International Risk Sharing with Endogenously Segmented Asset Markets

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Abstract

Asset price data imply a large degree international risk sharing, while aggregate consumption data do not. We evaluate how well a model with fixed costs of exchanging money for assets can account for this discrepancy. In our model, households receive idiosyncratic income shocks, and only a fraction of households adjust their asset holdings each period. These households share risk within and across countries, and their marginal utilities price assets, so asset prices imply high international risk sharing. Inactive households do not share risk, so aggregate consumption reflects low risk sharing. Quantitatively, this mechanism depends on the degree of asset market segmentation, which we choose so that the cross-sectional dispersion of consumption relative to income matches that in US data.

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1 Introduction

How much do countries share risk through international financial markets, and how big are the gains from doing so? The answers to these questions depend on how we measure the degree of risk sharing: statistics based on asset market data give significantly different answers from aggregate consumption data. For example, Brandt, Cochrane, and Santa-Clara (2006) show that stochastic discount factors derived from stock prices are very similar across countries, showing significant international risk sharing, while stochastic discount factors derived from aggregate consumption (i.e., intertemporal marginal rates of substitution) are weakly correlated across countries, and so display a lack of risk sharing. Similarly, Lewis (2000) argues that the high volatility of stochastic discount factors derived from stock data implies higher gains from international risk sharing than suggested by the low volatility of marginal utility growth derived from consumption data.

In this paper, we evaluate the extent to which frictions that limit participation in asset markets can account for the discrepancy between the asset-market-based view and the consumption-based view of international risk sharing. We consider a two-country model with international trade in goods and financial assets along the lines of Alvarez, Atkeson, and Kehoe (2002), in which households must pay a fixed cost to transfer money into or out of interest-bearing assets. Households face idiosyncratic and aggregate income shocks, and asset markets are endogenously segmented because only a fraction of households at any point in time find it beneficial to pay the fixed cost associated with adjusting their asset holdings. Households that actively adjust their asset holdings share risk among each other - both within and across countries. Since these households' expected marginal utility growth determines asset prices, the behavior of asset prices implies a high degree of international risk sharing. On the other hand, these households account for only a (time-varying) fraction of aggregate consumption in each country, so measures of consumption risk-sharing imply a low degree of international risk sharing at the aggregate level.

We quantify this mechanism by calibrating our model to match facts on the cross-sectional variance of household income and consumption in US data. With standard parameters for preferences and the stochastic process governing aggregate shocks, the model predicts a high correlation of the stochastic discount factors that would be measured from asset price data (that is, the intertemporal marginal rates of substitution of active households) and a correlation of aggregate consumption across countries that is much lower.

Our model also has implications for the relationship between consumption and real exchange rates. As Alvarez, Atkeson, and Kehoe (2002) point out, asset market segmentation in principle breaks the link between aggregate consumption and real exchange rate fluctuations. However, our results show that this asset market friction does not solve the Backus and Smith (1993) puzzle; that is, the ratio of aggregate consumption is highly correlated with the real exchange rate as well. Risk sharing among active households directly relates the ratio of their consumption in each country to fluctuations in the real exchange rate, but in practice, fluctuations in the relative price of different goods transmits risk sharing benefits of trade even to households that do not participate in asset markets (as pointed out by Cole and Obstfeld (1991)).

Recent papers that address international risk sharing, and the Backus-Smith puzzle using segmented asset markets include Sungur (2004) and Kollmann (2009). Kollmann builds a model with an exogenous, time-varying fraction of households participating in asset markets. Our model, which builds on Alvarez, Atkeson, and Kehoe (2002), endogenizes this fraction and its behavior. Sungur tests the prediction of the Alvarez, Atkeson, and Kehoe (2002) model in Italian regional data and finds support for the model's relationship between the real exchange rate and active households' consumption. Our results are also related to studies of the differences between stockholders and non-stockholders in Vissing-Jørgensen (2002), or between active and inactive stock market participants in Bonaparte and Cooper (2009).

2 Model

2.1 Summary

The model is a variant of the two-country environment in Alvarez, Atkeson, and Kehoe (2002). We consider an infinite horizon pure-exchange economy with three goods: one internationally tradable good, and two nontradable goods. We refer to the two countries as "home" and "foreign", and label foreign variables with an asterisk ($*$). In each country, there is a continuum of households who receive endowments of tradable and nontradable goods. Each household's endowment of each good consists of an idiosyncratic component, which is i.i.d. across households and over time, and an aggregate component. Exogenous fluctuations in the aggregate components of endowments are exogenous are the source of uncertainty in the economy. We introduce tradable and nontradable goods to generate real exchange rate fluctuations, while allowing trade in goods at the same time. As pointed out by Brandt, Cochrane, and Santa-Clara (2006), if there were no trade in goods, consumption would be constrained by domestic resources, and there could be no risk sharing, even through trade in financial assets.

Households value consumption of both tradable goods and nontradable goods, and they can buy and sell internationally traded assets to insure against idiosyncratic and aggregate fluctuations. However, they must pay a fixed cost to transfer goods into or out of these assets. This segmentation of households into active participants and non-participants in the asset market disconnects asset prices from aggregate consumption. In Alvarez, Atkeson, and Kehoe (2002), a similar separation of goods and assets accounts is specified through a cash-in-advance restriction with a fixed cost motivated as in Baumol (1952) and Tobin (1956). We abstract from money, and simply require households to pay a fixed cost whenever they consume more or less than their current period income. One motivation for such a cost is that there is a fixed cost to ensuring repayment of private debt, as described by Chatterjee and Corbae (1992). A fixed cost like this is also related to the stock market participation cost considered by Luttmer (1999).

2.2 Timing and Uncertainty

Time is discrete and labeled $t = 0, 1, \ldots$. At the beginning of period t, the aggregate endowments of tradable goods, Y_{Tt}, Y^*_{Tt}, and nontradable goods, Y_{Nt}, Y^*_{Nt}, are realized, and each household receives a draw y_t of an idiosyncratic shock from a distribution with density function f. This idiosyncratic shock determines the household's endowment of each good, $y_t Y_{Tt}$ and $y_t Y_{Nt}$ (and similarly in the foreign country). The mass of households in each country is normalized to 1, and the distribution of idiosyncratic endowments has mean 1, so that the aggregate tradable home endowment is in fact Y_{Tt}, and so on.

We refer to the aggregate event in period t as the realization of the four aggregate endowments, $s_t = (Y_{Tt}, Y_{Nt}, Y^*_{Tt}, Y^*_{Nt})$, and define $s^t = (s_0, s_1, \ldots, s_t)$ as the history up to date t of these events, with s_0 given. A household's state in period t is (s^t, y^t), where $y^t = (y_0, y_1, \ldots, y_t)$ is its history of idiosyncratic shocks. Let $g(s^t)$ denote the density of the aggregate state and $f(y^t)$ denote the density of the individual history (this is an abuse of notation, since f is also the density of the shock in each period. When used below, the argument of f will make it clear whether it refers to the density over histories or over current realizations.)

2.3 Households

Households have preferences given by:

$$\sum_{t=0}^{\infty} \int \int \beta^t U\left(C\left(s^t, y^t\right)\right) g\left(s^t\right) f\left(y^t\right) ds^t dy^t \tag{1}$$

with $\beta \in (0,1)$ and $U(C) = C^{1-\eta}/(1-\eta)$. The quantity $C(s^t, y^t)$ is the amount of a composite good consumed by a household in state (s^t, y^t). The composite good is given by a constant elasticity of substitution aggregate of tradable and nontradable consumption,

$$C\left(s^t, y^t\right) = \left(a c_T\left(s^t, y^t\right)^{\frac{\sigma-1}{\sigma}} + (1-a)\, c_N\left(s^t, y^t\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$

where $\sigma > 0$ is the elasticity of substitution between tradable and nontradable goods, and $a \in (0,1)$ is the weight on tradable goods in consumption. We normalize the price of the tradable good to 1, and denote the price of the nontradable good in the home country by $p_N\left(s^t\right)$. The price index for one unit of home country composite consumption as well as the demands for tradable and nontradable goods given a level C of composite consumption are given by the cost-minimization problem:

$$\begin{aligned} P\left(s^t\right) C &= \min_{c_T, c_N} c_T + p_N\left(s^t\right) c_N \\ & \quad \text{subject to:} \\ & \quad \left(a\left(c_T\right)^{\frac{\sigma-1}{\sigma}} + (1-a)\left(c_N\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \geq C \end{aligned}$$

which gives demands for tradable and nontradable consumption as functions of composite consumption:

$$\begin{aligned} c_T\left(s^t, y^t\right) &= \left(aP\left(s^t\right)\right)^{\sigma} C\left(s^t, y^t\right) \\ c_N\left(s^t, y^t\right) &= \left(\frac{1-a}{p_N\left(s^t\right)} P\left(s^t\right)\right)^{\sigma} C\left(s^t, y^t\right) \end{aligned} \tag{2}$$

and the price index for composite consumption purchases:

$$P\left(s^t\right) = \left(a^{\sigma} + (1-a)^{\sigma} p_N\left(s^t\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$$

Households have two budget constraints: one that constrains current consumption and saving by income, and one that describes the evolution of their asset balances. We refer to the first as the "goods market budget constraint" and the second as the "asset market budget constraint". In the goods market,

$$P\left(s^t\right) C\left(s^t, y^t\right) \leq y_t\left[Y_{Tt} + p_N\left(s^t\right) Y_{Nt}\right] + z\left(s^t, y^t\right) \tau\left(s^t, y^t\right) \tag{3}$$

where $z\left(s^t, y^t\right) = 1$ if the household consumes more or less than current income, and $\tau\left(s^t, y^t\right)$ is the amount transferred into or out of the asset market account. If $\tau > 0$, then the household withdraws resources from the asset market account and consumes more than current income, and if $\tau < 0$, the household saves some of its current income.

In the asset market, all households start in period 0 with some amount $b(s^0)$ of initial asset holdings. In any period, they can purchase a full set of one-period securities with payoffs contingent on the aggregate and idiosyncratic state in the next period, denominated in tradable goods. These transactions are carried out with a competitive financial intermediary. The price of a claim to one unit of tradable goods if the future event is (s_{t+1}, y_{t+1}) and the household's current state is (s^t, y^t) is $q(s^t, s_{t+1}, y^t, y_{t+1})$. A household with history (s^t, y^t) purchases $b(s^t, s_{t+1}, y^t, y_{t+1})$ amount of these securities. The budget constraint for this household is:

$$\begin{aligned} &\int\int q\left(s^t, s_{t+1}, y^t, y_{t+1}\right) b\left(s^t, s_{t+1}, y^t, y_{t+1}\right) ds_{t+1} dy_{t+1} \\ &\qquad\qquad\qquad\qquad + z\left(s^t, y^t\right)\left[\tau\left(s^t, y^t\right) + \gamma\right] \quad \leq \quad b\left(s^t, y^t\right) \end{aligned} \tag{4}$$

so that the current payoff from asset holdings, $b(s^t, y^t)$, is allocated toward purchases of new securities and transfers to the goods market account, if any. Transferring to or from the goods market (i.e., choosing $z(s^t, y^t) = 1$) requires the payment of a fixed amount γ of tradable goods out of asset balances.

For a foreign country in state (s^t, y^t), the goods market budget constraint is:

$$P^*\left(s^t\right) C^*\left(s^t, y^t\right) \leq y_t\left[Y^*_{Tt} + p^*_N\left(s^t\right) Y^*_{Nt}\right] + z^*\left(s^t, y^t\right) \tau^*\left(s^t, y^t\right)$$

and the asset market budget constraint is:

$$\begin{aligned} &\int\int q\left(s^t, s_{t+1}, y^t, y_{t+1}\right) b^*\left(s^t, s_{t+1}, y^t, y_{t+1}\right) ds_{t+1} dy_{t+1} \\ &\qquad\qquad\qquad\qquad + z^*\left(s^t, y^t\right)\left[\tau^*\left(s^t, y^t\right) + \gamma\right] \quad \leq \quad b^*\left(s^t, y^t\right) \end{aligned}$$

For foreign households, the price index $P^*(s^t)$ and the consumption levels $c^*_T(s^t, y^t)$ and $c^*_N(s^t, y^t)$ are defined the same way they are for the home country, given composite consumption level $C^*(s^t, y^t)$ and the nontradable goods price $p^*_N(s^t)$.

2.4 Asset market

There is a world financial intermediary that buys and sells assets from households. The intermediary has no wealth of its own, so total purchases of assets from households must equal sales of assets to other households. Net revenues of the intermediary when the aggregate state is s^t are given by adding up the transactions of (s_{t+1}, y_{t+1})-contingent assets to households of all histories y^t:

$$\int\int\int q\left(s^t, s_{t+1}, y^t, y_{t+1}\right)\left[b\left(s^t, s_{t+1}, y^t, y_{t+1}\right) + b^*\left(s^t, s_{t+1}, y^t, y_{t+1}\right)\right] f\left(y^t\right) dy^t dy_{t+1} ds_{t+1}$$

The intermediary maximizes these net revenues subject to the constraint that at all future states s^{t+1}, net payments on s^{t+1}-contingent claims must be zero:

$$\int\int \left[b\left(s^t, s_{t+1}, y^t, y_{t+1}\right) + b^*\left(s^t, s_{t+1}, y^t, y_{t+1}\right)\right] f\left(y^t\right) f\left(y_{t+1}\right) dy^t dy_{t+1} = 0$$

That is, adding up the payments made on y_{t+1}-contingent purchases across households that had histories y^t must equal zero.

The intermediary's problem yields the following no-arbitrage condition:

$$q\left(s^t, s_{t+1}, y^t, y_{t+1}\right) = q\left(s^t, s_{t+1}\right) f\left(y_{t+1}\right) \tag{5}$$

where $q\left(s^t, s_{t+1}\right) > 0$. This condition states that the value of one unit of tradable goods for a household in state (s^{t+1}, y^{t+1}) must equal the value of one unit of tradable goods for any household in aggregate state s^{t+1}, weighted by the probability of receiving the idiosyncratic shock y_{t+1} in period $t+1$.

2.5 Market Clearing and Equilibrium

In the goods market, home households' consumption plus foreign households's consumption of tradable goods plus the fixed costs of transferring between accounts equals the world endowment of tradable goods:

$$\int \left[c_T\left(s^t, y^t\right) + \gamma z\left(s^t, y^t\right)\right] f\left(y^t\right) dy^t + \int \left[c_T^*\left(s^t, y^t\right) + \gamma z^*\left(s^t, y^t\right)\right] f^*\left(y^t\right) dy^t = Y_{Tt} + Y_{Tt}^*$$

The market clearing conditions for nontradable goods are:

$$\int c_N\left(s^t, y^t\right) f\left(y^t\right) dy^t = Y_{Nt}$$

$$\int c_N^*\left(s^t, y^t\right) f\left(y^t\right) dy^t = Y_{Nt}^*$$

In the asset market, at each aggregate state s^{t+1}, bond holdings summed across all households equal zero:

$$\int\int\left[b\left(s^t,s_{t+1},y^t,y_{t+1}\right)+b^*\left(s^t,s_{t+1},y^t,y_{t+1}\right)\right]f\left(y_{t+1}\right)dy_{t+1}f\left(y^t\right)dy^t=0$$

An equilibrium consists of goods prices and asset prices along with consumption quantities and asset holdings that solve households' problems and the financial intermediary's problem taking prices as given, and that satisfy the market clearing conditions.

2.6 Characterizing Equilibrium

We follow a similar procedure as in Alvarez, Atkeson, and Kehoe (2002) to show that an equilibrium is characterized by a few simple, static conditions determining consumption allocations and asset market participation decisions. The set of households that is active in asset markets (i.e., those for whom $z\left(s^t,y^t\right)=1$) is characterized by a static threshold rule: households with a current idiosyncratic income shock in a certain range transfer, and others do not. Active households pool their income within a period, and have equal consumption, while inactive households consume the value of their income.

To solve a household's problem, we write a date-0 budget constraint. Letting $Q\left(s^t\right)=q\left(s^0,s_1\right)q\left(s^1,s_2\right)\cdots q\left(s^{t-1},s_t\right)$ denote the price of tradable goods at state s^t in terms of tradable goods at date 0, and using the no-arbitrage condition (5), the sequence of budget constraints for home country households (4) can be written:

$$\sum_{t=0}^{\infty}\int_{s^t}\int_{y^t}Q\left(s^t\right)f\left(y^t\right)z\left(s^t,y^t\right)\left[\tau\left(s^t,y^t\right)+\gamma\right]ds^tdy^t\leq b\left(s^0\right) \qquad (6)$$

The household's problem is then to choose consumption, $C\left(s^t,y^t\right)$, transfer decisions $z\left(s^t,y^t\right)$, and transfers $\tau\left(s^t,y^t\right)$ to maximize expected utility (1) subject to (6) and the goods market budget constraint, (3).

The first order conditions for C and τ are:

$$\begin{aligned}\beta^tU'\left(C\left(s^t,y^t\right)\right)g\left(s^t\right)f\left(y^t\right) &= P\left(s^t\right)\mu\left(s^t,y^t\right)\\ z\left(s^t,y^t\right)\mu\left(s^t,y^t\right) &= \lambda Q\left(s^t\right)f\left(y^t\right)z\left(s^t,y^t\right)\end{aligned}$$

where λ is the multiplier on the date-0 budget constraint and $\mu\left(s^t,y^t\right)$ is the multiplier on the goods market budget constraint in state $\left(s^t,y^t\right)$.

In states $\left(s^t,y^t\right)$ for which $z\left(s^t,y^t\right)=1$, these first order conditions become:

$$\beta^tU'\left(C\left(s^t,y^t\right)\right)g\left(s^t\right)=P\left(s^t\right)\lambda Q\left(s^t\right)$$

which says that $C(s^t, y^t)$ is independent of y^t if $z(s^t, y^t) = 1$. That is, idiosyncratic risk is pooled among all active households, and they all consume the same level. Call this consumption level $C_A(s^t)$, for "active" households' consumption.

Now, we consider the choice of z. We know that if $z(s^t, y^t) = 1$, then $C(s^t, y^t) = C_A(s^t)$ and the amount transferred into or out of the asset market account is whatever it needs to be: $\tau(s^t, y^t) = P(s^t) C_A(s^t) - y_t (Y_{Tt} + p_N(s^t) Y_{Nt})$. So we can write the household's problem:

$$\max \sum_{t=0}^{\infty} \int_{s^t} \int_{y^t} \beta^t \left[z\left(s^t, y^t\right) U\left(C_A\left(s^t\right)\right) + \left(1 - z\left(s^t, y^t\right)\right) U\left(C\left(s^t, y^t\right)\right)\right] g\left(s^t\right) f\left(y^t\right) ds^t dy^t$$

subject to:

$$\sum_{t=0}^{\infty} \int_{s^t} \int_{y^t} Q\left(s^t\right) f\left(y^t\right) z\left(s^t, y^t\right) \left[P\left(s^t\right) C_A\left(s^t\right) - y_t \left[Y_{Tt} + p_N\left(s^t\right) Y_{Nt}\right] + \gamma\right] ds^t dy^t \leq b\left(s^0\right)$$

If we consider the Lagrangian of this problem (again with multiplier λ on the date-0 budget constraint), the value in state (s^t, y^t) of setting $z(s^t, y^t) = 1$ is:

$$\beta^t U\left(C_A\left(s^t\right)\right) g\left(s^t\right) f\left(y^t\right) - \lambda Q\left(s^t\right) f\left(y^t\right) \left[P\left(s^t\right) C_A\left(s^t\right) - \left[y_t Y_{Tt} + p_N\left(s^t\right) Y_{Nt}\right] + \gamma\right]$$

And the value of setting $z(s^t, y^t) = 0$, using the fact that $C(s^t, y^t) = \frac{y_t\left(Y_{Tt} + p_N\left(s^t\right) Y_{Nt}\right)}{P(s^t)}$ when $z(s^t, y^t) = 0$, is:

$$\beta^t U\left(\frac{y_t \left(Y_{Tt} + p_N\left(s^t\right) Y_{Nt}\right)}{P\left(s^t\right)}\right) g\left(s^t\right) f\left(y^t\right)$$

The value of λ is given by the first order condition when $z = 1$:

$$\lambda = \frac{\beta^t U'\left(C_A\left(s^t\right)\right) g\left(s^t\right)}{P\left(s^t\right) Q\left(s^t\right)} \tag{7}$$

So the net gain of setting $z(s^t, y^t) = 1$ versus setting $z(s^t, y^t) = 0$ is positive whenever:

$$\begin{aligned} & U\left(C_A\left(s^t\right)\right) - U\left(\frac{y_t\left(Y_{Tt} + p_N\left(s^t\right) Y_{Nt}\right)}{P\left(s^t\right)}\right) \\ -\frac{U'\left(C_A\left(s^t\right)\right)}{P\left(s^t\right)} & \left[P\left(s^t\right) C_A\left(s^t\right) - y_t \left[Y_{Tt} + p_N\left(s^t\right) Y_{Nt}\right] + \gamma\right] \quad > \quad 0 \end{aligned} \tag{8}$$

The first two terms in (8) give the increase in consumption for a household in state (s^t, y^t) that switches from being inactive to being active. The third term gives the net cost of the change in asset balances necessary to get to the active consumption level $C_A(s^t)$: an active household increases or reduces asset balances, which has an effect on future lifetime utility.

So, define the function

$$
\begin{aligned}
h\left(y; C_A, Y_T, Y_N, p_N, P\right) &= U\left(C_A\right) - U\left(\frac{y\left(Y_T + p_N Y_N\right)}{P}\right) \\
&\quad -\frac{U'\left(C_A\right)}{P}\left[P C_A - y\left(Y_T + p_N Y_N\right) + \gamma\right]
\end{aligned}
$$

It is straightforward to verify that h has a minimum when $y = \frac{PC_A}{(Y_T + p_N Y_N)}$, is decreasing for $y < \frac{PC_A}{(Y_T + p_N Y_N)}$ and increasing for $y > \frac{PC_A}{(Y_T + p_N Y_N)}$, and is convex. For the utility function we use, $U\left(C\right) = C^{1-\eta}/\left(1-\eta\right)$ with $\eta > 0$, $\lim_{y \to 0} h = \lim_{y \to \infty} h = \infty$, so that h is U-shaped, with two zeros. We'll refer to the two zeros of h as $y_L\left(s^t\right)$ and $y_H\left(s^t\right)$, with $y_L < y_H$. For households with $y_t \in \left[y_L\left(s^t\right), y_H\left(s^t\right)\right]$, the cost of transferring outweigh the benefit, so they consume their current income in period t. For households with $y_t < y_L\left(s^t\right)$ or $y_T > y_H\left(s^t\right)$, the benefit of being active and consuming $C_A\left(s^t\right)$ outweighs the cost.

The characterization of this decision is analogous in the foreign country, where active households consume $C_A^*\left(s^t\right)$. Combining the first order condition (7) with its foreign analogue, yields the following risk-sharing condition:

$$
\frac{P^*\left(s^t\right)}{P\left(s^t\right)} = \frac{\lambda^*}{\lambda}\frac{U'\left(C_A\left(s^t\right)\right)}{U'\left(C_A^*\left(s^t\right)\right)} \tag{9}
$$

This condition relates the ratio of marginal utilities to the real exchange rate, the ratio of consumption price indices in the two countries. The marginal utility of home country active households relative foreign country active households' rises in proportion to the appreciation of the home real exchange rate. Active households therefore share the risk associated with national endowment shocks internationally.

An equilibrium allocation is characterized by active consumption levels and cutoffs determining the set of active households in each country, along with the implied consumption levels for tradable and nontradable goods. The market clearing conditions can be written:

$$
\begin{aligned}
Y_{Tt} + Y_{Tt}^* &= \int_{y_L(s^t)}^{y_H\left(s^t\right)} c_T\left(s^t, y^t\right) f\left(y\right) dy + \left[F\left(y_L\left(s^t\right)\right) + 1 - F\left(y_H\left(s^t\right)\right)\right]\left(c_{TA}\left(s^t\right) + \gamma\right) \\
&\quad + \int_{y_L^*(s^t)}^{y_H^*\left(s^t\right)} c_T^*\left(s^t, y^t\right) f\left(y\right) dy + \left[F\left(y_L^*\left(s^t\right)\right) + 1 - F\left(y_H^*\left(s^t\right)\right)\right]\left(c_{TA}^*\left(s^t\right) + \gamma\right)
\end{aligned}
$$

$$
Y_{Nt} = \int_{y_L(s^t)}^{y_H\left(s^t\right)} c_N\left(s^t, y^t\right) f\left(y\right) dy + \left[F\left(y_L\left(s^t\right)\right) + 1 - F\left(y_H\left(s^t\right)\right)\right] c_{NA}\left(s^t\right)
$$

$$Y_{Nt}^{*}=\int_{y_L^{*}(s^t)}^{y_H^{*}(s^t)} c_N^{*}\left(s^t,y^t\right) f\left(y\right) dy+\left[F\left(y_L^{*}\left(s^t\right)\right)+1-F\left(y_H^{*}\left(s^t\right)\right)\right] c_{NA}^{*}\left(s^t\right)$$

where F is the cdf associated with the density f, and $c_{TA}\left(s^t\right), c_{NA}\left(s^t\right)$, and $c\left(s^t,y^t\right)$ follow from the demand functions in (2) (and the foreign analogues).

All equilibrium variables depend only on the current realization of $s_t=\left(Y_{Tt},Y_{Nt},Y_{Tt}^{*},Y_{Nt}^{*}\right)$ and not on it's history. For each s_t, we solve the three market clearing conditions along with the risk-sharing condition (9) and the conditions $h\left(y_L\left(s^t\right)\right)=h\left(y_H\left(s^t\right)\right)=\left(y_L^{*}\left(s^t\right)\right)=\left(y_H^{*}\left(s^t\right)\right)=0$ for the active consumption levels, the thresholds for households to make transfers, and the equilibrium prices of nontradable goods, $p_N\left(s^t\right)$ and $p_N^{*}\left(s^t\right)$. We solve for an equilibrium in which all home and foreign households are identical in period 0, so that $\lambda=\lambda^{*}$ in (9).

3 Numerical Results

3.1 Parameterization

We set the parameters governing preferences to standard values in the international trade and business cycle literatures. We set $\beta=0.96$ and $\eta=2$. We set the elasticity of substitution σ between tradable and nontradable goods to 0.5, and we set the share a on tradable goods in consumption so that the fraction of expenditures on tradable goods is 50%. These are both close to the values estimated in Stockman and Tesar (1995).

We choose the distribution of idiosyncratic income shocks and the fixed cost of making a transfer to match statistics on income and consumption inequality in the US. Using data from the Consumer Expenditure Survey (CEX), we estimate residual variances of income and consumption unexplained by household characteristics. We regress income and consumption on the following characteristics of the reference person: sex, race, education, experience (proxied by age), interaction terms between experience and education and dummies for region of residence. From 1980 to 2006, these characteristics explain, on average, about 23 percent of the cross-sectional variance of income and consumption. The variance of the log residual income is 0.37. We choose the distribution of income in our model to be lognormal with a mean of 1 and a variance of log income equal to 0.37.

Varying the fixed cost γ allows us to match a given cross-sectional variance of consumption: an arbitrarily low value of γ implies that all households are active, and hence the variance of consumption is zero, while an arbitrarily high value of γ means that no households are active, so that the variance of consumption equals the variance of income. Using the CEX data from 1980 to 2006, the variance of consumption unexplained by household characteristics is, on average, 0.23. The implied value of γ that generates this degree of dispersion in the model is 0.77 units of consumption. About 12% of households are active per period in the steady state, and about 9% of steady state real income is spent on transaction costs per period.

While income and consumption inequality have both risen in the US, our model assumes a time-invariant cross-sectional variance of income, so we pick average measures of inequality in the CEX sample, and plan to examine how the results change when we vary the parameters to match different targets.

The stochastic process of shocks is given by:

$$\log \begin{bmatrix} Y_{Tt+1} \\ Y_{Nt+1} \\ Y^*_{Tt+1} \\ Y^*_{Nt+1} \end{bmatrix} = \begin{bmatrix} \rho_T & & & \\ & \rho_N & & \\ & & \rho_T & \\ & & & \rho_N \end{bmatrix} \log \begin{bmatrix} Y_{Tt} \\ Y_{Nt} \\ Y^*_{Tt} \\ Y^*_{Nt} \end{bmatrix} + \begin{bmatrix} \varepsilon_{Tt+1} \\ \varepsilon_{Nt+1} \\ \varepsilon^*_{Tt+1} \\ \varepsilon^*_{Nt+1} \end{bmatrix}$$

where the persistence parameters, ρ_T and ρ_N, and the covariance matrix of the ε's are chosen to match second moments in growth rates of the U.S. and a trade-weighted aggregate of 19 OECD countries. We use GDP of the manufacturing, mining, agriculture, and utilities sectors as a measure of the tradable endowment Y_T, and the remainder (services and construction) as a measure of the nontradable endowment Y_N. We calculate the persistence parameters $\rho_T = 0.49$, and $\rho_N = 0.63$. The standard deviation of growth rates of tradable output is 3.6 percent, and of nontradable output is 1.6 percent. The cross-country correlations are 0.25 for Y_T and 0.23 for Y_N. The within-country correlation between Y_T and Y_N is 0.60, and the correlation between tradable output in one country and nontradable output in the other is 0.01.

3.2 Implications for International Risk Sharing

The data in Brandt, Cochrane, and Santa-Clara (2006) suggest that stochastic discount factors computed from asset price data are highly correlated across countries, while the intertemporal marginal rates of substitution computed from aggregate consumption are not very correlated. A standard model in which all households actively participated in asset markets in each period would not generate such a disparity. In addition, international business cycle models such as Backus, Kehoe, and Kydland (1992) and Stockman and Tesar (1995) generate correlations in consumption across countries that are too high relative to output when compared to the data.

Table 1 presents results for our baseline parameterization, and for two alternative asset market structures. We compute the following statistics for all households and for active households alone: the cross-country correlation of consumption, the correlation of the intertemporal marginal rate of substitution, and a risk sharing index developed by Brandt, Cochrane, and Santa-Clara (2006). This index (labeled BCS risk sharing index in the table) is:

$$1 - \frac{var\left(m_{t,t+1} - m^*_{t,t+1}\right)}{var\left(m_{t,t+1}\right) + var\left(m^*_{t,t+1}\right)}$$

where $m_{t,t+1}$ is a measure of the intertemporal marginal rate of substitution (e.g. $\beta^t \left(C_{At+1}/C_{At}\right)^{-\eta}$ for active households). This index lies between -1 and 1, with a value of 1 implying that $m_{t,t+1} = m^*_{t,t+1}$, and therefore there is perfect risk sharing.

The significance of constructing statistics with two different measures of consumption (active vs. all households) is that active households price assets, so their intertemporal marginal rate of substitution is reflected in asset prices. Hence, the analogue of Brandt, Cochrane, and Santa-Clara (2006)'s construction of stochastic discount factors from stock market data in our model is to use the marginal rate of substitution (MRS) of active households. Statistics based on aggregate consumption in our model correspond to measures of risk sharing based on aggregate consumption.

Table 1: Model Results, and Alternative Asset Market Structures

	Benchmark Model	All households active	Fixed fraction active
STANDARD DEVIATION (%)			
income	2.81	2.81	2.81
real exchange rate	1.75	2.54	2.81
INTERNATIONAL CORRELATIONS			
real income	0.17	0.17	0.17
Aggregate variables			
consumption	0.62	0.84	0.30
intertemporal MRS	0.65	0.87	0.32
BCS risk sharing index	0.62	0.86	0.26
Active households' variables			
consumption	0.93	n/a	0.99
intertemporal MRS	0.95	n/a	0.99
BCS risk sharing index	0.94	n/a	0.99
CORRELATION BETWEEN $\frac{C^*}{C}$ AND $\frac{P}{P^*}$			
all households	0.62	1.00	-0.59
active households	1.00	n/a	1.00

Note: averages of statistics of logged series from 100 simulations of 100 periods each.

We see in the first column of Table 1 that the model generates a substantial difference between the asset-price based risk sharing measures (those for active households) and the measures based on aggregate consumption. The correlation of consumption, the correlation of the intertemporal MRS, and the BCS risk sharing index are all close to one-and-a-half times as high for active households as for all households. Active households in each country trade a complete set of state-contingent assets, and thus are able to share country-specific risk with active households in the other country. Thus, the model goes some way toward explaining the discrepancy that asset prices imply high risk sharing while aggregate consumption suggests low risk sharing.

Our model partly resolves the Backus and Smith (1993) puzzle as the correlation between the real exchange rate and the cross-country ratio of aggregate consumption in the benchmark model is significantly lower than 1 (see the last two rows of Table 1). For consumption among active households, this high correlation is dictated by the condition (9). However, even for households that do not participate in asset markets, and therefore for whom an analogue of (9) does not hold, a moderate degree of risk sharing is achieved; indeed, the table shows that aggregate consumption is significantly more correlated than output. This is a result of the movements in the relative price of nontraded goods. For example, a positive shock to the endowment of nontraded goods in the home country lower the price of the nontraded good; this increased endowment must be consumed domestically, and both active and inactive households find it optimal to increase their consumption in response to the change in relative prices.

In the second column of Table 1, we solve the model assuming that all households or active, or equivalently that $\gamma = 0$. The measures of risk sharing are all higher than the measures for aggregate consumption in the benchmark model, consistent with a high degree of risk sharing implied by asset prices in the data, but inconsistent with a low degree of risk sharing implied by aggregate consumption data.

In the last column of Table 1, we solve the model with the fraction of active households fixed at the steady state level of the full model, to evaluate how important endogenous segmentation is. In this model with a fixed fraction of households active, we assume that the idiosyncratic shock y is distributed i.i.d. across both active and inactive households, according to the same distribution f. We see that this model generates more volatility in the real exchange rate, and there is essentially perfect risk sharing among the set of active households. In addition, this model generates a negative correlation between relative consumption and real exchange rates, which indicates that the movements in the fraction of active households in our benchmark model with endogenous segmentation provides a significant amount of risk sharing from the perspective of aggregate consumption.

In Table 2, we consider two different goods market structures. The first column is our benchmark model with tradable and nontradable goods; the second is for a model with only one good that is freely traded across countries; and the last column is a model with no trade in goods, which is closest to the original model in Alvarez, Atkeson, and Kehoe (2002).

Table 2: Alternative Goods Market Structures

	Benchmark Model	One good	No trade in goods
STANDARD DEVIATION (%)			
income	2.81	1.89	1.89
real exchange rate	1.75	0	5.24
INTERNATIONAL CORRELATIONS			
real income	0.17	0.16	0.16
Aggregate variables			
consumption	0.62	0.74	0.16
intertemporal MRS	0.65	0.74	0.16
BCS risk sharing index	0.62	0.72	0.09
Active households' variables			
consumption	0.93	1.00	0.16
intertemporal MRS	0.95	1.00	0.16
BCS risk sharing index	0.94	1.00	0.09
CORRELATION BETWEEN $\frac{C^*}{C}$ AND $\frac{P}{P^*}$			
all households	0.62	0.00	1.00
active households	1.00	0.00	1.00

Note: averages of statistics of logged series from 100 simulations of 100 periods each.

With only one good that is traded, there are no movements in the real exchange rate, and there is perfect risk sharing among active households. With no trade in goods at all, there is essentially no risk sharing, even among active households, because consumption of all households within a country is constrained by domestic endowments.

3.3 Suggestive Evidence of our Mechanism

The model yields a high cross-correlation in consumption of households actively transferring into or out of assets. We aim to investigate whether this prediction is supported by data. Ideally, we would need panel datasets with consumption, income and changes in asset balances for multiple countries and multiple years. Due to lack of panel data availability, we resort to examining cross-country correlations in consumption from cross-sectional data.

We use U.S. data from the Consumer Expenditure Survey, provided by Heathcote, Perri, and Violante (2010), and U.K. data from the Family Expenditure Survey provided by Blundell and Etheridge (2010). Within these datasets, we identify households who hold financial assets. We find that, from 1980 to 2005, the cross-country correlation in consumption of asset holders was 0.22. In addition, when we exclude the households with the top 5 percent of assets to income ratio, the correlation rises to 0.28. Over the same time period, the cross-country correlations for aggregate consumption and consumption of households with no financial assets were 0.21 and -0.02, respectively. This provides suggestive evidence that households participating in asset markets share risk across countries, while the consumption of households with no financial assets is not correlated across countries. We plan to enrich this analysis by refining the group of "active households" in the data, and we also aim to add additional countries to the analysis.

4 Conclusions and Future Work

A simple extension of the segmented asset markets model in Alvarez, Atkeson, and Kehoe (2002) has the potential to resolve the puzzle highlighted by Brandt, Cochrane, and Santa-Clara (2006), namely that the behavior of asset prices in different countries suggests a high degree of international risk sharing, while the behavior of aggregate consumption suggests the opposite.

Our contributions are in quantifying the degree to which segmented asset markets explain this discrepancy. Firstly, we can calibrate the model's degree of asset market segmentation to features of the cross-sectional distribution of consumption and income. Second, with more than one good, relative price fluctuations provide consumption insurance even in the absence of asset markets, so it is not obvious that segmenting markets will lead to poor observed risk sharing for inactive households. In addition, there can be no actual risk sharing without trade in goods, so the structure of the goods market matters, and it is necessary to go beyond one-good models without trade in goods.

Using microdata from the US and the UK from 1980 to 2005, we provide suggestive evidence for the mechanism of our model. Namely, we find that US and UK households participating in asset markets share risk, while the consumption of households with no financial assets are not correlated across these two countries.

Extensions to be done include a better choice of the idiosyncratic income process, that allows for persistent differences across households. Moreover, including production and physical capital investment would allow us to evaluate the performance of the model at matching other international business cycle statistics.

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