Dynamic Dependence in Corporate Credit*

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Abstract

Characterizing the dependence in credit spreads and default intensities across companies is a central problem in credit risk. Existing practice typically relies on factor models or simple static Gaussian copula models. We instead use genuinely dynamic copula models which can capture univariate and multivariate non-normalities and asymmetries for large cross-sections of firms. Using weekly data for over nine years on 223 firms, we find that the dependence in default intensities and CDS spreads is highly time-varying and persistent, and it increases significantly in the financial crisis. The dependence between equity returns also increases in the financial crisis, but this increase is much less persistent. We document substantial multivariate non-normalities for CDS spreads and default intensities, but multivariate asymmetries are less important for credit than they are for equities. The increase in cross-sectional dependence during the financial crisis has substantially reduced diversification benefits from selling credit protection, and it is critical to take non-normality into account when computing these benefits. These findings have important implications for the management of portfolio credit risk, the pricing of structured credit products, and the role of credit-risky securities in diversified portfolios.

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1 Introduction

Characterizing the dependence between credit-risky securities is of great interest for portfolio management and risk management, but not necessarily straightforward because multivariate modeling is notoriously difficult for large cross-sections of securities. In existing work, computationally straightforward techniques such as factor models are often used to model correlations for large portfolios; alternatively, simple rolling correlations or exponential smoothers are used.

We use explicitly multivariate econometric models for the purpose of modeling credit correlation and dependence. We use genuinely dynamic copula techniques that can capture univariate and multivariate deviations from normality, as well as multivariate asymmetries. We are able to apply these techniques to large cross-sections of firms by using recently proposed econometric innovations. We perform our empirical analysis using weekly data on 5-year Credit Default Swap (CDS) contracts for 223 constituents of the first 18 series of the CDX North American investment grade index, which cover the period from March 20th, 2003 to September 20th, 2012. Our 223 firms enter and leave the sample at different times, but this can easily be accommodated by the estimation methodology we employ. We investigate the dependence between CDS spreads as well as default intensities. We also analyze the dependence in equity prices for comparison.

We proceed in three steps. The two first steps are univariate. In the first, we remove the short-run dynamics from the raw data by estimating firm-by-firm ARMA models on weekly log differences. In a second step, we estimate firm-by-firm variance dynamics on the residuals from the first step. We use an asymmetric NGARCH model with an asymmetric standardized t-distribution following Hansen (1994).[1] Finally, in a third step we provide a multivariate analysis using the copula implied by the skewed t-distribution in DeMarta and McNeil (2004). Dynamic copula correlations are modeled based on the linear correlation techniques developed by Engle (2002) and Tse and Tsui (2002).[2] To alleviate the computational burden, we rely on the composite likelihood technique of Engle, Shephard, and Sheppard (2008) and the moment matching from Engle and Mezrich (1996). See Patton (2012) for a recent survey of copula models.

We find that copula correlations in CDS spreads and default intensities vary substantially over our sample, with a significant increase following the financial crisis in 2007. Equity correlations also increase after 2007, but less significantly so, and the increase is not as persistent. The increase in cross-sectional dependence is clearly important for the management of port-

[1] Engle (1982) and Bollerslev (1986) developed the first ARCH and GARCH models. Bollerslev (1990) first combined the GARCH model with a t-distribution.

[2] See Engle and Kroner (1995) for an early multivariate GARCH model and Engle and Kelly (2012) for a simplified dynamic correlation model.

folio credit risk and the relative pricing of CDO tranches with different seniority levels. Our estimates indicate fat tails in the univariate credit distributions, but also multivariate non-normalities for CDS spreads and default intensities. Multivariate asymmetries seem to be less important for credit than for equities, confirming the results from threshold correlations.

We use our estimates to compute time-varying diversification benefits from selling credit protection. We find that the increase in cross-sectional dependence following the financial crisis has substantially reduced diversification benefits, similar to what happened in equity markets. When computing diversification benefits, taking non-normality into account is more important for credit than for equity.

The paper proceeds as follows. In Section 2 we briefly discuss CDS markets. We discuss how to extract default intensities from CDS spreads, ass well as existing techniques for modeling credit dependence. Section 3 summarizes stylized facts in the CDS and equity data. Section 4 presents the models and Section 5 discusses the empirical results. Section 6 concludes.

2 CDS Markets, Default Intensity and Dependence

In this section, we first briefly discuss CDS markets. We then explain how to extract default intensities from CDS spreads, and we discuss existing techniques for modeling default dependence.

2.1 CDS Markets

A CDS is essentially an insurance contract, where the insurance event is defined as default by an underlying entity such as a corporation or a sovereign country. Which events constitute default is a matter of some debate, but for the purpose of this paper it is not of great importance. The insurance buyer pays the insurance provider a fixed periodical amount, expressed as a "spread" which is converted into dollar payments using the notional principal–the size of the contract. In case of default, the insurance provider compensates the insurance buyer for his loss.

The CDS market exploded in size between 2000 and 2007, standing at over 55 Trillion $US in notional principal in late 2007, according to the Bank for International Settlements. While the CDS market has subsequently been reduced to approximately 27 Trillion $US in notional principal as of June 2012, market size seems to have stabilized over the last two years after a sharp drop during the financial crisis. Also, the decline in CDS market size is much less significant than the decline for more complex credit derivatives, in particular structured credit products. This suggests that CDS markets have survived the financial crisis, highlighting the importance of a market for single-name default insurance.

One element of the success and resilience of CDS markets has been the creation of market indexes consisting of CDSs, the CDX index in North America and the iTraxx index in Europe. In our empirical work, we use all 226 constituents of the first 18 series of the CDX North American investment grade index. The list of firms we use in our empirical analysis is provided in Table 1 and we discuss the data in more detail in Section 3.

2.2 Extracting Default Intensities from CDS Premia

When studying dependence in the CDS market, the question arises what the dependence analysis should focus on. The most obvious object of interest are the CDS spreads themselves. However, the purpose of our paper is to better understand correlation between credit names, as part of any portfolio consisting of CDS or other credit-risky securities, or underlying a structured product such as a collateralized debt obligation (CDO). From this perspective, the analysis of default intensities is of equal or greater importance than the analysis of spreads themselves. For example, the standard valuation tool for CDOs is a Gaussian copula, and the most important input into this analysis is the correlation between the default intensities of the CDO constituents. We therefore analyze the dependence between intensities as well as spreads. To better motivate the analysis of default intensities, it is necessary to understand how they can be extracted from the CDS contracts based on the valuation formulas.

The valuation of CDS contracts, and the estimation of default intensities that relies on this valuation, has been studied in several papers.[3] Consider a given risk-neutral survival probability $q(t,T)$. The premium on a CDS is the spread paid by the protection buyer that equates the expected present value of default costs to be borne by the protection seller ("floating leg") to the expected present value of investing in the CDS ("fixed leg"). The value of the fixed leg is the present value of the spread payments the protection seller receives from the protection buyer, while the unknown floating leg comprises the potential payment by the protection seller to the buyer.

Consider now a CDS contract with payment dates $T=(T_1,...,T_N)$, maturity T_N, premium P and notional 1. Denote the value of the fixed leg by $V_{Fixed}(t,T,P)$, the value of the floating leg by $V_{Floating}(t)$, and the discount factors by $D(t,T_i)$. At each payment date T_i, the buyer has to pay $\tau(T_{i-1},T_i)P$ to the seller, where $\tau(T_{i-1},T_i)$ represents the time period between T_{i-1} and T_i (T_0 is equal to t). If the reference entity does not default during the life of the contract, the buyer makes all payments. However, if default occurs at time $s \leq T_N$, the buyer has made

[3] The literature on CDS contracts has expanded rapidly. For theoretical work, see Das and Sundaram (2000) and Hull and White (2000). For empirical studies, see Berndt, Douglas, Duffie, Ferguson, and Schranz (2004), Blanco, Brennan, and Marsh (2005), Ericsson, Jacobs and Oviedo (2009), Houweling and Vorst (2005), Hull, Predescu and White (2006), Longstaff, Mithal and Neiss (2004), and Zhang, Zhou and Zhu (2009).

$I(s)$ payments, where $I(s) = \max(i = 0, ..., N : T_i < s)$, and has to pay an accrual payment of $\tau(T_I(s), s)P$ at time s. Denote the probability density function associated with the default intensity process ψ_t by $f_\psi(t)$, and let the recovery rate be δ, then

$$V_{Fixed}(t, T, P) = P_t \left[\sum_{i=1}^{N} D(t, T_i)\tau(T_{i-1}, T_i)q(t, T_i) + \int_t^{T_N} D(t, s)\tau(T_{I(s)}, s)f_\psi(s)ds \right] \tag{2.1}$$

$$V_{Floating}(t) = \int_t^{T_N} D(t, s)(1 - \delta)f_\psi(s)ds. \tag{2.2}$$

At initiation of the contract, the premium P_0 is chosen in such a way that the value of the default swap is equal to zero. Equating the value of the two legs, the premium should be chosen as $P_0 = V_{Floating}(0)/V_{Fixed}(0, T, 1)$.

In our empirical application, we compute the integrals by numerical approximations. We define a daily grid of maturities $s_0, ..., s_m$, where $s_0 = t$ and $s_m = T_N$. Premium payments fall on the 20th of March, June, September and December, and we set T_N to correspond to the 20th payment (quarterly payments with a maturity of 5 years). The integrals in equations (2.1) and (2.2) are approximated by permitting default on days s_1 to s_m via

$$\int_t^{T_N} D(t, s)(1 - \delta)f_\psi(s)ds \approx \sum_{i=1}^{m} D(t, s_i)(1 - \delta)(q(t, s_{i-1}) - q(t, s_i))$$

$$\int_t^{T_N} D(t, s)\tau(T_{I(s)}, s)f_\psi(s)ds \approx \sum_{i=1}^{m} D(t, s_i)\tau(T_{I(s_i)}, s_i)(q(t, s_{i-1}) - q(t, s_i)$$

We use this valuation framework to back out time paths of default intensities that are subsequently used as inputs in an econometric analysis of the correlation of default intensities across companies. The estimated paths will depend on the assumptions made regarding the intensities. We start with the simplest possible approach. We back out a time path of default intensities assuming that the default intensity is constant at every time t. The link between the risk-neutral survival probability $q(t, T)$ and the default intensity is then simply

$$q(t, T) = \exp(-\psi_t(T - t))$$

The time-subscript on the ψ indicates that a new value for ψ in the constant-intensity model is extracted on each day. This method gives economically plausible results for all companies.

It is well-known that the default intensities extracted using a constant-intensity model are closely related to spreads when a fixed recovery rate is used. Of course, a high time-series correlation between spreads and default intensities may still imply different inferences regarding cross-sectional dependence. Nevertheless, our constant-intensity assumption could be criticized as overly simplistic. We therefore need to investigate the robustness of our results to the specification of dynamic models for default intensities in a reduced-form setup. Alternatively, one could also back out default intensities from CDS premia using a structural model. We plan to conduct these robustness exercises in future work.

2.3 Modeling Credit Dependence

Measuring default dependence has always been a problem of interest in the credit risk literature. For instance, a bank that manages a portfolio of loans is interested in how the borrowers' creditworthiness fluctuates with the business cycle. While the change in the probability of default for an individual borrower is of interest, the most important question is how the business cycle affects the value of the overall portfolio, and this depends on default dependence. An investment company or hedge fund that invests in a portfolio of corporate bonds faces a similar problem. Over the last decade, the measurement of default dependence has taken on added significance because of the emergence of new portfolio and structured credit products, and as a result new methods to measure correlation and dependence have been developed.

We now discuss different available techniques for estimating default dependence. First, default correlation can be computed using historical data. Second, the Merton (1974) model–or any of its offspring–can be used in conjunction with a factor model to model default dependence using equity prices. Third, there are different ways to estimate and model default correlation in the context of intensity-based credit risk models. Finally, we discuss the modeling of default correlation using copula-based models.

The oldest and most obvious way to estimate default correlation is the use of historical default data. In order to reliably estimate default probabilities for an individual firm, typically a large number of historical observations are needed which are not often available. Nevertheless, historical data on default are a rich source of information. See for instance deServigny and Renault (2002).

For publicly traded corporates, a second source of data on default correlation is the use of Merton (1974) type structural models model that link equity returns or the prices of credit-risky securities to the underlying asset returns.[4] The use of a factor model for the underlying equity return implies a factor model for the value of the credit risky securities, and it also determines the default dependence. Clearly the reliability of the default dependence estimate

[4] The structural approach goes back to Merton (1974). See Black and Cox (1976), Leland (1994) and Leland and Toft (1996) for extensions. See Zhou (2001) for a discussion of default correlation in the context of the Merton model.

is determined by the quality of the factor model.

A third way to estimate default dependence is in the context of intensity-based models, which have become very popular in the academic credit risk literature over the last decade.[5] This approach typically models the default intensity using a jump diffusion, and is also sometimes referred to as the reduced-form approach. Within this class of models, there are different approaches to modeling default dependence. One class of models, referred to as conditionally independent models or doubly stochastic models, assumes that cross-firm default dependence associated with observable factors determining conditional default probabilities is sufficient for characterizing the clustering in defaults. See Duffee (1999) for an example of this approach. Das, Duffie, Kapadia and Saita (2007) provide a test of this approach and find that this assumption is violated. Other intensity-based models consider joint credit events that can cause multiple issuers to default simultaneously, or they model contagion or learning effects, whereby default of one entity affects the defaults of others. See for example Davis and Lo (2001) and Jarrow and Yu (2001). Jorion and Zhang (2007) investigate contagion using CDS data.

Finally, modeling default correlation using copula methods has become extremely popular, especially among practitioners and for the purpose of CDO modeling. The advantage of the copula approach is its flexibility, because the parameters characterizing the multivariate default distribution, and hence the correlation between the default probabilities, can be modeled in a second stage, after the univariate distributions have been calibrated. In many cases the copulas are also parsimoniously parameterized and computationally straightforward, which facilitates calibration. The most commonly used model is the Gaussian copula, and calibration of the correlation structure is mostly performed using CDO data.

Note that while copula modeling is sometimes interpreted as an alternative to the structural and reduced-form approaches, this is strictly speaking not the case. Structural and reduced-form models describe how default occurs. Both factor and copula models are, on the other hand, tools to capture the dependence between either asset values within the structural approach, or times-to-default within the reduced-form framework. Below we propose and implement copulas that exhibit non-normality and dynamic dependence, and therefore constitute empirically relevant alternatives to factor models.

[5] See Jarrow and Turnbull (1995), Jarrow, Lando and Turnbull (1997), Duffee (1999), and Duffie and Singleton (1999) for early examples of the reduced form approach. See Lando (2004) and Duffie and Singleton (2003) for surveys.

3 CDS and Equity Data: Stylized Facts

In our empirical work we rely on data for 223 U.S. firms. Using data from Markit, we consider 5-year CDS contracts on all the 226 firms included in the first 18 series of the CDX North American investment grade index dating from March 20th, 2003 to September 20th, 2012. We remove 3 firms which do not have a consecutive 52-week history. We construct weekly data by using one day each week. We use Wednesdays, which is the weekday that is least likely to be a holiday. Equity data for the 223 remaining firms are obtained from Bloomberg. The 226 firms are listed in Table 1 and the three firms we have omitted (Beam, Duke Power, XLIT) are highlighted in bold italics.

Figure 1 plots over time the across-firm median CDS spread (Panel A), default intensity (Panel B), stock price (Panel C), and realized volatility (Panel D). The realized weekly volatility in Panel D is computed simply as the square root of the average daily squared returns within the week.

The vertical lines in Figure 1 denote six major events during our sample period:

- The WorldCom bankruptcy on July 19, 2002
- The Ford and GM downgrades to junk on May 5, 2005
- The Delphi bankruptcy on October 8, 2005
- The Quant meltdown on August 6, 2007
- The Bear Stearns bankruptcy on March 16, 2008
- The Lehman bankruptcy on September 15, 2008
- The stock market bottom on March 10, 2009
- The U.S. sovereign debt downgrade on August 5, 2011

Figure 1 indicates that the median CDS spread and the median default intensity are very similar. Both in turn are closely related to the median equity volatility in Panel D. The relationship between credit spreads and equity volatility is of course suggested by structural credit risk models such as Merton (1974). As expected, these three series reach their maximums in the peak of the financial crisis in 2008. Turbulence during the dot-com bust in 2002 and the US sovereign debt downgrade in 2011 is evident as well. Figure 1 also shows that CDS spreads and default intensities, like volatility, are highly persistent over time. Finally, Panel C of Figure 1 indicates that credit spreads and default intensities are negatively related to equity prices. This stylized fact may be due to the well-documented negative relation between stock returns and volatility. Figure 1 clearly indicates that the relation between volatility and spreads is tighter than the relation between spreads and stock prices.

The shaded areas in Figure 2 show the interquartile range (IQR) across firms of the spreads, intensities, stock prices and equity volatilities. It is interesting to note that the cross-sectional range of spreads is wider during and after the financial crisis compared to the pre-crises years. The high post-crisis range in spreads suggests that investors may be able to at least partly diversify credit risk which is a key topic of interest for us.

The dashed lines in Figure 2, which use the right-hand scale, report the number of firms available in the sample each week. The dynamic models we implement below allow for firms to enter and exit over the sample period. Notice that we have more equity than credit data in the early part of the sample. As pointed out by Patton (2006A), the dynamic multivariate modeling approach we employ below allows for individual series to begin (and end) at different time points. We make full use of this and include a firm if it has at least one year of consecutive weekly data points. As noted above we only eliminate three of our initial 226 firms.

Figure 3 plots the CDS spreads for 9 of the firms with a CDS spread history that spans our entire dataset. The 226 firms in our sample are distributed along the following 11 sectors: Financials (36 firms), Telecommunications Services (16), Utilities (9), Healthcare (11), Consumer Services (58), Basic Materials (11), Consumer Goods (34), Energy (13), Industrials (23), Technology (13), Government (2). For ease of exposition we merge the consumer goods and consumer services sectors and we also merge the Government sector which only has two firms (Freddie Mac and Fannie Mae) with Financials. Figure 3 reports on one firm in each of the resulting nine industries. While the financial crisis is apparent in many of the subplots, the magnitude of the firm-specific variation across the sample period is quite remarkable. This again bodes well for the potential diversification benefits of investors exposed to corporate credit risk.

Figure 4 plots the median CDS spread in each industry and reports the interquartile ranges as well. Compared to Figure 3, the financial crisis becomes more apparent when we aggregate to industry levels in Figure 4, although it is clear that the crisis affected different industries quite differently. It is also interesting to note how certain industries (see Technology, Telecom, and Utilities in the bottom row of panels in Figure 4) were affected relatively more by the 2001-2003 upheaval versus the 2007-2009 crises. Finally, note that the IQR widens substantially following the financial crisis for several industries.

In Table 2 we report sample averages across firms for spreads, default intensities and equity prices. Panel A of Table 2 shows the first four sample moments of the series in levels along with the IQR for each moment. Panel B reports the same four sample moments for the weekly log differences. We also report the Jarque-Bera tests for normality as well as the first two autocorrelation coefficients. Note the strong evidence of non-normality as well as some evidence of dynamics in the weekly returns. We will model both of these features below.

In Panel C we report the firm-median sample correlations between log differences in spreads, intensities and equity prices. On the diagonal we report the median and IQR across the correlations between each firm and all other firms. On the off-diagonal we report the median and IQR of the correlation between the different series (spread, intensity and equity return) for the same firm. Not surprisingly, the spreads and intensities are very highly correlated. The relatively high and robust negative correlation between weekly equity returns and weekly spreads (and intensities) is interesting. Note that the log-difference in spreads can be viewed as the return on *buying* credit protection and thus reducing credit risk. The negative correlation between spreads and equity returns is thus evidence of a positive correlation between the exposure to credit and equity risk. Modeling potential dynamics in credit and equity the correlations is the key modeling challenge in our paper.

Below, we will work solely with the weekly log-differences in CDS spreads, default intensities and stock prices. For simplicity we will refer to them generically as returns and denote them by R_t.

In order to further explore the dependence across firms we compute threshold correlations as done for example in Ang and Chen (2002) and Patton (2004). We define the threshold correlation $\bar{\rho}_{ij}(x)$ with respect to deviations of standardized returns $\overline{R}_i$ and $\overline{R}_j$ from their means as

$$\bar{\rho}_{ij}(x) = \begin{cases} Corr(\overline{R}_i, \overline{R}_j \mid \overline{R}_i < x, \overline{R}_j < x) & \text{when } x < 0 \\ Corr(\overline{R}_i, \overline{R}_j \mid \overline{R}_i \geq x, \overline{R}_j \geq x) & \text{when } x \geq 0, \end{cases}$$

where x is the number of standard deviations away from the mean, and returns are standardized using their sample mean and standard deviation. The threshold correlation reports the linear correlation between two assets for the subset of observations lying in the bottom-left or top-right quadrant defined with respect to their mean. In the bivariate normal distribution the threshold correlation approaches zero when the threshold goes to plus or minus infinity.

Figure 5 reports the median and IQR of the bivariate threshold correlations computed across all possible pairs of firms. Panel A and B show that the spread and intensity threshold correlations are high and almost symmetric. The equity threshold correlations in Panel C are also high but show some evidence of asymmetry: Large downward moves are more highly correlated than large up-moves. Altogether, Figure 5 shows strong evidence of multivariate non-normality. This is evidenced by the large deviations of the solid line (empirics) from the dashed lines (normal distribution) in Figure 5. Adequately capturing this non-normality motivates the non-normal copula approach below.

4 Dynamic Models of Credit Dependence

Our modeling strategy proceeds in three steps. In the first step, we model the mean dynamics on the univariate time series of each CDS spread, default intensity, and stock return. In the second step, we model the variance dynamics and the return residual distribution for each firm. In the third step, we develop dynamic copula models for spreads, default intensities, and equity returns using all the firms in our sample.

4.1 Mean Dynamics

The log-differencing on the raw data is partly done to remove long memory in the data. However, the weekly data we analyze contain short-run dynamics as well. In order to obtain white-noise innovations required for consistent models of correlation dynamics, we fit univariate $ARMA$-$NGARCH$ models to the weekly log-differenced time series. We first fit all possible $ARMA$ models with AR and MA orders up to two. The $ARMA$ order for each time series is then chosen using the finite sample corrected Akaike criterion (AIC).

To be specific, in a first step, we estimate the following nine possible models nested within the $ARMA(2,2)$ model on the first differences for spreads, intensities, and equity prices for each firm

$$R_t = \mu + \phi_1 R_{t-1} + \phi_2 R_{t-2} + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \varepsilon_t \tag{4.1}$$

where ε_t is assumed to be uncorrelated with R_s for $s < t$. The conditional mean for R_t constructed at the end of week $t-1$ is then simply

$$\mu_t = \mu + \phi_1 R_{t-1} + \phi_2 R_{t-2} + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2}$$

The $ARMA$ models in (4.1) are estimated using Gaussian quasi-maximum likelihood (QMLE) on a firm-by-firm basis for weekly log differences in default spreads, intensities and stock prices.

4.2 Variance Dynamics

In a second step we fit the Engle and Ng (1993) $NGARCH(1,1)$ model to the $ARMA$ filtered residuals ε_t

$$\begin{aligned}
\varepsilon_t &= \sigma_t z_t \\
\sigma_t^2 &= \omega + \alpha \left(\varepsilon_{t-1} - \gamma \sigma_{t-1}\right)^2 + \beta \sigma_{t-1}^2 \\
z_t &\sim i.i.d.ast(z; \lambda, \nu)
\end{aligned} \tag{4.2}$$

where we constrain $\omega > 0$, $\alpha > 0$, $\beta > 0$ in order to ensure that the conditional variance is positive on every day. The i.i.d. return residuals, z_t, are assumed to follow the asymmetric standardized t distribution from Hansen (1994) which we denote $ast(z; \lambda, \nu)$. The skewness and kurtosis of the distribution are nonlinear functions of the parameters λ and κ. When $\lambda = 0$ the symmetric standardized t distribution is obtained. The corresponding cumulative return probabilities are now given by

$$\eta_t \equiv \Pr_{t-1}(R < R_t) = \sigma_t^{-1} \int_{-\infty}^{\sigma_t^{-1}(R_t - \mu_t)} ast(z; \lambda, \nu) dz \tag{4.3}$$

Note that in our approach the individual return residual distributions are constant through time but the individual return distributions do vary through time because the return mean and variance are dynamic.

Using time series observations on ε_t, the parameters, ω, α, β, γ, λ and ν are estimated using a likelihood function based on (4.2) and $ast(z; \lambda, \nu)$. For each firm we again estimate three sets of parameters on spreads, default intensities, and equity prices, respectively.

4.3 Dynamic Copula Functions

From Patton (2006B), who builds on Sklar (1959), we can decompose the conditional multivariate density function of a vector of returns for N firms, $f_t(R_t)$, into a conditional copula density function, c_t, and the product of the conditional marginal distributions $f_{i,t}(R_{i,t})$

$$\begin{aligned} f_t(R_t) &= c_t(F_{1,t}(R_{1,t}), F_{2,t}(R_{2,t}), ..., F_{N,t}(R_{N,t})) \prod_{i=1}^{N} f_{i,t}(R_{i,t}) & (4.4) \\ &= c_t(\eta_{1,t}, \eta_{2,t}, ..., \eta_{N,t}) \prod_{i=1}^{N} f_{i,t}(R_{i,t}), & (4.5) \end{aligned}$$

where R_t is now a vector of N returns at time t, $f_{i,t}$ is the density and $F_{i,t}$ is the cumulative distribution function of $R_{i,t}$.

Following Christoffersen, Errunza, Jacobs, and Langlois (2012), and Christoffersen and Langlois (2013) we allow for dependence across the return residuals using the copula implied by the skewed t distribution discussed in Demarta and McNeil (2004). The skewed t copula cumulative distribution function, C_t, for N firms can be written

$$C_t(\eta_{1,t}, \eta_{2,t}, ..., \eta_{N,t}; \Psi, \lambda_C, \nu_C) = t_{\Psi,\lambda,\nu}(t_{\lambda,\nu}^{-1}(\eta_{1,t}), t_{\lambda,\nu}^{-1}(\eta_{2,t}), ..., t_{\lambda,\nu}^{-1}(\eta_{N,t})), \tag{4.6}$$

where λ_C is a copula asymmetry parameter, ν_C is a copula degree of freedom parameter, $t_{\Psi,\lambda,\nu}$ is the multivariate skewed Student's t density with correlation matrix Ψ, and $t_{\lambda,\nu}^{-1}$ is the inverse cumulative distribution function of the corresponding univariate skewed t distribution.

Note that the copula correlation matrix Ψ is defined using the correlation of the copula residuals $z_{i,t}^* \equiv t_{\lambda,\nu}^{-1}(\eta_{i,t})$ and not of the return residuals $z_{i,t}$. If the marginal distribution in (4.3) is close to the copula $t_{\lambda,\nu}$ distribution, then z_t^* will be close to z_t.

We now build on the linear correlation techniques developed by Engle (2002) and Tse and Tsui (2002) to model dynamic copula correlations. We use the copula residuals $z_{i,t}^* \equiv t_{\lambda,\nu}^{-1}(\eta_{i,t})$ as the model's building block instead of the return residuals $z_{i,t}$. In the case of non-normal copulas, the fractiles do not have zero mean and unit variance, and we therefore standardize the z_i^* before proceeding.

The copula correlation dynamic is driven by

$$\Gamma_t = (1 - \beta_C - \alpha_C)\Omega + \beta_C \Gamma_{t-1} + \alpha_C \bar{z}_{t-1}^* \bar{z}_{t-1}^{*\top} \tag{4.7}$$

where β_C and α_C are scalars, and $\bar{z}_t^*$ is an N-dimensional vector with typical element $\bar{z}_{i,t}^* = z_{i,t}^* \sqrt{\Gamma_{ii,t}}$. The conditional copula correlations are defined via the normalization

$$\Psi_{ij,t} = \Gamma_{ij,t} / \sqrt{\Gamma_{ii,t}\Gamma_{jj,t}}.$$

Below we refer to the model using (4.6) and (4.7) as the Dynamic Asymmetric Copula (DAC) model. The special case where $\lambda_C = 0$ we denote by the Dynamic Symmetric Copula (DSC). When further $1/\nu_C = 0$ we obtain the Dynamic Normal Copula (DSC).

Following Engle, Shephard and Sheppard (2008) we estimate the copula parameters α_Γ, β_Γ, λ_C, and ν_C using the composite likelihood (CL) function defined by

$$CL(\alpha_C, \beta_C, \lambda_C, \nu_C) = \sum_{t=1}^{T} \sum_{i=1}^{N} \sum_{j>i} \ln c_t(\eta_{i,t}, \eta_{j,t}; \alpha_C, \beta_C, \lambda_C, \nu_C), \tag{4.8}$$

where c_t is the copula density from (4.4). Note that in the CL function is built from the bivariate likelihoods so that the inversion of large-scale correlation matrices is avoided. In a sample as large as ours, relying on the composite likelihood approach is imperative. The matrix of unconditional correlations is estimated by unconditional moment matching (See Engle and Mezrich, 1996)

$$\widehat{\Omega} = \frac{1}{T} \sum_{t=1}^{T} \bar{z}_t^* \bar{z}_t^{*\top}. \tag{4.9}$$

which is another crucial element in the feasible estimation of large-scale dynamic models.

As discussed above, the estimation of dynamic dependence models using long time series and large cross-sections is computationally intensive. In our case, estimating the dynamic copula models for 223 firms in possible only because we implement unconditional moment matching and the composite likelihood approach. An additional advantage of the composite likelihood approach is that we can use the longest time span available for each firm-pair when estimating the model parameters, thus making the best possible use of a cross-section of CDS time series of unequal length.

5 Credit Dependence and Diversification Dynamics: Empirical Results

In this section we first report on the ARMA conditional mean models and the NGARCH conditional volatility models estimated using weekly log-differences of CDS spreads, default intensities, and equity prices for the CDX constituents listed in Table 1. We then discuss the dynamic copula correlation estimates. Finally, we compute conditional diversification benefits measures for credit and equity portfolios.

5.1 Dynamic Mean and Variance Estimates

Table 3 reports in Panel A the percentage of firms for which each of the nine estimated $ARMA(p, q,)$ models were favored by the Akaike criterion for spreads, intensities, and equity prices. The percentages are quite even across the nine possible models. The $ARMA(2, 2)$ is the single-most selected model, suggesting that perhaps higher lags should be considered. Panel A also shows the median and interquartile range across firms of each $ARMA$ coefficient estimate. The parameter values vary considerably across firms. The Ljung-Box test on the z_t residuals show that the test fails to reject the null that the residuals are serially uncorrelated 99% of the times for CDS spreads and default intensities, and 96% of the times for equity returns. This suggests that the ARMA models are able to adequately capture conditional mean dynamics across firms and markets.

Panel B in Table 3 shows the median and interquartile range across firms of for each of the four NGARCH parameters as the two parameters in the asymmetric t distribution. Weekly volatility persistence, defined by $(\alpha (1 + \gamma^2) + \beta)$, is fairly tightly distributed around the median values of 0.954 for CDS spreads, 0.941 for default intensities, and 0.978 for equity returns. Volatility is clearly highly persistent in all three sets of data. The γ parameter captures the asymmetric volatility response to positive and negative return residuals. For equities, the median γ value is 1.313 and the interquartile range is entirely positive. For CDS spreads and default intensities the γ is negative and smaller in magnitude. Recall that the CDS spreads and default intensities capture the returns to *buying* credit protection.

The Ljung-Box test of serial correlation in the z_t^2 shows that the NGARCH model is able to adequately capture variance dynamics. Equity returns, which have the highest volatility persistence, have 8% of NGARCH models rejected by Ljung-Box at the 5% level which is clearly not dramatic.

The ν parameter has medians of 3.6 (CDS spreads), 3.7 (default intensity), and 6.5 (equity returns), indicating fat tails in the conditional distribution. The asymmetry parameter, λ, is negative for equities and positive for CDS spreads and default intensities. Recall again that the CDS spreads and default intensities capture the returns to buying credit protection.

Figure 6 shows the median and IQR in the conditional variance over time. Panel A in Figure 6 shows that while most of the spikes in CDS spread volatility match the spikes in spread levels in Figure 1, this is not always the case. Note for example that one of the highest peaks in CDS spread volatility occur at the time of the quant meltdown in August 2007, which coincides with only a minor uptick in spreads in Figure 1. The default intensity volatility dynamics in Panel B are similar to the CDS spread dynamics in Panel A. The NGARCH equity volatility in Panel C is much smoother than the realized weekly volatility in Figure 1 but the general dynamic patterns are of course similar. Comparing the volatility IQRs in Figure 6 with those in Panels A, B, and C of Figure 2, the cross-sectional dispersion in volatilities is clearly much smaller. Finally, note that the volatility patterns in spreads and default intensities in Panels A and B of Figure 6 are somewhat different from the volatility in equity in Panel C of Figure 6. An obvious example is May 2005, around the time of the Ford and GM downgrade, when equity volatility in Panel C does not spike up, but median CDS spreads and default intensities do. Note that the realized volatility evidence in Panel D of Figure 1, which is not model-dependent, confirms the conclusion from Panel C of Figure 6.

Figure 7 plots the weekly NGARCH dynamic in CDS spreads for the 9 firms from Figure 3. Note that the spread volatility in Figure 7 has many more spikes than the spreads themselves in Figure 3. Spread volatility clearly does not seem to be a simply deterministic function of the spreads themselves. Note also in Figure 7 that the variation of spread volatility across firms is quite dramatic.

Figure 8 plots the median and IQR of NGARCH volatility within each of the 9 industries. Note the commonality in conditional volatility within each industry, unlike the credit spreads in Figure 4, which vary considerably within each industry. The high levels of CDS spread volatility in the recent financial crisis is apparent as well.

Table 4 contains descriptive statistics of the ARMA-NGARCH model residuals. Panel A shows that skewness and kurtosis is still present after standardizing by the NGARCH model. As expected, the residual correlations in Panel B are not materially different from the raw return correlations in Panel C of Table 2.

Finally, Figure 9 plots the median and IQR threshold correlations on the weekly ARMA-NGARCH residuals. Comparing with the threshold correlations on raw returns in Figure 5, we see that the median threshold correlations in residuals are often lower but still higher than the bivariate Gaussian distribution (dashed lines) would suggest. It is interesting to note that the left side correlations are lower in Figure 9 for credit markets and the right-side correlations are lower for equity markets when comparing with Figure 5. This suggests that the ARMA-NGARCH models are able to remove some of the multivariate non-normality in the data.

5.2 Copula Correlation Estimates

Panel A of Table 5 contains the Dynamic Asymmetric Copula (DAC) parameter estimates and composite likelihoods from fitting a single model to the 223 firms in our sample. We again present separate results for estimation on the ARMA-NGARCH residuals from weekly log-differences in CDS spreads, default intensities, and equity prices, respectively. The copula correlation persistence is highest for CDS spreads and default intensities (0.980 in both cases) and somewhat lower at 0.949 in the case of equity prices. Comparing with the volatility persistence in Table 3, it is interesting to note that equities have relatively higher volatility persistence and lower correlation persistence when compared with credit spreads and default intensities. This result shows the importance of modeling separate dynamics for volatility and correlation.

Panel B in Table 5 reports the parameter estimates for the Dynamic Symmetric Copula model where $\lambda_C = 0$ and Panel C reports on the Dynamic Normal Copula where we also impose $1/\nu_C = 0$. While we do not have asymptotic distribution results available for testing differences in composite likelihoods, the results suggest that the improvements in fit are largest when going from the normal copula in Panel C to the symmetric t copula in Panel B. When going from the symmetric t copula in Panel B to the asymmetric t copula in Panel A the improvement in fit seems to be largest for equities. This result matches the threshold correlations in Figures 5 and 9 which show the strongest degree of bivariate asymmetry for equities. Comparing the correlation persistence across Panels A-C we see that the equity correlation persistence is highest in all three copula models.

In Figure 10 we plot the median and IQR of the DAC copula correlations for the CDS spreads, default intensities and stock returns. Comparing the copula correlations in Figure 10 with the volatilities in Figure 6 we see that the correlations appear to be smoother and more persistent than the volatilities. The credit correlations in Panel A and B show a pronounced and persistent uptick in mid 2007, whereas the equity correlations in Panel C show less persistent upticks in late 2008 following the Lehman bankruptcy and again in mid 2011 following the US sovereign downgrade.

In Figure 11 we plot the median and interquartile range in DAC correlations of CDS spreads for the 9 firms from Figures 3 and 7. For each firm we plot the median and IQR of the pairwise correlations with all other firms in the sample. While the uptick in correlation during 2007 is evident in many cases, the variation across firms is large. Note for example the relatively flat time path of correlations for AT&T.

Figure 12 reports on the median and IQR in CDS spread correlations in the 9 industries using the DAC model again. For each industry we report the median and IQR using all pairwise correlations within the industry. The increase in correlations in 2007 is now evident in most industries, although not in Telecom. The IQRs indicate that within industries, there are substantial differences in correlation between firms. While this is also the case for spreads in Figure 4, the differences are larger after the financial crisis in those cases, whereas in Figure 12 they obtain over the entire sample. Note that for volatility in Figure 8, the IQRs are very narrow. This again emphasizes the need for separate models for volatilities and correlations.

5.3 Conditional Estimates of Diversification Benefits

Consider an equal-weighted portfolio of the constituents of the on-the-run CDX investment grade index in any given week. We want to assess the diversification benefit of the portfolio using the dynamic, non-normal copula model developed above. As in Christoffersen, Errunza, Jacobs and Langlois (2012), we define the conditional diversification benefit by

$$CDB_t(p) \equiv \frac{\overline{ES}_t\,(p) - ES_t(p)}{\overline{ES}_t(p) - \underline{ES}_t(p)}, \tag{5.1}$$

where $ES_t(p)$ denotes the expected shortfall with probability threshold p of the portfolio at hand, $\overline{ES}_t(p)$ denotes the average of the ES across firms, which is an upper bound on the portfolio ES, and where $\underline{ES}_t(p)$ is the portfolio VaR which is a lower bound on the portfolio ES. The $CDB_t(p)$ measure takes values on the $[0, 1]$ interval, and is increasing in the level of diversification benefit. Note that by construction CDB does not depend on the level of expected returns. Expected shortfall is additive in the conditional mean which thus cancel out in the numerator and denominator in (5.1).

The CDB measure depends on the threshold probability p. Below we consider $p = 5\%$ and $p = 50\%$. The CDB measure is not available in closed form for our dynamic copula model and so we compute it using Monte Carlo simulation. We also report on a volatility-based measure which is defined by

$$VolCDB_t = 1 - \frac{\sqrt{\mathbf{1}^\top \Sigma_t \mathbf{1}}}{\mathbf{1}^\top \sigma_t}, \tag{5.2}$$

where $\mathbf{1}$ denotes a vector of ones, and where Σ_t denotes the usual matrix of linear correlations computed in our case via simulation from the DAC model. One can show that under conditional normality, $VolCDB_t$ will coincide with $CDB_t(50\%)$ so that the difference between these two measures indicates the degree of non-normality from a diversification perspective.

Figure 13 shows the $CDB(5\%)$ measure for an equal-weighted portfolio *selling* credit protection as well as for an equal-weighted portfolio of equity returns. Note that we have included the VIX index from CBOE in grey on the right-hand axis for reference. Consider first Panel A: Diversification benefits for CDS swaps have declined from above 70% at the end of 2003 to below 50% at the end of our sample. The majority of the decline took place during the mid 2007 to mid 2008 period and was relatively gradual. Panel B shows that the decline in diversification benefits in equity markets have been roughly similar in magnitude from just below 80% in 2007 to just above 60% at the end of our sample. The majority of the decline in equity market diversification benefits took place from early 2007 to early 2009 and it was relatively gradual as well.

It is interesting to note that the majority of the decline in diversification benefits in both credit and equity markets took place well before the peak in the VIX. The credit market CDB actually increased a bit during late 2008 and early 2009 when the equity market turmoil was at its highest.

In Figure 14 we plot the $CDB(50\%)$ and $VolCDB$ measures for CDS spreads in Panel A and for equities in Panel B. Comparing Figures 13 and 14 (note the scales are different) we see that the dynamic patterns are broadly similar, which is not surprising.

When comparing Panel A with Panel B in Figure 14 we see that the difference between the $CDB(50\%)$ and $VolCDB$ is larger for the credit portfolio than for the equity portfolio. These results suggest that non-normality plays a large role even in a well-diversified credit portfolio, and that relying on $VolCDB$ would thus exaggerate the benefits from credit diversification.

Finally, in Figure 15 we analyze to which extent portfolio optimation can be employed to avoid the decline in diversification benefits over time evident in the equal-weighted portfolio in Figures 13-14. Each week we choose portfolio weights by maximizing CDB on a set of nine industry portfolios each of which is equally weighted across firms in the industry. For comparison we also show the equal-weighted portfolio CDB from Figure 13. Figure 15 shows that it is possible to at least partially avoid the decline in diversification benefits when optimally rebalancing each week across industries. However, part of the decline in diversification benefits remain. Rebalancing across individual firms may of course generate even better results but this is computationally much more demanding as we compute the optimal portfolio weights via simulation of the DAC model.

6 Conclusion

This paper documents cross-sectional dependence in CDS spreads, CDS-implied default intensities, and equity prices. Our results are largely complementary to existing correlation estimates, which typically are based on historical default rates or equity returns, and to existing intensity-based studies, which characterize observable macro variables that induce realistic correlation patterns in default probabilities (see Duffee (1999) and Duffie, Saita and Wang (2007)).

We obtain three main findings. First, copula correlations in CDS spreads and default intensities vary substantially over our sample, with a significant increase following the financial crisis in 2007. Equity correlations also increase in the financial crisis, but somewhat later and the increase is less significant and not as persistent. This increase in cross-sectional dependence is clearly important for the management of portfolio credit risk and the pricing of structured credit products, in particular the relative pricing of CDO tranches with different seniority levels. Second, our estimates indicate fat tails in the univariate distributions, but also multivariate non-normalities. Multivariate asymmetries seem to be less important for credit than they are for equities. Finally, when considering the diversification benefits from selling credit protection, the increase in cross-sectional dependence following the financial crisis has reduced diversification benefits, not unlike what happened in equity markets. When computing diversification benefits, taking non-normalities into account is more important for credit than for equity.

Our results suggest a number of extensions. First, given the richness and complexity of the equity-implied and default intensity-implied dependence, it may prove interesting to explore the implications for CDO valuation. In particular it would be interesting to investigate if the resulting CDO pricing model would remove some of the observed correlation smile in CDO tranches. See Berd, Engle, and Voronov (2007) for an example of this type of approach. Second, an investigation of how the time-varying dependence is related to firm-specific and economywide observables would be insightful. Third, alternative measures of credit portfolio risk should be investigated (Vasicek 1987, 1991, 2002). Finally, we need to investigate the robustness of the dependence patterns for default intensities when the intensities are extracted using more sophisticated models.

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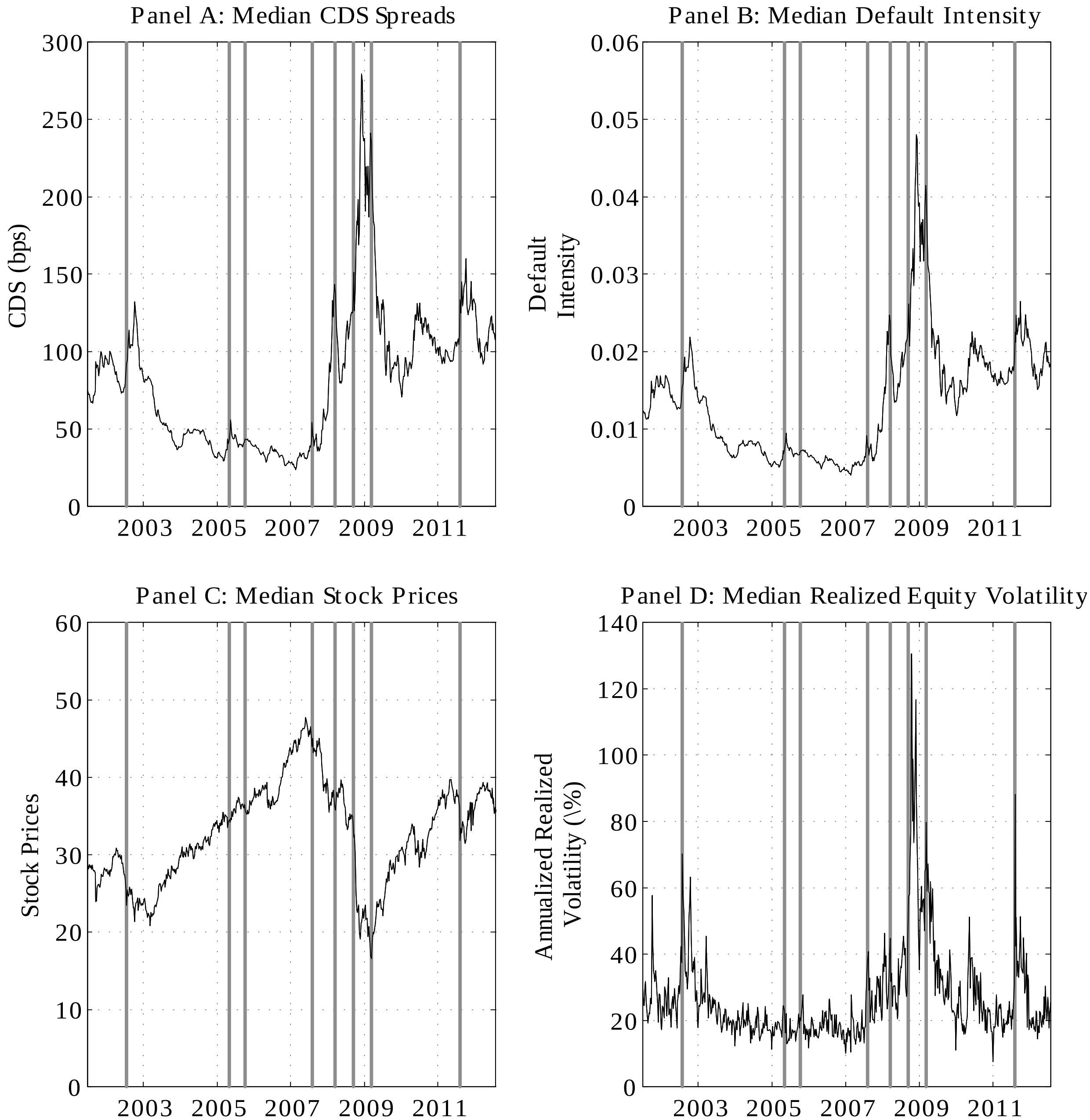
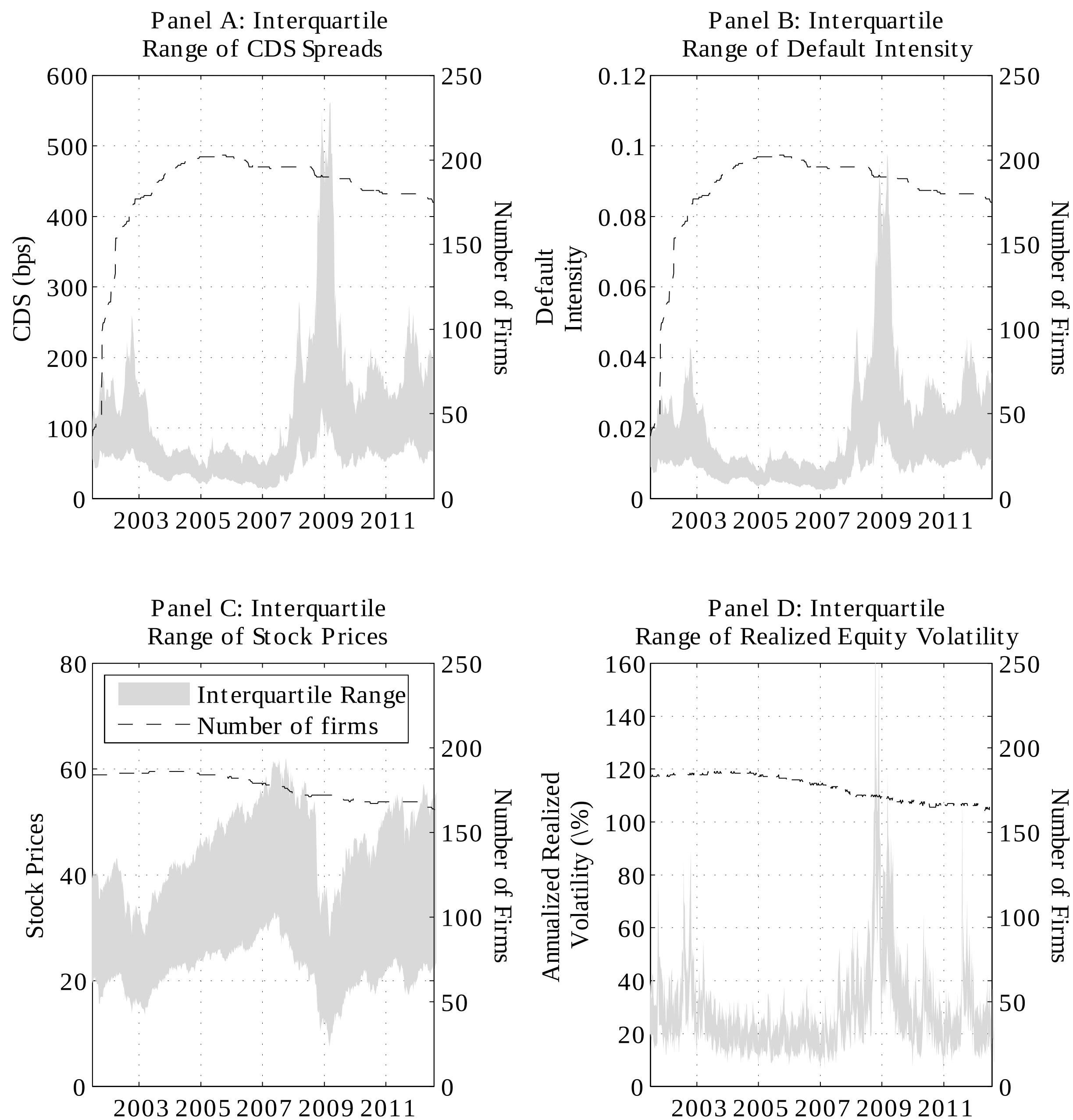
Figure 1: Weekly Median CDS Spreads, Default Intensities and Realized Equity Volatility.



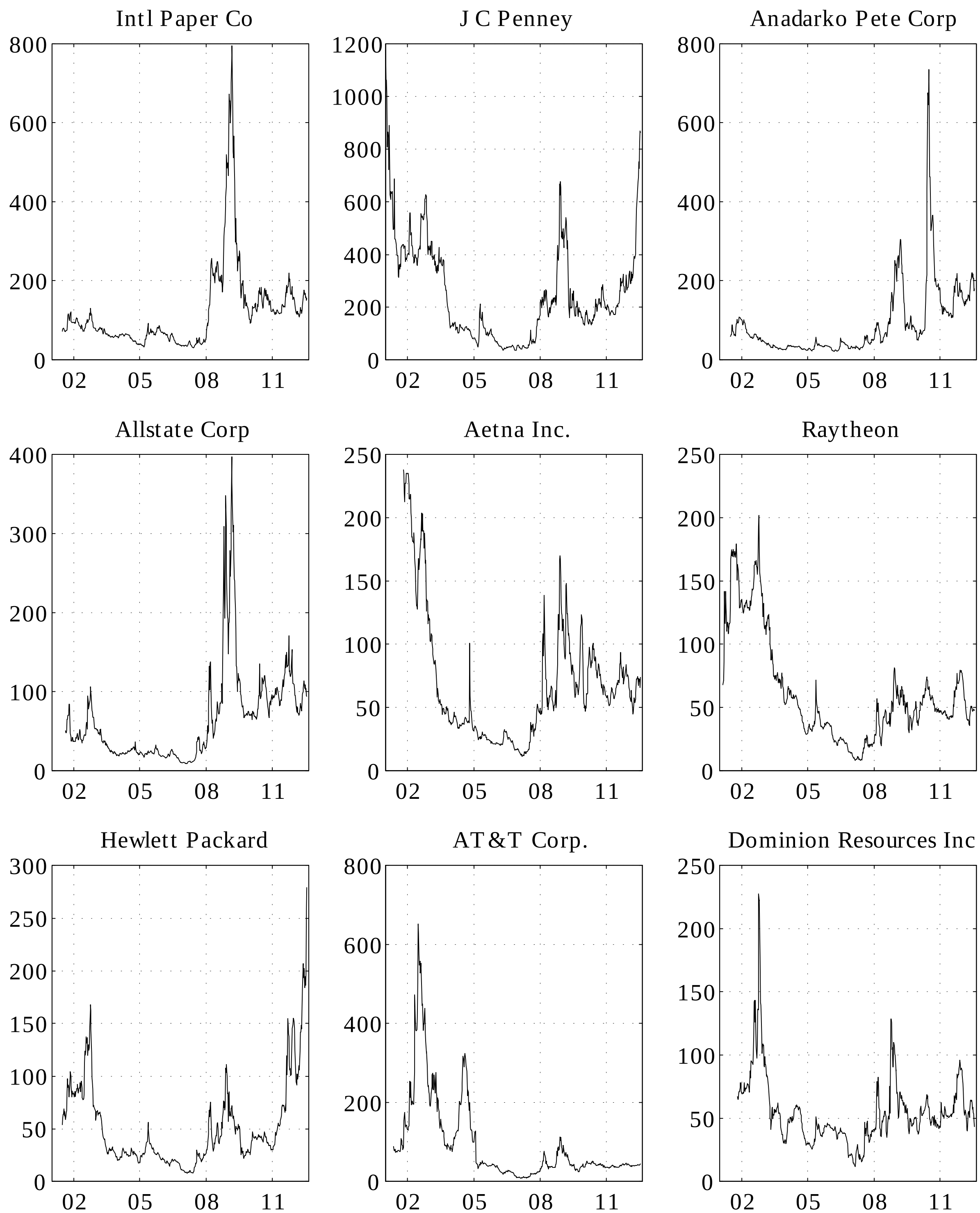
Notes to Figure: We plot the median values across the 223 equities listed in Table 1. Realized volatility is constructed from the sum of squared daily returns during each week. The vertical lines indicate major events during the sample period.

Figure 2: Interquartile Range of CDS Spreads, Default Intensities, and Realized Equity Volatility.



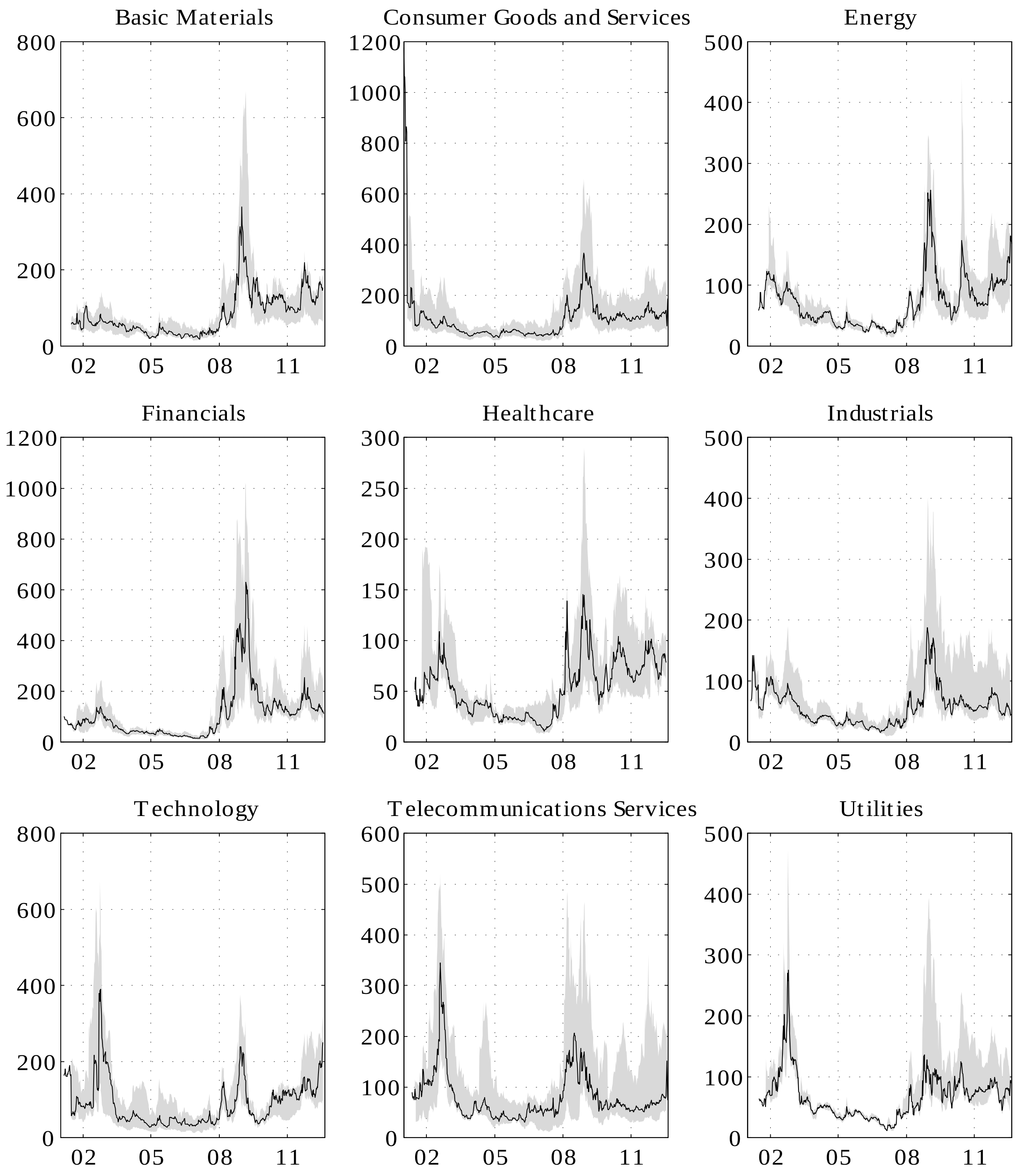
Notes to Figure: The grey area denote the interquartile range of the 223 equities listed in Table 1. Realized volatility is constructed from the sum of squared daily returns during each week. The dashes lines show the number of firms available in each sample each week.

Figure 3: CDS Spreads for Nine Firms.



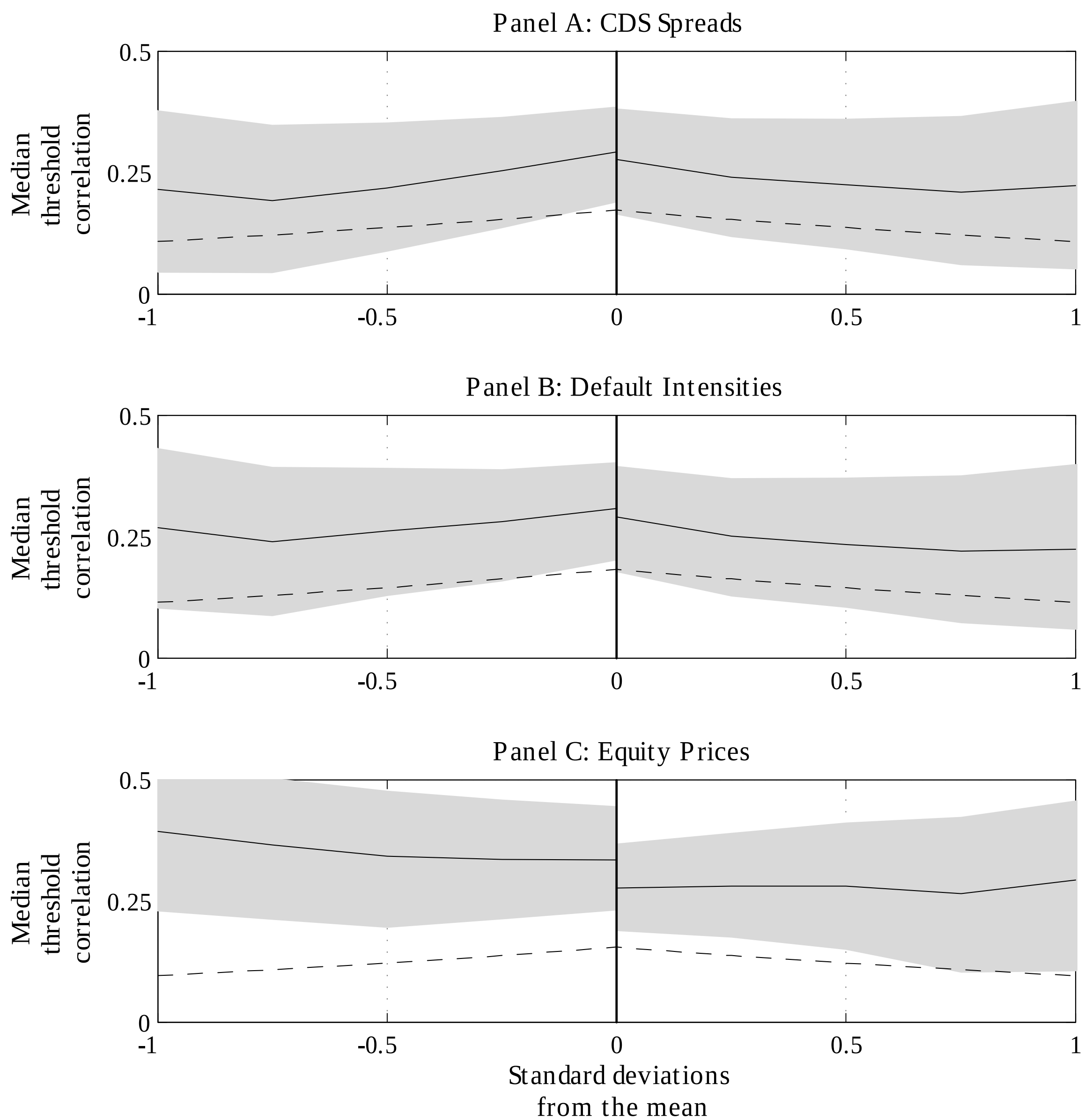
Notes to Figure: We plot the CDS spreads for 9 firms in our sample, one firm for each of the 9 industries.

Figure 4: Median CDS Spreads and Interquartile Range for Nine Industries



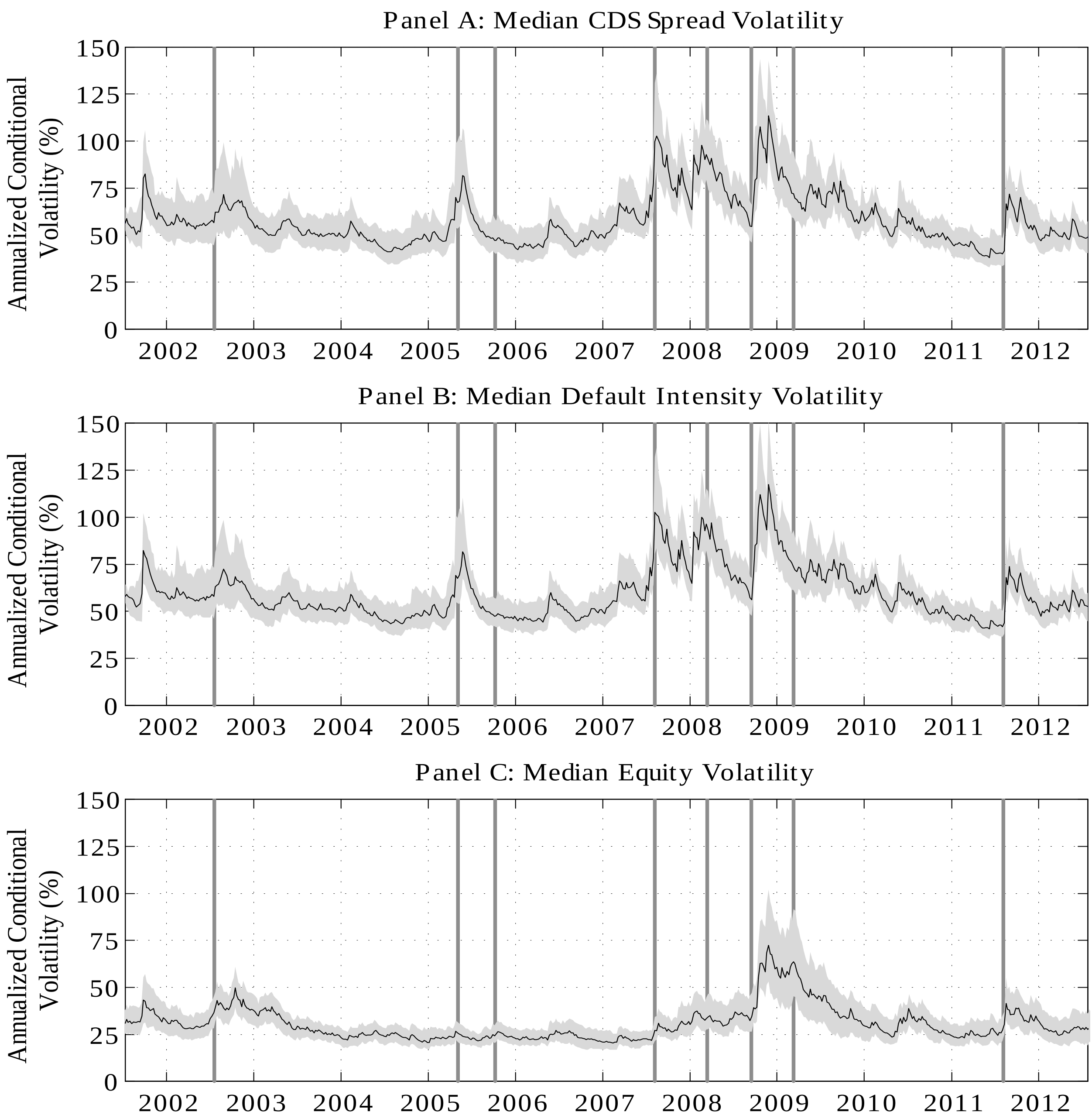
Notes to Figure: From the 11 industries in Markit we merge consumer goods and consumer services. We also merge government (Freddie Mac and Fannie Mae) with financials. The figure shows the median and interquartile range of CDS spreads within each of the remaining 9 industries.

Figure 5: Threshold Correlations for Weekly Log Differences:
CDS Spreads, Default Intensities and Equity Prices



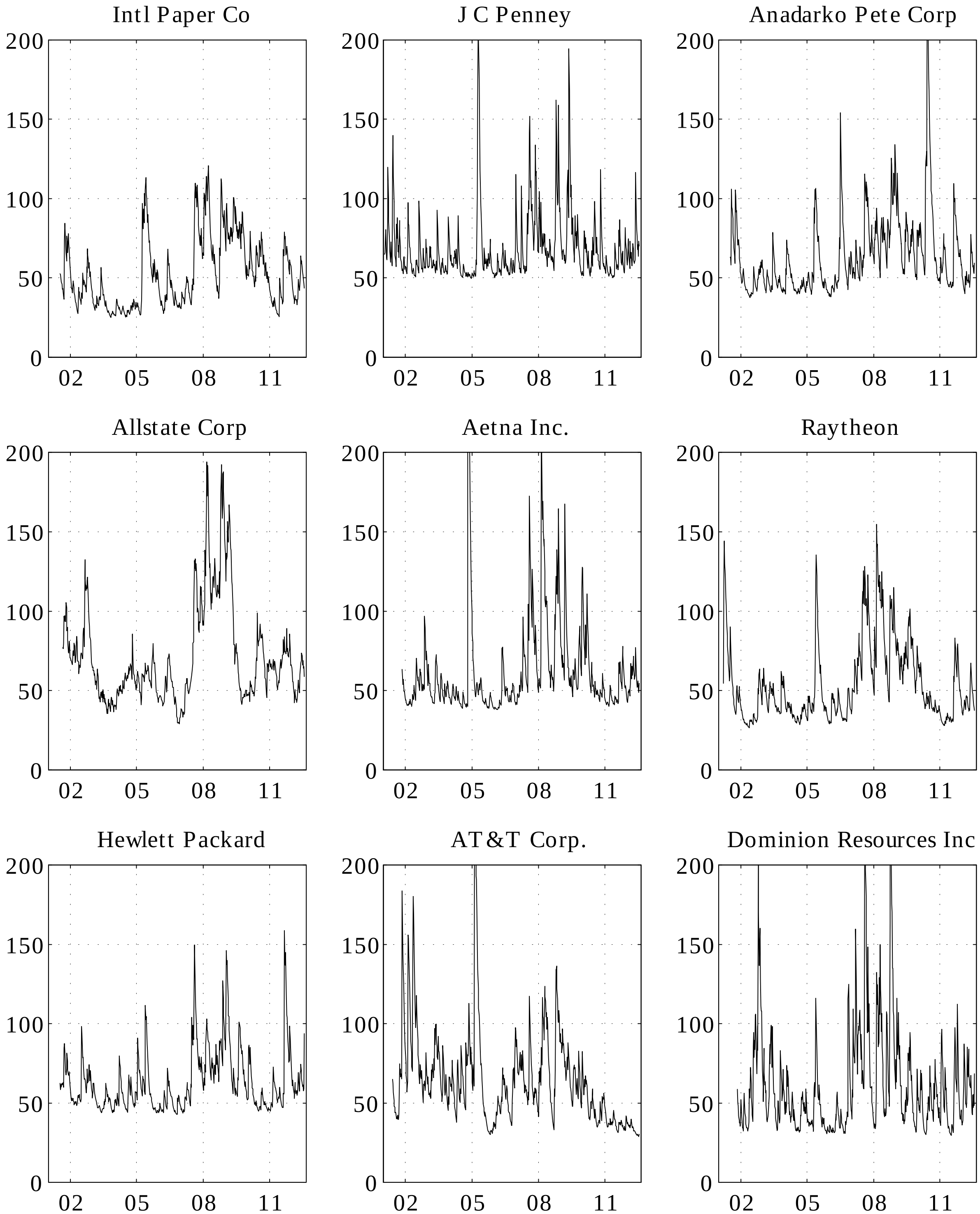
Notes to Figure: For each pair of firms we compute threshold correlations on a grid of thresholds defined using the standard deviation from the mean for each firm (horizontal axis). The solid lines show the median threshold correlations across firm pairs, the grey areas mark the interquartile ranges and the dashed lines show the threshold correlations from a bivariate Gaussian distribution with correlation equal to the average for all the pairs of firms.

Figure 6: Median and Interquartile Range of Conditional Volatility:
CDS Spreads, Default Intensities, and Equity Prices



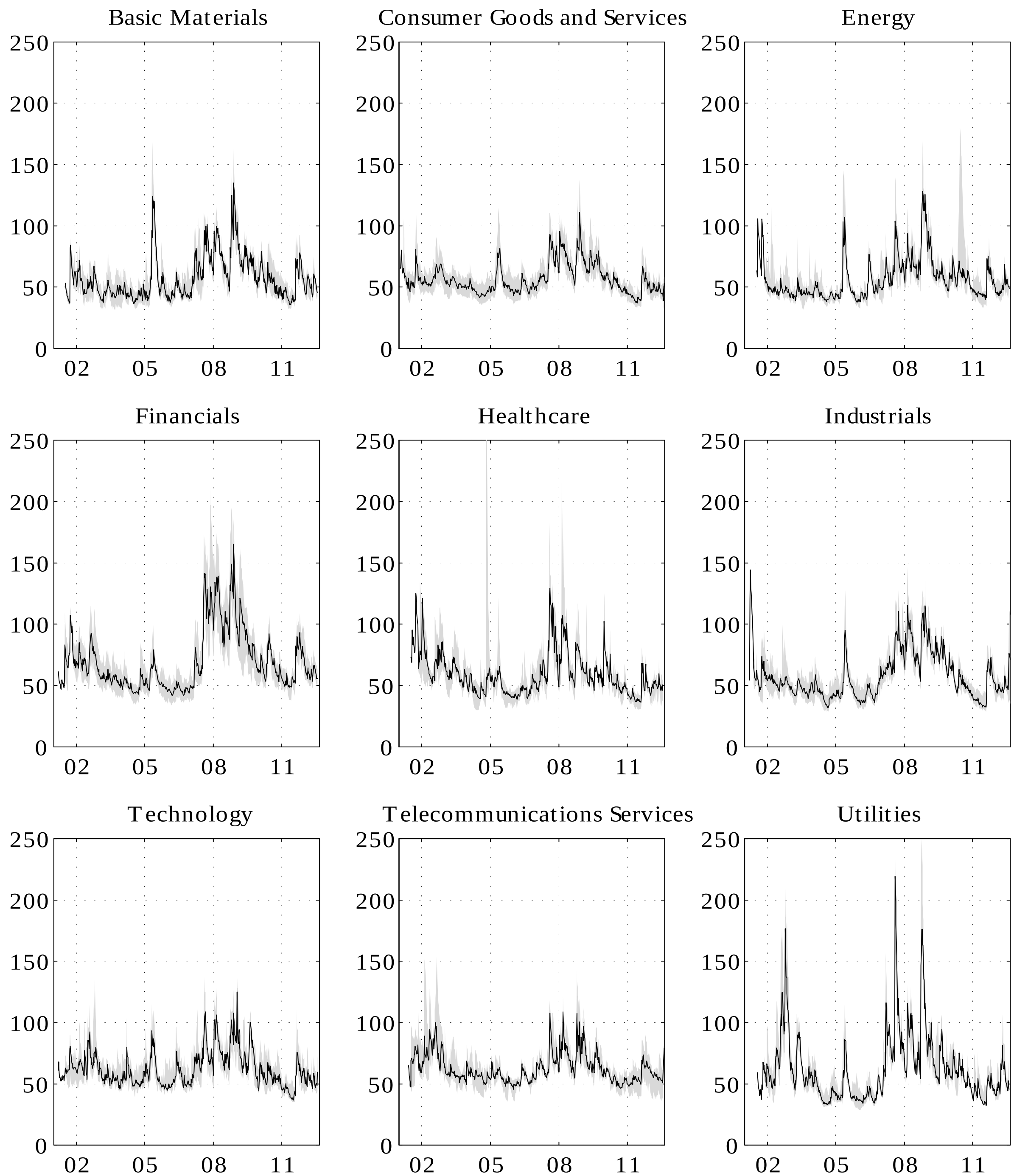
Notes to Figure: For each firm we estimate an NGARCH model on the weekly log differences in CDS spreads, default intensities and equity prices. The plot shows the median and interquartile range across firms of the weekly conditional volatility.

Figure 7: Conditional Volatility of CDS Spreads for Nine Firms.



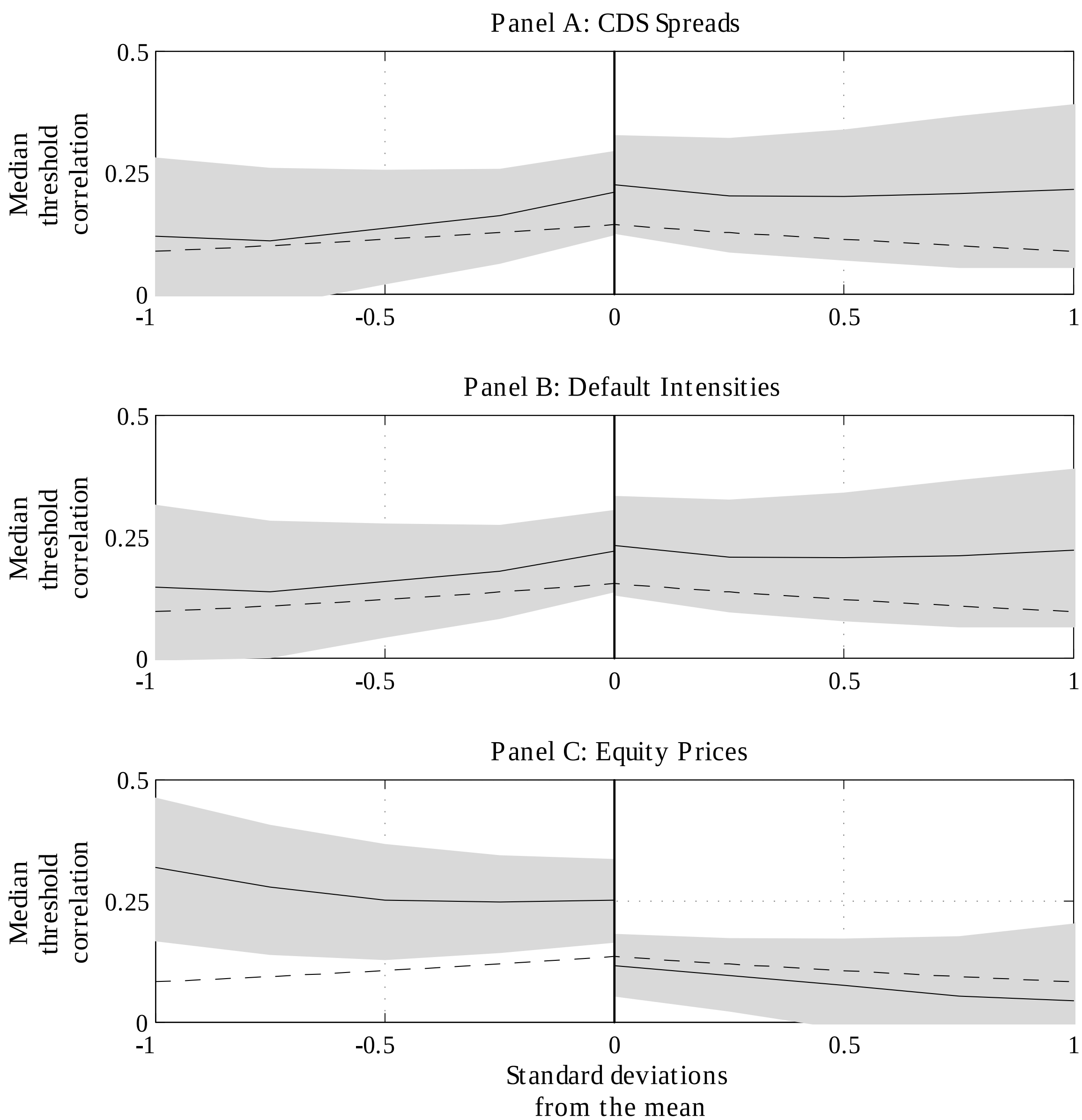
Notes to Figure: We plot the conditional volatility of CDS spreads for 9 firms in our sample, one for each industry.

Figure 8: Median and Interquartile Range of Conditional Volatility within Nine Industries.



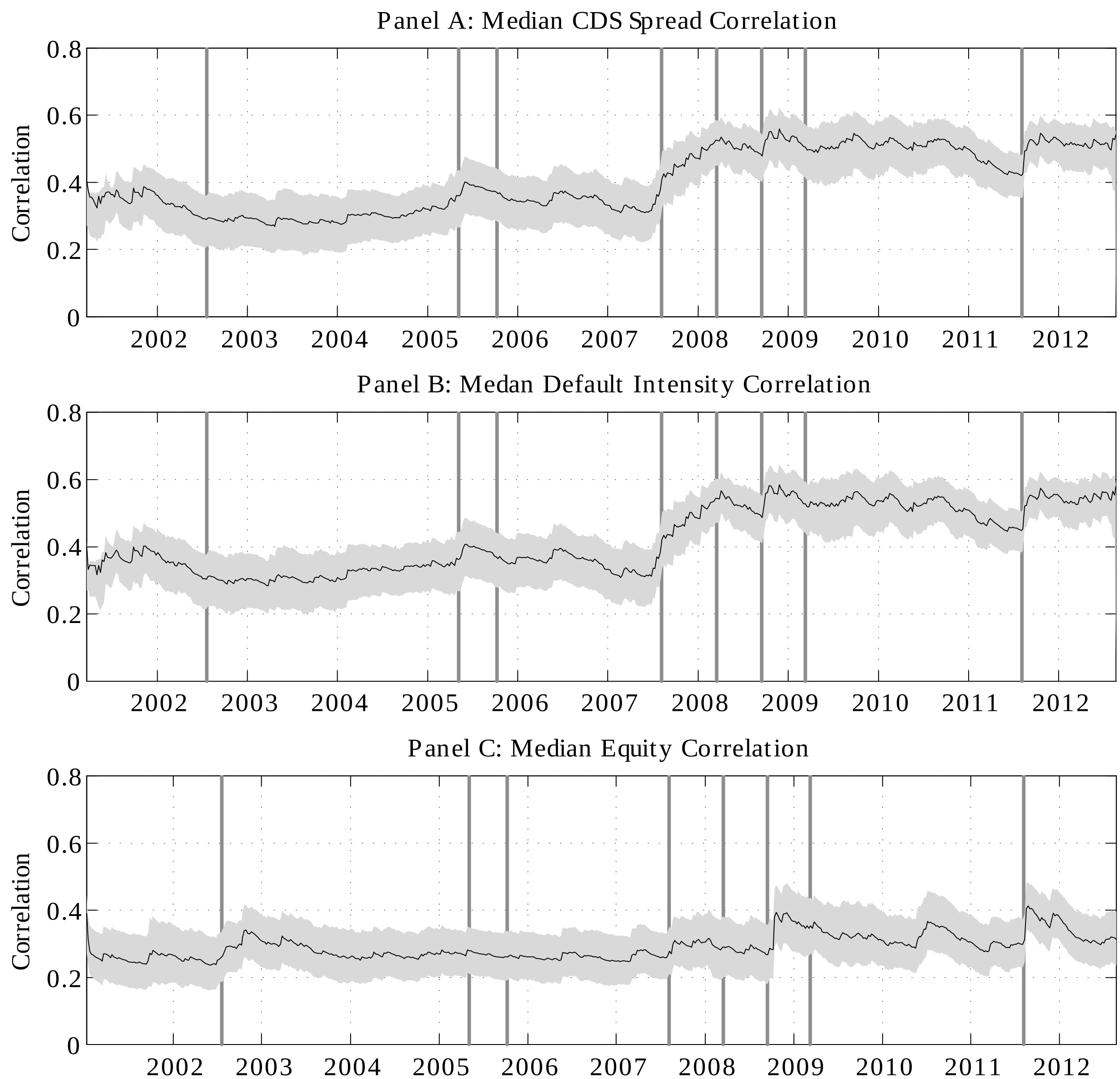
Notes to Figure: From the 11 industries in Markit we merge consumer goods and consumer services. We also merge government (Freddie Mac and Fannie Mae) with financials. The figure shows the median and interquartile range of conditional volatility of CDS spreads within each of the remaining 9 industries.

Figure 9: Threshold Correlations for Weekly Residuals:
CDS Spreads, Default Intensities and Equity Prices



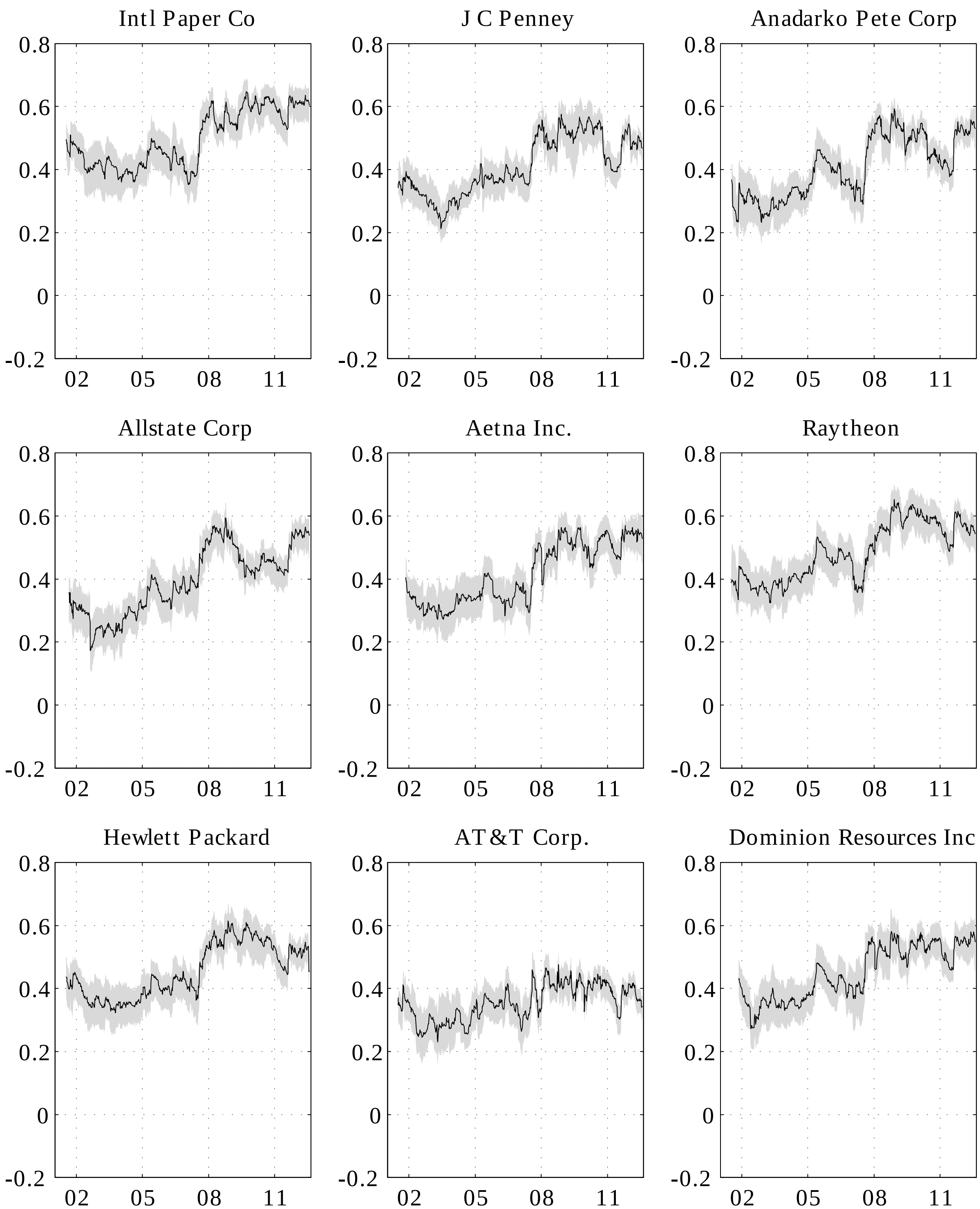
Notes to Figure: For each pair of firms we compute threshold correlations on the ARMA-NGARCH residuals. The solid lines show the median threshold correlations across firm pairs, the grey areas mark the interquartile ranges and the dashed lines show the threshold correlations from a bivariate Gaussian distribution with correlation equal to the average for all the pairs of firms.

Figure 10: Median and Interquartile Range of Copula Correlations:
CDS Spreads, Default Intensities and Equity Prices



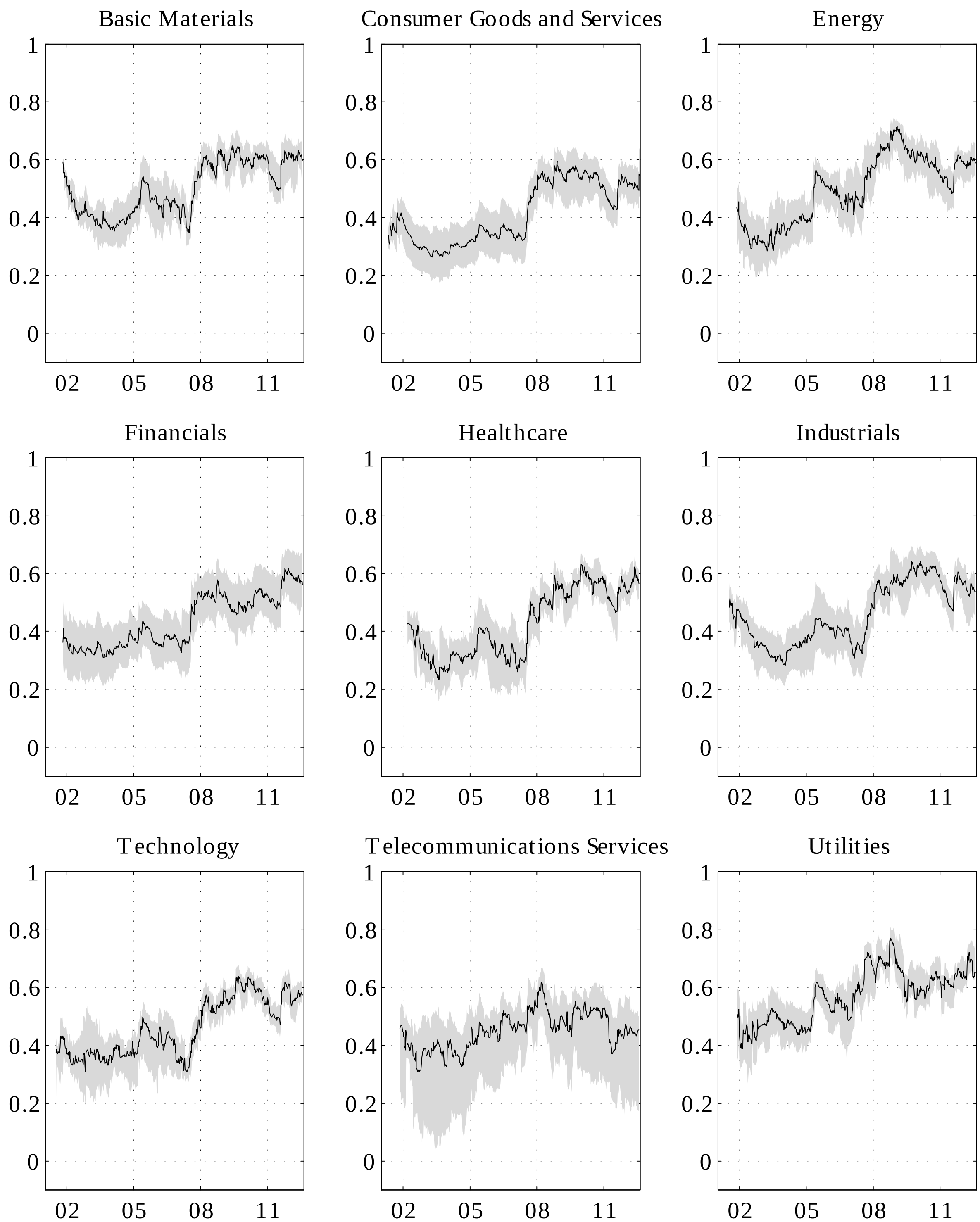
Notes to Figure: Using all pairs of firms we report the median and interquartile range of the weekly dynamic copula correlations from the DAC model.

Figure 11: Median and Interquartile Range of Copula Correlations for Nine Firms.



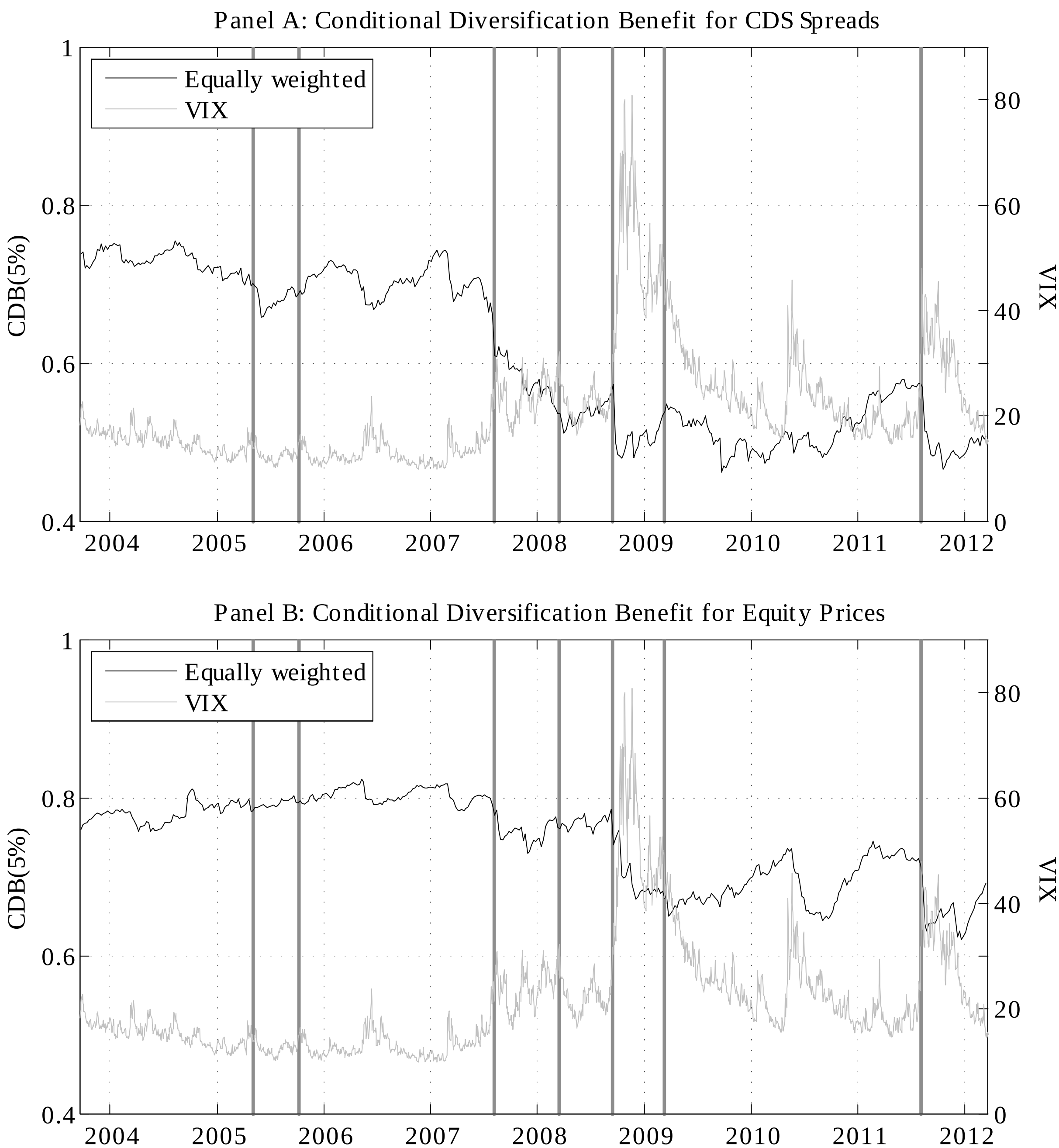
Notes to Figure: For each firm we report its median and interquartile range of correlation with the other firms in the sample.

Figure 12: Median and Interquartile Range of Copula Correlations within Nine Industries



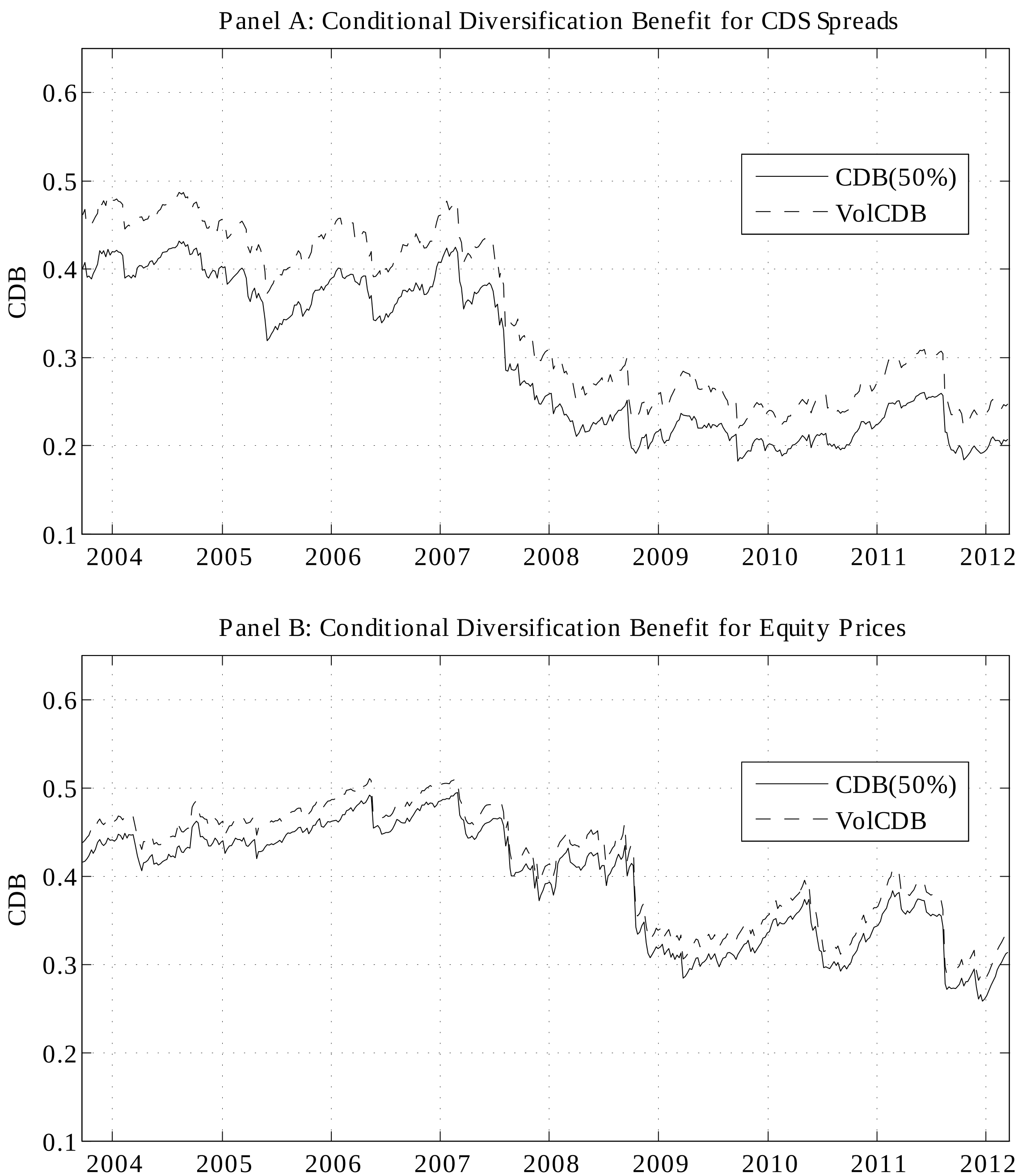
Notes to Figure: We report the median and interquartile range of the pairwise DAC copula correlation for all the firms in the industry with all other firms in the same industry.

Figure 13: Conditional Diversification Benefits.
Equal-Weighted Credit and Equity Portfolios. 5% Tail.



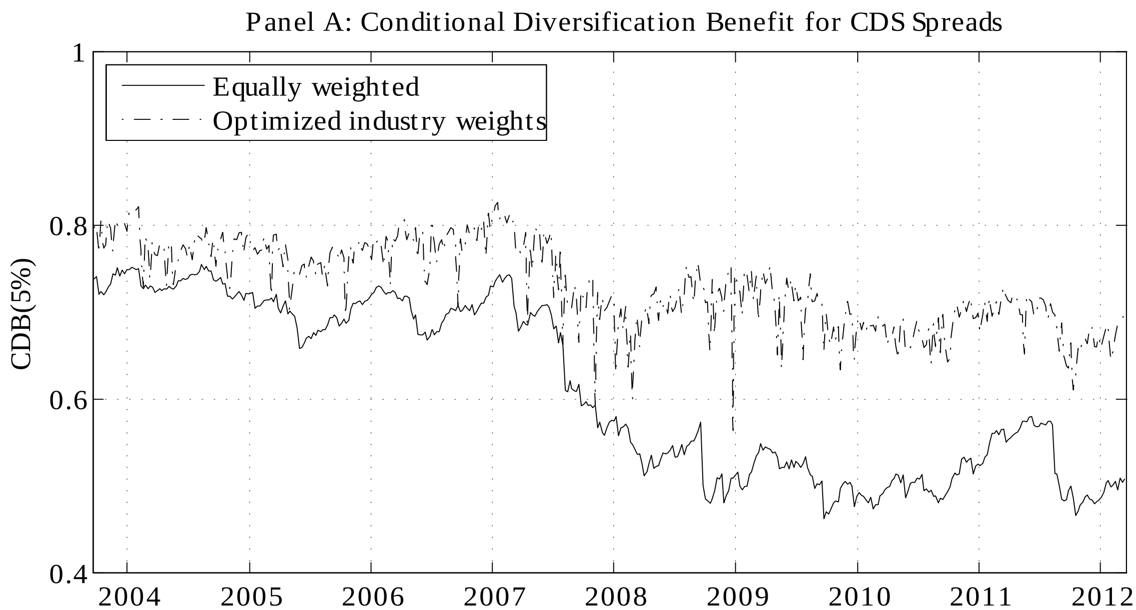
Notes to Figure: Using an equal-weighted portfolio of available on-the-run CDX firms in a given week we compute the 5% conditional diversification benefit (CDB) using our DAC model estimated first on CDS spreads and then on equity returns. The grey line shows the VIX on the right-hand axis. The credit portfolio is selling credit protection by shorting CDS contracts.

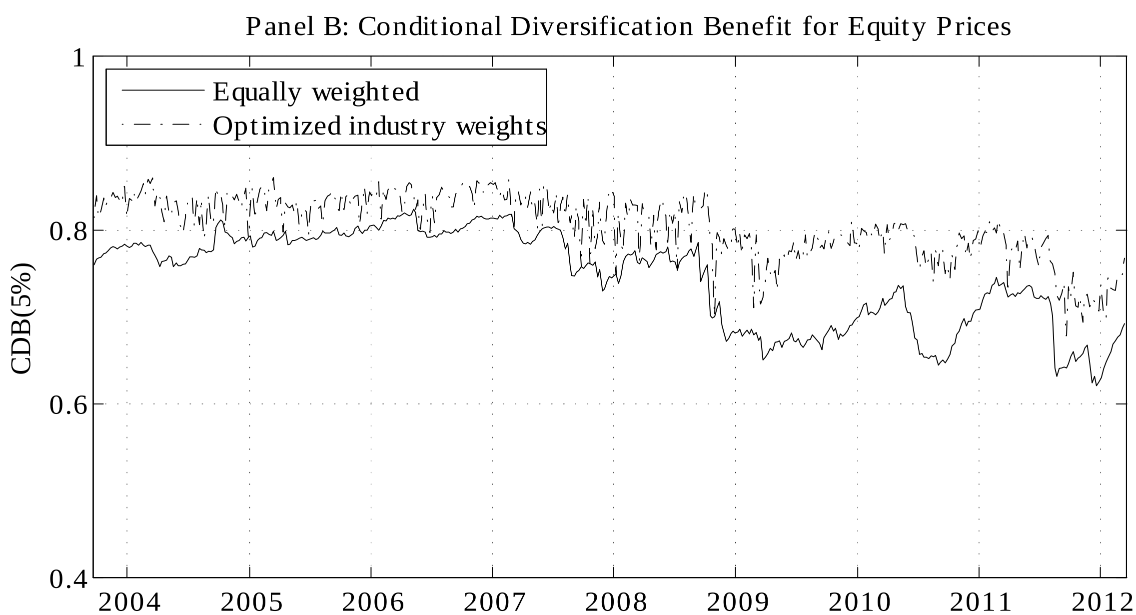
Figure 14: Conditional Diversification Benefits.
Equal-Weighted Credit and Equity Portfolios. 50% CDB and Volatility CDB Measures.



Notes to Figure: Using an equal-weighted portfolio of available on-the-run CDX firms in a given week we compute the 50% conditional diversification benefit (CDB) using our DAC model estimated first on CDS spreads and then on equity returns. We also show the volatility-based VolCDB measure which takes into account only volatilities and linear correlations. The credit portfolio is selling credit protection by shorting CDS contracts.

Figure 15: Conditional Diversification Benefits.
Optimized Industry Weights and Equal-Weighted Credit and Equity Portfolios. 5% Tail.





Notes to Figure: We show the 5% CDB measure for a optimized portfolio of the 9 industry portfolios along with an equal-weighted portfolio of individual firms. We compute the conditional diversification benefit using our DAC model estimated first on CDS spreads and then on equity returns. The grey line shows the VIX on the right-hand axis. The credit portfolio is selling credit protection by shorting CDS contracts.

Table 1: Company Names

ACE Limited	Cap One Bk USA Natl	Fortune Brands, Inc.	McDONALDS Corp	The Kroger Co.
ALLTEL Corporation	Cap One Finl Corp	Freeport McMoran	McKesson Corporation	NY Times Co
Amern Elec Pwr Co Inc	Cardinal Health Inc	GATX Corporation	MeadWestvaco Corp	The TJX Companies, Inc.
AT&T Corp.	Caterpillar Inc.	Gannett Co., Inc.	MetLife, Inc.	TOYS "R" US, INC.
AT&T Inc.	Cendant Corporation	Gen Elec Cap Corp	Mohawk Inds Inc	Target Corporation
AT&T Mobility LLC	CenturyLink, Inc.	General Mills, Inc.	Motorola Solutions Inc	Temple-Inland Inc.
Aetna Inc.	CenturyTel, Inc.	Gen Mtrs Accep Corp	Norfolk Sthn Corp	Textron Finl Corp
Albertson's, Inc.	Cingular Wireless LLC	Goodrich Corporation	Nabors Inds Inc	Allstate Corp
Rio Tinto Alcan Inc.	Cisco Systems, Inc.	H. J. Heinz Company	Natl Rural Utils Coop	Black & Decker Corp
Alcoa Inc.	Ctzns Comms Co	Halliburton Co	Newell Rubbermaid Inc	The Chubb Corporatio...
Altria Group, Inc.	Clear Channel Comms Inc	Harrahs Oper Co Inc	News America Inc	Dow Chem Co
Amerada Hess Corp	Comcast Cable Comms LLC	Hewlett Packard Co	Nordstrom, Inc.	The Home Depot, Inc.
Amern Axle & Mfg Hldgs	Comcast Corporation	Hilton Hotels Corp	Northrop Grumman Corp	May Dept Stores Co
Amern Express Co	Computer Assoc Intl Inc	Honeywell Intl Inc	Olin Corporation	Sherwin Williams Co
Amern Intl Gp Inc	Computer Sciences Corp	IAC InterActiveCorp	Omnicom Group Inc.	Walt Disney Co
Amgen Inc.	ConAgra Foods Inc	Ingersoll Rand Co	Pfizer Inc.	Time Warner Cable Inc
Anadarko Pete Corp	ConocoPhillips	Intelsat, Ltd.	Pitney Bowes Inc.	Time Warner Inc
Arrow Electrs Inc	Constellation Engy Gp Inc	Intl Business Machs	Progress Engy Inc	Toll Bros Inc
Autozone Inc	Ctrywde Home Lns Inc	Intl Lease Fin Corp	Pulte Homes, Inc.	Transocean Inc.
Avnet, Inc.	Cox Comms Inc	Intl Paper Co	Quest Diagnostics Inc	Tribune Company
Barrick Gold Corp	DIRECTV Holdings LLC	J C Penney Co Inc	R R Donnelley & Sons	Tyson Foods, Inc.
Bombardier Inc	Darden Restaurants Inc	Johnson Ctls Inc	Radian Group Inc.	Union Pacific Corpor...
Brunswick Corp	Deere & Company	Jones Apparel Gp Inc	RadioShack Corp	United Parcel Servic...
Baxter Intl Inc	Dell Inc.	KerrMcGee Corp	Raytheon Co	UnitedHealth Group I...
Beam Inc.	Delphi Corporation	Kinder Morgan Engy	Residential Cap Corp	Universal Health Ser...
BellSouth Corp	Devon Engy Corp	Knight Ridder Inc	Reynolds Amern Inc	Valero Energy Corp
Belo Corp.	Dominion Res Inc	Kohls Corp	Rohm & Haas Co	Verizon Comms Inc
Berkshire Hathaway Inc	Duke Energy Carolinas LLC	Kraft Foods Inc.	Ryder System, Inc.	Verizon Global Fdg Corp
Boeing Cap Corp	Duke Engy Corp	Lear Corporation	SBC Comms Inc	Viacom Inc.
Bombardier Cap Inc	***Duke Pwr Co LLC***	Lennar Corporation	Sears Roebuck Accep	Visteon Corporation
Boston Pptys Ltd	E I du Pont de Nemours	Liberty Media Corp	SLM Corporation	Vornado Realty L.P.
Boston Scientific Corp	EOP Oper Ltd Pship	Limited Brands, Inc.	Sprint Nextel Corp	Wal Mart Stores Inc
Bristol Myers Squibb Co	ERP Oper Ltd Pship	Liz Claiborne, Inc.	SUPERVALU INC.	Washington Mutual
Burlington Northern	Eastman Chem Co	Lockheed Martin Corp	Sabre Holdings Corpo...	Wells Fargo & Co
CA, Inc.	Eastman Kodak Co	Loews Corporation	Safeway Inc.	Wendys Intl Inc
Cdn Nat Res Ltd	Electr Data Sys Corp	Lowes Cos Inc	Sara Lee Corporation	Weyerhaeuser Co
Carnival Corp	Embarq Corporation	M.D.C. Holdings, Inc.	Sempra Energy	Whirlpool Corp
CBS Corporation	Exelon Corporation	Marriott Intl Inc	Simon Ppty Gp Inc	Wyeth
CENTEX CORP	Expedia, Inc.	Marsh & Mclennan Cos	Southwest Airls Co	XL Cap Ltd
CIGNA Corporation	Ford Mtr Cr Co	MBIA Ins Corp	Sprint Corporation	XL Group Ltd.
CIT Group Inc.	Fed Home Ln Mtg Corp	MBNA Corporation	Staples, Inc.	***XLIT Ltd.***
CSX Corporation	Fed Natl Mtg Assn	Motorola Solutions Inc	Starwood Hotels	XTO Energy Inc.
CVS Caremark Corp	Fedt Dept Stores Inc	Macy's, Inc.	Sun Microsystems Inc	Xerox Corporation
CVS Corporation	1st Data Corp	Masco Corporation	GAP INC	YUM! Brands, Inc.
Campbell Soup Co	FirstEnergy Corp.	Maytag Corporation	Hartford Financial	iStar Financial Inc.
Capital One Bank				

Notes to Figure: Using data from Markit, we consider all the 226 firms included in the first 18 series of the CDX North American investment grade index dating from March 20, 2003 to September 20, 2012. We start with 226 firms and remove the 3 firms (in bold italics) which do not have a consecutive 52-week history.

Table 2: Descriptive Statistics on CDS Spreads, Default Intensities and Equity Prices

Panel A: Sample Moments on Weekly Levels

	Average	Standard Deviation	Skewness	Kurtosis
CDS Spreads				
Median	0.01	0.01	1.62	6.44
Interquartile Range	[0.01, 0.02]	[0.00, 0.01]	[1.09, 2.35]	[3.92, 9.69]
Default Intensities				
Median	0.02	0.01	1.65	6.51
Interquartile Range	[0.01, 0.03]	[0.01, 0.02]	[1.09, 2.34]	[3.92, 9.34]
Equity Prices				
Median	32.71	9.28	0.14	2.47
Interquartile Range	[23.89, 45.37]	[6.17, 13.82]	[-0.19, 0.59]	[2.02, 3.18]

Panel B: Sample Moments on Weekly Log-Differences

	Annualized Average (%)	Annualized Standard Deviation (%)	Skewness	Kurtosis	Jarque-Bera p-value	AR(1) Coefficient	AR(2) Coefficient
CDS Spreads							
Median	3.25	66.97	0.81	10.23	0.00	0.08	0.05
Interquartile Range	[-23.13, 12.84]	[52.58, 73.39]	[-0.87, 1.44]	[5.02, 15.23]	[0.00, 0.00]	[-0.05, 0.13]	[-0.06, 0.09]
Default Intensities							
Median	3.28	67.82	0.74	10.12	0.00	0.08	0.05
Interquartile Range	[-23.85, 12.89]	[53.39, 74.49]	[-0.72, 1.37]	[5.05, 14.96]	[0.00, 0.00]	[-0.05, 0.12]	[-0.05, 0.08]
Equity Prices							
Median	3.25	34.02	-0.47	7.40	0.00	-0.04	-0.02
Interquartile Range	[-20.78, 8.08]	[21.00, 42.28]	[-2.51, -0.15]	[4.31, 10.94]	[0.00, 0.00]	[-0.15, 0.01]	[-0.10, 0.03]

Panel C: Correlations of Weekly Log-Differences

	CDS Spreads	Default Intensities	Equity Prices
CDS Spreads			
Median	0.37	0.99	-0.32
Interquartile Range	[0.12, 0.46]	[0.97, 0.99]	[-0.50, -0.25]
Default Intensities			
Median		0.39	-0.32
Interquartile Range		[0.12, 0.47]	[-0.50, -0.25]
Equity Prices			
Median			0.32
Interquartile Range			[0.15, 0.42]

Notes to Table: We report sample moments on weekly CDS spreads, default intensities and equity prices across available firms. Panel A computes the moments for spreads, intensities and prices in levels. Panel B reports sample moments computed on the weekly log-differences. Panel C reports average sample correlations across firms using weekly log-differences. On the diagonal we report the median and IQR across the correlations between each firm and all other firms. On the off-diagonal we report the median and IQR of the correlation between different series for the same firm.

Table 3: Summary of ARMA-NGARCH Estimation on Weekly Log-Differences

Panel A: Conditional Mean Dynamics

Proportion of Model Chosen by AICC Criterion		CDS Spreads	Default Intensities	Equity Prices
ARMA(0,0)		10%	12%	6%
ARMA(0,1)		10%	8%	7%
ARMA(0,2)		9%	12%	10%
ARMA(1,0)		14%	17%	5%
ARMA(1,1)		14%	11%	7%
ARMA(1,2)		4%	4%	8%
ARMA(2,0)		11%	12%	9%
ARMA(2,1)		7%	5%	11%
ARMA(2,2)		20%	19%	37%
Parameter Estimates				
μ	Median	0.0004	0.0005	0.0004
	Interquartile Range	[-0.0006, 0.0022]	[-0.0007, 0.0023]	[-0.0005, 0.0017]
AR(1)	Median	0.099	0.091	-0.018
	Interquartile Range	[-0.185, 0.594]	[-0.181, 0.560]	[-0.816, 0.472]
AR(2)	Median	-0.142	-0.337	-0.621
	Interquartile Range	[-0.772, 0.097]	[-0.843, 0.088]	[-0.833, -0.101]
MA(1)	Median	0.072	0.080	-0.032
	Interquartile Range	[-0.519, 0.353]	[-0.514, 0.300]	[-0.507, 0.776]
MA(2)	Median	0.449	0.362	0.597
	Interquartile Range	[0.100, 0.894]	[0.087, 0.917]	[0.059, 0.847]
L-B(4) p-value > 5%	Proportion	99%	99%	96%

Panel B: Conditional Volatility Dynamics and Return Distribution

Parameter Estimates		CDS Spreads	Default Intensities	Equity Prices
ω	Median	0.0003	0.0005	0.0000
	Interquartile Range	[0.0002, 0.0010]	[0.0002, 0.0010]	[0.0000, 0.0001]
β	Median	0.769	0.760	0.794
	Interquartile Range	[0.635, 0.844]	[0.641, 0.843]	[0.703, 0.855]
α	Median	0.139	0.136	0.048
	Interquartile Range	[0.094, 0.203]	[0.088, 0.198]	[0.027, 0.084]
γ	Median	-0.168	-0.211	1.313
	Interquartile Range	[-0.431, 0.028]	[-0.479, -0.028]	[0.819, 2.263]
Volatility Persistence	Median	0.954	0.941	0.978
	Interquartile Range	[0.894, 0.980]	[0.894, 0.977]	[0.961, 0.990]
ν	Median	3.604	3.653	6.549
	Interquartile Range	[3.166, 4.159]	[3.232, 4.305]	[5.117, 8.425]
λ	Median	0.076	0.067	-0.109
	Interquartile Range	[0.042, 0.121]	[0.025, 0.107]	[-0.159, -0.061]
L-B(4) p-value z^2 > 5%	Proportion	98%	97%	92%

Notes to Table: For each firm we estimate an ARMA(p,q)-NGARCH(1,1) model where the p and q are chosen by the AICC criterion. The residual distribution is asymmetric t with parameters ν and λ. L-B(4) denotes a Ljung-Box test that the residuals (Panel A) or squared residuals (Panel B) are serially uncorrelated.

Table 4: Descriptive Statistics of Model Residuals

Panel A: Sample Moments

		Skewness	Kurtosis
CDS Spreads	Median	0.88	9.84
	Interquartile Range	[0.40, 1.67]	[6.60, 20.37]
Default Intensities	Median	0.84	9.18
	Interquartile Range	[0.37, 1.49]	[6.30, 18.37]
Equity Prices	Median	-0.45	5.23
	Interquartile Range	[-0.74, -0.22]	[4.25, 7.15]

Panel B: Correlation Between CDS, Default Intensity and Equity

		CDS Spreads	Default Intensities	Equity Prices
CDS Spreads	Median	0.33	0.98	-0.30
	Interquartile Range	[0.23, 0.42]	[0.97, 0.98]	[-0.37, -0.20]
Default Intensities	Median		0.34	-0.29
	Interquartile Range		[0.25, 0.43]	[-0.36, -0.20]
Equity Prices	Median			0.29
	Interquartile Range			[0.23, 0.37]

Notes to Table: Using the ARMA-NGARCH residuals, z, we compute the skewness, kurtosis and cross-correlation and report the median and interquartile range across firms in Panel A. We also compute for each firm the correlation between CDS spreads, default intensities and equity residuals and report the median and interquartile range in Panel B. On the diagonal we report the median and IQR across the correlations between each firm and all other firms. On the off-diagonal we report the median and IQR of the correlation between different series for the same firm.

Table 5: Dynamic Copula Parameter Estimation

Panel A: Dynamic Asymmetric Copula Estimation

	CDS Spreads	Default Intensities	Equity Prices
β_C	0.958	0.956	0.930
α_C	0.021	0.024	0.019
Correlation Persistence	0.980	0.980	0.949
ν_C	13.572	13.548	13.354
λ_C	0.048	0.045	- 0.380
Composite Log-likelihood	1,074,513	1,167,893	617,594

Panel B: Dynamic Symmetric Copula Estimation

	CDS Spreads	Default Intensities	Equity Prices
β_C	0.960	0.960	0.930
α_C	0.021	0.022	0.019
Correlation Persistence	0.982	0.983	0.949
ν_C	13.590	12.888	13.217
Composite Log-likelihood	1,073,786	1,167,612	604,583

Panel C: Dynamic Normal Copula Estimation

	CDS Spreads	Default Intensities	Equity Prices
β_C	0.960	0.960	0.924
α_C	0.020	0.021	0.018
Correlation Persistence	0.981	0.981	0.942
Composite Log-likelihood	1,025,863	1,115,747	573,347

Notes to Table: We estimate the dynamic asymmetric copula (DAC), the dynamic symmetric copula (DSC), and the dynamic normal copula (DNC) models on the 223 firms in our sample. Each of the three models is estimated on each of our three data sets containing ARMA-NGARCH residuals from weekly log-differences on CDS spreads, default intensities and equity prices.