Buyout Funds Started in Recessions: How Different Are They?

by

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Abstract

This paper examines US buyout funds with vintage years in recession periods to find the differences and similarities between these funds and funds started in periods of expansion. CFNAI is used as an indicator of economic cycles. Fund data comes primarily from VentureXpert (also known as Venture Economics), and spans over three decades. I examine funds between 1987 and 2009, and am specifically interested in buyout funds started in 1990-1991, 2001-2002, and 2008-2009, which are the three most recent recession periods of the US economy. I examine each stage of a fund's life, and found better performance, less dilution of returns, quicker draw-downs and less stock distribution in the group of recession funds. This comparative study can be useful for private equity investors when they make investment decisions during economic downturns.

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1. Introduction

Private equity investments have increased considerably in the past three decades and have developed into one of the most important types of alternative investment. Nowadays, private equity funds around the world have over two trillion dollars of holdings on their books and another trillion dollars of committed capital waiting to be invested (Bain report 2013). In 2012, the industry worldwide raised a total of $315 billion. North America alone raised $99 billion, in which over half of the amount was raised by buyout funds (Bloomberg Brief 2013).

Almost all private equity funds are partnerships, in which limited partners (LPs), consisting of institutional investors and high net-worth individuals, provide most of the capital, and general partners (GPs) manage the funds and make investments. Most of these partnerships last around ten years. The first five years are usually the investment period, and historically the average holding period of investments is around six years (Kaplan, Stromberg 2008).

The US buyout industry, which is a major component of the private equity industry, has been travelling on a bumpy road since the beginning of 1980s. The industry as a whole has been through three cycles so far, roughly one cycle every ten years. The 1980s witnessed the rise of leveraged buyouts (LBOs) and the flourishing of junk bonds. A well-known example of LBOs in this period is the massive buyout of RJR Nabisco. At that time, the general public became aware of the power of private equity firms in affecting large public companies, and started calling them "corporate raiders". This boom ended in late 1980s when some of the largest buyout deals, such as the $11 billion deal of Federated Department Stores, ended up in bankruptcy. The late 1980s and early 1990s witnessed the collapse of Drexel Burnham Lambert, which directly resulted in the lack of available financing for the LBO market. This time period also witnessed the real

estate market collapse as well as the savings and loan crisis, which contributed to the 1990 – 1991 economic recession.

Private equity entered into another cycle after 1992, and started putting emphasis on the long-term development of portfolio companies. Some of the most notable transactions include Domino's Pizza and Sealy Corporation. This boom, however, ended with the internet bubble in late 1990s and the early 2000s recession.

The most recent boom turned out to be the largest one, which started in 2003 and ended in 2008. The reduction of cost of borrowing and the loosening lending standards made financing available for private equity firms. Facing low interest rate and seeking for yields, investors again turned to the high-yield debt, which revitalized junk bond market as well as large acquisition activities. This time period witnessed some of the largest leveraged buyouts ever done, and some of the most notable transactions include Toys "R" Us ($6.6 billion in 2005), and Chrysler ($7.4 billion in 2007). This cycle ended with the credit crunch in 2007 and the recession period of 2008 -2009.

While the cyclicality of the buyout industry is not exactly the same thing as the cyclicality of the macro-economy and they could be caused by different reasons, the two cycles do coincide. After all, an optimistic economic outlook is the prerequisite of the booming of private equity industry, because the profitability of this type of investment depends largely on the market condition five to seven years later when the investment is exited. When the economy is growing, optimistic investors are willing to invest, resulting in the rapid growth of the industry. On the other hand, allocation of capital during bad times is more difficult, and investors are reluctant to invest because the market is filled with uncertainty. Therefore it is worth understanding how differently

funds start in recession periods tend to perform and behave compared to funds start in other times.

The paper is structured as follows: Section 2 discusses the data that is used in this paper. Section 3 looks at returns, both the overall vintage year performance of the industry and the difference in performance between big and small funds. Theory is explained in 3.1, and regression results are shown in 3.2. Section 4 examines the capital deployment rate of funds, and Section 5 looks at stock distribution behaviors. Section 6 discusses the change in fund size distribution, which shows how the industry landscape changes with the macro-economy. Section 7 concludes with a summary of these results.

2. Data

VentureXpert

Fund data mainly comes from VentureXpert, a database provided by Thomson Economics. The database provides information on private equity firms, funds, and portfolio companies from the 1970s to the present day, which covers all three business cycles we are interested in. In order to protect funds' privacy, individual fund performance is not available. Aggregate information, such as aggregate cash flows, allows us to understand how the entire industry performs in a certain period of time. Information on portfolio companies and exiting strategies is not uniform and relatively limited, especially in the early days, therefore not quite suitable for analysis.

Harris, Jenkinson, Kaplan (2013), as well as some earlier studies such as Stucke (2011), find that buyout performance is lower in VentureXpert compared with other commercial databases, such

as Preqin, Pitchbook and Burgiss. This downward bias should be less important in this paper, since we mainly look at the cyclicality of returns instead of the absolute level of returns.

CFNAI

Because the purpose of this paper is to compare recession funds and non-recession funds, the intuitive thing to do in the regression models is to assign a dummy variable to funds, i.e. funds in recessions =1 and others = 0. This dummy variable, however, ignores the complexity of the private equity industry. For instance, a fund that was started in 2000 is still well positioned to invest in distressed companies in the period of 2001- 2002, and a fund that was started in 2003 can still find sectors which recover slower than the overall economy. Yet all these funds are categorized as non-recession funds if we simply assign a dummy variable. Because of this oversimplification, I use the Chicago Fed National Activity Index (CFNAI), which is a continuous variable instead of a dummy variable, as an indicator of the economic environment.

CFNAI is a monthly economic report that is a weighted average of 85 indicators of national economic activity, which cover four areas: production and income; employment; consumer spending and housing; sales, orders, and inventories. The index has an average value of 0 and a standard deviation of 1. A positive CFNAI indicates economic growth above trend and a negative value indicates growth below trend. Historically, CFNAI is a coincident indicator of economic expansions and contractions. In particular, the US economy is likely to enter a recession when the three month moving average of the index (CFNAI-MA3) falls below -0.7, and it is likely to exit a recession when CFNAI-MA3 rises above +0.2.

I define two variables using CFNAI. Because CFNAI is a monthly index, and the analysis in this paper mainly examine year-by-year change, I calculate the average of 12 CFNAI values of that

year, and call it CFNAI Year Average. On the other hand, since the analysis emphasizes on recessions, I define another variable called CFNAI Recession Spread. To do so, I set values for recession years to zero, and calculate the spread between the CFNAI Year Average of a particular non-recession year and the CFNAI of the previous recession period, which is an average of two years if the recession period is two years. CFNAI Recession Spread can be expressed as the following:

in which CFNAI(R) is the average of CFNAI values of the nearest recession period prior to Y.

This variable normalizes recession periods in the sense that it brings all recession periods to the same level of CFNAI. Both variables (shown in Table 1 and Figure 1) will be used separately in each regression models, and repeatedly in the next few sections.

3. Returns

3.1 Hypothesis

Numerous studies based on both commercial and private databases have suggested that private equity funds start in tough macroeconomic environments tend to perform better. This cyclicality could be due to a few reasons. First of all, to buy low and sell high is the essence of the buyout business model, and companies are, in general, cheaper in recession periods. When the economy is in trouble, stock prices fall and it costs less to take companies private than it would when economy is booming. Second, the buyout industry is particularly sensitive to herding effects. Unlike the public market, it appears to be more difficult to make a profit when there are many

players in the private market. This could be because having more bidders chasing the same deal pushes up the price of a target company and also reduces the availability of high quality deals. During a recession, there are fewer funds in the industry due to the overall lack of capital, and the herding effect is weakened. Kaplan and Schoar (2003) support this argument and found that the number of new entrants is negatively and significantly related to the returns of buyout funds.

Assessing the returns of private equity funds is always a challenge. Simply accepting the reported IRR at face value could cause tremendous bias in the analysis of the industry. Gompers and Lerner (1997) discuss several reasons why performance assessment of private equity funds is particularly difficult. For instance, when an investment is still private, some funds have the incentive to aggressively overstate their value in order to raise the next fund. Furthermore, funds sometimes ignore bad news coming from the portfolio companies, or the illiquidity of the shares when it comes to the exiting stage, thereby overestimating the performance of the funds. Even after the funds are completely liquidated, what the investors actually receive (net IRR) can still differ greatly across funds because of different fee structures and the existence of stock distributions (more on stock distribution in Section 5).

Moreover, most past studies used capital weighted IRR as an indicator of the industry performance, which means the IRR of the entire collection of funds is the average IRR weighted by asset under management. This means the IRR of a fund with $5 billion under management is weighted about five times as that of a fund with $1 billion. However, the resulting IRR calculated this way is by no means the real IRR, because the IRR of each fund involve complicated computation, and simply weighted-averaging them does not make mathematical sense. Appendix A creates two fictitious funds to illustrate this point.

Cash flow information provides a less biased way to assess performance. In this section, I propose using cash flow data to test the cyclicality of buyout returns. On the one hand, IRR calculated using aggregate cash flows reflects the real return on money put into the private equity industry in a particular year. On the other hand, cash flow based IRR is net IRR instead of gross IRR, because fees are already deducted from every cash or stock distribution, which gives a better picture of what investors actually receive. Besides, cash flow data automatically takes care of the "weight" problem, because bigger transactions involve bigger cash flows, and have bigger impact on the resulting IRR. This, however, does not mean returns calculated this way is completely unbiased. Stock distributions are also included in the cash flow data with the assumption that investors would sell the stocks as soon as they receive them. This largely ignores the liquidity issue. Besides, returns of interim funds remain a problem, because lots of funds after 2003 are still holding some of their investments now. These investments appear as the last entry of Net Asset Value (NAV) in the cash flow data, and the valuation of these investments is potentially biased (see sample cash flow data in Appendix B).

To illustrate the cyclicality of returns, I extract cash flow data from VentureXpert to calculate vintage year performance (see Table 2). Vintage year performance means the return of a fund at the end of its life (or return calculated at present day if it is an interim fund).This database does not show cash flow data of individual funds, so aggregated cash flows of all funds with the same vintage year are used. The vintage year performances are measured in both IRR and multiple of capital (a.k.a. MOC or cash on cash). Figure 2 Panel A plots both performance measures against each vintage year. For example, the 10% IRR and 1.5x multiples of capital of 2005 means that putting all funds started in 2005 together, the combined fund has an IRR of 10% and cash on cash of 1.5x. Panel B uses PitchBook data. Instead of cash flow data, here I use performance

reported by funds and use the median as an indicator of the vintage year performance. Both databases generate a similar pattern which shows the ups and downs of private equity returns. For example, it is clear from the graphs that funds started in 2000 and 2001 performed much better than funds started in 1996 and 1997. I will test the following hypothesis in the next section to see if it is statistically significant.

Hypothesis 1: *Returns of buyout funds started in recession periods are higher than returns of funds started in expansion periods.*

Kaplan and Schoar (2003) also examines whether old funds are affected by the new entrants. While experienced venture capitalists still perform well when a large number of new entrants enter the market, the buyout industry seems to be a different case. During the booms, lots of new buyout funds create noise by crowding into the market. Overall returns in the buyout industry are significantly diluted, and both old funds and young funds are affected. I speculate that in these years, performance tends to be homogenous across funds because the noise brought by the entrants drags everyone down to a similar level. In the recession periods, I am interested in whether this noise is reduced. If it is, performance of recession funds should be less homogenous. Based on this argument, I therefore state:

Hypothesis 2: *Returns of buyout funds started in recession periods are less homogeneous than returns of funds started in expansion periods.*

Due to the fact that cash flows of individual funds cannot be separated from the aggregate cash flows, test of homogeneity has to be done indirectly. First, for each vintage year, I find the median fund size of all funds, and then divide them into two groups, one with AUMs higher than the median, and one with AUMs lower than the median. The new entrants are unlikely to raise

higher-than-median capital no matter what year they are in. For the sake of simplicity, we call these two groups of funds as “bigger funds” and “smaller funds”. Next I calculate the IRR and multiples based on aggregate cash flows of these two groups separately. Based on Kaplan and Schoar’s result and my speculation, the difference in performance is smaller when there is significant dilution. In the recessions, however, noise is almost eliminated, therefore the difference in returns between them should be bigger. If this turns out to be true, it provides evidence supporting the hypothesis.

3.2 Results

For Hypothesis 1, I regress vintage year IRR from 1987 to 2009 against the CFNAI recession spread. The result is significant with a T-statistic of -2.32 and p-value of 0.031 (see Appendix C). The coefficient of the variable is -0.0602, which means IRR decreases when CFNAI recession spread widens. Because we set CFNAI of recession years to 0, and the spreads of all other years over the previous recession year are positive, this result shows that the vintage year returns would be lower if the economy that year is better compared to the earlier recession. Therefore the data strongly support the first hypothesis.

If instead of regressing IRR, I regress MOC against CFNAI recession spread, the regression result is not significant. This is expected because the cyclicality of the industry does not affect cash on cash so much as it does IRR. An investment could have a low IRR but a decent MOC since the manager can determine what time the fund should exit the investment. If the economic environment is depressing at the time, the manager can wait till later to sell the company. This is why lots of businesses that were bought in a private equity boom are held for a long time and sold in the next boom, with a recession between the two booms. This causes some funds to have

high cash on cash but low IRR because IRR calculation takes into account the time frame. These funds usually started in the expansion periods, which is why the IRR of the industry displays cyclicality but cash on cash does not.

For Hypothesis 2, IRR is calculated respectively for big and small funds. The difference in performance is simply the absolute value of the difference between two IRRs of the same vintage year (see Table 3). Similar to the test of the first hypothesis, I regress this difference in IRR against the CFNAI recession spread. The result is statistically significant (see Appendix D). In this case, the T statistic is -2.71 and the p-value is 0.013, and the coefficient of variable is -0.032, which shows that the difference increases as CFNAI recession spread decreases, that is, performance of recession funds is less homogenous (see Figure 3 for scatterplots). This supports the hypothesis and suggests less noise in recession periods.

4. Capital Deployment

4.1 Hypothesis

When a private equity calls capital, it asks the limited partners to transfer committed capital to the fund so that it can be put to use. Usually a fund does not rely on a timetable to draw down capital, but only do so when making an investment, and requires the limited partners to provide cash on short notice. To the general partners, cash would lower the fund IRR if it cannot be deployed quickly, which is why they prefer the LPs to hold the cash before a new investment is made. Jain (2012) discusses the complications raised from this uncertainty of capital draw-downs. To the limited partners, the capital is committed at the inception of the fund. This amount of money could be in cash generating no returns so that it is surely available when needed. It can

also be invested in certain assets, which creates the possibility that the investment is not liquid enough at the time of the draw-down. Even if it is liquid, the market then could be depressed, causing big losses that cannot be remedied by the private equity investment. As a result, investors would avoid this risk and would most likely invest in low-volatility and low-return assets, which is why they prefer the capital to be called quickly and start generating higher returns early instead of staying in low return assets. How fast private equity funds deploy its capital, therefore, becomes a matter of concern.

The tough economic environment that recession funds would face can have both positive and negative effects on capital deployment rate, but overall it should be positive. After all, the value of something is higher when it is scarce, and the value of cash is high in recession periods. Private equity funds that have cash in a tough economy would have the opportunity to invest in distressed companies which are desperate for cash without many competitors bidding for the same deal. This would contribute to a shorter negotiation period, allowing funds to deploy their capital faster. Based on this argument, I state:

Hypothesis 3: *Recession funds have higher capital deployment rate.*

4.2 Results

There is no standardized way to quantify how fast a fund deploys its capital. In this section, I use the linear regression coefficient of the cumulative takedown cash flow data points as an approximation of the capital deployment rate. Figure 4 uses funds started in 2000 as an example to illustrate this method. The purpose of the regression is to see how cumulative takedown changes through time. For funds in each vintage year, the aggregate takedown cash flow data is

available in VentureXpert, and these cash flows are denoted by CF_1, CF_2, … , CF_n, assuming there are a total of n cash flows. Then the k th data point y_k is defined as:

$$y_k = \frac{\sum_{i=1}^{k} CF_i}{\sum_{j=1}^{n} CF_j}$$

The numerator is the cumulative takedown at a certain point of time, and the denominator is the total takedown of this group of funds. The fraction is a percentage, and each data point y_k is the percentage of capital already taken down at a certain time. The regression variable x_k, in this case, is the number of days between the first takedown and the k th takedown. For example, for funds started in 2000, 44.05% of capital was drawn in 790 days, therefore there is a data point (790, 0.4405) on the scatterplot.

A complication arises here. Since we are looking at the entire industry, there are bound to be some special situations in which very small amounts of capital are drawn later than usual, either because a few funds are particularly slow at deploying money, or because some of the deals are closed at a very late date. These takedowns would still be recorded as separate data points, and they drastically reduce the takedown rate, causing the linear approximation of the cumulative takedown curve to be a poor approximation (see Figure 4 Panel B). To address this long-tail problem, I truncate the data set to examine only the first 80% of capital draw-down, that is, $y_k<0.8$ in all cases (see Panel A). 80% is an arbitrary number, but should be a good indicator of capital deployment rate of the entire industry.

Another complication is that, funds started in 2008 and 2009 have only just deployed most of their capital, and the long-tail problem does not exist in these cases by looking at their graphs. If

their cash flow data is truncated again, I would really be evaluating only their first 60% of capital deployment, which could cause some bias in the outcome. Therefore the regression analysis includes all the cash flow data that is available for these two vintage years.

The next step is to run the regression and find the slope of the regression line. I first divide all funds into bigger funds and smaller funds again like in the last section because the size of the fund could also contain information about how fast the capital can be used. For each vintage year, there are two sets of y_k and x_k. Regressing y_k on x_k gives a slope α, which is the approximation of the capital deployment rate, and a β, which is the y-intercept. Each year has two sets of α and β (see Table 4), one for bigger funds of that year, and one for smaller funds. The "Takedown Rate" column shows all α.

Next we regress α on recession spread to see if α has some relationship with the economic environment. The result is not significant with a T-statistic of -1.05 and p-value of 0.299. However, if the regression is on CFNAI year average instead of recession spread, the result is significant, with T-statistic of -2.32 and p-value of 0.025 (Appendix E). Because the regression coefficient is negative, this means lower CFNAI year average would imply faster deployment rate. Although α is significant when regressed on only one of the variables, it still shows that deployment rate is sensitive to the economic environment at fund inception, and recession funds put their capital to use faster.

On the other hand, the size of a fund does not seem to matter, because the dummy variable of above or below median size does not return significant results when it is added to either regression.

5. Stock Distribution

5.1 Hypothesis

Buyout funds sometimes have the choice between distributing cash or stocks to investors after an investments is exited. In certain circumstances this decision can cause some tension between the general partners and the limited partners. For example, in an illiquid stock market, general partners may prefer to distribute shares of the portfolio company, and use the price on the day of distribution to calculate returns to investors. When the limited partners sell these shares in the market, the price could drop, causing the actual returns to be lower than the calculated returns. If the fund's performance is below the high watermark, the general partners are even more incentivized to distribute stocks, because overstated returns bring them closer to the high watermark, which allows them to receive performance bonus sooner. Fenn, Liang and Prowse (1995) discusses this tension between limited partners and general partners, and the attempts made by the limited partners to insert clauses into their contracts to prevent the general partners from doing so. How successful these attempts are may vary from fund to fund.

Because of the cyclicality of the business, recession funds are most likely to be in a better position than boom funds in the divesting period. The holding period of an investment is usually five to seven years, which is why recession funds can exit most of their investments during expansion times, while funds started in booms tend to hold investments longer to exit them in a less depressed market. With fewer write-offs and better performing investments, it is less likely for a recession fund to be below the high watermark, which makes them less incentivized to distribute stocks. I suggest the following hypothesis:

Hypothesis 4: *Recession funds distribute less stock when they exit investments.*

5.2 Results

The test is similar to the last section, and much simpler. The funds are still divided into bigger and smaller funds in case there is any useful information related to fund size. I calculate what percentage of total distribution is made in form of stocks for each vintage year and regress it on both CFNAI and Vintage Year. The result shows a marginally significant relationship between stock distribution and CFNAI year average (T=-1.97, P=0.055), and a significantly negative relationship with Vintage Year (Appendix F). This means funds appear to distribute much less stock today than they did in the past. As the private equity industry matures, the general trend seems to be less and less stock distribution. Whether the fund started in a good year or a bad year has relatively little impact on this behavior. Size of fund does not matter as well.

6. Fund Size

This section aims to examine the change of industry landscape and is written in an exploratory manner without an explicit hypothesis. I examine how the characteristics of fund size distribution, including mean, variance and skewness, change over time and whether it correlates with economic cycles.

VentureXpert has both fund size and fundraising target information for over 2000 US buyout funds between 1987 and 2010. The mean, standard deviation and skewness of fund size distribution of each vintage year are plotted separately on vintage year (see Table 5 and Figure 5). It is obvious that all of them increases as vintage year increases, that is, the distribution is shifting to the right, with higher standard deviation and more positive skewness as the industry grows older. The mean reflects the robustness of the industry, the standard deviation reflects

dispersion of fund sizes, and the skewness highlights the number of big funds (since the outliers in this case only lie on the right-hand side in this case). Regressing these three dependent variables on vintage year also yields significant result. For instance, the test on skewness returns t=4.00, coefficient=0.1, meaning as time goes by the buyout fund industry has more and more big players. These three moments are then regressed separately on CFNAI data. However, the tests bring insignificant results either using recession spread as variable or using year average CFNAI. For example, the test on skewness returns a T-statistic of -0.28 and -0.9 respectively, Appendix G).

Overall, change of fund size distribution is more sensitive to the industry growth over time, not so much to the economic cycle. With investors pumping capital into the private equity industry, the impact of economic cycles on fund size distributions is obscured by the constantly increasing size of pie. This is not the case in the other sections. Industry growth does not necessarily imply higher returns or faster deployment, but it does have the most direct impact on fund sizes.

It is possible to minimize the impact of industry growth if comparing two period of time that are short and relatively close to each other. For the sake of simplicity, I only compare the most recent recession period with the most recent expansion period. In this case, the recession group includes funds with vintage years in 2008, 2009 and 2010 (sample size=501), while the expansion group includes 2005, 2006, 2007 (sample size=636). A histogram of both distributions is shown in Figure 6, although it is hard to tell immediately how different they are by simply looking at the histogram.

Making both sample sizes exceed 500 is important here, as skewness is particularly sensitive to sample size. Studies have shown that skewness of random datasets from the same population

could vary considerably when the sample size is small, and only stabilizes when the sample size exceeds a couple hundred. The calculation of skewness of each vintage year, as it is done in the previous test, is based on very small samples. For instance, there are only 28 funds with available fund size information in 1988. The skewness of these 28 data points is highly unreliable. Even in booming years such as 2006, there are only 139 funds with available information, which is a barely acceptable sample size for calculating skewness. Therefore pooling three vintage years into a group to obtain a decent sample size is important in this case.

Descriptive statistics shows big differences between the two groups. Calculation of mean and variance gives that:

$$\text{mean}_{\text{expansion}} = 993.2552$$

$$\text{mean}_{\text{recession}} = 605.3872$$

$$\text{var}_{\text{expansion}} = 5.7182\text{e+}06$$

$$\text{var}_{\text{recession}} = 2.4276\text{e+}06$$

To test the significance of these differences, three different types of tests are needed. First a two sample t-test is used to test whether the difference in mean of the two sample sets is significant. The null hypothesis is that $\text{mean}_{\text{expansion}} = \text{mean}_{\text{recession}}$. The t-statistic is calculated as:

$$\frac{}{\frac{}{}\quad\frac{}{}}$$

This t-statistic shows significant evidence that the two means are different, and the average fund size in expansion years is higher than in recession years.

The test on equal variance is done by conducting Levene's test, which is less sensitive to non-normal distribution (as in this case) compared to most other standard tests. Test result is significant with P=0.003 and is shown in Figure 7. This means the fund sizes are a lot less diverse in recession years.

The test on equal skewness is more complicated. Calculation of skewness shows that,

$$\gamma_{recession}=7.4164 \qquad \gamma_{expansion}=5.1225 \qquad \gamma_{difference}=\gamma_{recession}-\gamma_{expansion}=2.2939$$

There is a difference of 2.3 in skewness, and the question is whether this difference is statistically significant or simply caused by randomness.

Unlike mean or variance, there is no standardized way to test whether two sample sets have the same skewness. Therefore I wrote a Matlab program to conduct a permutation test in order to test the null hypothesis that $\gamma_{difference}=0$ (Code attached in Appendix H). The permutation test first combines two sample sets together and randomly shuffles all the data points, and then divides them into two groups again with the same sample sizes as before. Then it calculates the skewness of both sets and the new difference in skewness. The program does this for ten thousand times and then compare the resulting skewness differences with the original one to see if that one lies in the top 5 percentile. If it does, the test suggests that the difference in skewness of two sample sets is not caused by randomness, which rejects the null hypothesis, and shows $\gamma_{difference}\neq 0$. The result of this test on our sample sets is as follows:

Skew_diff =2.2939

Prob_observation =0.9951

Significant_diff =1.4642

This result suggests that the skewness difference of 2.29 is significant at an alpha level of 0.5%. If it is truly caused by randomness, the probability of observing such a difference would be (1-99.51%) = 0.49%. In fact, a difference of 1.46 would be significant at an alpha level of 5%. This test rejects the null hypothesis and suggests that recession funds have a more positively-skewed distribution of fund size than non-recession funds, at least in recent years. The increase in skewness would suggest more outliers on the right. Accompanied by the lower mean in this case, the change in skewness tells us that the entire distribution is shifted to the left, but the right tail of the distribution is shifted less.

Based on the analysis above, we can see that the most recent recession period has witnessed lower average fund sizes, lower variance and more skewness to the right. This shows a rough picture of the private equity industry. On the one hand, it is harder to raise capital in an economic downturn as investors lose both confidence and capital in the market, causing average fund sizes to shrink. On the other hand, the combination of a lower mean and a more skewed distribution suggests that experienced fund managers are relatively less affected by the shrinkage of the pie in terms of fundraising, which increases the proportion of all capital in the industry that end up in their funds.

7. Conclusion

This paper analyzes the performance and behaviors of US buyout funds with vintage years in recessions, and uses CFNAI as an indicator of the economic condition. Section 3 shows higher vintage year returns for recession fund, along with less noise and less dilution in the buyout industry. Section 4 shows that capital deployment rate of private equity funds is sensitive to the macro-economy, and recession funds tend to deploy capital faster. Section 5 discusses the stock

distribution behavior, which turns out to have declined overall in the past two decades, with lower correlation with the economic cycles. Section 6 considers the changing landscape of the industry in terms of fundraising, and put emphasis on the most recent business cycle. The result shows that the distribution of fund sizes shifted to the left with lower variance and more positive skewness when the economy entered into the 2008-2009 recession.

The analysis in this paper is far from comprehensive, but it does show the general idea about private equity investment, that is, the private equity game is easier to play when it is more private. Investing in a buyout fund during the recession is probably a better idea than following everyone else in the boom.

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Tables

Table 1 - CFNAI Index (1987-2009)

	CFNAI Year Average	CFNAI Recession Spread
1987	0.5926	1.4931
1988	0.3636	1.2640
1989	-0.2211	0.6794
1990	-0.6596	0
1991	-0.6313	0
1992	0.0693	0.7148
1993	0.2007	0.8461
1994	0.6384	1.2839
1995	-0.0545	0.5910
1996	0.2320	0.8774
1997	0.5526	1.1980
1998	0.2035	0.8489
1999	0.3528	0.9982
2000	-0.1017	0.5437
2001	-1.0835	0
2002	-0.2619	0
2003	-0.0989	0.5738
2004	0.3686	1.0414
2005	0.2740	0.9467
2006	0.0713	0.7441
2007	-0.2589	0.4138
2008	-1.9405	0
2009	-1.6109	0

NBER recession years are in grey. A negative CFNAI does not necessarily indicate a recession. Empirically a -0.7 CFNAI indicates a recession. CFNAI recession spread sets each recession years to zero, and calculate the spread between each vintage year CFNAI and the average CFNAI of the previous recession period.

Table 2 – Vintage Year Performance (Calculated using cash flow data from VentureXpert 1987-2009)

Year	Number of funds	IRR	MOC
1987	27	11%	1.9
1988	23	12%	2.0
1989	27	22%	3.1
1990	15	13%	1.9
1991	7	24%	3.1
1992	20	19%	2.5
1993	25	18%	2.6
1994	29	8%	1.7
1995	32	10%	1.8
1996	37	4%	1.4
1997	54	8%	1.6
1998	72	3%	1.2
1999	53	1%	1.1
2000	69	11%	2.0
2001	39	18%	2.3
2002	32	18%	2.2
2003	27	18%	2.2
2004	37	13%	1.8
2005	46	10%	1.5
2006	51	2%	1.1
2007	55	7%	1.2
2008	47	11%	1.2
2009	25	16%	1.2
Total	822		

Table 3 – Returns (Bigger Funds vs. Smaller Funds)

	Number of Funds	Median Fund Size (in millions)	Bigger Funds		Smaller Funds		Absolute
			IRR	MOC	IRR	MOC	IRR Difference
1987	27	120	11.15%	1.96	11.81%	2.12	0.66%
1988	23	150	11.81%	1.74	12.24%	1.83	0.43%
1989	27	150	23.07%	2.54	15.15%	2.21	7.92%
1990	15	150	14.88%	1.81	5.48%	1.52	9.40%
1991	7	100	25.03%	3.03	17.53%	2.01	7.50%
1992	20	180	18.55%	1.94	20.61%	2.17	2.06%
1993	25	270	18.91%	2.10	13.28%	1.70	5.63%
1994	29	270	7.58%	1.47	11.67%	1.51	4.09%
1995	33	270	9.87%	1.54	10.00%	1.61	0.13%
1996	37	250	4.73%	1.34	2.09%	1.15	2.64%
1997	54	350	8.29%	1.47	5.26%	1.28	3.03%
1998	73	460	2.79%	1.17	8.63%	1.55	5.84%
1999	54	427	5.76%	1.34	5.93%	1.38	0.17%
2000	72	380	11.38%	1.61	10.78%	1.64	0.60%
2001	44	650	18.93%	1.90	12.02%	1.55	6.91%
2002	33	500	18.54%	1.72	17.69%	1.84	0.85%
2003	29	500	18.14%	1.75	10.98%	1.52	7.16%
2004	39	550	13.77%	1.60	10.34%	1.43	3.43%
2005	49	700	9.76%	1.45	10.03%	1.37	0.27%
2006	52	1000	1.90%	1.06	3.83%	1.13	1.93%
2007	56	1500	6.62%	1.18	13.11%	1.31	6.49%
2008	51	1150	9.03%	1.18	15.56%	1.30	6.53%
2009	25	900	15.73%	1.20	20.08%	1.32	4.35%

Table 4 – Capital Deployment Rate

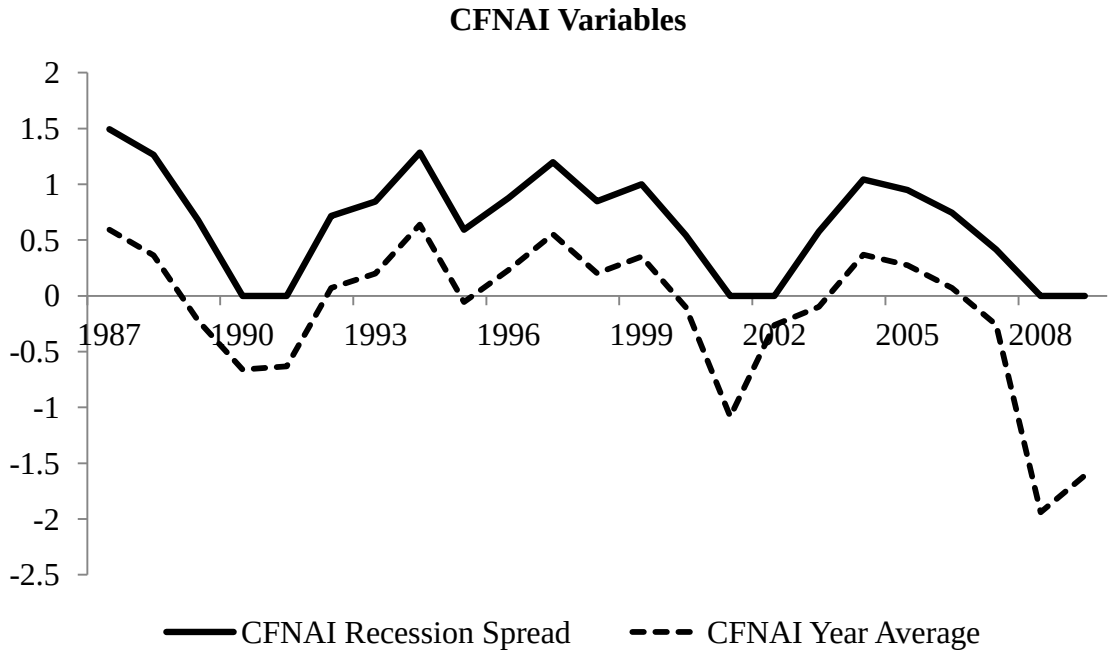
Vintage Year	Takedown Rate α	Y-axis intercept β	80% Take-down Days	Above Median Size?
1987	0.00065800	-0.030	1277	1
1987	0.00042700	0.145	1764	0
1988	0.00056700	0.012	1521	1
1988	0.00045700	0.167	1430	0
1989	0.00046100	0.035	1611	1
1989	0.00059200	0.058	1247	0
1990	0.00058500	0.164	1096	1
1990	0.00071300	0.001	1218	0
1991	0.00064200	0.100	1188	1
1991	0.00064600	-0.028	1430	0
1992	0.00061100	-0.053	1339	1
1992	0.00057400	-0.088	1492	0
1993	0.00062600	-0.054	1308	1
1993	0.00057400	-0.011	1461	0
1994	0.00053900	0.054	1520	1
1994	0.00064500	-0.065	1433	0
1995	0.00065700	-0.080	1246	1
1995	0.00059900	-0.069	1492	0
1996	0.00067600	-0.006	1247	1
1996	0.00073100	0.035	1127	0
1997	0.00058000	-0.032	1492	1
1997	0.00073600	-0.047	1185	0
1998	0.00080300	-0.078	1155	1
1998	0.00076300	-0.023	1155	0
1999	0.00054900	0.031	1612	1
1999	0.00048800	0.101	1612	0
2000	0.00042500	0.092	1705	1
2000	0.00043800	0.096	1674	0
2001	0.00049600	-0.005	1551	1
2001	0.00054400	-0.031	1581	0
2002	0.00050800	-0.033	1492	1
2002	0.00067200	-0.045	1369	0
2003	0.00060200	-0.059	1339	1
2003	0.00066300	-0.054	1369	0
2004	0.00075900	-0.077	1124	1
2004	0.00062200	-0.115	1430	0
2005	0.00076400	-0.085	1126	1
2005	0.00052200	-0.029	1704	0
2006	0.00078700	-0.067	1247	1
2006	0.00065100	0.011	1339	0
2007	0.00057400	0.079	1308	1
2007	0.00058800	-0.020	1339	0
2008	0.00065700	0.034	1430	1
2008	0.00071400	0.011	1430	0
2009	0.00105800	-0.025	974	1
2009	0.00095400	0.026	1064	0

Table 5 – Fund Size Distribution

Vintage Year	Mean Size (in millions)	Variance	Standard Deviation	Skewness
1987	281.18	212536.83	461.02	3.24
1988	209.18	78803.44	280.72	2.45
1989	219.86	76197.16	276.04	2.19
1990	124.25	21752.42	147.49	1.39
1991	73.17	2184.17	46.74	0.55
1992	269.96	164250.08	405.28	2.14
1993	283.25	80574.61	283.86	1.65
1994	267.64	124204.96	352.43	2.55
1995	494.58	555748.30	745.49	3.01
1996	304.04	173599.01	416.65	3.41
1997	489.55	765086.36	874.69	4.20
1998	543.06	662679.91	814.05	2.77
1999	488.12	400651.31	632.97	2.16
2000	793.38	1255590.22	1120.53	2.34
2001	661.32	1094641.13	1046.25	2.93
2002	502.15	700907.44	837.20	4.85
2003	443.82	777924.09	882.00	4.88
2004	515.02	504964.02	710.61	2.74
2005	690.05	1366670.82	1169.05	4.14
2006	1363.19	9958586.80	3155.72	4.11
2007	1525.87	10404058.67	3225.53	3.70
2008	924.61	5831867.05	2414.93	5.55
2009	698.04	1409142.72	1187.07	3.50

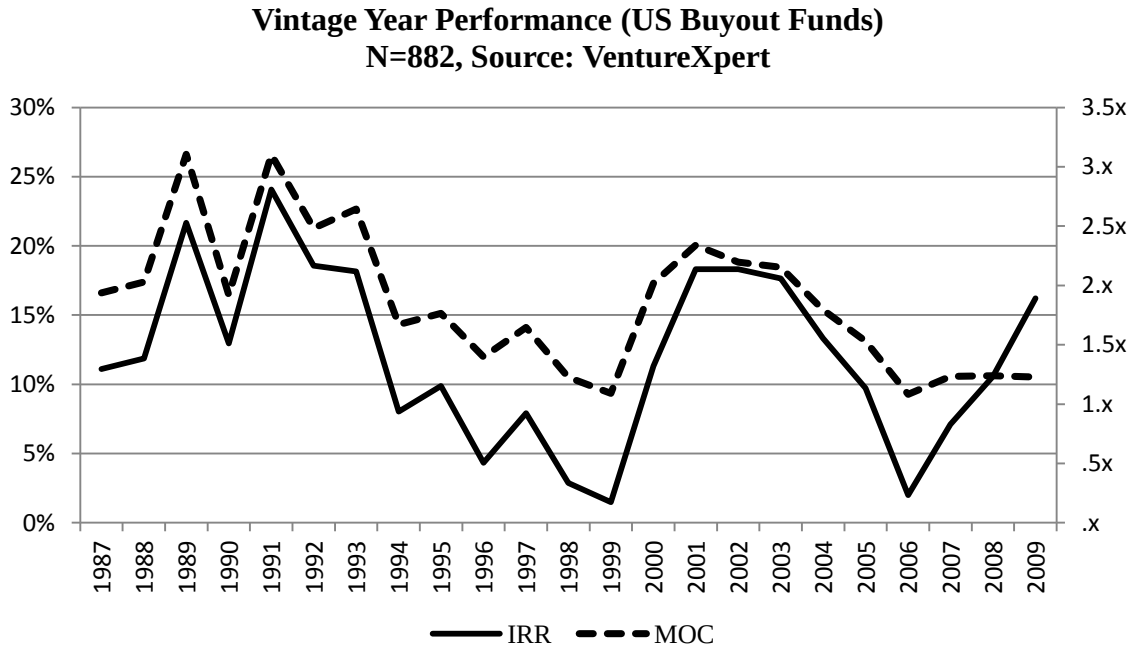
Figures

Figure 1 – CFNAI



The CFNAI recession spread before 1990 is calculated based on the earlier recession (the early 1980s) that is not shown in this graph.

Figure 2 Panel A



Panel B

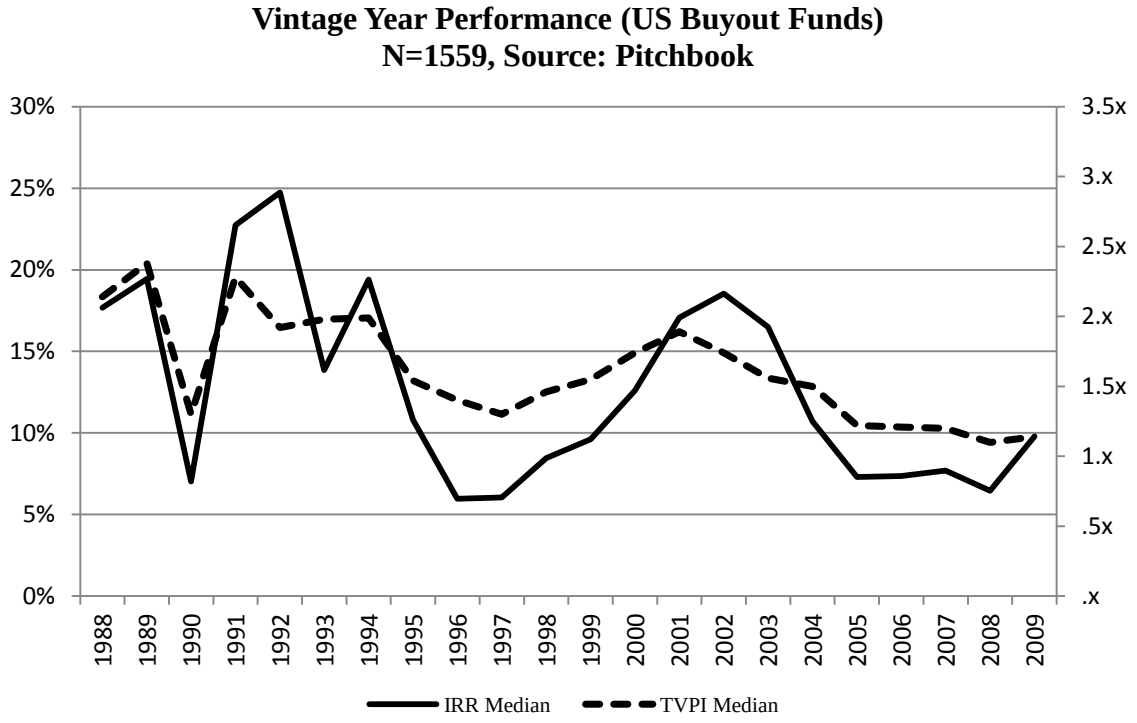
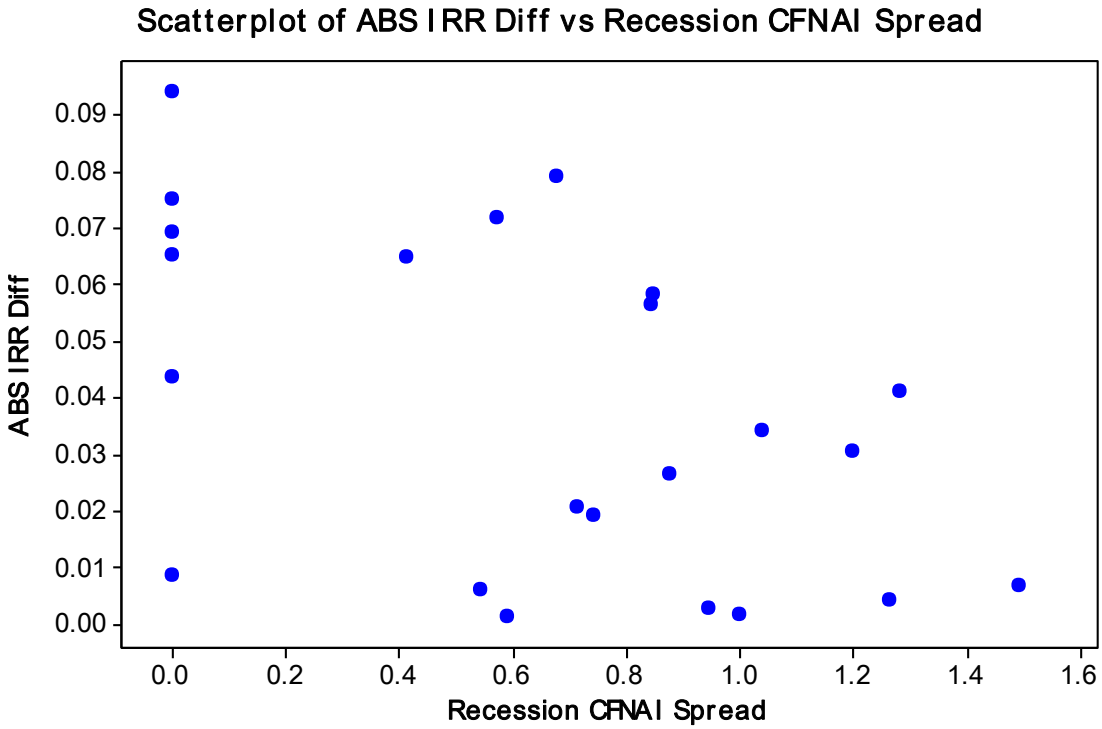


Figure 3 – Absolute IRR difference between big funds and small funds



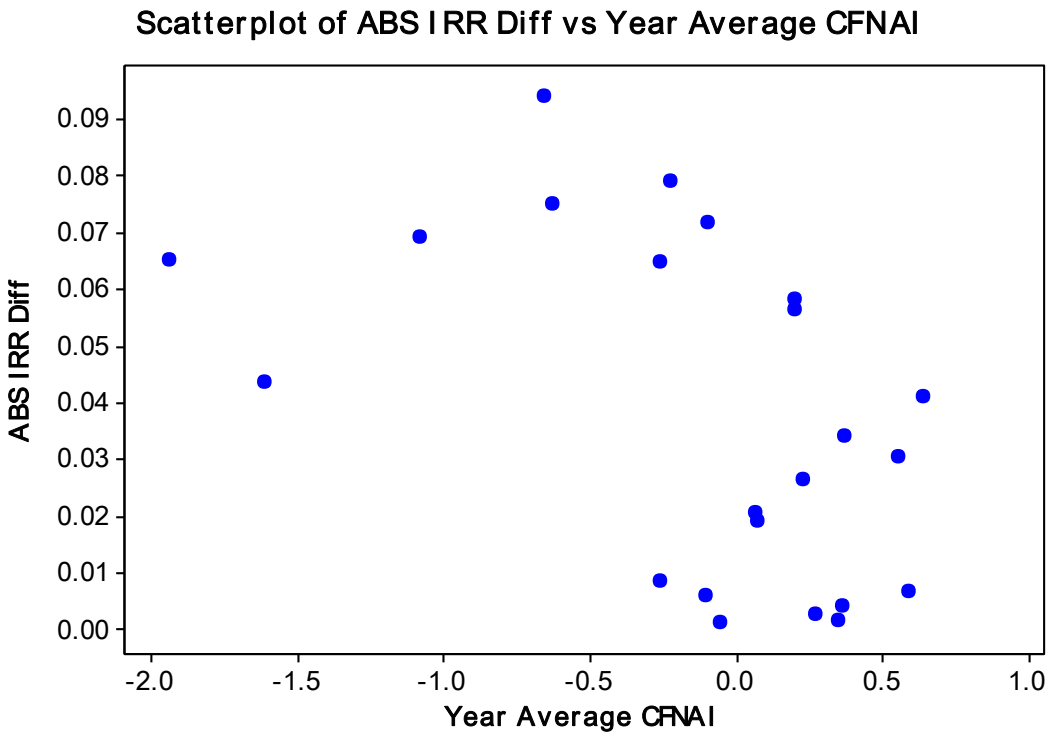
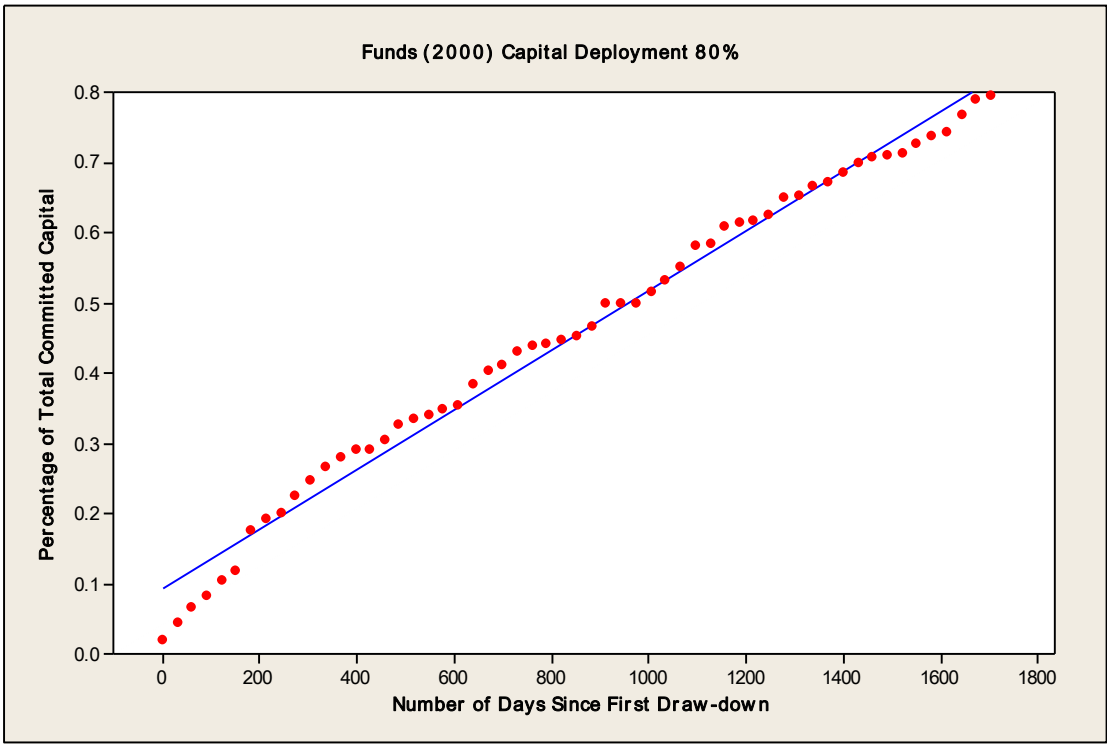

Scatterplot of ABS IRR Diff vs Year Average CFNAI
ABS IRR Diff
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0.00
-2.0
-1.5
-1.0
-0.5
0.0
0.5
1.0
Year Average CFNAI

Figure 4 - Sample Capital Deployment Rate - Funds Started in 2000

Panel A



Panel B

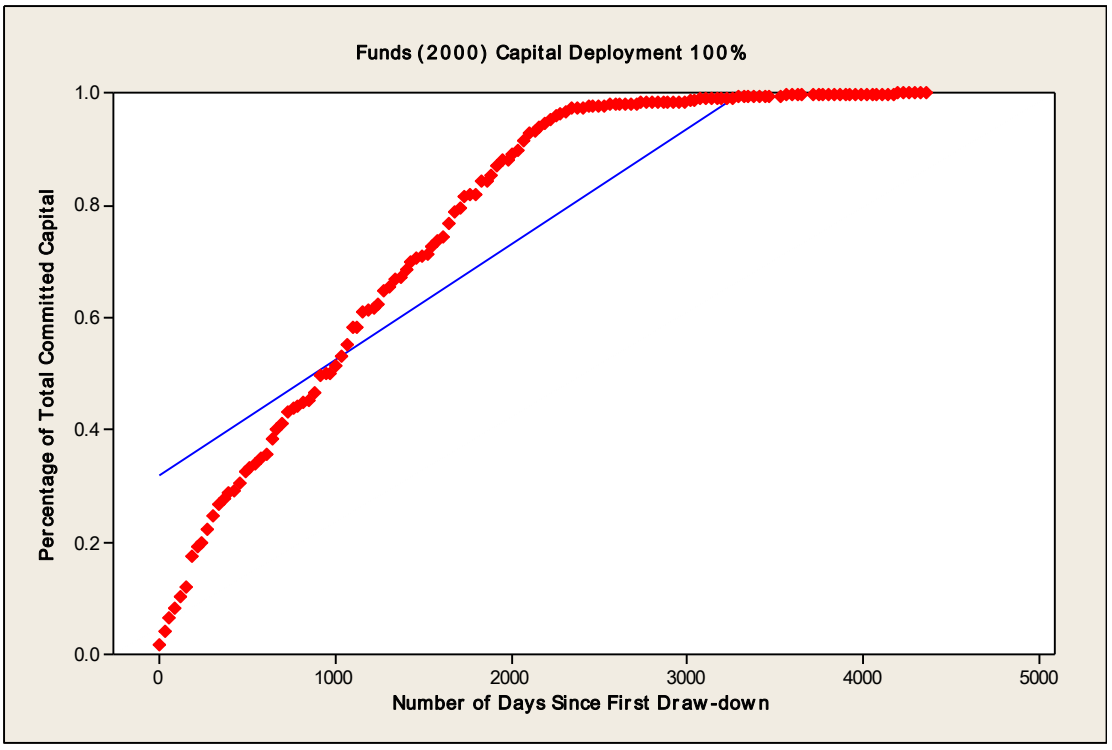
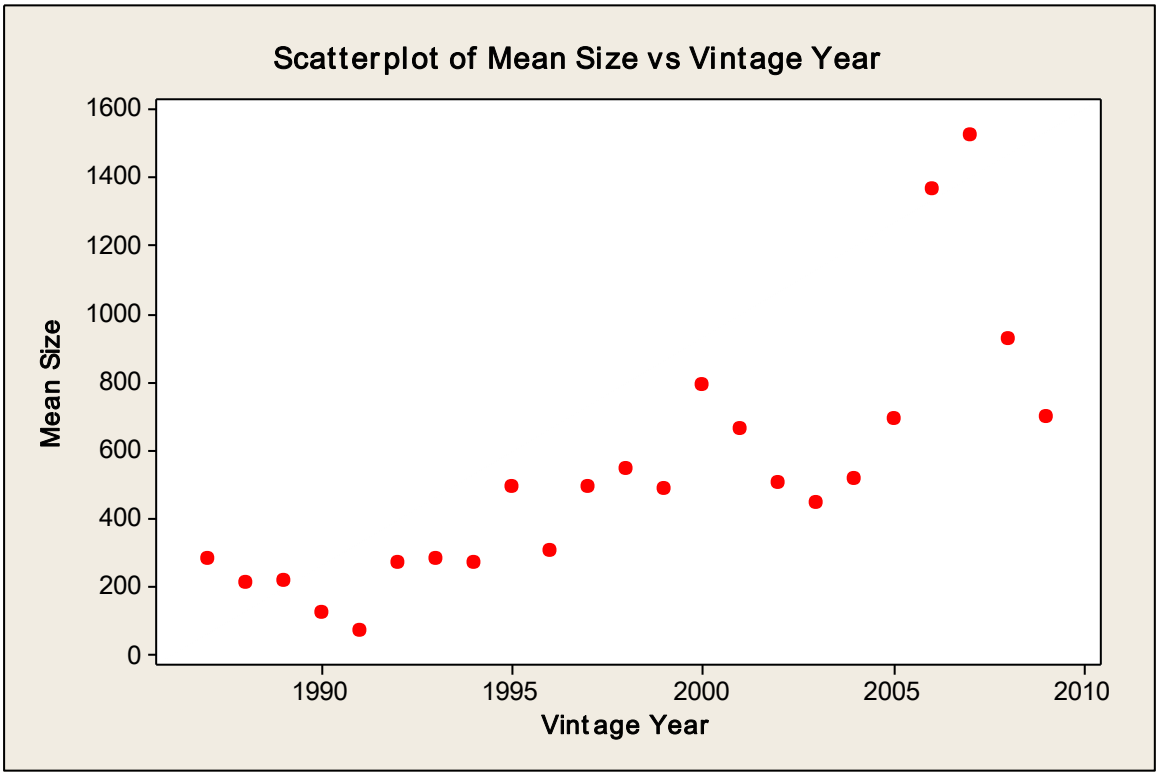
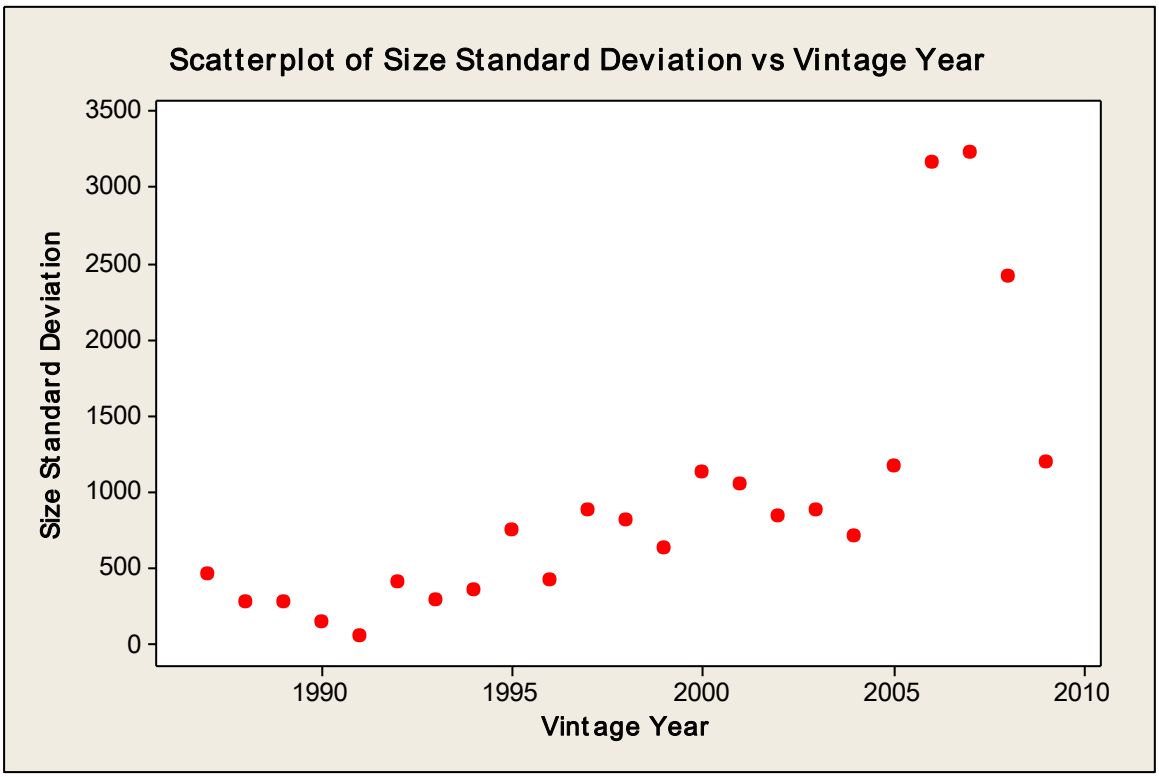
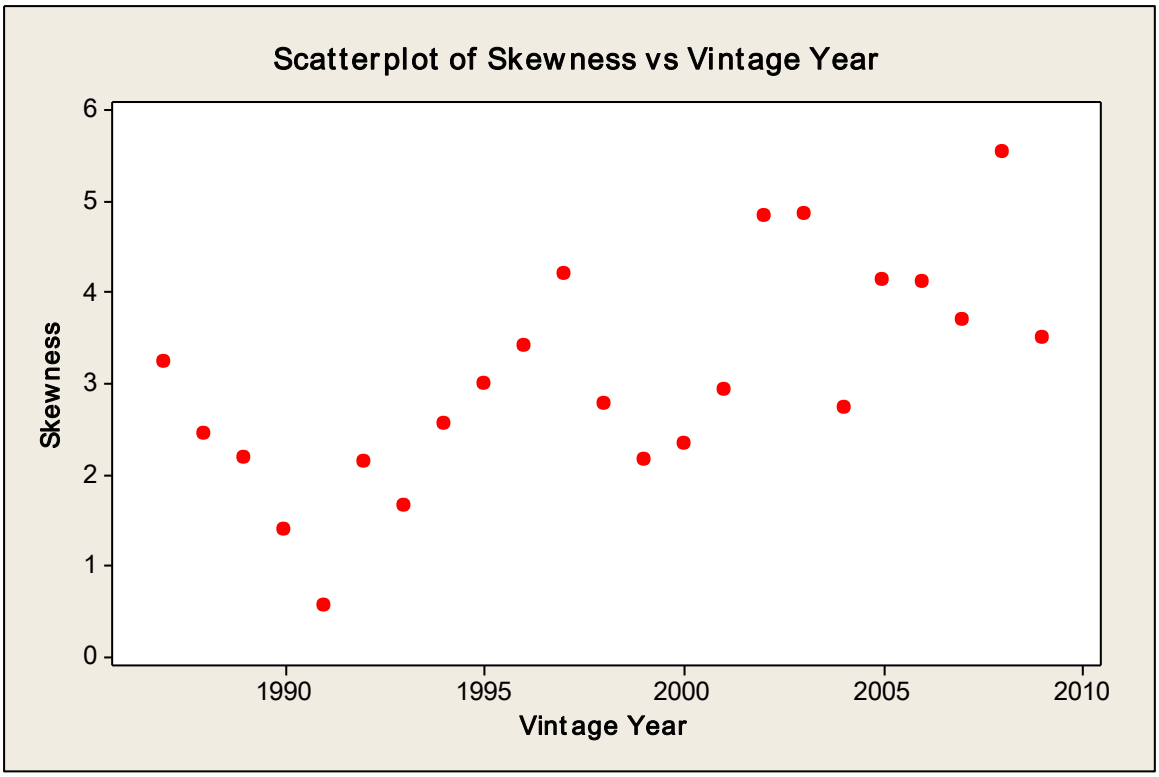


Figure 5 – Fund Size Descriptive Statistics

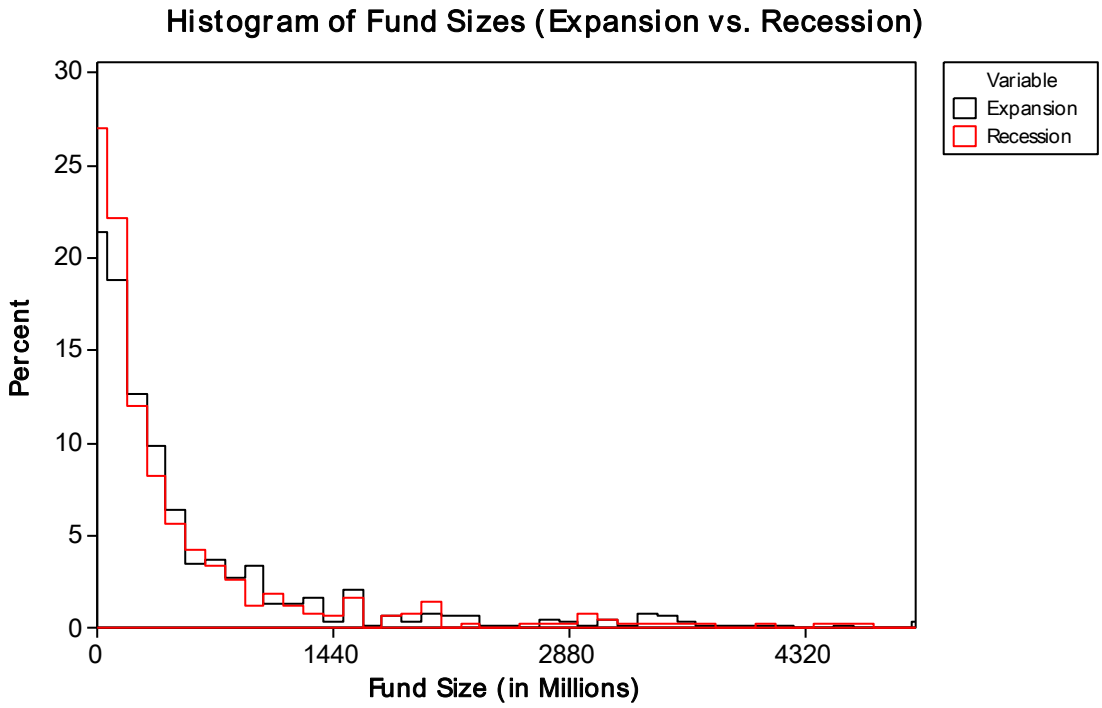






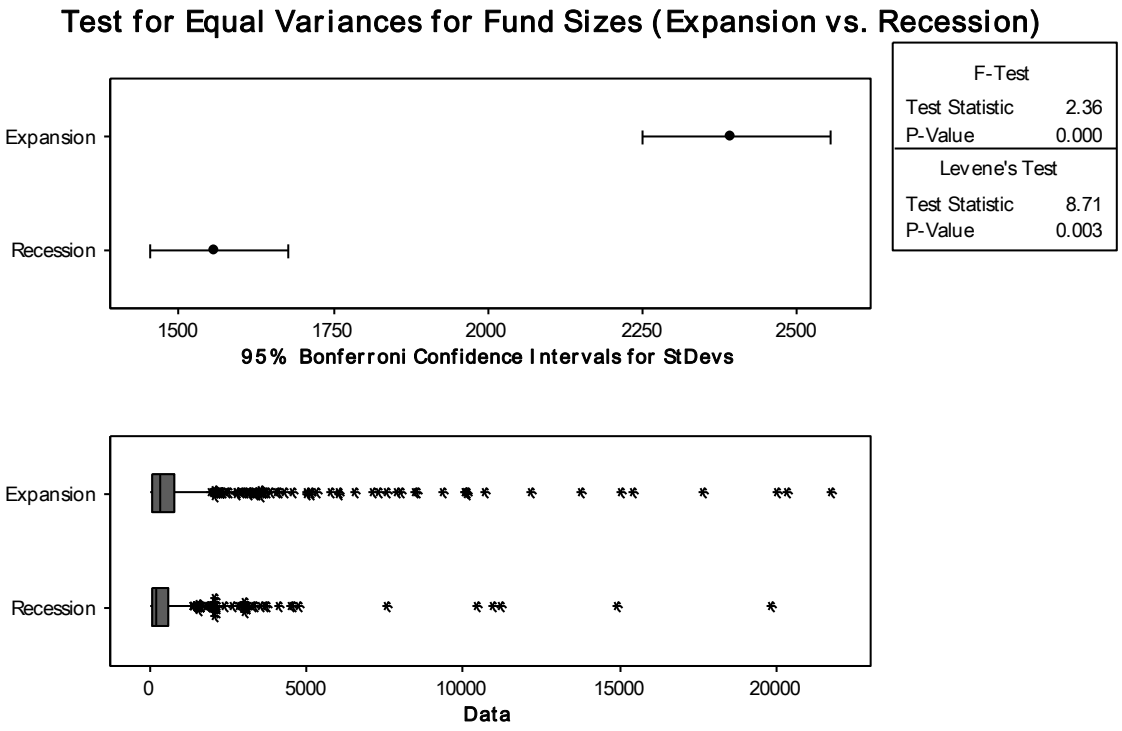
As Kaplan and Schoar (2003), I exclude all funds with a size below $5 million (about 100 funds in total). Buyout funds with such small sizes are unlikely to be the ones we are interested in. These numbers could also be typos.

Figure 6 – Fund Size Distribution



This histogram only shows funds below $5 billion because there are a few mega-funds that lie far away on the x-axis, which squeezes the chart and makes the left side of the distribution hard to observe. These mega-funds are included in the analysis though.

Figure 7 – Variance



APPENDICES

APPENDIX A

Capital-weighted IRR does not equal cash flow-based IRR

	Fund A	Fund B	Combined
Cash Flow 1	-35	-300	-335
CF 2	-20	-600	-620
CF 3	-35	170	135
CF 4	15	330	345
CF 5	25	190	215
CF 6	8	190	198
CF 7	9	10	19
CF 8	25	20	45
CF 9	70	30	100
CF 10	25	190	215
CF 11	125	150	275
Cash flow based IRR	**19.818%**	**8.621%**	**10.534%**
AUM	-90	-900	
AUM weight	9%	91%	
Capital-weighted IRR	**9.639%**		

Two fictitious funds A and B have IRRs of 19.8% and 8.6% respectively. The capita-weighted IRR is 9.6%. Note that this calculation only needs IRRs that funds report voluntarily, making capital-weighted IRR very easy to obtain. Combining the two funds together, that is, adding each cash flow together, the combined IRR is 10.5%. The latter makes more sense, because it reflects the real IRR of all the money in these two funds, making this method a better way to assess the performance of the whole industry. Capital-weighted IRR is merely an approximation of the real IRR. How good this approximation is remains unclear, and it is reasonable to assume that it gets worse when hundreds of funds with vastly different features are put together. But the latter is also more difficult to calculate, as cash flow data is not always available. VentureXpert provides us just the way to do it.

APPENDIX B - Sample Cash Flow Data (Vintage 1988)

PERIOD	TAKE DOWN	CASH DIST	STOCK DIST	TOTAL DIST	NAV
1988-01	53.219	.000	.000	.000	.000
1988-03	1.744	.000	.000	.000	54.964
1988-04	2.616	.000	.000	.000	54.964
1988-06	546.429	.000	.000	.000	604.009
1988-07	71.303	.766	.000	.766	604.009
1988-08	200.695	11.474	.000	11.474	604.009
1988-09	40.643	16.737	.000	16.737	916.649
1988-10	100.822	52.980	.000	52.980	916.649
1988-11	392.899	31.793	.000	31.793	916.649
1988-12	103.324	152.025	.000	152.025	1571.663
1989-01	423.413	11.685	.000	11.685	1571.663
1989-02	241.588	28.052	.000	28.052	1571.663
1989-03	190.762	9.645	.000	9.645	2348.431
1989-04	27.044	105.977	.000	105.977	2348.431
1989-05	27.631	13.790	.000	13.790	2348.431
1989-06	99.398	6.679	.000	6.679	2395.133
1989-07	336.416	17.045	.000	17.045	2395.133
1989-08	290.297	248.514	.000	248.514	2395.133
1989-09	54.946	40.922	.000	40.922	3062.467
1989-10	28.732	29.880	.000	29.880	3062.467
1989-11	14.621	5.180	.000	5.180	3062.467
…	…	…	…	…	…

This is part of the cash flow data of funds with vintage year of 1988 (the complete record is too long to fit in one page). Data include in which month a cash flow occurs, whether the distribution is in cash or stock, and what the net asset value is at the time. NAV is the estimated value of the remaining investments held by those funds. After a fund is liquidated, the NAV is supposed to be zero, but that is not always the case in the database. It could simply be because the fund fails to report write-offs.

APPENDIX C

Regression Analysis: IRR versus Recession CFNAI Spread

The regression equation is
IRR = 0.160 - 0.0602 Recession CFNAI Spread

Predictor	Coef	SE Coef	T	P
Constant	0.16002	0.02082	7.69	0.000
Recession CFNAI Spread	-0.06022	0.02598	-2.32	0.031

S = 0.0575566 R-Sq = 20.4% R-Sq(adj) = 16.6%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	1	0.017796	0.017796	5.37	0.031
Residual Error	21	0.069568	0.003313		
Total	22	0.087364			

APPENDIX D

Regression Analysis: ABS IRR Diff versus Recession CFNAI Spread

The regression equation is
ABS IRR Diff = 0.0592 - 0.0320 Recession CFNAI Spread

Predictor	Coef	SE Coef	T	P
Constant	0.059212	0.009446	6.27	0.000
Recession CFNAI Spread	-0.03199	0.01179	-2.71	0.013

S = 0.0261145 R-Sq = 26.0% R-Sq(adj) = 22.4%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	1	0.0050208	0.0050208	7.36	0.013
Residual Error	21	0.0143213	0.0006820		
Total	22	0.0193421			

Unusual Observations

Obs	Recession CFNAI Spread	ABS IRR Diff	Fit	SE Fit	Residual	St Resid
16	0.00	0.00850	0.05921	0.00945	-0.05071	-2.08R

R denotes an observation with a large standardized residual.

Scatterplot of ABS IRR Diff vs Recession CFNAI Spread

APPENDIX E

Regression Analysis: Capital Deployment Rate versus Recession CFNAI

The regression equation is
Capital Deployment Rate = 0.000655 - 0.000042 Recession CFNAI Spread

Predictor	Coef	SE Coef	T	P
Constant	0.00065474	0.00003152	20.77	0.000
Recession CFNAI Spread	-0.00004243	0.00004038	-1.05	0.299

S = 0.000126985 R-Sq = 2.4% R-Sq(adj) = 0.2%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	1	1.78010E-08	1.78010E-08	1.10	0.299
Residual Error	44	7.09504E-07	1.61251E-08		
Total	45	7.27306E-07			

Unusual Observations

Obs	Recession CFNAI Spread	Capital Deployment Rate	Fit	SE Fit	Residual	St Resid
45	0.00	0.001058	0.000655	0.000032	0.000403	3.28R
46	0.00	0.000954	0.000655	0.000032	0.000299	2.43R

R denotes an observation with a large standardized residual.

Regression Analysis: Capital Deployment Rate versus Year Average CFNAI

The regression equation is
Capital Deployment Rate = 0.000620 - 0.000064 Year Average CFNAI

Predictor	Coef	SE Coef	T	P
Constant	0.00061980	0.00001824	33.98	0.000
Year Average CFNAI	-0.00006352	0.00002732	-2.32	0.025

S = 0.000121332 R-Sq = 10.9% R-Sq(adj) = 8.9%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	1	7.95614E-08	7.95614E-08	5.40	0.025
Residual Error	44	6.47744E-07	1.47215E-08		
Total	45	7.27306E-07			

Unusual Observations

Obs	Year Average CFNAI	Capital Deployment Rate	Fit	SE Fit	Residual	St Resid
43	-1.94	0.000657	0.000743	0.000053	-0.000086	-0.78 X
44	-1.94	0.000714	0.000743	0.000053	-0.000029	-0.27 X
45	-1.61	0.001058	0.000722	0.000044	0.000336	2.97RX
46	-1.61	0.000954	0.000722	0.000044	0.000231	2.05RX

R denotes an observation with a large standardized residual.
X denotes an observation whose X value gives it large leverage.

APPENDIX F

Regression Analysis: PercentStock versus Vintage Year, Recession CFNAI

The regression equation is
PercentStock = 16.2 - 0.00806 Vintage Year - 0.0456 Recession CFNAI Spread

Predictor	Coef	SE Coef	T	P
Constant	16.179	3.238	5.00	0.000
Vintage Year	-0.008056	0.001618	-4.98	0.000
Recession CFNAI Spread	-0.04559	0.02314	-1.97	0.055

S = 0.0665538 R-Sq = 36.6% R-Sq(adj) = 33.6%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	2	0.109849	0.054924	12.40	0.000
Residual Error	43	0.190465	0.004429		
Total	45	0.300313			

Source	DF	Seq SS
Vintage Year	1	0.092663
Recession CFNAI Spread	1	0.017186

APPENDIX G

Regression Analysis: Skew versus Recession Spread

The regression equation is
Skew = 3.17 - 0.155 Rec Spread

Predictor	Coef	SE Coef	T	P
Constant	3.1652	0.4449	7.11	0.000
Rec Spread	-0.1552	0.5553	-0.28	0.783

S = 1.23001 R-Sq = 0.4% R-Sq(adj) = 0.0%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	1	0.118	0.118	0.08	0.783
Residual Error	21	31.771	1.513		
Total	22	31.890			

Unusual Observations

Obs	Rec Spread	Skew	Fit	SE Fit	Residual	St Resid
5	0.00	0.551	3.165	0.445	-2.614	-2.28R
22	0.00	5.554	3.165	0.445	2.388	2.08R

R denotes an observation with a large standardized residual.

Regression Analysis: Skew versus CFNAI Year Average

The regression equation is
Skew = 3.02 - 0.351 CFNAI Year Average

```
Predictor        Coef  SE Coef      T      P
Constant       3.0178   0.2570  11.74  0.000
Year Average  -0.3509   0.3849  -0.91  0.372


S = 1.20860   R-Sq = 3.8%   R-Sq(adj) = 0.0%


Analysis of Variance

Source          DF      SS     MS     F      P
Regression       1   1.214  1.214  0.83  0.372
Residual Error  21  30.675  1.461
Total           22  31.890


Unusual Observations

        Year
Obs  Average   Skew    Fit  SE Fit  Residual  St Resid
  5    -0.63  0.551  3.239   0.317    -2.689     -2.31R
 22    -1.94  5.554  3.699   0.741     1.855      1.94 X
 23    -1.61  3.501  3.583   0.623    -0.082     -0.08 X

R denotes an observation with a large standardized residual.
X denotes an observation whose X value gives it large leverage.
```

Regression Analysis: Skew versus Vintage Year

```
The regression equation is
Skew = - 197 + 0.100 Vintage Year


Predictor     Coef  SE Coef      T      P
Constant   -197.00    50.03  -3.94  0.001
Year       0.10010  0.02502   4.00  0.001
```

S = 0.956904 R-Sq = 40.0% R-Sq(adj) = 37.5%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	1	14.655	14.655	16.01	0.001
Residual Error	24	21.976	0.916		
Total	25	36.631			

APPENDIX H – Matlab Permutation Test (Fund Size Shuffle)

```
clear all
format compact
format short
Recess= xlsread('Fundsize080910.xlsx');
Expansion= xlsread('Fundsize050607.xlsx');


trial=10000;
skew_expansion=skewness(Expansion)
skew_recess=skewness(Recess)
skew_diff=skew_recess-skew_expansion
allskew(trial,1)=skew_diff;     %calculate observed skewness, put it
temporarily at the end of the outcome vector


% put two samples together into one big sample set A
Ntotal=length(Expansion)+length(Recess);
A(1:length(Expansion),1)=Expansion;
A(length(Expansion)+1:Ntotal,1)=Recess;


for n=1:trial-1
    Ashuffle=A(randperm(length(A)));    %shuffle the big sample set
    expansion_temp=Ashuffle(1:length(Expansion),1);   %seperate into two
sample sets of the original sizes
    Recess_temp=Ashuffle(length(Expansion)+1:Ntotal,1);
    skew_diff_temp=skewness(expansion_temp)-
skewness(Recess_temp);   %calculate the new skewness
    allskew(n,1)=skew_diff_temp;    %put calculated skewness into the outcome
vector one by one
end


sortall=sort(allskew);
position=find(sortall>skew_diff);   % find where the observed skewness is
Prob_observ=position(1,1)/trial
% probability that the skewness of shuffled sample sets is bigger than the
observed skewness
Significant_diff=sortall(trial*0.95,1)
% The skewness difference needs to be this big to be statistically
significant
```