Dynamic Dependence in Corporate Credit

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Discussion by Andrew Patton

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Motivation

- The CDS market has grown enormously over the last decade
  - \$0.6 trillion of gross notional principal in 2001
  - \$25.9 trillion at the end of 2011 (ISDA)
- The authors motivate their analysis of the dependence between CDS spreads via potential applications to:
  1. Managing portfolio credit risk
  2. Pricing structured credit products
  3. Including credit risk products in an investment portfolio

Overview of the paper

- The authors use weekly CDS spread data on 223 North American firms, March 2003–Sep 2012.
  - These are all firms that appeared in the first 18 CDX IG series
  - Their estimation method allows them to easily handle missing and non-overlapping data
- The authors propose a ~~DCC correlation model~~ skew *t* copula with dynamics governed by a DCC-type structure. This enables them to exploit two neat results from the DCC literature:
  1. Correlation targeting: the $N \times N$ "intercept" matrix can be estimated analytically rather than numerically
  2. The model can be estimated using "composite maximum likelihood"
  - These make estimation of a 223-dimensional dynamic copula feasible.

From the lit review section of my own paper

- Papers using time-varying and high (N$\geq$10) dimension copulas:

Authors	N	Copula	Dynamics	Estim
Zhang, *et al.* (2011, wp)	10	Skew t	GAS	ML
Christoffersen, *et al.* (2012, RFS)	33	Skew t	DCC	ML
Almeida, *et al.* (2012, wp)	30	Vine	SV	SML
Stöber and Czado (2012, wp)	10	Vine	RS	Bayes
Oh and Patton (2013, wp)	**100**	**Factor**	**GAS**	**ML**

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Oh and Patton (2013, wp)	100	Factor	GAS	ML
Christoffersen, et al. (2013, wp)	**233**	**Skew t**	**DCC**	**CML**

Main empirical findings of this paper

1. The conditional dependence (copula) between CDS spreads varies substantially through time
   - In addition to week-to-week variation, the authors find that overall dependence rose after the financial crisis
   - This reduces potential diversification benefits across these contracts
2. The conditional copula of CDS spreads is non-Normal
   - Strong evidence of non-zero tail dependence $\Rightarrow$ not Normal copula
   - Milder evidence of asymmetric dependence, and less evidence for CDS spreads than for equities

Outline of this discussion

1. **Motivation and main contributions of paper**

2. Suggestions for the authors

    1. The CDS Big Bang
    2. Cross-contract spill-overs
    3. CDS spreads vs. implied probabilities

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2. **Suggestions for the authors**
   1. The CDS Big Bang
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   3. CDS spreads vs. implied probabilities

The CDS "Big Bang" I

- With the growth of the CDS market through the 2000s, participants wanted more homogeneous contracts to increase liquidity
- On April 8, 2009, the North American CDS market underwent changes to contract conventions
  - CDS coupons were fixed to be 100 or 500 bp, with upfront payments adjusted accordingly
  - More rigid rules on triggers for "credit events" and auctions that follow such events
  - Move towards central clearing, away from OTC trading
- These changes could potentially change the dynamics of CDS spreads

The CDS "Big Bang" II

- In my paper using similar data (100 firms, **daily** speads over 2006-2012) we found breaks in the conditional mean and variance for 39 and 66 firms respectively
- I tested for a structural break in an AR(1)-GARCH(1,1) model for these 100 firms using **weekly** data, to match this paper and found:
  - Breaks in the conditional mean for 11 firms
  - Breaks in the conditional variance for 55 firms

★ The authors have a larger cross-section and use slightly different mean and variance models, but this is suggestive that allowing for a break may be important

Cross-contract spill-overs I

- An important aspect of applying Sklar's theorem to conditional distributions is making sure the information sets used for the margins and conditional copula are the same:

$$\mathbf{F}(y_1, ..., y_N | \mathcal{F}) = \mathbf{C}(F_1(y_1|\mathcal{F}), ..., F_N(y_N|\mathcal{F})|\mathcal{F})$$

- If we use $\mathcal{F}_1$ for the 1^{st} margin, $\mathcal{F}_N$ for the N^{th} margin, and $\mathcal{F}_c$ for the copula, then the resulting function $\mathbf{F}(\cdot|\cdot)$ will *not*, in general, be a conditional joint distribution.
  - The authors use ARMA(2,2) - NGARCH models for each marginal distribution, which is potentially a problem. But:

Cross-contract spill-overs II

- If we define $\mathcal{F} = \sigma(\mathcal{F}_1, ..., \mathcal{F}_N, \mathcal{F}_c)$, **and** it is true that

$$Y_1|\mathcal{F}_1 \overset{d}{=} Y_1|\mathcal{F}$$
$$\vdots$$
$$Y_N|\mathcal{F}_N \overset{d}{=} Y_N|\mathcal{F}$$
$$(U_1, ..., U_N)\,|\mathcal{F}_c \overset{d}{=} (U_1, ..., U_N)\,|\mathcal{F}$$

  then we can effectively use the information sets $(\mathcal{F}_1, ..., \mathcal{F}_N, \mathcal{F}_c)$ but interpret the results as having used $\mathcal{F}$ throughout, satisfying the condition on the information set.

  - Practically, this means we should *test* whether including information from "other" variables is needed. If not, we may exclude it.

Cross-contract spill-overs III

- Of course, using a 233-dimensional VAR is not feasible (or sensible).
- One practical alternative is to include a lagged "market" return to capture some cross-asset spillover.
- In Oh and Patton's study of **daily** CDS spreads we found:
  - 98 / 100 firms had significant spill-over in the mean
  - 13 / 100 firms had significant spill-over in the variance
- I replicated this on **weekly** data (to match the authors) using an AR(1)-GARCH(1,1) and found:
  - 6 / 100 firms had significant spill-over in the mean
  - 7 / 100 firms had significant spill-over in the variance

★ So I think the authors are OK here

CDS spreads vs. implied default probabilities I

- The authors do a very nice job contrasting the properties of the dependence structure of their 233 CDS spreads log-differences with that of the equity returns of the corresponding firms
  - eg: less asymmetric dependence in CDS spreads than equity returns, but a greater change in the level of dependence around the crisis
- The authors also contrast the dependence structure of CDS spreads log-differences with those of *default intensities*, i.e., CDS-implied probabilities of default.
  - They find very similar results. This is not so surprising:

CDS spreads vs. implied default probabilities II

- Under some simplifying assumptions (see Carr and Wu, RFS, for eg) it is possible to show that a CDS spread (S_t) is given by:

$$S_t = P_t \times LGD_t$$

where P_t is the implied probability of default and LGD_t is the loss-given-default.

  - This simple expression can also be obtained as a first-order approximation of more complicated formulas when $P_t \approx 0$.

- The LGD is often taken as fixed (at 0.60) and so:

$$\Delta \log S_t = \Delta \log P_t$$

- Thus these two series will be identical.

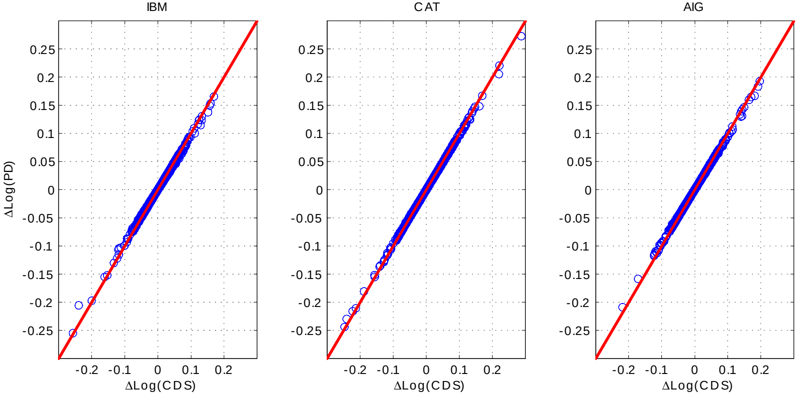
CDS spreads vs. implied default probabilities III

- What if we use a more serious pricing formula?
  - Conrad, Dittmar and Hameed (2011, wp):

$$S_t = \frac{\sum_{j=1}^{20} d_{t+j|t} \left(1 - P_t\right)^j P_t LGD_t}{\frac{1}{4}\sum_{j=1}^{20} d_{t+j|t} \left(1 - P_t\right) + \frac{1}{8}\sum_{j=1}^{20} d_{t+j|t} \left(1 - P_t\right)^j P_t}$$

- It is not clear what the relation between $\Delta \log S_t$ and $\Delta \log P_t$ will be for this formula. Here are three scatter plots:

CDS spreads vs. implied default probabilities IV



★ $\Delta \log S_t \approx \Delta \log P_t \Rightarrow$ study either $\Delta \log S_t$ *or* $\Delta \log P_t$?

Summary

- A very interesting paper:
  1. A large and relatively novel data set
  2. A new dynamic copula model, the largest to date
  3. Interesting contrasts between the dependence between CDS spreads and dependence between corresponding equity returns