Dynamic Dependence in Corporate Credit
Peter Christoffersen, Kris Jacobs, Xisong Jin and Hughes Langlois

Discussion by Andrew Patton

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Motivation

- The CDS market has grown enormously over the last decade
  - $0.6$ trillion of gross notional principal in 2001
  - $25.9$ trillion at the end of 2011 (ISDA)

- The authors motivate their analysis of the dependence between CDS spreads via potential applications to:
  1. Managing portfolio credit risk
  2. Pricing structured credit products
  3. Including credit risk products in an investment portfolio
The authors use weekly CDS spread data on 223 North American firms, March 2003–Sep 2012.

- These are all firms that appeared in the first 18 CDX IG series
- Their estimation method allows them to easily handle missing and non-overlapping data

The authors propose a DCC correlation model skew t copula with dynamics governed by a DCC-type structure. This enables them to exploit two neat results from the DCC literature:

1. Correlation targeting: the $N \times N$ “intercept” matrix can be estimated analytically rather than numerically
2. The model can be estimated using “composite maximum likelihood”

These make estimation of a 223-dimensional dynamic dynamic copula feasible.
Papers using time-varying and high ($N \geq 10$) dimension copulas:

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Main empirical findings of this paper

1. The conditional dependence (copula) between CDS spreads varies substantially through time
   - In addition to week-to-week variation, the authors find that overall dependence rose after the financial crisis
   - This reduces potential diversification benefits across these contracts

2. The conditional copula of CDS spreads is non-Normal
   - Strong evidence of non-zero tail dependence $\Rightarrow$ not Normal copula
   - Milder evidence of asymmetric dependence, and less evidence for CDS spreads than for equities
Outline of this discussion

1 Motivation and main contributions of paper

2 Suggestions for the authors
   1 The CDS Big Bang
   2 Cross-contract spill-overs
   3 CDS spreads vs. implied probabilities
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1 Motivation and main contributions of paper

2 Suggestions for the authors
   1 The CDS Big Bang
   2 Cross-contract spill-overs
   3 CDS spreads vs. implied probabilities
With the growth of the CDS market through the 2000s, participants wanted more homogeneous contracts to increase liquidity.

On April 8, 2009, the North American CDS market underwent changes to contract conventions:

- CDS coupons were fixed to be 100 or 500 bp, with upfront payments adjusted accordingly.
- More rigid rules on triggers for “credit events” and auctions that follow such events.
- Move towards central clearing, away from OTC trading.

These changes could potentially change the dynamics of CDS spreads.
In my paper using similar data (100 firms, *daily* spreads over 2006-2012) we found breaks in the conditional mean and variance for 39 and 66 firms respectively.

I tested for a structural break in an AR(1)-GARCH(1,1) model for these 100 firms using *weekly* data, to match this paper and found:

- Breaks in the conditional mean for 11 firms
- Breaks in the conditional variance for 55 firms

The authors have a larger cross-section and use slightly different mean and variance models, but this is suggestive that allowing for a break may be important.
An important aspect of applying Sklar’s theorem to conditional distributions is making sure the information sets used for the margins and conditional copula are the same:

\[ F(y_1, \ldots, y_N | \mathcal{F}) = C(F_1(y_1 | \mathcal{F}), \ldots, F_N(y_N | \mathcal{F}) | \mathcal{F}) \]

If we use \( \mathcal{F}_1 \) for the 1\(^{st}\) margin, \( \mathcal{F}_N \) for the \( N^{th}\) margin, and \( \mathcal{F}_c \) for the copula, then the resulting function \( F(\cdot | \cdot) \) will \textit{not}, in general, be a conditional joint distribution.

The authors use ARMA(2,2) - NGARCH models for each marginal distribution, which is potentially a problem. But:
If we define $\mathcal{F} = \sigma(\mathcal{F}_1, ..., \mathcal{F}_N, \mathcal{F}_c)$, and it is true that

$$Y_1|\mathcal{F}_1 \overset{d}{=} Y_1|\mathcal{F}$$
$$\vdots$$
$$Y_N|\mathcal{F}_N \overset{d}{=} Y_N|\mathcal{F}$$

$$(U_1, ..., U_N)|\mathcal{F}_c \overset{d}{=} (U_1, ..., U_N)|\mathcal{F}$$

then we can effectively use the information sets $(\mathcal{F}_1, ..., \mathcal{F}_N, \mathcal{F}_c)$ but interpret the results as having used $\mathcal{F}$ throughout, satisfying the condition on the information set.

Practically, this means we should test whether including information from “other” variables is needed. If not, we may exclude it.
Cross-contract spill-overs III

- Of course, using a 233-dimensional VAR is not feasible (or sensible).

- One practical alternative is to include a lagged “market” return to capture some cross-asset spillover.

- In Oh and Patton’s study of daily CDS spreads we found:
  - 98 / 100 firms had significant spillover in the mean
  - 13 / 100 firms had significant spillover in the variance

- I replicated this on weekly data (to match the authors) using an AR(1)-GARCH(1,1) and found:
  - 6 / 100 firms had significant spillover in the mean
  - 7 / 100 firms had significant spillover in the variance

⭐ So I think the authors are OK here
The authors do a very nice job contrasting the properties of the dependence structure of their 233 CDS spreads log-differences with that of the equity returns of the corresponding firms.

- eg: less asymmetric dependence in CDS spreads than equity returns, but a greater change in the level of dependence around the crisis.

The authors also contrast the dependence structure of CDS spreads log-differences with those of default intensities, i.e., CDS-implied probabilities of default.

- They find very similar results. This is not so surprising:
Under some simplifying assumptions (see Carr and Wu, RFS, for eg) it is possible to show that a CDS spread \( (S_t) \) is given by:

\[
S_t = P_t \times LGD_t
\]

where \( P_t \) is the implied probability of default and \( LGD_t \) is the loss-given-default.

This simple expression can also be obtained as a first-order approximation of more complicated formulas when \( P_t \approx 0 \).

The LGD is often taken as fixed (at 0.60) and so:

\[
\Delta \log S_t = \Delta \log P_t
\]

Thus these two series will be identical.
What if we use a more serious pricing formula?

Conrad, Dittmar and Hameed (2011, wp):

\[
S_t = \frac{\sum_{j=1}^{20} d_{t+j|t} (1 - P_t)^j P_t LGD_t}{\frac{1}{4} \sum_{j=1}^{20} d_{t+j|t} (1 - P_t) + \frac{1}{8} \sum_{j=1}^{20} d_{t+j|t} (1 - P_t)^j P_t}
\]

It is not clear what the relation between \( \Delta \log S_t \) and \( \Delta \log P_t \) will be for this formula. Here are three scatter plots:
\[ \Delta \log S_t \approx \Delta \log P_t \Rightarrow \text{study either } \Delta \log S_t \text{ or } \Delta \log P_t? \]
A very interesting paper:

1. A large and relatively novel data set
2. A new dynamic copula model, the largest to date
3. Interesting contrasts between the dependence between CDS spreads and dependence between corresponding equity returns