [17].

Repurchase agreements are an example of collateralized short term borrowing, while federal funds borrowing is unsecured. This implies that beginning-of-next-period cash and securities holdings are given by

$$\tilde{a}^{\theta}_{i,t+1} = a^{\theta}_{i,t+1} - (1 + r^B_t) \cdot B^{\theta}_{i,t+1} \geq 0. \tag{8}$$

Bank dividends at the end of the period are

$$\mathcal{D}^{\theta}_{i,t+1} = \pi^{\theta}_{i,t+1} + B^{\theta}_{i,t+1} \geq 0 \tag{9}$$

which are constrained to be positive since we assume that new equity financing is prohibitively expensive. A bank with positive cash flow $\pi^{\theta}_{t+1} > 0$ which chooses to pay that cash flow as dividends, chooses $B^{\theta}_{i,t+1} = 0$ otherwise it can lend or store cash $B^{\theta}_{i,t+1} < 0$ thereby raising beginning-of-next period's assets. A bank with negative cash flow $\pi^{\theta}_{t+1} < 0$ can borrow $B^{\theta}_{i,t+1} > 0$ against assets to avoid exit but beginning-of-next-period assets will fall.

There is limited liability on the part of banks. This imposes a lower bound equal to zero in the case that the bank exits. In the context of our model, limited liability implies that upon exit, the bank gets:

$$\max\left\{\xi\big[\{p(R_t, z_{t+1})(1 + r^L_t) + (1 - p(R, z_{t+1}))(1 - \lambda) - c^{\theta}_i\}\ell^{\theta}_{i,t} \right. \tag{10}$$
$$\left. +(1 + r^a_t)a^{\theta}_{i,t+1}\big] - d^{\theta}_{i,t}(1 + r^D_t) - \kappa^{\theta}, 0\right\}$$

where $\xi \in [0, 1]$ measures liquidation costs in the event of exit.

The fact that $a^{\theta}_{i,t+1} \in \mathbb{R}_+$, the capital requirement constraint (5), the collateral constraint (7), and limited limited liability (10) combine to imply that there exists a value of net securities $\underline{a}$ such that if $\tilde{a}^{\theta}_{i,t} < \underline{a}$ the only feasible option for the bank is to exit.[10] Thus, in order to avoid exit due to what amounts to an empty constraint set, any bank must hold (at least) a small amount of net securities.

Entry costs for the creation of the dominant bank are denoted by $\Upsilon^b \geq \Upsilon^f \geq 0$. Every period a large number of potential entrants make the decision of whether or not to enter the market. The value of initial deposits δ is drawn from probability distribution $G^{\theta,e}(\delta)$.

The industry state is defined as follows. Let $\mu_t(\tilde{a}, \delta)$ denote the distribution over matched deposits δ and net assets $\tilde{a}$ for fringe banks after entry and exit decisions are made. We define the variable $\hat{a}$ to be equal to the asset level of the big bank $\tilde{a}^b_{i,t}$ if the big bank is active and

[10] To see this, for simplicity, consider the case where $\tilde{a}^{\theta}_{i,t} = 0$. Then, given that there is no positive value of loans that satisfies the capital requirement constraint and that in general $r^a \geq r^D$, the bank will choose $a^{\theta}_{i,t+1} = d^{\theta}_{i,t}$. The continuation value of profits are $\pi^{\theta}_{i,t+1} = r^a_t a^{\theta}_{i,t+1} - (1 + r^D_t)d^{\theta}_{i,t} - \kappa^{\theta} < 0$ as long as $\kappa^{\theta} \geq d^{\theta}_{i,t}(r^a_t - r^D_t - 1)$. But $\kappa^{\theta} \geq d^{\theta}_{i,t}(r^a_t - r^D_t - 1)$ holds since $(r^a_t - r^D_t - 1) < 1$ for reasonable parameter values. The bank could cover the negative profits with borrowings, that is using $B^{\theta}_{i,t+1} = (1+r^D_t - r^a_t)d^{\theta}_{i,t} + \kappa^{\theta}$, but since $B^{\theta}_{i,t+1}(1 + r^B_t) \leq a^{\theta}_{i,t+1} = d^{\theta}_{i,t}$ has to hold, this option is not feasible. Thus, continuing is not a feasible option when assets are below a certain low threshold.

equal to $\emptyset$ if it is not. Similarly, define the variable $\hat{\delta}$ to be equal to the level of matched deposits of the big bank δ if it is active and equal to $\emptyset$ if it is not. The aggregate industry state is then denoted by $\zeta_t = \{\hat{a}_t, \hat{\delta}_t, \mu_t\}$.

3.4 Information

There is asymmetric information on the part of borrowers and lenders. Only borrowers know the riskiness of the project they choose (R_t) and their outside option (ω_t). All other information (e.g. project success or failure) is observable.

3.5 Timing

At the beginning of period t,

1. Liquidity shocks δ_t are realized.
2. Given the beginning of period state (ζ_t, z_t), borrowers draw ω_t.
3. The dominant bank chooses how many loans to extend, how many deposits to accept given depositors choices, and how many assets to hold $(\ell^b_{i,t}, d^b_{i,t}, a^b_{i,t+1})$.
4. Each fringe bank observes the total loan supply of dominant banks $(\ell^b_{i,t})$ and all other fringe banks (that jointly determine the loan interest rate r^L_t) and simultaneously decide how many loans to extend, deposits to accept, and how many assets to hold $(\ell^f_{i,t}, d^f_{i,t}, a^f_{i,t+1})$. Borrowers choose whether or not to undertake a project, and if so a level of technology R_t.
5. Aggregate return shocks z_{t+1} are realized, as well as idiosyncratic project success shocks.
6. Banks choose whether to borrow short term $(B^\theta_{i,t+1})$ and dividend policy. Exit and entry decisions are made in that order.
7. Households pay taxes τ_{t+1} to fund deposit insurance and consume.

4 Industry Equilibrium

Since we will use recursive methods to define an equilibrium, let any variable n_t be denoted n and n_{t+1} be denoted n'.

4.1 Borrower Decision Making

Starting in state z, borrowers take the loan interest rate r^L as given and choose whether to demand a loan and if so, what technology R to operate. Specifically, if a borrower chooses to participate, then given limited liability his problem is to solve:

$$v(r^L, z) = \max_R E_{z'|z}\Big[p(R, z')\left(z'R - r^L\right)\Big]. \tag{11}$$

Let $R(r^L, z)$ denote the borrower's decision rule that solves (11). We assume that the necessary and sufficient conditions for this problem to be well behaved are satisfied. The borrower chooses to demand a loan if

$$v(r^L, z) \geq \omega. \tag{12}$$

In an interior solution, the first order condition is given by

$$E_{z'|z}\Big\{\underbrace{p(R, z')z'}_{(+)} + \underbrace{\frac{\partial p(R, z')}{\partial R}}_{(-)}\left[z'R - r^L\right]\Big\} = 0 \tag{13}$$

The first term is the benefit of choosing a higher return project while the second term is the cost associated with the increased risk of failure.

To understand how bank lending rates influence the borrower's choice of risky projects, one can totally differentiate (13) with respect to r^L and re-arrange to yield

$$\frac{dR^*}{dr^L} = \frac{E_{z'|z}\left[\frac{\partial p(R^*, z')}{\partial R^*}\right]}{E_{z'|z}\left\{\frac{\partial^2 p(R^*, z')}{(\partial R^*)^2}\left[z'R^* - r^L\right] + 2\frac{\partial p(R^*, z')}{\partial R^*}z'\right\}} > 0 \tag{14}$$

where $R^* = R(r^L, z)$. Since both the numerator and the denominator are strictly negative (the denominator is negative by virtue of the sufficient conditions), a higher borrowing rate implies the borrower takes on more risk. Boyd and De Nicolo [7] call $\frac{dR^*}{dr^L} > 0$ in (14) the "risk shifting effect". Risk neutrality and limited liability are important for this result.

An application of the envelope theorem implies

$$\frac{\partial v(r^L, z)}{\partial r^L} = -E_{z'|z}[p(R, z')] < 0. \tag{15}$$

Thus, participating borrowers are worse off the higher are borrowing rates. This has implications for the demand for loans determined by the participation constraint. In particular, since the demand for loans is given by

$$L^d(r^L, z) = N \cdot \int_{\underline{\omega}}^{\overline{\omega}} 1_{\{\omega \leq v(r^L, z)\}} d\Omega(\omega), \tag{16}$$

then (15) implies $\frac{\partial L^d(r^L, z)}{\partial r^L} < 0$.

4.2 Depositor Decision Making

If $r^D = \overline{r}$, then a household would be indifferent between matching with a bank and using the autarkic storage technology so we can assign such households to a bank. If it is to match directly with a borrower, the depositor must compete with banks for the borrower. Hence, in successful states, the household cannot expect to receive more than the bank lending rate r^L but of course could choose to make a take-it-or-leave-it offer of their unit of a good for a return $\widehat{r} < r^L$ and hence entice a borrower to match with them rather than a bank. Given state contingent taxes $\tau(\zeta, z, z')$, the household matches with a bank if possible and strictly decides to remain in autarky otherwise provided

$$\begin{aligned} U \equiv \quad & E_{z'|z}\left[u(1+\overline{r}-\tau(\zeta,z,z'))\right] > \\ & \max_{\widehat{r}<r^L} E_{z'|z}\Big[p(\widehat{R},z')u(1+\widehat{r}-\tau(\zeta,z,z')) \\ & +(1-p(\widehat{R},z'))u\left(1-\lambda-\tau(\zeta,z,z')\right)\Big] \equiv U^E. \end{aligned} \tag{17}$$

Condition (17) makes clear the reason for a bank in our environment. By matching with a large number of borrowers, the bank can diversify the risk of project failure and this is valuable to risk averse households. It is the loan side uncertainty counterpart of a bank in Diamond and Dybvig [11].

If this condition is satisfied, then the total supply of deposits is given by

$$D^s = d^b(\tilde{a},\delta,z,\zeta) + \int d^f(\tilde{a},\delta,z,\zeta)\mu(d\tilde{a},d\delta) \leq H \tag{18}$$

4.3 Incumbent Bank Decision Making

After being matched with δ deposits, an incumbent bank i of type θ chooses loans ℓ_i^θ, deposits d_i^θ, and asset holdings $a_i^{\theta'}$ in order to maximize expected discounted dividends/cash flows. We assume Cournot competition in the loan market. Following the realization of z', banks can choose to borrow or store $B_i^{'\theta}$ and whether to exit $x_i(\theta)$.

Let σ_{-i} denote the industry state dependent balance sheet, exit, and entry strategies of all other banks. Given the Cournot assumption, the big bank takes into account that it affects the loan interest rate and its loan supply affects the total supply of loans by fringe banks. Differentiating the bank profit function π_i^θ defined in (3) with respect to ℓ_i^θ we obtain

$$\frac{d\pi_i^\theta}{d\ell_i^\theta} = \Big[\underbrace{pr^L-(1-p)\lambda-c^\theta}_{(+)\text{ or }(-)}\Big] + \ell_i^\theta\Big[\underbrace{p}_{(+)} + \underbrace{\frac{\partial p}{\partial R}\frac{\partial R}{\partial r^L}(r^L+\lambda)}_{(-)}\Big]\underbrace{\frac{dr^L}{d\ell_i^\theta}}_{(-)}. \tag{19}$$

The first bracket represents the marginal change in profits from extending an extra unit of loans. The second bracket corresponds to the marginal change in profits due to a bank's influence on the interest rate it faces. This term will reflect the bank's market power; for dominant banks $\frac{dr^L}{d\ell_i^b} < 0$ while for fringe banks $\frac{dr^{L,j}}{d\ell_i^f} = 0$.

Let the total supply of loans by fringe banks as a function of the aggregate state and the amount of loans that the big bank makes ℓ^b be given by

$$L^f(\zeta, \ell^b) = \int \ell_i^f(\tilde{a}, \delta, \zeta, \ell^b)\mu(d\tilde{a}, d\delta). \tag{20}$$

The loan supply of fringe banks is a function of ℓ^b because fringe banks move after the big bank.

The value of a big bank at the beginning of the period but after overnight borrowing has been paid is:

$$V^b(\tilde{a}, \delta, z, \zeta) = \max_{\ell \geq 0, d \in [0,\delta], a' \geq \gamma^b d} \beta E_{z'|z} W^b(\ell, d, a', \delta, \zeta, z') \tag{21}$$

s.t.

$$\tilde{a} + d \geq a' + \ell \tag{22}$$

$$\ell(1-\varphi^b) + a'(1 - w\varphi^b) - d \geq 0 \tag{23}$$

$$\ell + L^f(\zeta, \ell) = L^d(r^L, z) \tag{24}$$

where $W^b(\ell, d, a', \zeta, z')$ is the value of the bank at the end of the period for given loans ℓ, deposits d, net securities a', and realized shocks. Equation (24) is the market clearing condition which is included since the dominant bank must take into account its impact on prices. Changes in ℓ affect the equilibrium interest rate through its direct effect on the aggregate loan supply (first term) but also through the effect on the loan supply of fringe banks (second term). For any given ζ, $L^f(\zeta, \ell)$ can be thought of as a "reaction function" of fringe banks to the loan supply decision of the dominant bank.

The end-of-period function (that determines if the bank continues or exits and its future net securities position) is given by

$$W^b(\ell, d, a', \delta, \zeta, z') = \max_{x \in \{0,1\}} \left\{ W^{b,x=0}(\ell, d, a', \delta, \zeta, z'), W^{b,x=1}(\ell, d, a', \delta, \zeta, z') \right\} \tag{25}$$

where

$$W^{b,x=0}(\ell, d, a', \delta, \zeta, z') = \max_{B' \leq \frac{a'}{(1+r^B)}} \left\{ \pi^b(\ell, d, a', \zeta, z') + B' + E^b_{\delta'|\delta} V^b(\tilde{a}', \delta', z', \zeta') \right\} \tag{26}$$

s.t.

$$\tilde{a}' = a' - (1 + r^B)B' \geq 0 \tag{27}$$

$$\pi^b(\ell, d, a', \zeta, z') + B' \geq 0 \tag{28}$$

$$\zeta' = H(z, z', \zeta). \tag{29}$$

where $E^b_{\delta'|\delta}$ is the conditional expectation of future liquidity shocks for a big bank (i.e. based on the transition function $G^b(\delta', \delta)$). If the non-negativity of dividends constraint (28) is violated, we set $W^{b,x=0}(\ell, d, a', \zeta, \delta, z') = -\infty$ since we assume that banks have access to external funds only through B. In this case a bank that cannot borrow enough to stay afloat will exit. Equation (29) corresponds to the evolution of the aggregate state.

The value of exit is

$$W^{b,x=1}(\ell, d, a', \delta, \zeta, z') = \max \Bigg\{ \xi \big[\{p(R, z')(1 + r^L) + (1 - p(R, z'))(1 - \lambda) - c^b\}\ell + (1 + r^a)a' \big] - d(1 + r^D) - \kappa^b, 0 \Bigg\} \tag{30}$$

The lower bound on the exit value is associated with limited liability.

The solution to problem (21)-(30) provides big bank decision rules $\ell^b(\tilde{a}, \delta, z, \zeta)$, $a^{b'}(\tilde{a}, \delta, z, \zeta)$, $d^b(\tilde{a}, \delta, z, \zeta)$, $B^{b'}(\ell, d, a', \delta, z', \zeta)$, $x^b(\ell, d, a', \delta, \zeta, z')$ as well as value functions.

Next we turn to the fringe bank problem. The fringe bank takes as given the aggregate loan supply (and thus the interest rate). The value of a fringe bank at the beginning of the period but after any borrowings or dividends have been paid is:

$$V^f(\tilde{a}, \delta, z, \zeta) = \max_{\ell \geq 0, d \in [0,\delta], a' \geq \gamma^f d} \beta E_{z'|z} W^f(\ell, d, a', \delta, \zeta, z'), \tag{31}$$

s.t.

$$\tilde{a} + d \geq a' + \ell \tag{32}$$

$$\ell(1 - \varphi^f) + a'(1 - w\varphi^f) - d \geq 0 \tag{33}$$

$$\ell^b(\zeta) + L^f(\zeta, \ell^b(\zeta)) = L^d(r^L, z) \tag{34}$$

where $W^f(\ell, d, a', \zeta, \delta, z')$ is the value of the bank at the end of the period for given loans ℓ, deposits d, net securities a', and realized shocks. Even though fringe banks take the loan interest rate as given, that rate is determined by the solution to equation (34) which incorporates the loan decision rule of the big bank. The solution to this problem provides $\ell^f(a, \delta, z, \zeta)$,$d^f(a, \delta, z, \zeta)$ and $a'^f(a, \delta, z, \zeta)$.

The end of period function is given by

$$W^f(\ell, d, a', \delta, \zeta, z') = \max_{x \in \{0,1\}} \left\{ W^{f,x=0}(\ell, d, a', \delta, \zeta, z'), W^{f,x=1}(\ell, d, a', \delta, \zeta, z') \right\} \tag{35}$$

where

$$W^{f,x=0}(\ell, d, a', \delta, \zeta, z') = \max_{B' \leq \frac{a'}{(1+r^B)}} \left\{ \pi^f(\ell, d, a', \zeta, z') + B' + E^f_{\delta'|\delta} V^f(\tilde{a}', \delta', z', \zeta') \right\} \tag{36}$$

s.t.

$$\tilde{a}' = a' - (1 + r^B)B' \geq 0, \tag{37}$$

$$\pi^f(\ell, d, a', \zeta, z') + B' \geq 0, \tag{38}$$

$$\zeta' = H(z, z', \zeta). \tag{39}$$

As in the dominant bank case, if the non-negativity of dividends constraint (38) is violated, we set $W^{f,x=0}(\ell, d, a', \zeta, \delta, z') = -\infty$ since we assume that banks have access to external funds only through B'. In this case a bank that cannot borrow enough to stay afloat will exit. The value of exit is

$$W^{f,x=1}(\ell, d, a', \delta, \zeta, z') = \max\Big\{\xi\big[\{p(R,z')(1+r^L) + (1-p(R,z'))(1-\lambda) - c^f\}\ell \\ +(1+r^a)a'\big] - d(1+r^D) - \kappa^f, 0\Big\}. \quad (40)$$

At the end of every period after the realization of z' and exit occurs, there is a large number of potential entrants of type θ. In order to enter, they have to pay the entry cost Υ^θ and decide on their initial level of securities a' (equal to initial bank equity capital since there are no other liabilities). The value of entry net of entry costs for banks of type θ is given by

$$V^{\theta,e}(z, \zeta, z') \equiv \max_{a'} \left\{-a' + E^\theta_{\delta'} V^\theta(a', \delta', z', H(z, \zeta, z'))\right\} - \Upsilon^\theta. \quad (41)$$

Potential entrants will decide to enter if $V^{\theta,e}(z, \zeta, z') \geq 0$. The argmax of equation (41) for those firms that enter defines the initial equity distribution of banks.[11] Note that the new industry distribution is given by $\zeta' = H(z, \zeta, z')$. The total number of entrants will be determined endogenously in equilibrium.

We denote by E^f the mass of fringe entrants. Recall that, for simplicity, we assumed that there is at most one big active bank. Thus, the number of big bank entrants E^b equals zero when there is an incumbent big bank and it is at most one when there is no active big bank in the market. In general, free entry implies that

$$V^{\theta,e}(z, \zeta, z') \times E^\theta = 0. \quad (42)$$

That is, in equilibrium, the value of entry is zero, the number of entrants is zero, or both.

4.4 Evolution of the Cross-Sectional Bank Size Distribution

The distribution of fringe banks evolves according to

$$\mu'(a', \delta') = \int \sum_\delta (1 - x^f(\cdot)) I_{\{a' = \tilde{a}^f(\cdot))\}} G^f(\delta', \delta) d\mu(a, \delta) + E^f \sum_\delta I_{\{a' = a^{f,e}(\cdot))\}} G^{f,e}(\delta). \quad (43)$$

Equation (43) makes clear how the law of motion for the distribution of banks is affected by entry and exit decisions.

[11] After the initial injection, we do not allow outsiders to inject equity into banks.

4.5 Funding Deposit Insurance

Across all states (ζ, z, z'), taxes must cover deposit insurance in the event of bank failure. Let post-liquidation net transfers be given by

$$\Delta^\theta = (1+r^D)d^\theta - \xi\Big[\{p(1+r^L) + (1-p)(1-\lambda) - c^\theta\}\ell^\theta + a^{\theta'}(1+r^a)\Big]$$

where $\xi \leq 1$ is the post liquidation value of the bank's assets and cash flow. Then aggregate taxes are given by

$$\tau(z, \zeta, z') \cdot \Xi = \int \sum_\delta x^f \max\{0, \Delta^f\} d\mu(a, \delta) + x^b \max\{0, \Delta^b\}. \tag{44}$$

4.6 Definition of Equilibrium

Given government policy parameters $(r^a, r^B, \varphi^\theta, w, \gamma^\theta)$, a pure strategy Markov Perfect Industry Equilibrium (MPIE) is a set of functions $\{v(r^L, z),\ R(r^L, z)\}$ describing borrower behavior, a set of functions $\{V_i^\theta,\ \ell_i^\theta,\ d_i^\theta,\ a_i^{\theta'},\ B_i^{\theta'},\ x_i^\theta,\ \chi_i^\theta\}$ describing bank behavior, a loan interest rate $r^L(\zeta, z)$, a deposit interest rate $r^D = \bar{r}$, an industry state ζ, a function describing the number of entrants $E^\theta(z, \zeta, z')$, and a tax function $\tau(z, \zeta, z')$ such that:

1. Given a loan interest rate r^L, $v(r^L, z)$ and $R(r^L, z)$ are consistent with borrower optimization (11) and (12).

2. At $r^D = \bar{r}$, the household deposit participation constraint (17) is satisfied.

3. Given the loan demand function, $\{V^\theta,\ \ell^\theta,\ d^\theta,\ a_i^{\theta'},\ B_i^{\theta'},\ x^\theta,\ \chi^\theta\}$ are consistent with bank optimization (21)-(40).

4. The entry asset decision rules are consistent with bank optimization (41) and the free entry condition is satisfied (42).

5. The law of motion for the industry state (29) induces a sequence of cross-sectional distributions which are consistent with entry, exit, and asset decision rules in (43).

6. The interest rate $r^L(\zeta, z)$ is such that the loan market clears. That is,

$$L^d(r^L, z) = \ell^b(\zeta) + L^f(\zeta, \ell^b(\zeta))$$

   where aggregate loan demand $L^d(r^L, z)$ given by (16).

7. Across all states (z, ζ, z'), taxes cover deposit insurance transfers in (44).

5 Calibration

At this stage, we have not finished calibrating parameters. Some parameters come from the calibration of our model in Corbae and D'Erasmo [10]. As in that paper, a model period is set to be one year.

We begin with the parametrization of the four stochastic processes: $F(z', z)$, $G^{\theta}(\delta', \delta)$, $p(R, z')$, and $\Omega(\omega)$. To calibrate the stochastic process for aggregate technology shocks $F(z', z)$, we use the NBER recession dates and create a recession indicator. More specifically, for a given year, the recession indicator takes a value equal to one if two or more quarters in that year were dated as part of a recession. The correlation of this indicator with HP filtered GDP equals -0.87. Then, we identify years where the indicator equals one with our periods of $z = z_b$ and construct a transition matrix. In particular, the maximum likelihood estimate of F_{kj}, the $(j, k)th$ element of the aggregate state transition matrix, is the ratio of the number of times the economy switched from state j to state k to the number of times the economy was observed to be in state j. We normalize the value of $z_g = 1$ and choose z_b to match the variance of detrended GDP.

We identify "big" banks with the top 1% banks (when sorted by loans) and the fringe banks with the bottom 99% of the bank loan distribution. As in Pakes and McGuire [23] we restrict the number of big banks by setting the entry cost to a prohibitively high number if the number of incumbents after entry and exit exceeds a given number. In our application, we choose one (i.e. there will be at most one big dominant bank).

We make the following assumptions when parameterizing the stochastic deposit matching process. We assume that the support of δ for big banks is large enough that the constraint never binds, so we do not need to estimate a process for it. On the other hand, the law of motion for the deposit matching technology for fringe banks is parameterized using our panel of commercial banks in the U.S. In particular, we estimate the following autoregressive process for log-deposits in bank i in period t

$$\log(\delta_{it}) = (1 - \rho_d)k_0 + \rho_d \log(\delta_{it-1}) + k_1 t + k_2 t^2 + k_{3,t} + a_i + u_{it} \tag{45}$$

where t denotes a time trend, $k_{3,t}$ are year fixed effects, a_i are bank fixed effects and u_{it} is iid and distributed $N(0, \sigma_u^2)$.[12] Since this is a dynamic model we use the method proposed by Arellano and Bond [4]. To keep the state space workable, we apply the method proposed by Tauchen [25] to obtain a finite state Markov representation $G^f(\delta', \delta)$ to the autoregressive process in (45). To apply Tauchen's method, we use the estimated values of $\rho_d = 0.4735$ and $\sigma_u = 0.66$ from (45). Since we work with a normalization in the model (i.e. $z_g = 1$), the mean k_0 in (45) is not directly relevant. Instead we choose to calibrate the mean of the finite state markov process, denoted μ_d, to match the observed deposit market share of the fringe sector.

[12] Note that since the problem of the fringe bank is linear, the solution to our problem implies that the capacity constraint binds in almost all cases and we can approximate the constraint using information on deposits.

We parameterize the stochastic process for the borrower's project as follows. For each borrower, let $y = \alpha z' + (1-\alpha)\varepsilon_e - bR^{\psi}$ where ε_e is drawn from $N(\mu_\varepsilon, \sigma_\varepsilon^2)$. The borrower's idiosyncratic project uncertainty is iid across agents. We define success to be the event that $y > 0$, so in states with higher z or higher ε_e success is more likely. Then

$$\begin{aligned} p(R, z') &= 1 - \Pr(y \leq 0 | R, z') \\ &= 1 - \Pr\left(\varepsilon_e \leq \frac{-\alpha z' + bR^{\psi}}{(1-\alpha)}\right) \\ &= \Phi\left(\frac{\alpha z' - bR^{\psi}}{(1-\alpha)}\right) \end{aligned} \tag{46}$$

where $\Phi(x)$ is a normal cumulative distribution function with mean μ_ε and variance σ_ε^2.

The stochastic process for borrower outside options, $\Omega(\omega)$, is simply taken to be the uniform distribution $[\underline{\omega}, \overline{\omega}]$ where $\underline{\omega} = 0$.

We calibrate $\bar{r} = r^D$ using data from banks' balance sheets. We target the average cost of funds computed as the ratio of interest expense on funds (deposits and federal funds) over total deposits and federal funds for commercial banks in the US from 1976 to 2008.[13] Similarly, we calibrate r^a to the ratio of interest income from securities over the total securities.

Depositor preferences are given by $u(x) = \frac{x^{1-\sigma}}{1-\sigma}$ with $\sigma = 2$, a standard value in the macro literature. At this level of risk aversion the depositor participation constraint is satisfied. The mass of borrowers is normalized to 1.

We estimate the marginal cost of producing a loan c^θ and the fixed cost κ^θ using our panel of U.S. commercial banks following the empirical literature on banking (see for example Berger et. al. [6]).[14] The value of c^θ is derived from the estimated marginal Net Non-interest Expenses that, in place, are defined to be Marginal Non-interest Expenses minus Marginal Non-interest Income. Marginal Non-interest Income is estimated as the ratio of total non-interest income over assets. Marginal Non-interest Expenses is derived from the following trans-log cost function:

$$\begin{aligned} \log(T_{it}) &= a_i + k_1 \log(w_{it}^1) + h_1 \log(\ell_{it}) + k_2 \log(y_{it}) + k_3 \log(w_{it}^1)^2 \\ &\quad + h_2 [\log(\ell_{it})]^2 + k_4 [\log(y_{it})]^2 + h_3 \log(\ell_{it}) \log(y_{it}) + h_4 \log(\ell_{it}) \log(w_{it}^1) \\ &\quad + k_5 \log(y_{it}) \log(w_{it}^1) + k_6 \log(x_{it}) + \sum_{j=1,2} k_{7,j} t^j + k_{8,t} + \epsilon_{it} \end{aligned} \tag{47}$$

where T_{it} is total non-interest expense minus expenses on premises and fixed assets, w_{it}^1 corresponds to input prices (labor), ℓ_{it} corresponds to real loans (one of the two bank j's output), y_{it} represents securities and other assets (the second bank output measured by real assets minus loans minus fixed assets minus cash), x_{it} is equity (a fixed netput), the t

[13]Source: FDIC, Call and Thrift Financial Reports, Balance Sheet and Income Statement (http://www2.fdic.gov/hsob/SelectRpt.asp?EntryTyp=10). The nominal interest rate is converted to a real interest rate by using the CPI.

[14]The cost structure estimated is also used to compute our measure of Markups and the Lerner Index.

regressor refers to a time trend and $k_{8,t}$ refer to time fixed effects. We estimate this equation by panel fixed effects with robust standard errors clustered by bank. Marginal non-interest expenses is then computed as:

$$\frac{\partial T_{it}}{\partial \ell_{it}} = \frac{T_{it}}{\ell_{it}} \Big[h_1 + 2h_2 \log(\ell_{it}) + h_3 \log(y_{it}) + h_4 \log(w_{it}^1) \Big] \tag{48}$$

Finally, the fixed cost κ^θ is estimated as the total cost on expenses of premises and fixed assets. We present the estimates of κ^θ scaled by total loans at the bank level. Table 3 shows the estimated parameters.

Table 3: Bank's Cost Structure

Moment (%)	Non-Int Inc.	Non-Int Exp.	Net Exp. (c^θ)	Fixed Cost ($\kappa^\theta/\ell^\theta$)
Top 1 % Banks (%)	$2.32^\dagger$	$3.94^\dagger$	$1.62^\dagger$	$0.72^\dagger$
Bottom 99 % Banks (%)	0.89	2.48	1.60	0.99

Note: † Denotes statistically significant difference with Bottom 99% value. Data corresponds to commercial banks in the US. Source: FDIC, Call and Thrift Financial Reports. Net expense is calculated as Non-Interest Expense minus Non-Interest Income. Fixed cost κ^θ scaled by loans.

In our benchmark parametrization, we use values associated with current regulation. Thus we set the minimum level of bank equity risk-weighted capital ratio for both type of banks to 6%. That is, $\varphi^b = \varphi^f = 0.06$ and $w = 0$.

We are left with fifteen parameters to estimate: $\{\alpha, b, \mu_\varepsilon, \sigma_\varepsilon, \psi, \mu_d, \lambda, \overline{\omega}, \beta, \xi, r^B, \kappa^b, \kappa^f, \Upsilon^f, \Upsilon^b\}$. We will estimate the parameters of the model by Simulated Method of Moments. Since we are interested in the standard errors of the parameters the number of moments needs to be larger than the number of parameters. Except for one data moment, we use the data for commercial banks described in Section 2 and in our companion paper. The extra moment - the average real equity return (12.94%) as reported by Diebold and Yilmaz [13] - is added to shed light on the borrower's return R^*. The set of targets from commercial bank data includes the average default frequency (2.15%), the average entry rate (1.60%), average loan return (5.17%), average charge-off rate (0.79%), the loan market share of Bottom 99% banks (37.9%), the deposit market share of the Bottom 99% (35.56%), the capital ratio of the Bottom 99% banks (11.37%), the capital ratio of the Top 1% banks (7.5%), the securities to asset ratio of the bottom 99% banks (20.75%), the securities to asset ratio of the top 1% banks (13.41%), fixed cost to loan ratio of the top 1% banks (0.72%) and the fixed cost to loan ratio of the bottom 99% (0.99%),the average loan markup (102.73%), the ratio of profit rates of Top 1% banks to Bottom 99% banks (63.79%).

We use the following definitions to connect the model to some of the variables we presented in the data section. In particular,

- Default frequency: $1 - p(R^*, z')$.

- Borrower return: $p(R^*, z')(z'R^*)$.
- Bank Entry Rate: $E^f / \int \mu(a, \delta)$
- Loan return: $p(R^*, z')r^L$.
- Loan Market Share Bottom 99%: $L^f(\zeta, \ell^b(\zeta)) / \left(\ell^b(\zeta) + L^f(\zeta, \ell^b(\zeta))\right)$
- Loan Charge-off rate $(1 - p(R^*, z')\lambda$.
- Deposit Market Share Bottom 99%:

$$\frac{\int_{\tilde{a},\delta} d^f(\tilde{a}, \delta, z, \zeta) d\mu(\tilde{a}, \delta)}{\int_{\tilde{a},\delta} d^f(\tilde{a}, \delta, z, \zeta) d\mu(\tilde{a}, \delta) + d^b(\tilde{a}, \delta, z, \zeta)}$$

- Capital Ratio Bottom 99%: $\int_{\tilde{a},\delta} [e^f(\tilde{a}, \delta, z, \zeta)/\ell^f(\tilde{a}, \delta, z, \zeta)] d\mu(\tilde{a}, \delta) / \int_{\tilde{a},\delta} d\mu(\tilde{a}, \delta)$
- Capital Ratio Top 1%: $e^b(\tilde{a}, \delta, z, \zeta)/\ell^b(\tilde{a}, \delta, z, \zeta)$
- Securities to Asset Ratio Bottom 99%:

$$\int_{\tilde{a},\delta} [\tilde{a}^f(\tilde{a}, \delta, z, \zeta)/(\ell^f(\tilde{a}, \delta, z, \zeta) + \tilde{a}^f(\tilde{a}, \delta, z, \zeta))] d\mu(\tilde{a}, \delta) / \int_{\tilde{a},\delta} d\mu(\tilde{a}, \delta)$$

- Securities to Asset Ratio Top 1%: $\tilde{a}^b(\tilde{a}, \delta, z, \zeta)/(\ell^b(\tilde{a}, \delta, z, \zeta) + \tilde{a}^b(\tilde{a}, \delta, z, \zeta))$
- Profit Rate: $\frac{\pi_{\ell_i(\theta)(\cdot)}}{\ell_i(\theta)}$.
- Total Output / GDP :

$$L^s(z, \zeta)\Big\{p(z, \zeta, z')(1 + z'R) + (1 - p(z, \zeta, z'))(1 - \lambda)\Big\}.$$

Table 4 shows the calibrated parameters.

Table 4: Model Parameters

Parameter		Value	Targeted Moment
Mass of Borrowers	B	1	Normalization
Mass of Households	Ξ	B	Assumption
Depositors' Preferences	σ	2	Participation Const.
Aggregate Shock in Good State	z_g	1.0	Normalization
Aggregate Shock in Bad State	z_b	0.975	Std. Dev. GDP
Transition Probability	$F(z_g, z_g)$	0.86	NBER data
Transition Probability	$F(z_b, z_b)$	0.43	NBER data
Autocorrelation Deposits	ρ_d	0.47	Call Reports
Std. Dev. Error Dep.	σ_u	0.66	Call Reports
Dep Int. Rate (%)	$\overline{r}$	0.86	Interest Expense
Sec. Return (%)	r^a	1.2	Return Securities
Net Exp. Top 1% (%)	c^b	1.62	Net Expenses Top 1%
Net Exp. Bottom 99% (%)	c^f	1.60	Net Expenses Bottom 99%
Capital Req. Top 1% (%)	(φ^b, w)	(6.0,0)	Regulation
Capital Req. Bottom 99% (%)	(φ^f, w)	(6.0,0)	Regulation
Liquidity Req. (%)	$\gamma^b = \gamma^f$	0.0	Regulation
Weight Aggregate Shock	α	0.88	Default frequency
Success Probability Parameter	b	3.77	Borrower return
Mean Entrep. Dist.	μ_ε	-0.85	Bank entry rate
Volatility Entrep. Dist.	σ_ε	0.10	Loan return
Success Probability Parameter	ψ	0.78	Loan mkt share bottom 99%
Loss Rate	λ	0.21	Charge off rate
Max. Reservation Value	$\overline{\omega}$	0.25	Avg. Loan Markup
Discount Factor	β	0.95	Deposit mkt share bottom 99%
Mean Deposits	μ_d	0.04	Capital ratio bottom 99%
Asset Recovery Rate at exit	ξ	0.70	Capital ratio top 1%
Cost over night funds (%)	r^B	1.0	Sec. to asset ratio bottom 99%
Fixed Cost Top 1% (%)	κ^b	0.001	Fixed cost to loan ratio top 1%
Fixed Cost Bottom 99% (%)	κ^f	0.001	Fixed cost to loan ratio bottom 99%
Entry Cost Bottom 99%	Υ^f	0.006	Sec. to asset ratio top 1%
Entry Cost Top 1%	Υ^b	0.05	Ratio profit rate top 1% to bottom 99%

The finite state Markov representation $G^f(\delta', \delta)$ obtained using the method proposed by

Tauchen [25] and the estimated values of μ_d, ρ_d and σ_u is:

$$G^f(\delta', \delta) = \begin{bmatrix} 0.26 & 0.43 & 0.25 & 0.05 & 0.00 \\ 0.12 & 0.36 & 0.37 & 0.12 & 0.01 \\ 0.04 & 0.24 & 0.43 & 0.24 & 0.04 \\ 0.01 & 0.12 & 0.37 & 0.36 & 0.12 \\ 0.00 & 0.05 & 0.25 & 0.43 & 0.26 \end{bmatrix},$$

and the corresponding grid is $\delta \in \{0.009, 0.019, 0.040, 0.085, 0.179\}$. The distribution $G^{e,f}(\delta)$ is derived as the stationary distribution associated with $G^f(\delta', \delta)$.

Table 5 provides the moments generated by the model for the above parameter values relative to the data. Once again we note that the calibration is preliminary and so several model moments are relatively far from their targets.

Table 5: Model and Data Moments

Moment (%)	Model	Data
Default Frequency $1 - p(R^*, z')$	2.88	2.15
Borrower Return $p(R^*, z')(z'R^*)$	12.32	12.94
Entry Rate	0.50	1.60
Exit Rate	0.50	1.65
Loan Return $p(R^*, z')r^L$	6.85	5.17
Net Interest Margin	5.96	5.08
Charge-Off Rate $(1 - p(R^*, z'))\lambda$	0.60	0.79
Loan Market Share Bottom 99%	68.90	37.90
Deposit Market Share Bottom 99%	43.84	35.56
Capital Ratio (risk-weighted) Top 1%	6.46	7.50
Capital Ratio (risk-weighted) Bottom 99%	26.60	11.37
Securities to Asset Ratio Top 1%	5.95	15.79
Securities to Asset Ratio Bottom 99%	6.57	20.74
Fixed cost to loan ratio top 1%	1.1	0.72
Fixed cost to loan ratio bottom 99%	0.66	0.99
Avg. Loan Markup	124.86	102.73
Ratio profit rate top 1% to bottom 99%	99.98	63.79

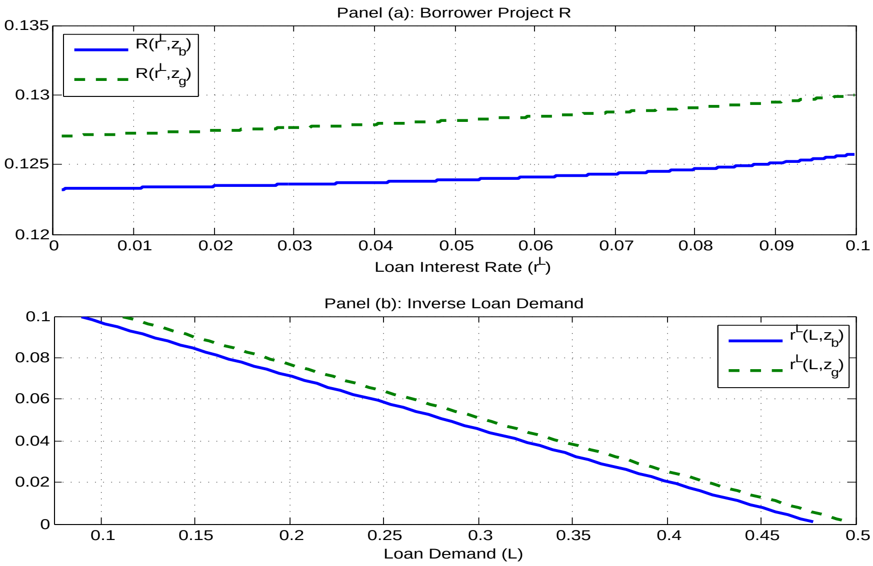
6 Results

For the parameter values in Table 4, we find an equilibrium where the dominant bank exit is not frequent (along the equilibrium path). On the other hand, exit occurs along the equilibrium path by fringe banks with small to median deposit holdings and low asset levels (i.e. $\delta \leq \delta_M = 0.04$ and $\tilde{a} \leq 0.004$) as well as fringe banks with bigger than median deposit holdings but even smaller asset levels (i.e. $\delta > \delta_M$ and $\tilde{a} \leq 0.002$) if the economy heads into bad times (i.e. $z = z_g$ and $z' = z_b$).[15] On the equilibrium path, fringe banks that survive the arrival of a bad aggregate shock accumulate securities in order to avoid exit.

6.1 Equilibrium Decision Rules

To understand the equilibrium, we first describe borrower decisions. Figure 3 shows the borrower's optimal choice of project riskiness $R^*(r^L, z)$ and the inverse demand function associated with $L^d(r^L, z)$. The figure shows that the borrower's optimal project R is an increasing function of the loan interest rate r^L. This is what Boyd and DeNicolo [7] call the "risk shifting" effect; that is, higher interest rates lead borrowers to choose more risky projects. Moreover, given that the value of the borrower is decreasing in r^L, aggregate loan demand is a decreasing function of r^L. The figure also illustrates that loan demand is pro-cyclical; that is, for a given interest rate, loan demand is higher in state z_g than z_b.

Figure 3: Borrower Project and Inverse Loan Demand

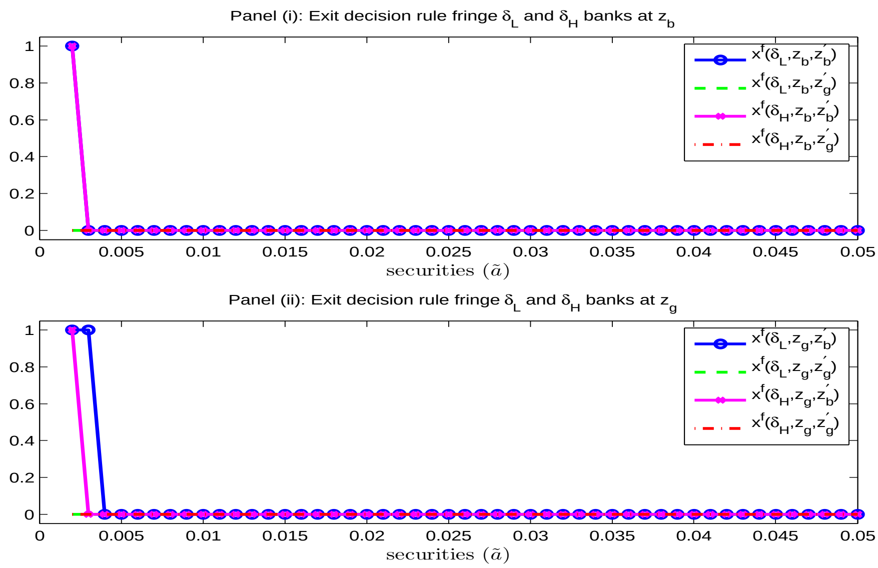


Next we turn to characterizing bank decision rules. Note that while these are equilibrium functions not every state is necessarily on-the-equilibrium path. It is best to work backwards and start with the exit decision rule. Except for the case where $\tilde{a}^b_{i,t} < \underline{a}$, we find the big bank does not exit (so we do not picture it). The big bank does not exit in equilibrium since it accumulates enough assets to avoid exit. The fringe banks do, however, exit as can be seen

[15] We also find that fringe banks with low asset levels ($\tilde{a} \leq 0.013$) exit if the economy stays in a recession (i.e. $z = z_b$ and $z' = z_b$) but this is off-the-equilibrium path behavior.

in Figure 4. Panel (i) graphs the smallest δ_L and largest δ_H fringe bank exit rules starting in the recession state z_b. With low assets, both types exit when the economy stays in a recession (off-the-equilibrium path). Panel (ii) shows that both small and large fringe banks exit when the economy transits from a boom to a recession if they have low assets. Notably, larger fringe banks are less likely than smaller ones to exit (i.e. their exit asset threshold is lower).

Figure 4: Fringe Banks Exit Rule (for different values δ)



Banks try to start the next period with sufficient assets to avoid exit (since exit means it loses its charter value). In Figure 5 we plot beginning-of-next period's asset choices by the big bank and the median fringe bank (what we called $\tilde{a}^{\theta}_{i,t+1}$ in (8)). Note that the big bank augments future net assets at low current levels in all states except when the economy enters a recession from a boom. The latter arises because the big bank chooses to borrow in that state. The figure also shows that the median fringe bank is more likely to save at higher asset levels than a big bank.

Figure 5: Big Bank and Median Fringe Bank Future Securities Rule $\tilde{a}^{\theta}$

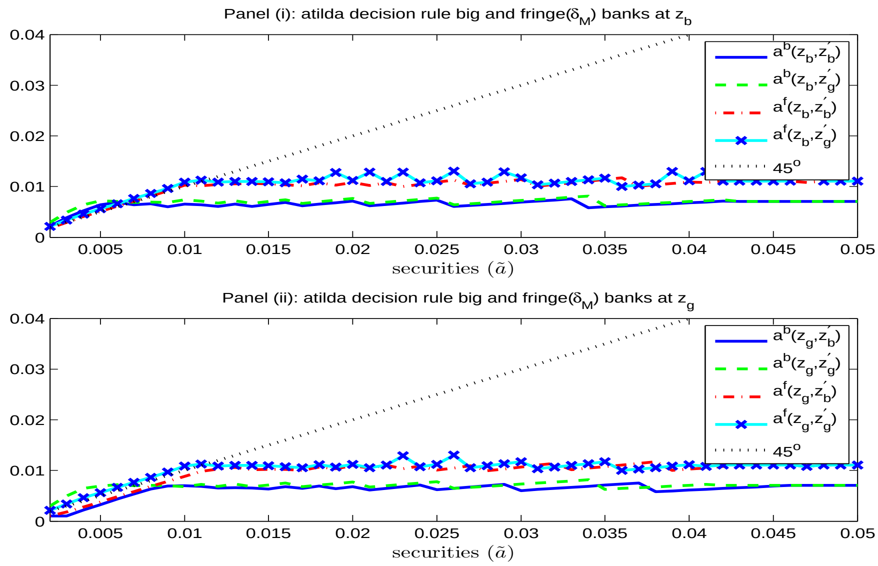
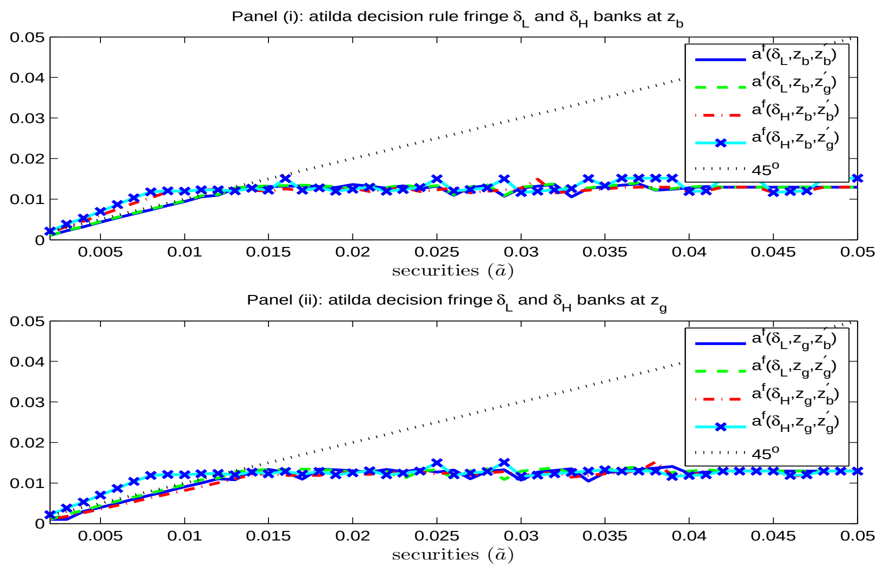


Figure 6 plots beginning-of-next period's asset choices by the smallest and largest fringe bank types. The figure shows that the smallest fringe bank is more constrained and unable to raise future securities like the largest fringe bank.

Figure 6: Fringe Banks Future Securities Rule $\tilde{a}^{\theta}$ (for different values δ)



The big and median fringe bank borrowing decision rules are illustrated in Figure 7. It is evident from panel (ii) that both the big banks (at almost all asset levels) and fringe banks (at low asset levels) borrow when entering a recession from good times. At all other times the banks store cash and/or lend short term.

Figure 7: Big Bank and Median Fringe Bank Borrowing Rule B^{θ}

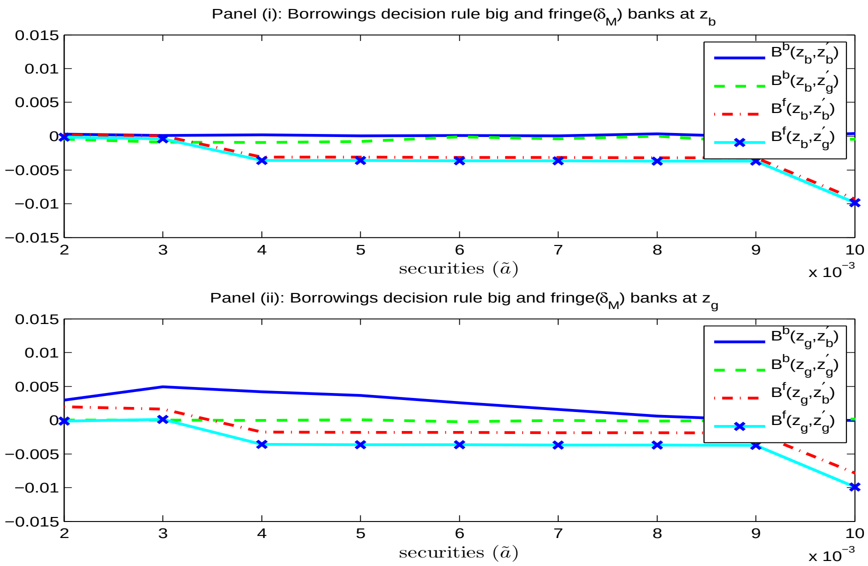
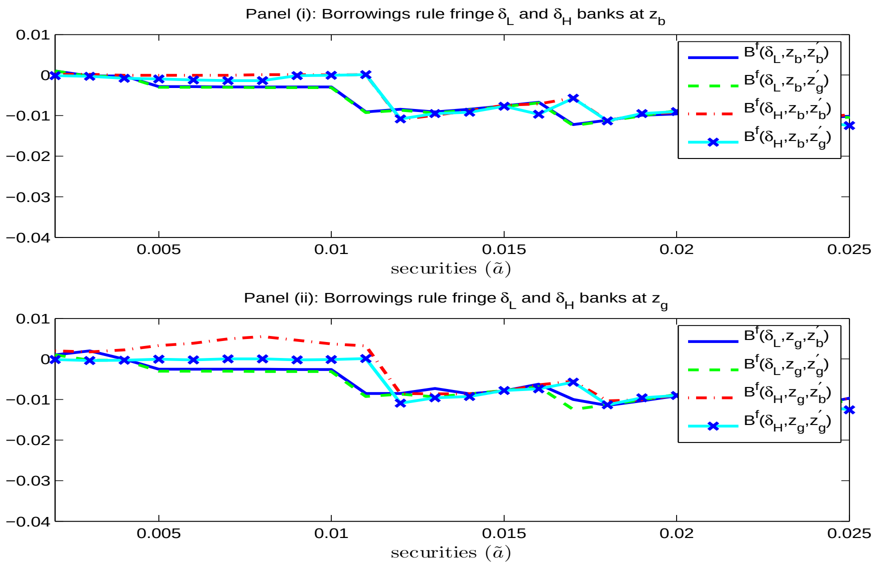


Figure (8) shows the borrowing decision rules for the smallest and largest fringe banks. As evident, both sizes of fringe bank store about the same amounts, except that the largest fringe stores significantly less as the economy enters a recession.

Figure 8: Fringe Banks Borrowing Rule B^{θ} (for different values δ)



The big and median fringe bank dividend decision rules are illustrated in Figure 9. While dividends are constrained to be non-negative in (9), strictly positive payouts arise only if the bank has sufficiently high assets. Note that the big banks tend to make higher payouts.

Figure 9: Big Bank and Median Fringe Bank Dividend Rule $\mathcal{D}^{\theta}$

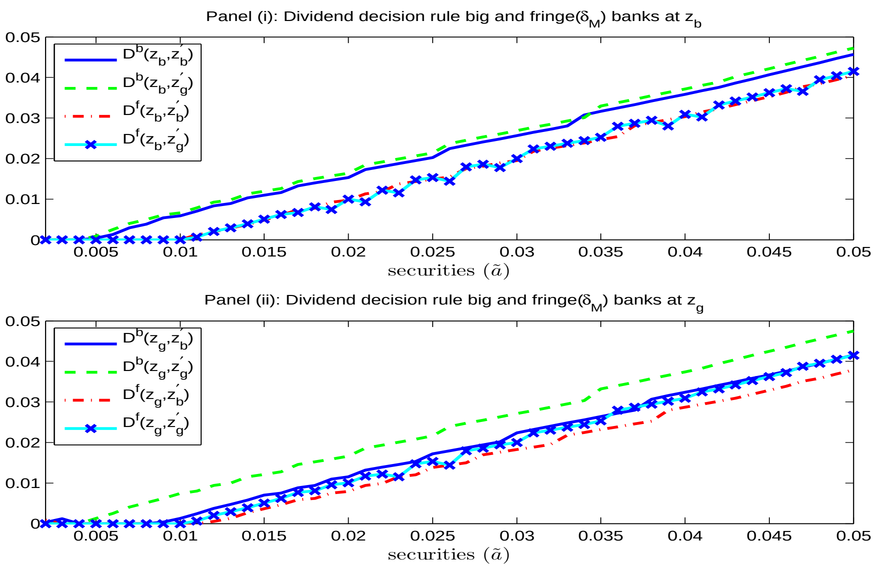
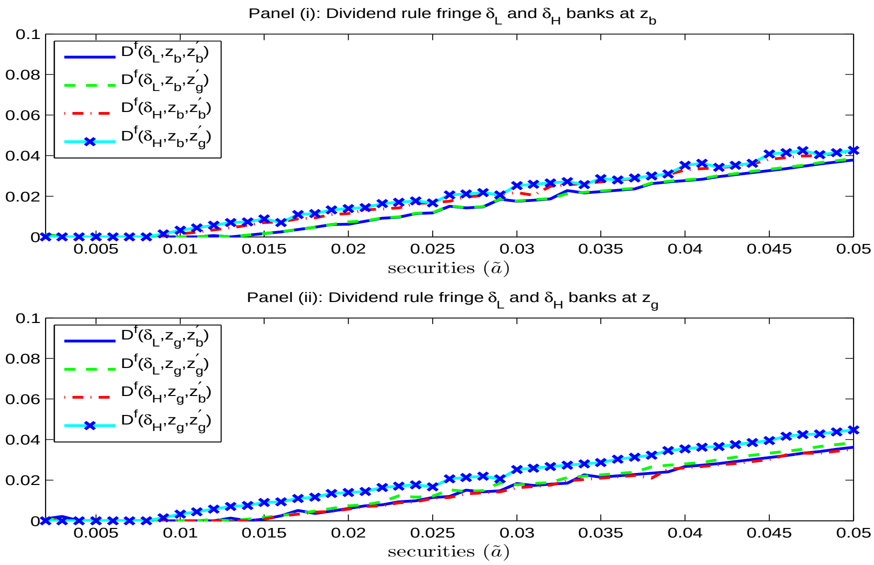


Figure (10) suggests that the biggest fringe banks are more likely to make dividend payouts than the smallest fringe banks.

Figure 10: Fringe Banks Dividend Rule $\mathcal{D}^{\theta}$ (for different values δ)



The beginning-of-period equity ratio $\frac{e^{\theta}}{\ell^{\theta}}$ is illustrated in Figure 11.[16] Recall from (4) that at the beginning of the period, equity is given by $e^{\theta} = a^{\theta'} + \ell^{\theta} - d^{\theta}$ and that capital requirements with $w = 0$ are given by $e^{\theta} \geq \varphi^{\theta}\ell^{\theta}$ in (5). The figure also plots the capital requirement $\varphi^{\theta} = 0.06$. As evident, the capital requirement is binding on the big bank for low asset levels.

[16]Note: We need to add more grid points to smooth this figure.

Figure 11: Big Bank and Median Fringe Bank Equity Ratios $e/\ell = (a' + \ell - d)/\ell$

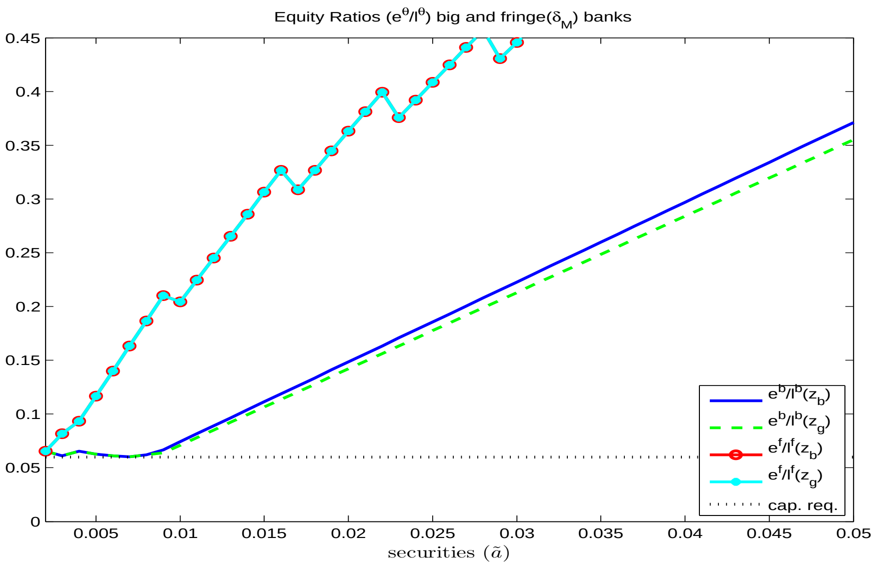
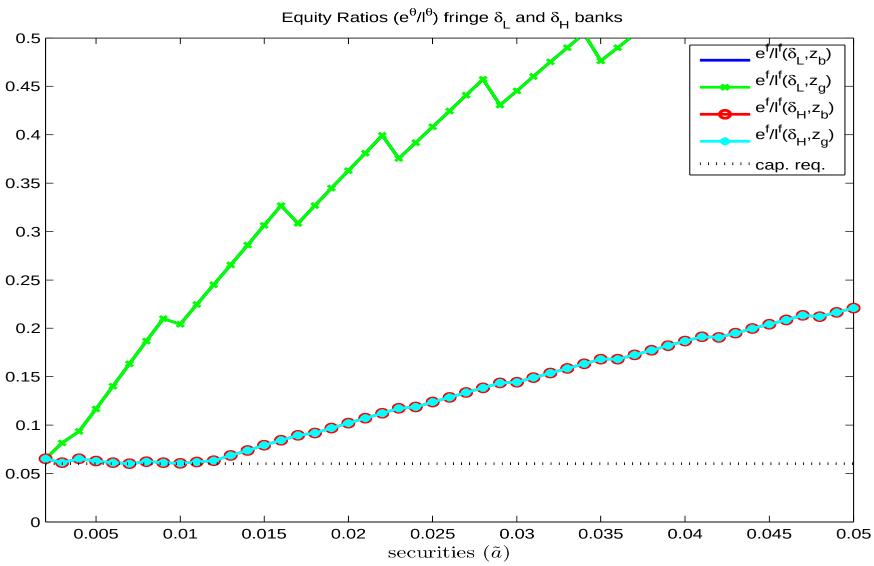


Figure (12) shows that small fringe banks have much higher equity ratios than large fringe banks across all asset levels. In particular, the figure provides evidence of the same type of ranking of capital ratios across big and small fringe banks as evidenced between the median fringe and dominant bank.

Figure 12: Fringe Banks Equity Ratios $e/\ell = (a' + \ell - d)/\ell$ (for different values δ)



The beginning-of-period loan decision rules for dominant and median fringe banks are illustrated in the top panel of Figure 13. If the dominant bank has sufficient assets, the figure shows that it extends more loans in good than bad times. However at low asset levels, loans are increasing in asset levels in both good and bad times. The same is true for its deposit decision. The figure also shows the effects of the capacity constraint on fringe banks. In particular, since the matching function is independent of aggregate state and asset holdings, so are deposit holdings in Panel (ii). Panel (i) shows that fringe banks which have more assets can make more loans. Since there is a simple ranking of loans and deposits among fringe banks, we do not graph that case.

Figure 13: Big Bank and Median Fringe Bank Loan and Deposit Decision Rules ℓ^θ and d^θ

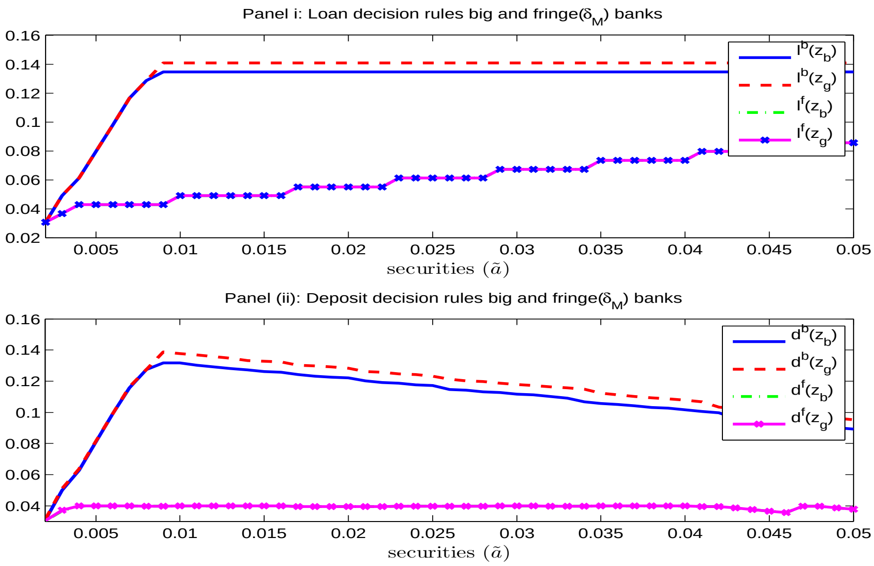


Figure (14) graphs the value function for a potential entrant over the fraction of incumbents M. What is important is that it is decreasing in the mass of incumbents; that is, the benefit of entering is smaller the larger the mass of incumbents. Since the probability of getting out of a recession are relatively high (57%), the value of entering is higher in bad times than good.

Figure 14: Value Fringe Bank Potential Entrant

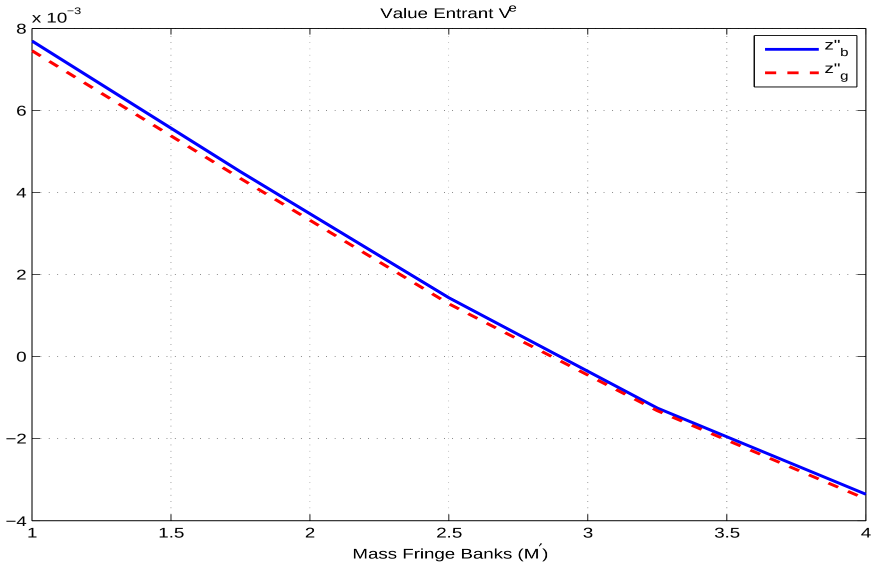
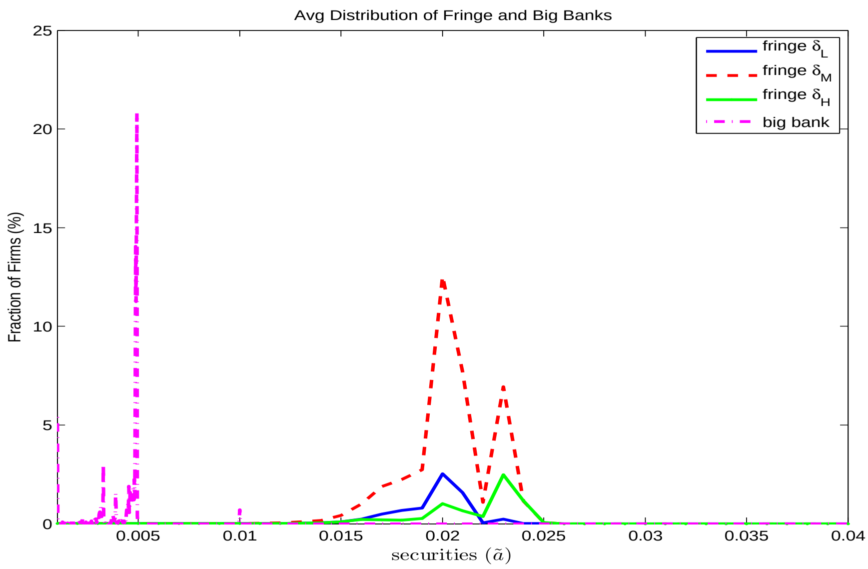


Figure 15 graphs the long run average distribution of bank assets for the three different "liquidity" constrained small banks as well as the dominant bank. Recall that there is no invariant distribution since there is aggregate uncertainty. In this figure, we show the average distribution that arises along the equilibrium path. More specifically, each period the model generates a distribution of fringe banks $\mu_t(a, \delta)$. This figure presents the average of fifty simulated panels of $\bar{\mu}(a, \delta) = \sum_{t=1}^{T} \mu_t(a, \delta)/T$, where $T = 2000$ is the number of simulated periods.[17] The values presented for the big bank correspond to the fraction of time that the big bank spends along the equilibrium path in each asset level (i.e. the histogram of securities). It is evident from the figure that the distribution of security holdings of the big bank is lower than that of the fringe banks.

[17]We discard the first 500 periods of the simulation to avoid dependence on initial conditions.

Figure 15: Avg. Distribution of Fringe and Big Banks



6.2 Business Cycle Correlations

We now move on to moments that the model was not calibrated to match, so that these results can be considered a simple test of the model. Table 6 provides the correlation between key aggregate variables with GDP.[18] We observe that, as in the data, the model generates countercyclical loan interest rates, exit rates, default frequencies, loan returns, charge-off rates, price-cost margins, markups and capital ratios across bank sizes. Moreover, the model generates procyclical entry rates as well as aggregate loans and deposits.

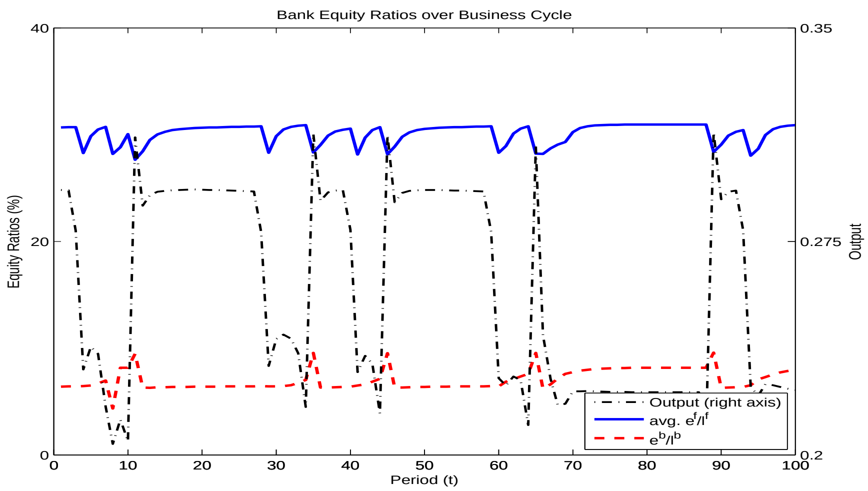
[18]We use the following dating convention in calculating correlations. Since some variables depend on z and ζ (e.g. loan interest rates $r^L(z,\zeta)$) and some other variables depend on z, ζ, and z', (e.g. default frequency $1-p(R(r^L(z,\zeta)),z')$), Table 6 displays $corr(GDP(z,\zeta,z'),x(z,\zeta))$ and $corr(GDP(z,\zeta,z'),y(z,\zeta,z'))$ where $x(z,\zeta)$ is any variable x that depends on (z,ζ) and $y(z,\zeta,z')$ is any variable y that depends on (z,ζ,z').

Table 6: Model and Data Business Cycle Correlations

Variable Correlated with GDP	Model	Data
Loan Interest Rate r^L	-0.96	-0.18
Exit Rate	-0.09	-0.25
Entry Rate	0.16	0.62
Loan Supply	0.98	0.58
Deposits	0.96	0.11
Default Frequency	-0.24	-0.08
Loan Return	-0.52	-0.49
Charge Off Rate	-0.24	-0.18
Price Cost Margin Rate	-0.52	-0.47
Markup	-0.96	-0.19
Capital Ratio Top 1% (risk-weighted)	-0.52	-0.75
Capital Ratio Bottom 99% (risk-weighted)	-0.60	-0.12

Figure 16 plots a simulation of capital ratios for big and fringe banks across a 100 period sample realization of business cycle shocks.

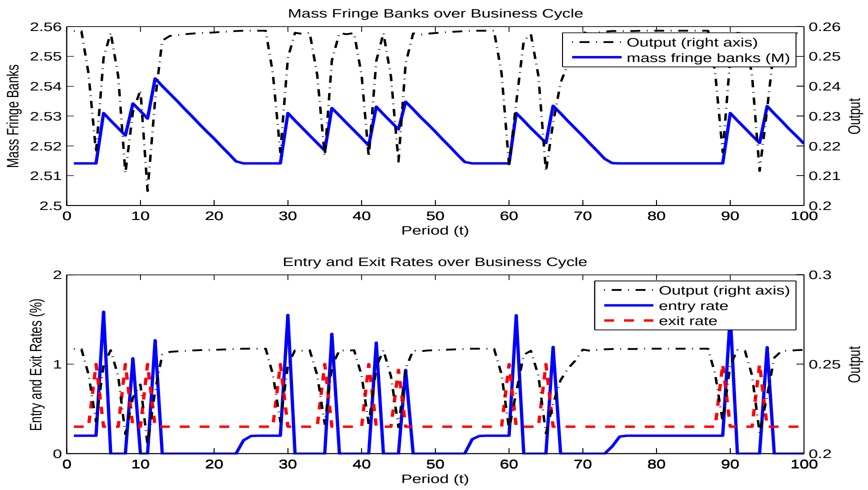
Figure 16: Capital Ratios over the Business Cycle



It is clear from this figure that equity ratios are countercyclical. The countercyclicality is mainly driven by changes in equity ratios in periods where $z \neq z'$. Intuitively, expansions (i.e. periods where $z = z_b$ and $z' = z_g$) are preceded by periods where banks reduced their level of securities in order to cover negative profits. The end of the recession is accompanied by an increase in the number of loans at a low level of securities generating a drop in the bank capital ratio. Similarly, before heading into a recession banks accumulate securities in order to cover possible losses. Thus, the beginning of a recession is associated with high capital ratios.

Figure 17 presents the evolution of the mass of fringe banks as well as entry and exit rates over the business cycle. When the economy enters into a recession, a fraction of fringe banks exit.

Figure 17: Competition over the Business Cycle



7 Counterfactuals

7.1 A perfectly competitive environment

In this subsection, we assess the role of imperfect competition in the model. Since our model nests a perfectly competitive environment (our fringe banks), we simply increase the entry cost for the big bank to a value that prevents entry. All other parameters remain identical to those used for the benchmark model. The spirit of this exercise is to endogenously generate an environment where all banks are perfectly competitive (i.e. all banks take prices as given). Table 7 presents a comparison of the moments in the Benchmark model and the model with Perfect Competition for the benchmark case where capital requirements are set to 6%.

Table 7: Benchmark vs Perfect Competition

Moment	Benchmark Model	Perfect Competition (higher Υ^b)	Change (%)
Capital Requirement φ (%)	6.00	6.00	0.00
Default Frequency (%)	2.88	2.29	-20.62
Exit Rate (%)	0.50	0.56	10.51
Loan Interest Rate (%)	7.06	5.97	-15.51
Borrower Project (%)	12.73	12.70	-0.27
Avg. Markup (%)	124.87	104.73	-16.13
Avg. Markup Fringe (%)	137.41	104.73	-23.79
Capital Ratio Fringe (%)	26.60	23.35	-12.24
Measure Fringe Banks	2.53	4.22	66.77
Output	0.24	0.29	19.71
Loan Supply	0.22	0.26	19.55
Taxes/GDP (%)	0.06	0.26	356.47

Table 7 makes evident that without competition from big banks, there is a large inflow of fringe banks (+66.77%) which results in a higher loan supply (19% change) and lower loan interest rates (-15% change). This results in slightly less risk taking by borrowers (-0.2% change) and a lower default frequency (-20% change). The increase in the number of banks and the reduction in interest rates results in an increase in output (+19% change).

Table 7 also shows that there is an important reduction in capital ratios (-12% change) between the benchmark and the competitive environment. Recall that the bank capital ratio is given by $e/\ell = 1 + (a' - d)\ell$. Banks' portfolio composition is driven by the valuable smoothing role that securities provide in cases of bank distress (negative profits) and the cost arising from differences in the expected loan spread of loans over securities. In the competitive environment, lower interest rates make it harder for banks to accumulate equity through retained earnings. Moreover, as evident from Table 8, in the perfectly competitive environment the volatility of all aggregates is lower. Thus, since the incentives to self-insure are reduced, the shadow value of an extra unit of securities also decreases, generating the lower capital ratios and the difference in portfolio composition between the perfectly competitive economy than in the benchmark.

Table 8: Volatility in Benchmark vs Perfect Competition

Coefficient of Variation (%)	Benchmark Model	Perfect Competition ($\uparrow \Upsilon^b$)	Change (%)
Loan Interest Rate	6.26	0.94	-85.01
Loan Return	8.62	5.15	-40.29
Borrower Return	7.06	5.91	-16.33
Default Frequency	209.27	214.41	2.46
Output	8.99	2.29	-74.50
Loan Supply	8.71	1.66	-80.92
Capital Ratio Fringe	3.16	0.73	-76.94
Measure Banks	0.87	0.64	-26.15
Markup	6.78	1.33	-80.41

7.2 Higher capital requirements with imperfect competition

Here we ask the question "How much does a 33% increase (from 6% to 8%) in capital requirements affect bank exit and outcomes?" Table 9 presents the results of this counterfactual.

Table 9: Capital Regulation Counterfactual

Moment	Benchmark ($\varphi = 6\%$)	Higher Cap. Req. ($\varphi = 8\%$)	Change (%)
Default Frequency (%)	2.88	2.85	-1.28
Exit Rate (%)	0.505	0.503	-0.30
Loan Interest Rate (%)	7.06	6.94	-1.77
Borrower Project (%)	12.734	12.732	-0.02
Loan Market Share Bottom 99% (%)	68.90	80.87	17.37
Deposit Market Share Bottom 99% (%)	43.84	56.07	27.90
Capital Ratio Top 1% (%)	6.46	7.27	12.54
Capital Ratio Bottom 99 % (%)	26.60	30.03	12.89
Probability Exit Big Bank (%)	0.70	0.60	-14.29
Avg. Markup	124.87	127.31	1.96
Avg. Markup Fringe (%)	137.41	133.68	-2.72
Measure Banks Bottom 99 %	2.53	2.95	16.58
L^s to output ratio	0.90	0.89	-0.03
Output	0.244	0.250	2.29
Loan Supply	0.218	0.223	2.26
Taxes/Output (%)	0.06	0.03	-44.14

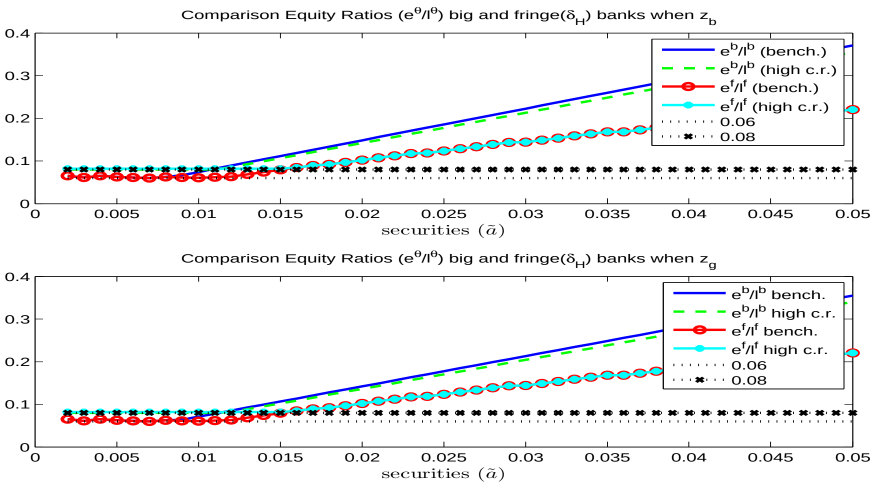
The capital requirement constraint is more often binding (or close to binding) for large fringe banks and dominant banks. The increase in the capital requirement constraint induces these banks to substitute into securities from loans. Substituting securities for loans reduces the profitability of these banks since loans dominate securities in expected return. Lower loan supply by these big banks also creates an opening for fringe banks to enter leading to an increase in loan market share of the bottom 99% as well as a higher measure of fringe banks. The higher aggregate loan supply leads to a lower loan interest rate, default frequency, and higher output.

As Table 9 makes clear, increasing capital requirements has the intended effect of reducing exit rates by 15% for large banks. The increase in capital requirements (everything else equal) reduces the continuation value of the bank (since their profits are lower) which is consistent with the lower entry rate. One of the benefits of higher capital requirements is the decrease in the exit rate which results in lower taxes (over GDP) to cover deposit insurance (-44% change).

One novelty of our model is that the level of competition is endogenous. We observe a large increase in the loan market share of the bottom 99%, compensating for the fact that the big bank is making less loans.

Figure 18 presents a comparison of the equity ratios for big banks and large fringe banks (i.e. those with δ_H) in the benchmark economy (bench.) and in the model with higher capital requirements (c.r.). As the figure makes clear, nearly all banks are constrained at low levels of securities.

Figure 18: Higher Capital Requirements and Equity Ratios for Big and Fringe Banks



7.3 Interaction between capital requirements and competition

In this subsection, we ask "How much does a 33% increase in capital requirements affect bank exit and outcomes under an assumption that all banks are perfectly competitive?" This experiment is meant to assess the interaction between market structure and changes in government policy and provide a measure of the contribution of our work against models with perfect competition and an indeterminate bank size distribution (such as Van Den Heuvel [26], Aliaga-Diaz and Olivero [1] and Repullo and Suarez [24]). Table 10 compares the responses to capital requirement changes in both the benchmark imperfect competition environment to the same policy change in the perfectly competitive model.

Table 10: Higher Capital Requirements and Competition

	Benchmark Model			Perfect Competition		
Moment	$\varphi = 6\%$	$\varphi = 8\%$	Change (%)	$\varphi = 6\%$	$\varphi = 8\%$	Change (%)
Default Frequency (%)	2.88	2.85	-1.28	2.29	2.33	1.90
Exit Rate (%)	0.50	0.50	-0.30	0.56	0.50	-9.51
Loan Interest Rate (%)	7.06	6.94	-1.77	5.97	6.07	1.67
Borrower Project (%)	12.73	12.73	-0.02	12.70	12.70	0.02
Capital Ratio Fringe (%)	26.60	30.03	12.89	23.35	27.79	19.01
Measure Fringe Banks	2.53	2.95	16.58	4.22	3.96	-6.21
Avg. Markup (%)	124.87	127.31	1.96	104.73	107.70	2.84
Avg. Markup Fringe (%)	137.41	133.68	-2.72	104.73	107.70	2.84
Output	0.24	0.25	2.29	0.29	0.29	-1.50
Loan Supply	0.22	0.22	2.26	0.26	0.26	-1.49
L^s to output ratio	89.51	89.48	-0.03	89.38	89.39	0.01
Taxes/GDP (%)	0.06	0.03	-44.14	0.26	0.22	-15.73

The policy results in the intended effect of reducing bank exit (in this case by a much larger amount (-9%). As in the benchmark, an increase in capital requirements (everything else equal) reduces the continuation value of the bank in the perfectly competitive model (since profitability is lower than when capital requirements are higher). In equilibrium, this results in a reduction in the average measure of incumbent fringe banks (-6% change). This is very different from the benchmark case where the measure of fringe banks actually rises. A lower mass of banks implies a higher loan interest rate (nearly +2% change) and a default frequency that is larger than that of the benchmark (nearly +2% change). The higher loan interest rate also results in less projects being operated and a lower GDP (-1.5% change).

A 33% increase in capital requirements results in an increase of 19% in the average capital ratio in the competitive environment, larger than that for banks in the benchmark. Since the perfectly competitive case is less volatile (as was shown in Table 8), a larger fraction of banks are closer to the minimum level of required capital and this results in the observed differential change in capital ratios for banks across the models. The increase in capital ratios and markups results in a large reduction in the bank exit rate (-9% change).

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8 Computational Appendix

We solve the model using an extension of the algorithm proposed by Krusell and Smith [21] or Farias et. al. [16] adapted to this environment. This entails approximating the distribution of banks $\zeta = \{\hat{a}, \hat{\delta}, \mu(\tilde{a}, \delta)\}$ by a finite number of moments. The moments we use are the mean asset $\overline{A}$ and deposit $\overline{\delta}$ level of fringe banks as well as the mass of fringe banks M, along with the asset level of the dominant bank. To keep the state space simple, we also assume that $\overline{\delta}$ is the unconditional mean of the $G(\delta', \delta)$ process (after checking whether it was a good assumption). Unlike the competitive framework in Krusell and Smith, when making loan decisions, the dominant bank needs to take into account how changes in its behavior affects the total loan supply of fringe banks. This reaction function also depends on the industry distribution. While Farias et. al. also have a reaction function, they base theirs only on the average firm's static profit function. For the same reasons as stated above, in the reaction function we approximate the behavior of the fringe segment of the market with the dynamic decision rules (which unlike Farias et. al. includes exit) of the "average" fringe bank, i.e. a fringe bank that holds the mean asset and deposit levels.

More specifically, in the decision problem of the dominant bank, instead of the state vector being given by $V^b(\tilde{a}, \delta, z, \zeta)$ and $W^b(\ell, d, a', \zeta, \delta, z')$, recognizing that we are approximating the fringe part of ζ by $\overline{A}$ and M, we use $V^b(\tilde{a}, z, \overline{A}, M)$ and $W^b(\pi, a', z, z', \tilde{a}, \overline{A}, M)$, respectively. We do not include δ since it is never binding for the big bank. Further, it is sufficient to know π rather than (ℓ, d) economizing on one state variable. Instead of the law of motion for the distribution $\zeta' = H(z, z', \zeta)$ in (29) we approximate the fringe part by $\bar{A}' = H^A(z, z', a^b, \bar{A}, M)$ and $M' = H^M(z, z', a^b, \bar{A}, M)$. Finally, we approximate the equation defining the "reaction function" $\ell + L^f(\zeta, \ell) = L^d(r^L, z)$ in (24) by $\ell + L^f(z, a^b, \overline{A}, M, \ell) = L^d(r^L, z)$ with

$$L^f(z, a^b, \bar{A}, M, \ell) = \ell^f(z, a^b, \bar{A}, M, \ell) \times M. \tag{49}$$

The mass of fringe banks depends on entry and exit decisions, which is why our "reaction function" must consider dynamic decisions unlike that in Farias, et. al.

A similar set of changes to the state vector need to be made to the problem of fringe banks (except that the deposit capacity constraints almost always bind so we must keep that state variable). In particular, $V^f(\tilde{a}, \delta, z, \zeta)$ is replaced by $V^f(\tilde{a}, \delta, z, a^b, \bar{A}, M)$ and $W^f(\ell, d, a', \zeta, \delta, z')$ is replaced by $W^f(\pi, a', \delta, z, z', a^b, \bar{A}, M)$. As before, the law of motion for the distribution $\zeta' = H(z, z', \zeta)$ in (29) is approximated by $\bar{A}' = H^A(z, z', a^b, \bar{A}, M)$, $M' = H^M(z, z', a^b, \bar{A}, M)$, and $a^{b'} = a^{b'}(a^b, z, \bar{A}, z')$. Finally, the reaction function in equation (24) uses the decision rule that solves the big bank loan choice problem; in particular $L^d(r^L, z) = \ell^b(a^b, a, \bar{A}, M) + L^f(z, a^b, \bar{A}, M, \ell^b(a^b, a, \bar{A}, M))$.

In order for the dominant bank to know how the fringe banks will react to its decisions, it must know how fringe banks will behave when it takes off-the-equilibrium path actions. To that end, we must introduce an auxiliary problem for the fringe banks where they choose optimally across any possible action of the big bank ℓ. The statement of the auxiliary problem is the same as for the fringe bank above except that the equation defining the reaction function in equation (24) is given by $L^d(r^L, z) = \ell + L^f(z, a^b, \bar{A}, M, \ell)$.

The algorithm is given by:

1. Guess **aggregate functions**. That is, guess $\{h_i^a\}_{i=0}^5$ and $\{h_i^m\}_{i=0}^5$ to get

$$
\begin{aligned}
\log(\overline{A}') &= h_0^a + h_1^a \log(z) + h_2^a \log(a^b) + h_3^a \log(\overline{A}) + h_4^a \log(M) + h_5^a \log(z'), \quad (50) \\
\log(M') &= h_0^m + h_1^m \log(z) + h_2^m \log(a^b) + h_3^m \log(\overline{A}) + h_4^a \log(M) + h_5^a \log(z') (51)
\end{aligned}
$$

   Make an initial guess of $\ell^f(\bar{A}, z, a^b, M, \ell; \bar{\delta})$ (i.e. the solution to the auxiliary problem) that determines the reaction function

$$
L^f(z, a^b, \bar{A}, \ell) = \ell^f(\bar{A}, z, a^b, \ell) \times M. \quad (52)
$$

2. Solve the **dominant bank** problem to obtain the big bank value function and decision rules: V^b, ℓ^b, $a^{b'}$, d^b, $B^{b'}$ and x^b.

3. Solve the problem of **fringe banks** to obtain the fringe bank value function and decision rules: V^f, ℓ^f, $a^{f'}$, d^f, $B^{f'}$ and x^f.

4. Using the solution to the fringe bank problem V^f, solve the **auxiliary problem** to obtain $\ell^f(\bar{A}, z, a^b, M, \ell; \bar{\delta})$.

5. Solve the **entry problem** of the fringe bank and big bank to obtain entry decision rules.

6. **Simulation**

   (a) Guess distribution of fringe banks over $\tilde{a}$ and δ, $\mu_0(a, \delta)$. Compute $\bar{A}_0 = \sum_{i,j} a_i \mu_0(a_i, \delta_j)$ and and $M_0 = \sum_{i,j} \mu_0(a_i, \delta_j)$.

   (b) Guess initial a^b.

   (c) Simulate a path of $\{z_t\}_{t=0}^T$.

   (d) Using decision rules for big banks obtain ℓ_t^b, d_t^b, a_{t+1}^b, B_{t+1}^b and $\tilde{a}_t^b$.

   (e) Solve for value of M_{t+1} such that the free entry condition for fringe banks is satisfied with equality.

   (f) Find $\mu_{t+1}(a, \delta)$ using decision rules for fringe banks. That is.

$$
\begin{aligned}
\mu_{t+1}(\tilde{a}, \delta') &= \sum_{i,j} (1 - x^f(a_i, \delta_j, z_t, a_t^b, \overline{A}_t, M_t, z_{t+1})) I_{\{\tilde{a}^f(a_i, \delta_j, z_t, a_t^b, \overline{A}_t, M_t, z_{t+1}) = \tilde{a}\}} G(\delta', \delta) \mu(a_i, \delta_j) \\
&\quad + G(\delta', \delta) E_t \sum_{\delta} I_{\{a' = a^{f,e}(\cdot))\}} G^{f,e}(\delta)
\end{aligned}
$$

   Compute $\bar{A}_{t+1} = \sum_{i,j} a_i \mu_{t+1}(a_i, \delta_j)$.

   (g) Continue for T periods and collect $\{a_t^b, \bar{A}_t, M_t\}_{t=1}^T$.

(h) Estimate equations (50) and (51) to obtain new aggregate functions.

(i) If the new aggregate functions are close enough to those used to solve the bank problems and along the equilibrium path the distance between the solution to the auxiliary problem ($\ell^f(\bar{A}_t, z_t, a_t^b, M_t, \ell_t^b; \bar{\delta})$) and the average loan of fringe banks ($\sum_{i,j} \ell_t^f \mu_t(a_i, \delta_j)/M_t$) are close enough you are done. If not, return to 2.

Table 11 presents the aggregate functions in the benchmark economy.

Table 11: Equilibrium Aggregate Functions

Function	$\log(\bar{A}')$	$\log(M')$
cons.	-0.753	0.012
$\log(z)$	-1.225	-0.108
$\log(a^b)$	-0.040	-0.002
$\log(\bar{A})$	-0.824	0.001
$\log(M)$	-0.202	0.580
$\log(z')$	3.439	0.276
R^2	0.981	0.930